

# 2

## CONCEPTS AND NOTATIONS RELATED TO MECHANISMS

### 2-1 MECHANISMS

A mechanism is a device to transform one motion into another. If the device also transmits substantial forces, it is a machine, which means that all machines are mechanisms in spirit. If forces are associated with the conversion of the energy of high-temperature fluids (as steam or gas) to shaft power, then the aggregate *may* be called an engine. At any rate, it is recognized that the parts comprising the device—mechanism, machine, or engine—must be resistant to deformation, i.e., the parts must approximate rigid bodies. We may then say that a mechanism is an assemblage of rigid or resistant bodies connected together for the purpose of transforming motion.

Two general groups of mechanisms exist, since the motion may go from either uniform to uniform or from uniform to nonuniform. Circular gears, chains, belts, and the like, comprise most of the uniform motion converters; their many problems will not be discussed in this book. Nonuniform conversions are made with noncircular gears, cams, ratchets, and linkages, both planar

and spatial. This book will direct itself exclusively to the design of linkages to meet certain motion-conversion specifications.

## 2-2 CLASSIFICATION SYSTEMS

Classification implies some sort of systematic grouping of factors that will show the relation of one thing to a group of things: it is a search for order. A complete system of mechanisms would show the genealogy of motion; i.e., it would show relations and lines of common descent when they exist. Individual mechanisms would no longer be separate mysteries; each would be part of a larger group having somewhat similar characteristics modified by personal idiosyncrasies. Furthermore, an important property of a classification system would be the aid it could furnish a designer in finding the forms and arrangements best suited to satisfying certain specifications. Thus far no completely unified and general classification scheme for all mechanisms has been found, although several attempts have been made.

In the work to date, two groups may be recognized: we distinguish between the functional and the structural. The functional classification system considers the *complete* mechanism needed to transform a given motion into another, as the conversion of uniform rotation into reciprocation. The complete mechanism is like a "black box," with provision for accepting one kind of motion at the input and producing another kind of motion at the output. Since it is in the nature of things to be able to accomplish a given task in more than one way, there would be a number of black boxes to choose from—their "insides," or "works," availability, and cost would be different, but the overall effect, i.e., motion transformation, would be the same. Monge's scheme belongs to this group. The shortcoming here is that no general principles exist to guide the problem of transforming a given motion into another. All that can be done is to run down a list of complete mechanisms that will do the specified job and choose from among the several the one that best fits additional specifications of available space and manufacture. There is really no unity, and the system is a mere collection of mechanisms.

The second, or structural, classification system deals with the nature of the *parts*, considering them from the standpoint of their relative motions. Willis followed this idea by regarding how the motion transformation between input and output members was achieved—as by rolling or sliding contacts, flexible connectors, barlike links, or tackle. Reuleaux was much more intimate: he considered, not the big input-output span, but only the immediate *connection between parts*. Here the shapes of the surfaces in contact—the working surfaces—impose particular and unique motion restrictions; i.e., they allow only a particular

type of relative motion, as rotation about a single axis, rectilinear translation, rotation about a point, etc.

Even though none of the known classification systems answers all needs, each one has some merit that can be exploited with profit for particular areas of interest.

### 2-3 RIGID AND RESISTANT BODIES

A *rigid body* is a material body in which the distance between any two points is invariable. Thus, if  $A$ ,  $B$ , and  $C$  are three points of a rigid body (Fig. 2-1), the three distances  $AB$ ,  $BC$ ,  $AC$  remain constant no matter how the rigid body is moved as from position 1 to position 2. If three noncollinear points  $A$ ,  $B$ ,  $C$  on a rigid body are known, any other point  $D$  may be identified by specifying the three distances  $DA$ ,  $DB$ ,  $DC$ . Even if the point  $D$  lies outside the apparent boundaries of the rigid body, it may still be considered as a point of the body as long as the three distances  $DA$ ,  $DB$ , and  $DC$  are invariable: when the rigid body moves from position 1 to position 2, point  $D$  moves with it.

When a rigid body is moved from an initial position 1 to a final position 2, it is said to have been given a displacement. As in the case of a point, the displacement of a rigid body depends only on the initial and final positions and is independent of the way in which it is carried out; i.e., the actual path followed during the displacement is of no consequence. As a rigid body moves from an initial to a final position, each of its points traces a particular path; at each instant of time a point has a particular velocity and a particular acceleration.

To be honest about it, there is no such thing as a rigid body, since all known materials deform somewhat under any stress no matter how

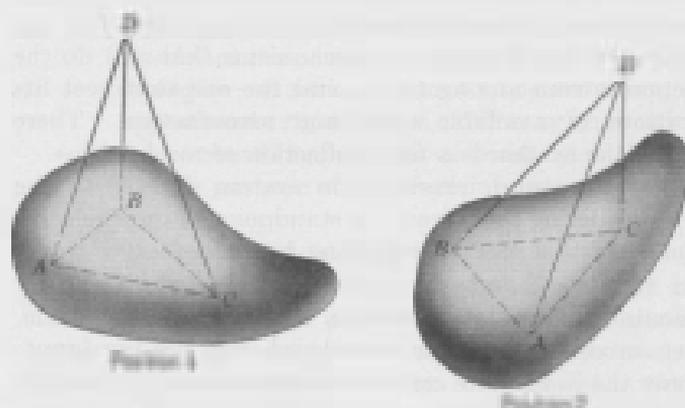


FIGURE 2-1. Two positions of a rigid body.

small. If we assume, as we must at least in the beginning, that the geometry of the body does not change as it moves—that any two points always bear the same relationship to each other—then we use “rigid” in the mathematician’s sense, overlooking the inevitable small deformations that are held to acceptable values by proper cross sections of the bodies. Realistically, we might better speak of resistant bodies, bodies so proportioned that their deformations are acceptably small. Under resistant bodies we may also include belts, chains, and hydraulic lines, for these have a one-way resistance to deformation. As a matter of fact, elastic deformations may become troublesome when machinery operates at high speed, but they cannot be established until the kinematic work is finished and the parts have acquired mass by having been given physical shapes and cross sections.

## 2-4 MOTION—RELATIVE AND ABSOLUTE

The very fact that motion exists implies reference frames of some sort on the moving parts. If the reference frame of one machine part moves with respect to the reference frame of another, we speak of relative motion. When the reference frame of one part is fixed with respect to the earth and the motions of the other parts are referred to it, then these particular relative motions are termed absolute motions: absolute motion is thus a special case of relative motion, the case in which the absolute-motion reference frame has no motion.<sup>6</sup> Of course, any rigid body may be chosen as a reference to which the motion of other bodies is referred; the operation of an aircraft landing gear is better referred to the fuselage than to the distant earth.

## 2-5 CONNECTIONS, AND THE PARTICULAR MOTIONS THEY PERMIT

A mechanism has been defined as a number of rigid bodies so connected that each moves with respect to another. The clue to the nature of a mechanism lies in how the parts are connected and what kind of relative motion the connection allows. In kinematics, a connection is a joint between two members permitting a particular kind of motion. We should note that the term connection, in fields different from kinematics, may mean an immovable connection, as a structural joint or splice, shrink fit, and the like.

Considerations based on the relative motions permitted by various mechanical connectors lead to a recognition of three broad classes of

<sup>1</sup> That the earth itself is in motion around the sun has no bearing on any problems to be discussed here.

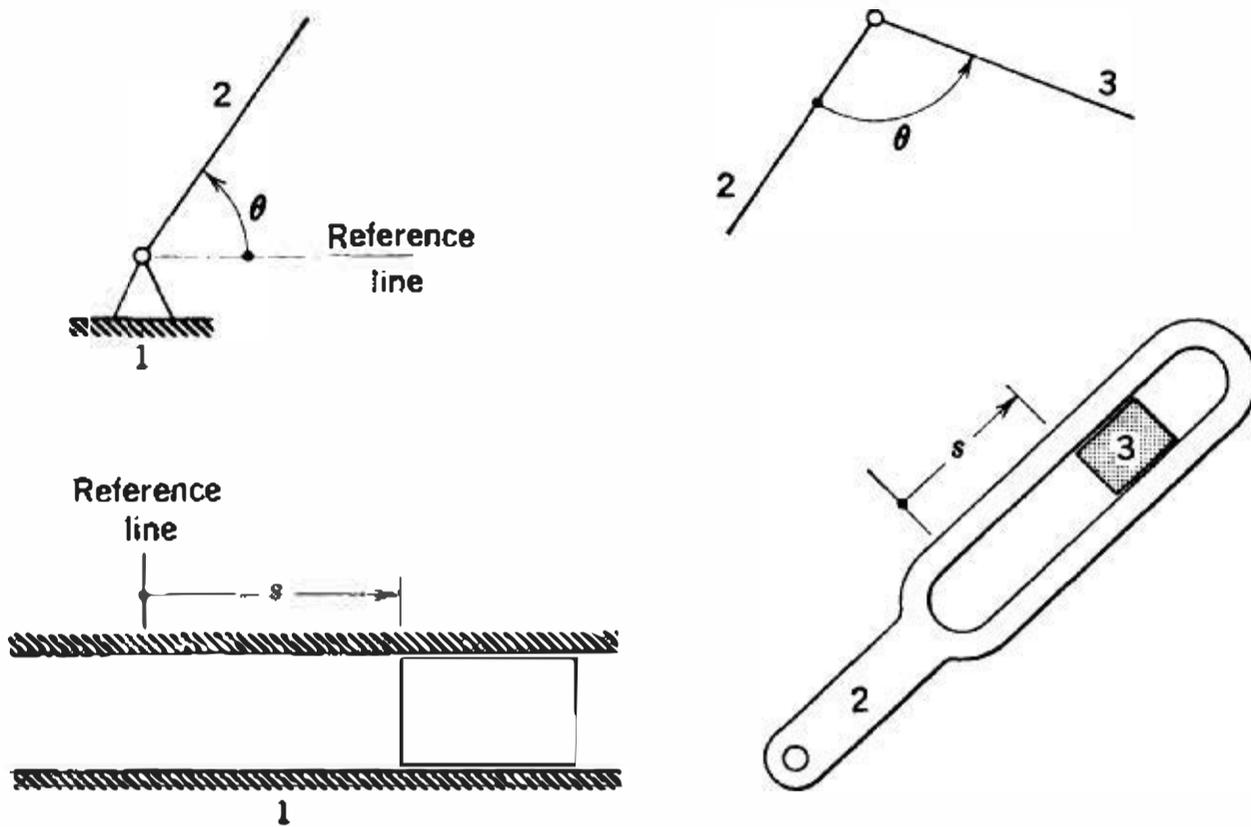


FIGURE 2-2 Conventions for showing pair variables.

connectors. These are the lower-pair connectors, the higher-pair connectors, and the wrapping connectors. The last are comprised of belts and chains with their one-way rigidity and will not be discussed here.

## 2-6 LOWER-PAIR CONNECTORS

Consider two links in parallel planes, connected in such a way that one may turn with respect to the other. The upper sketches in Fig. 2-2 show the two links in a conventional way: the small circle denotes the ability to turn, and the center of the circle represents the axis of rotation about which the angle of rotation is measured. Nothing has been indicated about the physical make-up of the joint: it could be made in many ways, as using a simple pin, a ball or roller bearing, etc. However, no matter how constructed, an angle such as  $\theta$  is always the definitive variable of motion and will be called the motion variable of such a joint.

Another situation is that of a crosshead in its guide. Here only (rectilinear) translation is possible, and the relative motion is described by means of a linear displacement such as  $s$  measured from some convenient origin. The actual form of the crosshead and guide sections, as laid out by the machine designer, may take any of a large number of shapes, having in common the property of restricting the relative motion to translation only. No matter what the form, the relative motion is described by a single variable such as  $s$ .

With the intuitive background of these two examples we may make a systematic approach to the problems of movable connections. Consider a set of  $x$ ,  $y$ , and  $z$  axes fixed to ground link 1 (Fig. 2-3). Link 2,

which is somehow suspended in space, also carries a set of rectangular axes  $u$ ,  $v$ , and  $w$ , with origin at  $A$ . The two sets of axes are parallel. If link 2 is moved to any other location, we could describe this by saying that  $A$ 's new location is at  $x_2$ ,  $y_2$ ,  $z_2$ : each axial displacement of  $A$ , that is,  $x_2 - x_1 = \Delta x$ ,  $y_2 - y_1 = \Delta y$ , and  $z_2 - z_1 = \Delta z$ , represents a motion of translation. The vector sum of these displacements,  $\Delta x + \Delta y + \Delta z = \Delta s$ , is the real displacement of  $A$ . It is convenient to keep track of the motion of  $A$  by means of three orthogonal motions (translations), which may be taken in any order.

In addition to the motion of  $A$ , there may also be a motion about  $A$ , as when link 2 has turned about the point  $A$ . This turning is conveniently described by separate rotations about each of the axes  $u$ ,  $v$ , and  $w$ , or, what amounts to the same thing, about the  $x$ ,  $y$ , and  $z$  axes. The sequence of rotations is important for spatial mechanisms. With regard to the motion about  $A$ , this may come either before or after the consideration of the motion of  $A$ ; the order of these two operations is immaterial.

We see from the foregoing that completely to define the position of a link requires the knowledge of six variables, three giving the translation of a point, and three giving the rotation of the link about that point. Each of these motion variables is also said to be associated with a degree of freedom, i.e., each is identified with a motion of either translation or rotation. To form a clearer picture of all this, we shall examine in detail the possible motions of link 2 with respect to link 1 by observing the nature of the connection:

1. If link 2 is permitted only a rotation about its  $w$  axis (the other

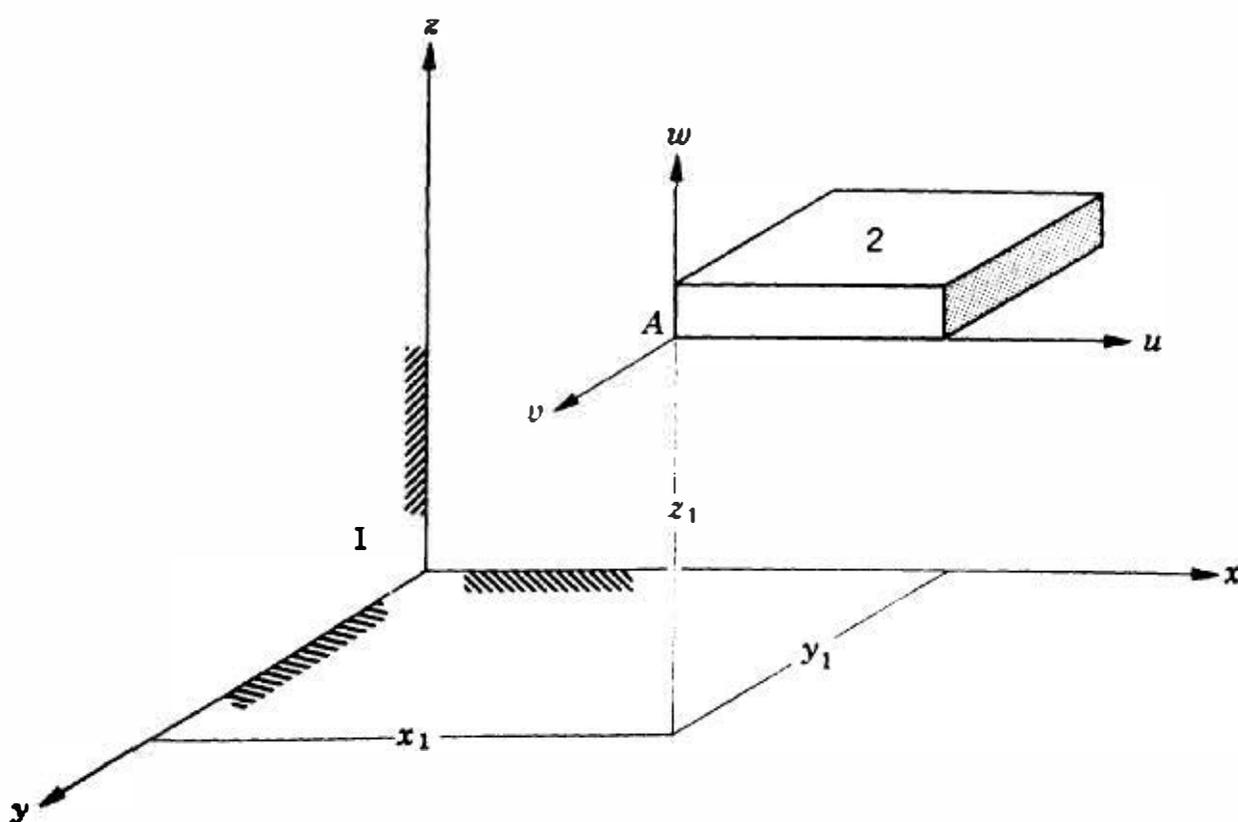


FIGURE 2-3 Reference frames for moving link.

five possible motions being suppressed), a variable sufficient to describe the relative motion is an angle  $\theta$ , measured in a plane perpendicular to the  $w$  axis. Following Reuleaux, we designate this connection as a revolute and give it the symbol  $R$ . The degree of freedom  $f$  of this connection is expressed by  $f = 1$ .

2. Were only translation along the  $w$  axis permitted (again with the other five possible motions suppressed), the two links would remain parallel to each other and the variable describing the relative motion would be the perpendicular distance, say  $s$ , between planes  $xy$  and  $uv$ . This type of motion, a rectilinear translation, is commonly associated with a crosshead and its guide. It is a prismatic connection with the symbol  $P$ , and degree of freedom  $f = 1$ .

3. Were both rotation about and translation along the  $w$  axis permitted, two independent variables—one for the translation, the other for the rotation—would be needed to describe the relative motion. Such a connection is called cylindrical: it is the motion of a shaft in a journal bearing if there is no axial restraint. In symbolic notation we write  $C$ ; the degree of freedom  $f = 2$ .

4. Suppose the  $w$  axis to be threaded, as a bolt, and the corner  $A$  tapped as a nut. As link 2 turned, it would remain parallel to link 1, although undergoing a translation along the  $w$  axis. Since the angle of rotation  $\theta$  and the translation  $s$  are related by the (constant) lead  $L$  of the screw, there is but one variable for the relative motion of the links. For the screw connection we write  $S_L$ ; the degree of freedom is  $f = 1$ .

5. Suppose that link 2 lay directly on the  $xy$  plane of link 1 ( $z = 0$ ) and were allowed to slide on that plane. We would then recognize three possible motions—two translations, and a rotation about the  $w$  axis. Such a planar connection (it occurs rarely) would have the symbol  $F$  (think of flat) and of course has three variables, which means that the degree of freedom  $f = 3$ .

6. To suppose again, assume that there is a ball-and-socket joint at  $A$ , thus joining links 1 and 2 with a spheric connection. We immediately recognize the complete suppression of any linear motions: the only possible motion of link 2 with respect to link 1 is spherical motion, which is to say that all points of link 2 move in concentric spheres referred to the center of the ball. The motion is best described by successive rotations about the three mutually perpendicular axes: the sequence of the rotations is important. This connection has three variables, whence  $f = 3$ . The symbol will be  $G$  (for globular).

The foregoing six types of connections, when reduced to simple forms of construction, are shown in Fig. 2-4. The common denominator of these connections appears to be the area contact between links. Each of the identical surfaces of contact, the working surfaces, is called an

*element*: taken together, the two elements constitute a *pair*, one element lying on one link, the second element lying on the second link. It is apparent that the relative motion of the two links is the relative motion of the pair elements and that this relative motion is defined by the variable of the connection, which we now call the *pair variable*.

The particular forms chosen for the screw, revolute, and prismatic pairs illustrated in Fig. 2-4 follow Reuleaux's suggestion that the revolute and prismatic pairs may be considered as special limiting cases of the screw pair, with the lead either zero or infinity. This observation will be put to use in designing a more complete symbolic notation to describe mechanisms in which all connections are made by lower pairs. With  $S_L$  representing a screw of lead  $L$ , then the symbols for the revolute and prismatic pairs follow logically as  $S_0$  and  $S_\infty$ , i.e., screw pairs  $S_L$  with  $L = 0$  or  $L = \infty$ , respectively.

These six element pairs, whose appearance in their simple constructional form is dominated by area contact, were called *lower pairs* by Reuleaux; the term came into the English language with Professor Kennedy's translation of 1876. It is unfortunate that this apparent dominance of area contact has often made area contact the criterion for lower pairs: *the real concept of lower pairs lies in the particular kind of relative motion permitted the connected links*; the particular motion of each pair is defined by an obviously associated pair variable or by a simple grouping of functionally unrelated pair variables.

A moment's reflection will show that identical relative motions are possible with various joint constructions having neither area contact nor geometrically identical elements. For example, a turning connection or revolute may be constructed with a ball or pivot bearing, and for small angles of rotation a knife-edge or a flexure pivot may even be considered: no area contact in the first, no elements in the second. However, the relative motion permitted the connected parts is a rotation, defined by an angle such as  $\theta$ . A typewriter carriage moves along its ways on small rollers with a motion of rectilinear translation: this means a prismatic pair without area contact; a distance  $s$  will be the pair variable. A ball spline is similar. A ball bushing is a practical way of constructing a low-friction cylindric pair, just as a ball-bearing screw is a screw pair of minimal friction. The various constructional forms have advantages for the machine designer, but they do not change the geometric relation of the connected links. They are merely different ways of physically achieving a specific kind of relative motion between the connected parts.

For all but the planar pair (symbol  $F$ ) we may speak of a hollow element and a full element. Thus, for the revolute pair  $R$  of Fig. 2-4a, the bearing surface of link 2 is the hollow element, written  $R^-$ ; and the surface of the shaft, which lies on link 1, is the full element, written  $R^+$ .

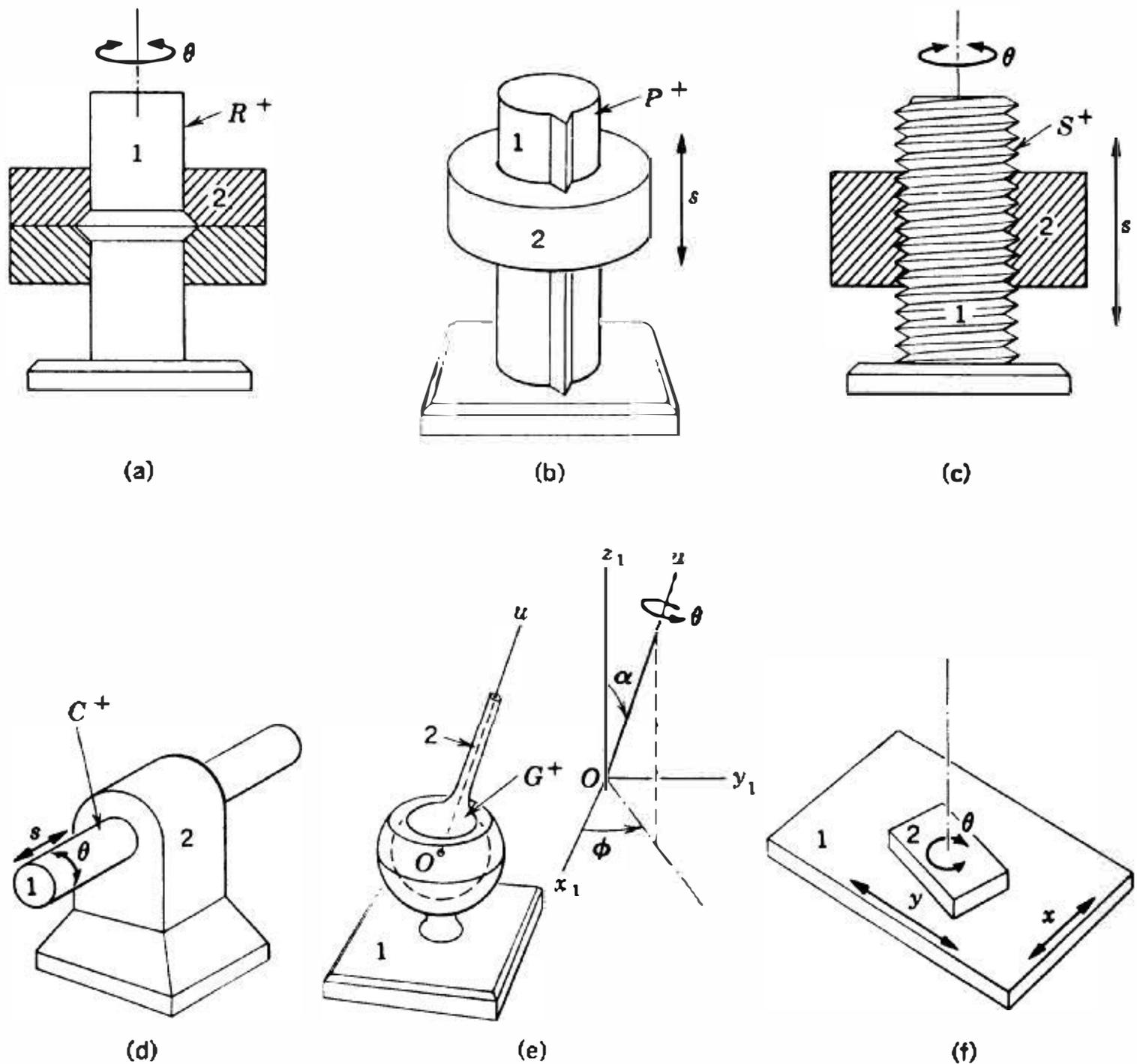


FIGURE 2-4 (a) *Revolute pair* (turning pair),  $f = 1$ . The relative motion is rotation about the axis and is defined by a single variable  $\theta$ . (b) *Prismatic pair*,  $f = 1$ . The relative motion is translation and is defined by a single variable  $s$ . (c) *Screw pair*,  $f = 1$ . The relative motion is helical and is defined by either the rotation  $\theta$  or translation  $s$  related through  $\Delta\theta/2\pi = \Delta s/L$ , where  $L$  is the lead of the screw (advance per revolution). (d) *Cylindric pair*,  $f = 2$ . The relative motion is a combination of a rotation  $\theta$  about an axis and a translation  $s$  parallel to the same axis; there is no relation between  $\theta$  and  $s$ . (e) *Spheric pair* (ball-and-socket joint),  $f = 3$ . The relative motion is spherical and is defined by three variables: two angles  $\alpha$  and  $\phi$  to define the direction  $Ou$  and the angle  $\theta$  of rotation about  $Ou$ . (f) *Planar pair*,  $f = 3$ . The relative motion is planar and is defined in terms of two translations  $x$  and  $y$  and a rotation  $\theta$ .

Since the hollow and full elements, when visualized as areas, not only are geometrically identical but the hollow element is “wrapped around” the full element, the five pairs are also known as *wrapping pairs*. We should note that the hollow and full elements of the wrapping pairs may be interchanged without affecting their relative motion. Thus, the relative motion between links 1 and 2 of the first five pairs of Fig. 2-4 will be the

same no matter whether link 1 or link 2 is the moving link. The word *wrapping* must be viewed with caution; inconsistent as it may seem, usage denotes chains and belts as wrapping *connectors*, but not as wrapping pairs.

Machine parts as actually constructed may have interrupted, or noncontinuous, elements (Fig. 2-5a and b). This feature does not change the character of the motion but is useful to the machine designer in distributing loads and stresses.

The same machine part may also carry elements of different pairs along the same axis (Fig. 2-5c). In the figure links 3 and 4 connect to link 2; each is a separate connection and must be so treated. There are two distinct coaxial revolutes,  $R_3$  and  $R_4$ . The full elements  $R_3^+$  and  $R_4^+$  both lie on link 2, and the pair variables  $\theta_3$  and  $\theta_4$  are measured from the same reference line of link 2. The difference between  $\theta_4$  and  $\theta_3$  is the pair variable between links 4 and 3.

For mechanical convenience an intermediate pin (Fig. 2-5d) is generally used to fashion a revolute connection. Although the actual

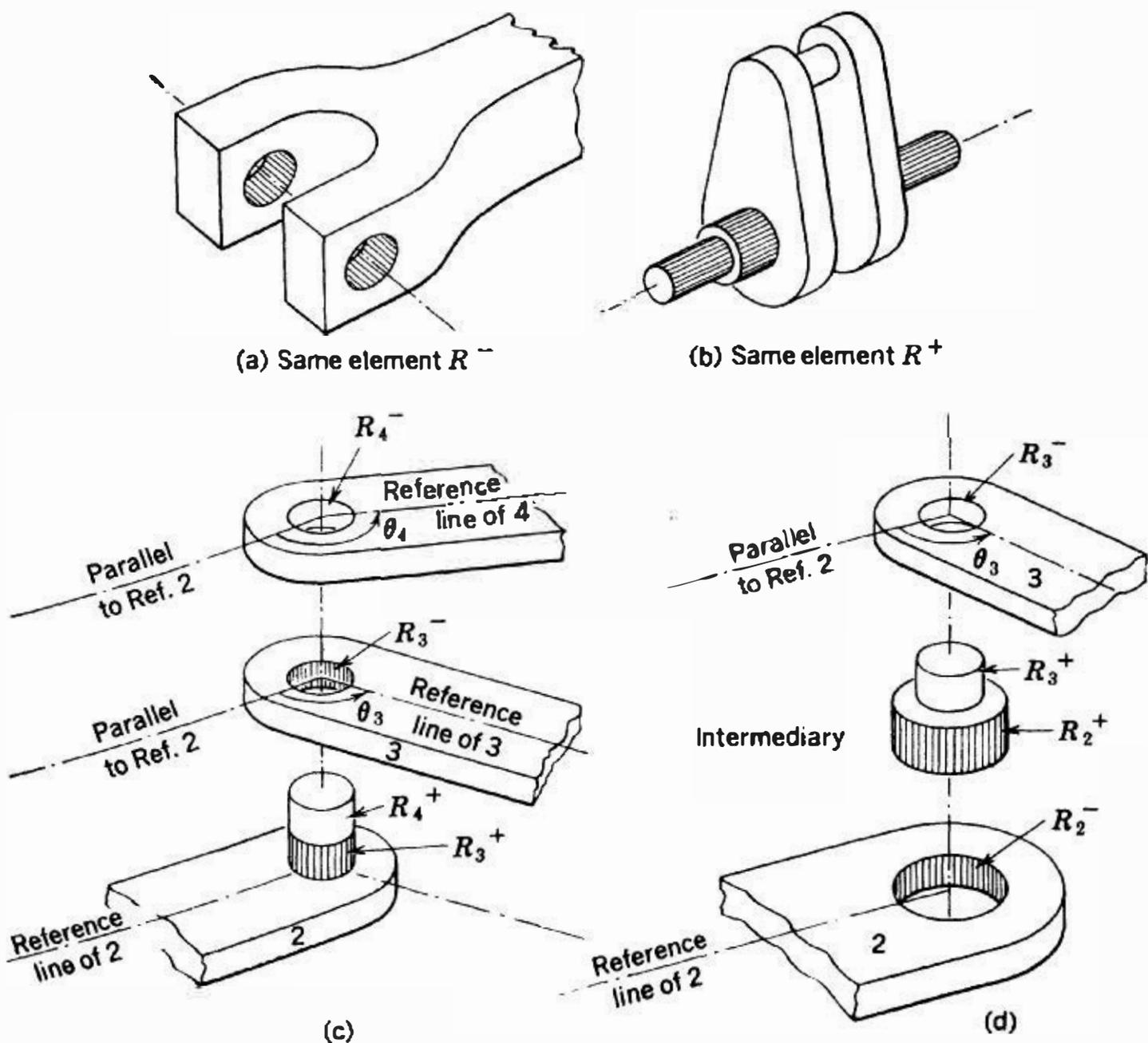


FIGURE 2-5 Interrupted and coaxial elements.

number of elements has been augmented by the introduction of the intermediary, the intermediary does not have to be counted if its own motion is not of interest, since its presence does not affect the relative motion between links 2 and 3.

## 2-7 HIGHER-PAIR CONNECTORS

We have seen that the connection between parts may be considered in terms of pair elements, the contacting surfaces of the parts. We identified lower-pair connectors as those permitting six different kinds of specific motions.

A second type of connection must be described for the sake of completeness: it is what Reuleaux called the higher-pair connection. In the higher pairs the surface elements are so shaped that only line or point contacts are possible between elements. Point contact is found in ball bearings, as well as between the teeth of helical gears on nonparallel shafts. Line contact is characteristic of cams, roller bearings, and most gears. The relative motion of the elements of higher pairs is generally quite complicated. The involved functional relationship between translation and rotation allows no succinct statement, and an infinite number of higher pairs exist. Under the circumstances, higher pairs do not follow a simple classification scheme, as do the lower pairs. Nor are they conveniently described by means of symbols.

Higher-pair connections may on occasion be replaced by a combination of lower pairs, to reduce unit contact pressures and allow take-up for wear. Thus, the pin element of link 2 riding in link 4 (Fig. 2-6) is not a very practical construction; the same relative motion between links 2 and 4 is retained on interposing link 3. We note that the two degrees of freedom of the higher-pair connection (translation and rotation) are maintained with the substitution of the two lower pairs and that another link has been added.

## 2-8 FOUR-BAR LINKAGES

A versatile example of mechanism is known as the *four-bar linkage* (Fig. 2-7). It consists of four rigid members: the *frame*, or fixed member,

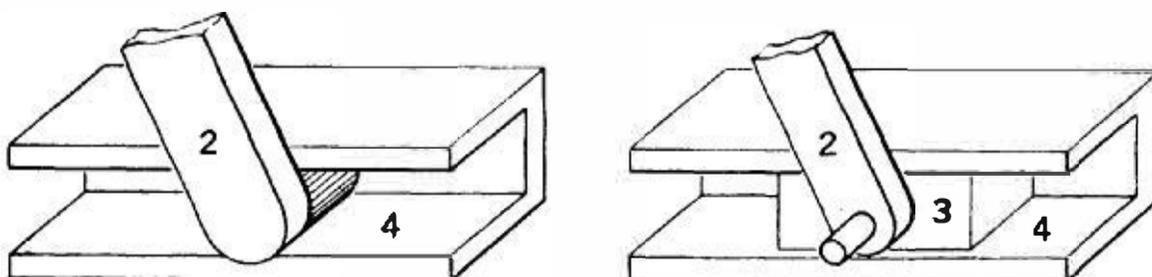


FIGURE 2-6 Substitution of two lower pairs for a higher pair; note the additional link.

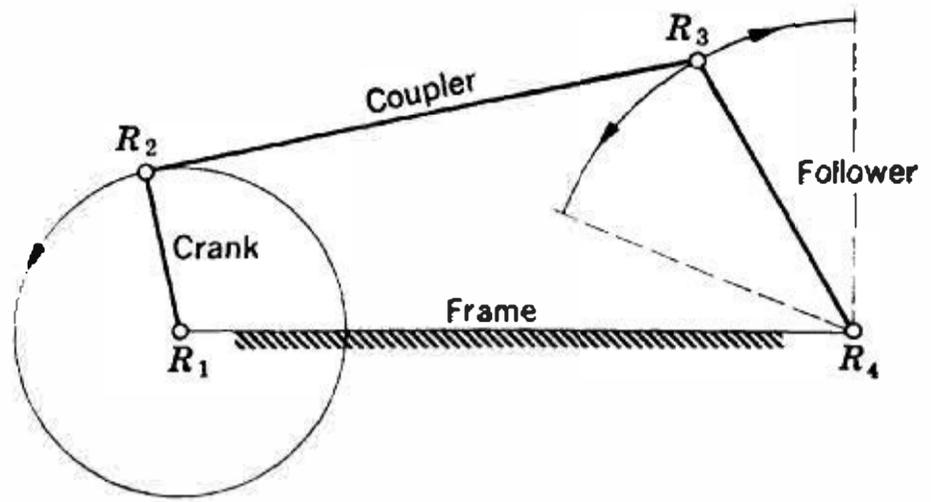


FIGURE 2-7 Four-bar linkage. With the link proportions shown, a continuous rotation of the crank is converted into an oscillation of the follower.

which is assumed stationary, and to which are pivoted the *crank* and *follower*, whose intermediary is aptly termed *coupler*. These members are connected by four revolute pairs,  $R_1, R_2, R_3, R_4$ , allowing relative rotation between adjacent members; all four revolute axes are parallel. The word linkage implies that all connections in the mechanism are lower pairs (here they all happen to be revolutes).

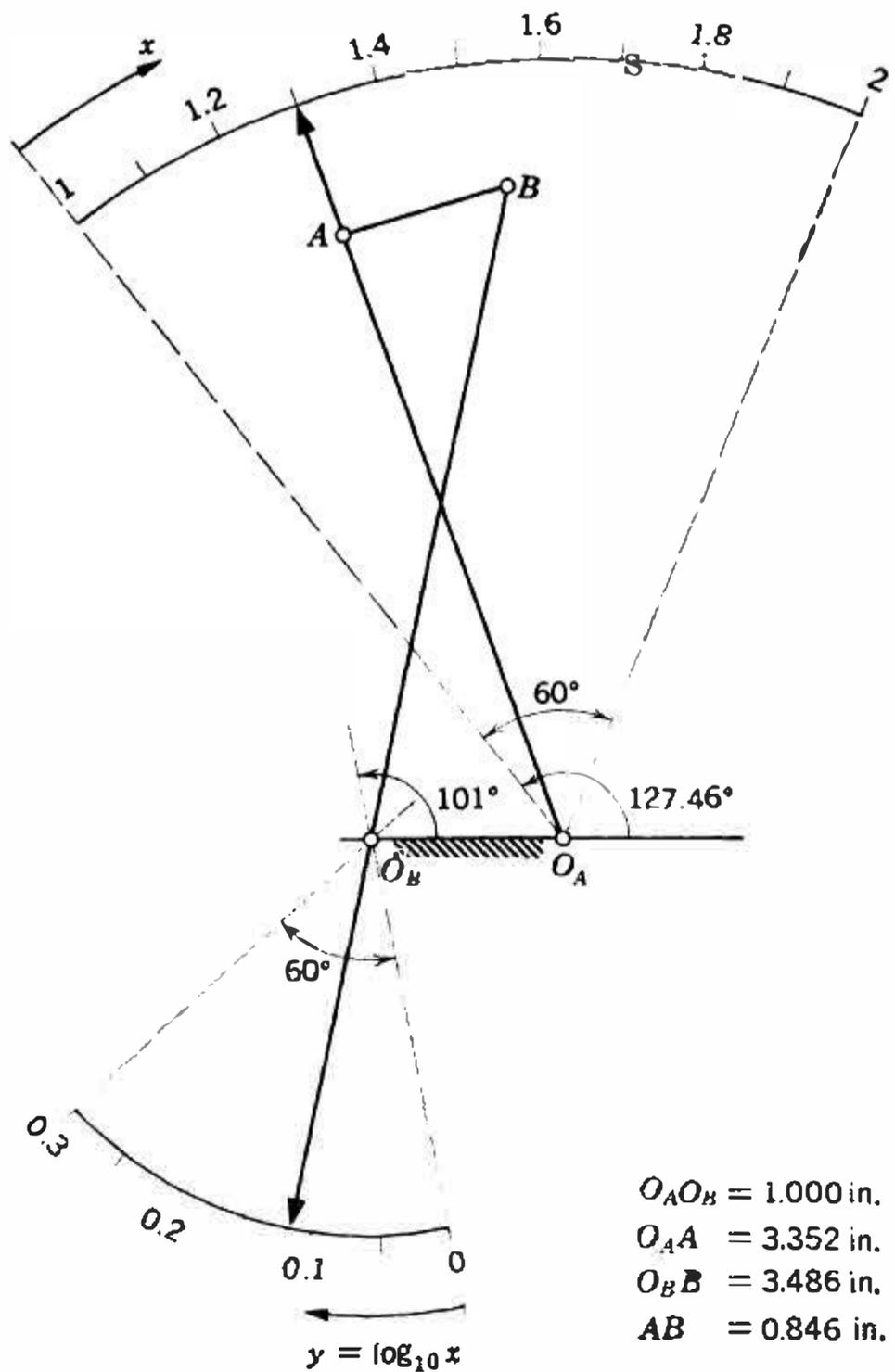


FIGURE 2-8 Four-bar linkage as generator of the logarithmic function  $y = \log x$  over specified ranges of the input variable  $x$  and the arcs of oscillation.

Older usage, when the four-bar linkage was known as a quadric-crank mechanism, implied that a "crank" could or could not rotate continuously, depending on its position in the mechanism. It will be convenient to use the word crank to designate (1) the input link, whether or not it is able to rotate completely (continuously in the same direction), or (2) a continuously rotating link which may or may not be the input.

As part of an instrument, the four-bar linkage may be used for scale conversion. Such devices are called function generators. To convert, for example, a linear scale into a logarithmic scale, the linkage shown in Fig. 2-8 may be used with an error which is less than  $0.0037^\circ$  for a  $60^\circ$  range of rotation of both crank and follower.

A point on the coupler of a four-bar linkage is called a *coupler point*; and its path when the crank is rotated is known as a *coupler-point curve* (or coupler curve) (Fig. 2-9), and the number of such curves is infinite. However, by proper choice of link proportions and coupler-point locations—this is one of the problems of synthesis—useful curves may be found. A curve's usefulness depends (1) on the particular shape of a segment—does it, for example, approximate a straight line or a circular arc?—or (2) on a peculiar shape of either the whole curve or parts of it. The coupler point, because of its motion characteristic, is now the output of the linkage. The coupler curve of the four-bar linkage

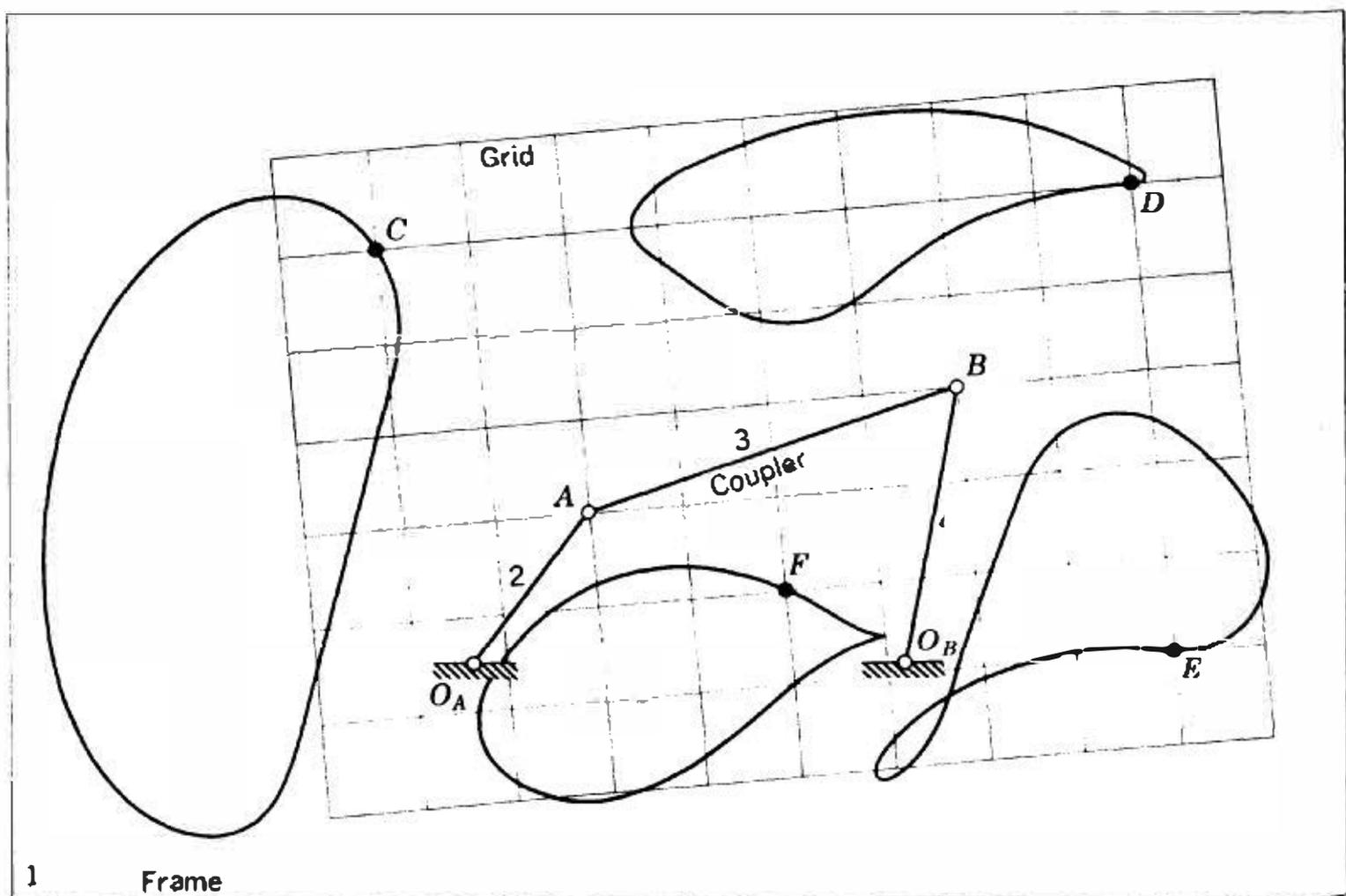


FIGURE 2-9 Four-bar coupler-point curves. The transparent grid is part of the coupler plane, link 3. The curves are traced on the plane of link 1, the frame.

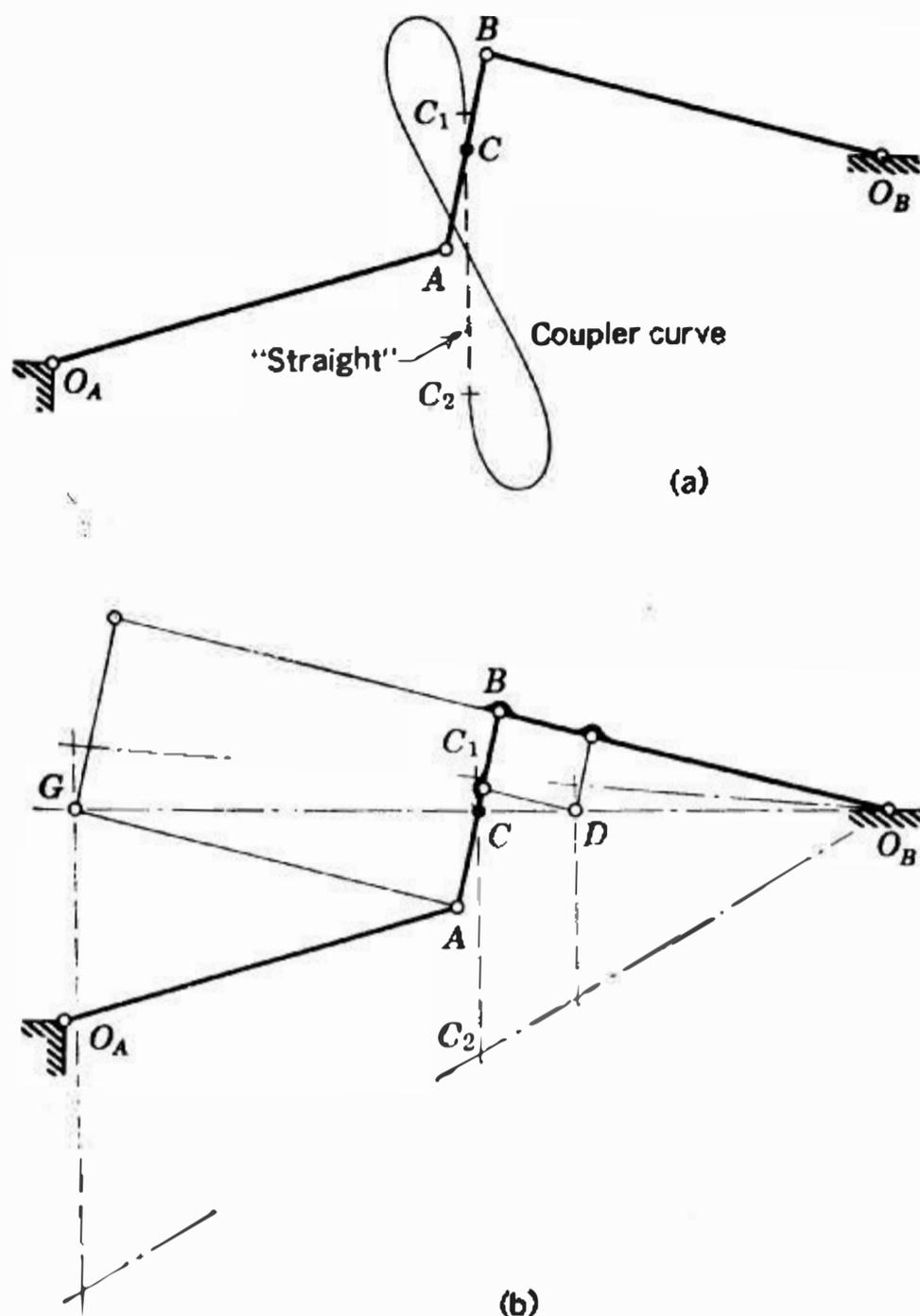


FIGURE 2-10 (a) Watt linkage, giving approximate straight-line motion ("parallel motion") at  $C$ ; (b) pantographs, to give similar outputs at  $G$  and  $D$ .

is of the sixth order.<sup>1</sup> The classical and best-known coupler-point linkage is the "straight-line," or "parallel-motion," linkage devised by Watt in 1784<sup>2</sup> to guide the upper end of a piston rod along a good-enough straight line. This was before the invention of the planer (1817); without the planer it was impractical to make straight surfaces 4 ft long as needed for the crosshead guides of the earliest double-acting engines. Linkwork was simple. Watt's linkage, in about the original proportions, is shown in Fig. 2-10. The coupler point  $C$  traces a curve of figure-eight shape. Watt used only the middle portion, which in this case is a very good approximation to a straight line. The addition of a pantograph allowed further exploitation of the "straight" coupler-curve segment. The Watt linkage is currently used for axle and differential suspensions of some high-performance cars.

<sup>1</sup> Special configurations of the four-bar linkage may generate coupler curves of the fourth or second order; e.g., the coupler curves of a parallelogram linkage are circles, i.e., of order 2.

<sup>2</sup> In referring to his many inventions, Watt remarked that this was the one of which he was proudest.

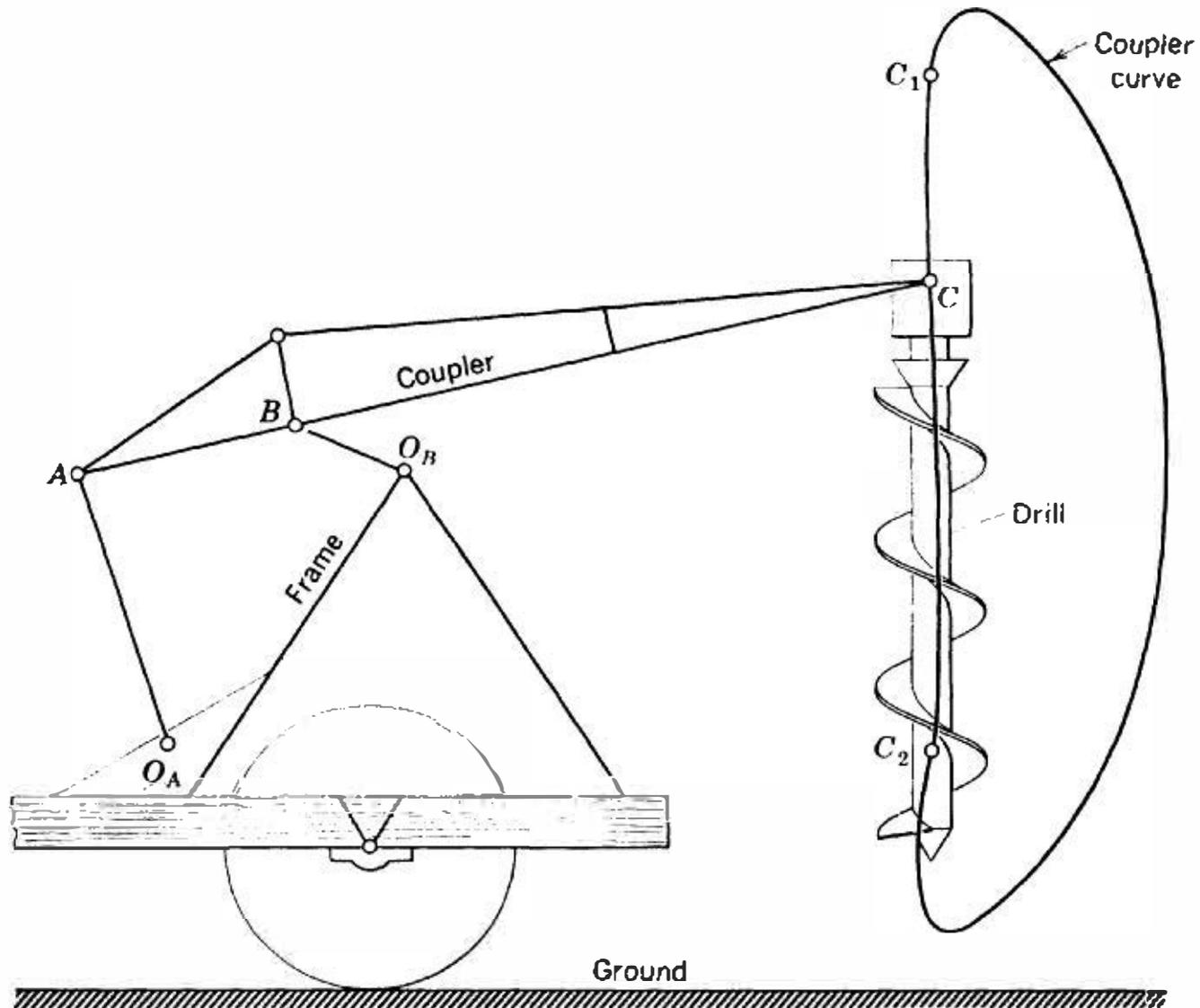


FIGURE 2-11 Portable posthole borer, or setter. (From Volmer, *VDI Ber.*, vol. 12, 1956.)

The Watt rotative engines retained the “great beam,” or “lever,” of the Newcomen engines, which made them just as bulky (see Figs. 1-8 and 1-9): their utility lay in the fact that they were rotative, giving power directly to a shaft. The famous “lap”<sup>1</sup> engine of 1787 had a great beam of about 1.5 ft length pinned to a connecting rod over 13 ft long; the engine was rated at 10 hp when running at 25 strokes per minute. Because of the one-to-one gearset of the sun-and-planet this gave a shaft speed of 50 rpm. Engines such as this required enormous enginehouses, whose cost sometimes equaled that of the engines.

Although the direct-connected engine, now called the slider-crank type, was explored around 1800 in association with early high-pressure<sup>2</sup>

<sup>1</sup> The engine was used for driving the machinery for lapping, or polishing, steel ornaments. It was taken out of service in 1858 and is now in the Science Museum, London.

<sup>2</sup> The adjective *high* is always associated with an age of development, and it is the age that sets any number. In 1800 a high-pressure engine was one dispensing with the vacuum, working with positive steam pressures of 2 or 3 atm, and exhausting at atmospheric pressure. Around 1860 a speed of 125 rpm was exciting enough to evoke the mistrust of old-timers accustomed to half that speed.

engines, the slow beam engines dominated the power scene for many decades. Much effort was expended in searching for less bulky linkages that would produce straight-line guidance for piston-rod ends, for engine-building tradition demanded vertical cylinders. Of these we may mention the side-lever engines, used principally in boats and ships, and the grasshopper, or Evans, linkwork of small stationary engines. The latter is of particular interest, since it is related to the Watt linkage in a not directly obvious manner, as we shall see later.

Another example of the use of an approximately straight-line segment is the posthole borer shown in Fig. 2-11, where point  $C$  is a coupler point of the four-bar linkage  $O_A A B O_B$ . The path of  $C$  approximates the straight vertical segment  $C_1 C_2$ . This device, of German design, is capable of boring a vertical hole 6 ft deep.

Coupler-point curves having segments approximating circular arcs can be invoked to produce linkages having a dwell or two sufficiently complete for many practical purposes (Fig. 2-12). The figure shows a

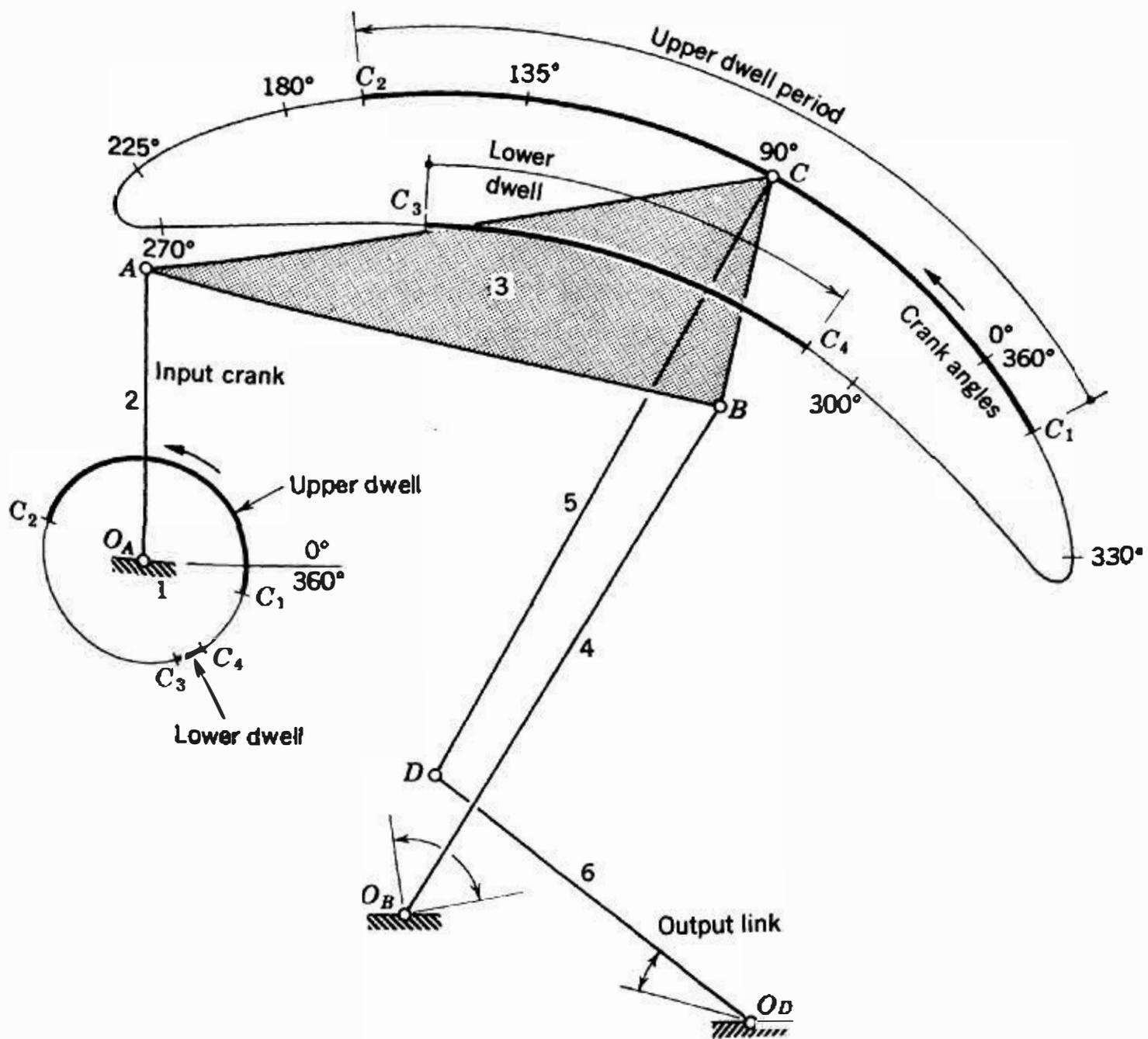


FIGURE 2-12 Double-dwell linkage.

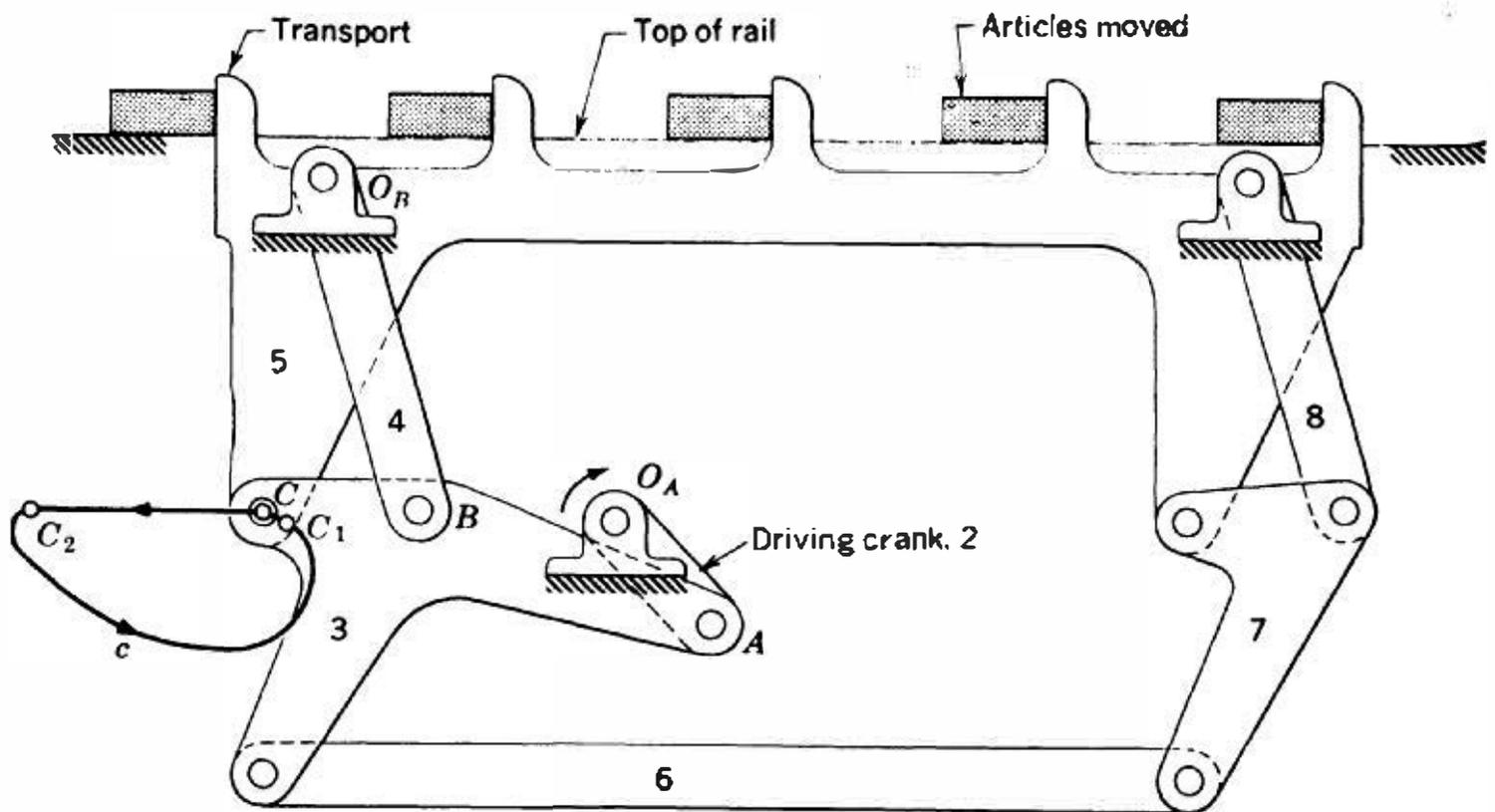


FIGURE 2-13 Transport mechanism. (From C. W. Ham, E. J. Crane, and W. L. Rogers, "Mechanics of Machinery," 4th ed., p. 439, McGraw-Hill Book Company, New York, 1958.)

coupler curve having two nearly circular segments  $C_1C_2$  and  $C_3C_4$  of nearly the "same" radius. Two bars—a dyad—are added at the coupler point  $C$ . The second bar, link 6, is the output link: it will be at dwell (rest) while the coupler point is traversing the "circular" arcs  $C_1C_2$  and  $C_3C_4$ . It must be noted that the dwells are only as good as the approximation between the actual curve and circular arcs; the dwells will not be quite complete but nevertheless will be adequate for many applications. If the output oscillation of link 6 is followed through the complete excursion of  $C$  around the curve, it will be seen to have the characteristic of the oscillating follower of a rise-dwell-drop-dwell cam.

As another example of the use of a whole coupler curve, we may consider the piece of materials-handling equipment shown in Fig. 2-13. Here point  $C$  on the coupler of the four-bar linkage  $O_AABO_B$  describes the path  $c$  as crank 2 rotates through  $360^\circ$ . This motion is communicated to the transport member 5 by means of parallelogram linkages; member 5 moves horizontally for the line segment  $C_1C_2$ , then drops out of the way, to reappear later at the right, rising nearly vertically before moving to the left. A similar mechanism is used for the film transport in some motion-picture cameras (Fig. 2-14).

## 2-9 SLIDER-CRANK MECHANISMS

Another versatile linkage is the slider-crank mechanism (Fig. 2-15), familiar from reciprocating engines and pumps. Here the translation

of the piston is transformed into rotation of the crank shaft, or vice versa. Like the four-bar linkage, this mechanism consists of four rigid members, *frame* (fixed member), *crank*, *coupler* (connecting rod), and *follower* (slider). It differs from the four-bar linkage only in that one revolute pair  $R_4$  has been replaced by a prismatic pair  $P_4$ , and many of the properties and applications of the four-bar linkage may be transposed to the slider-crank mechanism. The slider-crank mechanism is properly a linkage, since both revolute and prismatic pairs are lower pairs.

The dimensions of the slider-crank mechanism as used in reciprocating engines are approximately those shown in Fig. 2-15, and the path of the center of rotation of revolute  $R_2$ , the wrist pin, usually goes through the center of the main bearings, revolute  $R_1$ . When this is the case, the mechanism is called a *central* slider crank; it is otherwise an *offset*, or *eccentric*, slider crank (see Fig. 2-16).

The points of the coupler of a slider-crank mechanism are, as in

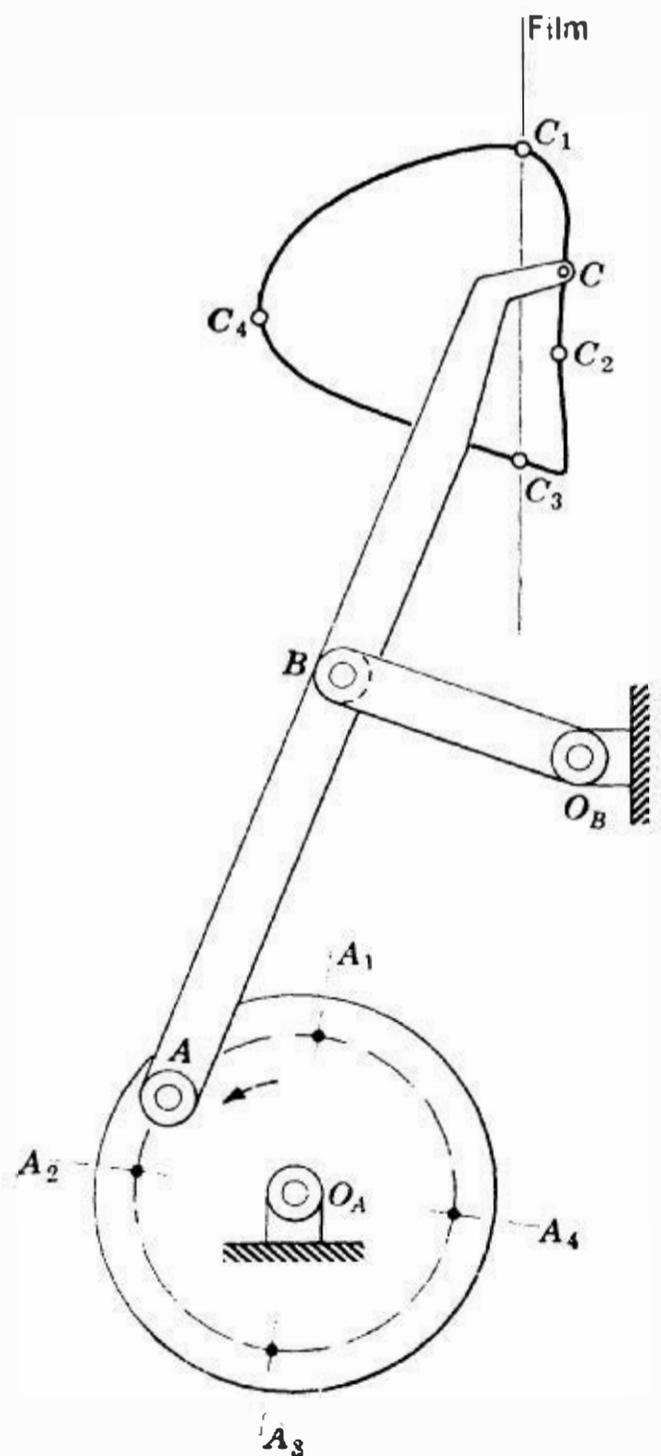


FIGURE 2-14 Film feed. (From "Die Wissenschaftliche und Angewandte Photographie," ed. by Kurt Michel, vol. 3, Harald Weise, "Die Kinematographische Kamera," p. 202, Springer-Verlag OHG, Vienna, 1955.)

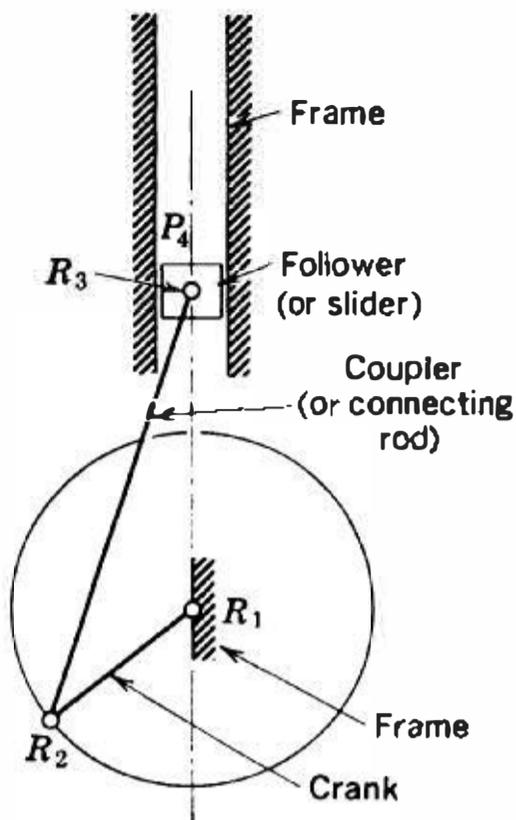


FIGURE 2-15 Central slider-crank mechanism, the conventional case.

the case of the four-bar linkage, also called coupler points, and their paths as the crank is rotated are coupler-point curves, but of the fourth order (Fig. 2-17). As with the four-bar linkage, slider-crank coupler curves may also be put to work. One application is shown in Fig. 2-18.

## 2-10 TRANSMISSION, DEVIATION, AND PRESSURE ANGLES

It would be useful to have a measure, criterion, or index of how well a mechanism might "run" while it is still in kinematic skeleton form on the drawing board. "Run" is a term that more formally means the effectiveness with which motion is imparted to the output link; it implies smooth operation, in which a maximum force component is available to produce a torque or a force, whatever the case might be, in an output member. Generally speaking, torque and force are not at all compatible with only the kinematics and statics of a given situation. As is well known, the magnitude of the dynamic forces may be several times as large as the static forces and may in addition possess quite different directions. Even a test on a kinematic model will check out only an approximation of the static forces and will tell nothing about the dynamic forces.

However, some evaluation of the state of affairs is better than

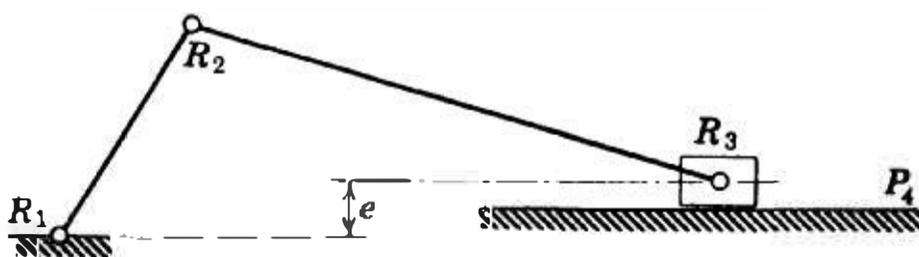


FIGURE 2-16 Offset, or eccentric, slider-crank mechanism.

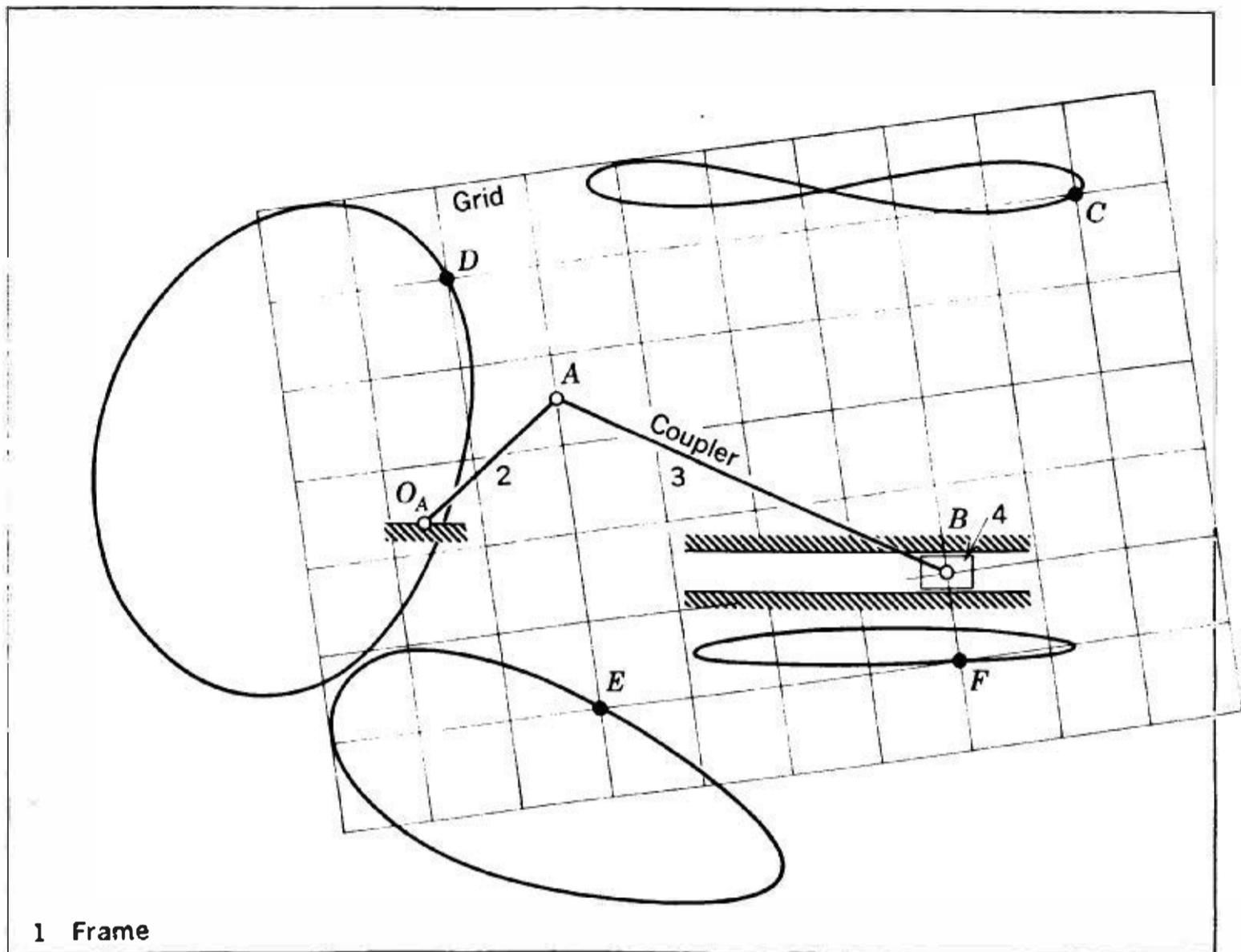


FIGURE 2-17 Slider-crank coupler-point curves. The transparent grid is part of the coupler plane, link 3. The curves are traced on the plane of link 1, the frame.

none. Alt<sup>1</sup> defined the aptness of motion transference from the driving link (not the input link of the mechanism) to the output link in terms of the transmission angle: the transmission angle  $\gamma$  is the smaller angle between the direction of the velocity difference vector  $\mathbf{v}_{B,A}$  of the driving link and the direction of the absolute velocity vector  $\mathbf{v}_B$  of the output link, both taken at the point of connection. This is shown in Fig. 2-19a for a four-bar linkage. Since the velocity vectors are perpendicular to their respective links, the transmission angle is also given by the angle between link centerlines,  $b$ . Clearly the optimum value of  $\gamma$  is  $90^\circ$ ; the recommended tolerance is about  $\pm 50^\circ$ . Alt<sup>2</sup> recognized that this kinematically determined transmission angle does not reflect the action of gravity or dynamic forces. Thus, in a single-cylinder piston engine the transmission angle (which is here between the connecting rod and crank) becomes zero at the dead-center positions, requiring the dynamic action of a flywheel to further the motion. Linkages with more than four bars have peculiar difficulties.

<sup>1</sup> H. Alt, *Werkstattstech.*, vol. 26, pp. 61-64, 1932.

<sup>2</sup> H. Alt, *Getriebetechnik, VDI Tagungsheft*, vol. 1, p. 197, 1953.

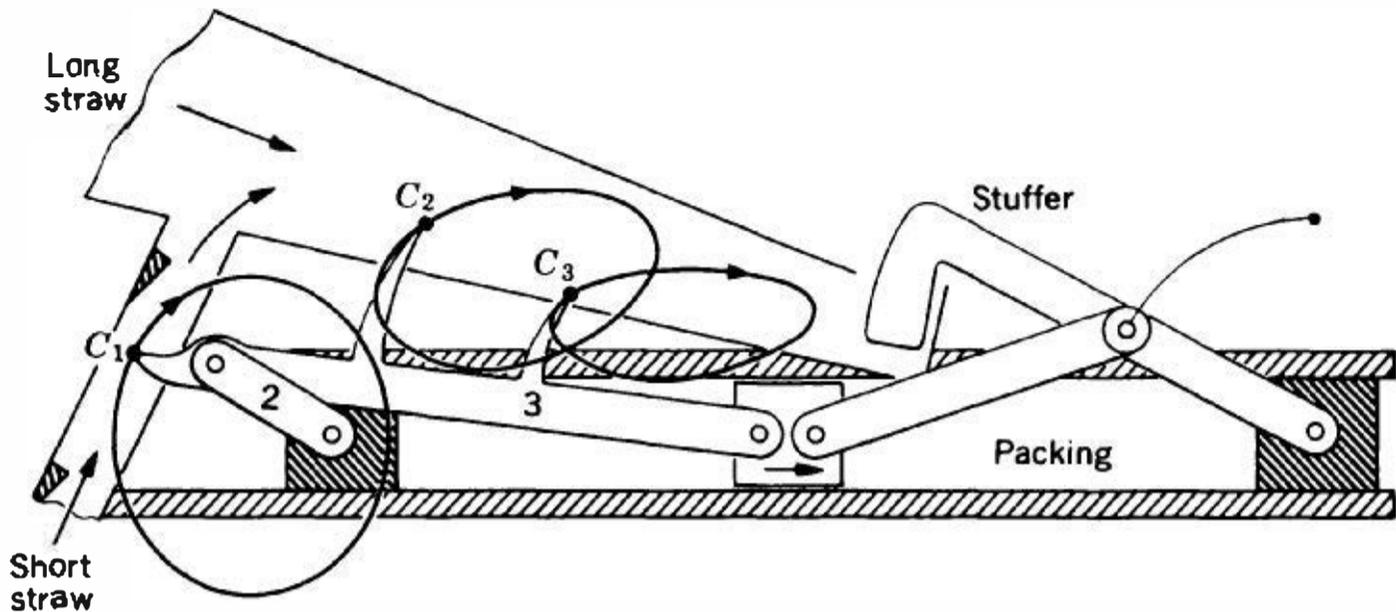


FIGURE 2-18 Straw packer making use of the coupler curves of a slider-crank mechanism. The crank is link 2; the coupler is link 3, with coupler points  $C_1$ ,  $C_2$ , and  $C_3$ . (After Kurt Rauh, "Praktische Getriebelehre," 2d rev. ed., vol. 1, fig. 299, Springer-Verlag OHG, Berlin, 1951.)

Another approach was taken by A. Bock,<sup>1</sup> who suggested working with the directions of the static force and velocity at the point of connection, terming the angle between the directions the deviation angle  $\delta$ . The deviation angle is shown in Fig. 2-19b; its optimum value is  $0^\circ$ . When the driving link (in this case link 3) is a two-force member,  $\gamma + \delta = 90^\circ$ . This relation fails when the driving link has more than two forces acting on it, as may be seen from Fig. 2-20 (adapted from Bock). Nerge<sup>2</sup> inclines toward Bock's view, although he does not name the angle. The pressure angle of a disk cam with roller follower—the angle between the common normal at the point of contact and the follower motion of the roller center—is recognized as the deviation angle (Fig. 2-21).

<sup>1</sup> A. Bock, *VDI Ber.*, vol. 29, p. 158, 1958.

<sup>2</sup> G. Nerge, *VDI Ber.*, vol. 29, p. 157, 1958.

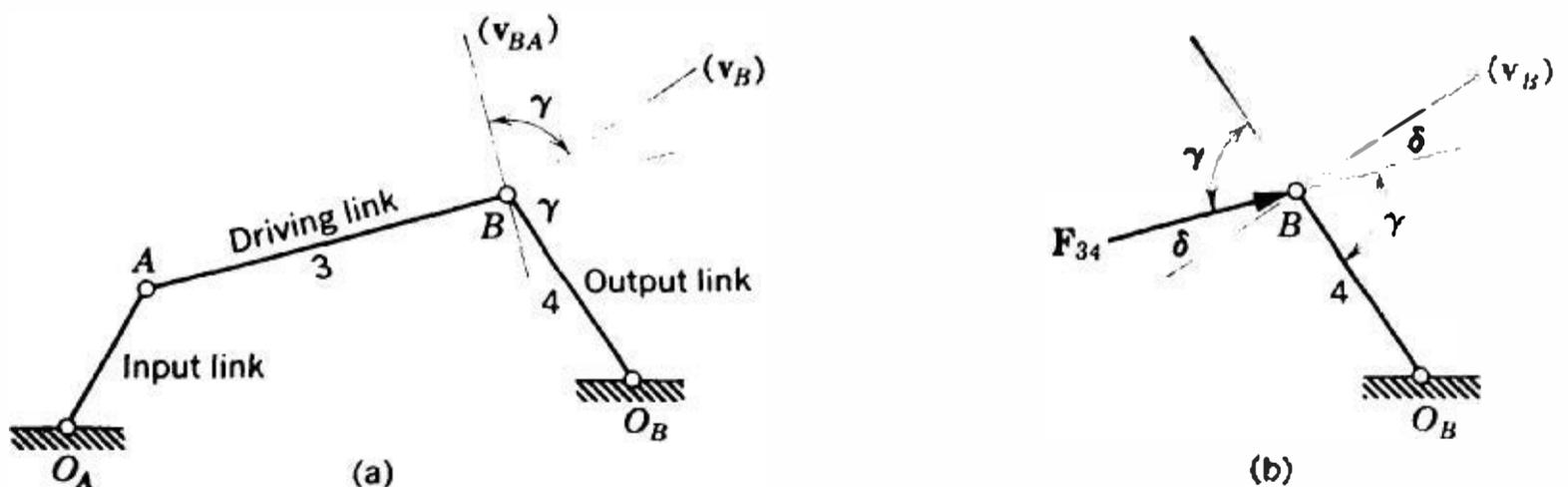


FIGURE 2-19 (a) Transmission angle  $\gamma$  of a four-bar linkage; (b) transmission angle  $\gamma$  and deviation angle  $\delta$  of a four-bar linkage.

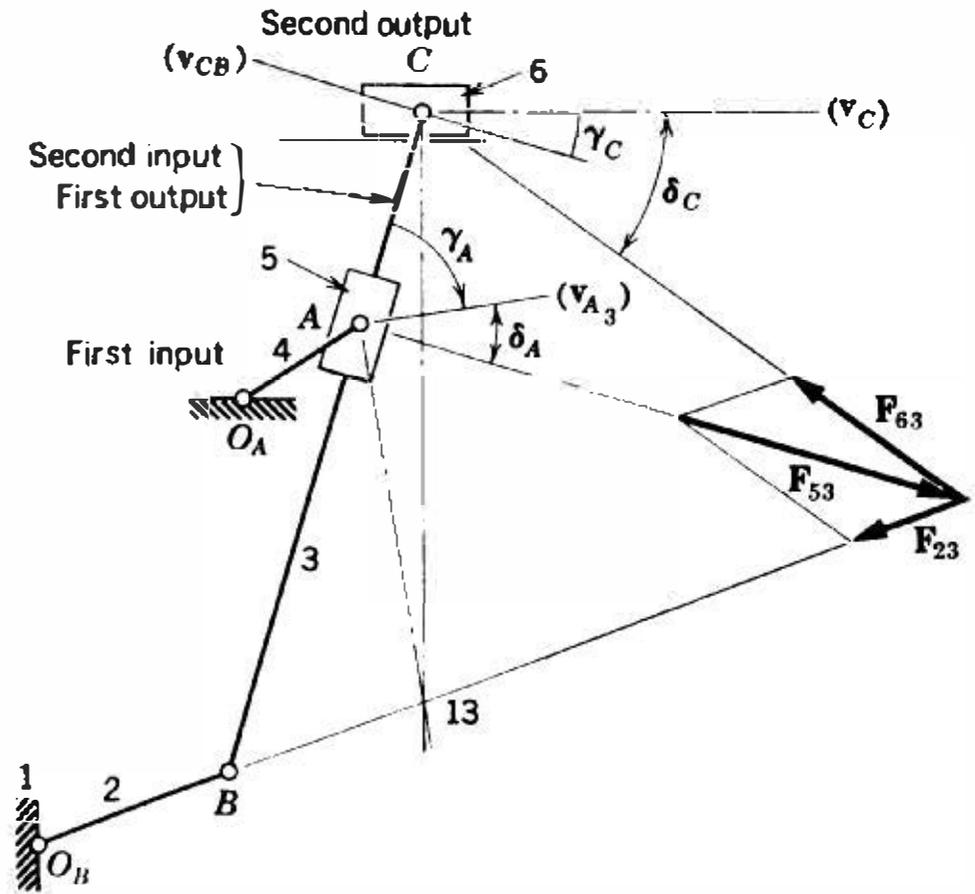


FIGURE 2-20 Transmission and deviation angles of a six-link mechanism.

2-11 PLANAR AND SPATIAL:  
THE MOTIONS AND THE MECHANISMS

Planar and spatial motions of bodies are distinguished from each other by noting the motions of *all* particles of the bodies. A body is said to have planar motion if *all* its particles move in parallel planes, i.e., when the true paths of *all* its particles can be represented on a single plane parallel to the planes of the moving particles. A body rotating about a *fixed* axis, for example, has planar motion, and any plane perpendicular to the axis may be considered as the plane of motion, for the true paths of all particles can be projected into this plane. Other bodies may be referred to this plane, provided that their motions are a combination of rotations about axes that are parallel to the fixed axis and translations along axes perpendicular to the fixed axis. A mechanism whose links have planar motions all parallel to the same plane is called a

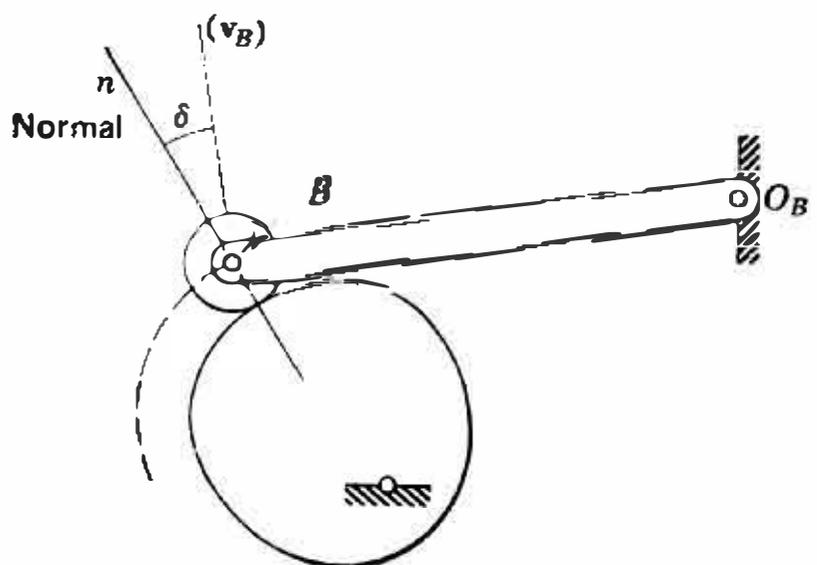


FIGURE 2-21 Disk cam, in which pressure and deviation angles are identical.

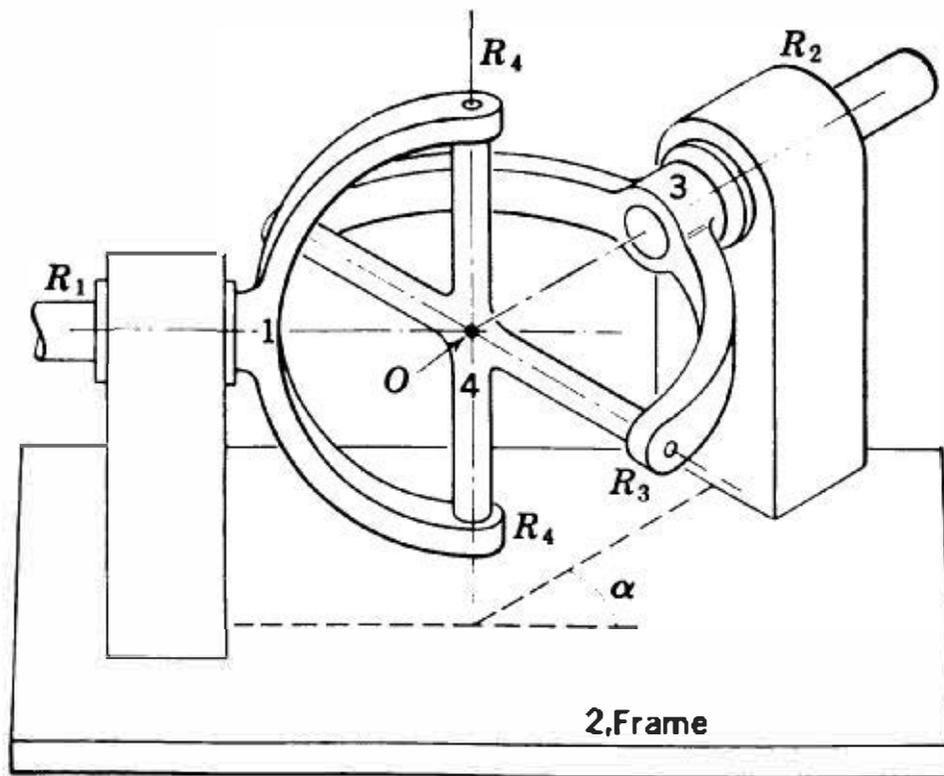
planar mechanism: the true paths of all particles of all links may be shown in one plane, "the plane of the paper." The four-bar linkage, the slider-crank mechanism, gears on parallel shafts, the disk cam with reciprocating follower, and so on, are typical examples.

A body has spatial motion if all its particles do not move in parallel planes. A screw turning in its nut, and hence also moving axially, has spatial motion, since the angle of rotation and the axial translation cannot be depicted on the same plane: any particle of the screw describes a helical path in space. A mechanism which is not planar is said to be spatial. A spatial mechanism may contain but one link with spatial motion (as a screw); or it may have a number of links whose planar motions are not parallel to a common plane. Among the characteristics of spatial mechanisms are the presence of nonparallel axes of rotation and cylindric and ball-and-socket joints. The Hooke universal joint is a familiar spatial linkage; it is also a representative of the special case of spherical mechanisms.

The Hooke coupling is commonly called a universal joint because of its ability to transmit motion between two intersecting but noncollinear shafts. It should be remarked that there are a number of universal joints and that the Hooke type is but one of the lot. In continental Europe it is known as the Cardan (also Kardan) joint. As it happens, neither Cardan nor Hooke invented it; Hooke's name is associated with it since he put it to use in the seventeenth century.

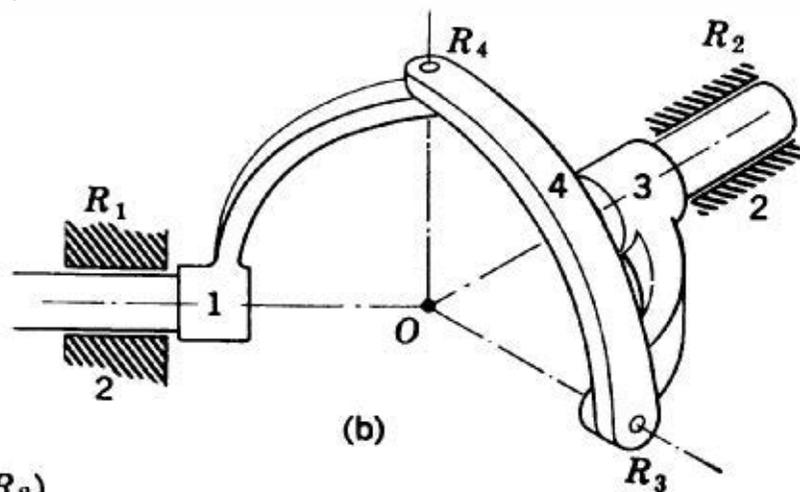
A recognizable Hooke joint is shown in Fig. 2-22*a*. The two shafts misaligned by an angle  $\alpha$  are represented by the revolutes  $R_1$  and  $R_2$ . The central cross 4 carries the revolutes  $R_3$  and  $R_4$ , whose axes are at right angles. Furthermore, the axis of  $R_1$  is perpendicular to that of  $R_4$ , and the axes of  $R_3$  and  $R_2$  are also perpendicular. Lastly, all four revolute axes intersect at a common and fixed point  $O$ ; it is this mutual intersection of all revolute axes at a fixed point that declares this spatial mechanism to be also a spherical mechanism. We may go one step further and remark that the Hooke joint is itself a special case of a spherical mechanism by reason of the three right angles.

The Hooke joint is shown in one schematic form in Fig. 2-22*b*. We recognize that all particles of link 4 (no matter what its physical shape might be) move on spherical surfaces centered at the fixed point  $O$ , that is, all particles of link 4 move on concentric spheres whose center is the fixed point  $O$ . Such a motion is specifically called spherical, to distinguish it from less well regulated spatial motions that would occur if revolute axes did not intersect at a common point. Links 1 and 3, considered individually, have planar motion; the path of any particle is a circle lying in a plane perpendicular to the particle's axis of rotation. However, since the axes of  $R_1$  and  $R_2$  possess a common point at  $O$ , we

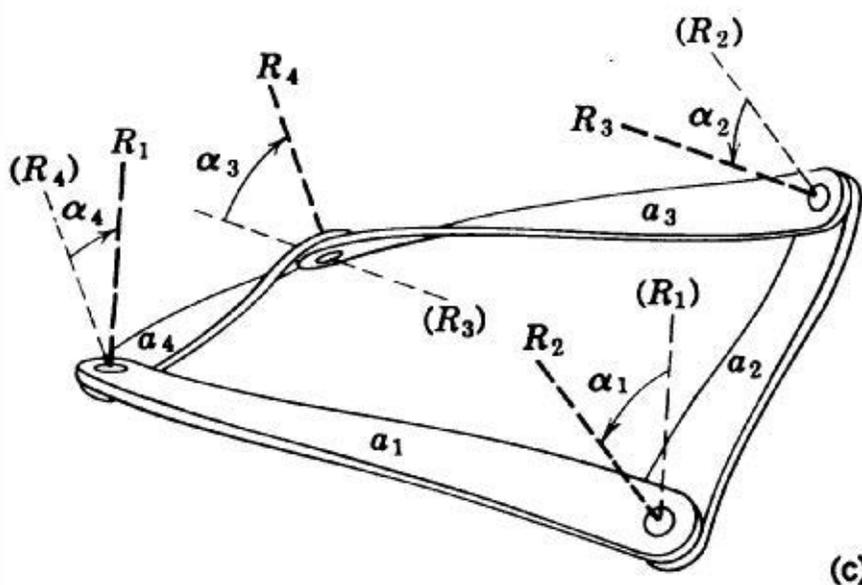


Hooke Joint (a, b)

(a)



(b)



(c)

FIGURE 2-22 Four-revolute spatial mechanisms.

can also imagine the particles of links 1 and 3 to move on spheres centered at  $O$ . The simplest case of a spherical mechanism would involve two bevel gears; the simplest spatial mechanism would be a worm-and-wheel or two crossed helical gears.

The Hooke joint is a spherical four-bar linkage; like the planar four-bar, it has four revolute connections. The difference between the two lies in the orientation of the revolute axes. In a spherical four-bar,

the definitive parameters are the four angles between axes; in a planar four-bar, the parameters are the four link lengths. One other four-revolute linkage, a spatial mechanism also, exists: it is the Bennett mechanism (Fig. 2-22c). In this, the opposite links have the same lengths and the same angles of twist, but the lengths and the twists are related.

For other and more complicated spatial mechanisms, see Chap. 12.

## 2-12 KINEMATIC CHAINS

A material body with two or more kinematic elements is called a *link*. Each element represents a place of contact with, or connection to, another link. A link carrying two elements is a *binary link*; if there are three elements, it is a *ternary link*, if four, a *quaternary link*; and so on. A planar four-bar linkage, a spherical four-bar, and a Bennett mechanism are each composed of four binary links. A cam in contact with only a single follower would also be a binary link.

The bell crank, link 2 in Fig. 2-23a, is a ternary link, for it connects, or "contacts," links 1, 3, and 4. Link 3 of the transport mechanism of Fig. 2-13, reproduced here as Fig. 2-23b, is a quaternary link,

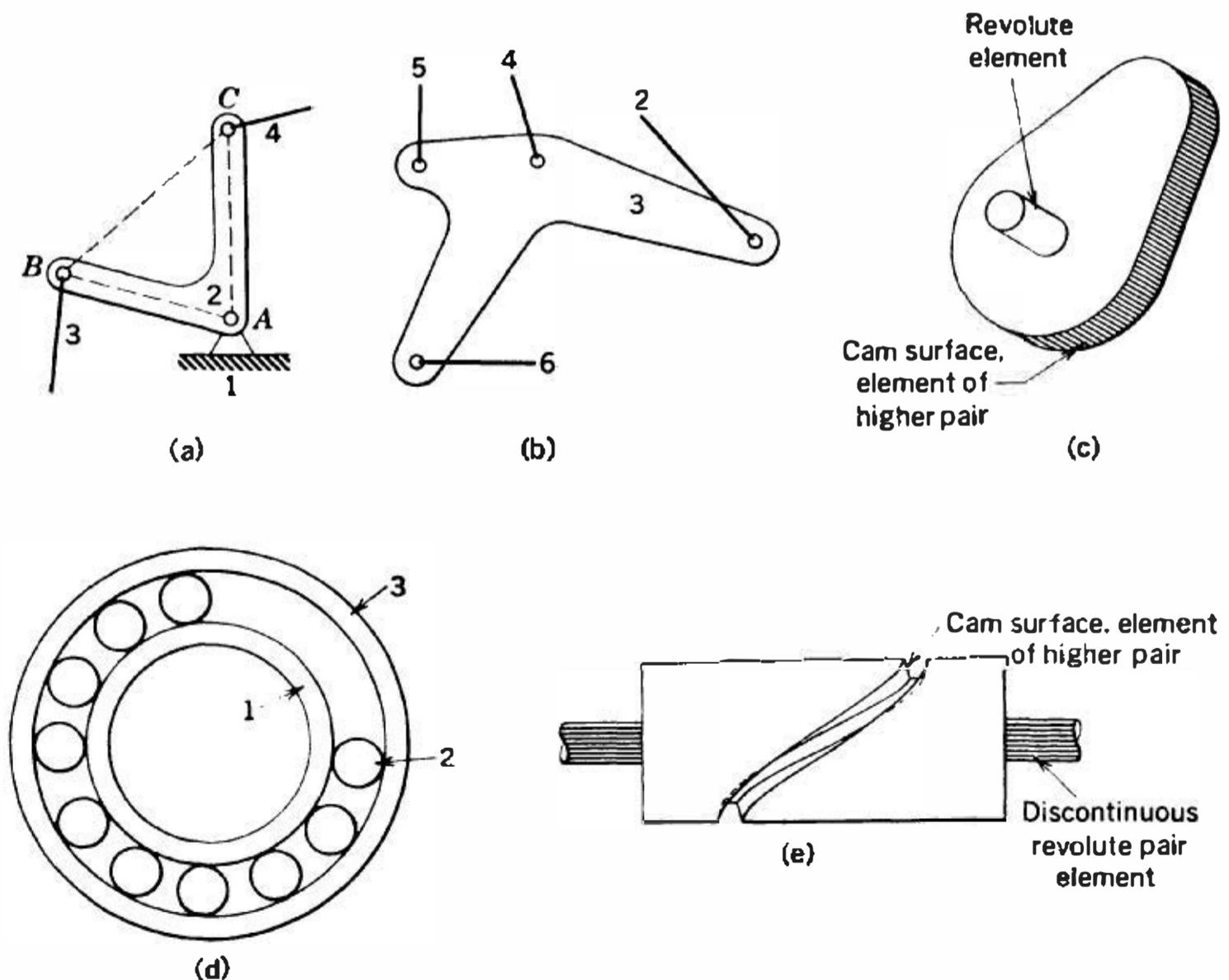


FIGURE 2-23 Examples of links.

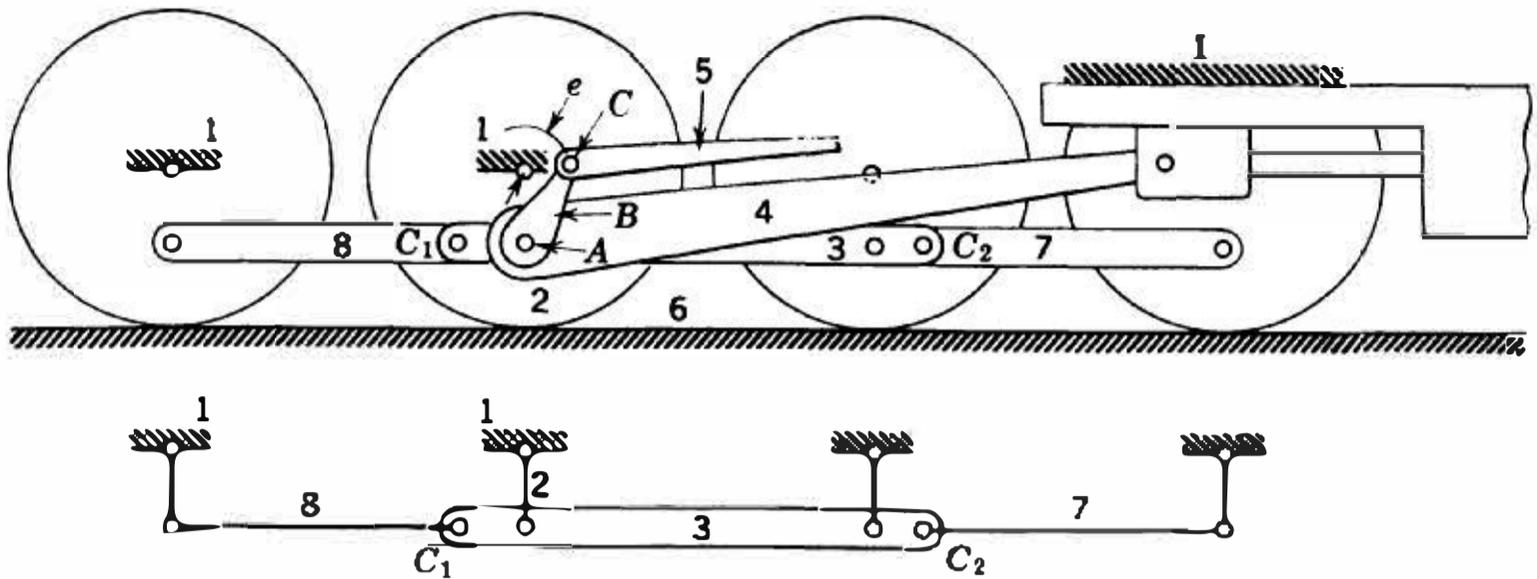


FIGURE 2-24 Example of quintary link. The wheel, link 2 (which includes *B*), connects with links 1, 3, 4, 5, and 6. Note that  $C_1$  and  $C_2$  are coupler points of a parallelogram linkage and describe circles of crank radius.

connecting as it does with the (four) links 2, 4, 5, and 6. Other examples of links, with higher-pair elements, are shown in Fig. 2-23*c*, *d*, and *e*.

The locomotive wheel of Fig. 2-24 presents an interesting situation: it not only is a quaternary link, or quintary when wheel-rail contact is considered, but also shows the construction of a short-throw crank. Wheel, axle, crankpin *A*, and “eccentric link” *B* constitute link 2. The axle is in contact with the frame (link 1); the crankpin carries the connecting rod (link 4) and side, or parallel, rod (link 3). In addition, the eccentric link *B* connects to the eccentric rod (link 5). This last connecting point, *C*, describes a circle of radius *e* about the center of the wheel.

Although the above definition of a link is very general and includes the possibility of one-way-rigid links such as bands, ropes, belts, and fluids, the present text will be concerned only with rigid links.

A kinematic *chain* is an assemblage of parts, or links, connected by pairs. Geometric considerations sometimes preclude motion of the chain after closure, in which case the chain is called a *structure*, and this may be statically determinate or indeterminate (Fig. 2-25). A chain is *closed* when all pairs are complete because of mated, or connected, elements, as in Fig. 2-26. Incomplete pairs indicate an *open* chain (Fig. 2-27). A *simple-closed* chain is composed of only binary links, each link connecting to but two others, as in Fig. 2-26*a* and *b*. *Compound-closed* chains contain ternary and higher-order links, each connecting to more

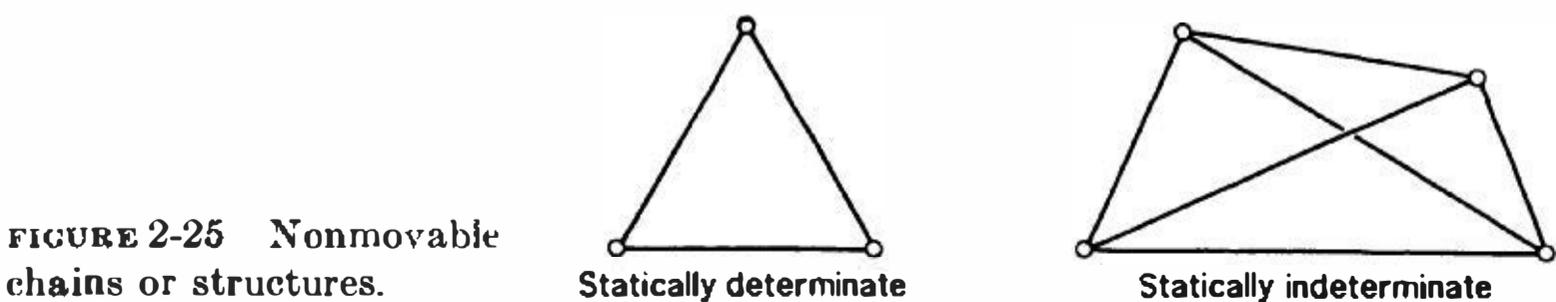


FIGURE 2-25 Nonmovable chains or structures.

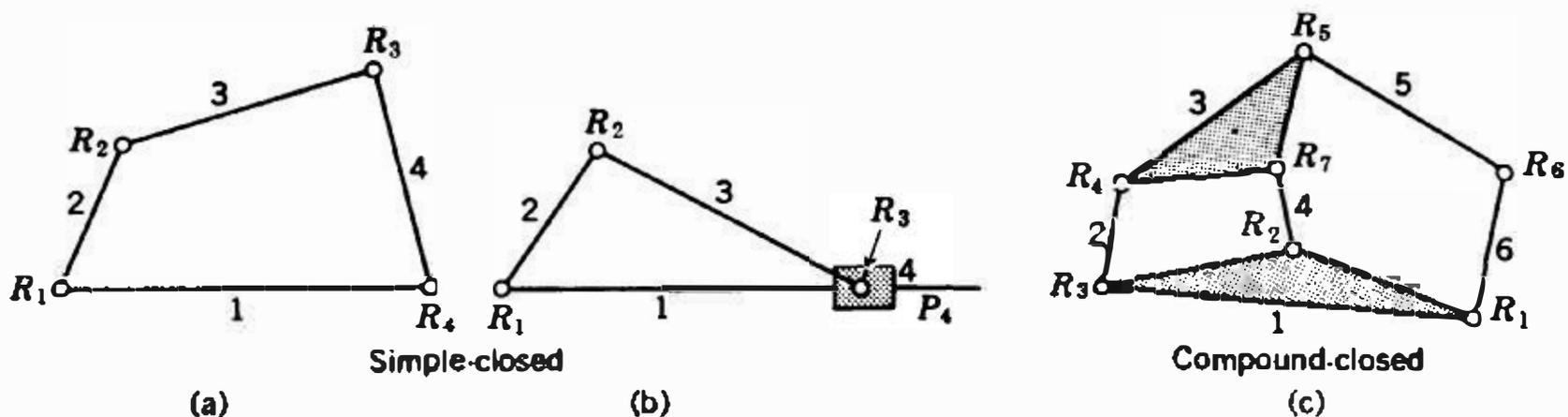


FIGURE 2-26 Movable closed chains.

than two other links, as in Fig. 2-26c, where each of the ternary links 1 and 3 connects to three other links.

With the aid of the notion of kinematic chain, a mechanism, considered earlier as a motion-transforming device, may now be given a new, more accurate, and perhaps more restricted definition: a mechanism is a movable closed kinematic chain with one of its links stationary.<sup>1</sup>

<sup>1</sup> Although we have gone from chain to mechanism by selecting a fixed link, no input link—the source of the motion to be transformed—has been designated. The Germans have a word for a mechanism in which the driving link has been selected: it is *Getriebe* and has caused much confusion. The German *Kette* means chain, and *Mechanismus* is the equivalent of mechanism, but there is no accepted translation for *Getriebe*, although drive and train have been used. In consequence the untranslatable *Getriebe* appears as mechanism when used as a noun. In adjective form, e.g., *Getriebelehre* (*Lehre* = theory or science of), it is translated as kinematics or mechanisms.

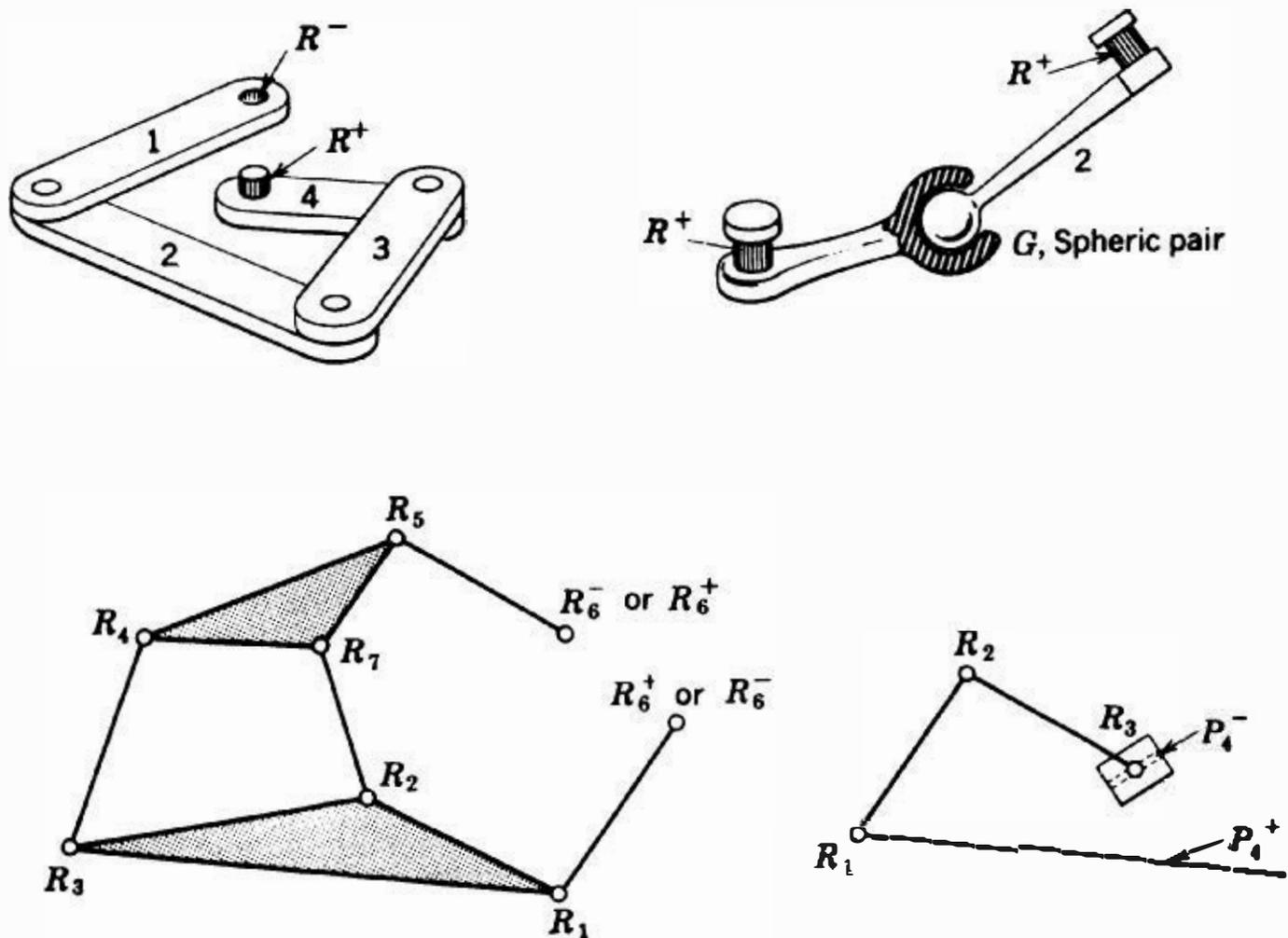


FIGURE 2-27 Open chains.

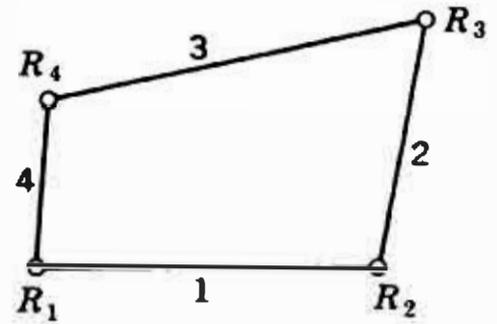


FIGURE 2-28 Four-revolute planar chain meeting Grashof condition (see Sec. 3-3).

Thus, in Fig. 2-26, the chain at (a) becomes a mechanism, a four-bar linkage, when one of its links (link 1, for example) is made stationary to form the frame of the mechanism. The chain at (b) gives a slider-crank mechanism when link 1 is made the frame.

As noted earlier, a mechanism (either planar or spatial) in which all connections are lower pairs is called a *linkage*. Linkage and lower-pair mechanism are therefore taken to be synonymous.

### 2-13 INVERSION

A mechanism is derived from a closed kinematic chain by making one of its links stationary by choosing different links as the stationary link or frame, the same closed chain will yield as many distinct mechanisms as it has links. The chain of Fig. 2-28, for example, yields four different four-bar linkages, as shown in Fig. 2-29. The four-bar linkage *a*, sometimes called the crank-and-rocker mechanism, gives an oscillation of the follower 2 for a continuous rotation of the crank 4. At *b*, a rotation of link 3 with constant angular velocity gives link 1 a continuous rotation with variable angular velocity. This is the drag-link mechanism; double-crank mechanism is also descriptive. At *c*, the situation is similar to that of *a*, but with different motion characteristics. At *d*, links 1 and 3 can rotate only through angles less than  $360^\circ$ ; this is the double-rocker mechanism.

The process of fixing different links of a chain to create different mechanisms is called *kinematic inversion*. The four-bar mechanisms *a*, *b*, *c*, and *d* (Fig. 2-29) are the four inversions of the four-bar chain (Fig. 2-28).

The mechanism shown in Fig. 2-30a, called the Scotch yoke, consists of four rigid links connected by two revolute pairs  $R_1$  and  $R_2$  and two prismatic pairs  $P_3$  and  $P_4$ . Crank rotation at constant angular velocity gives the yoke a translation that is a sinusoidal function of time. An inversion in which link 2, the crank, is chosen as the fixed link yields the mechanism of Fig. 2-30b. This, known as the Oldham coupling, transmits rotation between two parallel shafts with an angular velocity

<sup>1</sup> Distinct refers to the input-output relations of links attached to the frame; the relative motions of all links remain the same.

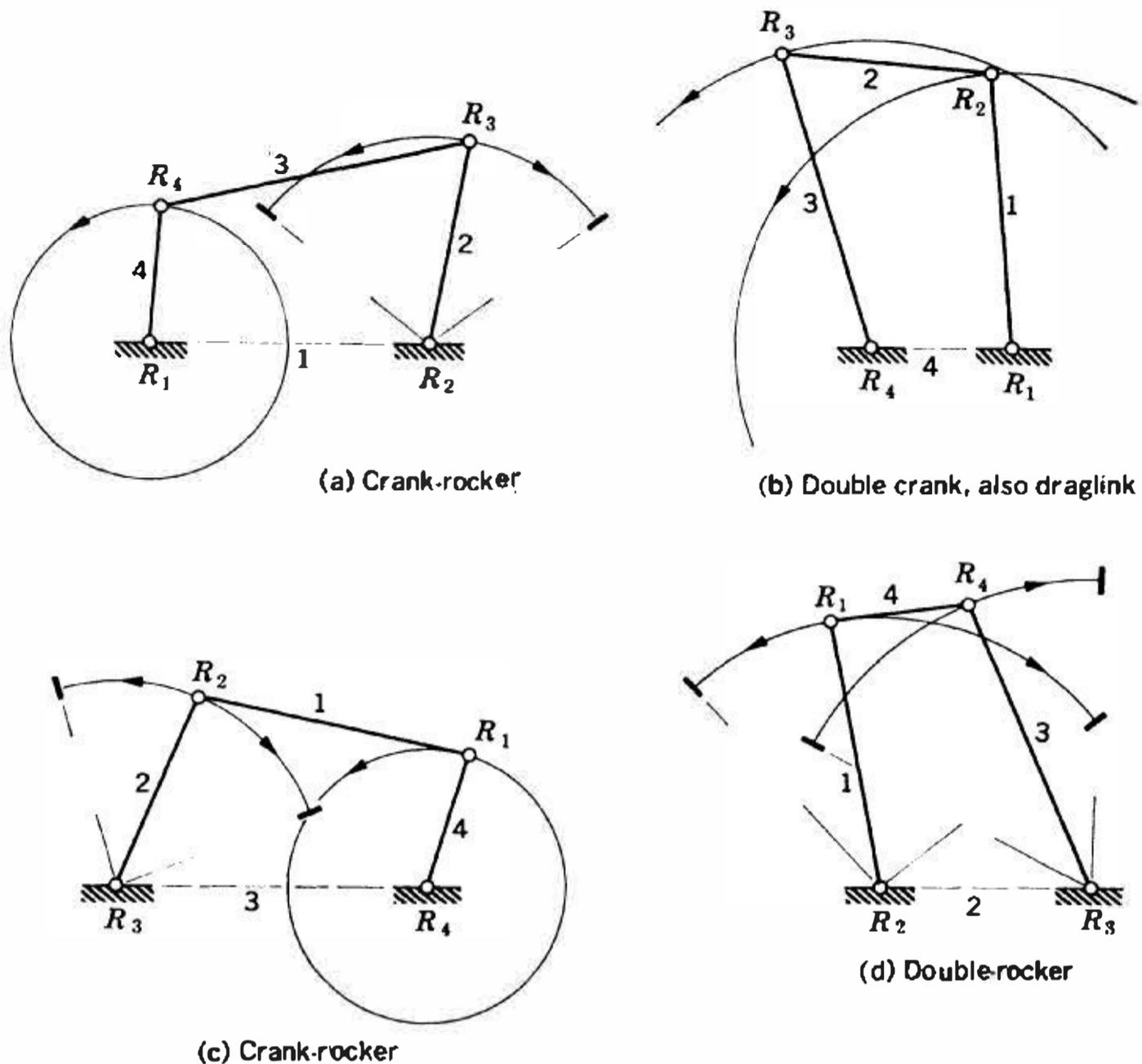


FIGURE 2-29 Inversions of the four-bar linkage.

ratio of unity. Another inversion, in which link 4 (yoke) is made the fixed link, yields the elliptic trammel (Fig. 2-30c); here the center point  $C$  of link 2 traces a circle, with all other points describing ellipses. A fourth and last inversion, in which link 3 (block) is fixed, gives a Scotch yoke different from the first.

As a further example of inversion we discuss the slider-crank mechanism (Fig. 2-31a). When crank 2 is fixed, mechanism  $b$  gives link 3 a continuous rotation of variable angular velocity when link 1 rotates at constant angular velocity. This motion transformation is similar to that of the drag-link inversion of the four-bar linkage; it has been applied to many machine tools, where it is called the Whitworth quick-return mechanism. With connecting rod 3 made the fixed link, mechanism  $c$  has found application in oscillating-cylinder steam engines<sup>1</sup>

<sup>1</sup> For example, the *Great Eastern*, built in 1858, had a four-cylinder oscillating engine (74-in. bore, 14-ft stroke) developing 3,410 hp at 11 rpm with steam at 24 psig. This engine, sitting low in the hull, drove overhead crankshafts to which 56-ft-diameter paddle wheels were directly connected. In addition, there was a conventional four-cylinder horizontally opposed engine of 4,890 hp (and 39 rpm) for a 24-ft screw. If all this failed, six masts could spread  $1\frac{1}{3}$  acres of sail!

and pumps. Mechanism *d* finds use as a hand pump: link 1 is oriented in the vertical, and link 2 is extended to form the pump handle.

Among the earliest aircraft power plants was the rotary engine. Spectacularly successful in 1910, its useful life extended well into World War I. This engine (Fig. 2-32) was the same inversion as the Whitworth mechanism: the crankshaft (link 2) was bolted to the fuselage to become the fixed link. The propeller was bolted to the crankcase (link 1), and this assembly, complete with cylinders, revolved.

2-14 EXPANSION OF REVOLUTE PAIRS AND OTHER DISGUISES

Too often the physical shape of the connection between links is such that the true character and function of the connection are not immediately apparent. The reason for the disguise may stem from practical design considerations such as strength requirements, ease of manufacture, or space limitation, all of which obscure the nature of the kinematic elements of the connection, although the relative motions of the links remain unaffected. The situation is simply that the center

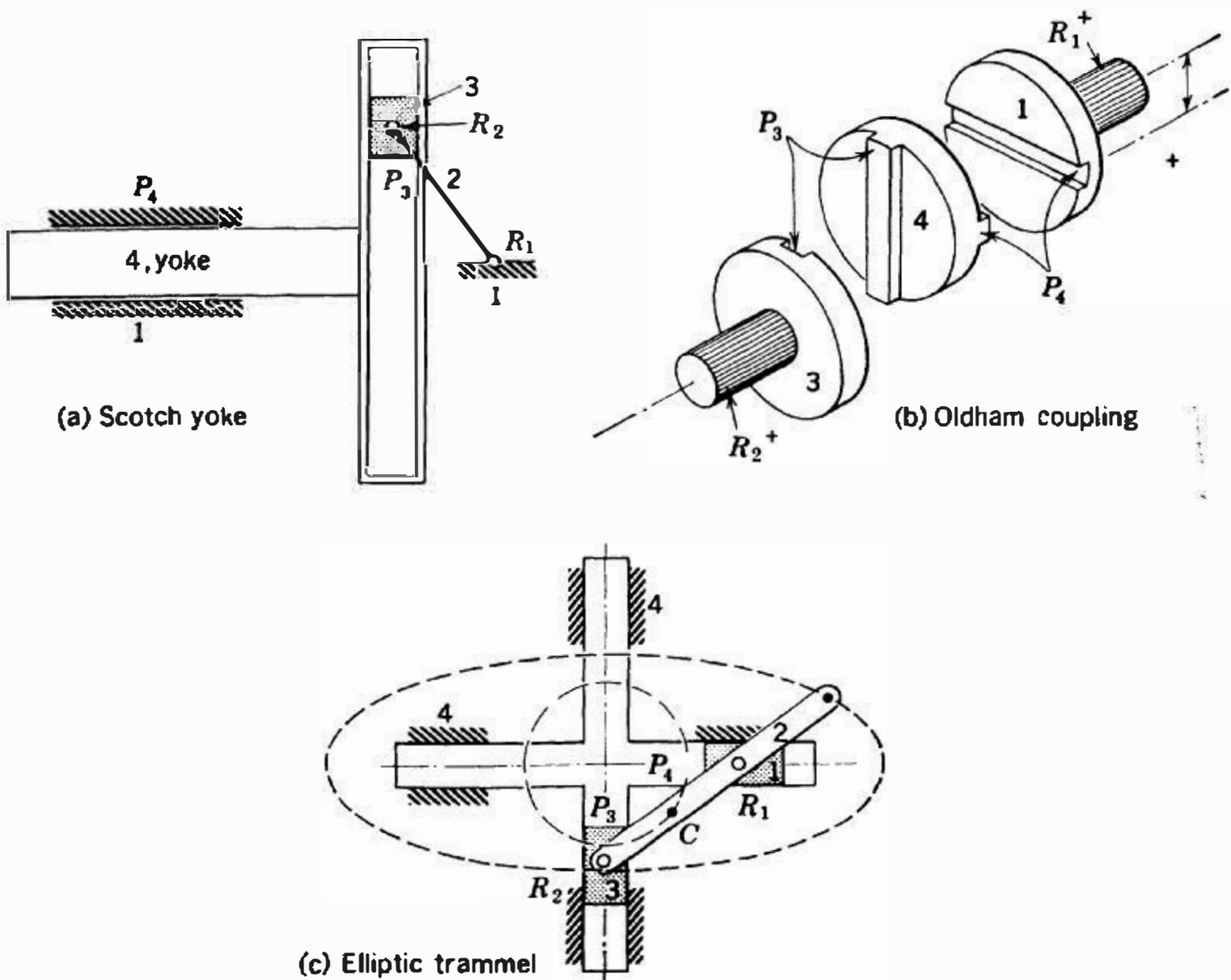


FIGURE 2-30 Inversions of the Scotch yoke.

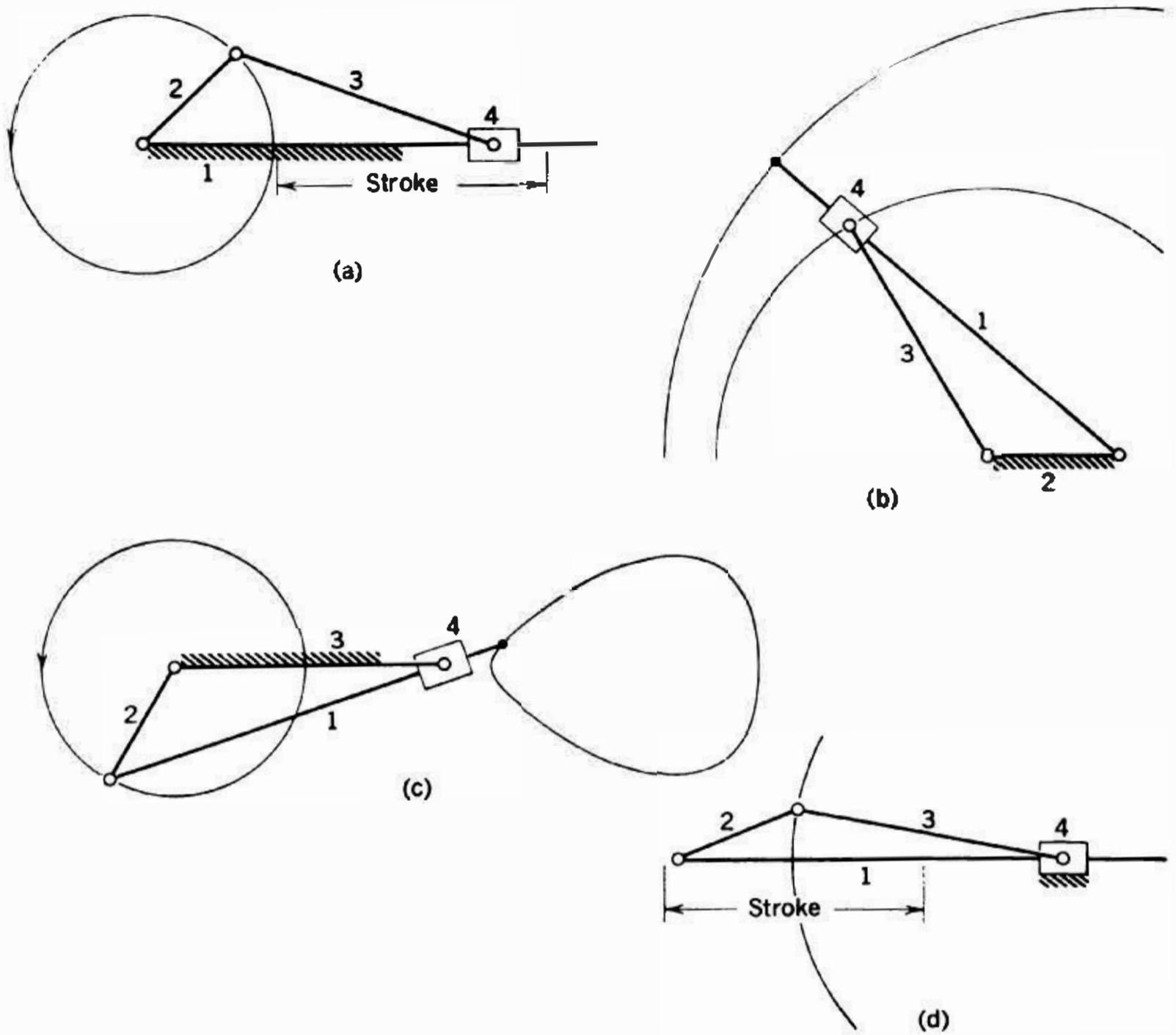


FIGURE 2-31 The slider-crank chain inversions.

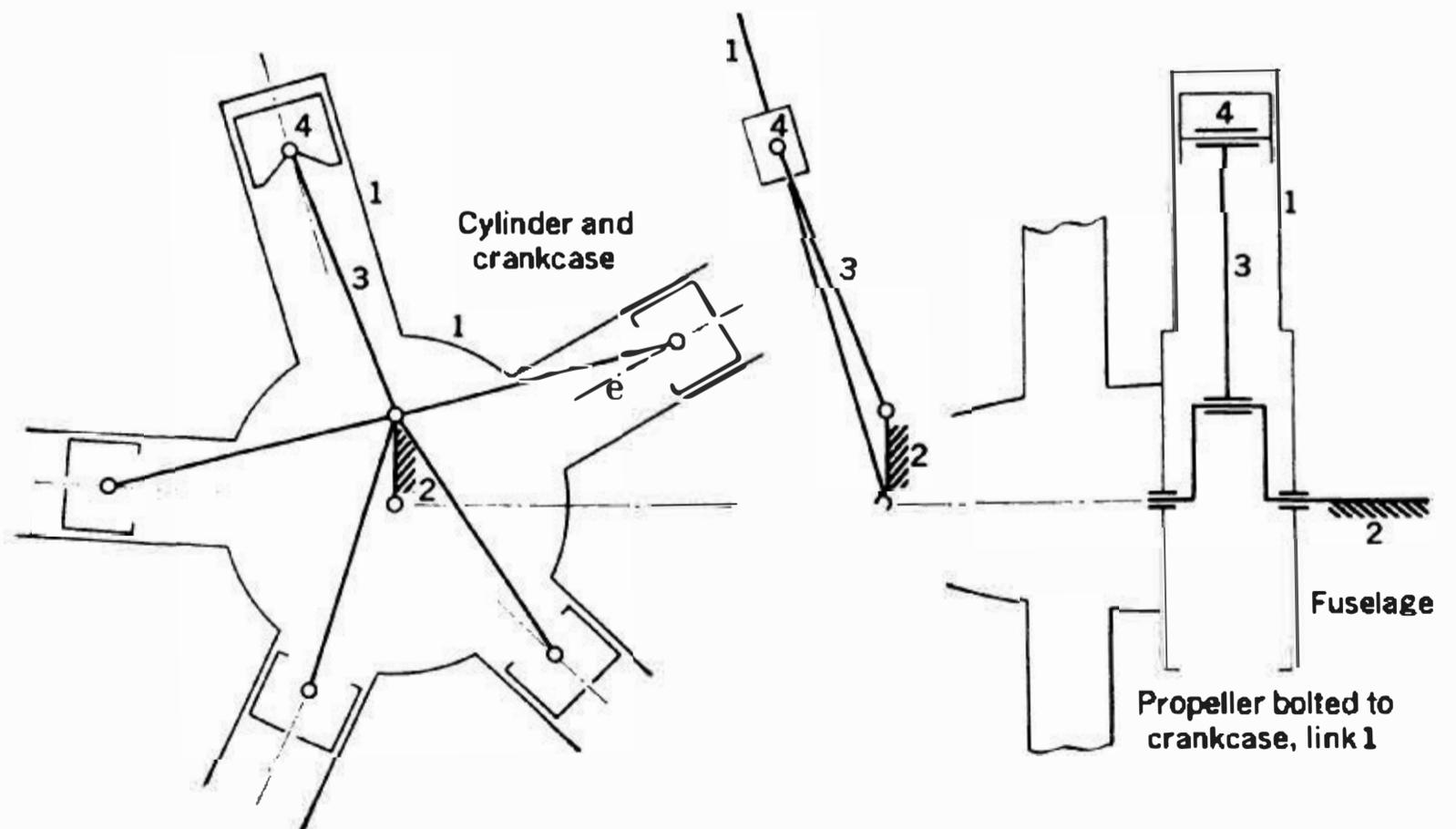


FIGURE 2-32 Rotary aircraft engine, a slider-crank inversion with link 2 fixed.

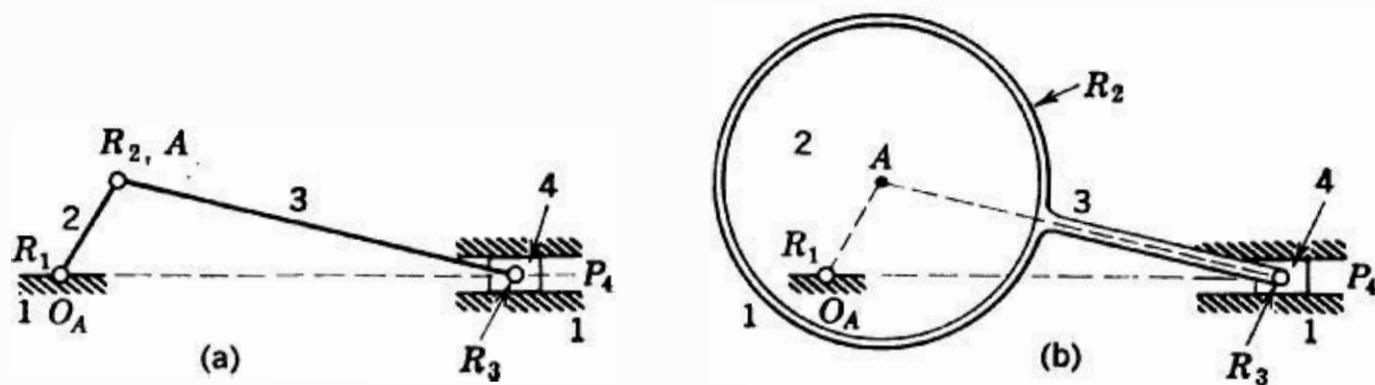


FIGURE 2-33 Expansion of revolute pair.

of what is kinematically a revolute pair is not directly discernible. A case in point is the common eccentric (Fig. 2-33b) examination shows it to be an oversized, or “expanded,” crankpin, evident on a comparison with the slider-crank mechanism of Fig. 2-33a. Inspection identifies the center  $A$  of revolute  $R_2$  in Fig. 2-33b and finds the crank of length  $O_A A$ . Revolute 2 in this expanded form is known as an “eccentric,” and the shaft constituting revolute  $R_1$  may now be continuous instead of interrupted by a crank. Obviously the physical diameters of the revolutes  $R_1$  and  $R_2$  are of no kinematic importance: what is significant is that the centers  $O_A$  and  $A$  are the same distance apart in the two mechanisms. The trick lies in discovering the disguised center of the “misproportioned” revolute  $R_2$ . Variations in form and size of a turning pair that do not alter the relative motion between connected members constitute an expansion of a revolute pair.

Further examples may be found. A four-bar linkage is shown in Fig. 2-34a in its familiar form; at  $b$  revolute pair  $R_3$  has been expanded, as was  $R_2$  in the slider crank, and the coupler has now become block 3. For a complete revolution of crank 2 block 3 traverses only the small arc  $E_1 E_2$  of the element of revolute  $R_3$  on member 4. The motion of block 3 would still be described by means of an angle referred to  $B$ . Link 4 may be given a still different physical form as at  $c$  without altering the relative motions of the links. It should be observed that the “curved slider” is a form of revolute: the motion of block 3 in its guide is a rotation defined by an angle, not by a linear distance. We note that the space requirements of forms  $a$ ,  $b$ , and  $c$  are quite different.

## 2-15 PRISMATIC PAIR AS THE LIMIT OF A REVOLUTE PAIR

Although differing in appearance from the “pin connection” of Fig. 2-34a, the “curved slider” of Fig. 2-34c remains a revolute pair as long as its radius of curvature is finite. The center of curvature  $B$  is part of the moving plane 4 from which the physical shape of link 4 has been cut. If, however, the radius of curvature of a revolute pair becomes

infinite, i.e., when its center of rotation goes to infinity, then, *and only then*, does the revolute pair become a prismatic pair, i.e., the pair variable changes from an angle to a linear distance.

This transition from a revolute to a prismatic pair is shown in Fig. 2-35. By expansion of the revolute pair  $R_4$ , the four-bar linkage shown at *a* takes the form *b*. Suppose now that the center of rotation  $O_B$  of the revolute  $R_4$  is moved down along the vertical by increasing the lengths of members 1 and 4 as shown at *c*. By expansion of the revolute  $R_4$ , this new four-bar linkage takes the form *d*, in which the radius of curvature of curved slider  $R_4$  is greater than at *b*. Moving the center  $O_B$  farther down simply increases the radius of curvature of  $R_4$ . At the limit, when  $O_B$  is at infinity on the vertical and members 1 and 4 have become infinitely long as at *e*, the four-bar linkage becomes a slider-crank mechanism *f*. The curved slider has now become straight, yielding a prismatic pair. Thus, a prismatic pair may be considered as a revolute pair whose center is at infinity in the direction perpendicular to the generatrix. Having arrived at this stage,  $O_B$  may be located at infinity, either "up" or "down."

In a previous section, the prismatic pair was considered as a limiting case of a screw pair with an infinite lead. The prismatic pair is here considered as a revolute pair with its center of rotation at infinity. These two interpretations of the prismatic pair, however, should not be

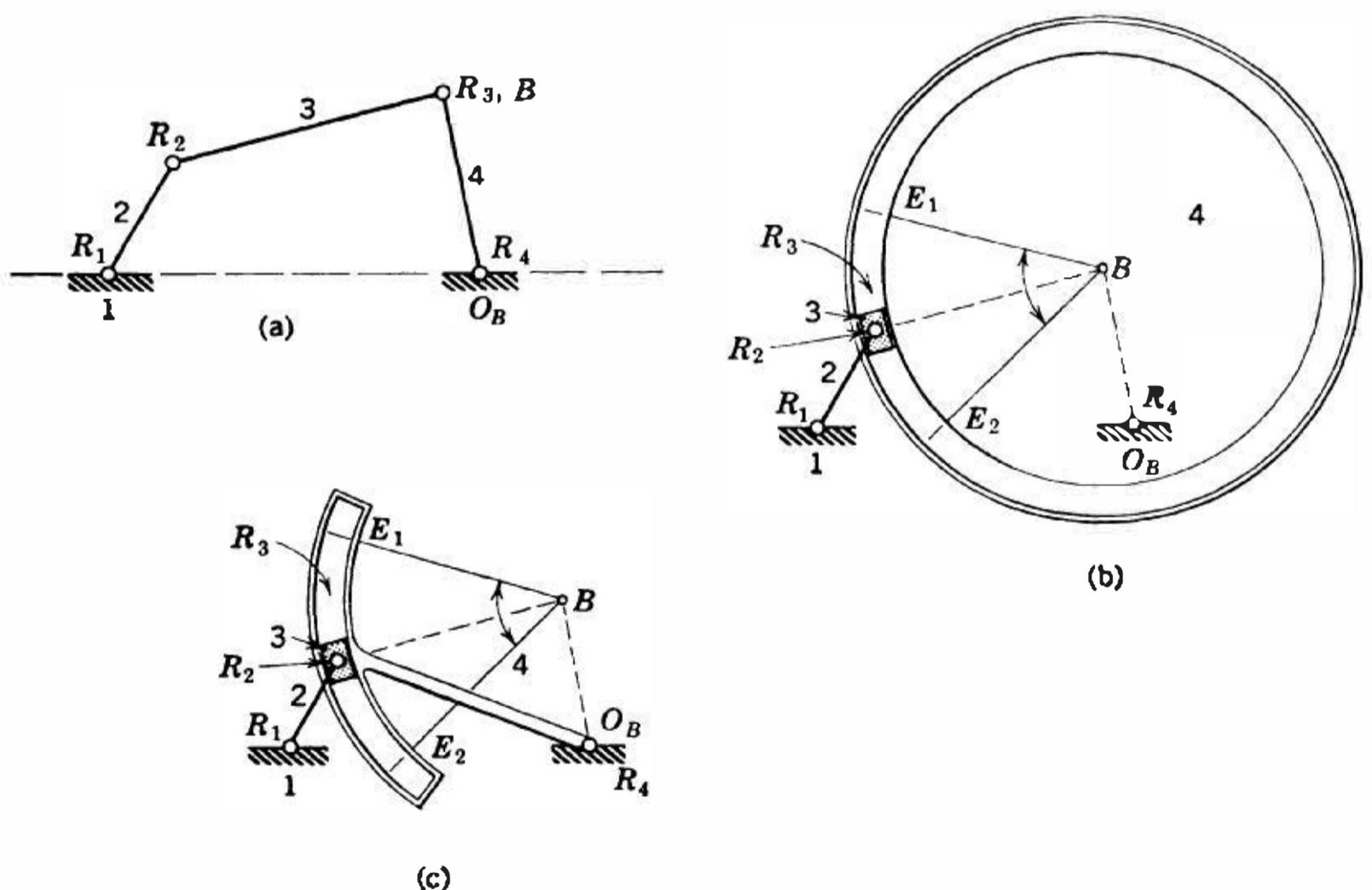


FIGURE 2-34 Expansion of revolute pair.

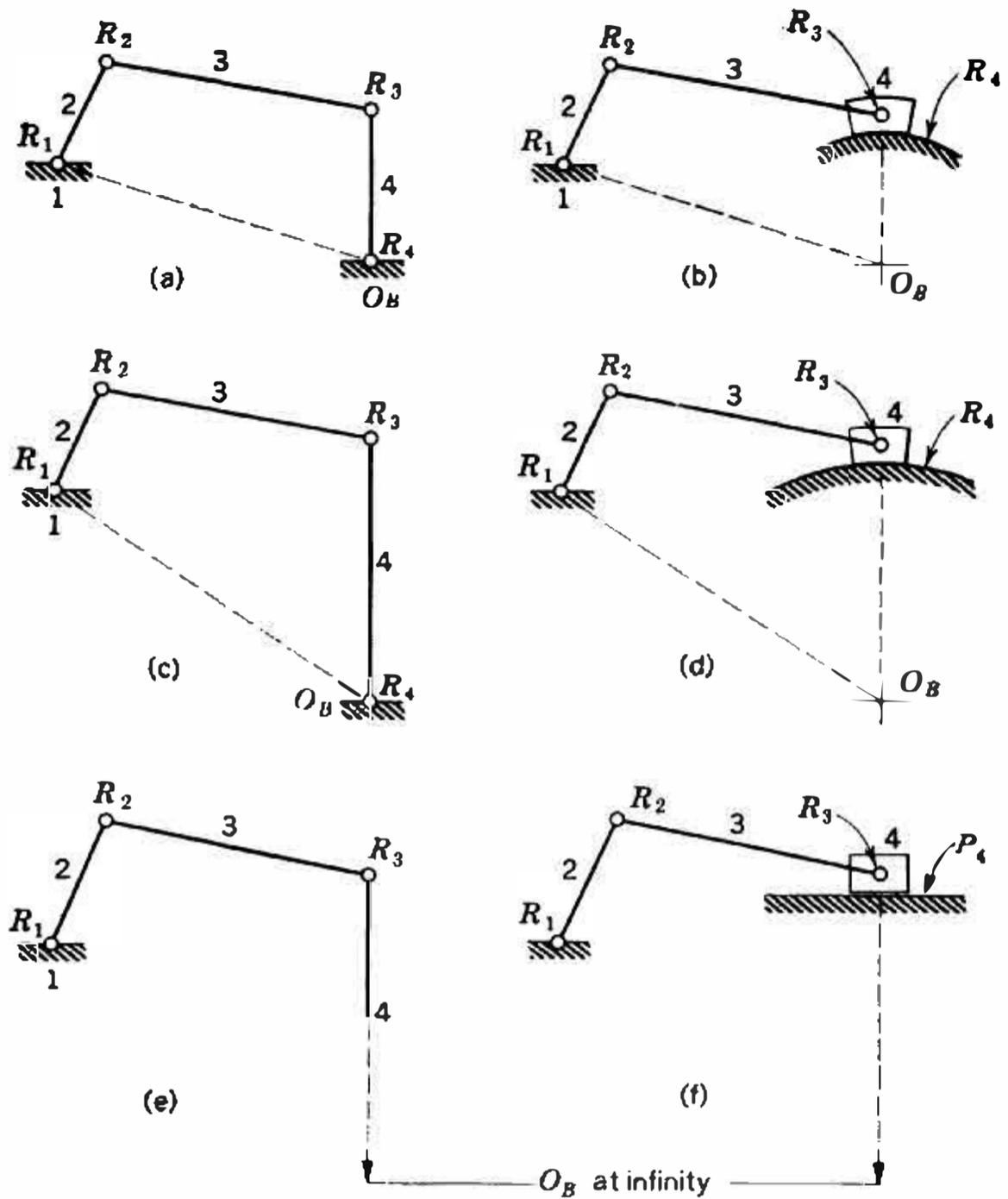


FIGURE 2-35 Prismatic pair as a limiting case of revolute pair with center at infinity.

considered as conflicting, for each view may serve a different purpose; it is a case of the end justifying the means—making use of the most convenient and legitimate argument for differing purposes. As noted before, the first interpretation is convenient in setting up a symbolic notation for lower-pair mechanisms. The second interpretation is put to use in the synthesis of planar mechanisms, allowing many of the properties of the four-bar linkage, when carried through the limit process shown in Fig. 2-35, also to become properties of the slider-crank mechanism.

## 2-16 EQUIVALENT LINKAGES

The complete kinematic analysis of a mechanism includes, among other things, the determination of velocities and accelerations. Difficulties may be encountered when links are connected by a higher pair,

as shown in Fig. 2-36a, where the relative motion between the two profiles consists in rolling coupled in uncertain fashion with sliding. On proceeding in the usual manner with vector equations (see Chap. 4), it would be necessary to apply the Coriolis theorem and to know the curvature of the path traced by a point of one link with respect to the other. If no easily recognized path is found, it may be difficult, or at the very least tedious, to establish the desired path curvature.

The equivalent linkage replaces the higher pair with properly disposed lower pairs. These will, for the instantaneous phase under consideration, give correct values of velocities and accelerations. Let

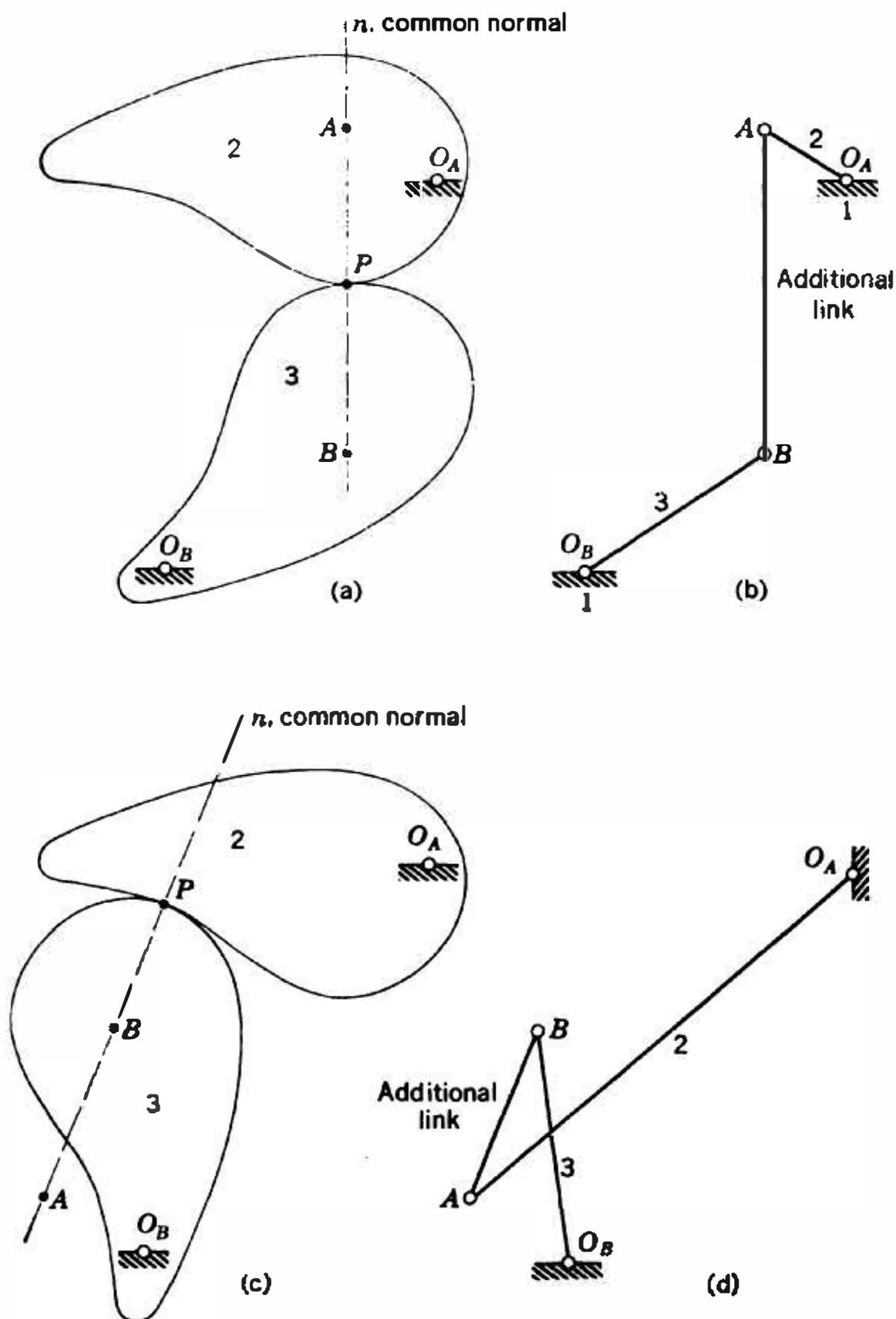


FIGURE 2-36 Equivalent linkages.

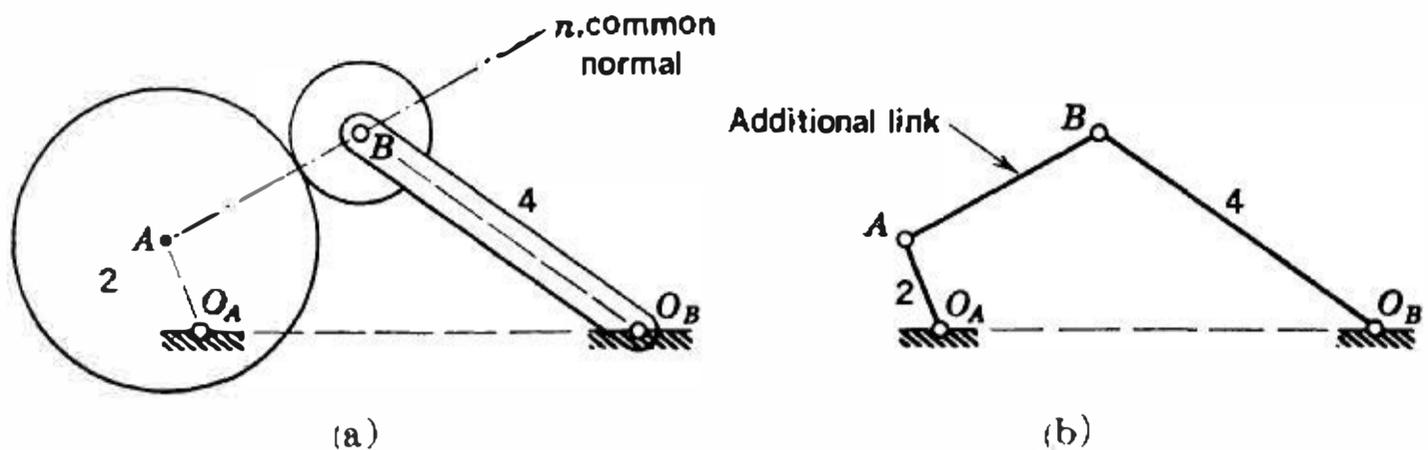


FIGURE 2-37 Cam with constant curvature profile and its invariant equivalent linkage.

$A$  and  $B$  be the centers of curvature of the profiles of the higher pair at their point of contact. Because of the properties of the center of curvature, the distance  $AB$  will remain constant for three infinitesimally close positions of the cam and follower, and for these three positions the same correlation in displacements between links 2 and 3 may be obtained by means of the four-bar linkage  $O_A A B O_B$  shown at  $b$ . This linkage is called an *equivalent linkage* of the higher-pair mechanism shown at  $a$ .

This equivalence, however, is valid only for three infinitesimally close positions of the mechanism. When the links are rotated through a finite angle, as at  $c$ , the dimensions of the equivalent linkage  $d$  are different. In other words, an equivalent linkage is generally valid only for a given instant or phase; it does not ordinarily apply to a complete cycle.

In general, two mechanisms are said to be equivalent if they give the same correlation between three infinitesimally close positions of their input and output members. The velocities and accelerations of the points of a moving link are defined by considering three infinitesimally close positions of the link. Such positions are preserved in equivalent linkages, which may therefore be used to evaluate velocities and accelerations of more complicated mechanisms. Equivalent linkages, however, cannot be used to evaluate displacements or the time rate of change of acceleration, sometimes called the jerk, or pulse, because these quantities can be defined only in terms of more than three infinitesimally close positions of a link.

In some few instances an equivalent linkage may be found that will duplicate the motion transformation between input and output links of a given mechanism throughout the motion cycle. Consider, for example, the cam mechanism shown in Fig. 2-37a, the cam profile being circular, with center at  $A$ . The roller has its center at  $B$ . The distance  $AB$  is now constant throughout the cycle, whence the equivalent linkage  $O_A A B O_B$  shown at  $b$  is valid for all instants (positions or phases). The motion transformation between links 2 and 4 of both mechanisms is the same throughout the complete cycle.

## 2-17 SYMBOLIC NOTATIONS

Language, either written or spoken, is a means of communicating an idea or thought. In written form, it involves the use of signs now so highly conventionalized that their original pictorial origins are not evident. Any system of signs may be called a language, e.g., the language of mathematics, in which the symbols stand for operations whose word descriptions are tedious and unhappy. Mathematical notation is the most powerful shorthand known: not only is it precise, but, more important, it is manipulative. Physics, chemistry, and electronics also have highly developed notations by means of which symbols other than only the mathematical allow the statement of ideas and complex situations in a compact manner.

No real symbolic notation is widely used in kinematics. Practically all that exists is a stylized representation of connections, useful for drawing skeleton diagrams. These diagrams sort out the basic geometry of a problem and are therefore indispensable, but they lead no further, for they do not represent a shorthand of the whole mechanism—they are only a quick picture of it, and no manipulation is possible.

Notwithstanding the controversial nature of the subject, the authors have been led to reconsider the matter of symbolic notations and devise an extension of Reuleaux's system to supplement the use of skeleton diagrams. Of this new symbolic notation, particularly convenient when applied to lower-pair mechanisms (both planar and spatial), an elementary form<sup>1</sup> will be presented in this section. It provides a concise qualitative method for identifying linkages by the number and nature of their pairs. This elementary form will be elaborated upon in Chap. 12 to make it quantitative by including the geometric relations between the pair elements of each link. The quantitative form of the symbolic notation allows an interpretation in terms of matrix algebra to make it manipulative; it is particularly useful for the study of spatial linkages. However, the complete form is not necessary for the planar studies forming the major parts of this book.

1. *Symbols for lower pairs* Each of the six lower pairs described in Sec. 2-6 will be represented by its symbol:

Spheric pair	$G$ (think of <i>Globe</i> —sphere)
Planar pair	$F$ (think of <i>Flat</i> )
Cylinder pair	$C$
Screw pair with lead $L$	$S_L$
Revolute pair	$R$
Prismatic pair	$P$

<sup>1</sup> This form is in the spirit of what Reuleaux called the "contracted formulæ" (Kennedy, "Reuleaux' Kinematics of Machinery," p. 263).

As already remarked in Sec. 2-6, a revolute pair may be considered as a screw pair with a lead equal to zero; similarly, a prismatic pair is a screw pair with an infinite lead. The symbols for the revolute and prismatic pairs may therefore be written respectively as  $S_0$  and  $S_\infty$  without the introduction of any new symbols. However, because the revolute and prismatic pairs are very common, it is convenient to give them their particular symbols of  $R$  and  $P$ .

2. *Symbolic description of simple-closed chains* A simple-closed chain consists of only binary links; i.e., each link is connected to two, and only two, other links. The chain is completely described if each connection is designated by its proper symbol, the connections being labeled in sequence, clockwise or counterclockwise. One connection is arbitrarily chosen as the starting point and the others noted in sequence while going around the loop in either sense. Thus, the symbolic descriptions of the chains shown in Fig. 2-26 might be written

For  $a$ :

$$R_1R_2R_3R_4 \quad R_1R_4R_3R_2 \quad R_2R_3R_4R_1 \quad \text{or} \quad R_2R_1R_4R_3 \quad \text{etc.}$$

For  $b$ :

$$R_1R_2R_3P_4 \quad R_2R_3P_4R_1 \quad \text{etc.}$$

3. *Symbolic description of compound-closed chains* A compound-closed chain consists of a combination of simple-closed chains, possible because some of the links connect to more than two other links, i.e., some of the links are ternary, quaternary, etc. Such a chain is completely described if each connection is designated by its proper symbol (as in the case of simple-closed chains) and enough simple-closed chains are described to include all the connections (pairs). This method is similar to the analysis of electrical circuits, in which the voltage equation is written for each independent loop of the circuit. Examination of the compound-closed chain of Fig. 2-26 discloses three different simple-closed chains:

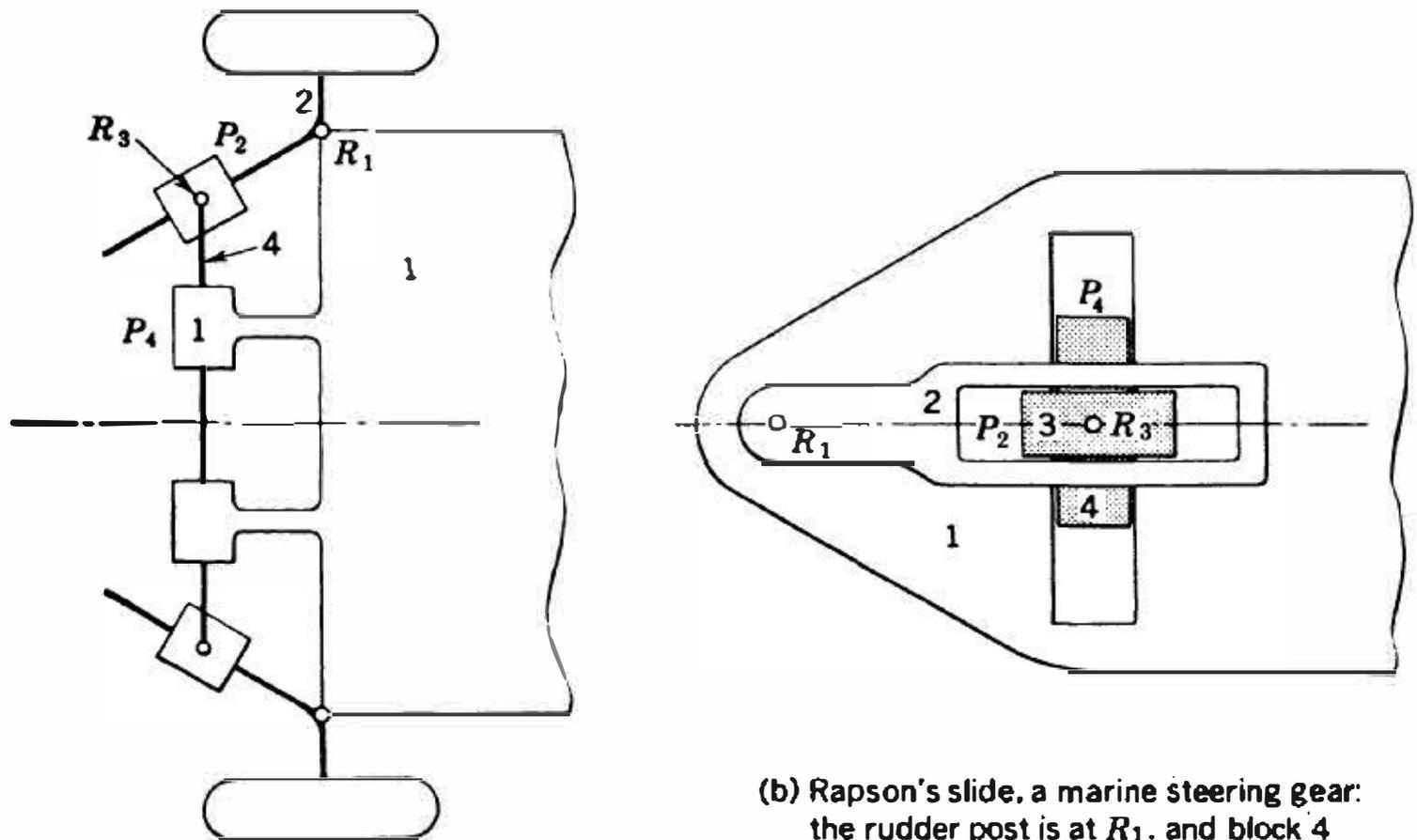
$$R_1R_2R_3R_4 \quad R_1R_2R_5R_6R_7 \quad \text{and} \quad R_4R_3R_5R_6R_7$$

Any two of the above chains, however, are sufficient to describe the compound-closed chain, since they include all the pairs. The compound-closed chain of Fig. 2-26 may therefore be written as

$$R_1R_2R_3R_4 \\ R_1R_2R_5R_6R_7$$

Two other combinations of simple-closed chains would also serve.

The utility of the symbolic notation is shown in rather striking fashion by Fig. 2-38. At first glance, and perhaps even at the second,



(a) Davis automobile steering gear

(b) Rapson's slide, a marine steering gear: the rudder post is at  $R_1$ , and block 4 is actuated by chains or wire ropes.FIGURE 2-38 Two kinematically identical mechanisms  $R_1P_2R_3P_4$ .

Rapson's slide and the Davis steering arrangement evidence no kinship. Identification and ordering of the kinematic pairs show the two mechanisms to be the same chain. This is all nicely obscured by the different shapes of the corresponding pieces of hardware (in part due to the exchange of hollow and full elements of the prismatic pairs) and, of course, the unrelated areas of application. Reuleaux, who was the first to appreciate the need for and the niceties of a symbolic notation by creating one, applied it to showing the kinematic similarities existing in the many rotary steam engines known to him.<sup>1</sup>

Mechanisms with higher pairs are subject to symbolic representation only in terms of their equivalent linkages. For certain cases, such as that of Fig. 2-37, a single invariant equivalent linkage may be found that will duplicate the motion transformation between input and output links of the mechanism for the complete cycle. Here the symbolic representation of the cam mechanism is given by that of its invariant equivalent linkage, a planar four-bar.

The foregoing symbolic descriptions are qualitative rather than quantitative: they identify the nature and number of the pairs involved in a mechanism and the order in which the pairs appear but give no information about angles and distances between pair axes. Such a description does not differentiate, for example, between a planar four-bar linkage, spherical four-bar, Hooke joint, and Bennett mechanism, for they are all written  $R_1R_2R_3R_4$ ; further qualification must be supplied

<sup>1</sup> *Ibid.*, pp. 342-384.

by an adjective. An idea of how the necessary quantitative information is added to the symbolic notation will be given in Chap. 12; until then it must be conveyed in words as part of the context.

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