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KINEMATIC SYNTHESIS OF LINKAGES

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McGraw-Hill Book Company
New York San Francisco Toronto London

Kinematic Synthesis of Linkages

Library of Congress Catalog Card Number 64-23251

26910

PREFACE

In the early 1940s, the idea of kinematic synthesis of linkages was, in a manner of speaking, mentioned for the first time in this country. After a decade of gestation, original articles on the subject began to appear in America. Now, with the passing of the second decade, it is appropriate to attempt a consolidated story of some of the many efforts of European and American kinematicians.

Conventional texts give an adequate presentation of the area of planar kinematic analysis. In these books, first contact is established with fundamental concepts, and known planar mechanisms are investigated for the displacements, velocities, and accelerations of their parts. In analysis, mechanisms are put on the psychiatrist's couch, so to speak, and are probed for their performance traits. The important and complementary area, that called kinematic synthesis—where mechanisms are created to meet certain motion specifications—is touched upon only by a consideration of cams and possibly a glimpse of the simpler aspects of planar linkage synthesis.

In the design of a mechanism for a given application, a decision must first be reached regarding the *type* of mechanism to be employed, as, for example, deciding between a cam or a linkage. The *number* of links and connections required to give the desired degree of freedom must then be determined. Finally, the required *dimensions* needed to bring about a particular motion must be deduced. In the broadest sense, kinematic synthesis thus consists of the three interrelated areas of type, number, and dimensional synthesis.

Of these areas, type synthesis is based largely on nonkinematic considerations. It depends, of course, upon the designer's kinematic experience, but, beyond that, other factors, such as machine-design problems and economic considerations, must be taken into account. While recognizing the problem of type synthesis, we make no effort to cope with it and go directly to linkages, with the avoidance of cams and gears. The present text focuses on the dimensional synthesis of linkages, an area amenable to mathematical treatments. Number synthesis, not a very real problem for the simple linkages that can be devised by synthetic procedures, is given a brief discussion.

This text is devoted to the synthesis of planar and spatial linkages and considers both basic approaches, the geometric and algebraic, allowing the reader to choose whichever method best suits his needs or available tools. The geometric methods are quick and have the advantage of staying close to the physical problem. They may have adequate accuracy for many tasks but tend to become tedious under repeated use. Algebraic methods give any desired degree of accuracy and through their functional relations indicate the relative importance of any parameter. They also make possible the study of errors in dimensions, as those associated with tolerance or wear. And of course the algebraic methods lead to the use of digital computers, especially convenient if the same calculations have to be repeated often for incremental inputs or parameter changes.

Recent years have seen the publication of a wealth of new material relating to synthesis, both in this country and abroad. In addition, there has been a growing awareness of the very considerable European effort, especially the German. This material is difficult to follow because of the scattered sources, terseness of presentation, and language barriers. The present text is an attempt to present a well-balanced and intelligible account of some of the geometric and algebraic procedures, filling in as necessary, making comparisons, and elaborating on the implications to give a well-rounded picture.

The bulk of the available European material seems to divide into two broad classes. One makes use of rather sophisticated geometry and mathematics, while, in the other, descriptions of techniques predominate over the scrutiny of principles. This book attempts a middle-of-the-road presentation having "rigor" without "mortis," integrating the European and American contributions.

The book studies, compares, and applies various procedures for handling the several types of problems constituting the synthesis of linkages. The start is made by reviewing or establishing definitions, concepts, and notations, with recapitulation of familiar methods of velocity and acceleration analysis. An account is given of type, number, and dimensional synthesis. This includes a discussion of the Chebyshev polynomials and their application to the determination of an optimal spacing of accuracy points. The properties and uses of four-bar coupler curves are discussed, including their equation, the implications of their geometry (cusps, symmetry, and crunodes), and the Roberts-Chebyshev theorem, with its extensions to the slider-crank mechanism and six-bar linkages. The Euler-Savary equation and the cubic of stationary curvature are derived analytically, by using complex numbers, the ambiguities and passage to the limit of the direct geometric approach being thus avoided. Two chapters are devoted to the geometric methods of synthesis with three and four accuracy points. Numerous sample problems are given here; some deliberately contain pitfalls to show what can be done when trouble arises. In the presentation of algebraic methods of synthesis using displacement equations or complex numbers, an attempt is made to include the latest developments on synthesis with five accuracy points and the use of digital computers in the solution of advanced problems.

The last chapter, Synthesis of Spatial Linkages, is an original contribution; this material is to be found nowhere else. Previous work of the authors on the analysis of spatial mechanisms by matrix algebra is summarized. This is followed by several sections on the synthesis of spatial linkages making use of displacement equations. Here again a number of examples are given.

In addition to the material which properly belongs under the title of kinematic synthesis, complementary material has been added to make the book more readable and self-contained. The reader is introduced to the history of kinematics, and the development of concepts and procedures from the great personalities of the past is traced. The chapter devoted to the construction of kinematic models is of the greatest aid in establishing a rapport between the static drawing of one phase and the many other positions a mechanism will assume. Models give immediate information about displacement relations, unfavorable transmission angles, dead points, binding, and friction. A review of complex numbers, determinants, and solutions of linear equations is included as an appendix. These tools, rust-covered for many readers (the voice of experience), are resharpened in an effort to make the book a self-sufficient unit suited for self-study.

Our spiritual guide and close friend was the late Professor Rudolf Beyer of Munich. His works and infectious enthusiasm have been the inspiration for this book and the source of strength for overcoming the many difficulties and discouragements inherent in the process that creates any book, especially one dealing with the development of deductive processes.

The contributions of many others—friends, colleagues, and students in overwhelming numbers—cannot be acknowledged individually, much as we would like to. We remain grateful to our benefactors even though their kindnesses and help must go unrecorded.

It is a last pleasant duty to acknowledge the suggestions of the reviewers of the manuscript.

Richard S. Hartenberg
Jacques Denavit

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