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Abstract

The value of the timing option implicit in CBOT corn futures contract is estimated. Separate estimates are obtained for the option without and with convenience yield. The effect of the option on basis behavior at day one of the maturity month is examined and is found to be statistically important.

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Net benefits of using a futures market as a risk management tool depend on hedging effectiveness. A perfect hedge can be completed if the futures price converges exactly to the spot price at maturity, i.e., the basis converges to zero. In practice, complexities of delivery specifications of futures contracts as well as arbitrage costs cause imperfect convergence. Delivery options embedded in contract specification introduce uncertainty about the relevant spot price to which the futures contract will converge at maturity, resulting in basis risk. Variability of the basis at maturity is a cost to hedgers and negatively influences hedging demand. Thus, from the point of view of an exchange as well as that of risk management demand, it is important to understand price and basis behavior at contract maturity.

This paper analyzes the effect of the timing option on basis behavior. The option permits delivery any time during the expiration month. Timing option's value on the first delivery day is estimated for the corn futures contract traded at the Chicago Board of Trade (CBOT) for all expiration months during 1989-97. We show that the timing option has a positive value even in the absence of convenience yield, contrary to the result of Boyle. When convenience yield is incorporated in the estimation procedure, the value of the option increases dramatically. The effect of the timing option on the basis is examined.

The rest of the paper is organized as follows. First, selected literature on the timing option and basis behavior is reviewed. Then the model and data are presented, followed by empirical results. Finally conclusions are drawn, and suggestions made for future research.

Timing Option and Basis Behavior

Timing Option

A futures contract creates an obligation to deliver (if short) or accept delivery (if long) of the underlying asset at time T for a price agreed upon at time t<T. Models of futures prices ordinarily assume the expiration date T to be fixed. Under the assumption of perfect frictionless markets, exact convergence of the futures and spot prices occurs at t = T. In reality, convergence is not perfect. First, for many contracts, no single expiration day exists. Shorts have an option to deliver any time during the delivery month, the so-called timing option. Second, many futures contracts' specifications include other options pertaining to quality, location, and time of delivery. The quality option gives the short the right to deliver non-par assets in place of the par asset at specified discounts or premia (Margrabe, Manaster and Gay). The location option allows delivery at various locations (Pirrong et al.). The wild card and end-of-the month options, present in the T-bond futures, give added flexibilities to the time of delivery (Hemler). Since the short will choose the cheapest asset and location for delivery, the futures price will converge to the price of this asset. If relative prices of deliverable assets and locations vary, basis risk arises as at time t uncertainty exists about the cheapest to deliver asset and location at time T.

The timing option embedded in futures contracts gives the seller choices: while trading persists, she/he can offset, deliver, or defer the choice. Boyle and Silk come to different conclusions in their analyses of the timing option. Boyle argues that the timing option has no value in the absence of quality and/or location options. With only one deliverable asset and one deliverable location, the asset will be delivered at the earliest date possible. Only the interaction effect of the timing option with quality/location option provides benefits to delaying delivery. Boyle assumes no dividend and no service flow (convenience yield).

Silk develops a model more suitable for agricultural commodity contracts. He assumes that there are costs as well as benefits in terms of convenience yield from holding the spot commodity. Both of them are taken to be constant and known when the futures position is initiated. His analysis implies that all deliveries occur immediately at the beginning of the delivery month, t=0, or on the last delivery day, t=T. The choice depends on the value of the convenience yield relative to the storage and opportunity costs of holding the commodity. Thus, convenience yield is the only source of potential benefit from delivering later.

Basis

Basis is defined here as the difference between the futures and the spot prices and as noted above, the lack of exact convergence implies basis risk. The larger the basis risk the lower the hedging demand, ceteris paribus. Heifner and Peck and Williams analyze hedging effectiveness in terms of the predictability of the change in the basis using the basis at the time of hedge initiation. Heifner evaluates gains from basing storage decisions on predicted basis changes. Peck and Williams focus on the effect of delivery timing during the expiration month on basis convergence. They find the effect insignificant in predicting the basis change.

Model

Timing Option Model

The model used in our analysis assumes perfectly competitive markets, no transaction costs, and no taxes. As we are interested in the value of the timing option on the first delivery date, the no transactions costs assumption is perhaps a reasonable abstraction. Most small hedgers offset their positions prior to the expiration month. Thus, traders who potentially might make delivery are likely those with low transaction costs. The other two assumptions are widely used in the theoretical as well as empirical literature on option pricing, but it is true that futures markets become more concentrated as the last day of trading approaches.

In this paper, the futures contract is assumed to have a timing option where no deliveries are allowed after the last trading day.[1] No quality, location, or other timing options are present in the contract. The delivery month runs from t = 0 to t = T, where T represents the last day of trading. Let F(t) be the price at day t of the currently deliverable futures contract and S(t) the spot price of the underlying asset at time t. Every day during the delivery month, the short has an option to deliver, offset, or hold the futures position open, and makes a decision based on maximizing the value of his/her position. Thus, the value of the option at any time t equals

$$Max[F(t) - S(t), 0] + CF(t),$$

where F(t) is the price of the currently deliverable futures contract, S(t) is the spot price of the underlying asset, both at day t and CF(t) is the cash flow to the short from marking

[1] Actual delivery can occur until the last day of the delivery month. The price for deliveries after trading stops is the futures price of the last trading day.

to market. The timing option has a character of an American put option on the underlying spot with a stochastic strike price equal to the futures price.

Note, the discrete nature and the implicit institutional structure of our timing option model make it possible for the futures price to exceed the spot price during the delivery month, even in the absence of convenience yield. The usual arbitrage argument relies on the possibility to buy spot, sell futures and deliver immediately. The strategy would yield a positive arbitrage profit if F(t) > S(t). Our implicit institutional framework prevents arbitrage using this strategy. A futures position is established at the beginning of the day t, and can only be closed by delivery at the end of the day t. Thus, in addition to the difference F(t)-S(t), a random cash flow occurs. The usual arbitrage strategy does not yield a certain profit.

The spot price is assumed to follow a continuous stochastic process of the form

$$dS(t) = \mu \cdot dt + \sigma \cdot dW(t)$$

where dS(t) denotes a change in spot price during a (infinitesimally small) time increment dt and dW(t) is a Wiener process with zero mean and variance dt. μ and σ are the diffusion drift and volatility parameters respectively.[2] As the probability distribution of the geometric Brownian motion describing the spot price is lognormal, a binomial approximation to the lognormal distribution can be applied to value the American option.

First, given the initial spot price and volatility, a binomial tree for the spot price is generated. Next, the futures price tree is constructed such that the value of the futures contract at every node is zero. Finally, the option value is estimated by backward induction, checking every state of the world in every time period for optimal early

[2] The spot price can have seasonal behavior, but this is not explicitly modeled in our paper.

exercise. A martingale pricing approach is used implying risk neutral evaluation. The risk-free interest rate, used in calculating present values in the binomial lattice, is assumed constant for the duration of each individual contract month, a reasonable assumption given the short horizon. As a result, the binomial lattice recombines (see Appendix A).

No convenience yield: In the first approach, convenience yield is ignored in the estimation of the timing option. The up and down factors for the binomial tree, U and D respectively, are determined as

$$U = e^{(r-0.5\cdot\sigma^2)\cdot h+\sigma\cdot\sqrt{h}} \text{ and } D = e^{(r-0.5\cdot\sigma^2)\cdot h-\sigma\cdot\sqrt{h}},$$

where r is the riskless interest rate, σ denotes volatility and h is the time increment here chosen to be one day, h =1. The value of the timing option is obtained for the first delivery day, t = 0. This approach separates the pure value of the choice of timing the delivery from that derived from the convenience yield.

Boyle argues that timing option without convenience yield and without any other delivery options present does not have a positive value. However, his analysis is valid for forward contracts only, as marking to market is ignored. He justifies his approach by the result that forward and futures contracts are equivalent under deterministic interest rates (Jarrow and Oldfield, 1981). However, this result is only valid for futures contracts absent timing or any other delivery options. Jarrow and Oldfield (1988) show that a futures price represents the price of an asset with a deteriorating present value and early exercise may be optimal.

Implicit Convenience Yield: In the second approach, convenience yield is estimated from the market data by inversion. The theoretical futures price with convenience yield equal to zero is compared to the observed futures price. By adjusting the value of the convenience yield in the tree generating process, the theoretical futures price that closely approximates the observed market price is arrived at iteratively[3]. The spot price tree resulting in this futures price is used to estimate the timing option value on the first delivery date. The up and down factors for this approach are

$$U = e^{(r-y-0.5\cdot\sigma^2)\cdot h+\sigma\cdot\sqrt{h}} \quad \text{and} \quad D = e^{(r-y-0.5\cdot\sigma^2)\cdot h-\sigma\cdot\sqrt{h}},$$

where y is the proportional convenience yield. The estimates now represent the joint effect of the convenience yield and the option of timing the delivery (see Appendix B).

Basis Model

As noted earlier, theoretical models of futures prices assume a single expiration day T and perfect convergence of the futures and spot prices on this date. For any date t < T , the futures price in perfect and frictionless markets equals

$$F(t,T) = S(t)\cdot e^{(r-y+c)(T-t)},$$

where c is storage cost, and F(t,T), S(t), r and y are as defined above. This is a result of a cash-and-carry no arbitrage argument. The timing option adds a value to the short and results in a lower futures price:

$$F(t,T) = S(t)\cdot e^{(r-y+c)(T-t)} + TO(t),$$

where TO(t) is the value at time t of the timing option. Thus, basis at time t is a function of the interest rate, convenience yield, storage cost, time to maturity and the timing option

$$\frac{F(t,T)}{S(t)} = e^{(r-y+c)(T-t)} + \frac{TO(t)}{S(t)}.$$

[3] Note, convenience yield is estimated residually and may capture other effects due to possible misspecification of the model. At a minimum, storage costs are ignored.

The following model for the basis is estimated by OLS,

$$LnB = \beta_0 + \sum_{i=1}^{4} \alpha_i D_i + \beta_1 Ln(\frac{TO}{S}) + \beta_2 FS + \beta_3 IntRate + e \ ,$$

where LnB is the log of the basis, IntRate is the 90-day T-bill rate, FS is the spread between the price of the currently deliverable futures contract and that of the next nearby on day t and is a proxy for carrying charges, and TO/S denotes the estimated timing option as a proportion of the spot price. In turn, TO is defined using the values of the timing option without and with convenience yield. D_i's are contract month dummies with December as a reference month. Storage costs, other than interest rates, are not included in the regression, but they are likely to have low variation within the sample period. The same model is also fitted as a linear equation, with observed basis as a function of TO/S.

Data

The value of the timing option is estimated for the corn futures contract traded at the CBOT. Daily data are used for each expiration month (March, May, July, September and December) in 1989 to 1997. The years before 1989 are influenced heavily by price support programs and substantial government stocks and are excluded from the analysis. Futures prices are the daily settlement prices. Cash prices are those reported for the Chicago terminal market. The 90 day T-bill rates obtained from CRSP database are used as risk-free rates. The number of trading days in individual delivery months ranges from 12 to16.

As the initial spot price, martingale probabilities and the up and down factors are the only lattice parameters needed to price an option under risk neutral valuation, just the initial spot price, riskless rate and volatility need to be known for the estimation method. The volatility for each contract month is estimated as a sample variance of the log of spot price returns, with the number of sample observations equal to the number of trading days in individual expiration months. This approach is equivalent to assuming perfect foresight. Martingale equivalent probabilities for a lognormal distribution are equal to 0.5 (Jarrow and Turnbull).

Empirical Results

The value of the timing option without convenience yield averaged 0.26 cent over the years 1989-97, ranging from zero to 0.7 cent (Table 1). This constitutes just 0.1% of the average futures price but 4% of the average basis, both as observed on the first delivery day. On average, the option has the lowest value for the expiration months of July and September prior to the new harvest (Figure 1). In July, prices are especially

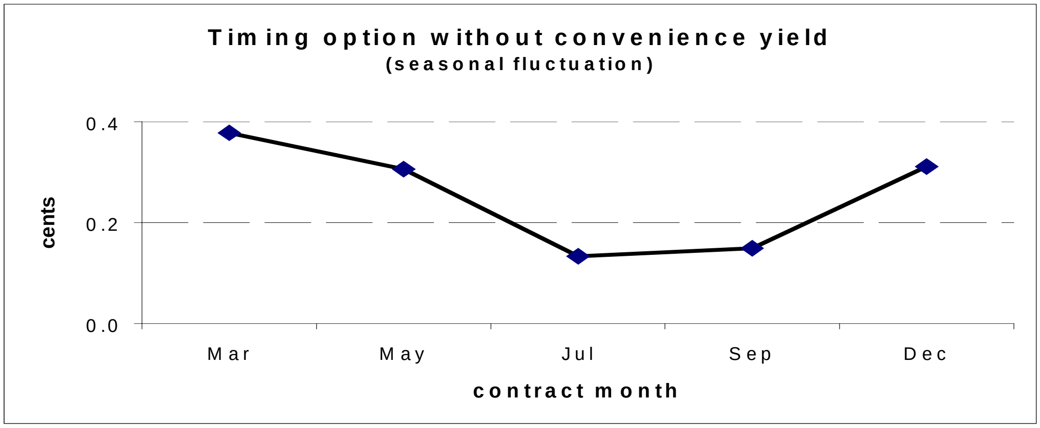
Table 1. Timing option with no convenience yield (in cents)[a]

Year / Month	**89**	**90**	**91**	**92**	**93**	**94**	**95**	**96**	**97**
March	0.8	0.6	0.3	0.3	0.3	0.2	0.5	0.4	0
May	0.7	0.6	0.4	0.2	0.05	0	0.5	0.2	0.1
July	0	0.4	0.2	0.2	0	0	0.4	0	0
September	0.4	0.3	0.2	0.04	0.1	0	0.1	0	0.2
December	0.5	0.4	0.2	0.1	0.2	0.3	0.4	0.4	0.3

a. Values are estimated for the first delivery day.

sensitive to changing information about the expected harvest and are characterized by the highest volatility. As vega ($\partial P / \partial \sigma$) for an option is positive, where P is the value of a put option, values of the timing option should increase during the months with high price volatility. However, corn prices are typically at their highest levels in July. The effect of the high prices through the negative delta of a put option ($\partial P / \partial S$) offsets the effect of the positive vega, resulting in low option values in July and September.

Figure 1



As indicated in Table 2, the value of the timing option increases dramatically with convenience yield incorporated in the estimation procedure. The average value is 5.7 cents, representing 2% of the futures price, but this option value represents a large proportion of the basis, 92% on average.

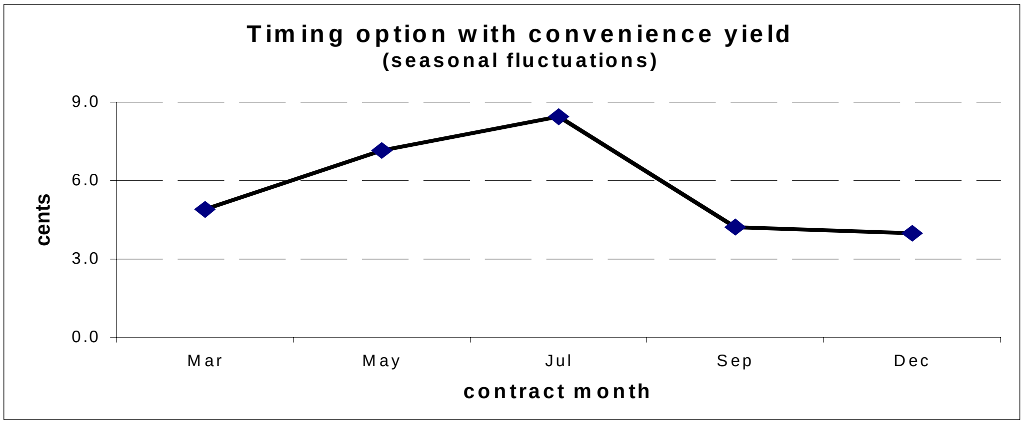
Table 2: Timing option with convenience yield (in cents)[a]

Year / Month	**89**	**90**	**91**	**92**	**93**	**94**	**95**	**96**	**97**
March	6.5	4.3	1.6	4.2	1.6	1.9	4.3	5.9	13.9
May	10.1	9.9	2.4	0.9	3.3	3.9	3.6	20.8	9.6
July	7.1	10.9	3.8	0.1	8.5	7.1	1.5	27.1	9.8
September	0.3	0.0	4.1	0.0	7.5	7.3	18.6	0.0	0.1
December	5.6	0.8	1.8	6.3	6.4	3.0	4.9	4.1	2.9

a. Values are estimated for the first delivery day.

The combined effect of the vega and delta of the timing option is reversed by the effect of a convenience yield, and consequently, on average the timing option value is largest in July. In its traditional interpretation, as a value to merchants of holding the spot commodity, convenience yield increases with decreasing aggregate stocks (Telser). Beginning stocks are lowest on September 1, but September is a transition month to the new crop when stocks will increase. December is the first month with the new crop fully in storage. As an increasing function of convenience yield, the timing option with convenience yield is lowest in December and highest in July (Figure 2).

Figure 2



Basis

Basis is calculated as the ratio of the futures price of the currently deliverable contract and the spot price on the first delivery day. Over years 1989-97, the basis averaged 2.7 percent, ranging from 0.1 percent to 7.9 percent. Results presented in Table 3 as well as Figure 3 illustrate, that the basis is lowest in December and rises over the

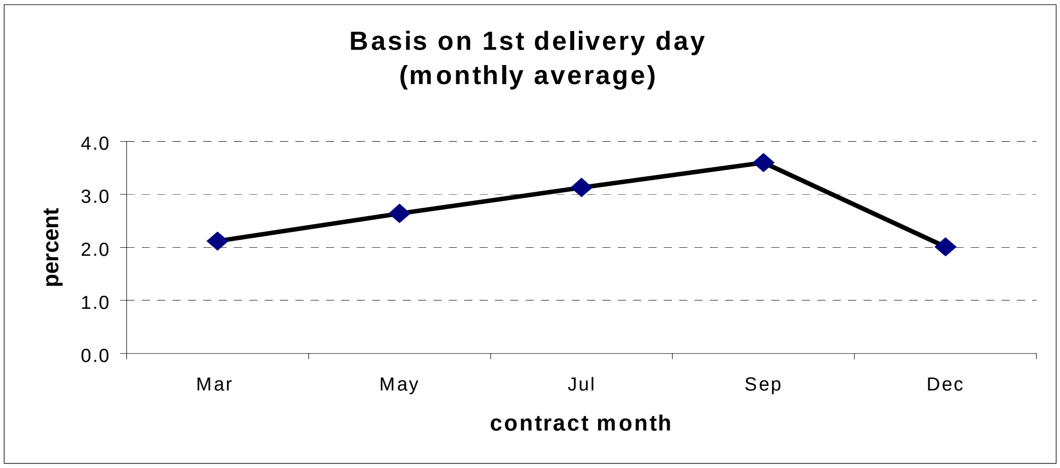
Table 3. Basis on the first delivery day (in percent)[a]

Month / Year	**March**	**May**	**July**	**September**	**December**
1989	2.7	4.2	3.0	0.1	2.9
1990	2.2	3.9	4.1	7.9	0.8
1991	1.0	1.3	2.1	2.1	1.1
1992	1.9	0.7	0.4	0.1	3.6
1993	1.2	1.9	4.3	3.9	2.7
1994	1.0	1.8	3.3	3.9	1.9
1995	2.3	1.8	0.9	7.1	1.8
1996	1.7	4.6	5.5	7.2	1.9
1997	5.1	3.6	4.6	0.1	1.4

a. Basis = F(t,T)/S(t)-1 and is calculated for the first delivery day.

crop year. On average, basis convergence is worst in September, the transition month and best in December, the first month with full new crop in storage.

Figure 3



Basis behavior

The estimated option values, obtained above, are used in regression models of basis behavior. These models are fitted to the data for the five delivery months in each of the years 1989 through 1997 using ordinary least squares (Tables 4 and 5). Since the basis is measured on the first day of the delivery month, the models help explain why the degree of convergence up to this point can vary from one delivery month to the next and from year to year.

The following conclusions can be drawn from the regression results. First, as measured by R^2, the explanatory power of the models is modest, but compares favorably with other attempts to model basis behavior at or near contract maturity (Leuthold). It perhaps should be noted that the R^2 coefficients for the linear and logarithmic models are not directly comparable, because the dependent variables differ in the two equations. Given that the costs of arbitrage (making and taking delivery) are important, it seems likely that any model of basis behavior at contract maturity will have a large random component.

Table 4: Basis behavior using timing option without convenience yield

	Log model		Linear model	
	Coefficient (t-stat)	Coefficient (t-stat)	Coefficient (t-stat)	Coefficient (t-stat)
Constant	-5.851 (-5.3)	-5.705 (-5.4)	0.014 (1.4)	0.015 (1.6)
D1 (Mar)	-0.085 (0.2)		0.001 (0.1)	
D2 (May)	-0.046 (0.1)		0.001 (0.1)	
D3 (Jul)	- 0.060 (-1.1)		- 0.006 (-0.6)	
D4 (Sep)	0.729 (-1.5)		0.007 (0.8)	
Futures Spread	- 5.279 (-1.2)	-3.779 (-1.0)	- 0.186 (-2.6)	-0.168 (-2.5)
Interest Rate	14.475 (1.5)	13.644 (1.4)	0.361 (1.6)	0.356 (1.6)
Timing Option	- 0.182 (-1.8)	-0.131 (-1.4)	- 5.905 (-1.1)	-5.698 (-1.2)
R^2	24%	21%	36%	32%

Second, the measure of the timing option value with convenience yield increases the explanatory power of the models and reverses the sign of the coefficient of this variable. Using the broader definition, the timing option effect is positively related to the size of the basis; the larger the value of the timing option, the larger the initial basis, ceteris paribus.

Third, the futures spread coefficient is consistently negative across the various models. In the logarithmic models, which are consistent with the underlying model of price behavior, using the timing option value with convenience yield has the effect of increasing the absolute size of the coefficient of the futures spread as well as its t-ratio relative to using the timing option value without convenience yield. The analogous change does not occur in the linear equations.

Fourth, the interest rate coefficient is consistently positive, as suggested by theory, though the t-ratios typically range between one or two. In general, the dummy variables for the delivery months are statistically unimportant. The other variables in the model apparently are successful in capturing the "seasonal" behavior of the basis.

Table 5: Basis behavior using timing option with convenience yield

	Log model		Linear model	
	Coefficient (t-stat)	Coefficient (t-stat)	Coefficient (t-stat)	Coefficient (t-stat)
Constant	-3.405 (-5.1)	-3.533 (-6.2)	0.007 (0.8)	0.009 (1.2)
D1 (Mar)	-0.057 (0.1)		-0.001 (-0.2)	
D2 (May)	-0.040 (-0.1)		-0.001 (-0.2)	
D3 (Jul)	- 0.303 (-0.6)		- 0.004 (0.7)	
D4 (Sep)	-0.153 (-0.3)		0.011 (1.7)	
Interest Rate	6.889 (1.5)	7.172 (0.8)	0.127 (1.0)	0.140 (1.1)
Futures Spread	- 8.817 (-2.3)	-7.632 (-2.4)	-0.131 (-2.3)	-0.099 (-1.5)
Timing Option	0.163 (2.1)	-0.166 (2.6)	0.648 (4.7)	0.001 (2.7)
R^2	26%	25%	58%	50%

These results suggest that measuring the value of the timing option contained in agricultural futures contracts can help explain the variability of the basis at contract maturity. Since the option value has a seasonal pattern, the basis at contract maturity also has a seasonal pattern.

Concluding Remarks

The value of the timing option in CBOT corn futures contract is estimated for all expiration months during years 1989-97. Estimates show that the value of the timing option without convenience yield averaged 0.26 cent per bushel, representing 0.1% of the futures price and 4% of the basis. The option value increases when convenience yield is incorporated in the estimation procedure. The value of the timing option then averages 5.7 cents per bushel, and represents 2% of the futures price and 92% of the basis.

The timing option has positive value even without taking account of convenience yield, i.e., it may be optimal to delay delivery even in the absence of convenience (dividend) yield and other delivery options. This result highlights the importance of taking institutional arrangements into account. Namely, Boyle's intuition that timing option is worthless in the absence of convenience yield and/or other delivery options ignores the marking to market feature of the futures markets. The daily marking to market cash flows may cause forward and futures contracts with the timing option to be different even under constant interest rates. As the futures price represents the price of a deteriorating asset, it may be optimal to delay delivery.

The timing option values without convenience yield is lowest during the months of July and September. These months, especially July, are characterized by the highest price levels as well as by relatively high price volatility and demonstrate the dominating effect of the delta over the vega effect of the put option in the corn futures contract. The combined delta and vega effect is, however, reversed by the effect of convenience yield. When convenience yield is incorporated in the estimation, values of the timing option attain their highest levels in the month with the lowest inventory levels and decline as aggregate stocks grow. This is a result of convenience yield being a decreasing function of inventory levels and the timing option value being an increasing function of the convenience yield.

The timing option has low explanatory power for basis variability when estimates without convenience yield are used. With convenience yield incorporated, a percent increase in the timing option value increases the proportional basis by 16%. Thus, timing option for commodities with convenience yield appears to be a significant factor in basis non-convergence.

Additional research should focus on incorporating other delivery options in the analysis of hedging effectiveness as well as analyzing other aspects of price and delivery behavior at contract maturity. The quality and location options should be included in a comprehensive study of delivery options and their effects on hedging effectiveness. While the literature has treated different options additively, present analysis offers a way to capture the joint effect of all three options by interacting the location and quality options with the timing option. Estimated values of delivery options may help explain timing of deliveries within the expiration month.

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APPENDIX A

Estimating Option Values without Convenience Yield

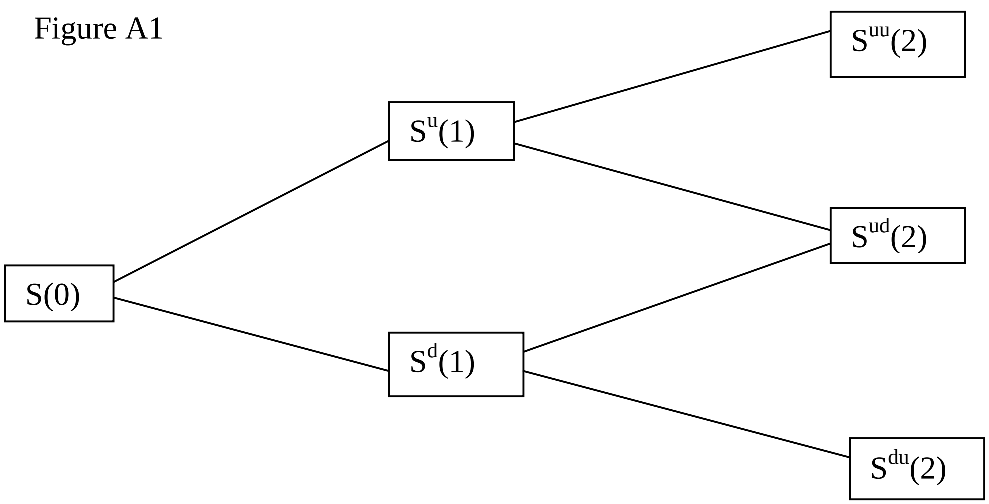
Spot Price Tree

The binomial tree approach assumes that price in one period can move in two possible directions, up or down. Prices in the two states of the following period are generated as products of the current price and the up and down factors,

$$S^u(t+1) = U*S(t) \text{ and } S^d(t+1)=D*S(t), \quad (1)$$

where the factors for a lognormal distribution are $U = e^{(r-0.5\cdot\sigma^2)\cdot h+\sigma\cdot\sqrt{h}}$ and $D = e^{(r-0.5\cdot\sigma^2)\cdot h-\sigma\cdot\sqrt{h}}$ respectively. r, σ, and h are the riskless interest rate, volatility, and time increment here defined as one, respectively. Figure A1 illustrates a spot-price tree generating process for two periods.

Figure A1



Given the initial price S(0), price in period one can either become $S^u(1)$ with equivalent martingale probability p or $S^d(1)$ with equivalent martingale probability (1-p). The prices in the two possible states of the world in period one are obtained from (1) as $S^u(1) = U*S(0)$ and $S^d(1) = D*S(0)$. From each node in period one, prices can again move up or down, resulting in three possible states of the world in period two

$S^{uu}(1) = U*S^{u}(1) = U*U*S(0)$,

$S^{ud}(1) = S^{du}(1) = D*S^{u}(1) = D*U*S(0)$,

$S^{dd}(1) = D*S^{d}(1) = D*D*S(0)$.

The tree recombines as a result of constant (deterministic) interest rates and the number of nodes in every time period t is t+1.

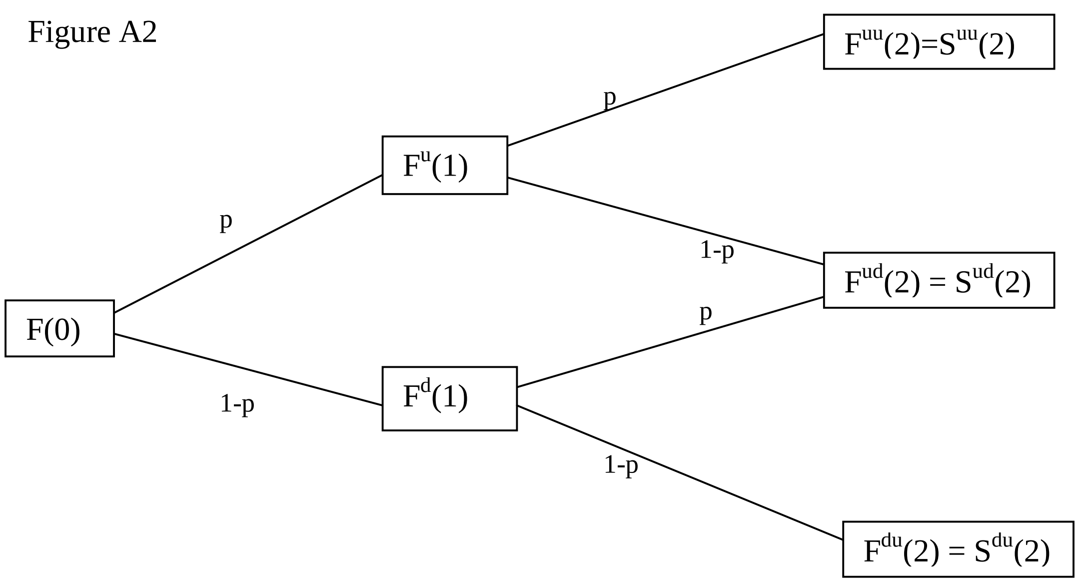
Futures price tree

The futures price on the last trading day is assumed to equal the spot price, F(T)=S(T), in all states. As the value of a futures contract is reset to zero every day through marking to market, the following relationship for the futures prices obtains under risk neutral valuation

$F(t-1) = pF^{u}(t)+(1-p)F^{d}(t)$,

where p is the equivalent martingale probability of the up-state, for a lognormal distribution equal to 0.5 (Jarrow and Turnbull). Backward induction is used to calculate futures prices down the tree. Thus, $F^{u}(1) = pF^{uu}(2) + (1-p)F^{ud}(2)$ and $F^{d}(1) = pF^{du}(2) + (1-p)F^{uu}(2)$. The futures price at period zero is $F(0) = pF^{u}(1) + (1-p)F^{d}(1)$, as illustrated in A2.

Figure A2



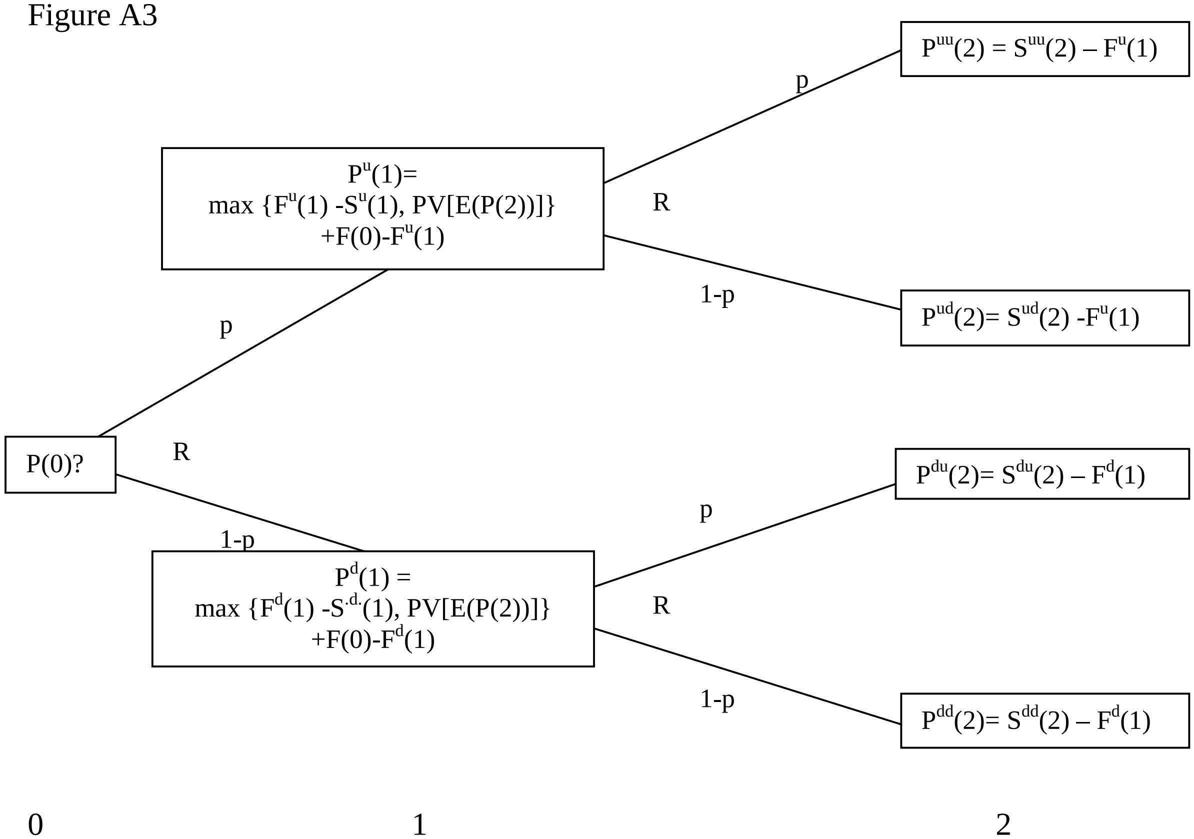
American option tree

Every day, the short decides whether to exercise the put option and deliver or keep the option alive and delay delivery. The boundary condition is

$$Max\{F(t)-S(t), PV[E(P(t+1))]\}+CF(t),$$

where PV[E(P(t+1))] is the present value of the expected value of the option and the expectation is under equivalent martingale probabilities. CF(t) is the cash flow to the short at date t, F(t-1)-F(t). Depending on the optimal exercise decision, the value of the option at date t is F(t)-S(t) if exercised, and $[pP^u(t+1)+(1-p)P^d(t+1)]/R + CF(t)$ if kept alive, where R=1/(1+r). The value of the option at the initial day is obtained by working down the tree (Jarrow and Turnbull). A two period example is given in Figure A3.

Figure A3



The calculations assume no cash flow at date zero and P(0) = max{F(0)-S(0), PV[E(P(1))]}.

APPENDIX B

Estimating Option Values with Convenience Yield

The spot price tree can be generated with convenience yield included. The up and down factors are now $U = e^{(r-y-0.5\cdot\sigma^2)\cdot h+\sigma\cdot\sqrt{h}}$ and $D = e^{(r-y-0.5\cdot\sigma^2)\cdot h-\sigma\cdot\sqrt{h}}$, where y is the proportional convenience yield. Convenience yield is estimated from the market data by inversion. The theoretical futures price with convenience yield equal to zero (as estimated in Figure A2) is compared to the observed futures price at date zero. If the theoretical futures price is above (below) the observed futures price, y in the spot price tree up and down factors is increased (decreased) by a fixed step value and new spot and futures price trees are generated. The new theoretical futures price at date zero is again compared to the observed futures price. By iteration, an implicit convenience yield is arrived at that equates the theoretical and observed futures prices. The corresponding spot and futures price trees are used in estimating the timing option value.