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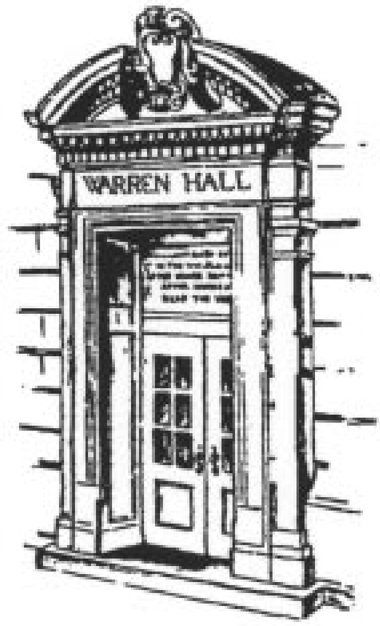

Working Paper

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Determining the Optimal Amount of Nitrogen to Apply to Corn Using the Box-Cox Functional Form

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Abstract

A Box-Cox functional form was estimated from corn-nitrogen data previously used to report optimal nitrogen use from a quadratic production function. Results suggest that less nitrogen should be applied than recommended using the quadratic function, but with more nitrogen being applied if poor growing conditions are expected.

In an article in the *Journal of Production Agriculture*, Vanotti and Bundy (VB) discuss an alternative rationale for corn nitrogen fertilizer recommendations. Instead of basing fertilizer N recommendations on corn yield goals, they suggest basing N recommendations on soil- and year-specific data. Essentially, they suggest estimating a production function from past observed yields and inputs, and then using the objective of profit maximization with relevant prices for N and corn. Using 24 years of data, they estimate a quadratic production function for the lowest yield years and separately for the highest yield years. Given a corn price of $2.00 and N price of $.15, they conclude that the optimal N of 170 lbs. per acre to apply during a low yield year is identical to the optimal N to apply during a high yield year. They argue that this result may be due to the fact that the corn plant is not as efficient in utilizing N in poor growing years. Although N intake by the corn is lower during those years, more soil N must be made available to the corn plant because of this reduced N intake efficiency.

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This result has very significant implications. The first is that it is not necessary to know what type of crop year will transpire in order to make an optimal economic decision as to the amount of N to apply. This is useful information since it is virtually impossible to forecast growing conditions for an upcoming growing year. A second implication that VB discuss is that since the corn plant will not utilize all of the fertilizer N applied in a poor crop year, there is the potential for higher levels of N leaching into ground water after a low yield year.

A limitation of the work by VB is that they estimated a quadratic production function which is rigid in it's ability to fit production input/output data. The purpose of this note is to estimate these production relations using the flexible Box-Cox functional form, and use those estimated functions to determine optimal N application. The results are numerically different from the results of VB, although statistically the quadratic functional form as the correct functional form could not be rejected. Less N should be applied than suggested by VB in both poor and good growing years, with ironically more N applied in poor growing years than in good growing years.

Materials and Methods

The Box-Cox functional form estimated is:

$$\frac{y^{\lambda}-1}{\lambda} = c + b\cdot\frac{x^{\lambda}-1}{\lambda} + a\cdot\left(\frac{x^{\lambda}-1}{\lambda}\right)^2$$

where y is the corn yield in bushels per acre, and x is the N applied in pounds per acre. Lamda along with a, b, and c, are to be estimated, and since lamda can assume any value, this function form is flexible. If lamda is equal to one, the equation collapses to the quadratic functional form that VB estimated. If the limit of lamda approaches zero, then the functional form becomes a double log form, often referred to as a translog form by economists.

Since the equation is non-linear in it's parameters, the Box-Cox must be estimated using a non-linear estimator. The computer software used here is the BOX command in SHAZAM. It maximizes the log-likelihood function.

In order to determine the optimal amount of N to apply, it is necessary to equate the first derivative of a production function to the ratio of the input price to output price. The first derivative of the Box-Cox was obtained from the symbolic mathematics software package DERIVE, and verified with the software Mathcad. The first derivative of the function is:

$$\frac{dy}{dx} = \lambda^{-\frac{1}{\lambda}} \cdot x^{\lambda - 1} \cdot \left(2 \cdot a \cdot x^{\lambda} - 2 \cdot a + b \cdot \lambda\right) \cdot \left[a \cdot x^{2 \cdot \lambda} + x^{\lambda} \cdot (b \cdot \lambda - 2 \cdot a) + a - \lambda \cdot (b - c \cdot \lambda - 1)\right]^{\frac{1-\lambda}{\lambda}}$$

After inserting the estimated values of λ, a, b, and c, this derivative is equated to the ratio of input to output price and then numerically solved for x using DERIVE.

The data used are from Table 1 of VB. To be consistent with VB, the year 1979 was excluded from the high yielding years and the year 1988 was excluded from the low yielding years. VB found no statistical response to N in either of those two years. The quadratic function results of VB were first replicated. Results for the high yielding years were identical to VB, but the low yielding year results were slightly different. My computed optimal N was 170 lbs. for the high years and 179 lbs. for the low years. Using the reported equations in VB, their optimal N was 169.5 in the high years and 170.4 in the low years.

Since the Box-Cox function cannot accommodate zero valued observations, the observations of zero N in Table 1 of VB were set equal to numerical ones. That modification did not significantly change my revised quadratic estimates; the optimal N differed by less that .5 of a lb. from those results stated above.

Results and Discussion

The Box-Cox estimates for the high and low years are shown in Table 1. The lamda value for both types of years is less than one. However, the null hypothesis that $\lambda=1$ could not be rejected in either equation using a chi-square test. This implies that the quadratic is the correct functional form for the data.

None-the-less, the optimal N calculated from the first derivative of the Box-Cox is N=103.5 lbs. during the high years and N=135.5 lbs. during the low years. Both of these amounts are considerably lower than the optimal N calculated from the quadratic functions, and demonstrate that since the price of N is low relative to the price of corn, the optimal area of the production surface should be relatively flat. If that is the case, slight variations in the estimation of the slope of the top part of the production surface can drastically alter the optimal amount of N to apply.

Table 1: Estimates of the Box-Cox and Quadratic Production Functions

	Box-Cox		Quadratic	
Coefficient	High Year	Low Year	High Year	Low Year
Intercept (t-statistic)	35.183 (28.47)	33.632 (17.97)	93.739 (20.96)	61.265 (14.77)
Linear (t-statistic)	.53970 (7.41)	.5914 (6.91)	.58443 (7.20)	.50653 (6.94)
Quadratic (t-statistic)	-.0059929 (-5.11)	-.0031521 (-4.72)	-.0014954 (-5.30)	-.0012038 (-5.01)
Lamda	.73	.82		
Chi-Square for lamda=1	2.4	1.0		
R-Square Adjusted			.62	.63

Although optimal N application rates are lower with the Box-Cox, the result shows that significantly more N should be applied during the low years than during the high years. If less N is utilized during the low years, the potential exists for more soil residual N to leach into water supplies. Given that farmers are not able to predict what type of growing year they will experience, most will opt to apply N based upon the low years' results.

References

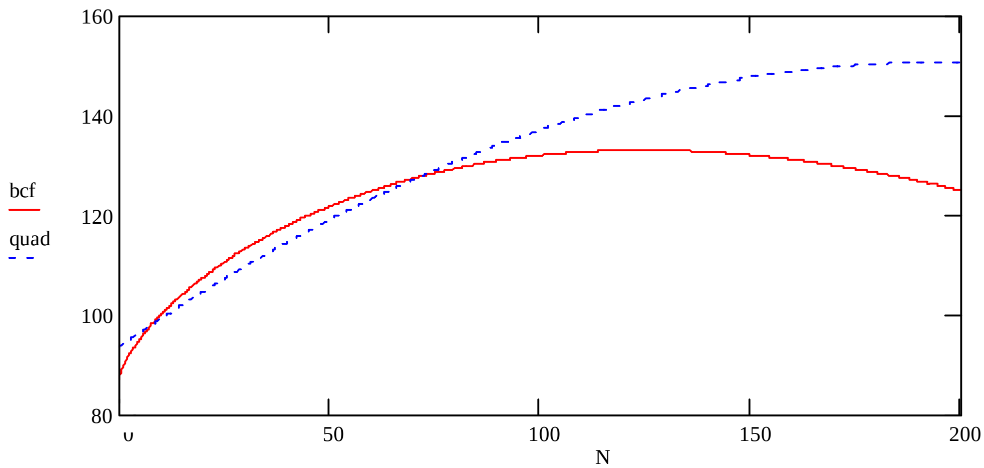
DERIVE, Soft Warehouse, Inc., Honolulu, Hawaii.

Mathcad, MathSoft Inc., Cambridge, Massachusetts.

SHAZAM User's Reference Manual Version 7.0, McGraw-Hill, 1993.

Vanotti, M. B. and L. G. Bundy, "An Alternative Rationale for Corn Nitrogen Fertilizer Recommendations." *Journal of Production Agriculture*, Vol. 7, No. 2, April-June 1994, pp. 243-249.

Figure 1: The Box-Cox (bcf) and Quadratic (quad) Production Functions for the High Yield Years



Appendix: Data in Table 1 of Vanotti and Bundy

NITH	NITH2	NITL	NITL2	YIELDH	YIELDL
0.000000	0.000000	0.000000	0.000000	115.0000	97.00000
75.00000	5625.000	75.00000	5625.000	128.0000	117.0000
150.0000	22500.00	150.0000	22500.00	136.0000	103.0000
300.0000	90000.00	300.0000	90000.00	135.0000	100.0000
0.000000	0.000000	0.000000	0.000000	97.00000	67.00000
75.00000	5625.000	75.00000	5625.000	150.0000	103.0000
150.0000	22500.00	150.0000	22500.00	154.0000	118.0000
300.0000	90000.00	300.0000	90000.00	156.0000	114.0000
0.000000	0.000000	0.000000	0.000000	95.00000	60.00000
75.00000	5625.000	75.00000	5625.000	121.0000	101.0000
150.0000	22500.00	150.0000	22500.00	120.0000	123.0000
300.0000	90000.00	300.0000	90000.00	134.0000	105.0000
0.000000	0.000000	0.000000	0.000000	91.00000	59.00000
75.00000	5625.000	75.00000	5625.000	124.0000	100.0000
150.0000	22500.00	150.0000	22500.00	145.0000	108.0000
300.0000	90000.00	300.0000	90000.00	135.0000	110.0000
0.000000	0.000000	0.000000	0.000000	105.0000	69.00000
50.00000	2500.000	75.00000	5625.000	140.0000	89.00000
100.0000	10000.00	150.0000	22500.00	138.0000	97.00000
200.0000	40000.00	300.0000	90000.00	139.0000	90.00000
0.000000	0.000000	0.000000	0.000000	47.00000	44.00000
50.00000	2500.000	75.00000	5625.000	140.0000	79.00000
100.0000	10000.00	150.0000	22500.00	132.0000	97.00000
200.0000	40000.00	300.0000	90000.00	151.0000	128.0000
0.000000	0.000000	0.000000	0.000000	66.00000	76.00000
50.00000	2500.000	50.00000	2500.000	109.0000	90.00000
100.0000	10000.00	100.0000	10000.00	136.0000	102.0000
200.0000	40000.00	200.0000	40000.00	144.0000	97.00000
0.000000	0.000000	0.000000	0.000000	86.00000	35.00000
50.00000	2500.000	50.00000	2500.000	135.0000	86.00000
100.0000	10000.00	100.0000	10000.00	139.0000	116.0000
200.0000	40000.00	200.0000	40000.00	150.0000	118.0000
0.000000	0.000000	0.000000	0.000000	100.0000	63.00000
50.00000	2500.000	50.00000	2500.000	146.0000	88.00000
100.0000	10000.00	100.0000	10000.00	148.0000	88.00000
200.0000	40000.00	200.0000	40000.00	168.0000	86.00000
0.000000	0.000000	0.000000	0.000000	68.00000	36.00000
50.00000	2500.000	50.00000	2500.000	116.0000	93.00000
100.0000	10000.00	100.0000	10000.00	146.0000	127.0000
200.0000	40000.00	200.0000	40000.00	122.0000	119.0000
0.000000	0.000000	0.000000	0.000000	104.0000	47.00000
50.00000	2500.000	50.00000	2500.000	142.0000	64.00000
100.0000	10000.00	100.0000	10000.00	140.0000	97.00000
200.0000	40000.00	200.0000	40000.00	141.0000	108.0000