CHAPTER XI.

CHAINS INVOLVING SCREW MOTION.

90. Formation of Screw Surfaces.—It has already been stated (§ 8) that lower pairs of elements can be constructed in which the surfaces in contact are screws of uniform pitch. Fig. 186 serves to illustrate the formation of

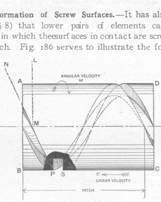

FIG. 186.

such surfaces. Imagine that a cylinder $ABCD$ is caused to rotate with uniform angular velocity, as indicated by the arrow, and let a cutting tool whose point is ground into the shape $PQRS$ be moved with uniform linear velocity v in a direction parallel to the axis of the cylinder, so as to cut out a continuous groove in the material of the cylinder. If now the tool is so set that the lines PQ and SR when produced pass through the axis of the cylinder, the surfaces forming the side of the groove will be screw or helical surfaces of uniform pitch. It will evidently be possible to form in a somewhat similar manner a hollow cylinder having the material of its inner surface removed in such a way as to leave a projecting thread of such a form as will exactly fit into the groove $PQRS$. The inner surface of this nut will be the exact counterpart of the outer surface of the screw, and when working together their relative motion must be a copy of the original relative motion of the cutting tool and the cylinder. In other words, the only possible relative motion of such a screw and its nut will be a motion of rotation, combined in a constant ratio with a motion of translation along the axis of rotation. By the term *pitch* we mean the distance (measured along the axis of rotation) through which the nut moves relatively to the screw during one complete relative rotation. Thus if ω be the angular velocity of the cylinder in radians per second, the time of one complete rotation will be $\frac{2\pi}{\omega}$ seconds. During this time the cutting tool will have moved a distance $\frac{2\pi v}{\omega}$; this expression therefore gives the numerical value of the pitch. If we imagine that a piece of paper wrapped round the cylinder has the outline of the screw-thread marked upon it, and is then unwrapped, the line representing the edge of the screw-thread will be found to be straight, and it will make with the line representing the edge AB of the cylinder an angle such as LMN. A little consideration will show that the tangent of this pitch-angle will be

$$\frac{\text{pitch of thread}}{\text{circumference of cylinder}}.$$

It is quite easy to arrange a mechanism which will cut a screw-thread of *variable pitch*. This is, in fact, often done in rifling guns. In this case, if the angular velocity of the screw is uniform, the linear velocity of the tool must be variable, and the pitch-angle changes as we go along the thread. A hollow surface the exact counterpart of the screw would then only fit exactly in one position, and no relative motion of such a pair of surfaces would be possible. It is for this reason that a screw pair composed of rigid elements must consist of screw surfaces of *uniform pitch*. The section of the thread, as governed by the form of the cutting tool producing it, may be of any convenient form, and a number of standard threads are described in text-books on machine design. The reader should note that screws are often made with two, three, or a larger number of threads by cutting the required number of independent grooves on the cylinder. These threads may further be either right- or left-handed. The thread in Fig. 186 is right-handed; Fig. 187 shows a left-handed screw having

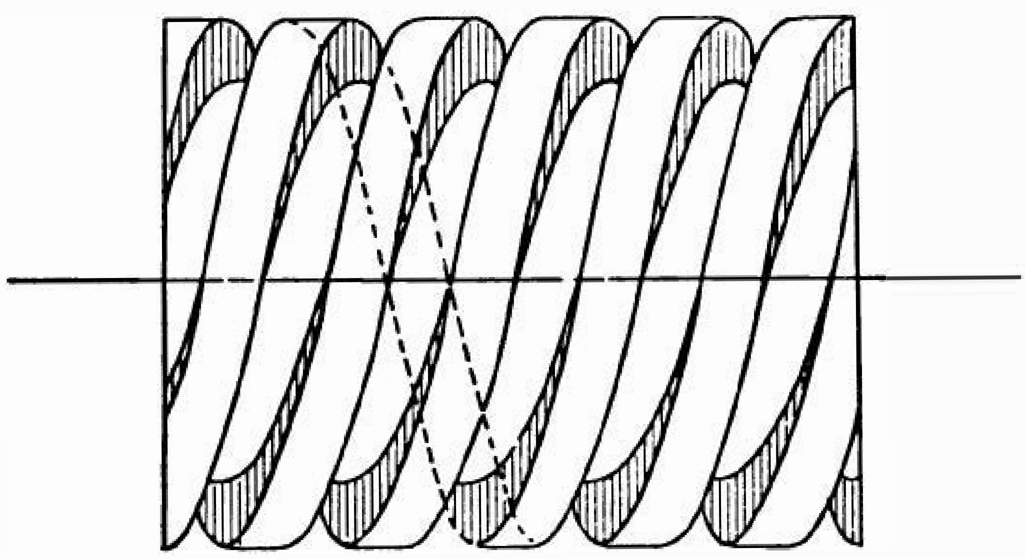

FIG. 187.

three threads. We shall see later that these multiple-threaded screws are of importance in screw mechanisms involving higher pairing, and we now consider certain cases in which lower pairing of screw surfaces is used in chains containing rigid links only.

91. Screw Mechanisms Involving Lower Pairing of Rigid Links.—The relative motion of screw links is in general non-plane. On examination it will be found that in a screw and its nut, while there is at any instant rotation about the axis of the screw, there is also a simultaneous linear movement along that line. In more complex cases of the screw motion of two bodies it has been pointed out * that there is at any instant a line common to the two bodies, called the *twist axis*, about and upon which each body is (at the instant

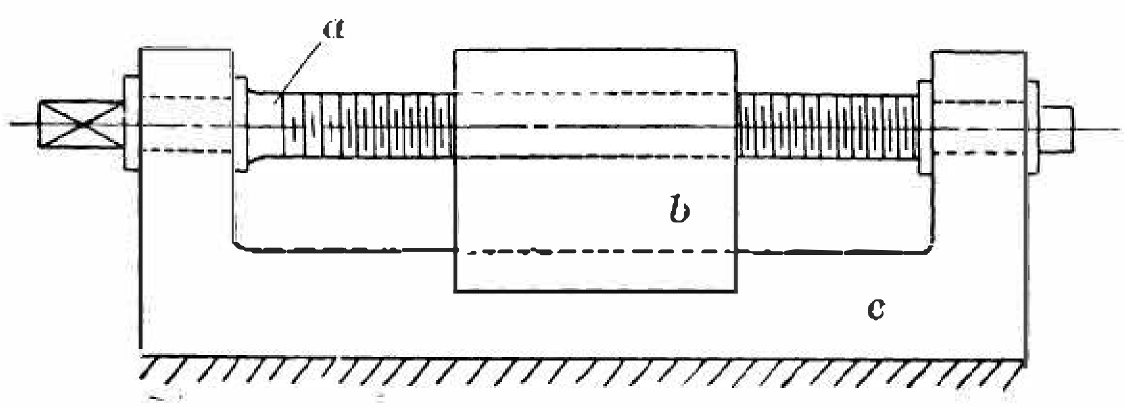

FIG. 188.

considered) turning and sliding relatively to the other body. In this work but little space can be devoted to the consideration of relative motion of this complex character, and in this section we shall discuss some of the simpler screw mechanisms involving lower pairing.

The simplest screw chain is shown in Fig. 188; it comprises three pairs—a screw pair *ab*, a turning pair *ac*, and a sliding pair *bc*. This chain is of common occurrence in the form of a screw press. By a suitable choice of the pitch of the screw we can obtain a machine in which a large angular motion of *a* gives us a comparatively small linear motion of *b*, so that in a copying-press, for instance, a large pressure is obtained by applying a relatively small force to the end of the screw arm.

If the screw has a sufficiently fine pitch this machine cannot be reversed; that is, it is not possible by the application of an axial force to the nut *b* to cause rotation of *a*. By making the pitch of the screw sufficiently great, however, this action becomes possible, as in the common Archimedean drill.

A little consideration will show the reader that we may look upon a sliding pair as a screw pair of infinite pitch, while a turning pair is also a special case of a screw pair in which the pitch is zero. Accordingly we may expect to

* Kennedy, Mechanics of Machinery, § 68.

find mechanisms of three links containing two screw pairs and a sliding pair, or two screw pairs and a turning pair (as shown in Figs. 189*a* and 189*b*), the pair *bc* or *ac* in Fig.

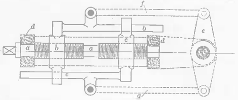

FIG. 189*a*.

188 having been modified into a screw pair. Further, it is possible to transform the last remaining turning pair of Fig. 189*b* into a screw pair and obtain a chain of three links and three screw pairs. The reader should have no difficulty in sketching for himself such a chain.

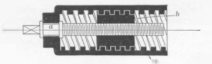

FIG. 189*b*.

Fig. 189*a* represents the chain containing two screw pairs and a sliding pair, as employed in a form of steering-gear; the complete gear forms a compound chain of seven links, four of which (shown by dotted lines) are added to the screw chain itself. The screw *a*, on which are cut two separate threads, right- and left-handed respectively, gears with two nuts *b* and *c* which evidently have a relative sliding motion, approaching or receding from each other as *a* rotates. The links *f* and *g* connect *b* and *c* to the arms of a yoke e secured to the rudder head. The frame or fixed link d is the hull of the ship, to which are fixed the bearings in which a and e rotate. Plainly, rotation of a will cause the rudder to turn.

Fig. 189b shows the chain containing two screw pairs and a turning pair. Its most important application in practice will be discussed when we deal with screw chains involving fluid links. (See § 92.)

A great variety of more complex screw chains are in practical use. Fig. 190 shows a *crossed screw chain* often employed as a portion of the reversing-gear of steam-engines. It consists of five links and contains a screw pair ab and

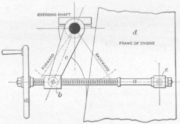

FIG. 190.

four turning pairs. The rotation of a hand-wheel on a moves the reversing-shaft from its position when the engine goes ahead to the position when the engine is in backward gear.

The mechanism of Fig. 190 is a simple example of a class of mechanisms involving *general screw motion*, in which the relative motions of the links are often very complex. The majority of such chains, in fact, have not been worked out kinematically, but the more complicated general screw mechanisms find so small a field of usefulness that we shall not devote any space to them here.

92. Screw Mechanisms containing Fluid Links. — One of the simplest screw mechanisms containing a fluid link is the rifled gun shown in longitudinal section in Fig. 191. This train consists essentially of three links. We have the gun itself, a, having traced upon the surface of its cylindrical bore the rifling, in the shape of a many-threaded hollow screw shown in cross-section at AB. The projectile or shell b is introduced at the breech of the gun, which is closed by a screw-plug or breech-block, and the projectile is provided at its base with one or more copper driving-bands, which are

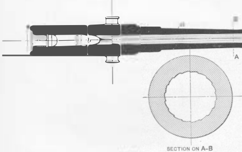

FIG. 191.

of such a size that when the projectile is forced through the bore the projecting portions of the rifling cut into the copper, and in this way cause the shell to rotate. The third link, c, is of course the gas which is produced by the combustion of the charge, and which exerts the pressure necessary to propel the shell. It should be noted that the pitch of the rifling has to be large, compared with the calibre or diameter of the bore of the gun. Frequently the pitch of the rifling is not uniform, but is so designed as to decrease from the breech to the muzzle in such a way as to give as nearly as possible uniform angular acceleration to the shell.

In Fig. 192 is represented a mechanism which is a special case of the screw-chain used for such important purposes as the propulsion of ships (screw propeller), the measurement of speed through fluid (anemometer, patent log), the

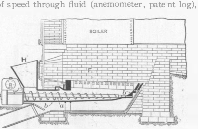

FIG. 192.

utilization of the energy of the wind (windmill), and so on. The figure shows diagrammatically a screw conveyor used for the purpose of forcing broken coal from the hopper H into the furnace F of a mechanical stoker. We have here a rotating propeller a of peculiar form; it is contained in a casing b, and acts on the powdery material enclosed by the casing. The reader will note that in the screw propeller of a ship we have exactly the same mechanism, except that the outer casing is used only under special circumstances,* and the material acted upon is fluid.

The mechanism of Fig. 189b, when modified by the substitution of a fluid link for the piece b, takes the form of the parallel-flow (Jonval) turbine of Fig. 193, and is used for purposes of motive power. Here c, the turbine-casing, carries a bearing for a, the hollow shaft, and also has upon it a number of fixed guide-blades corresponding kinematically to the hollow screw-thread of Fig. 189b. The fluid,

*Barnaby, Marine Propellers, Chapter VII.

rushing past these blades, encounters the blades of the turbine-wheel *a*, to which it communicates motion. The kinematic correspondence of the two mechanisms is evident.

It should be noted that the surfaces of the guide-blades and buckets of a turbine, or of the blades of a propeller,

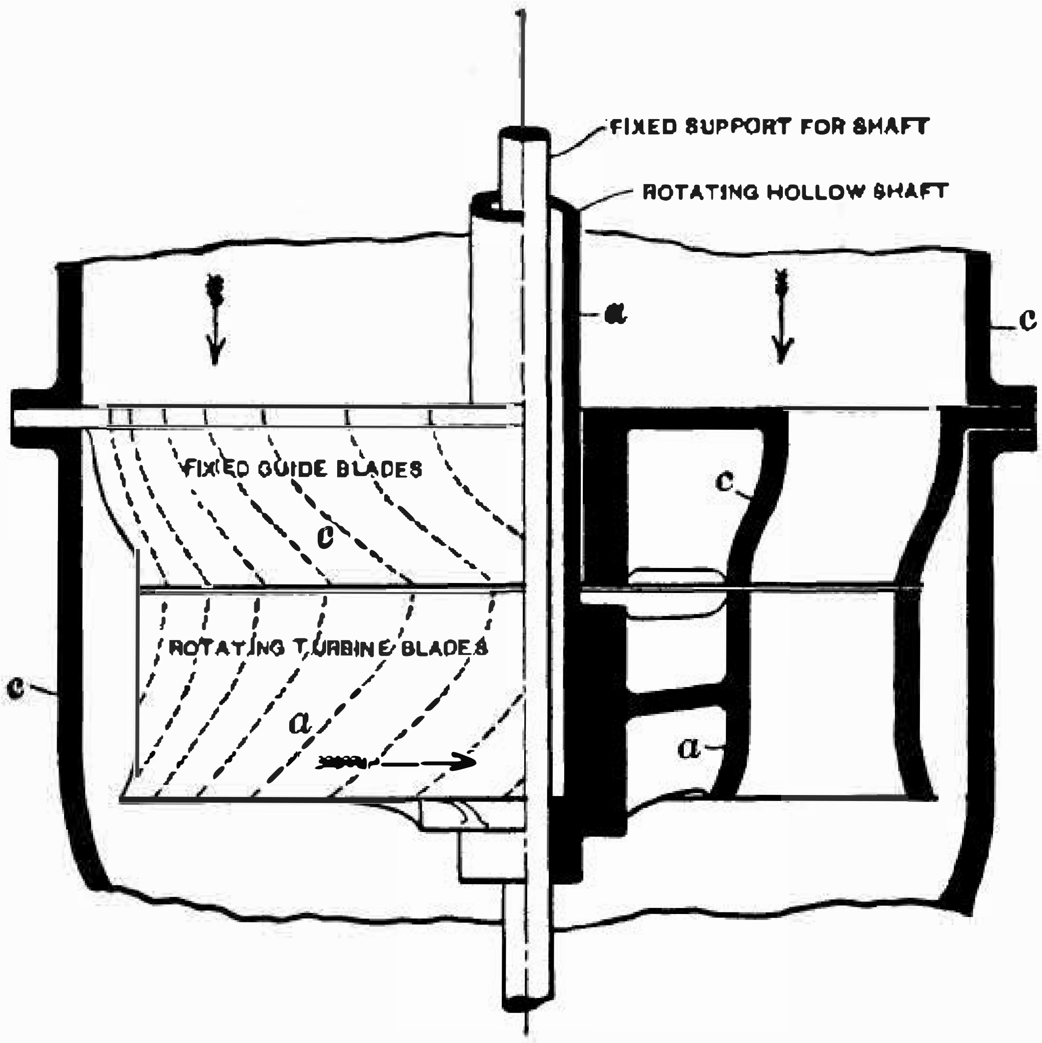

FIG. 193.

are not necessarily true helical surfaces. With solid links we have seen that in order to obtain lower pairing the screw surfaces must have uniform pitch. The adoption of a varying pitch in the rifled gun is only possible because the copper driving-band, which pairs with the rifling, is narrow and so soft as to be deformed with comparative ease. When we consider the pairing of fluid links with such surfaces, however, the mobility of the fluid permits of great latitude in the form of the curved surface over which it flows.

We have so far considered only screw-threads traced upon a cylinder, but there is no reason why such threads should not be formed on a conical surface, or indeed upon many surfaces of revolution. Fig. 194 shows a thread cut upon a globoid, for instance, Let us now imagine a screw-thread traced upon a conical surface, as is the case in some forms of self-centring chuck,* or in the breech-blocks of certain quick-firing guns.† From such a screw-thread it is

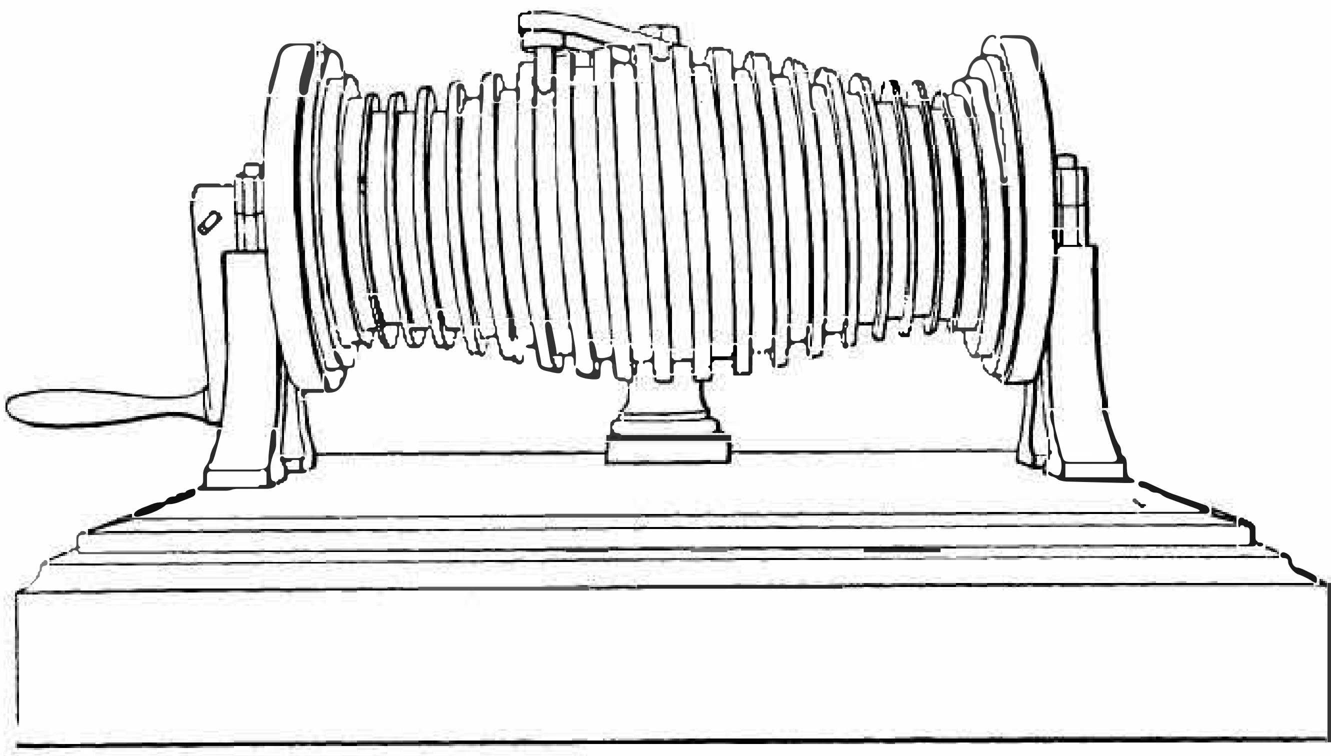

FIG. 194.

but a step to the formation of such a surface as that of the vane of the wheel of a centrifugal pump (Fig. 176) or the vane of a radial-flow turbine, where the blades form what may be termed a screw surface projected on a plane. The kinematic chain of Fig. 176 is then really a modification of that shown in Fig. 193, the guide-blades being suppressed, and the whole forming a pump instead of a motor. The curves of the blades in a centrifugal pump are formed in such a fashion that their rotation impels the fluid from the centre to the outside of the pump-casing. They are thus spiral in form, or may even take the shape of radial straight lines.

93. **Screw-wheels and Worm-gearing.**—In machine construction screws are employed not only in lower pairing for driving, or being driven by, rigid nuts, but also, in higher pairing, for gearing with rotating toothed wheels. In this case contact between the screw and the link with which it

* "Horton" chuck. † *Engineering*, Vol. LXI. p. 11.

pairs takes place either along a line or at a point. The ordinary worm and worm-wheel is the most familiar example of such gearing. Fig. 195 represents in plan and elevation

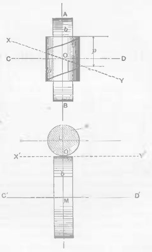

FIG. 195.

two cylindrical wheels, a and b, whose axes AB and CD do not intersect and are at right angles in plan. The wheels are in contact at the point O through which passes $LO'M$, the common perpendicular to AB and CD. The length of this common perpendicular is of course the sum of the radii of the two cylinders. Let a helical line or screw-thread be traced on the surface of a, so as to pass through the common point O, the pitch-angle of this helix being XOC. Also suppose that a second helical line, not shown on the diagram, of pitch-angle XOA, is traced on the surface of b, so as also to pass through the point O. The two helices will then touch at that point, and the line XOY will be their common tangent. If now the helix on b is replaced by a projecting thread, while that on a is converted into a corresponding groove into which the thread gears, any rotation of a about its axis AB will cause the rotation of b about its axis CD, and this relative motion of a and b will be continuous if we provide a series of projecting threads on b so spaced as to come into gear in succession with the thread or groove on a.

It will be noted that, in Fig. 195, a is a single-thread screw, while the wheel b is a portion of a many-threaded screw, the number of threads on b being equal to the number of times that the pitch p is contained in the circumference of b. We can, however, evidently make pairs of screw-wheels in which a as well as b is a portion of a many-threaded screw, and a pair of such wheels is shown in Fig. 196, the teeth or threads being represented by the inclined lines. In speaking of the pitch of the teeth of these wheels, we must distinguish between (1) the helical pitch, or pitch of the screw-thread (p in Fig. 195); (2) the normal pitch, or distance from centre to centre of teeth, measured at right angles to their length (q in Fig. 196); (3) the circumferential pitch (r, Fig. 196); (4) the axial pitch, or distance from centre to centre of teeth measured parallel to the axis of the wheel (s, Fig. 196). A little consideration will show that in a pair of screw-wheels the circumferential pitch of each must be equal to the axial pitch of the other, supposing that, as in the figure, the axes of the wheels are at right angles in plan.

We have now to find the angular velocity ratio of the wheels a and b. It is plain that since the teeth of a and b, while the wheels rotate, remain in continuous contact, their velocity measured along a line drawn perpendicular to their common tangent at the point of contact and lying in the plane which touches both wheels must be equal. In Fig. 197

this common velocity is represented by the line v_c. Now let ω_a and ω_b be the angular velocities of a and b respectively, while r_a and r_b are their radii. Then if v_a, v_b are the actual linear velocities of points on the pitch-circles of a and b, we have

$$v_a = \omega_a r_a \quad \text{and} \quad v_b = \omega_b r_b$$

The lines OL and OM in the figure are supposed to be

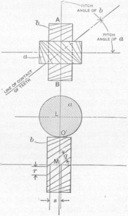

Fig. 196.

drawn in the plane touching both wheels and represent in magnitude and direction the linear velocities of their respective pitch surfaces. ON is drawn in the same plane, and represents v_c, the common velocity of the teeth of both wheels measured in a direction perpendicular to the common tangent of the teeth.

The velocity v_a may be resolved into two components, namely, ON, the velocity of a point on the tooth of a resolved in a direction normal to the line of the tooth, and NL, the velocity resolved along the line of the tooth. Similarly OM

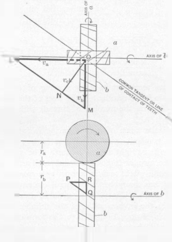

Fig.

may be resolved into two components at right angles, ON and NM; as we have seen, the component normal to the line of contact of the teeth must be the same for both wheels, because it represents the common velocity v_c. The line LM will represent the relative linear velocity (along the tooth) of the pitch surface of the wheel a relatively to that of the wheel b, or the speed with which the teeth slide endways over each other.

Now consider the circumferential and axial pitches of the two wheels. From the figure, by similar triangles

$$\frac{v_a}{v_b} = \frac{OL}{OM} = \frac{PR}{QR} = \frac{\text{axial pitch of } b}{\text{circumferential pitch of } b}$$
$$= \frac{\text{circumferential pitch of } a}{\text{circumferential pitch of } b},$$

and

$$\frac{\omega_a}{\omega_b} = \frac{v_a}{v_b} \cdot \frac{r_b}{r_a} = \frac{\text{circumf. pitch of } a}{\pi r_a} \times \frac{\pi r_b}{\text{circumf. pitch of } b}$$
$$= \frac{\text{number of threads on } b}{\text{number of threads on } a}.$$

It is thus seen that in screw gearing of this kind the velocity ratio is independent of the sizes of the wheels, and depends solely on the number of threads with which they are provided.

A particular form of screw-gearing is frequently employed to transmit motion with a high velocity ratio between shafts at right angles in plan. The smaller wheel has only one, two, or three threads, of small axial but large circumferential pitch, and is known as a *worm*, while the *worm-wheel* has many teeth, of small circumferential but large axial pitch. The velocity ratio is, as we have just seen, simply the inverse ratio of the number of threads. Worm-wheels of good design have the form of their pitch surfaces modified so that their teeth are no longer screw-threads traced on a cylindrical surface, but are formed so as to obtain a larger area of contact between the teeth than would be possible in the case of a cylindrical screw-wheel.* The teeth of a pair of accurately formed cylindrical screw-wheels of rigid material would only touch in a point; in practice there would of course be a very small but perceptible area of contact. Such wheels are therefore most suitable for light loads; and for heavy service, worm-gearing, in which the screw-thread and wheel-tooth may have line

* See § 94.

contact, is preferable. Fig. 198 shows the appearance of screw-gearing and worm-gearing as actually made.

The axes of a pair of screw-wheels may of course make any desired angle in plan with one another. In a given case, when this angle, the sum of the radii of the pitch surfaces, and the velocity ratio have been decided, a number of different pairs of wheels may be designed which will obtain

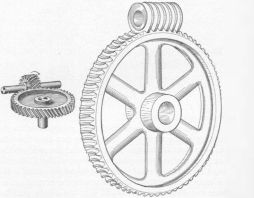

Fig. 198.

the intended result; the difference between them depending on the pitch-angles which are selected. A numerical example will make this clearer. Suppose that screw-gearing is to be designed to connect two shafts making an angle of 60° in plan, the shortest distance between the axes being 10 inches, and the velocity ratio three to one. Two solutions of this problem are shown in Fig. 199. In the first case a pitch-angle of 60° has been chosen for each wheel; in the second case the line of contact of the wheel-teeth is parallel to the axis of b, and the pitch-angles of the wheels a and b are respectively 30° and 90°. The wheel b is thus a spur-wheel. The velocity diagrams showing the relation between v_a, v_b, and v_c are drawn in the two cases. In order to determine the radii of the pitch surfaces of the wheels, we have in the first case

$$v_a = v_b.$$

But $$\frac{r_a}{r_b} = \frac{v_a}{v_b} \cdot \frac{\omega_b}{\omega_a} \quad \text{and} \quad \frac{\omega_b}{\omega_a} = \frac{1}{3}.$$ Hence

$$\frac{r_a}{r_b} = \frac{1}{3}.$$

The distance between the axes being 10 inches, plainly $r_a = 2.5''$ and $r_b = 7.5''$.

In the second case $v_a = 2v_b$; hence, since $\frac{\omega_b}{\omega_a} = \frac{1}{3}$ as before,

$$\frac{r_a}{r_b} = 2 \times \frac{1}{3} = \frac{2}{3},$$

so that $r_a = 4''$ and $r_b = 6''$.

In both cases the number of teeth on the wheels a and b must be in the ratio 1 : 3, but in the first case the wheels have the same circumferential pitch, and the radii are therefore inversely as the angular velocities; while in the second case the circumferential pitches of the wheels a and b are in the ratio 2 : 1, so that the radii are in the proportion 2 : 3. From the diagrams it is evident that the sliding velocity of the teeth will be in the first case

$$v_s = v_a = v_b,$$

and in the second case

$$v_s = v_a \times \frac{\sqrt{3}}{2} = v_b \times \sqrt{3}.$$

It is noteworthy that for the same speeds of the shafts in the two cases v_a will be greater in the second instance than in the first in the proportion of 4:2.5. Hence the sliding velocity of the teeth will be greater in the second design in the proportion $2\sqrt{3}$: 2.5, or nearly 1.39 : 1. This is shown in the figure, where the velocity diagrams, shown in heavy lines, are both drawn to the same scale. In screw-wheels the sliding velocity of the teeth is a minimum when the wheels have the same pitch angle.

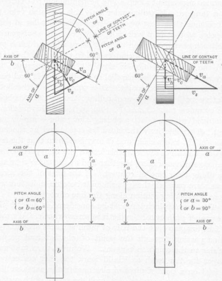

FIG. 199.

94. Forms of Teeth in Screw-gearing and Worm-gearing. —The cylinders shown in the previous figures are of course intended to represent the pitch surfaces of the actual screw-wheels. We have seen that the relative linear motion of these surfaces in the plane which touches them both is composed of a relative sliding motion along the line which is the common tangent to the teeth at the point of contact, and a common movement along a line perpendicular to that common tangent. If we consider only this common velocity, it becomes plain that we have here a case similar in some respects to that of a pair of spur-wheels. In spur-gearing the teeth have a common velocity along a line which is perpendicular to the line of centres and to the line of the teeth, and the relative motion is simply a rolling together, the virtual axis being a line parallel to the wheel axes and passing through the pitch-point. In a pair of screw-wheels the relative motion is also one of rolling; the virtual axis being the common tangent (XOY, Fig. 195), but this motion is combined with a relative sliding along the common tangent. This line may therefore be considered as the *twist axis** of the two wheels, and it may be shown that the *twist axodes* of a pair of screw-wheels form a pair of hyperboloids;† the two surfaces rolling together and at the same time sliding along their line of contact. If the axes of the screw-wheels are parallel, the hyperboloids become cylinders and there is no sliding motion along the line of contact.

The above principles must be considered in determining the proper profiles for the teeth in screw-wheels. We must in fact imagine that the pair of pitch surfaces are cut by a plane passing through the point of contact and perpendicular to the common tangent. We must then take such shapes for the teeth as would work together correctly if formed on a pair of pitch-circles having the same radii of curvature as the sections—in which the actual pitch surfaces are cut by our imaginary plane—have at their point of contact.

Evidently the traces of the cylindrical pitch surfaces of the screw-wheels on the imaginary plane will be a pair

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* See § 91. † See § 95.

of ellipses, always touching at the ends of their minor axes. The plane will make with the median plane of each wheel an angle $(90° - \alpha)$, where α is the pitch angle of the helical teeth or screw-threads, and the semi-minor axis of the ellipse will be r; the semi-major axis being $\frac{r}{\sin \alpha}$, where r is the radius of the pitch surface of the wheel. In Fig. 200 is shown the pitch surface of a screw wheel (radius r) having traced upon its surface a helix of pitch angle XOC. As in Fig. 195, the line XOY is a tangent to the helix at the point O, and if O were the point of contact of the wheel with another, XOY would represent the common tangent or line of contact of the teeth.

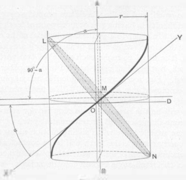

Fig. 200.

A plane passing through O and perpendicular to XOY will cut the cylinder in the ellipse $LMNO$, of which the major and minor axes will be evidently $\frac{2r}{\sin \alpha}$ and $2r$ respectively. In order to find the proper form of tooth profile (taken of course in the plane of $LMNO$ and at right angles to the centre line of the tooth), we must take as an imaginary pitch-circle the circle of curvature of the ellipse at the point O. The radius of curvature of the ellipse at this point is easily shown to be $\frac{r}{\sin^2 \alpha}$; hence in a pair of screw-wheels the teeth profiles should be designed as if they belonged to a pair of ordinary spur-wheels of radii

$$\frac{r_a}{\sin^2 \alpha_1} \quad \text{and} \quad \frac{r_b}{\sin^2 \alpha_2},$$

where α_1 and α_2 are the pitch-angles of the teeth of the screw-wheels a and b.

Screw-wheels are often used to connect shafts which are parallel. In this case the pitch-angles of the wheels are of course equal, and if the wheels are properly designed there is no difficulty in having two or more pairs of teeth in contact at once. In order to avoid the prejudicial effect of the end thrust developed when a pair of such wheels are doing heavy work, it is usual to make each wheel of two parts, similar in pitch, but one half right-handed and the other half left-handed. Fig. 201 shows such a double helical pinion of the form employed in a rolling-mill. The teeth profiles in such wheels, taken on a section by a plane perpendicular to the axes, will be the same as those required for spur-wheels of the same diameter and circumferential pitch of teeth; for in these wheels there is no sliding motion along the teeth. If properly shaped, the teeth will be in contact along short lines inclined more or less towards the pitch surfaces, according to the pitch-angle chosen for the helices. When this pitch-angle is 90° the wheels become spur-wheels, in which the lines of contact of the teeth are of course parallel to the pitch surfaces.

When worm-gearing is constructed with cylindrical pitch surfaces, and teeth of uniform cross-section, contact, as in the case of other screw-wheels, occurs at a point only, and the forms of teeth may be found by the method just described. It is not difficult, however, by modifying the form of the wheel, to obtain worm-gearing having linear contact. The method of doing this is fully explained in works on Machine Design.*

The section of the worm and wheel by a plane perpendicular to the axis of the wheel, and passing through the axis of the worm, is that of a rack in gear with a spur-wheel, and the form of the worm-thread and wheel-teeth in this plane may be drawn by the methods already discussed in §§ 66 and 67. The trace of such a plane is shown in Fig. 202

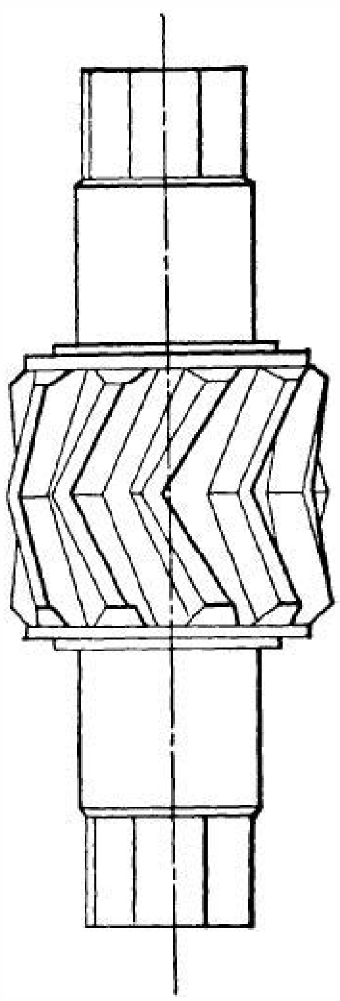

FIG. 201.

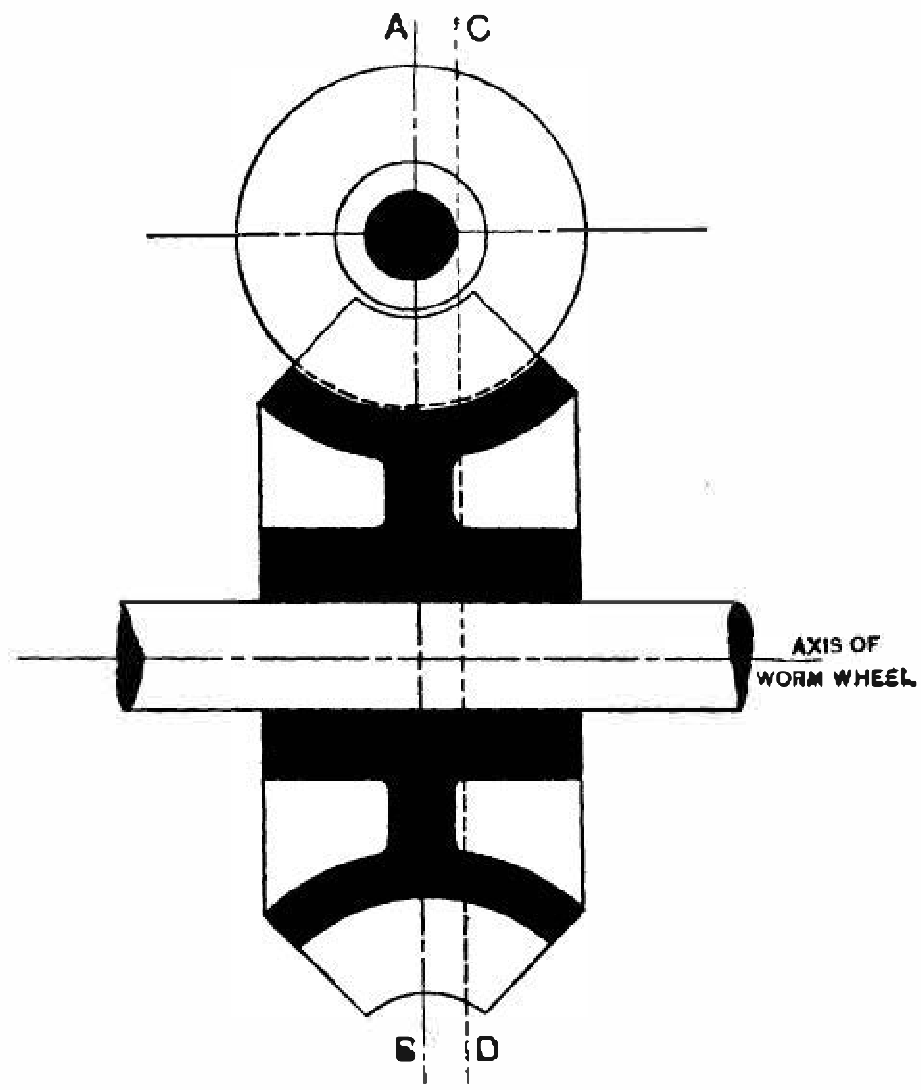

FIG. 202.

by the line AB; the figure represents the section of the worm and wheel by a plane containing the axis of the wheel and perpendicular to the axis of the worm. The form of a section of the worm by a plane parallel to the axis of the worm, and perpendicular to the axis of the wheel, is next to be determined; the trace of such a plane on the plane of the

* Unwin, "Machine Design," Vol. I, § 234.

figure is the line CD, and its intersection with the worm-thread will take the shape of a rack having curved and unsymmetrical teeth. The form of worm-wheel tooth required to gear correctly with such teeth must then be found by the proper method of construction, and the shape determined is to be used for the section of the worm-wheel tooth cut by the plane CD. A number of such sections, found for planes at different distances from the median plane AB, will enable a practically correct wheel-pattern to be made. As a rule such wheels are machine cut by being rotated in correct relation to a steel cutter or hob which is a duplicate of the worm to be used.

The Hindley worm* has a screw-thread of varying section traced on a non-cylindrical pitch surface whose outline is an arc of the pitch-circle of the wheel. This form of tooth,

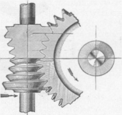

FIG. 103.

if correctly cut by means of a hob, has line contact: the teeth touching the thread at all points in the median plane.

95. Hyperboloidal Wheels. — It is possible to construct wheels which will transmit motion between inclined non-intersecting axes, and which are so formed that their teeth are straight and have line contact. The pitch surfaces of

* American Machinist, March 25, 1897.

such wheels are hyperboloids of revolution, as has already been stated. In Fig. 204 let AB and CD be the axes of a pair of such wheels; the line of contact of their teeth is to be the line XOY, passing through a point O on LOM the common perpendicular to AB and CD. In general XOY will be parallel neither to AB nor to CD.

If now we imagine the line XOY to be rotated around AB as an axis, while its position in relation to AB remains unaltered, XY will describe in space the hyperboloid a; and similarly, if we suppose the rotation to take place about CD, the hyperboloid b will be described. The two hyperboloids will of course touch along the line XY, which is in fact their twist axis when relative motion occurs. The smallest circular sections of the hyperboloids are known as the gorge-circles. We proceed to determine the relative angular velocity of a pair of such hyperboloidal surfaces, supposing that they roll together. It is to be particularly noted that hyperboloidal wheels differ from the cylindrical screw-wheels hitherto discussed, in that the pitch surfaces of the latter can touch only at a point, while those of the former are in contact along a line. The relative motions of the two kinds of wheels are, however, of the same kind, namely, a rolling together, combined with relative sliding along the line of the teeth. Let θ_1, θ_2 be the angles (in plan) made by the projection of the line of contact XY with the projection of the axis of a and the projection of the axis of b respectively. The angular velocity-ratio of the wheels a and b must evidently be the same as that for a pair of screw-wheels of the same size as the gorge-circles of the hyperboloids and having the same obliquity of teeth. The velocity diagram will therefore be that drawn in thick lines, by the method of § 93, and we shall have

$$\frac{\omega_a}{\omega_b} = \frac{v_a}{r_a} \cdot \frac{r_b}{v_b} = \frac{r_b \cdot v_c}{\cos\theta_1} \cdot \frac{\cos\theta_2}{r_a v_c} = \frac{r_b \cos\theta_2}{r_a \cos\theta_1}.$$

Now consider any point Y on the line of contact XOY. The normal to the curved surface of the hyperboloid at any point must pass through the axis; hence a straight line drawn through Y and normal to the curved surfaces which

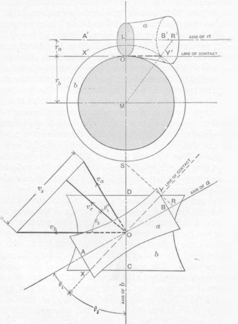

FIG. [illegible]

touch there must intersect both CD and AB. Such a common normal is shown in the figure, where RYS is its projection on a plane parallel to both axes AB and CD, and $R'Y'M$ is its projection on a plane perpendicular to the axis of b. In this view of course the points D, M, C, and S coincide.

By a well-known principle of projection the real lengths of the segments of the common normal are proportional to their projections RY, YS, and $R'Y'$, $Y'M$. Hence

$$\frac{r_a}{r_b} = \frac{LO}{OM} = \frac{R'Y'}{Y'M} = \frac{RY}{YS}$$

and

$$\frac{\omega_a}{\omega_b} = \frac{YS \cos \theta_2}{RY \cos \theta_1}.$$

Again, since the common normal is perpendicular to XOY, the line of contact, its projection RYS will be perpendicular to the projection of XOY on a parallel plane; so that in the lower figure OYS and OYR are right angles. Thus, finally,

$$\frac{\omega_a}{\omega_b} = \frac{DY}{BY}.$$

This shows that the angular velocities of a pair of hyperboloidal wheels are to each other in the inverse ratio of the lengths of the projections of the perpendiculars drawn from any point on the line of contact to the axes; these projections being upon a plane parallel to both axes and to the line of contact.

It should be noted, that in designing hyperboloidal wheels, if the angle between the axes (in plan) and the velocity ratio are given, the position of the line of contact (in plan) is determined. Thus in drawing such a pair of wheels we proceed as follows:

(1) Draw the axes in elevation and in plan.

(2) The velocity ratio being given, draw the line of contact XOY (in plan), determining the point Y by marking off DY and BY having lengths in the proper ratio.

(3) Draw SYR perpendicular to XOY, and also draw $MY'R'$, the projection of SYR on a plane perpendicular to the axis of b.

(4) Through Y' draw $X'O'Y'$; this determines the values of r_a and r_b and settles the sizes of the gorge-circles.

(5) Proceed to complete the projections of the hyperboloids, as shown.

As in the case of screw-wheels, the velocity diagram shows the rate at which the teeth of a and b slide along each other. This relative sliding velocity is shown as v_s in Fig. 204.

It is not necessary in practice to use more than a comparatively small portion of the hyperboloid for a working wheel. Fig. 205 shows a pair of hyperboloidal rollers, and a pair of skew-bevel wheels having the same velocity ratio. It will be seen that these bevel wheels correspond in fact to the end portions of the hyperboloids. The forms of the teeth of hyperboloidal wheels may be conceived as being marked out upon the cones whose surfaces are normal to those of the hyperboloids at the point of contact considered. Methods of doing this have been discussed by Willis,* Rankine,† and others. Here it will be sufficient to note that the teeth of such wheels will not be of uniform section throughout their length. In the comparatively narrow hyperboloidal wheels generally used there is but little variation in the form of the tooth in passing from one end of the wheel to the other. An approximately correct form of tooth may be determined for such wheels in the same way as for screw-wheels.

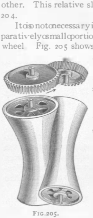

Fig. 205.

In Fig. 204, for example, we may imagine the two hyperboloids cut by a plane parallel to RYS and perpendicular

* Principles of Mechanism, p. 131. † Machinery and Millwork, p. 146.

to XOY. When the resulting sections are drawn out their circles of curvature may be approximately found, and the tooth-forms designed in the ordinary manner, remembering that the circumferential pitch, and therefore also the normal pitch, increases as we pass from the gorge to the ends of the wheels. It will be seen that this method is practically the same as that adopted in the case of ordinary bevel-wheels (see § 98), and is equivalent to drawing out the teeth on the development of the cones previously mentioned.

The subject of hyperboloidal wheels is treated at considerable length in MacCord's "Mechanical Movements," to which work the reader is referred for further information.