

CHAPTER VIII.

WHEEL-TRAINS AND MECHANISMS CONTAINING THEM. CAMS.

68. Simple and Compound Wheel-trains.—The determination of the velocity ratio in such a wheel-train as that of Fig. 130 involves no difficulty, for it is plain that one or

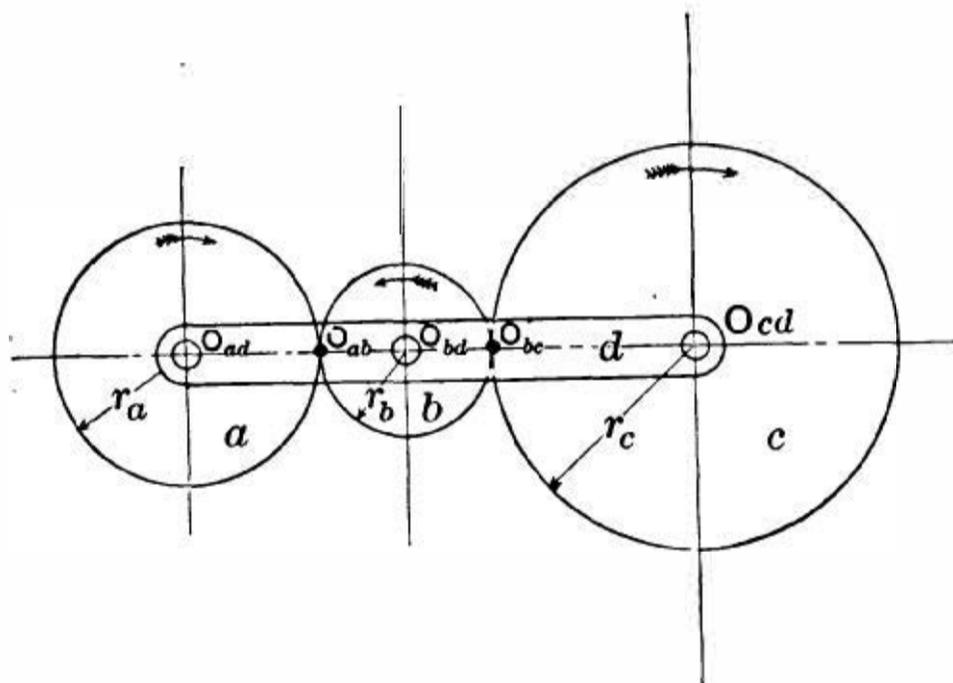


FIG. 130.

more intermediate wheels (as *b*) will not affect the numerical value of the velocity ratio of the first and last wheels. The linear velocity of the pitch-line of every wheel is the same, and the angular velocity ratio of the first and last, there-

fore, only depends on their own diameters, so that $\frac{\omega_{ad}}{\omega_{cd}} = \pm \frac{r_c}{r_a}$,

the sign depending on the number of idle wheels. Intermediate or *idle* wheels thus simply reverse the direction of motion. When all the wheels in the train have external contact, the angular velocity ratio of the first wheel to the last has a positive value (or, both wheels turn in the same

sense) if the number of axes is odd, while an even number of axes gives the velocity ratio a negative value. More complex wheel-trains, however, require further consideration. In Fig. 131 we have a compound spur-wheel mechanism of

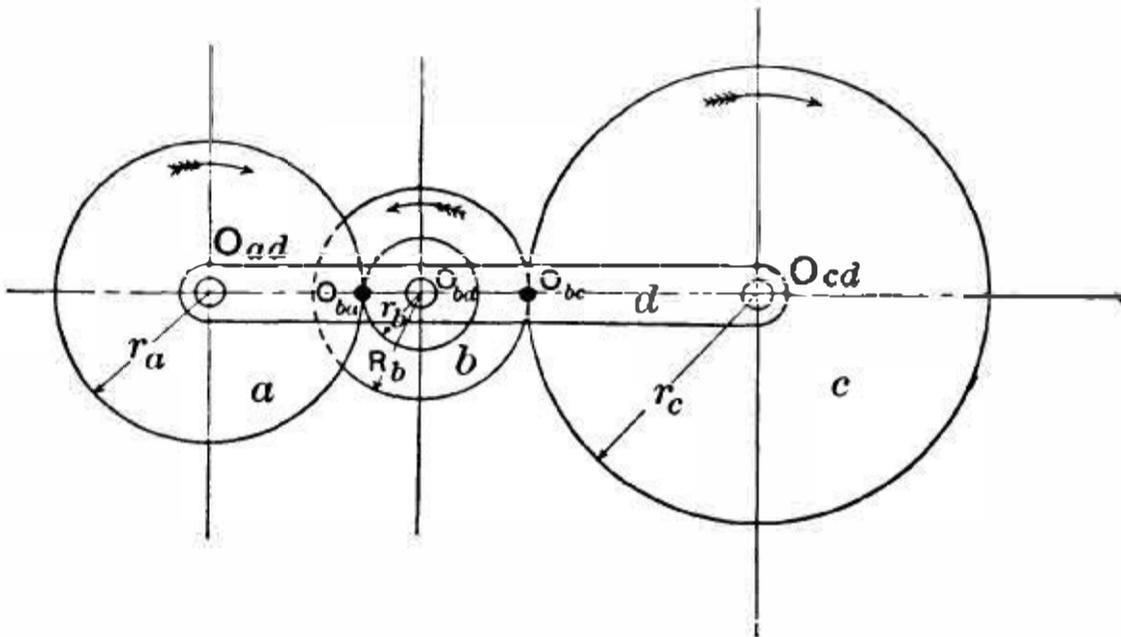


FIG. 131.

four links, d being fixed, while b consists of two wheels rigidly connected and turning on the same axis.

Let r_a , r_b , R_b , r_c , be the radii of the pitch-circles, then from § 64 we have

$$\frac{\omega_{ad}}{\omega_{bd}} = \frac{r_b}{r_a}.$$

Also,

$$\frac{\omega_{bd}}{\omega_{cd}} = \frac{r_c}{R_b}.$$

Hence

$$\frac{\omega_{ad}}{\omega_{cd}} = \frac{r_b \times r_c}{r_a \times R_b} = N.$$

Suppose a to be the driving-wheel, while c is the driven one; we see that the above result may be expressed by saying that

$$\begin{aligned} \text{velocity ratio} &= \frac{\text{revolutions of driving-wheel}}{\text{revolutions of driven wheel}} \\ &= \frac{\text{product of radii of followers}}{\text{product of radii of drivers}}. \end{aligned}$$

Instead of radii we might evidently put numbers of teeth.

It would be easy to find a single pair of wheels having the same velocity ratio as the given train. For example, if we had a pair of wheels, A and C , such that

$$r_A - r_C = r_a + r_b + R_b + r_c \quad \text{and} \quad \frac{r_A}{r_C} = \frac{r_a \times R_b}{r_b \times r_c},$$

these would have the same velocity ratio and the same distance from centre to centre. The point of contact of their pitch-circles would divide the distance $O_{ad} O_{cd}$ externally in the proportion of the angular velocities of a and c , and would in fact be the point O_{ac} .* Hence in Fig. 131 we have only to divide the line of centres, graphically or otherwise, in the proper ratio to find the sixth virtual centre.

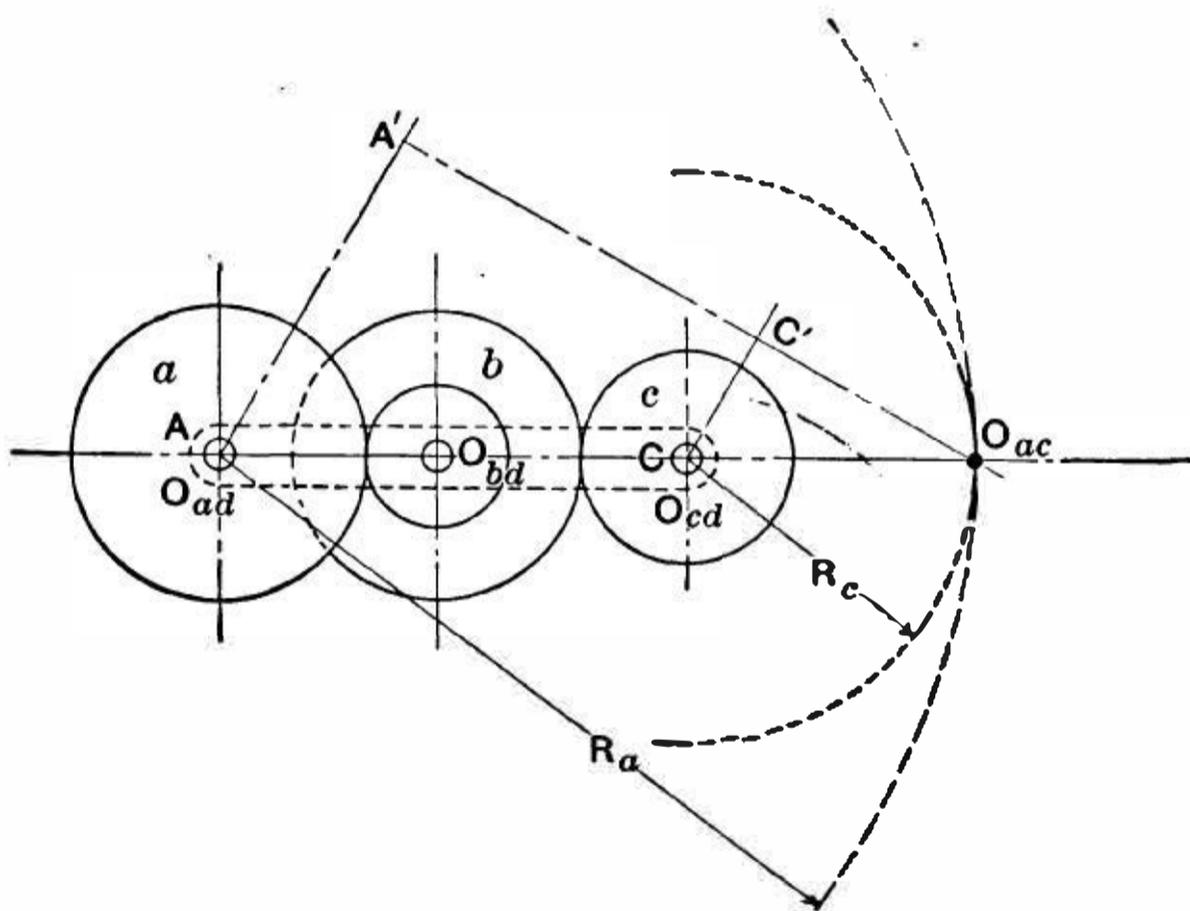


FIG. 132.

In doing this (as in working all problems connected with wheel trains), note must be taken of the sign of the velocity ratio, which depends on the presence or absence of *annular wheels* (i.e., wheels having internal contact), or of idle wheels, and also on the number of axes in the train. Take, for example, the two trains shown in Figs. 132 and 133, in

* For graphic methods of determining virtual centres of wheel-trains, see Kennedy, *Mech. of Machinery*, note to Chapter VI.

the first of which suppose $r_a = 2$, $r_b = 1$, $R_b = 2$, $r_c = 1.5$, so that the velocity ratio in Fig. 132 has the value

$$N = \frac{\omega_{ad}}{\omega_{cd}} = + \frac{1.5 \times 1}{2 \times 2} = + \frac{3}{8}.$$

In Fig. 133 we have a simple train having exactly the same numerical value for its velocity ratio (since $\frac{r_c}{r_a} = -\frac{3}{8}$), but in this case the negative value must be adopted, since the wheels a and c turn in opposite senses. In Fig. 132 O_{ac} may be found by drawing $AA'CC'$ parallel to one another, and of lengths 8 and 3 respectively, to any convenient scale. The intersection of $A'C'$ with the line of centres fixes O_{ac} . Evidently the given train might be replaced by a pair of wheels of radii R_a and R_c , the larger being annular, having their centres at A and C , and their pitch-circles touching at O_{ac} , as shown by the dotted arcs. Again, in Fig. 133

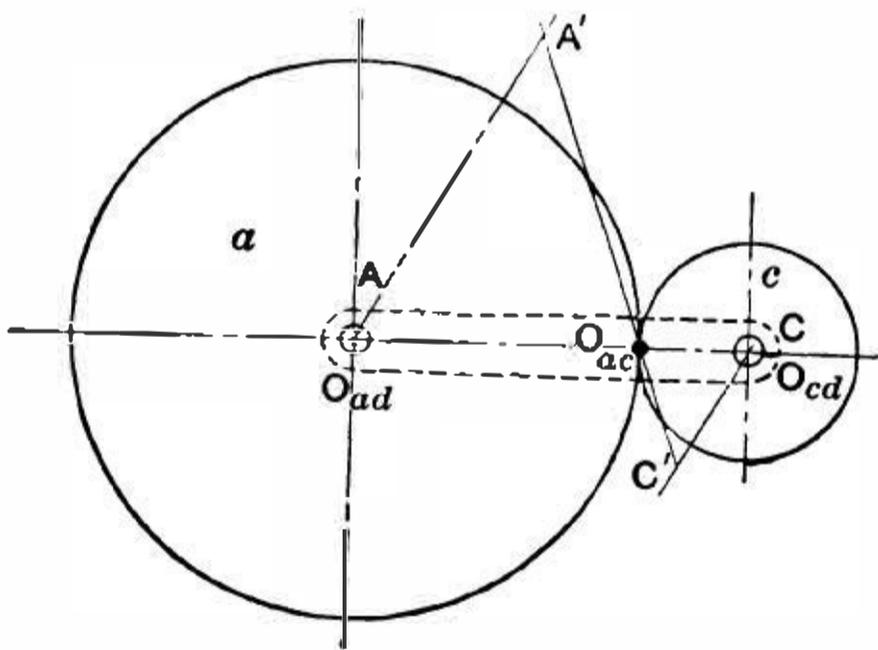


FIG. 133.

$AA'CC'$ must be drawn parallel, but in opposite senses, so as to allow for the negative velocity ratio, and O_{ac} is, of course, the point of intersection of AC and $A'C'$.

It is by no means necessary to have the centres of all the wheels of a train in one straight line. The back-gear of a lathe, for example, is an instance of a compound *reverted train* in which the centres of the first and last wheels coincide.

This arrangement makes no difference in the numerical value of the velocity ratio, and is simply adopted for convenience in construction.

• **69. Epicyclic Gearing.**—In the above examples of wheel-trains we have supposed the frame carrying the wheels to be the fixed link. Wheel gearing is often employed in which one of the wheels is the fixed link and the frame or arm carrying the remaining wheels is movable. Such gearing is called *epicyclic*, and we proceed to discuss some of its simpler cases.

We take first the mechanism of Fig. 133, but suppose a to be fixed, while d is rotated in a clockwise or positive sense (Fig. 134). Let N be the velocity ratio of the train, i.e., let

$$N = - \frac{r_a}{r_c} = \frac{\omega_{cd}}{\omega_{ad}}$$

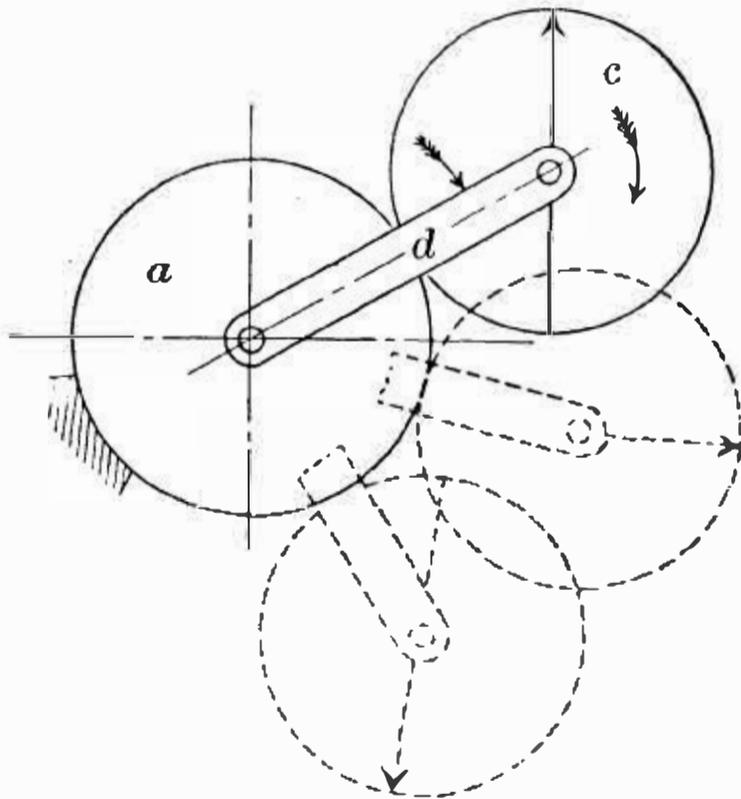


FIG. 134.

Plainly, if we consider ω_{da} as being positive in sign, then ω_{ad} must be negative, hence

$$\omega_{ad} = - \omega_{da}$$

Now in any case where two bodies, c and d , have motion relatively to a third, a , which is fixed, any angular movement of c relatively to a may be looked on as the algebraic

sum of the motions of c relatively to d and of d relatively to a . Thus

$$\begin{aligned}\omega_{ca} &= \omega_{cd} + \omega_{da} \\ &= +\omega_{cd} - \omega_{ad} \\ &= -\omega_{ad} \left(1 + \frac{r_a}{r_c} \right),\end{aligned}$$

or

$$\frac{\omega_{ca}}{\omega_{da}} = 1 - N,$$

where N is itself a negative quantity. A numerical example may, perhaps, make this clearer. Suppose the wheels a and c to have 100 teeth and 90 teeth respectively; these teeth have the same pitch, and we can, of course, take the ratio of the numbers of teeth instead of the ratio of the diameters or radii of the pitch-circles. Thus $\frac{r_a}{r_c} = \frac{10}{9}$; in other words, supposing d to be fixed, while a makes one revolution with

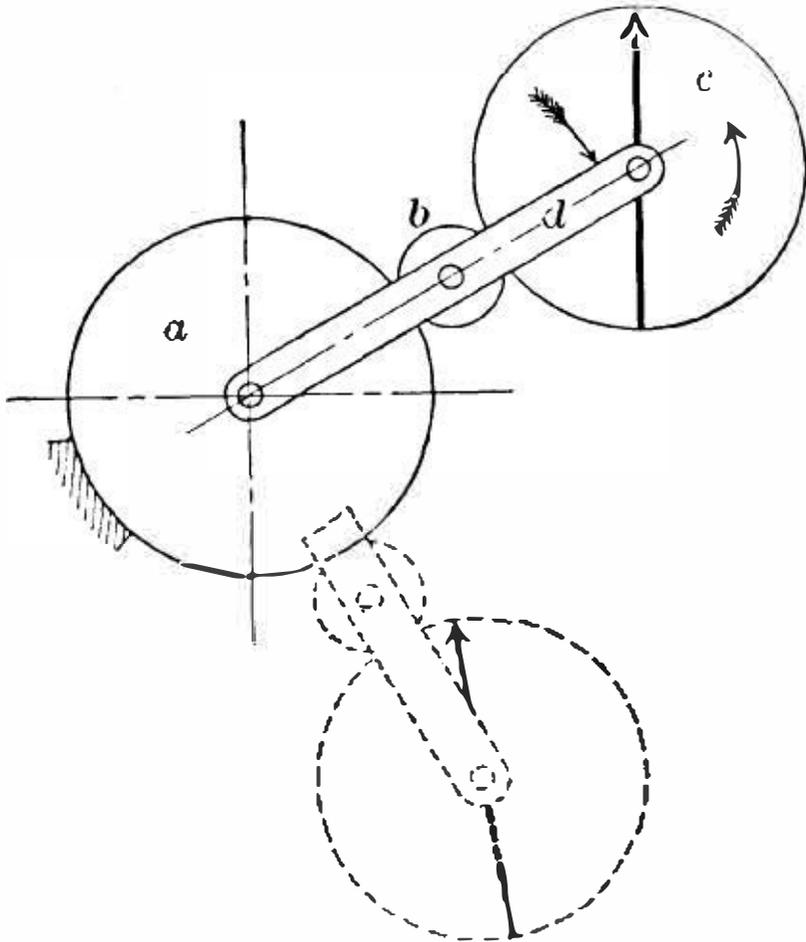


FIG. 135.

regard to d , c would make $1\frac{1}{9}$ in the opposite sense. Now suppose that in a certain time a makes -1 revolution, c making $+1\frac{1}{9}$, while d is at rest. Cause the whole mechanism to execute $+1$ rotation in the same time around O_{ad} ;

this brings a to rest, makes d perform $+1$ revolution, and therefore gives c $1\frac{1}{9} + 1 = 2\frac{1}{9}$ revolutions in the same sense as that of the arm.

If an idle wheel, b , had been interposed between a and c , as in Fig. 135, we should have had $N = +\frac{r_a}{r_c} = \frac{\omega_{cd}}{\omega_{ca}}$ a positive quantity, and $\omega_{ca} = \omega_{da} + \omega_{cd}$, whence $\frac{\omega_{ca}}{\omega_{da}} = 1 + \frac{\omega_{cd}}{\omega_{da}} = 1 - N$, as before. With the numbers of teeth, as in the example just given, and the train arranged as in Fig. 135, we should have, if $N = +1\frac{1}{9}$,

$$\frac{\omega_{ca}}{\omega_{da}} = 1 - 1\frac{1}{9} = -\frac{1}{9};$$

i.e., for each revolution of the arm, c makes $\frac{1}{9}$ revolution in the reverse sense.

Fig. 136 represents a compound epicyclic reverted train. Let $n_a, n_{b_1}, n_{b_2}, n_c$ be the numbers of teeth in the wheels

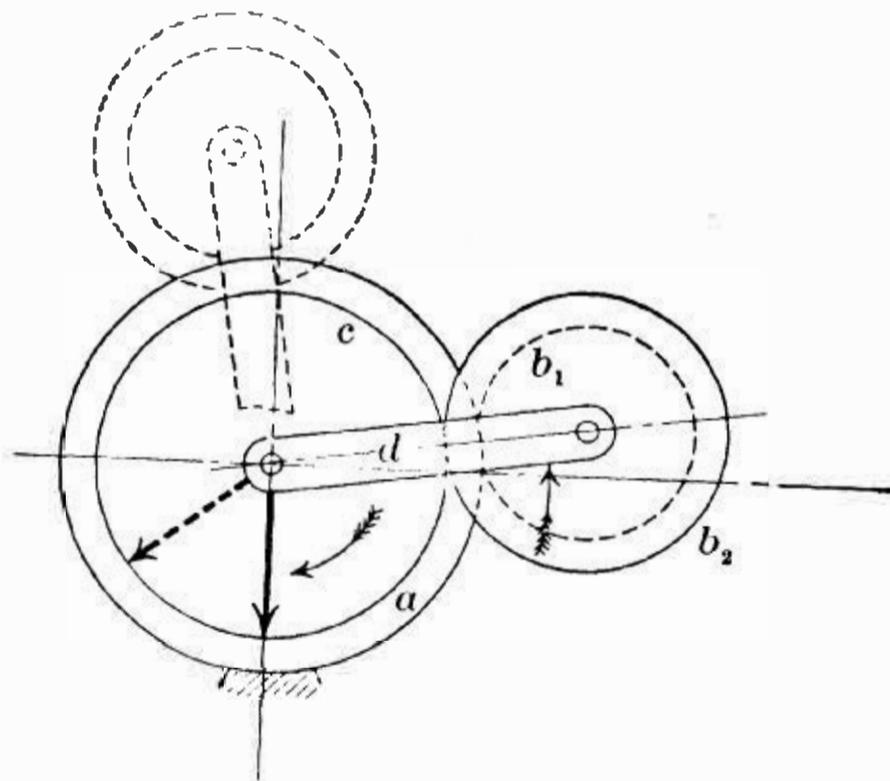


FIG. 136.

$a, b_1, b_2,$ and c respectively. Evidently, if the pitch of both pairs is the same, $n_a + n_{b_1} = n_{b_2} + n_c$. The velocity ratio of the train will be positive and has the value

$$N = \frac{\omega_{cd}}{\omega_{ad}} = \frac{n_a \times n_{b_2}}{n_{b_1} \times n_c}$$

hence $\omega_{cd} = -(N \times \omega_{da})$.

Further, when a is the fixed link

$$\omega_{ca} = \omega_{cd} + \omega_{da} = \omega_{da}(1 - N).$$

Thus, for instance, suppose a has 30 teeth, b_1 has 15, and b_2 and c have respectively 20 and 25; then

$$N = + \frac{30 \times 20}{15 \times 25} = + 1.6 = \frac{\omega_{cd}}{\omega_{da}},$$

and $\frac{\omega_{ca}}{\omega_{da}} = 1 - N = -0.6$.

Thus c will make 0.6 revolution for each revolution of the arm, but in the opposite sense. Such a train might evi-

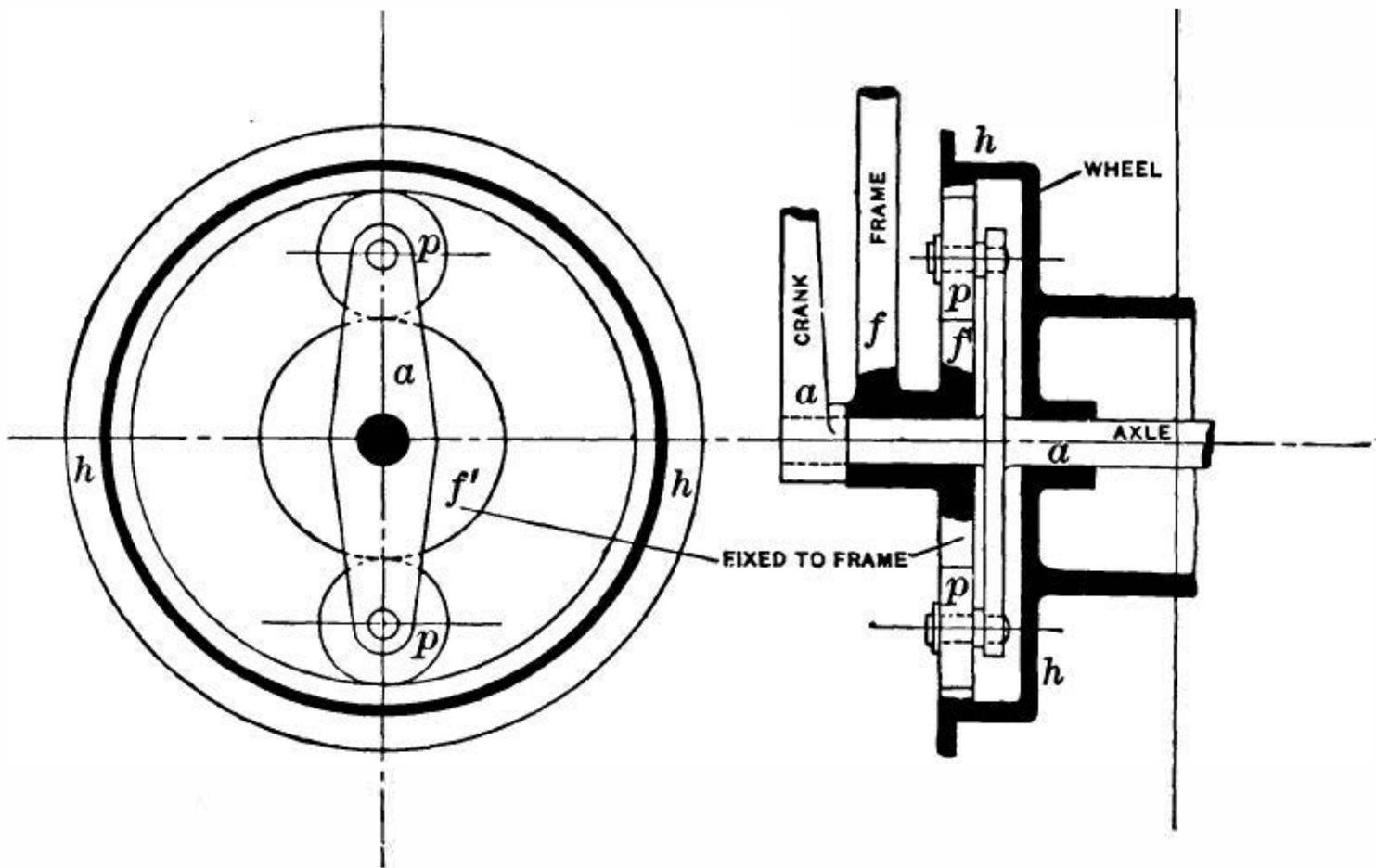


FIG. 137.

dently be arranged to give c a very slow rotary motion, say

$\frac{1}{10,000}$ revolution for each revolution of the arm d .

As an example of an epicyclic reverted gear containing an annular wheel the wheel-train used in certain front-driving bicycles* may be given. In Fig. 137 ff represents

* See also the Weston triplex pulley-block described in § 78, Chapter IX.

part of the front fork of the bicycle, to which is rigidly attached the central pinion f' . The arms a , secured to the axle and cranks, carry one or more planet-wheels, p , gearing with the central pinion and with an annular wheel formed on the inside of the hub, h , of the driving-wheel. Suppose this wheel h has 60 teeth, while p has 15 and f' has 30; it is plain that $n_h = n_f + 2n_p$ if the wheels are to gear together and the wheel h is to be coaxial with f' .

Now if a were the fixed link,

$$\frac{\omega_{ha}}{\omega_{fa}} = -\frac{30}{60} = -\frac{1}{2};$$

therefore

$$\omega_{ha} = -\frac{1}{2} \times \omega_{fa} = \frac{1}{2} \omega_{af}.$$

The ratio to be determined is the number of revolutions of the wheel h per revolution of the crank a ; this is the same quantity as

$$\frac{\omega_{hf}}{\omega_{af}} = \text{velocity ratio of } h \text{ and } a.$$

Now

$$\begin{aligned} \omega_{hf} &= \omega_{ha} + \omega_{af} \\ &= \left(\frac{1}{2} + 1\right) \omega_{af}. \end{aligned}$$

Hence

$$\frac{\omega_{hf}}{\omega_{af}} = +1.5;$$

in other words, the wheel will make $1\frac{1}{2}$ revolutions for each revolution of the crank, and in the same sense. A bicycle having a driving-wheel 44 inches diameter would therefore be geared to 66 inches with this arrangement.

70. Mechanisms Containing Wheel-trains.—Mechanisms are of common occurrence in which wheel trains form part of chains containing also sliding and turning pairs. Fig. 138 shows diagrammatically a "sun-and-planet" gear containing an annular wheel, forming part of a mechanism containing a slider-crank chain.

The crank a is able to rotate about the point O_{ad} with reference to a fixed frame d , and pairs with a link b , forming the connecting-rod in a slider-crank chain, of which c is the

sliding block. On b , however, is formed a spur-wheel whose pitch-circle has its centre at O_{ab} . The spur-wheel gears with an annular wheel e whose pitch-circle has its centre at O_{ad} . The virtual centres are marked on the dia-

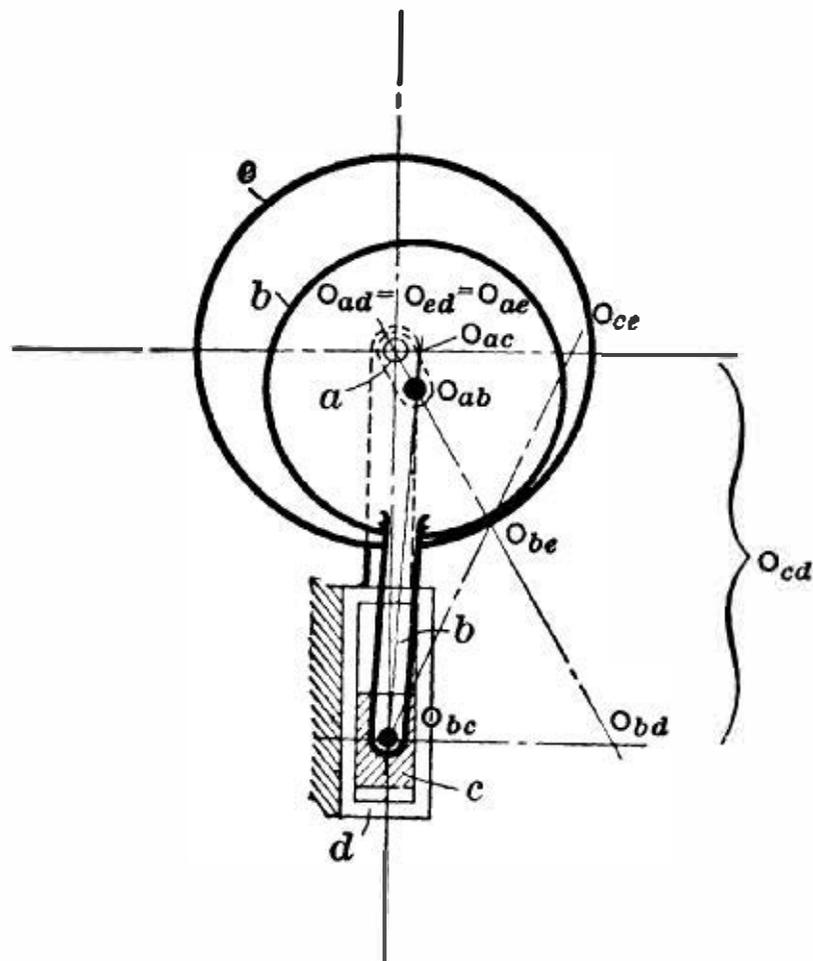


FIG. 138.

gram. We wish to find the number of revolutions of the annular wheel e for each revolution of the crank a .

As in previous examples, let N be the velocity ratio of the wheel-train; i.e., let

$$N = \frac{\omega_{ea}}{\omega_{ba}} = \frac{n_b}{n_e} = \frac{r_b}{r_e}.$$

Plainly N will be a positive fraction in this case. We note that during the action of the mechanism the average value of ω_{bd} is zero, for b simply swings to and fro, and a line marked on it describes equal angles right and left from its mid-position. Hence we may say that on the average

$$\omega_{ab} = \omega_{ad} + \omega_{db} = \omega_{ad};$$

that is, we may consider the angular velocity of a and b instead of that of a and d . The two would always be exactly

equal if b always remained parallel to itself, i.e., if the connecting-rod were infinitely long.

Now

$$\begin{aligned}\omega_{ed} &= \omega_{ad} + \omega_{ea} \\ \frac{\omega_{ed}}{\omega_{ad}} &= 1 + \frac{\omega_{ea}}{\omega_{ad}} \\ &= 1 - \frac{\omega_{ea}}{\omega_{da}} \\ &= 1 - \frac{\omega_{ea}}{\omega_{ba}} \\ &= 1 - N.\end{aligned}$$

For example, suppose e had 100 teeth while b had 95, so that $N = +0.95$; then for each revolution of the crank a , e would make $1 - 0.95 = 0.05$ revolution in the same sense. This mechanism is actually used as gearing for a capstan driven by a hydraulic engine, b being attached to the connecting-rod, while the capstan barrel is attached to e .

As another example of a mechanism containing a wheel-train we may take the wheel crank-chain of Fig. 139, which

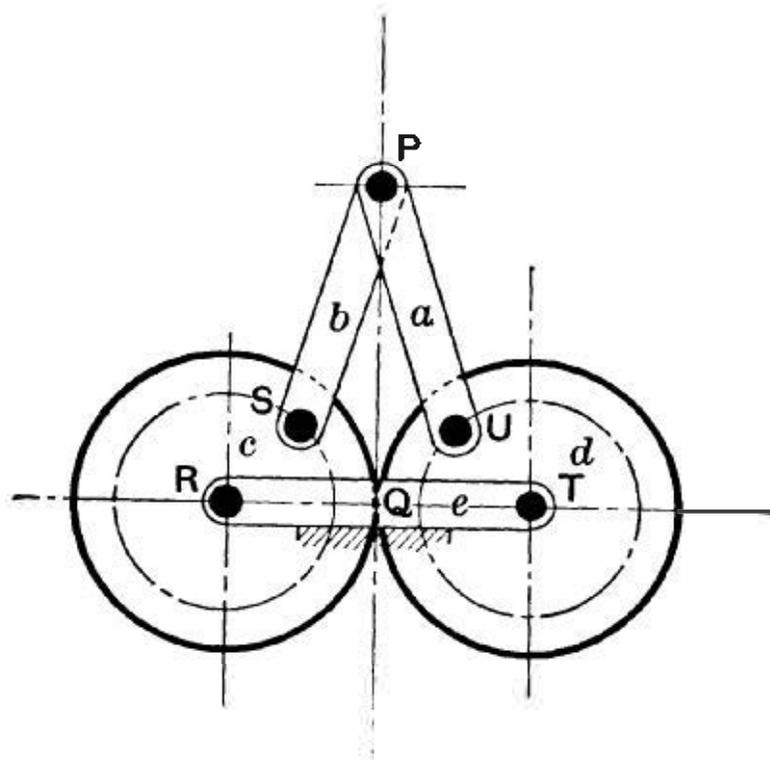


FIG. 139.

is formed by combining a simple wheel chain with an open crank-chain of five links.

If the lengths of the links a and b , and also those of c

and d , are equal, as in the figure, we obtain Cartwright's straight-line motion, in which the point P describes a straight-line path passing through Q . The purpose of the two spur-wheels is to close the five-link chain, whose motion would otherwise be unconstrained.

In Figs. 140a and 140b we have a slider-crank chain in which spur-wheels can be used for a somewhat similar purpose. Consider a slider-crank chain in which the connect-

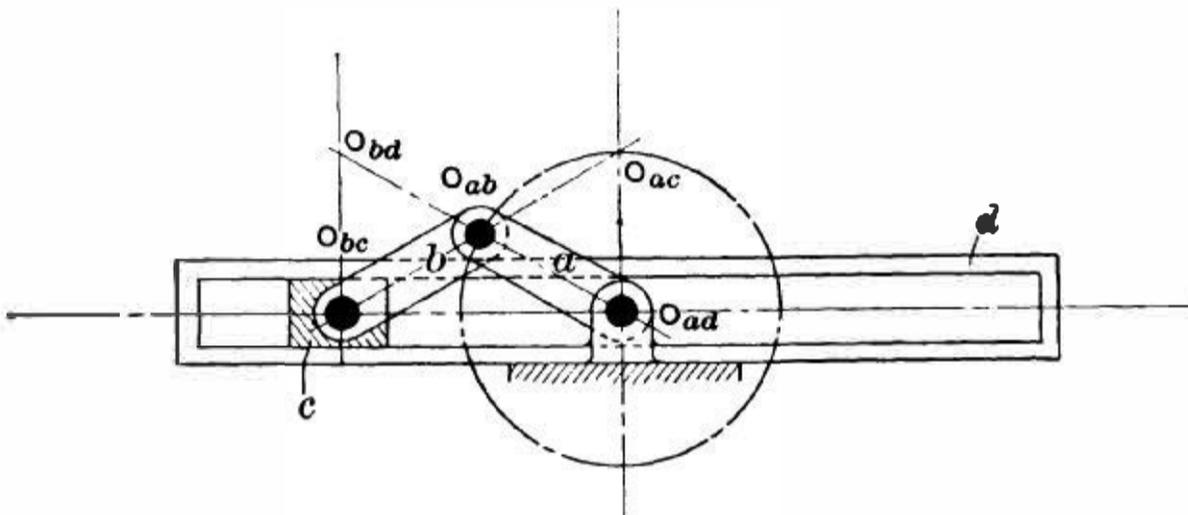


FIG. 140a.

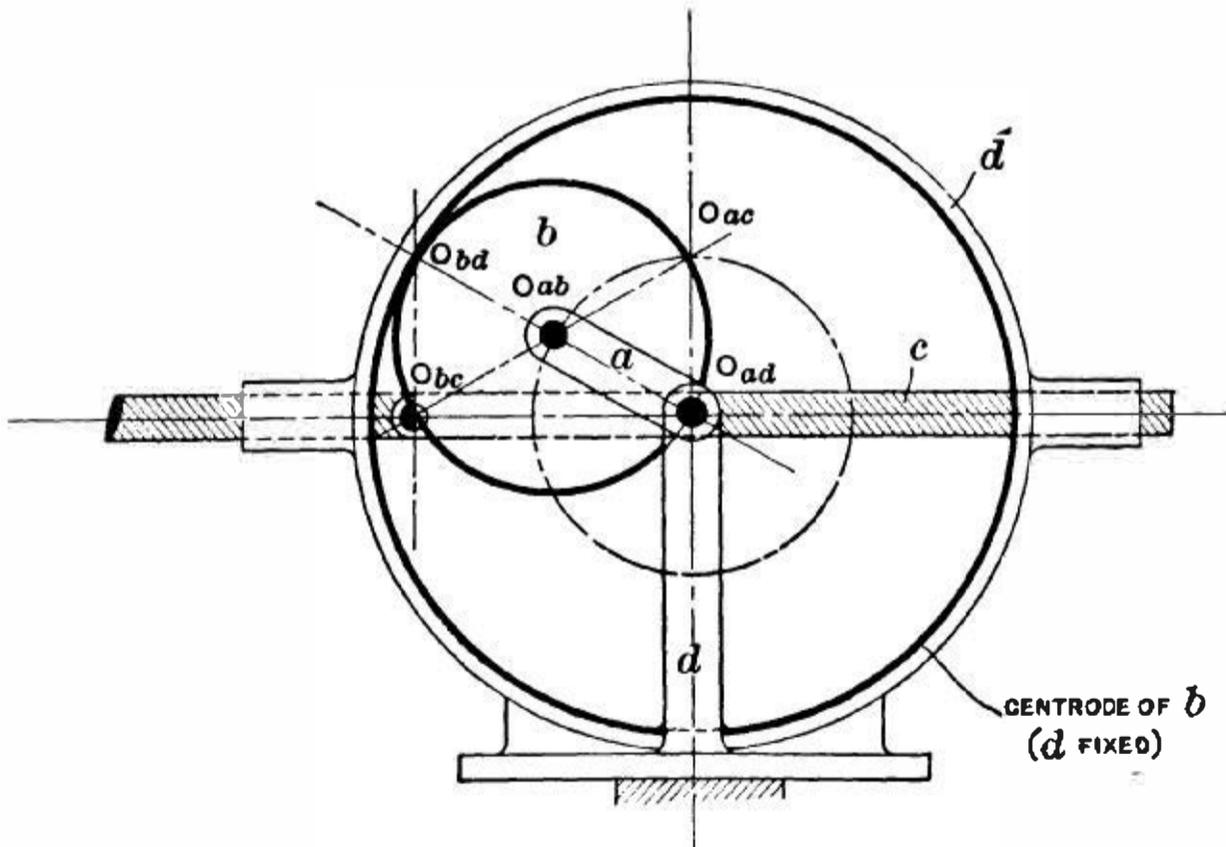


FIG. 140b.

ing-rod b is made equal in length to the crank a . (Compare Figs. 60 and 91.) With these proportions it is possible for the stroke of c to be (1) either twice the length of the crank,

as in the ordinary slider-crank chain, or (2) four times the length of the crank, in which case O_{bc} must travel on past O_{ad} . A third possibility is that O_{bc} and O_{ad} may remain coincident, in which case b and a move together, and the mechanism will reduce to a pair of elements.

Notice in Fig. 140a that since the length of b is equal to the length of a and the angle $O_{bd}O_{bc}O_{ad}$ is a right angle, the points $O_{bd}O_{ad}O_{cb}$ must lie on the circumference of a circle whose radius is the length of a or the length of b . Since $O_{bd}O_{ad}$ is a diameter of this circle, it follows that O_{bd} remains always at the same distance from O_{ad} , and the centrode of b with regard to d is a larger circle whose radius is twice the length of a . If now (Fig. 140b) we attach to d an annular wheel whose pitch-circle is the centrode of b with regard to d , and if we fix to b a spur-wheel whose pitch-circle is the centrode of d with regard to b , these wheels will gear together, and will compel O_{bd} to remain always at a fixed distance from O_{ad} . If these wheels were not provided we should have a change-point at the instant when O_{bc} passes O_{ad} , but if the virtual centre O_{bd} is compelled to remain at a fixed distance from O_{ad} by the action of the spur-wheels, O_{bc} is compelled to continue its travel, and the mechanism is not permitted to change. Obviously this arrangement is really a case of pair-closure at a change-point. (Compare the examples in § 59.) The only really essential portions of the wheels are therefore those teeth which are in gear while O_{bc} is passing O_{ad} .

71. Cam-trains.—The name cam-train is applied to mechanisms containing a rotating disc (generally non-circular) or a sliding plate, the profile of which forms one element of a higher pair and gives some desired periodic motion to the second element of the pair. Such a cam-pair may be closed by forming one element into the geometrical envelope of all possible positions of the other element. Mechanically cam-pairs usually possess the disadvantage of small wearing surface and rapid wear, common in higher pairs. Almost invariably force-closure is necessary to make up for the

looseness of fit following on wear. A cam-train is in general a mechanism of three links; such, for example, is the cam-train found in the stamp-mill used for crushing hard ores (Fig. 141).

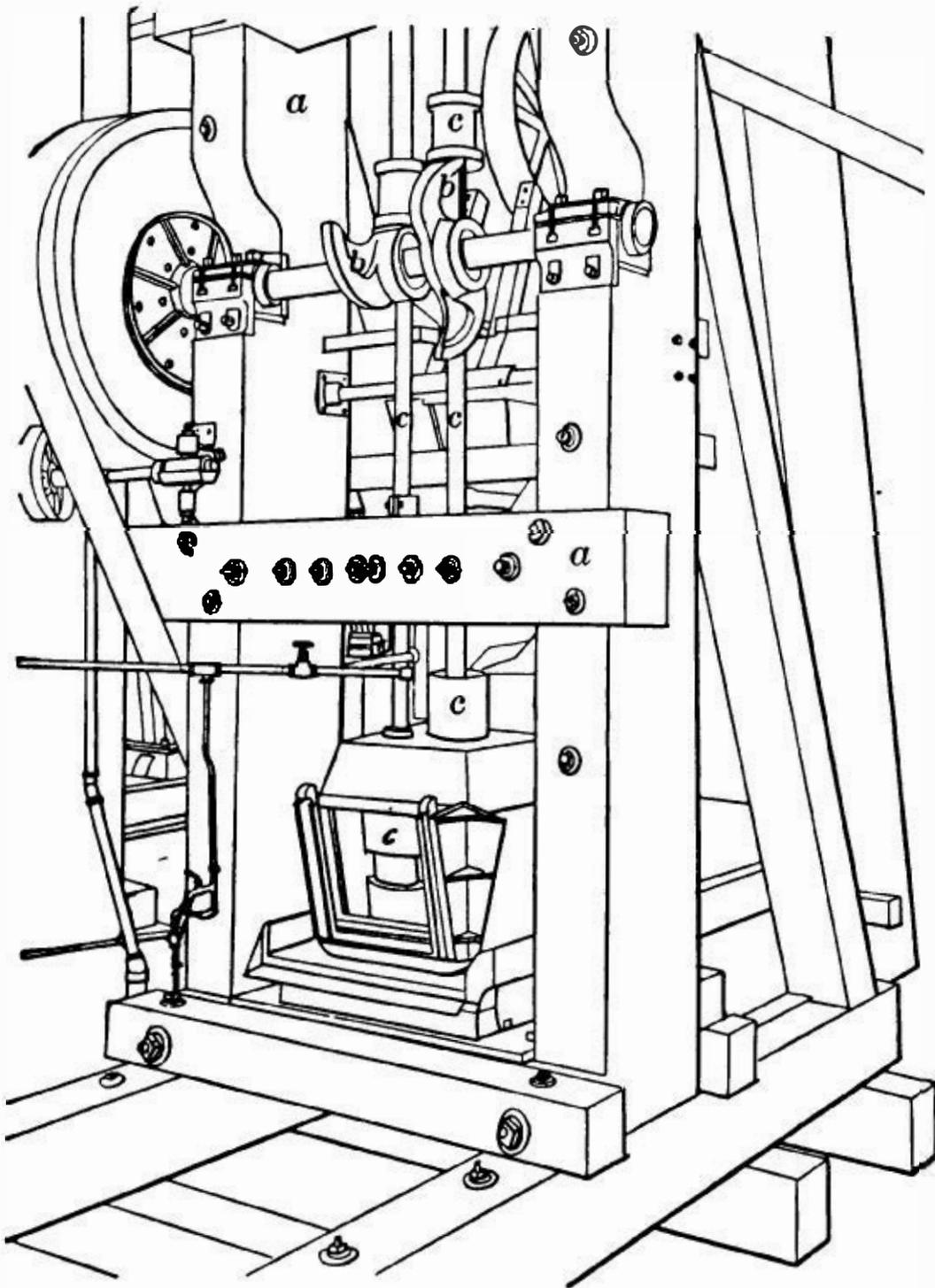


FIG. 141.

A rotating shaft carries the cams *bb*. These successively lift and let fall the stamps *cc*, which are guided by means of the framework *a*. It will be noted that the cam-pair *bc* is force-closed by the weight of the stamp itself, and also that the form of the cam is such that in any position during the upward stroke it is touched by the horizontal under surface of the collar on the stamp-rod. This fact has to be consid-

ered in selecting the form of the cam, for it is obvious that during all the upward movement the point of the cam surface which touches the collar must be at a higher level than any other point on the cam surface. It will be found that with such a cam it is not possible to give the collar any arbitrary position for any given angular position of the cam.*

In many cases the cam has to determine the position of a point, instead of a flat surface, on the *follower* or link driven by the cam. This point is then usually taken as the centre of a roller or pin with which the cam engages, and within certain limits imposed by constructive considerations, any desired continuous change of position can be given to the follower by suitably choosing the form of the cam profile.

72. Rotating Cams.—The action of cam-trains will be most easily understood by the study of a few examples. To produce a given form of reciprocating motion along a straight or curved line we may employ either a rotating, a sliding, or a cylindrical cam. The first example (Fig. 142) will be that of a rotating cam designed to give its follower a reciprocating motion along a straight line passing through the cam centre, the velocity being uniform throughout both strokes if the cam rotates with uniform angular velocity. The mechanism is somewhat similar to that of Fig. 141, and consists of a cam *c* (whose form is to be determined), a guiding frame *a*, and a follower *b*, which is to slide with the periodic motion specified above. The end of the follower is provided with a roller, for the sake of lessening friction.

Since the cam rotates uniformly, while the follower moves with uniform velocity, the cam describes equal angles while the follower traverses equal distances. Plainly the outline of the curve required will be such that successive radii making equal angles with one another have a constant difference in length. All we have to do, in fact, is to divide the path of *A* into any convenient number of equal parts,

* See Kennedy, *Mech. of Machinery*, p. 154.

say six, and to divide the half-revolution of the cam into the same number of equal angles. If CA_0 is the least distance of the centre of the roller from the centre of the cam, and CA_1, CA_2 , etc., are the distances after one, two, etc., twelfths of a revolution, we then make $Ca_1 = CA_1, Ca_2 = CA_2$, and so on. The curve drawn through a_1, a_2 , will be recog-

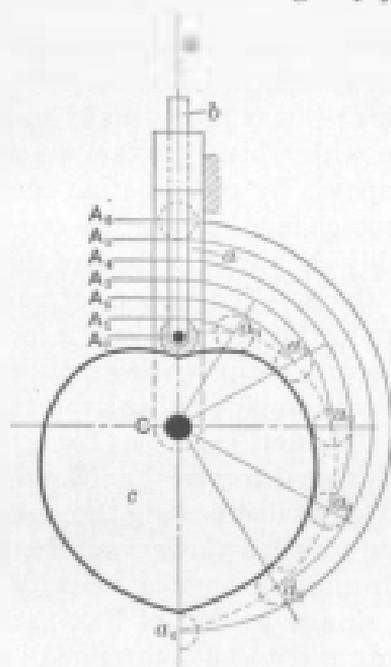


FIG. 142.

nized as an Archimedean spiral, and the same kind of curve will, of course, be found for the remaining half of the cam. In this curve let $CA_0 = r_0$, and let r be any radius-vector of the curve, then $r = r_0 + m\theta$, where θ is the angle the radius-vector makes with CB and m is a constant. The real outline of the cam itself is not the dotted line a_1, a_2, a_3 , but the full line drawn so as to touch a series of circles whose centres lie on a_1, a_2 , etc., and whose diameters are all equal to that of the roller on the follower.

A cam frequently has to actuate a point on a lever, which of course moves in the arc of a circle. An exactly

similar construction in this case gives the form of the cam, but the points A_0, A_1, \dots , will now be placed along a circular path instead of along a straight line.

As a more difficult case, let us consider the form to be given to a cam arranged to move a follower with uniform *acceleration* during one half of a revolution, after which the

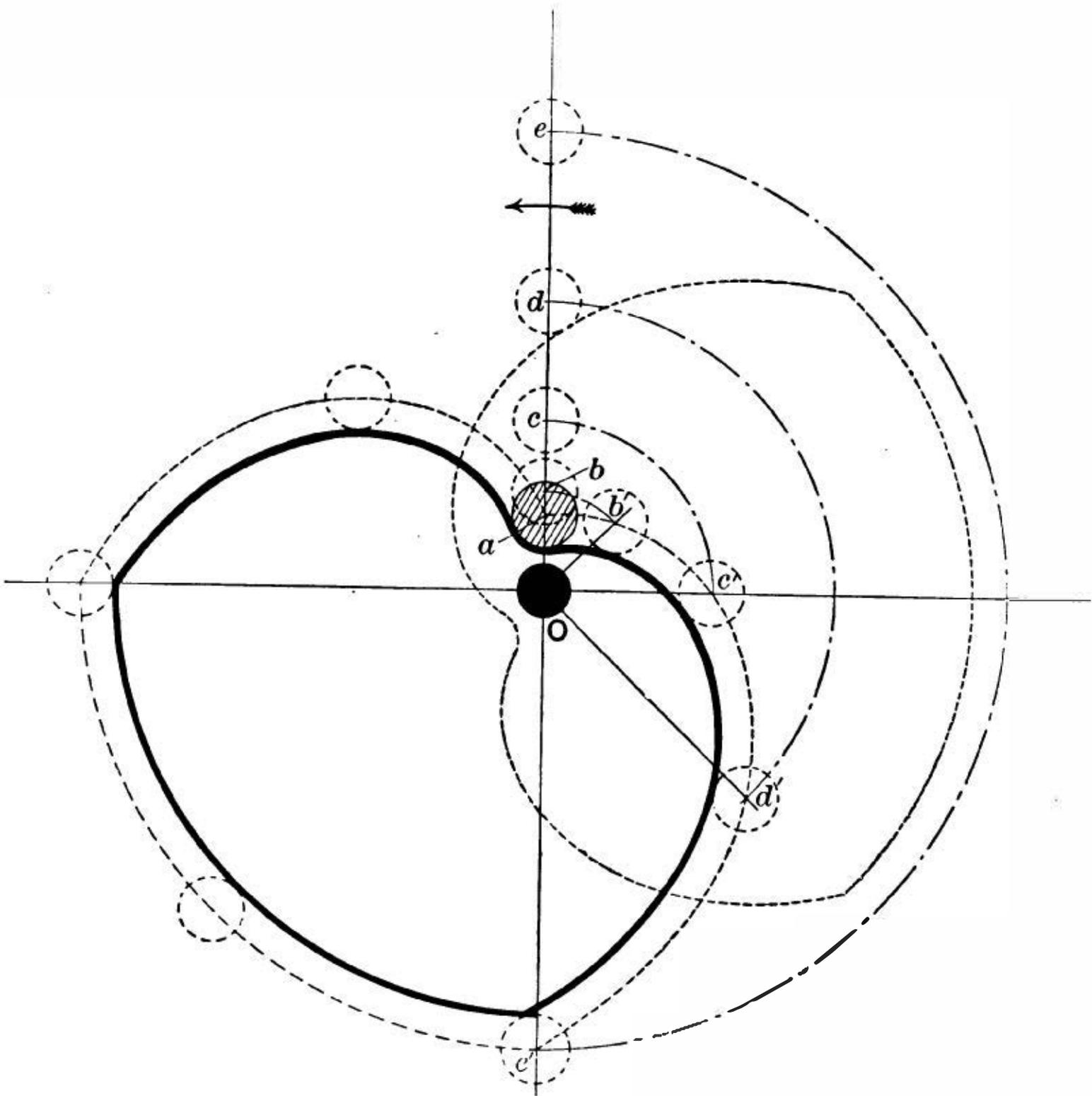


FIG. 143.

sliding piece remains at rest during a quarter of a revolution and returns with uniform velocity during the remaining quarter revolution, the cam rotating uniformly.

In Fig. 143 let the points a, b, c, d, e correspond to the positions of the centre of the pin on the reciprocating piece

at equal intervals of time during one half revolution, while the distance ae is the length of stroke of the reciprocating piece. Since the upward stroke is to be made with uniform acceleration, the distance $ac = 4ab$, while $ad = 9ab$, and $ae = 16ab$ (where ab is the distance moved in one eighth of a revolution). Let O be the centre of the cam. Then, starting with the sliding-piece in its lowest position, when the cam has turned through a quarter of a revolution, the line

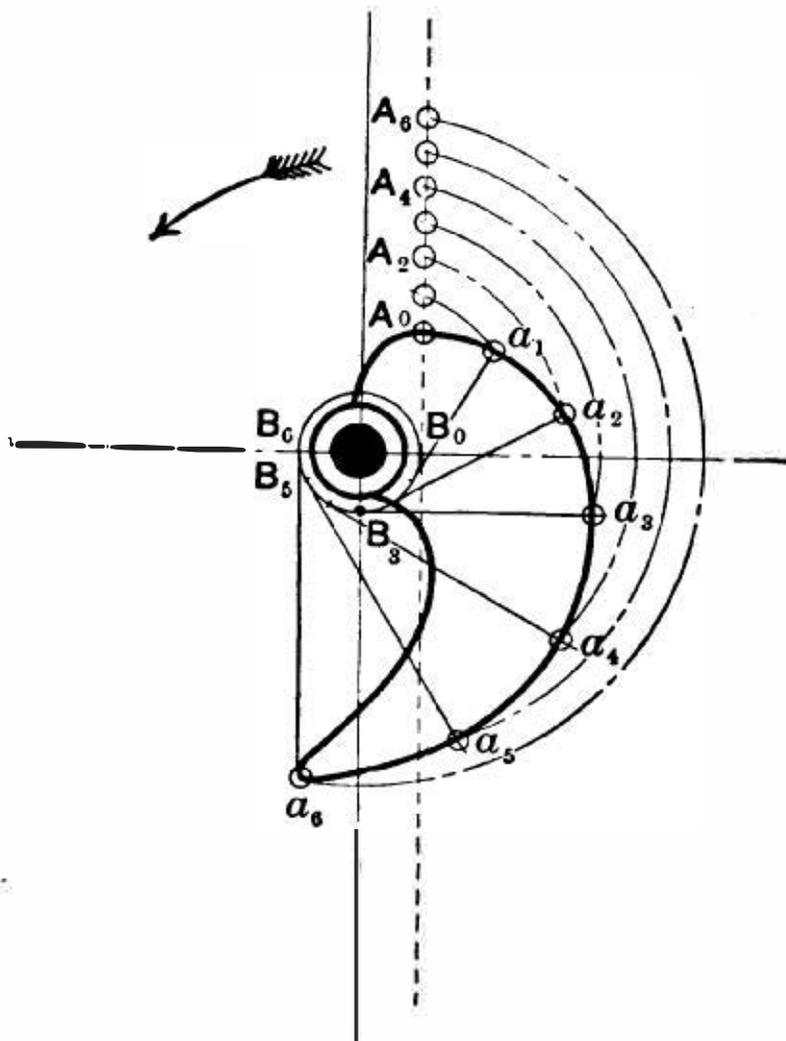


FIG. 144.

Oc' will coincide with Oc . Thus at the instant when the sliding-pin has its centre at c , the radius Oc' will be vertical, and we must set off a distance $Oc' = Oc$. By a similar construction other points on the curve, such as b' , d' , etc., are obtained. The profile of the cam itself is obtained by drawing a curve (shown in full lines) at a uniform distance from $ab'c'd'e'$ equal to the radius of the pin on the sliding-piece.

We may now consider the construction to be adopted if the line of motion of the follower-point does not pass through

the centre of rotation of the cam. In Fig. 144, let A_0, A_1, A_2, \dots be successive positions of the follower-point, corresponding to successive equal angles described by the cam-shaft, and let the line A_0, A_1, A_2, \dots produced be a tangent to a small circle $B_0, B_1, B_2, B_3, \dots$ described about the cam centre.

As the cam rotates it is seen that such a line as B_0, a_0 , drawn touching the small circle will take up the position B_1, A_1, A_2 , when it becomes vertical. Hence the point a_1 will be found by drawing a circle with O as centre, and radius O, A_0 , so as to cut B_1, a_1 ; then B_1, a_1 is the line which coincides with B_0, A_0 at the time when the follower-point is at A_1 .

If the distance A_1, A_2 is equal to the arc B_0, B_1 , and the distances A_1, A_2, B_1, B_2 , are equal, and so on, it is evident that the curve a_0, a_1 is an involute of the base-circle $B_0, B_1, B_2, B_3, \dots$. Such curves are generally used for the cams of an ore-crushing stamp-mill. In an involute the tangent to the base-circle is a normal to the curve (see § 61); hence in any position of an involute cam the point lying on the vertical line B_0, A_0 will touch a horizontal line corresponding to the under surface of a collar on the stamp-rod.

73. Sliding and Cylindrical Cams.—The form of a *slid-*

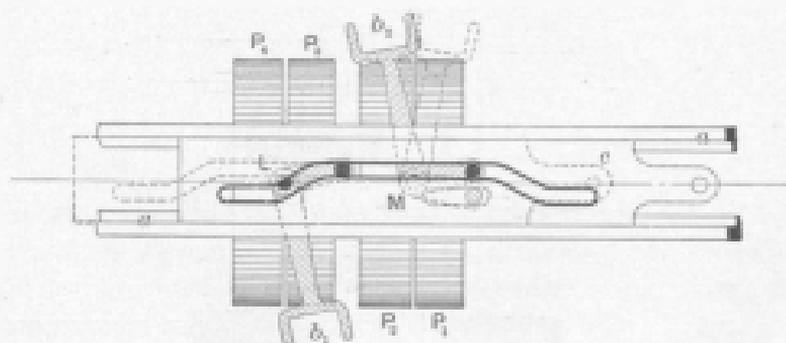


FIG. 144.

ing cam to obtain any desired kind of periodic motion is easily determined. Fig. 145 shows the arrangement of a cam of this kind used for giving the requisite motion to the

belt-shifting gear of a planing-machine. The cam c here takes the form of a slotted plate, sliding in a frame or guide a . Two bell-crank levers, b_1 and b_2 , pivoted to a at L and M , carry follower-pins which work on the slot in c . The longer arms of the levers are provided with forks for shifting the belts as required from the fast to the loose pulleys of the planing-machine. The length and form of the slot in the cam are such that when c is in its extreme position to the left, as shown by dotted lines, b_2 is thrown to the right, and its belt runs on the fast pulley P_1 , while the fork b_1 is also inclined to the left and its belt runs on a loose pulley P_3 . On moving the cam from left to right it will be seen that b_2 first moves its belt on to the loose pulley P_2 , and afterwards b_1 moves the second belt on to the fast pulley P_4 . In this way it is impossible for both belts to be on fast pulleys at the same time. The belts drive the pulleys P_1 and P_4 in opposite senses, hence the action of the gear is to reverse the motion of the shaft to which the pulleys are attached. The motion of the planing-machine table is thus also reversed.

Fig. 146 shows a *cylindrical cam*, in which the cam profile

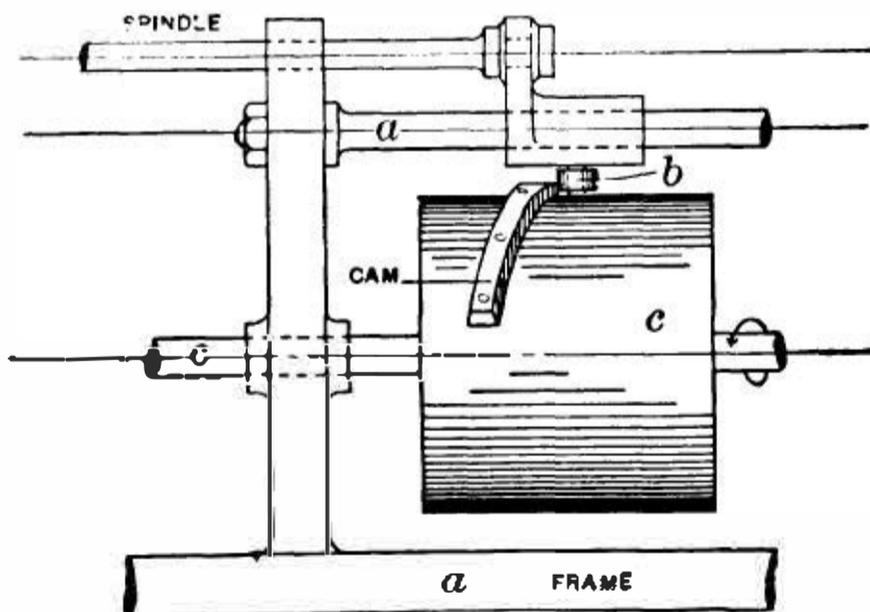


FIG. 146.

is traced on the surface of a rotating cylinder and the line of motion of the follower is parallel to the axis of the cylinder. The figure shows how such cams are employed in certain

automatic screw-making machines for the purpose of giving endwise motion to the rotating spindles carrying the work. The working profile of the cam is the edge of a strip secured to the surface of the cylinder c by screws; a series of these strips may evidently be arranged so as to give any desired periodic range of rest and motion to the carriage of the rotating spindle. In this case the roller on the follower b is kept pressed against the cam edge by a spring or other

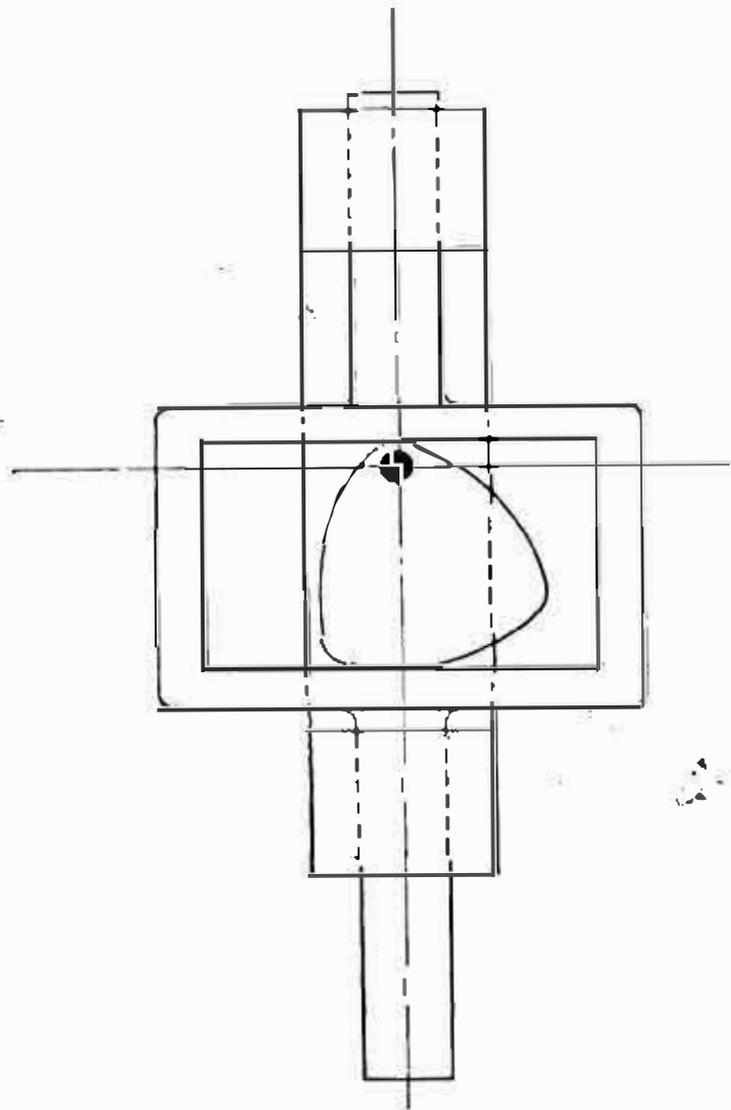


FIG. 147.

suitable means. The mechanism is thus *force-closed*. It would, however, be quite easy, by attaching two strips, to form a groove in which the follower-roller would work; the mechanism would then have *pair-closure*.

A rotating cam of the kind shown in Fig. 143 could be closed in the same manner if a groove were formed on the flat surface of the cam-plate, engaging with a pin or roller attached to the follower. Or, as an alternative, the cam may be formed with a figure of constant breadth, in which

case it can have constrained motion with relation to a follower provided with a pair of parallel faces, as in Fig. 147.

Cams which are pair-closed are often called positive-motion cams.

74. Velocity Ratio in Cam-trains.—Let Fig. 148 represent a cam-train of three links we wish to find the velocity ratio of the train, i.e., the ratio

$$\frac{\text{linear velocity of follower}}{\text{angular velocity of cam}}$$

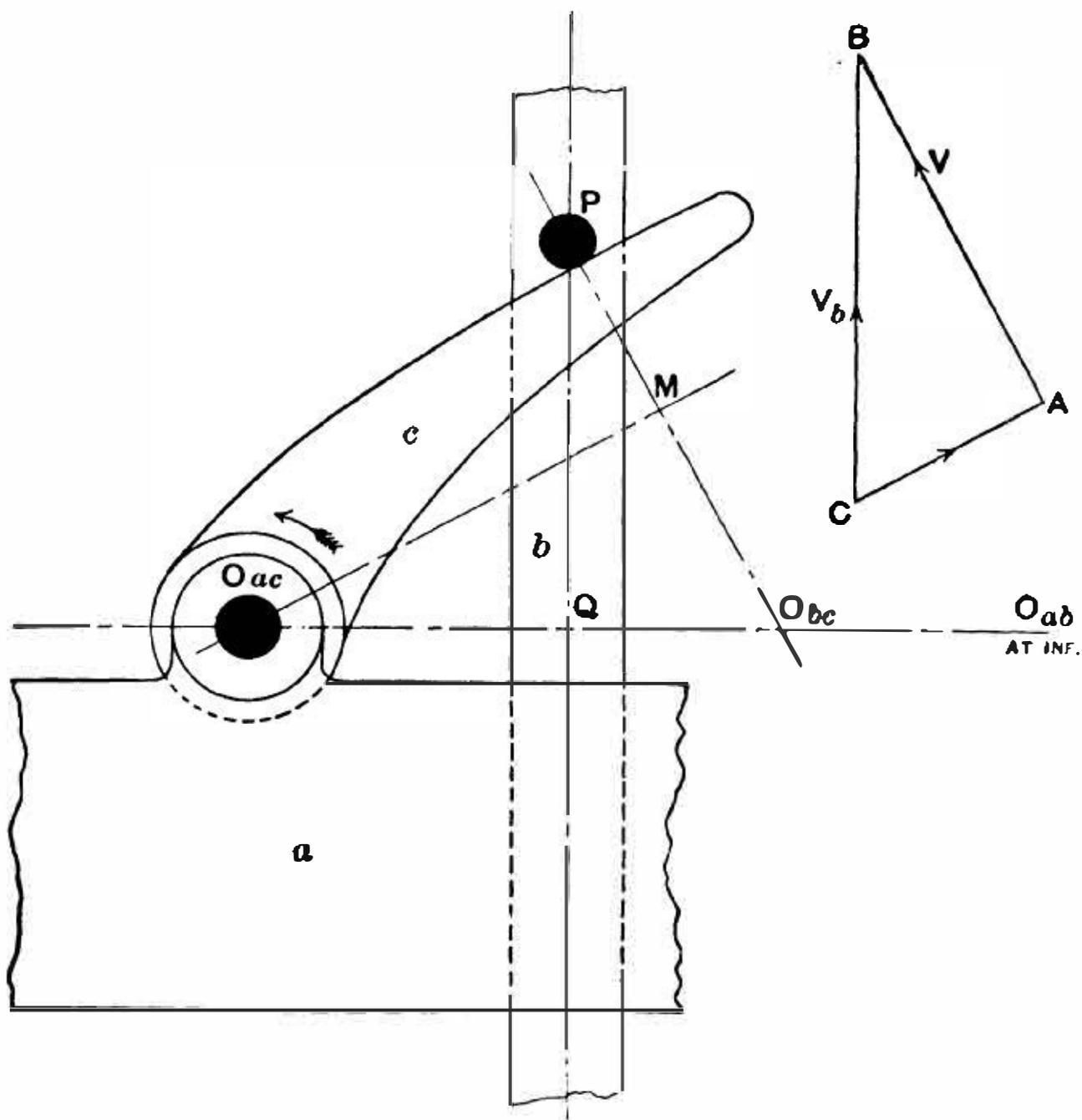


FIG. 148.

Find O_{ac} , the centre about which the cam rotates, and also O_{ab} , which in this case is at an infinite distance, since the follower moves in a straight line, QP . If the follower were a lever turning about a fixed centre, the line $O_{ac}O_{ab}$

would be as easily determined, The third virtual centre O_{bc} must be the point where the common normal to the cam and follower at their point of contact cuts this line. Let

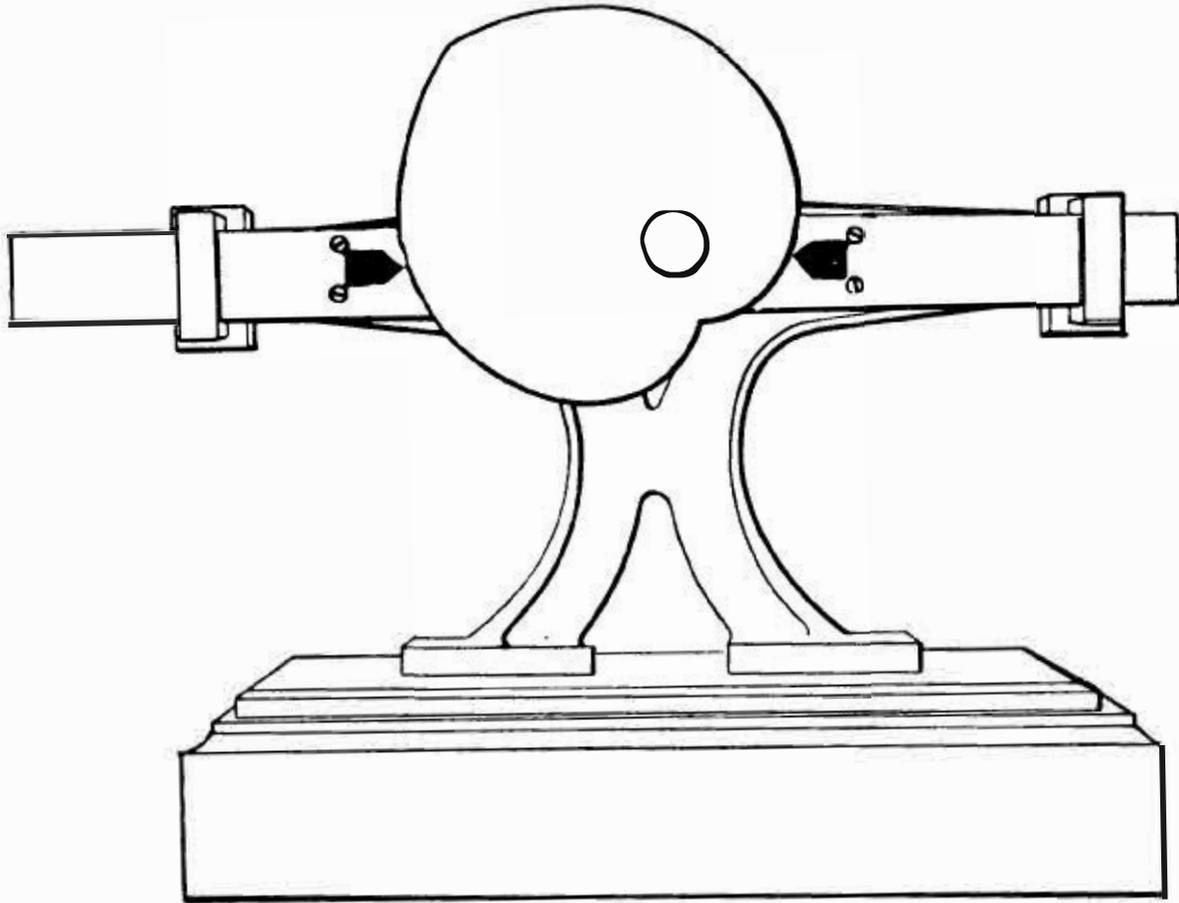


FIG. 149a.

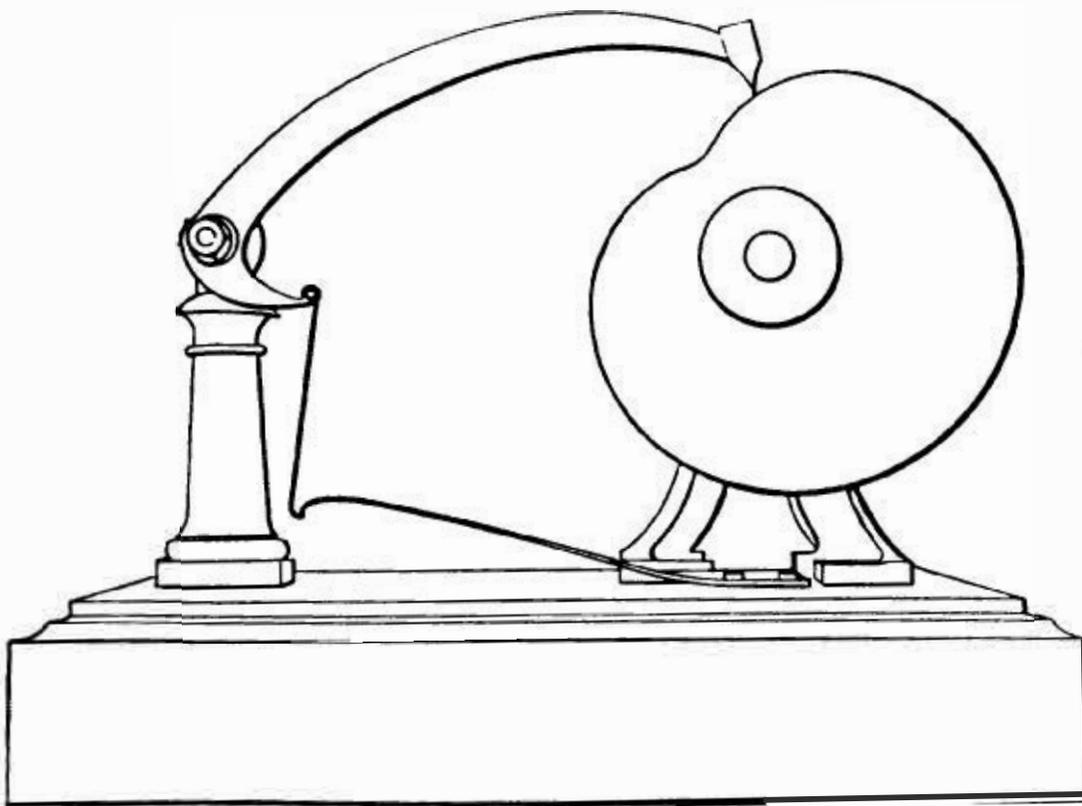


FIG. 149b.

V be the common velocity of the bodies b and c at their point of contact; its direction will be parallel to the line MP . Draw a triangle of velocities ABC , in which AB represents V , CB represents V_b , and AC represents the velocity of

sliding of c on b in a direction parallel to the common tangent at the point of contact. Draw $O_{ac}M$ parallel to this tangent, and therefore perpendicular to PM .

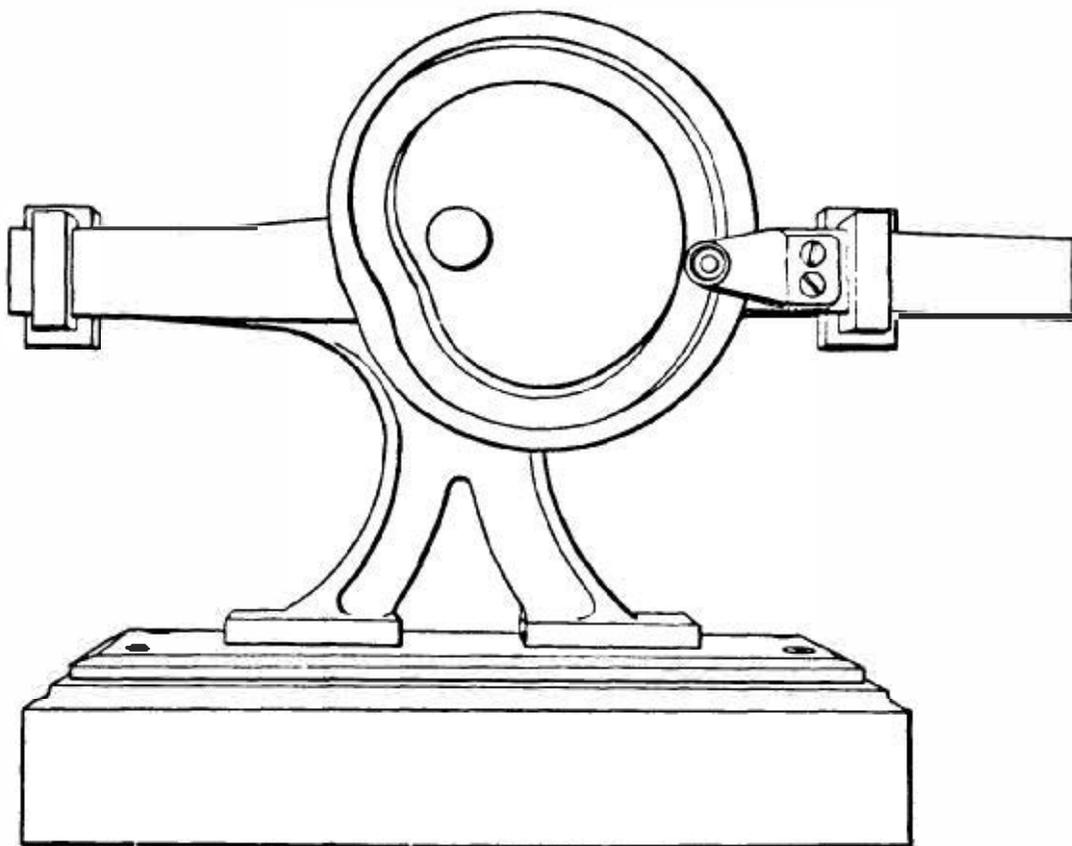


FIG. 149c.

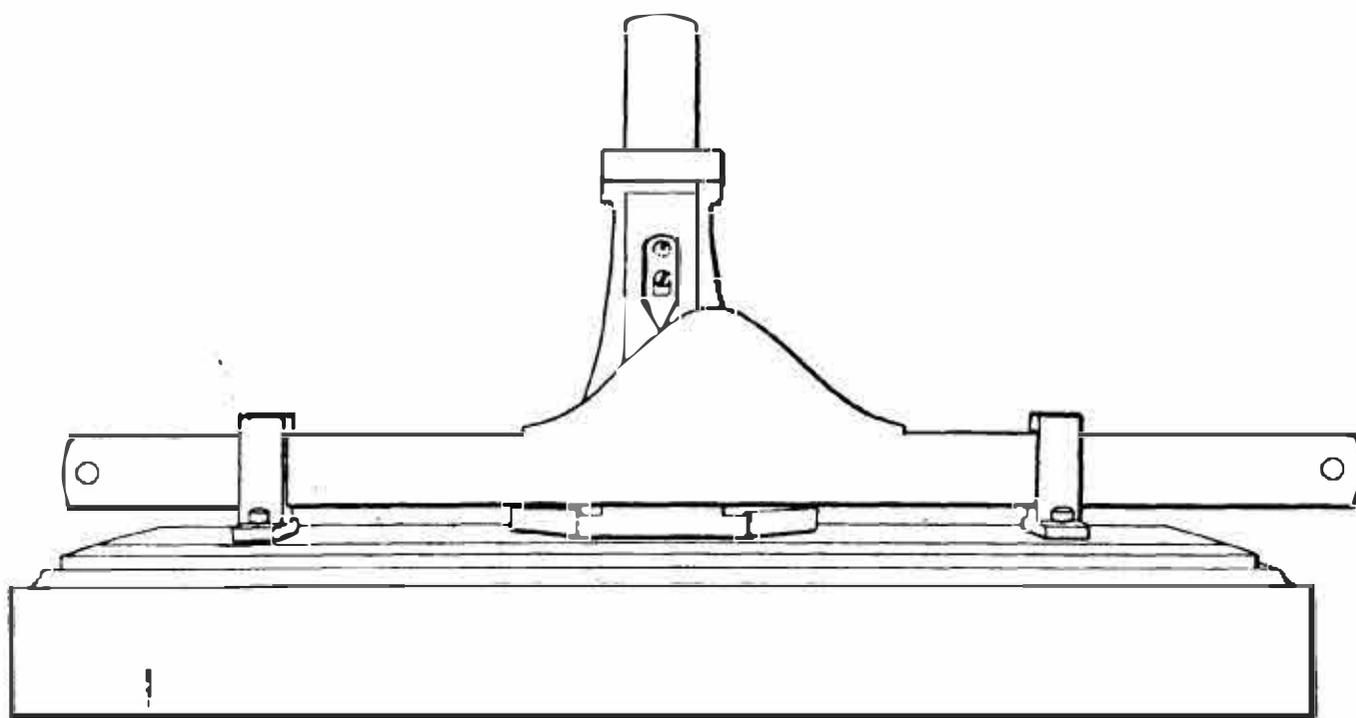


FIG. 149d.

Then the angular velocity of the cam

$$\omega_{ca} = \frac{V}{O_{ac}M}$$

and the linear velocity of the follower along QP

$$V_b = V \cdot \frac{BC}{BA} = V \frac{O_{ac}O_{bc}}{O_{ac}M}$$

Hence the velocity ratio of the pair is

$$\frac{V_b}{\omega_{ca}} = \frac{V \cdot \overline{O_{ac}O_{bc}}}{\overline{O_{ac}M}} \cdot \frac{\overline{O_{ac}M}}{V} = \overline{O_{ac}O_{bc}}$$

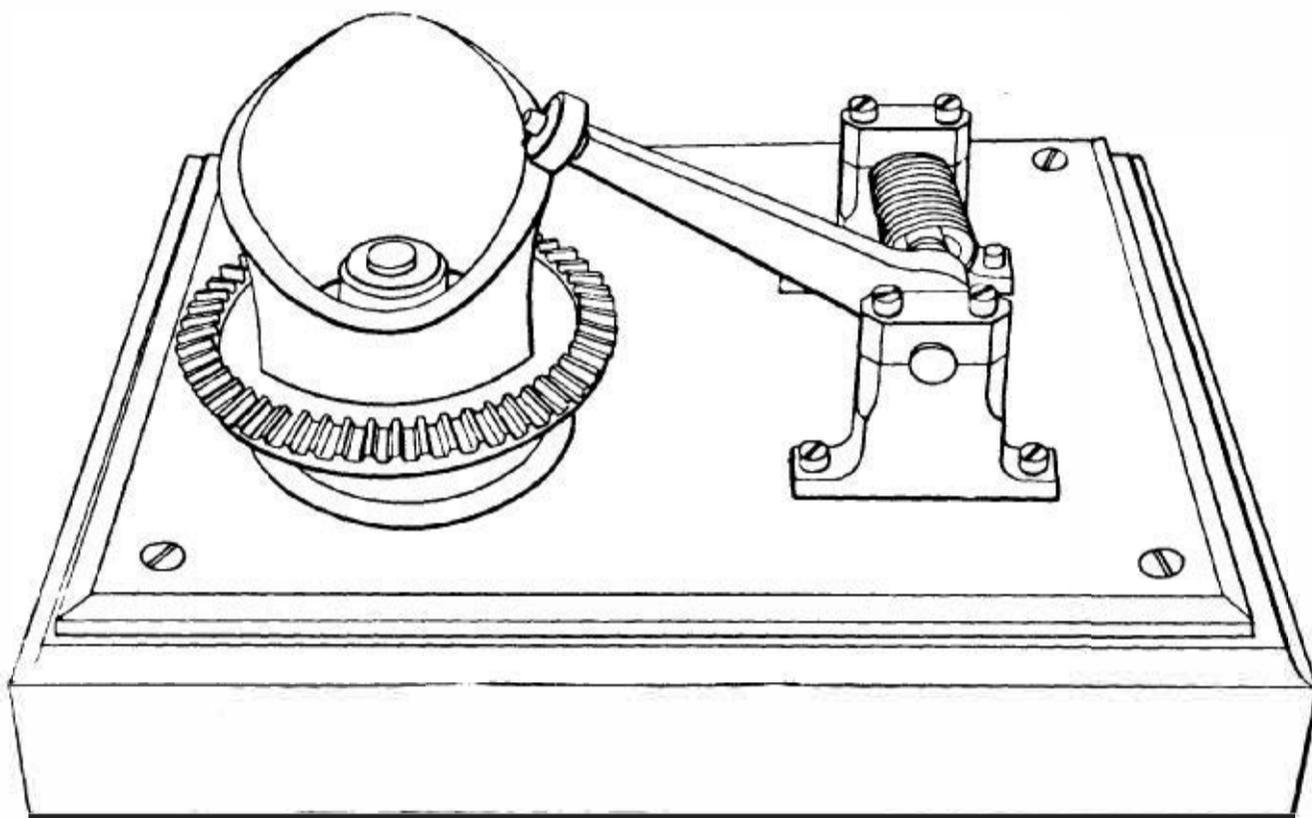


FIG. 149e.

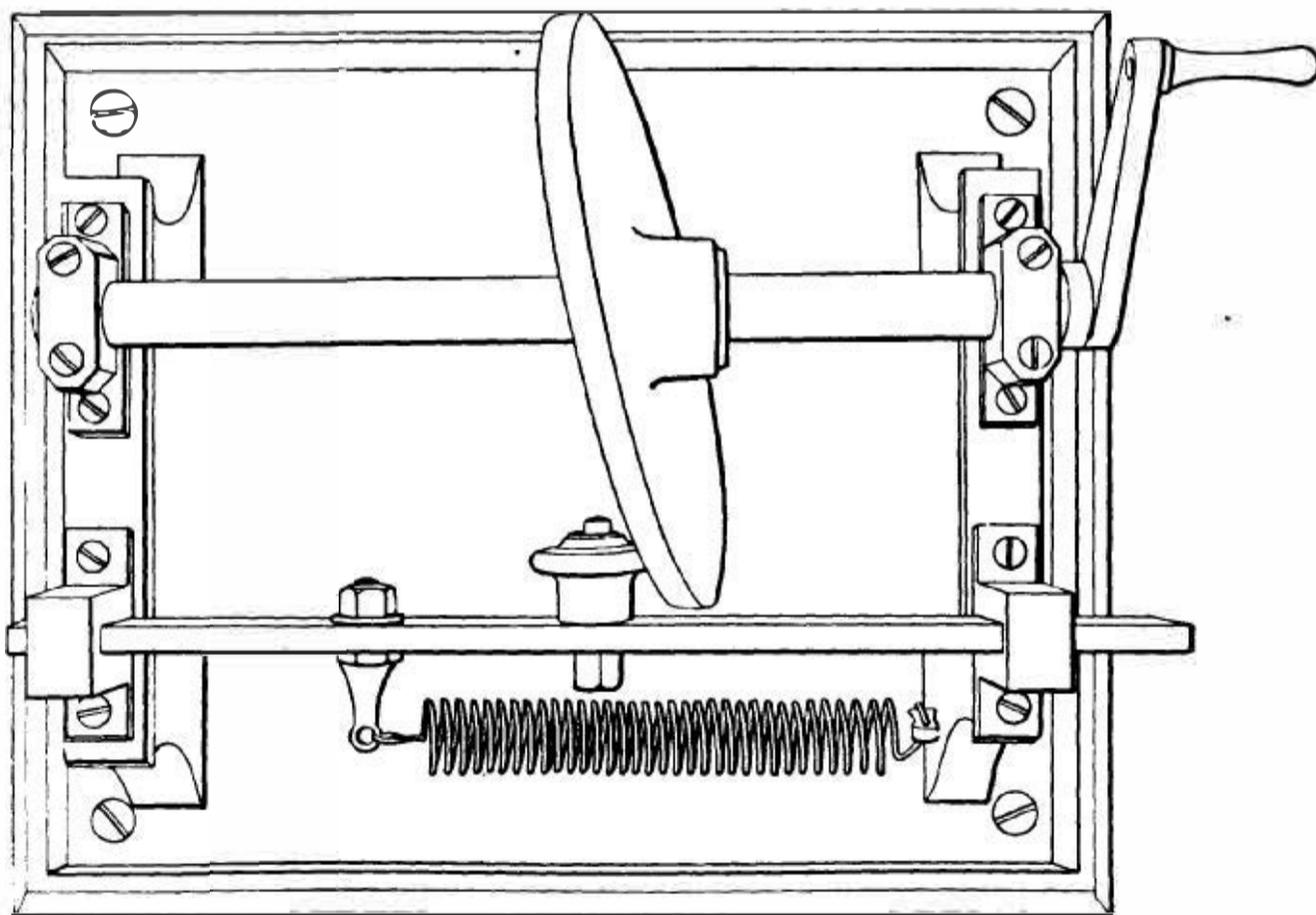


FIG. 149f.

In other words, if the cam rotates with uniform velocity, the linear velocity of the follower is proportional to the distance between the centre of rotation of the cam and the virtual

centre of the cam and follower. If the point P moves in an arc of a circle about a point O_{ab} as in Fig. 149*b*, then it may be shown, exactly as in § 63, that the angular velocity ratio of b and c will be the number

$$\frac{\overline{O_{ca}O_{bc}}}{\bullet_{ab}O_{bc}}$$

A number of the forms taken by cam-trains are illustrated by the models represented in Figs. 149*a*–149*f*. A positive-motion cam, of form somewhat similar to that of Fig. 142, is shown in Fig. 149*a*, while Fig. 149*b* is an example of a cycloidal cam whose follower point moves in a circular arc. Fig. 149*c* shows another positive-motion cam, where the follower-pin works in a groove in the cam, and Fig. 149*d* is a sliding cam. In Fig. 149*e* we have a rotating globoidal cam actuating a lever; Fig. 149*f* represents the form of cylindrical cam known as a “swash-plate.” The reader will notice in three of these cases the springs which close the pair.