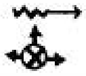
Appendix

I. TABLE OF SYMBOLS AND SIGNS

Symbols

The following symbols are used in this text (see Table, Appendix II for units not listed below).

A	=	Area
a	=	Linear acceleration
AC	=	Alternating current
B,b	=	Base of triangle used to measure slope of curve
C	=	Capacitance (microfarads)
C	=	Damping constant (lb-sec/ft)
Cc	=	Critical damping constant (lb-sec/ft)
cg	=	Center of gravity
cp	=	Center of percussion
D	=	Linear displacement
DC	=	Direct current
E	=	Voltage (volts)
F	=	Force
g	=	Acceleration due to gravity
H,h	=	Height of triangle used to measure slope of curve
I	=	Moment of inertia (slug-ft^2)
I	=	Electrical current (amperes)
J	=	Jerk (rate-of-change of acceleration—ft/sec^2/sec)
K	=	Spring constant (pounds/ft)
K	=	Coefficient of restitution (dimensionless)
L	=	Length (ft)
L	=	Inductance (henries)
m	=	Mass (slugs)
N	=	Gear ratio
Q	=	Energy lost during impact (ft-lbs)
R,r	=	Radius (ft)
R	=	Electrical resistance (ohms)
t	=	Time
Δt	=	Elapsed time
v	=	Linear velocity
v'	=	Linear velocity following a collision or impact between two or more bodies (ft/sec)
W	=	Weight (lbs)
X,Y,Z	=	Coordinate axes
α	=	Angular acceleration
ρ	=	Density (slugs/ft^3)
β	=	Angle (radians)
θ	=	Angle (radians)
θ	=	Angular displacement (radians)
τ	=	Torque (ft-lbs)
ω	=	Angular velocity (radians/sec)
ω'	=	Angular velocity following a collision or impact between two or more bodies (radians/sec)
∞	=	Infinity
●	=	Grounded pivot or bearing
○	=	Moving (translating) pin or bearing
$\rightsquigarrow$	=	Signifies motion in the direction indicated
	=	Differential gear

Sign Conventions

Linear vector quantities (linear force, acceleration, velocity, displacement, etc.), are considered positive if upward, or to the right, in the XY plane. They are considered negative if downward, or to the left.

Rotational vector quantities (angular torque, acceleration, velocity, displacement, etc.) are considered positive if operating in a clockwise direction (looking toward the body along the axis of rotation or torque). They are considered negative if counterclockwise.

II. TABLE OF COMPATIBLE UNITS*
(Stick to any one column and you will stay out of trouble.)

Quantity	English		CGS	SI
Force	lb	ounce	dyne	newton
Acceleration	ft/sec^2	in./sec^2	cm/sec^2	m/sec^2
Velocity	ft/sec	in./sec	cm/sec	m/sec
Displacement	foot	inch	cm	meter
Mass	slug	oz sec^2/in.	gram	kilogram
Torque	ft-lb	in.-oz	cm-dyne	newton-meter
Angular Acceleration	rad/sec^2	rad/sec^2	rad/sec^2	rad/sec^2
Angular Velocity	rad/sec	rad/sec	rad/sec	rad/sec
Angular Displacement	radian	radian	radian	radian
Moment of Inertia	slug-ft^2	inch-ounce-sec^2	g-cm^2	kg-m^2
Mass Density	slugs/ft^3	oz/in.^{3}	g/cm^3	kg/m^3
Volume	ft^3	in.^{3}	cm^3	m^3
Acceleration due to Gravity	32 ft/sec^2	384 in./sec^2	980 cm/sec^2	9.8 m/sec^2
Work	ft-lb	in.-oz	dyne-cm (erg)	joule
Potential Energy	ft-lb	in.-oz	dyne-cm (erg)	joule
Kinetic Energy	ft-lb	in.-oz	dyne-cm (erg)	joule
Impulse	lb-sec	oz sec	dyne-sec	newton-sec
Momentum	slug ft/sec	oz in./sec	g cm/sec	kg m/sec
Time	second	second	second	second
Voltage	volt	volt	volt	volt
Current	ampere	ampere	ampere	ampere
Spring Constant	lbs/ft	ozs/in.	dynes/cm	newtons/meter

* If engineers were as rigorous as physicists they would be more familiar with the "correct" units of force in the SI and C-G-S systems of units: these correct units being the newton and dyne respectively. Unfortunately, we use spring scales and balance weights that are calibrated in grams and kilograms (units of mass) on the one hand, or calibrated in pounds and ounces (units of force) on the other. I strongly recommend that the reader convert data from "gram of force" or "kilogram of force" units to dynes and newtons whenever he encounters them. You can use the following formulae for converting:

$$\frac{\text{Grams of force}}{980} = \text{dynes}$$

$$\frac{\text{Kilograms of force}}{9.8} = \text{newtons}$$

III. GRAPHICAL INTEGRATION

There are several ways to integrate a curve graphically. Integration, of course, merely consists of "finding the area under" the curve. One way to do it is to draw the original curve on a piece of graph paper and then count the squares under the curve to determine area. The "value" of each square is determined by the scale factors used to plot the original curve, as explained in Chapter 1. (See Fig. A–1.)

Another method is to use algebraic equations to calculate areas under larger portions of the curve. This, of course, is only possible if these areas can be broken down into simple shapes such as rectangles, triangles, squares, etc. (See Fig. A–2.)

A third method is to use the instrument shown in Fig. A–3, which is called a *planimeter.* A stylus on one arm of the instrument is moved along the curve and along the axis which bounds the area to be measured. A small counting device on the planimeter then gives the total enclosed area. Again, the units of area must be related to the scale factors of the original curve.

IV. GRAPHICAL DIFFERENTIATION

There are several ways to find the differential of a curve at a given point, graphically. The first step in each case is to draw a line which is tangent to the curve at the point where the slope of the curve is to

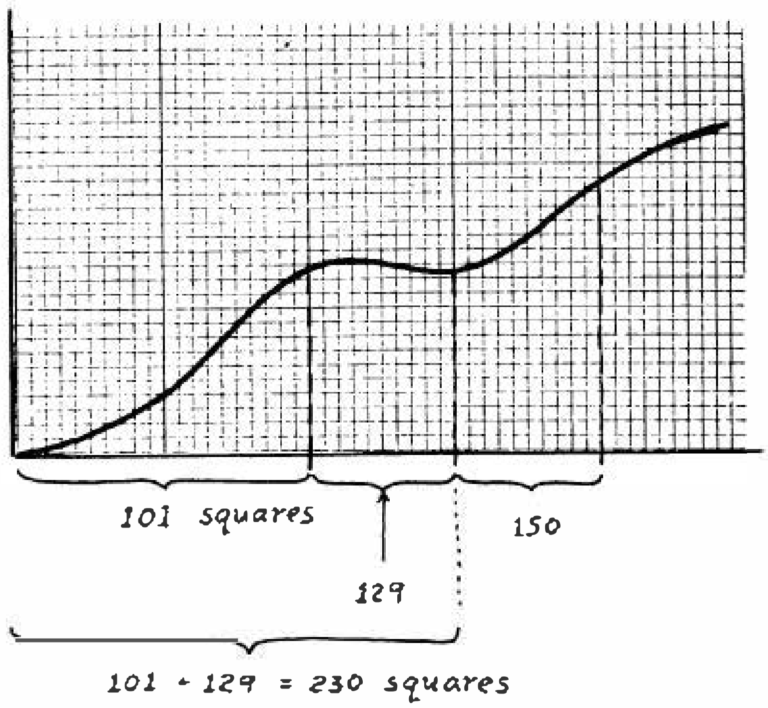
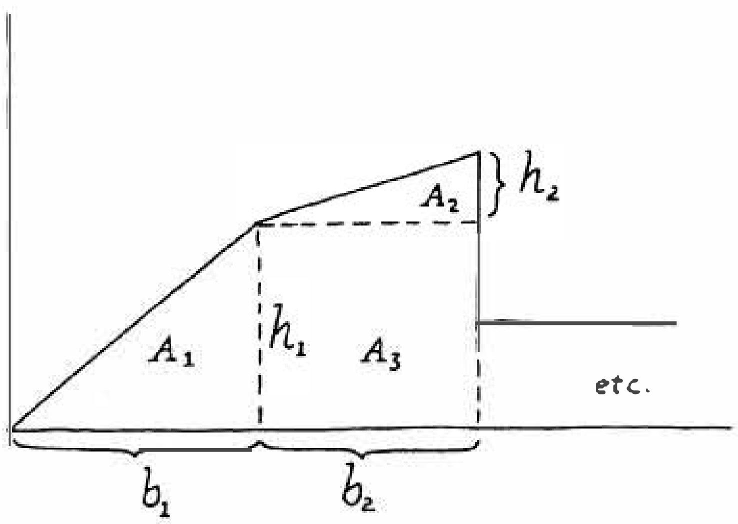


Fig. A-1. Counting the squares under a curve to integrate it graphically.



$$A_1 = \frac{1}{2}\, b_1 h_1$$
$$A_2 = \frac{1}{2}\, b_2 h_2$$
$$A_3 = b_2 h_1$$

Fig. A-2. Algebraic formulas can be used to plot the area under a curve.

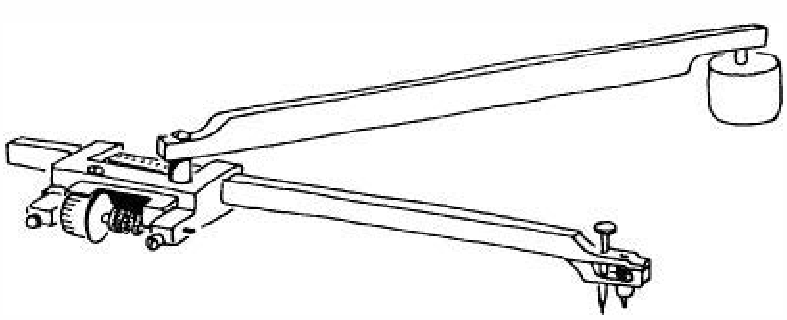


Fig. A-3. The planimeter, an instrument used to compute areas, can be used for graphical integration.

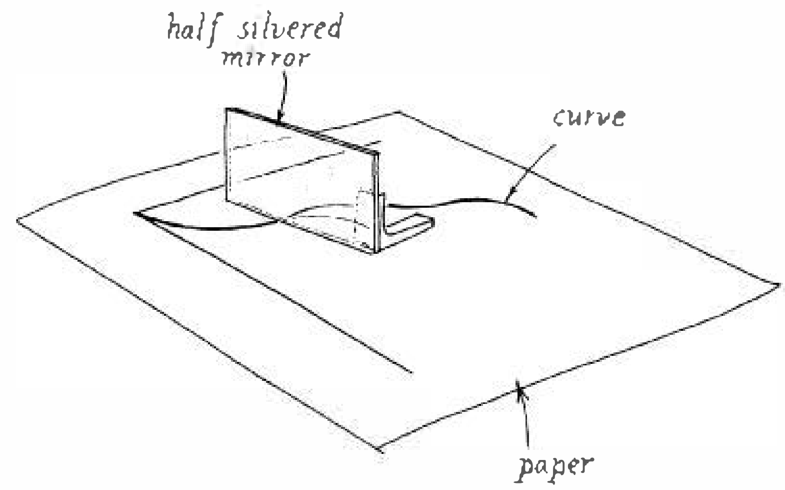


Fig. A-4. A half-silvered mirror, held perpendicular to the plane of the paper, will help you draw a line normal (perpendicular) to the curve. Draw a line along the bottom of the mirror after turning it so that the two images seen in (and through) the mirror coincide.

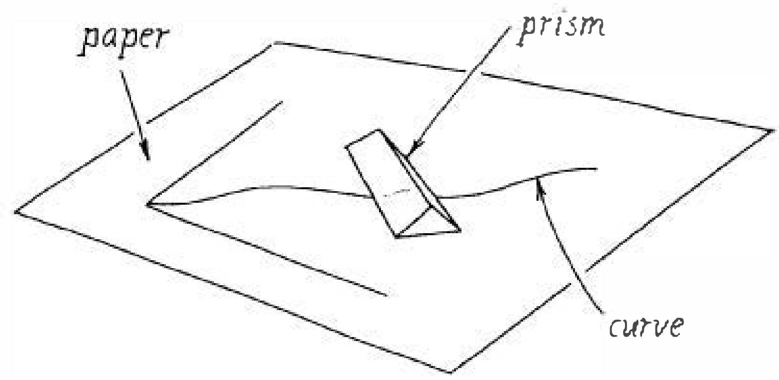


Fig. A-5. A prism can also be used to construct normal lines. The slope or tangent line, of course, is drawn at right angles to the normal line.

$$\text{tangent } \theta = \frac{1.60}{1.70}$$

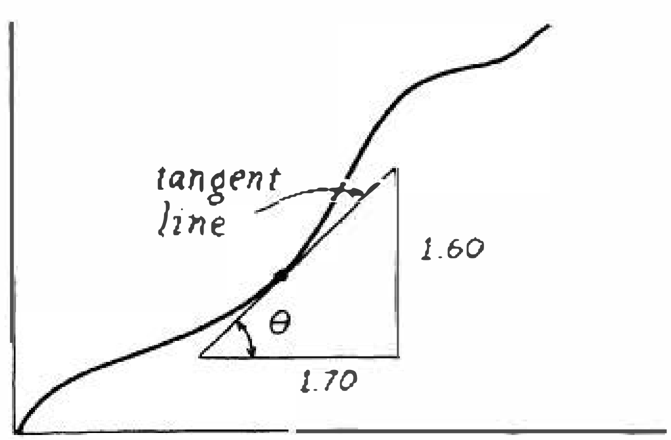


Fig. A-6. Slopes can be calculated algebraically. ("Slope" equals the tangent of the angle θ.)

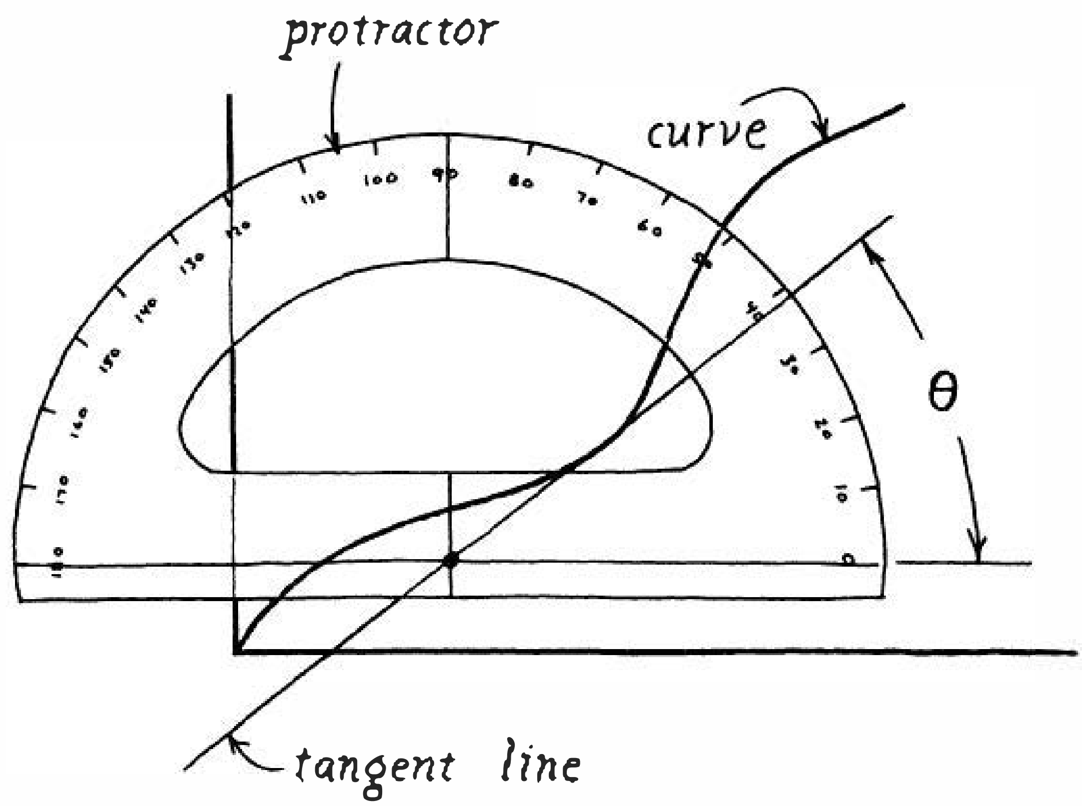


Fig. A-7. A protractor can be used to measure angle θ, whose tangent is then found in a trigonometric table.

be determined. There are three ways to construct such tangent lines which will be described here.

In many cases, you can merely draw the line in "by eye." If the curve is geometrically simple, and consists of a series of triangles, rectangles, or gentle curves, etc., then, of course, this is a very easy thing to do. If the curve is complex, the tangents you draw by eye may be in error to some extent, but this may not affect your final results too drastically if all you are after is an approximate "solution."

There are two instruments which can be used to improve the accuracy of the tangent line. One of them consists of a half-silvered mirror held perpendicular to the plane of the paper on which the original curve is drawn. (See Fig. A-4.) When the operator looks in this mirror he can see two lines. One of them is a reflection of the curve in front of the mirror, and dimly seen is the other, the remainder of the curve in back of (through) the mirror. The mirror is now rotated until these two images come together and appear to describe a single curve. A line is then drawn along the bottom of the mirror. This line is normal (or perpendicular) to the curve at the point of intersection. A drafting machine or a pair of triangles can now be used to draw a tangent line at this point, the tangent line of course, being perpendicular to the normal line.

A prism can also be used to find a normal to a curve. The prism is placed over the curve and the draftsman looks down into the prism at the image of the line which appears within it. In most orientations, the curve will appear to have been broken by the prism, which is now rotated until the curve once more appears to be a continuous line. A line drawn along the edge of the prism is again normal to the curve in this region and it can be used to construct a tangent line. (See Fig. A-5.) The prism technique, I think, is a little less accurate than the mirror technique where there are rapid changes in the curvature of the line being differentiated, but both have their place.

Once the tangent line has been drawn, we can proceed to determine the slope of that line; the slope being the tangent of the angle formed by that line and the horizontal axis. There are several ways to determine this slope.

The slope can be determined trigonometrically as shown in Fig. A-6 where we measure the height and base of the triangle formed by the tangent line and then divide one by the other to determine the tangent of the angle. Height and base, of course, must be measured in the units or scale factors used to plot the original curves.

Another technique is to use a conventional pro-

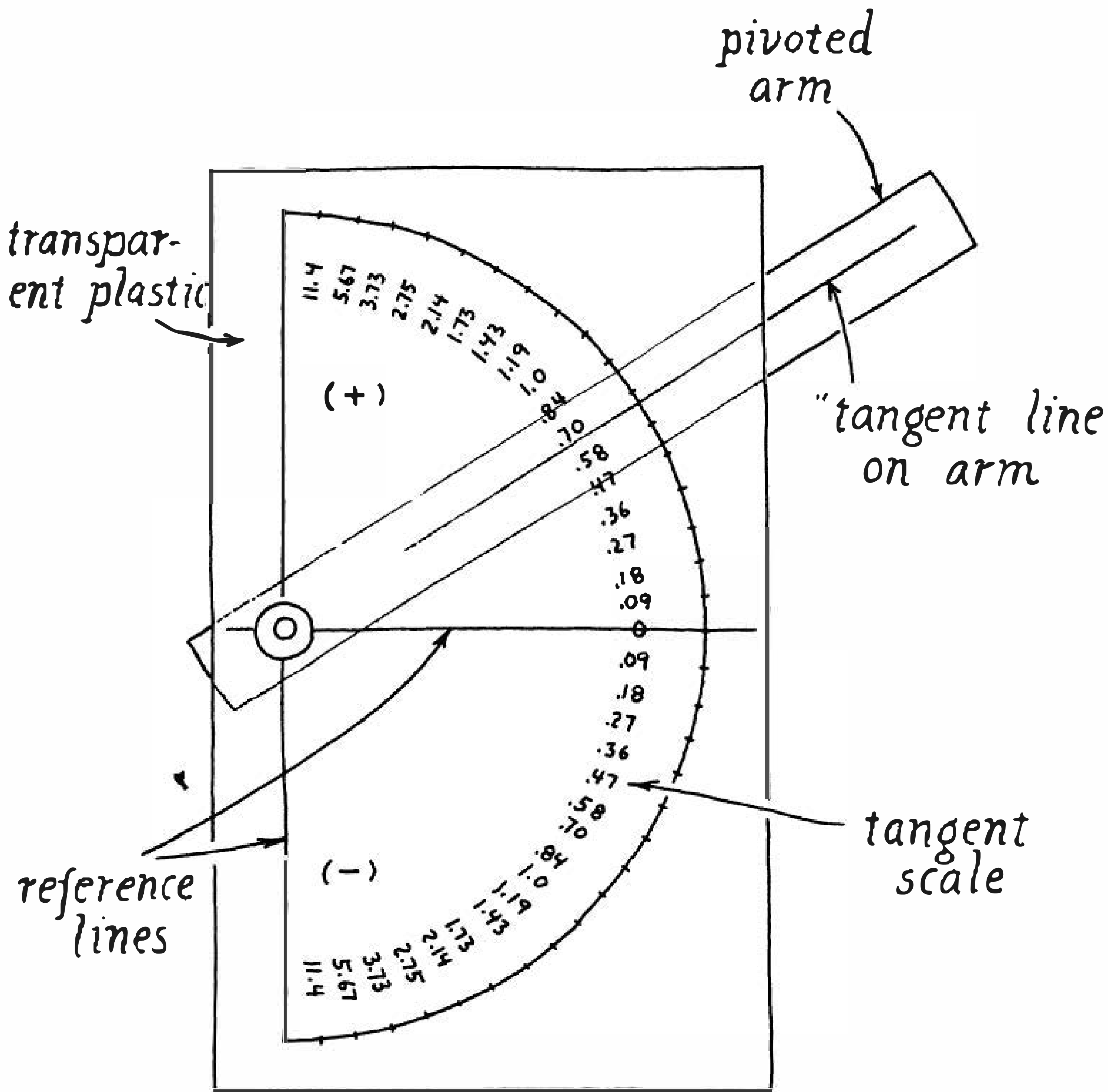


Fig. A-8. You can make a protractor of transparent plastic film (drafting film) and calibrate it in tangents rather than in angles. A line drawn on a pivoted arm is aligned with the curve, or with a previously drawn slope line. A great time saver.

tractor to measure the angle θ whose tangent is then found in a trigonometric table (slope = tangent θ). (See Fig. A–7.)

As a third method, you can make your own protractor from drafting film, calibrating it in tangents of the angles rather than in the angle itself. I have found this a very handy tool to make when there is a great deal of graphical differentiation to be done. It is even handier if the protractor is provided with a rotating arm as in Fig. A–8. A line drawn on this arm is aligned with the slope line of the curve and the tangent of the slope angle is read directly on the protractor. The reference lines on the protractor, of course, must first be aligned with the axes of the graph.

V. MOMENTS OF INERTIA

The moment of inertia of a body is, of course, a measure of its resistance to angular acceleration.

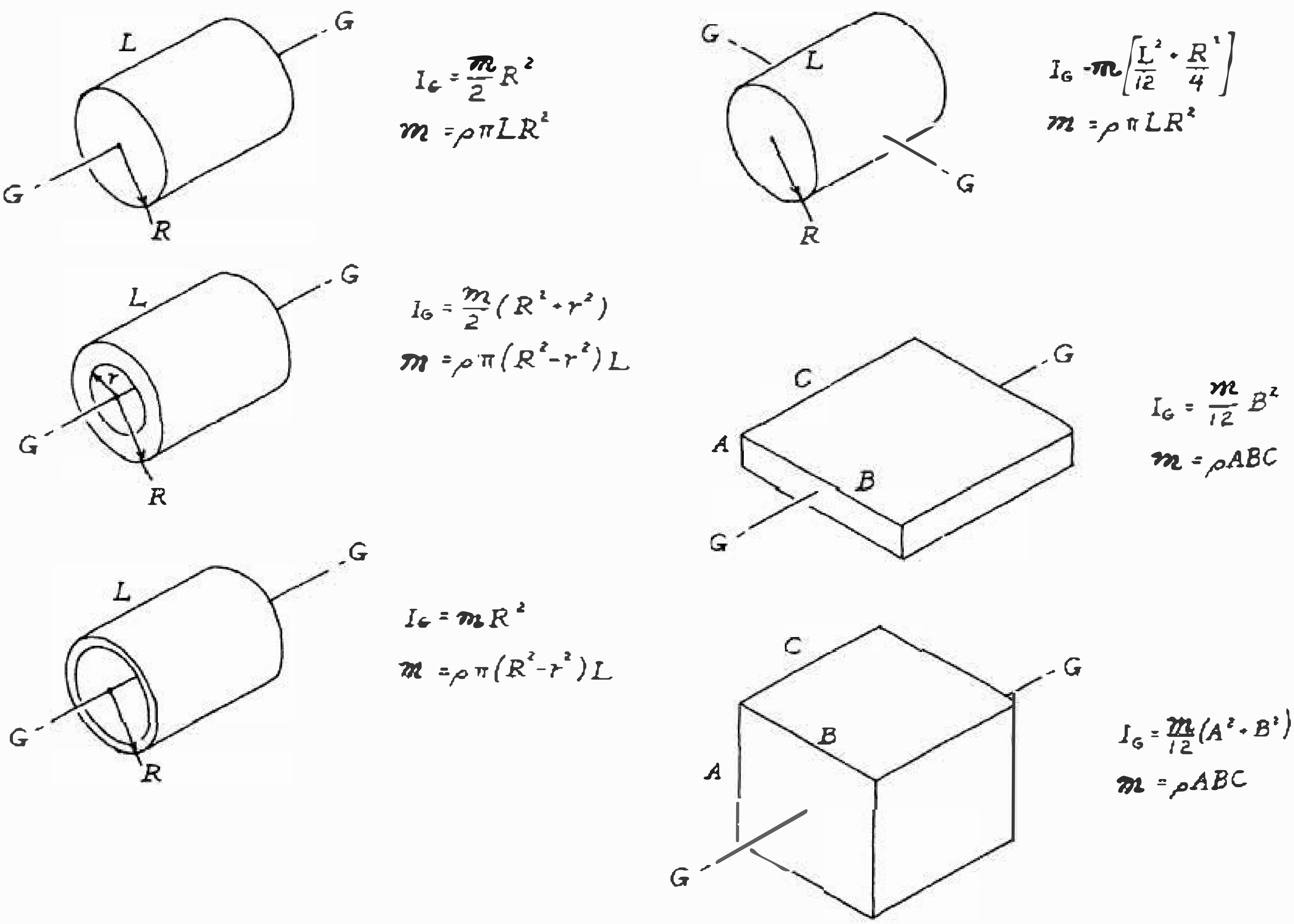
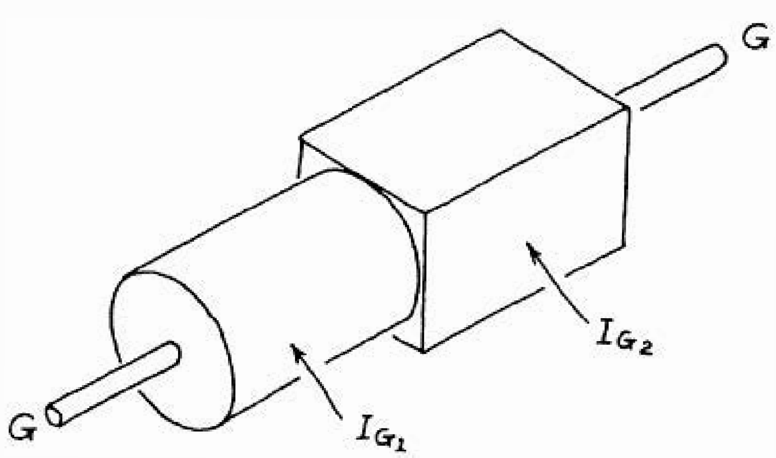


Fig. A-9. Moments of inertia for a few standard shapes. In each case the equation is for the moment about axis G–G.



$$I_{total} = I_{G_1} + I_{G_2}$$

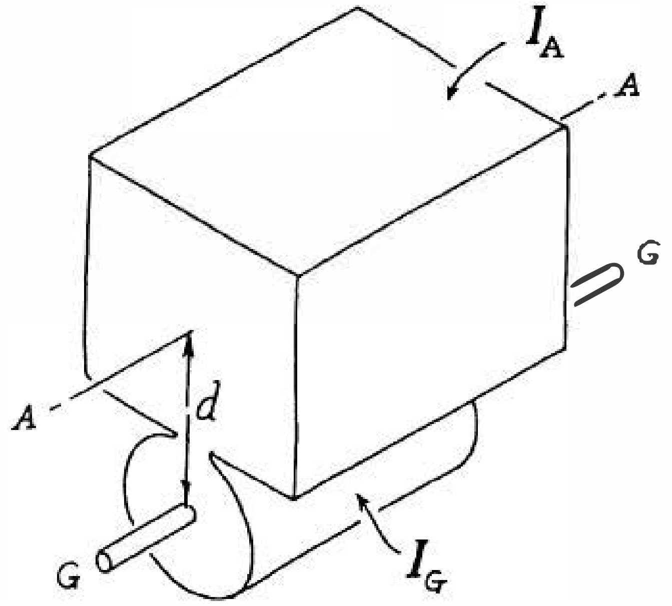
Fig. A-10. Adding inertias of simple shapes to determine the moment of inertia of a complex body. All shapes rotating about the same axis (G–G).

Moment of inertia is a function of the mass of a body and the distribution of this mass. In general, the greater the mass, the greater the moment of inertia; and the farther this mass is located from the center of rotation of the body, the greater the moment of inertia.

Figure A–9 shows several standard machine shapes and the equations for the inertias of these various bodies in terms of material density and body dimensions. More complete tables of this type are available in most engineering handbooks.

Figure A–9 shows the moments of inertia of elementary geometrical shapes only; however, most machine bodies are more complex. To obtain the inertia of a "compound" shape when all portions of the body are rotating about the same axis, the inertias of each portion are merely added together, as shown in Fig. A–10. If it is desired to know the moment of inertia of a body about some parallel axis which does not pass through the body, the procedure and equation shown in Fig. A–11 is used. If a complex body consists of one shape on the axis of rotation and another one fastened to it that is off the axis of rotation, we use the off-axis formula to compute the inertia of the second body before adding it to the first.

Frequently, we need to relate the inertias of several bodies to a single point in the machine, even though these bodies are separated from the point of interest (perhaps the drive shaft of the motor) by gearing, etc. In this case, the remote inertias must be multiplied by the square of the gear ratio between load and driver if the body is moving faster than the driver. If the body is moving slower than the driver, we divide by the square of the gear ratio as shown in Fig. A–12.



$$I_{total} = I_G + (I_A + m_A d^2)$$

Fig. A-11. Determining total moment of inertia of a complex body when portions of the body are centered on a parallel axis. Note that these "off axis" bodies are fastened to the cylinder and rotate with it, however, about axis (G–G). I_A equals the moment of inertia of the block about its own axis (A–A). M is the mass of the block.

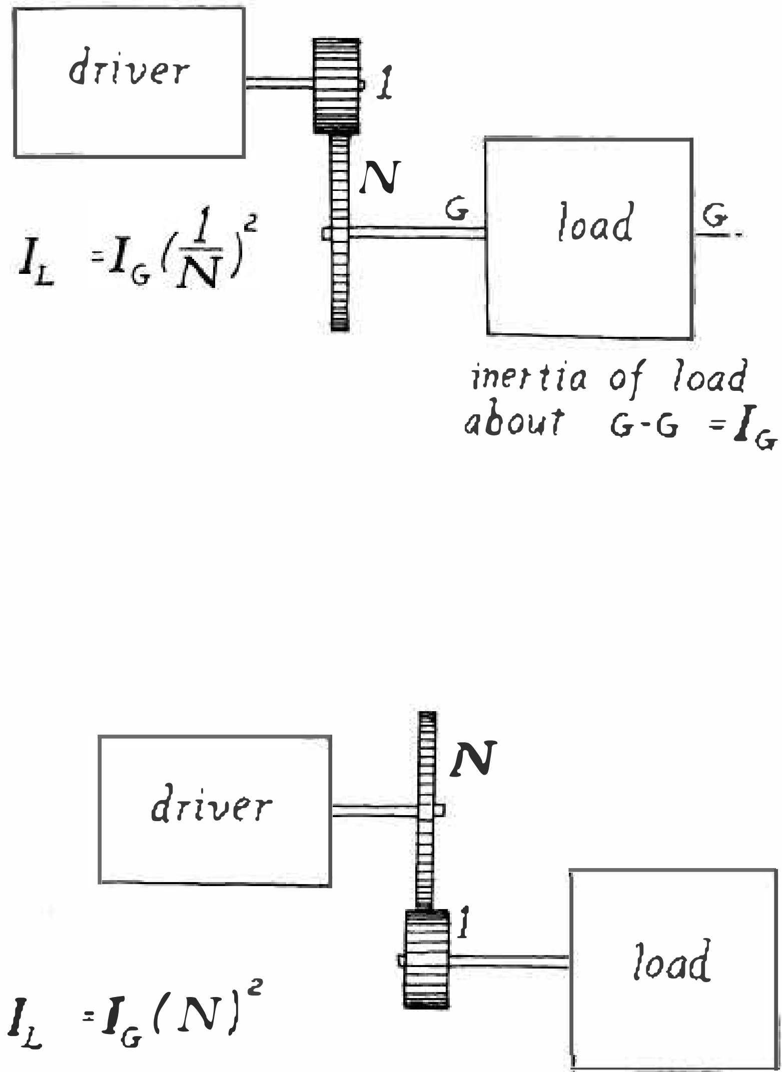


Fig. A-12. The effect of gearing on moment of inertia. In each case the calculation is for the inertia of the load as seen by the driver.

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