CHAPTER 1

Introduction and Some Basic Concepts of Mechanics

INTRODUCTION

Why We Need "Theories"

To design, evaluate, or understand any kind of mechanism it is helpful to be able to measure or predict the forces, accelerations, velocities, and displacements which occur in various parts of the mechanism. From this information the designer can then determine stress levels and performance characteristics. Will the mechanism do the job expected of it? Will it do it in the time available? Will it have the desired operating life, etc.?

But Theories Do Not Always Work

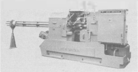
The engineer or designer acquires his understanding of force, acceleration, velocity, and displacement through his study of mechanics. Too often, however, his early struggles to apply his knowledge of mechanics to actual design problems defeat him. The abstract language of mathematics has not given him sufficient "feel" for the subject to allow him to apply it to a specific design situation, or the machine just does not seem to obey the rules which he was taught because the machine is not a collection of rigid bodies connected by ideal joints. It is, instead, a series of elastic bodies interconnected by links that introduce friction, backlash, and clearance, all subject to impact, vibration, and chatter; and it seems often to have a mind of its own and be determined never to complete the performance or life intended for it.

At this point, most beginners give up, usually decide that the theoretical approach is wrong, and that only a trial and error experimental approach gives the "correct" answer. Too frequently the result is a machine that works, but whose performance is so dependent on a delicate balance of forces and dimensions that the designer's employer has a long string of production problems ahead to contend with. Small variations in part dimensions, hardness, or material composition can lead to large and mysterious changes in machine performance and life.

Experimental Versus Theoretical Models

It would be far better if the designer had enough feel for, and patience with, theoretical design so that he could build a crude theoretical model of his machine in parallel with the construction of his experimental model. He should realize that both models are imperfect at the start—that both will require debugging and development. Thus, the theoretical model must be refined, when experiments with an actual prototype machine show that the theoretical model is inadequate, to describe the actual performance.

The physical model must be refined and debugged when its performance turns out to be inadequate, and particularly when the theoretical model shows that the performance of the actual machine will change drastically if there are slight variations made in the dimensions and in the composition of certain critical machine elements.

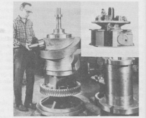


Courtesy of the New Britain Machine Company, Division of Litton Industries

Fig. 1-1. Multiple-spindle automatic bar machine uses a Geneva indexing mechanism.

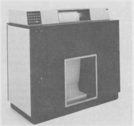
The designer who succeeds in simultaneously developing both a theoretical and physical model of a ne[illegible] reach the product[illegible] phase of the design process in full control of his new creation. He will be able t[illegible] the implicat[illegible] of [illegible]mensional, material, or configurational changes requested by tooling or manufacturing people without having to build a multiplicity of new models and subject them to life and performance tests. He will know which design tolerances to fight for and which to relax. He will know whether or not the sales department can [illegible] an import[illegible]t customer some modification in the published performance specific[illegible]ion or whether such modification [illegible] clearly impractical or even impossible. Figures 1-1, 1-2, and 1-3 show substantial end products of the successful [illegible]

Some engineers and designers now are able to do [illegible] the [illegible]ly [illegible] tools [illegible] they were taught [illegible] standard courses on [illegible] design. In my experience, however, the majority never reach th[illegible] happy state of aff[illegible], [illegible] because the ma[illegible] too abstract, or because the mathematics required for an actual design situation is, perhaps, too complex. By the time the designer has considered all the factors, his mathematical model may be so complicated that it can only be handled by large computer techniques. This is an e[illegible]ve, ti[illegible] process and only [illegible] sit[illegible]ations where huma[illegible] depend on the results (for example, in the aircraft industry) will the typical design boss tolerate the work required for such an [illegible]



Courtesy of Ferguson Machine Company

Fig. 1-3. Large cam used to drive a loaded indexing table weighing over 13,000 lbs, 45 degrees in ¾ second.



Courtesy of IBM

Fig. 1-2. High-speed [illegible] machine uses a variety of intermittent motion mechanisms

[illegible]

It is also possible, however, to build a rough, graphical theoretical model of a new design, rather than a mathematical one. This approach has several things to recommend it. First of all, you use basically one technique (plot a curve) to solve all types of problems, rather than a mathematical approach where you handle each type of equation, i.e., linear motion, harmonic motion, damped harmonic motion with shock [illegible]xcit[illegible]ion, etc., differently. With graphics you always follow the same step-by-step procedure regardless of the simplicity or complexity of the original data.

The graphical approach is started with a measurement or prediction of one of four quantities: force, acceleration, velocity, or displacement. It might, for example, be decided that the machine is to move through a certain pattern of motion in a certain time. This would provide a prediction of the displacement of the machine as a function of time. Simple graphical techniques are now used to estimate the velocity, acceleration, and forces which the machine must experience to perform the desired displacement cycle.

It should be emphasized that this is only a sketchpad procedure. It does not lend itself to exact answers unless graphs and charts are drawn with great precision. At first glance, then, it would appear that graphical models suffer in comparison with mathematical models, but I maintain that in most machine design situations, mathematical models do not produce exact results either. They can produce exact results only when they consider *all* factors and have exact inputs, but this is hardly ever possible in a real design situation. The mathematical model, then, can only give approximate answers or indicate trends as changes in dimensions of a design are explored. A graphical model can do this also, and, I think, do it more readily. Its obvious visual message is far more enlightening to most designers than a page full of formulas and a table of numerical results.

The graphical approach to mechanics, furthermore, is ideal for a study of intermittent motion mechanisms, because here an exact mathematical model is not only difficult to build, it is often virtually impossible. There are too many unknowns, too many variables, to allow anything but an approximate solution for certain special and highly simplified cases of little interest to a practical designer. The graphical approach, on the other hand, bolstered by certain fairly easy-to-acquire experimental data, can show the designer approximately where he stands and in approximately which direction he should change his design to improve its performance and production life.

SOME BASIC CONCEPTS OF MECHANICS

Before we tackle graphical mechanics in a serious way, we must understand a few basic concepts such as force, velocity, angular acceleration, etc., and

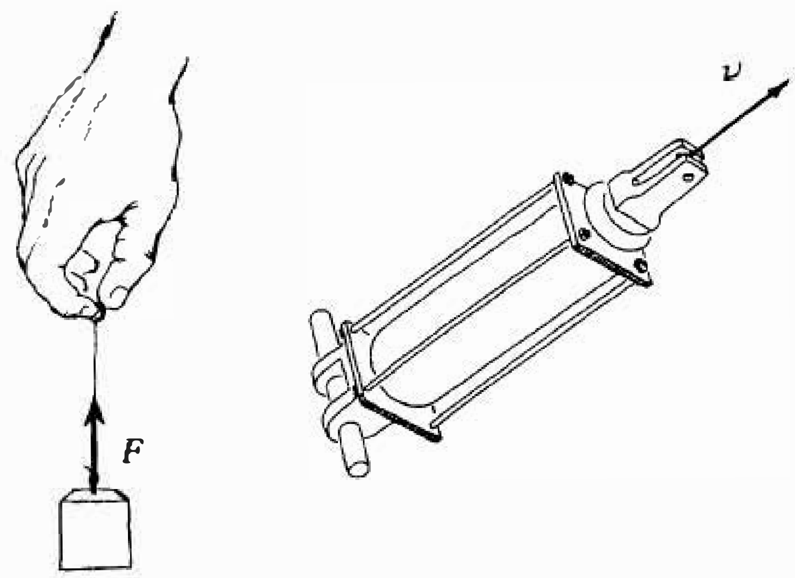


Fig. 1–4. Force and linear velocity vectors.

learn how to relate these to actual machine parts. This discussion will not be complete in a textbook sense; it is not intended to be a course in mechanics. It is expected that the reader will have had such a course and will use the material given here merely to refresh his understanding of some of the elementary ideas of mechanics needed in the discussion of intermittent motion and its mechanisms.

Vector Representation of Force and Velocity

Many things can be represented by *vectors*, which are mathematical quantities representing direction and magnitude, and are usually illustrated as small arrows. Linear and angular displacement; velocity and acceleration; force; torque; momentum; and impulse are all vector quantities. We will be concerned, however, with vector representation of only two of these: force and linear velocity. We will be using several of the others; angular velocity and torque, for example, but will be interested only with their scalar properties; that is, we will be concerned only with their magnitude and not with their vectorial *direction*. Figure 1–4 shows the vector representation of the force exerted on a block by a man, and the vector representation of the linear velocity of the piston of an hydraulic cylinder. Both vectors are shown as arrows whose directions indicate the direction of the force and velocity, respectively, and whose lengths represent the magnitude of those two parameters.

Resultants of Two Vectors

It is possible to add two or more vectors together to find what is called a *resultant vector*; a single

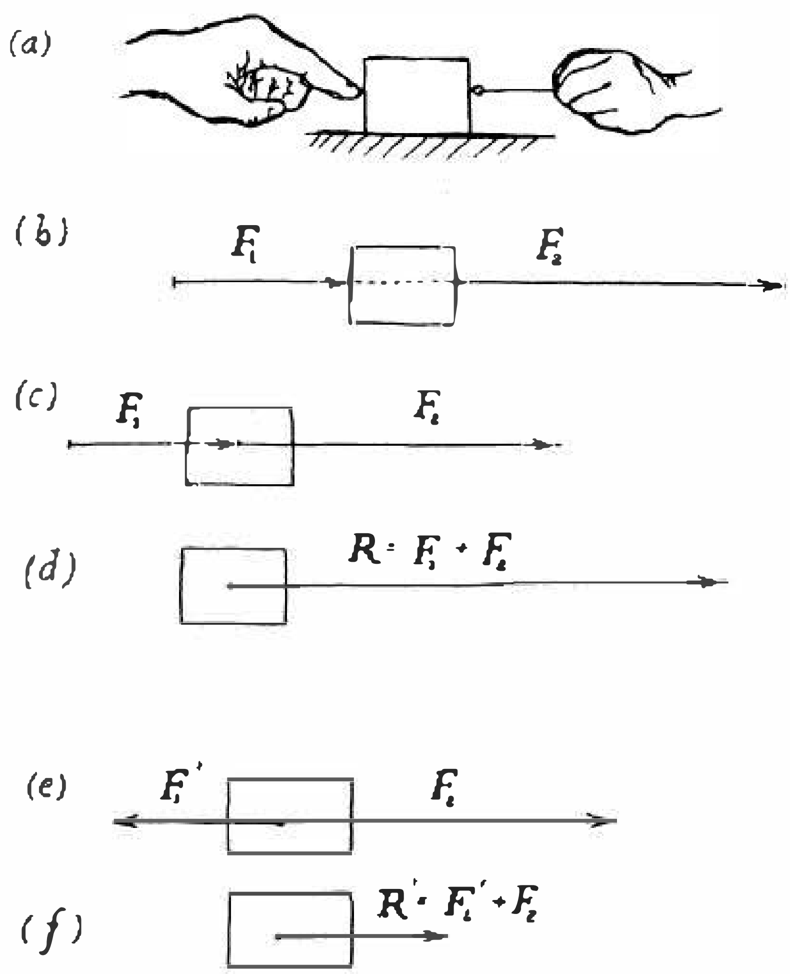


Fig. 1–5. Finding the resultant of co-linear vectors.

vector that combines or summarizes all the properties of the original group. In Fig. 1–5a, for example, one man is pushing on a block while a second is pulling on it. The two forces exerted on the block are shown in vector representation in Fig. 1–5b. Since both of these forces are: Fig. 1–5b, acting along the same line, and, Fig. 1–5c, pass through the same point in the block, they can be added together, Fig. 1–5d, by drawing a single vector, equal in length to the sum of the original two, and pointing in the same direction as the original two. If one of the two men pushed (or pulled) with a force of this magnitude and direction he would produce the same effects on the block as the two men produced in the original situation.

If both men pulled on the block in opposite directions, Fig. 1–5e, instead of one pushing and one pulling in the same direction, the resultant vector would be the difference between the two original vectors, and would be in a direction determined by the stronger man, as shown in Fig. 1–5f.

Vectors need not be acting along the same line to be added. In Fig. 1–6 we show the steps required to add two force vectors that are acting through a common point on a block but are not acting along a common line. Assume that the block is falling under the influence of its weight W, which acts through the center of gravity *cg*, as shown in Fig. 1–6a. This produces one vector force on the block in a vertical downward direction. As the block falls, a second force F_1 is exerted on the upper right-hand corner, as shown in the illustration. Notice that the line of action of force F_1 also passes through the center of gravity of the block. This is very important. The type of vector addition we are about to perform cannot be done unless all the forces involved have lines of action that pass through a common point.

Since these forces do have lines of action that pass through a common point, the center of gravity, we can add them together as shown in Figs. 1–6b and 1–6c. F is first "moved" along its line of action until its "tail" coincides with the tail of the vector W (Fig. 1–6b). A vector parallelogram is then constructed to find the resultant R. The net, or total force on the block is then represented by vector R, alone. The block would tend to move downward and to the left, as a result of the action of these two forces.

Again, this is just a refresher course on vectors; we assume the reader has already been exposed to these topics. If not, he is urged to consult a mechanics book if he cannot understand or remember the principles discussed here.

It is also possible to find the resultant of more than two vectors, or to find the resultant of two or more vectors whose lines of action do not pass through the same point, but these constructions will not be necessary for the purpose of this book.

Everything that has been said here about force vectors, of course, applies to linear velocity vectors as well. They can also be added, subtracted, etc., in the same way.

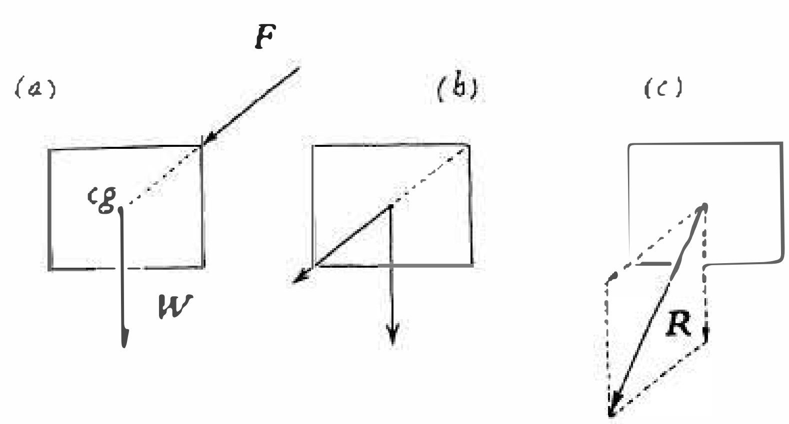


Fig. 1–6. Finding the resultant of two vectors whose lines of action pass through a common point.

Components of Vectors

We can reverse the process described above and convert a resultant, or single, vector into two or more components whose combined effect is equivalent to that produced by the original single vector quantity. This is frequently helpful in determining forces or velocities in certain directions that are of special interest in a given design problem. The component vectors do not have to be at right angles to each other. In Fig. 1–7, for example, five possible ways of breaking a single vector down into two components are shown. An infinite number of component pairs are possible and all are equally correct. It is only necessary that the original vector be the diagonal of the parallelogram defined by the two component vectors that replace it.

Again, the purpose of doing this is to determine what is happening in directions of special interest in a given design situation. In Fig. 1–7e, for example, the heavy vector (original resultant) describes the instantaneous linear velocity of a drive pin. This velocity vector has been broken down, for study purposes, into two components; one showing the instantaneous velocity of the pin along the axis of the slot; the second showing the instantaneous velocity of the pin at right angles to the slot. As we will see later, the ability to determine vector components such as these is essential if we are to study the various velocities of different parts of a machine.

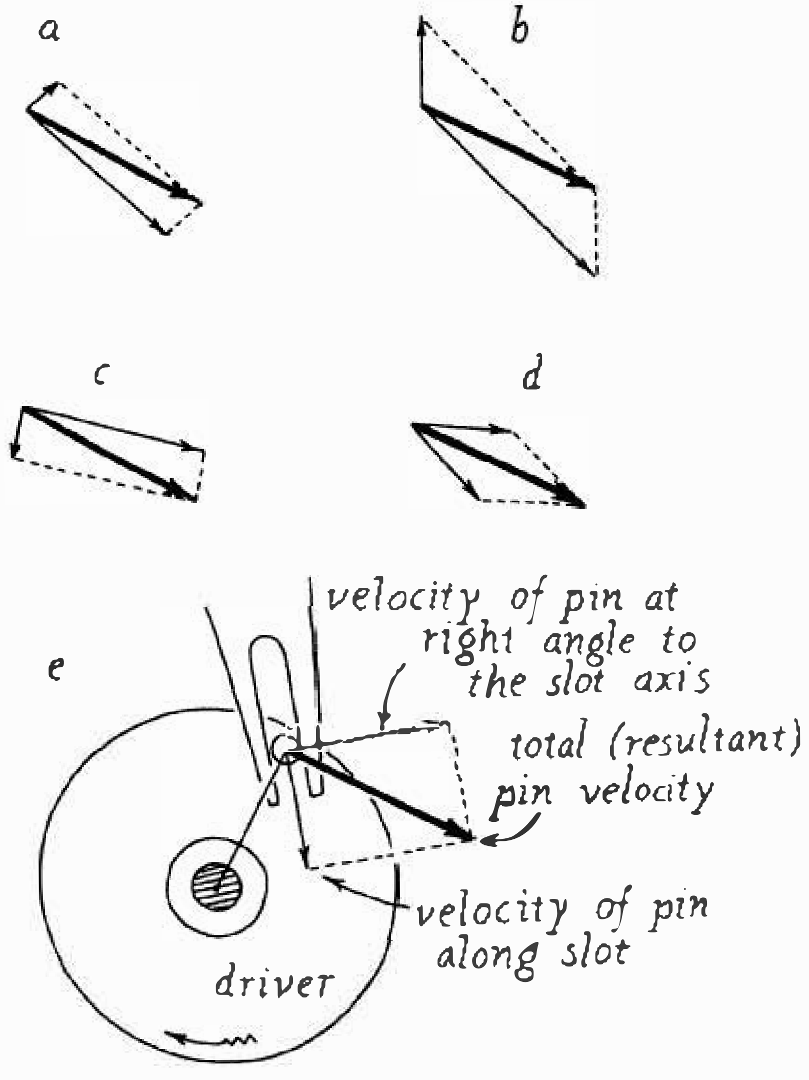


Fig. 1–7. Five ways to divide the same vector into two components. Each of these groups is correct, and there are many other available choices.

Free Bodies

Whenever we use vectors to describe the forces acting in a given design we must be careful not to confuse the forces acting *on* a body with the forces produced *by* the body on something else. To avoid this difficulty we make use of the concept of the *free body*, which can be either a single machine part or an assembly of parts, but which, in every case, is treated as a single item. In Fig. 1–8a, for example, an intermittent motion mechanism is shown, which consists of a heart cam, a spring and solenoid drive system, and a drive linkage. At the moment shown in the picture the solenoid has been actuated and the linkage and cam are moving in the directions shown by the small arrows with broken tails. (These arrows will be used throughout the text to show direction of motion; they are not vectors as they show direction only.) In Fig. 1–8b, c, d, and e are shown the heart cam, the upper drive roller, one bar of the linkage and the rest of the system, as four separate free bodies. Shown in each case are the forces acting *on* that free body only; never the forces produced *by* that body on something else. We could have treated the other linkage parts, the solenoid, the spring, etc., as separate free bodies instead of the assembly shown in Fig. 1–8e if we had wished.

Looking at Fig. 1–8b, we see the forces produced by the drive roller and the supporting shaft *on* the heart cam, which is the free body in this case. Each force is also accompanied by a small friction force represented by a vector at the base of the main vector and at right angles to it, since that will be the direction and relative magnitude of these friction forces. We could have shown each pair of forces, friction and main force, as a single resultant, of course, but chose to show them this way, since we are usually interested in the effects and directions of the friction forces separately.

The force produced by the roller on the cam is mirrored by the force produced by the cam on the roller as in Fig. 1–8c, where the roller is now the free body. Every force is accompanied by an equal

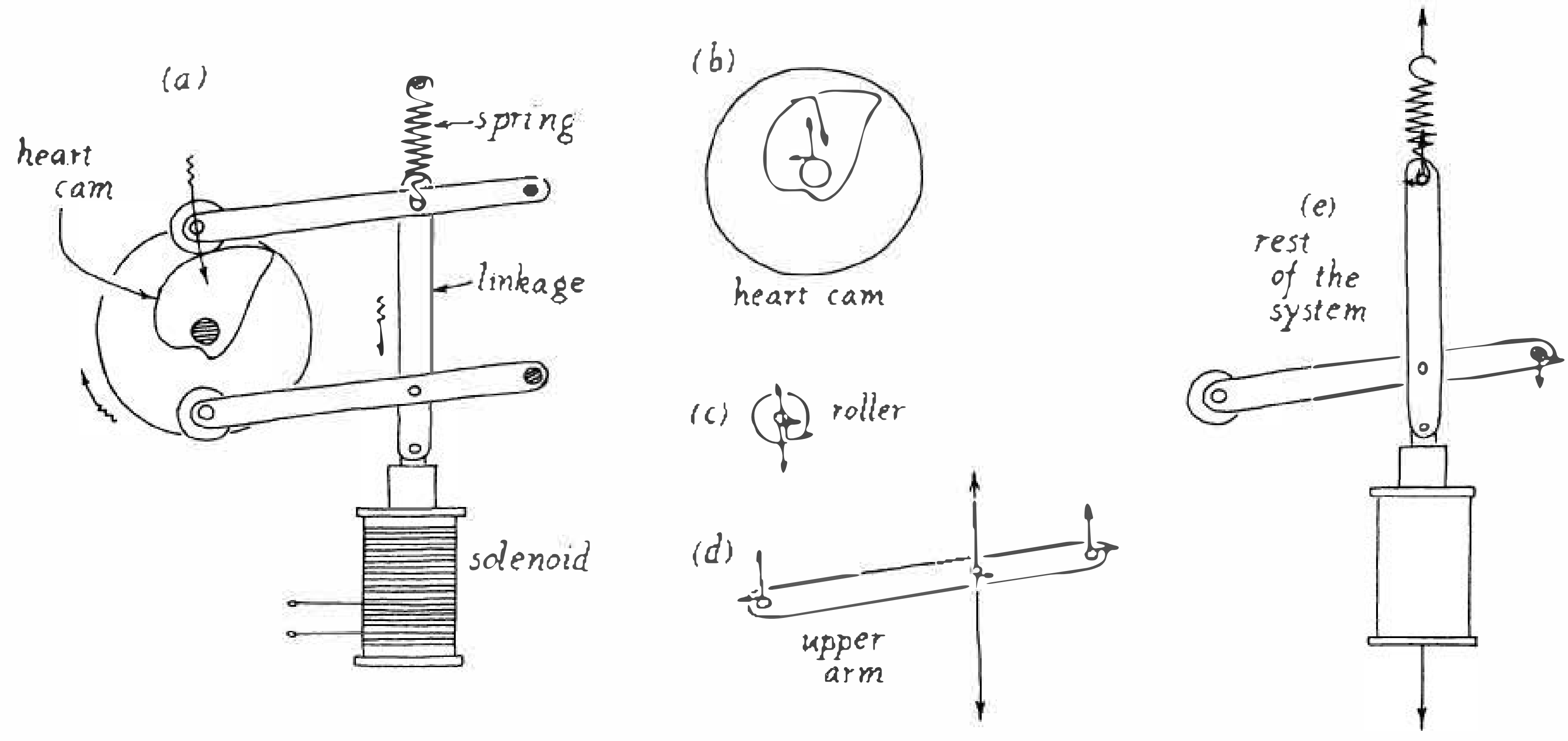


Fig. 1–8, Dividing an intermittent motion mechanism into free bodies for force or velocity analysis. The free bodies can consist of individual parts or groups of parts.

and opposite force somewhere in the system, as Newton taught us, and as you learned in your mechanics course. The drive roller is also acted upon by the upper arm.

The upper arm, in turn, is acted upon by the roller, the spring, the other parts of the linkage, and the upper-right pivot shaft. The rest of the system is acted upon by the table on which the solenoid is mounted, the pin which restrains the upper end of the spring, the upper arm, and the lower-right pivot shaft, as shown in Fig. 1–8e.

Again, in each case, we have shown the forces acting on the part or group of parts selected as the *free body*. We have never shown forces acting within the body, or produced *by* the body *on* something else.

Determining the Directions of Forces in Machine Bodies

Most designers could probably identify the locations of the forces acting on the various free bodies in Fig. 1–8, but many would have difficulty in determining the actual direction of these various forces. The direction of a force is not determined by the direction of motion of the body producing that force, but rather by the shapes of the bodies involved and the presence or absence of friction.

Shown in Fig. 1–9, for example, is the force exerted by a driver cam on a load cam at one instant in time. This force F_D is perpendicular to a line that is tangent to both surfaces at the point of contact. Notice that F_D is not tangent to the arc described by the instantaneuos contact point of the driver, nor the arc described by the instantaneous contact point of the load. Instead, the direction of F_D is determined by the shapes of these two surfaces. In the illustration the two surfaces are assumed to be

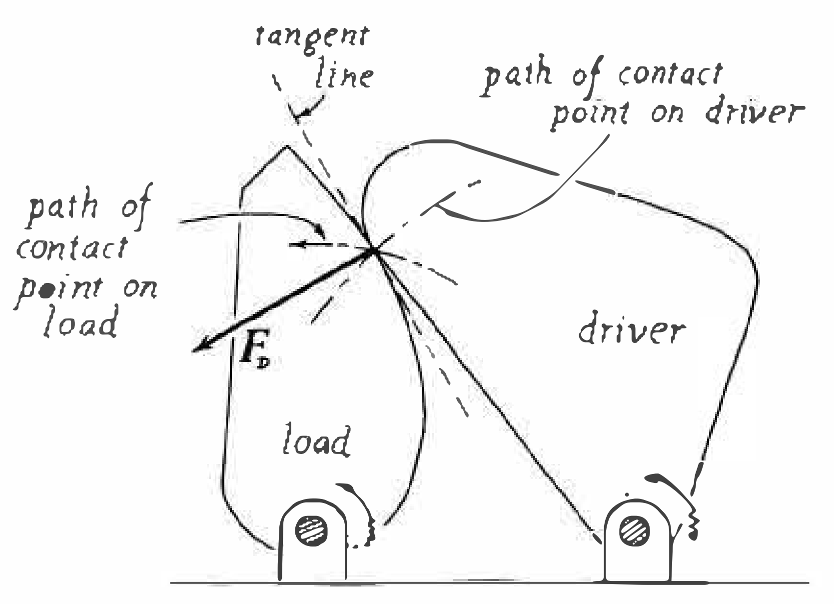


Fig. 1–9. Determining the direction of force exerted by a drive cam on a load cam, assuming that both cams have frictionless surfaces. The resultant force will be perpendicular to the cam surfaces at the point of contact, and its direction will not be influenced by the direction of motion of the contact points on the two cams.

frictionless and so the only force which can be produced by one cam on the other is a force normal to their surfaces at the point of contact.

It is also useful to consider the torque produced on the load by the driver in Fig. 1–9, and the torque required of the driver. These are not necessarily the same. Since torque is the product of force-times-radius (with the radius drawn perpendicular to the line of action of the force), we see from Fig. 1–10 that the torque generated by the driver is $F_D \times r_1$ and that the resulting torque produced on the load is $F_D \times r_2$ (units in both cases: ft-lbs). Assuming no friction, there is no loss of energy in the system. Due to the respective cam contours, the load will

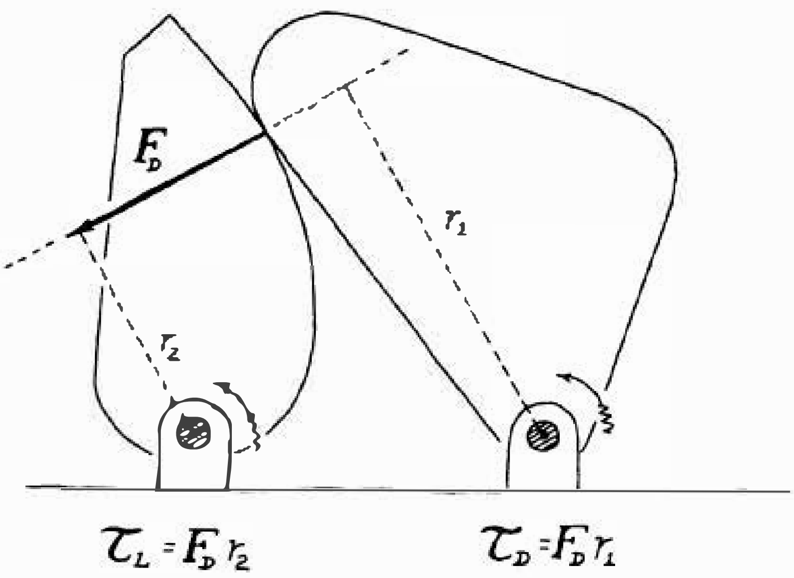


Fig. 1–10. Determining the torque required of the driver and produced by the driver on the load in a frictionless-drive situation.

move through a larger angle than the driver, during their operation, and the input work (torque times input displacement angle) will be equal to the output work (torque times output displacement angle).

Including Friction

Figure 1–11 shows the same pair of cams at the same instant of time, but we are now back in the real world and will consider the effect of the friction forces generated between the cams during their operation. The driver still produces a normal force F_D, on the load. This, in turn, produces a friction force F_F, as shown in Fig. 1–11. The direction of the friction force is determined by the direction of motion of the driver over the surface of the load cam. Remember that we are drawing the forces produced by the driver on the load. If you study the illustration

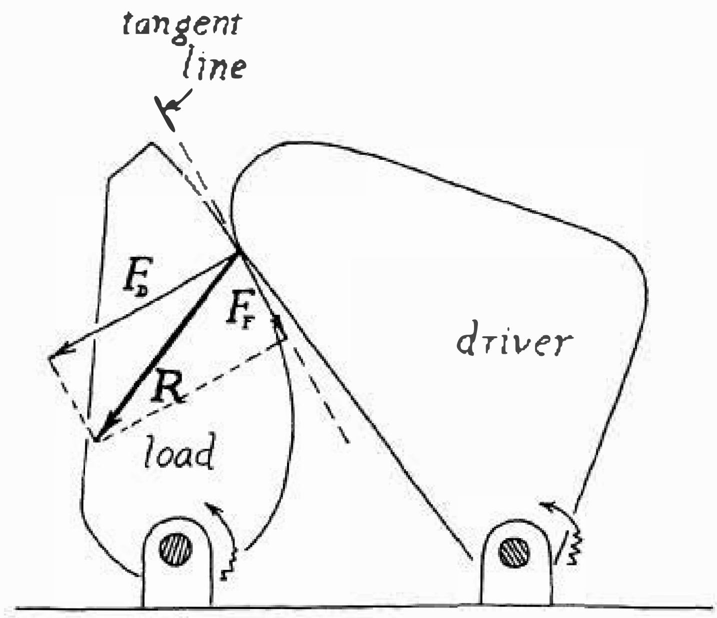


Fig. 1–11. Determining the direction of the resultant force produced by a drive cam on a load cam when friction is present.

a minute, you will see that as these two cams continue to turn, the driver will move down over the surface of the load cam, pulling or scraping on it in the direction shown by the friction force vector.

The resultant force produced by the driver on the load is the sum of the normal force F_D, and the friction force F_F, and is shown as resultant R in Fig. 1–11.

Calculations of drive torque and the torque produced on the load by the driver must also take the friction force into account. This is most easily done by basing the calculations on the resultant rather than on the friction and normal forces separately. In Fig. 1–12 we see that the torque on the load is

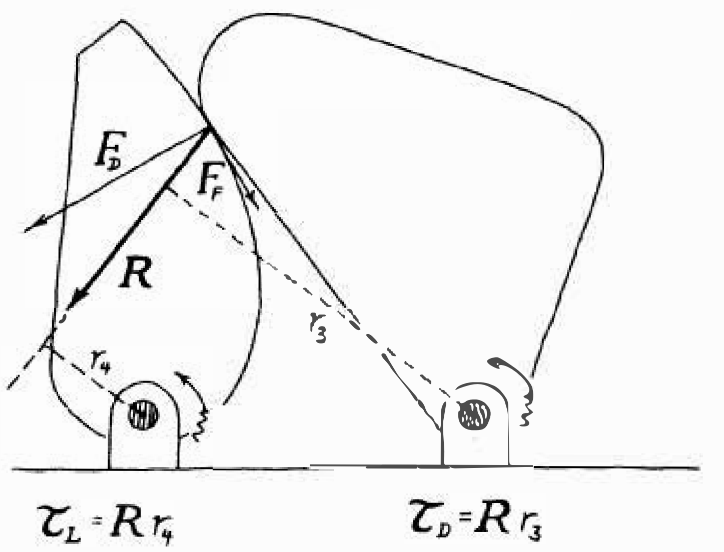


Fig. 1–12. Determining the torque required of the driver, and produced by the driver on the load when friction is present.

$R \times r_4$ and that the torque required of the driver is $R \times r_3$. (Again, the units for the torque are ft-lbs.)

Although net resultant R is slightly longer than the normal force F_D, you will find that the torque produced by the driver on the load, in this case, is less than that of the frictionless case discussed earlier, because r_4 in Fig. 1-12 is much smaller than r_3 in Fig. 1-10. The useful work transmitted to the load from the driver has been decreased by the friction, as we would expect. This is not always the case, however. It is possible to design a cam system in which the friction forces at the drive surface aid the drive force. So-called recess-action gears use this principle, for example.

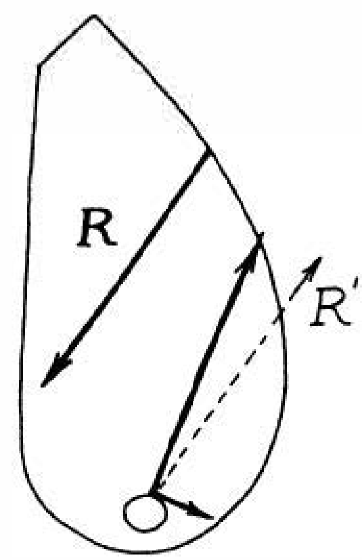


Fig. 1-13. Possible direction of pivot reaction forces on the load cam of the system shown in Fig. 1-12.

Direction of Pivot Forces

What about the direction of the forces produced by the pivot on the load cam in Fig. 1-12? How do we determine their direction? The answer again comes from basic mechanics. Remember the concept of equilibrium? If the net linear motion of a body in the horizontal direction is 0, as it would be in the case of a cam rotating about a pivot, then the net resultant of all the forces in the horizontal direction on the body must also be zero. Similarly, the forces in the vertical and perpendicular directions must be 0 if the body does not move in those directions either. From these facts we write "equations of motion" which will enable us to determine the so-called X, Y, and Z forces produced by the pivot on the body, and ultimately, the resultant force produced by the pivot on the body. The real-life situation is complicated by the fact that the pivot force consists of a reaction or supporting force and a friction force, and that we usually want to know these separately. The equilibrium equations only give us a resultant. In many cases, we must assume frictionless pivots to avoid what is called a "statically indeterminate situation" and arrive at any conclusion at all!

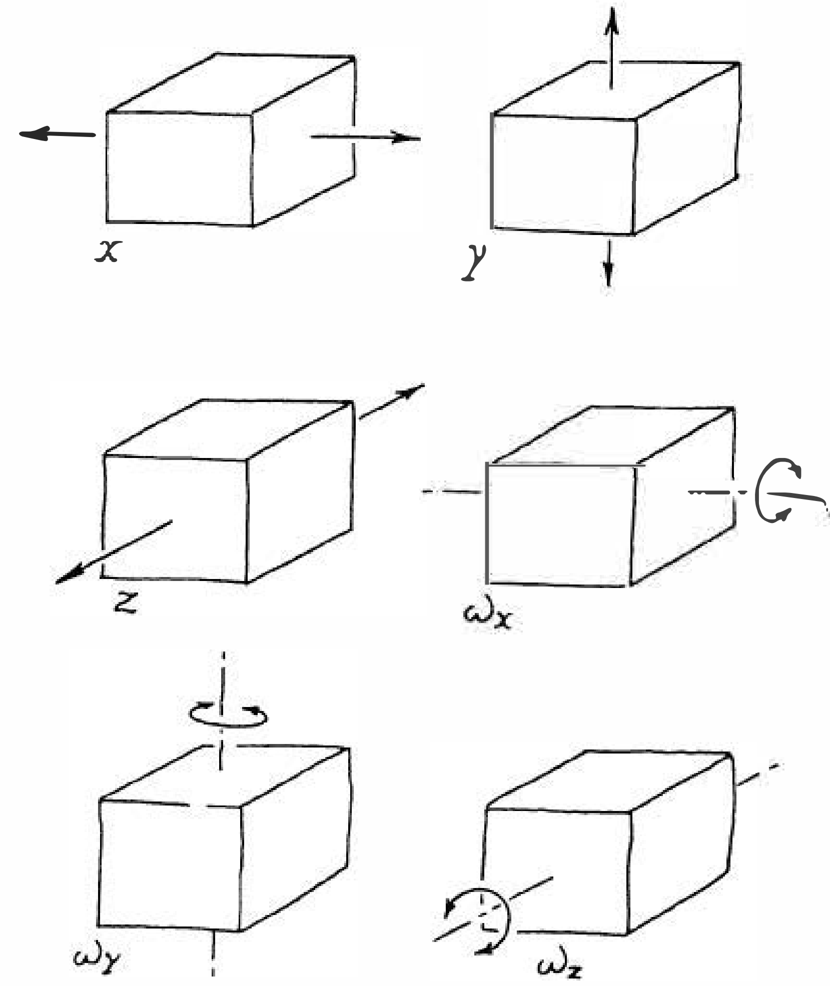


Fig. 1-14. A free body has six degrees of freedom; three in translation and three in rotation, as shown. Any set of mutually perpendicular axes could be chosen.

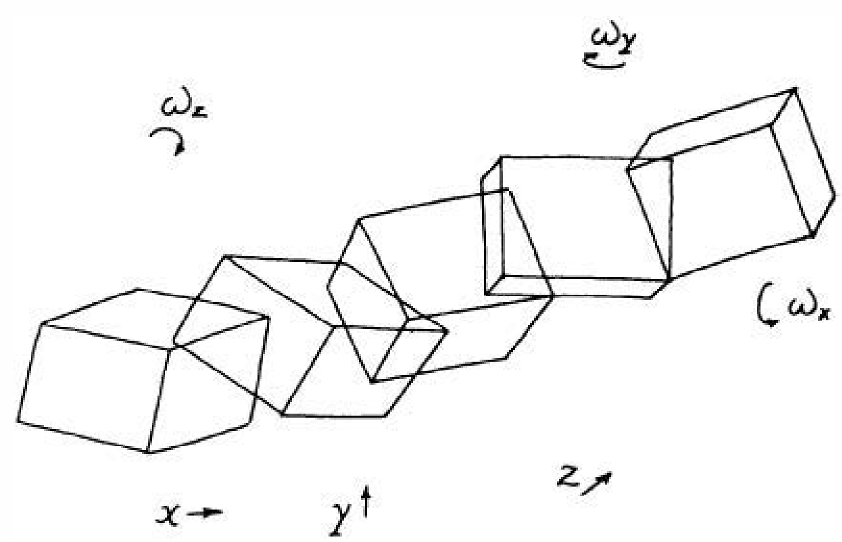


Fig. 1-15. A truly free body can have simultaneous motion in all six degrees of freedom.

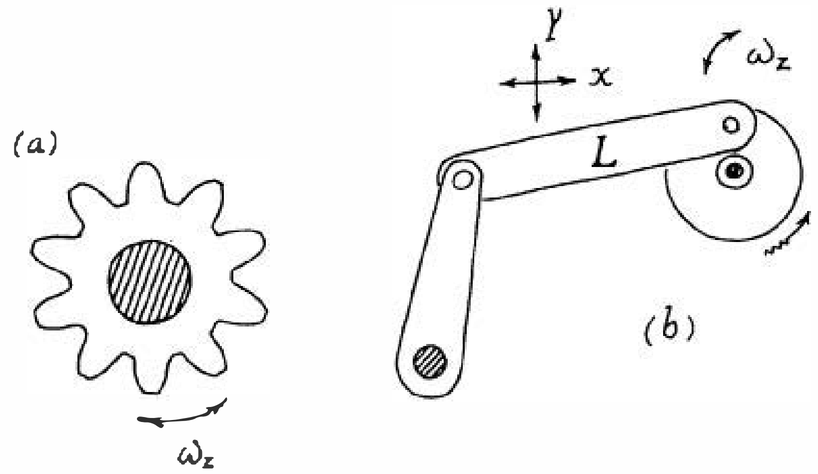


Fig. 1-16. Most machine bodies are constrained by bearings and structures and thus have limited numbers of degrees of freedom. The gear, above, can move in only one way. Linkage L, can move in three ways simultaneously, but only for a limited distance.

Fortunately, we will not need to get into all of this to understand intermittent motion mechanisms, nor will we be using equations of motion here. But the designer should be aware that pivots and supporting structures do, indeed, exert supporting and reactionary forces on bodies in response to those produced by the loads and driving members, and he will sometimes want to predict the approximate

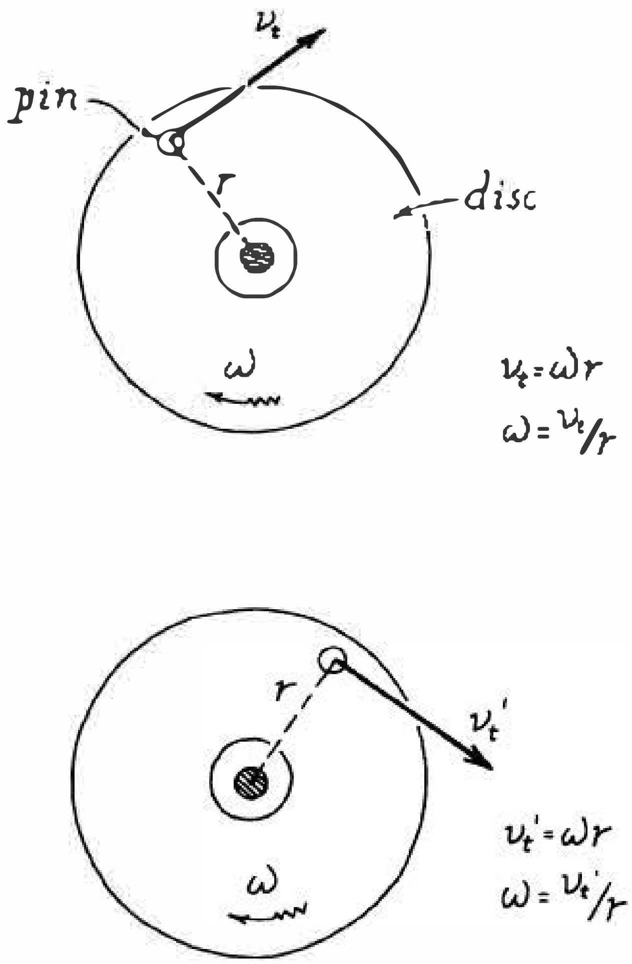


Fig. 1-17. Instantaneous tangential velocity of a pin on a disc. The disc is assumed to be rotating with a constant angular velocity, ω.

directions of such forces and reactions. Figure 1-13 shows a "guess" of the resultant pivot force R', and the normal and friction components of R', for the load cam of Fig. 1-12. If the discussion above is not a sufficient reminder for the designer, he must turn to his book on mechanics for a more detailed discussion.

Degrees of Freedom

Every free body has what is called six degrees of freedom, as shown in Fig. 1-14. The body can have

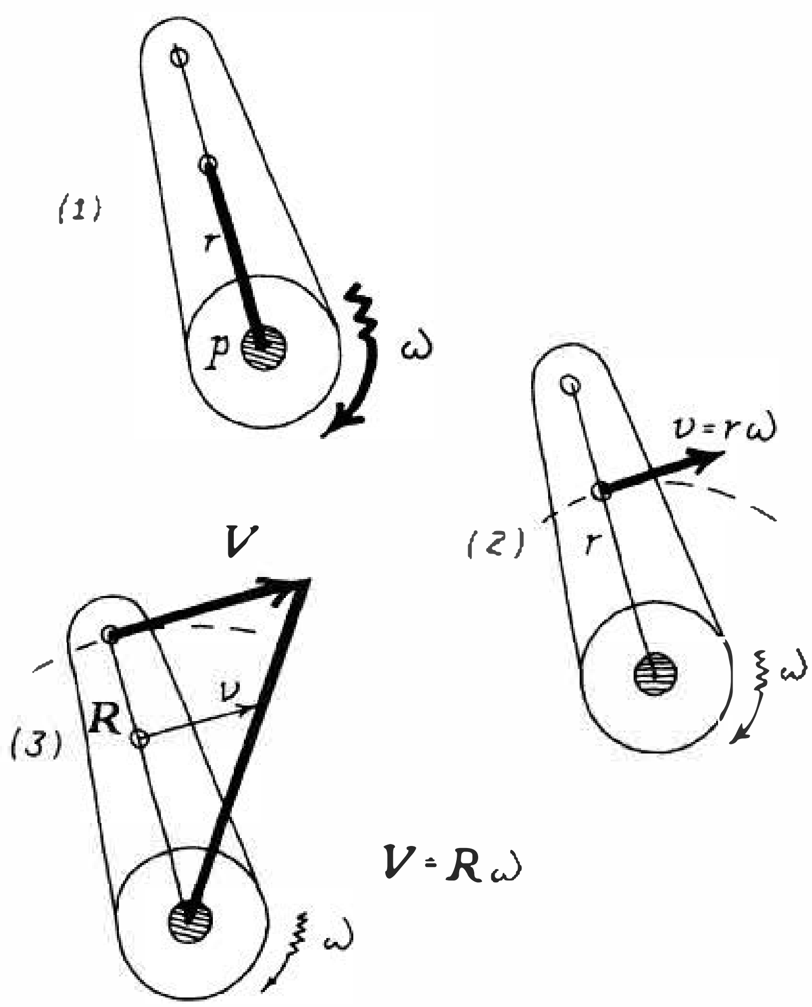


Fig. 1-18. Showing the relationship between the instantaneous tangential velocity of two pins located at different radii on a rotating link.

linear motion in the X, Y, or Z directions or it can rotate about the X, the Y, or the Z axes. The choice of these three axes is not fixed. You can choose any three directions that suit your problem, as long as they are mutually perpendicular.

A truly free body can move in a very complex fashion, as suggested by Fig. 1-15. At first glance, this would appear to be motion not defined by any of the six degrees of freedom shown in Fig. 1-14. Mathematically, however, complex motion is merely

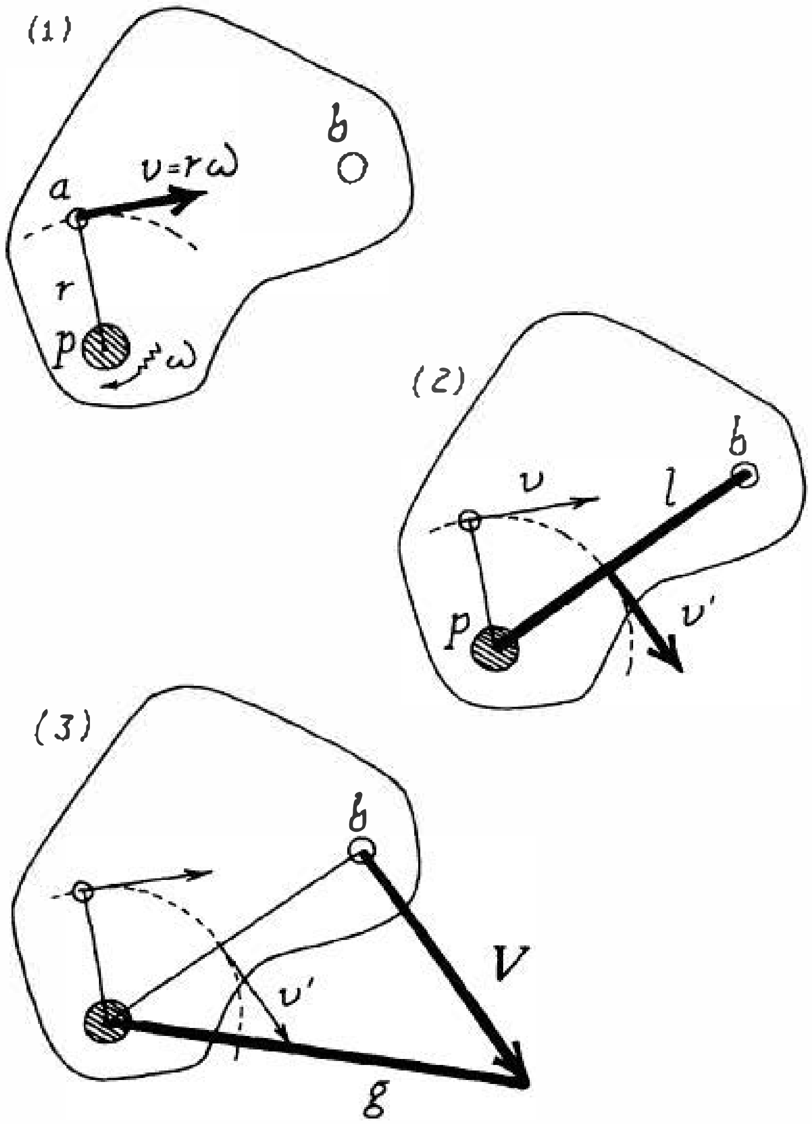


Fig. 1–19. Graphical construction to find the instantaneous linear velocity of pin *b*, on an irregular link, when the instantaneous linear velocity of pin *a*, is known. (Pin *a*, for example, might be driven by a crank whose velocity has previously been determined.)

(1) Known: v, r.

(2) Construct v' tangent to the arc (the dotted line describing the path of pin *a*), starting on line *l*, as shown. v' is equal to v, in length, since both are the same distance from center, *p*.

(3) Construct v parallel to v', using construction line, *g*. This is the same triangle construction used in Fig. 1–18.

a combination of two or more of the six basic motions allowed the body. In Fig. 1–15 the body is moving simultaneously in the X, Y, and Z directions, while rotating simultaneously about the X, Y, and Z axes.

In our analysis of machine motions, we will sometimes study the simultaneous motion of a machine link along or around more than one axis. Fortunately, however, machine bodies are always constrained to some extent, by bearings and supporting structure, and so can never (I suppose *never* is overly optimistic!) go through the gyrations suggested by Fig. 1–15. The gear in Fig. 1–16a, for example, can only rotate about the Z axis, which we assume is perpendicular to the page. It cannot translate (move linearly) along the X or Y axes without shearing the shaft on which it is mounted. It cannot move along that shaft (along the Z axis) without presumably breaking the pin or collar which holds it to the shaft. And it cannot rotate about the X or Y axes without breaking the shaft. Thus, we need only consider one degree of freedom to analyze its normal motion.

Link L, in Fig. 1-16b, is allowed a little more freedom of motion. It is permitted a little motion in the X and Y direction and can rotate through a small angle about the Z axis, but rotation about the X and Y axes and translation along the Z axis is prohibited by the way L is connected to the other members of the linkage train.

Velocities of Machine Members

The velocity of a body is officially defined as the rate-of-change of displacement of the body as a function of time. Velocity is a vector quantity, a measure of both magnitude and direction.

Linear velocity is relatively easy to understand and visualize. An automobile has a velocity of 60 miles per hour in a direction 23 degrees east of north, for example. Velocities associated with rotating parts are a little more difficult to understand, since both linear velocities (rate-of-change of linear displacement as a function of time) and angular velocities

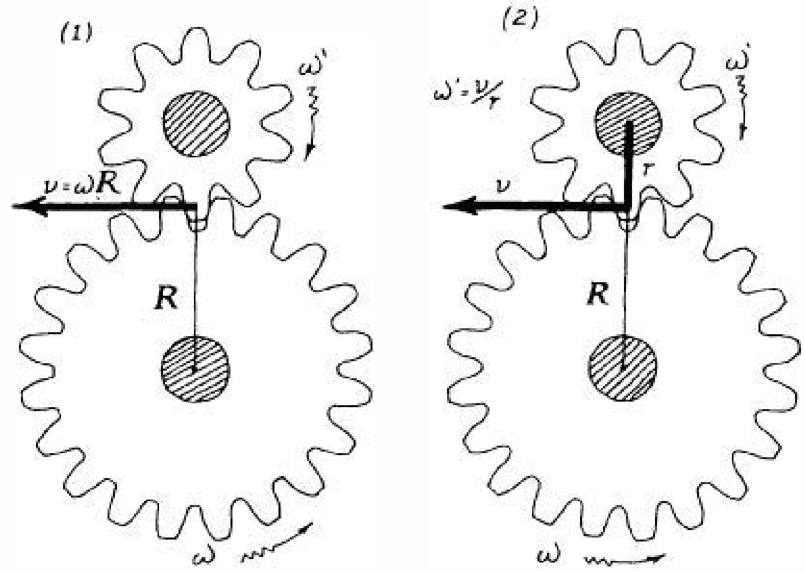


Fig. 1–20. Graphical construction for finding the angular velocity of one spur gear when the angular velocity of a mating gear is known.

(1) Known: ω, R.

(2) Construct v, as shown in (1), with a magnitude $v = \omega R$.

(3) Construct r as shown in (2).

(4) The angular velocity ω' of the small gear can now be found, since v is common to both gears. ω' equals v divided by r.

(rate-of-change of angular displacement as a function of time) are involved.

Consider, for example, the velocity of the pin on the disc shown in Fig. 1–17. When we first look at the pin (upper diagram), it has an instantaneous velocity v_t at right angles to the radius connecting pin and the shaft on which the disc turns, and is moving up and toward the right of the page. Let us assume the disc on which the pin is mounted has a constant angular velocity ω. A short time later, therefore, (lower diagram), the pin will have an instantaneous velocity v_t'. The length of velocity

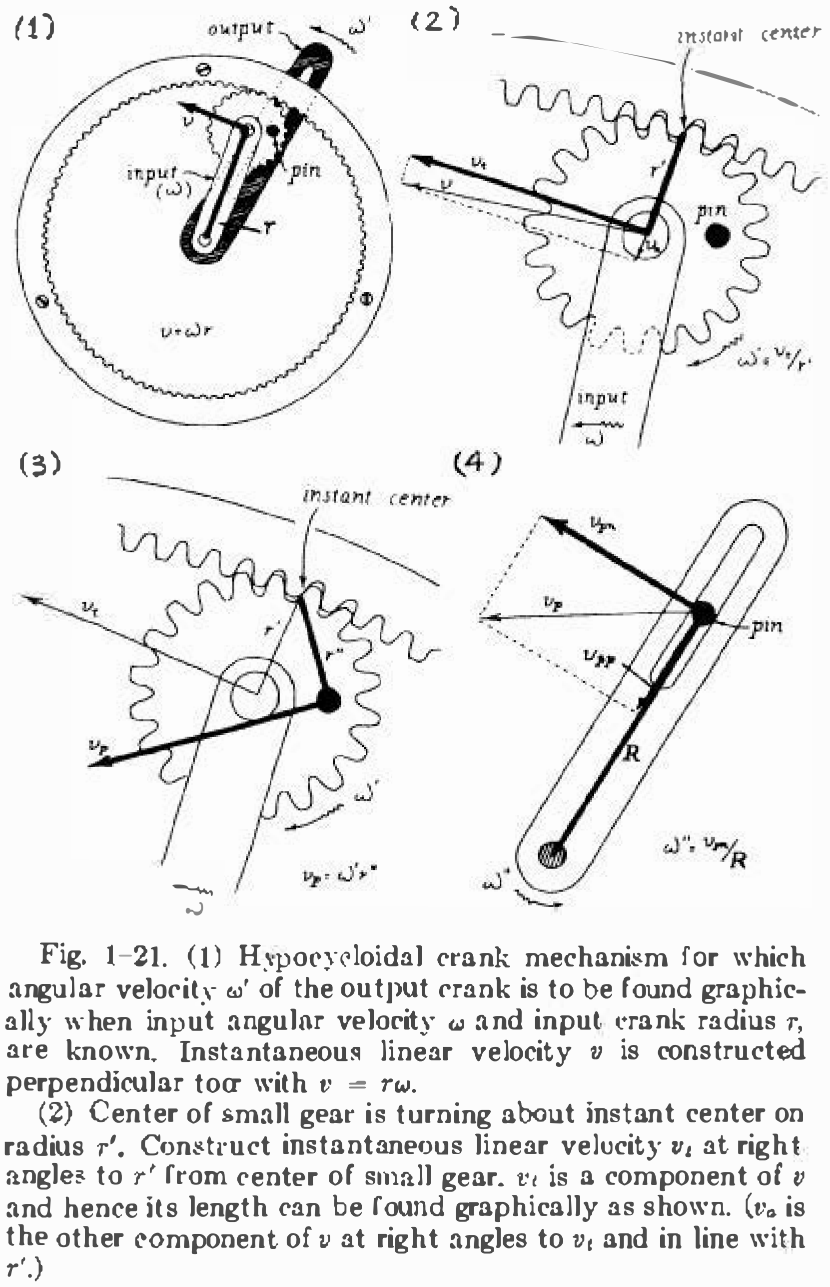


Fig. 1–21. (1) Hypocycloidal crank mechanism for which angular velocity ω' of the output crank is to be found graphically when input angular velocity ω and input crank radius r, are known. Instantaneous linear velocity v is constructed perpendicular to r with $v = r\omega$.

(2) Center of small gear is turning about instant center on radius r'. Construct instantaneous linear velocity v_t at right angles to r' from center of small gear. v_t is a component of v and hence its length can be found graphically as shown. (v_a is the other component of v at right angles to v_t and in line with r'.)

(3) Here the angular position of the pin is taken to be such that its radius r'' about the instant center is equal to r'. The instantaneous linear velocity v_p of the pin is now drawn at right angles to r'' and will be equal to v_t.

(4) Here the radius of the pin from the center of rotation of the output is shown as R. With instantaneous linear velocity v_p known, its component velocity v_{pn} at right angles to R can be found graphically as shown. (v_{pa} is the other component at right angles to v_{pn} and along radius R). The angular velocity ω'' of the output crank can now be found by the formula $\omega'' = v_{pa}/R$.

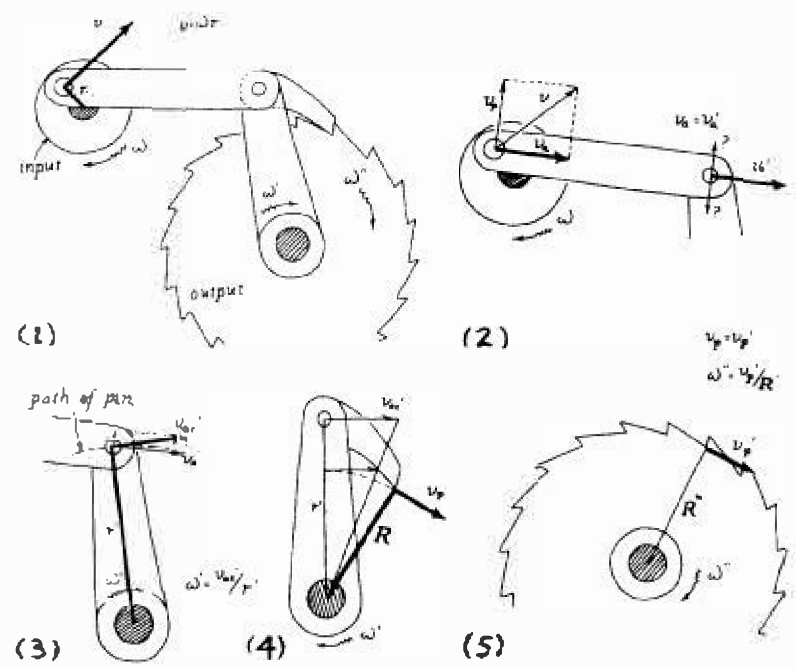


Fig. 1–22. (1) Ratchet wheel driven by crank with angular velocity ω and radius r of the input known. Instantaneous linear velocity v of the input pin is found by the formula $v = r\omega$.

(2) The components of v, one parallel to the axis of the floating link, v_a, and one at right angles to this axis, v_p, are then found graphically as shown. Instantaneous velocity, v_a', is the same as v_a since the two ends of the link cannot have different linear velocities along the axis without the link being either crushed or torn apart. Hence v_a' is constructed as shown. Note that the right-hand end of the link could be moving at right angles (and only at right angles!) to v_a as shown by the vectors bearing question marks.

(3) With v_a' known, the instantaneous linear velocity v_{at}' of the end of the next (vertical) link at right angles to its axis can be found graphically as shown. (Note that any other motion of this end of the link must be at right angles to v_a', in this case, upward.) With linear velocity v_{at}' and radius of rotation r' of the vertical link known, the angular velocity ω' of this link can be found from the equation $\omega' = v_{at}'/r'$.

(4) The vertical link and the ratchet pawl together can be assumed to be a single free body for the instant at which we are determining the linear velocities of the various members of this mechanism *train*. With v_{at}' known, the construction methods used in Fig. 1–19 can be used to find the velocity v_w of the tip of the drive pawl.

(5) Since the pawl engages the ratchet wheel they must have a common instantaneous linear velocity at the point of contact. Thus, v_w' is shown equal in length to v_w and at right angles to R', its *radius* to the center of rotation of the ratchet wheel. The angular velocity of the ratchet wheel ω'' can now be found from the equation $\omega'' = v_w'/R'$.

vector v_t' is the same as that of v_t, but the direction of motion of the pin has changed. If the angular velocity of the disc ω, had changed, then v_t' would have differed from v_t in both magnitude and direction. Note that the pin's linear velocity is always perpendicular to the radial line between pin and shaft.

There is a simple relationship between instantaneous linear velocity v_t or (v_t') and instantaneous

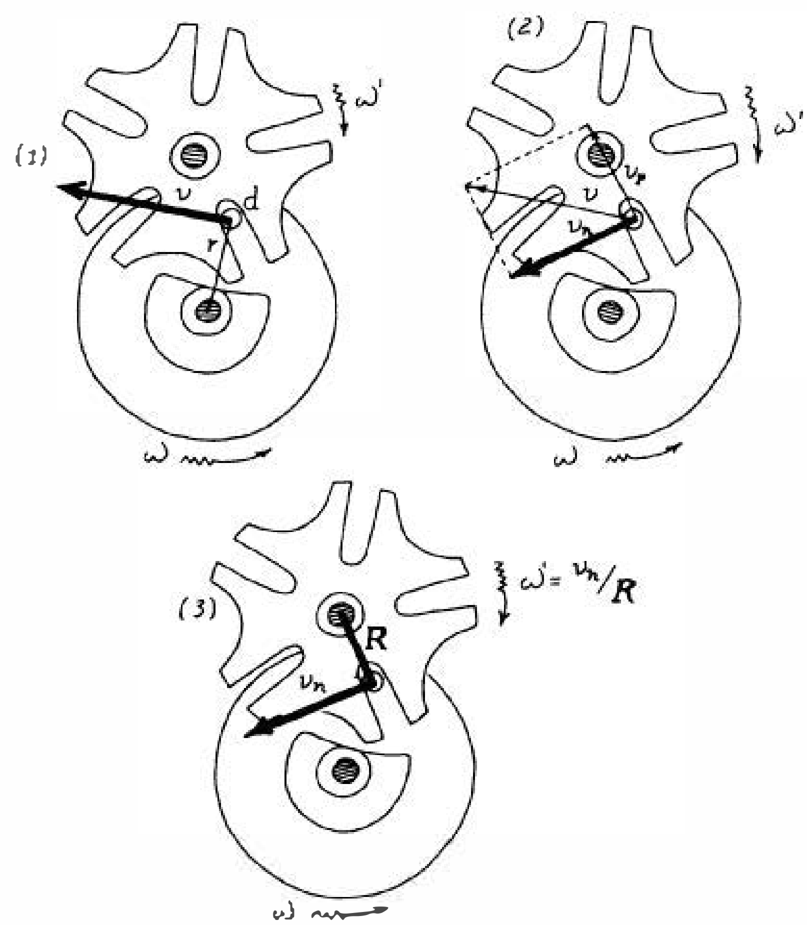


Fig. 1–23. (1) Geneva wheel for which instantaneous angular velocity ω' is to be found when angular velocity ω of input wheel and radius to drive pin r are known. This example illustrates the method of finding the velocity of a driven member when the drive pin moves in a slot in the driven member. Instantaneous velocity v of the drive pin d at right angles to r is found from the equation $v = \omega r$.
(2) Instantaneous linear velocity v is now divided into two components, one parallel to the axis of the Geneva wheel slot, v_p, and one at right angles to this axis, v_n.
(3) With v_n and radius R known, ω' can be found by the equation: $\omega' = v_n/R$. It should be noted that the selection of the components of velocity v was determined by the desire to find ω' using the formula $\omega' = v_n/R$.

angular velocity ω; it is:

$$v_t = \omega r$$

$$\text{Units: } \frac{\text{ft}}{\text{sec}} = \frac{\text{radians}}{\text{sec}} \times \text{ft}$$

where r is the radial distance between the pin and the center of rotation.

We see another illustration of this relationship in Fig. 1–18-(1) where we have a crank with two pins. To find the velocity of the inner pin, we first measure the radius (r), between that pin and the center of rotation, p. The instantaneous tangential velocity of that pin is then shown by vector v, in Fig. 1–18-(2). Vector v is perpendicular to radius r (or tangent to the arc described by the pin as the link turns), and is equal in length to the product of r and ω, the latter being the angular velocity of the link.

The instantaneous velocity of the outer pin can be found by multiplying its radial distance from p, by ω, or by constructing the triangle shown in Fig. 1–18-(3). Notice that large V and small v are parallel to each other, and that their magnitudes are proportional to their relative distances from the center of rotation to the link.

Other Constructions for Determining Machine Velocities

The triangle construction technique used to find V in Fig. 1–18 can help us visualize and determine velocities of machine members in much more complex design situations than that represented by the single link of Fig. 1–18. We will usually use algebra to get numerical values for the forces, velocities, etc., of interest in design situations, but should use graphics or sketches to visualize what is happening and, therefore, to determine the correct algebraic equations to use. Figures 1–19 through 1–23 show graphical determinations of velocities for irregular links, a pair of simple gears, an hypocycloidal gear-crank mechanism, a fourbar ratchet-drive mechanism, and a Geneva mechanism, respectively. If you can understand each of these constructions you should be able to determine the velocities of any mechanism described in this book.

A Word About Units

A detailed discussion of units occurs later, in Chapter 2. In the illustrations of this first chapter we can use:

torque (τ) ft-lbs
force (F) lbs
radius (R, r) ft
linear velocity (v), ft/sec
angular velocity (ω), radians/sec.