SLIP COMPLEXITY IN A CRUSTAL-PLANE MODEL OF AN EARTHQUAKE FAULT

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ABSTRACT: We study numerically the behavior of a two-dimensional elastic plate (a crustal plane) that terminates along one of its edges at a homogeneous fault boundary. Slip-weakening friction at the boundary, inertial dynamics in the bulk, and uniform slow loading via elastic coupling to a substrate combine to produce a complex, deterministically chaotic sequence of slipping events. We observe a power-law distribution of small to moderately large events and an excess of very large events. For the smaller events, the moments scale with rupture length in a manner that is consistent with seismological observations. For the largest events, rupture occurs in the form of narrow propagating pulses.

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The discovery of dynamic complexity in the uniform, one-dimensional Burridge- Knopoff (BK) model of an earthquake fault [Burridge and Knopoff, 1967; Carlson and Langer, 1989] has brought new urgency to some questions about models of seismic sources. Perhaps the most pressing of these questions concerns the role of elasticity in the crustal plane – an ingredient that is necessarily missing in any one-dimensional model but which must be important for an understanding of the dynamics of slipping events. We report here on our recent efforts to address this issue.

Previous studies [Carlson, Langer, Shaw, and Tang, 1991; Langer and Tang, 1991; Myers and Langer, 1993; Shaw, 1994a; Carlson, Langer and Shaw, 1994] indicate the following: The completely uniform, one-dimensional BK model, with velocity-weakening stick-slip friction, is a deterministically chaotic dynamical system that exhibits a broad range of earthquake-like events. The frequency-magnitude distribution for these events includes a scaling region of small to moderately large localized events that is qualitatively similar to a $b \approx 1$ Gutenberg-Richter (GR) law [Gutenberg and Richter, 1954], and a region of large delocalized events whose frequency exceeds that of the extrapolated GR law and which account for most of the moment release. The large events propagate along the fault at roughly the sound speed in the form of "Heaton pulses" [Heaton, 1990]. In order to be well posed mathematically, the model requires a cut-off at very small length scales; but none of the features of its behavior listed above are sensitive to the precise nature of this cut-off. If the cut-off is introduced by adding a small viscous force, as opposed to simply relying upon the discretization length in a numerical solution, then the model has a well defined continuum limit.

Our purpose in the investigations reported here has been to test each of the above features of the one-dimensional BK model in a two-dimensional model that includes elasticity in the crustal plane. Off-fault elasticity is relevant to many features of real earthquake faults such as stress concentrations at rupture fronts, long-range elastic interactions, and seismic radiation. We find that the crucial features are indeed preserved in the new model, and we also find some interesting new properties.

Our model is a semi-infinite elastic plate that occupies the upper half of the x, y plane (the "crustal plane") and terminates at a fault on the x-axis. For simplicity, we load the plate in mode III and describe its anti-plane displacement by the field $U(x, y, t)$. The plate is pulled slowly and uniformly at speed ν via an elastic coupling to a rigid substrate. Thus, for $y > 0$, U satisfies a massive wave equation:

$$\frac{\partial^2 U}{\partial t^2} = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} - U + \nu t. \tag{1}$$

The displacement U is measured here in units of a slipping distance (of order meters in earthquakes) that is determined by the coupling to the substrate and the maximum drop in friction during sliding. The loading rate ν is measured in units of the corresponding slipping speed (of order meters/second); thus $\nu \ll 1$. The position variables x, y are expressed in units of a length (of order 10 kms.) that we identify roughly as the thickness of the crust, *i.e.,* the distance between the plate and the substrate. Accordingly, our unit of time t is the time taken for a sound wave to traverse this distance or, equivalently, the period of the slowest vibrational mode of the system. The mass in (1), *i.e.,* the coefficient (unity) of $-U$ on the right-hand side, causes the vibrational spectrum to have a lower bound associated with some large length scale in the system, in this case, the crust depth. We believe that this property is an essential feature of any realistic model of an earthquake fault.

To complete the model, we write the boundary condition at $y = 0$ in the form:

$$\left.\frac{\partial U}{\partial y}\right|_{y=0} = \Phi \tag{2}$$

where Φ is the stick-slip friction that provides the traction on the fault surface. Here we depart from our previous practice and focus primarily on a slip-weakening rather than a velocity-weakening version of the friction law. One reason for doing this, as we shall discuss in more detail, is that the slip-weakening version is better behaved in the continuum limit than the velocity-weakening model. A second reason for focusing on the slip-weakening model is that we believe it to be a more realistic picture of frictional weakening in the Earth. In a scenario proposed by Sibson [1973], frictional heating raises the temperature and pressure of pore fluids, thereby reducing the effective normal stress and friction. This scenario leads to slip-weakening friction when heat dissipation is slow compared to the rupture timescale [Lachenbruch, 1980; Shaw, 1994b], and to velocity-weakening friction when heat dissipation is correspondingly fast [Shaw, 1994b]. Given the time scales involved, this mechanism suggests that slip-weakening is the case relevant to earthquakes.

The form of slip-weakening friction that we use is

$$\Phi = \begin{cases} (-\infty\ ,\ \phi(S - S_0)] & \frac{\partial S}{\partial t} = 0, \\ \phi(S - S_0) & \frac{\partial S}{\partial t} > 0, \end{cases} \tag{3}$$

with

$$\phi(S - S_0) = \frac{1}{1 + \alpha(S - S_0)}. \tag{4}$$

Here, $S(x,t) = U(x,0,t)$ is the displacement of the crust along the fault, $S_0(x) = U[x, 0, t_0(x)]$ is the value of S at the beginning of an event, and $t_0(x)$ is the time when slip starts at the point x. For numerical convenience, we have modified the function ϕ in (4) so that it drops sharply but continuously by a small amount at the onset of slipping. This device allows us to compute efficiently in the limit $\nu \to 0$ [Carlson, Langer, Shaw, and Tang, 1991]. The resulting complete separation between the loading and slipping time scales means that there is no ambiguity in our definition of an "event" as a sequence of motions that takes place with no change in the load νt.

Equation (3) specifies the stick-slip nature of the friction; it resists motion up to a threshold and decreases continuously once slipping starts. Note that $S(x) - S_0(x)$ is the *total* slip at x starting from the beginning of an event, as is consistent with the relatively slow rate of heat dissipation in our physical picture of slip weakening. In a complex event, the material at a point x may slip and restick more than once, but Φ continues to decrease throughout this motion according to (3). Once the event is over, the fault reheals, and the slipping threshold is reset to $\phi(0) = 1$ everywhere.

There are two overall symmetries in the equations which allow subtraction of a constant from Φ and overall multiplication by another constant. These symmetries have been used to set to unity both the maximum sticking friction and the maximum drop in friction due to slip weakening. The reduction of friction ϕ with slip $S - S_0$ in (4) is the source of the crucial instability that leads to slip complexity, and the slip-weakening parameter α determines the strength of this instability. For α large compared to unity we see a generic complex behavior. We have used $\alpha = 3$ throughout the computations described here, as a representative value. The exact functional form of ϕ appears also to be unimportant. For the one- dimensional case, we know that exponential and piecewise linear forms of ϕ give results which are essentially the same as those obtained with the algebraic form (4) used here [Shaw, 1994b]; we see no reason to think that the two-dimensional case might be different in this regard.

In the velocity-weakening case, the slip $S - S_0$ in (4) would be replaced by the slip rate $\frac{\partial S}{\partial t}$. Interestingly, the velocity-weakening model as defined in this way fails to have a well-defined continuum limit. The difficulty can be understood by examining the linear growth rates for sinusoidal deformations on a free fault surface with $\Phi = 0$. For a perturbation of the form $U \sim e^{ikx - \kappa y + \Omega t}$, we find that

$$\kappa = \alpha, \quad \Omega = \sqrt{\alpha^2 - k^2 - 1} \tag{5}$$

for slip weakening, and

$$\kappa = \alpha\Omega, \quad \Omega = \sqrt{\frac{k^2 + 1}{\alpha^2 - 1}} \tag{6}$$

for velocity weakening. There is no apparent problem in going to a continuum limit for the case of slip-weakening friction; the smallest wavelengths remain marginally stable. In contrast, for velocity-weakening, the smallest wavelengths are strongly unstable. A viscous force of the form $\eta \frac{\partial^3 S}{\partial^2 x \partial t}$ cures this difficulty in the one-dimensional BK model

[Myers and Langer, 1993; Shaw, 1994a] and also is useful in a related two-dimensional model of ordinary fracture [Langer and Nakanishi, 1993]. The analogous regularization in the present case, however, would require fully two- dimensional viscoelasticity near the fault, which would complicate our numerical analysis. Thus there are both computational and physical reasons for adopting the slip-weakening model in the studies presented here.

In our numerical integrations, we have used a finite-difference scheme with a variety of grid spacings and have satisfied ourselves that the behavior of the system becomes grid-independent in the limit of small grid size. There is generally an advantage to working with models that possess well defined continuum limits; one usually is in better control of the mathematics in such situations. We emphasize, however, that our recent experiences with dynamical earthquake models, both here in two dimensions and elsewhere in one dimension, indicate that the qualitative features of the large-scale behavior are not much affected by the way in which we regularize the system at small length scales [Shaw, 1994a]. In contrast, some quasi-static models are sensitive to the treatment of continuum limits [Rice, 1993].

We have performed our numerical integrations using a finite, rectangular grid of physical size L_x by L_y and grid spacings δx and δy. We impose periodic boundary conditions in the x direction (along the fault) and a zero-normal- derivative (Neumann) condition along the boundary at $y = L_y$. Because the plate is necessarily finite in the y direction, we need to minimize the extent to which elastic waves reflect back upon the fault from the system's outer edge at $y = L_y$. To accomplish this, we have added a layer of viscous damping to the equation of motion (1) near the outer edge (from $3L_y/4 \leq y \leq L_y$). The damping has the form $\eta(y)\nabla^2\frac{\partial U}{\partial t}$ with $\eta(y) = ay^3 + by^2 + cy + d$; and we fix the parameters a, b, c and d so that $\eta(3L_y/4) = 0$, $\eta(L_y) = 0.5$, $\frac{\partial \eta}{\partial y}(3L_y/4) = 0$, and $\frac{\partial \eta}{\partial y}(L_y) = 0$. The damping strength $\eta(y)$ therefore rises smoothly from zero at $y = 3L_y/4$ and saturates at a value of 0.5 at the outer boundary.

Our finite-difference scheme steps forward in time using an explicit Euler method which is first order accurate in the time step δt. Spatial derivatives, both in the bulk and on the boundary, are accurate to $O((\delta x)^2, (\delta y)^2)$. Our computations have been performed on IBM RS6000 workstations.

Beginning from an arbitrary nonuniform initial configuration, our system evolves into a statistically steady state with a rough configuration and a wide range of event sizes. Fig. 1 shows a typical displacement field U in a fully stuck configuration. The displacements are inhomogeneously locked by the friction on the fault and decay exponentially into the bulk over length scales of order unity.

Fig. 2 shows a sequence of stuck configurations at the fault boundary as the plate moves forward. Note that there are many small events (most of which are not visible) and fewer large ones, but almost all of the forward motion occurs in the large events. One way to characterize this complexity is to look at the differential distribution R of event magnitudes $\mu = \log_{10} M$, which is shown in Fig. 3. The moment M is the total slip on the fault during an event:

$$M = \int (S_f(x) - S_0(x))dx \tag{7}$$

where S_0 and S_f are the initial and final configurations. Just as in the one-dimensional cases, $\log_{10} R(\mu)$ has a straight-line scaling or power-law region with slope approximately -1 for small to moderately large events, and a distinct peak for the very large events. We also show in Fig. 3 that these distributions are not very sensitive to changes in the grid spacings. Aside from a trivial shift at the low-magnitude end of the scale, the curves lie on top of one another to within our statistical uncertainty. The apparent grid-size dependence of the smallest events, which themselves are on the scale of the grid spacing, does not imply failure of convergence to a grid-independent limit for motions on larger scales. We also have checked that there is no appreciable dependence of these distributions on the dimensions of the system, L_x and L_y. In this regard, note that the biggest events in Fig. 2 are far from big enough to span the system.

One respect in which the two-dimensional model differs from the one-dimensional version is the correlation between the moment M and the source dimension Δ (the size of the region that slips in an event). In the Earth, this correlation is well fit over a wide range of source dimensions by an assumption of constant stress drop which implies that the average slip scales linearly with Δ. Therefore, for our two-dimensional model, we expect $M \sim \Delta^2$. The dashed lines in Fig. 4 indicate that we see this behavior throughout the scaling region. The average slip M/Δ continues to increase with slip-zone size Δ up to and beyond $\Delta = 1$, which is our analog of the crust depth. A similar phenomenon has been reported for real earthquakes by Scholz [1982; 1994].

For the very largest events in Fig. 4, we see $M \sim \Delta$, which means that the average slip becomes independent of slip-zone size. These are the delocalized events in which slip occurs in the form of propagating pulses. One such delocalized event is shown in Fig. 5, where we show instantaneous slip rate $\partial S(x,t)/\partial t$ as it evolves in time. We see two pulses triggered at an epicenter ($x \approx 52$) and travelling outward at about the sound speed. As a pulse passes any point on the fault, the slip rate rises rapidly and then falls more slowly. The fault stays open for a time (the dislocation rise time), but it then closes behind the pulse so that the whole region of the rupture is generally not open at any instant in time. This picture of a propagating, "self-healing pulse of slip" fits the scenario advocated by Heaton [1990].

The analytic theory of this pulse and its relation to theories of ordinary fracture is too large a topic to address in this report. (Recent developments in this area have been described by Langer and Tang [1991], Myers and Langer [1993], and Langer and Nakanishi [1993].) For present purposes, we need only remark that we have used the present model in a careful study of a pulse propagating at constant speed along a uniformly loaded fault and have assured ourselves that there are no pathologies in either the slip $S(x,t)$ or the friction

Φ. We had not used a two-dimensional model with purely slip-weakening friction in our earlier work and, therefore, we needed specially to be sure that the stress concentrations are properly controlled at the slipping and resticking points. Both S and Φ are smooth and well behaved at these points. The dynamic length scale required to enable resticking is provided by the crust depth via the mass in the wave equation (1).

In conclusion, we have demonstrated that two-dimensional elastodynamics with slip-weakening friction along a one-dimensional fault boundary produces slip complexity. The model used here has a well defined continuum limit. Many aspects of the behavior of this model are similar to what is observed for real earthquake faults. In particular, the model exhibits a power-law distribution of small earthquake-like events and an excess of large events. In the small events, the average slip scales linearly with slip-zone size. In the large events, narrow "Heaton pulses" of slip propagate along the fault. We believe that these results support the case that inertial dynamics and frictional weakening are contributing in fundamental ways to earthquake complexity.

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Figure Captions

FIG 1. The displacement field $U(x, y)$ in a fully stuck configuration. The variable x is the distance along the fault and y is the distance perpendicular to the fault. The fault boundary is the x axis, along which the frictional stresses are applied.

FIG 2. A sequence of stuck configurations of the displacement at the fault boundary. The area between subsequent configurations is the moment M of an event. The lattice parameters used in this figure are $\delta x = 0.15$, $\delta y = 0.075$, $L_x = 60$, and $L_y = 3.75$.

FIG 3. The differential magnitude distribution and a demonstration that these results are insensitive to changes in the grid spacing. $R(\mu)$ is the number of events with magnitudes between μ and $\mu + d\mu$ per unit fault loading per unit fault length. The three curves differ only in their grid spacings as shown. $L_x = 60$ and $L_y = 3.75$ for all curves. Also plotted is a line of slope -1.

FIG 4. The moment M as a function of the slip zone size Δ. The dots indicate individual events. The lower two dashed lines have slope 2; the upper two lines have slope 1.

FIG 5 The slip rate at the fault boundary dS/dt, as a function of time, during a large event. Both pulses accelerate to approximately the sound speed (unity). Note that the velocity of the larger pulse increases as its amplitude increases.