Kemble & Present

to explain 0.000 cm⁻¹ raising shift of S level
cutting potential off flat, or replace by infinite
repulsion. Needs range 10⁻¹² cm of repulsion.

Houston Williams f Deuterium spectrum

interpreted by Pasternak: Raise S level by 0.30
Richards, Drinkwater & Williams:

Could be H impurity in D

Fission theory Veblen 1936

in region 5·10⁻¹¹ cm, potential changed; effectively increased
(because decrease by polarization is less at high frequency than
for slowly varying fields), so S state lower than P; also
magnitude is much less than required.

Lifetime of S½ level about 2 x life of P levels

Apparatus: \( H² \rightarrow H \rightarrow H⁺ \rightarrow \) Interaction space with RF \( \rightarrow \) Detector

Detection: Secondary electrons can be ejected from W by metastable
H atoms. About 40% efficiency.

Excitation by a current of electrons

Magnetic field (few 100 gauss) to split S and P levels
Current of secondary electrons

Frequency set; observe current as fn. of $H$

$\lambda$ between 2.6 and 3.6 cm

Expected Raman frequencies

Result: S level shifted up by about 0.037 cm$^{-1}$ (1100 Me)

Present width of lines probably increased by RF interaction
Oppy
Positron theory (wave length dependence)
( high field )
Nuclear interaction
Electrodynamic term shift $e^2$

\[ \sum \frac{s_i^+ s_+^i}{p_i^+ (E_p + E_x - E_i) E_p} \]

$E_p = \text{enero}$

First term $\frac{s_i^+ s_+^i}{E_p^2} = \frac{s_i^+}{E_p}$ same for all levels
Second term $\frac{s_i^+ s_+^i (E_x - E_i)}{E_p^2}$
Third term $\frac{(E_x - E_i)^2}{E_p^4}$

Converges and might explain Lamb's result.
Deuterium ground state

\[ \nu = \nu_D \left(1 + \frac{g}{3} x^2\right) \]

\[ x = \frac{g \mu_0 H}{\nu_D} \]

\[ \nu_D = 327.38 \pm 0.05 \text{ Mc} \]

Hydrogen

\[ \nu_1 = \nu_H + \Delta \nu_1 + \Delta \nu_0 \]

\[ \nu_2 = \Delta \nu_1 + \Delta \nu_0 \]

\[ \nu_1 - \nu_2 = \nu_H \]

\[ \nu_H = 1421.3 \pm 0.2 \text{ Mc} \]

\[ h \nu_{\text{theor.}} = \frac{8 \pi}{3} \frac{2 I + 1}{I} \frac{g N}{m_n} \mu_0 \gamma^2(0) \]

\[ \nu = \frac{g}{3} \frac{2 I + 1}{I} \frac{g N}{1836.5} \left(\frac{m_n}{m_0}\right)^3 \alpha^2 R_n \]

\( g N \) = nuclear moment in nuclear magnetons

Calculated \( \nu_H = 1416.9 \pm 0.54 \)

\[ \nu_D = 326.53 \pm 0.16 \]

\[ \nu_H = 4.3393 \pm 0.0014 \]

\[ \frac{\nu_H}{\nu_D} = 4.3416 \pm 0.0007 \]

\[ \frac{4.3416}{4.3000} = 0.0005 \]
\[ \Psi^* \rho_0 + e \cdot \rho e + \alpha \cdot \rho P + \rho_0 \cdot m c + \beta \cdot M c - \frac{e^2}{2\gamma^2} \left[ -\frac{\alpha e \cdot \alpha P}{r} + \frac{\alpha e \cdot \alpha P}{r^3} \right] + \gamma \rho = 0 \]

**Felter**  
**Neutron - Electron**

Interaction potential over range \( \frac{e^2}{\gamma c^2} \)

0 ± 5 keV

First expr. Kr, later Xe

Corrections: von der Waals Force

Interaction of electric field of atom with neutron magnetic moment

**Rabi**  
Change of \( \sigma \) in liquid Pb about \( \leq 0.3 \) b

out of 11 b. total nuclear \( \sigma \).

\[ \sigma_c = \left( \frac{0.3}{2 \times 82 \times \sqrt{11}} \right)^2 \leq 3 \times 10^{-7} \text{ barns} \]

Acc. to Schwinger, this means \( \approx 10 \) keV interaction over \( \frac{e^2}{\gamma c^2} \)
Mesons Capture. (Rossi)

Capture! Orpy suggests

Production of mesons
Wataghin production of meson showers as fn. of height
Jansky barometer effect

Meson showers

apparently double showers frequent
showers not produced by mesons but by probably nuclear particles

a) Multiplicity cannot be proportional to energy because then altitude dependence should be the same at all latitudes

b) Multiplicity cannot be independent of energy because then altitude dependence should be stronger (?)
Kramers

\[ H = \left( \frac{p - e A}{\hbar} \right)^2 + e \phi + \frac{1}{8\pi} \int (E^2 + H^2) \, dt \]

In external field

\[ m \dot{R} = e E + \frac{e}{c} \dot{R} \times H \]

Elimination of longitudinal field:

\[ E = E_\| - \nabla \phi \]

\[ \nabla^2 \phi = -4\pi \rho \quad \text{div} \, A = 0 \]

1. Replace \( m \) by \( m_0 \) (mechanical mass)
2. \( \frac{\dot{A}}{c} \) "at electron" by \( \int \rho \, A \, dt \)
3. Try to get structure - independent part of theory
   \[ \Delta = \Delta' + \Delta_0 \]
   \[ \Delta_0 = \text{prop field}, \Delta' = \text{external field} \]
   \[ A_0(p) = Tr \left( \frac{i}{c} \int \frac{\rho A}{\nabla} \right) \]
   \[ T_r = \text{transverse part} \]
   \[ \text{so that} \quad \nabla^2 A_0 = -\frac{4\pi}{c} \left( \rho \frac{\dot{R}}{c} \right)_{\text{transverse}} \]

4. Don't integrate field equations yet (retarded & advanced potentials)

At large distance, \( A_0 = \frac{e}{2\pi c} \left( \frac{\dot{R}}{c} + \frac{\dot{R}}{r} \right) \)

At electron, \( \dot{A}_0 = \frac{e}{c} \mu \frac{\dot{R}}{c} \)

[Definition \( \mu \frac{\dot{R}}{c} = \frac{i}{4\pi c} \int E \times H \, dt \)]

\[ m_0 \ddot{R} = -\frac{e}{c} \frac{\partial A}{\partial t} + \frac{e}{c} \dot{R} \times \text{curl} \dot{A} - \nabla U \]

\[ = -\frac{e}{c} \frac{\partial A'}{\partial t} + \frac{e}{c} \dot{R} \times \text{curl} A' - \nabla U - \frac{e}{c} \frac{\partial \Delta_0}{\partial t} + \frac{e}{c} \dot{R} \times \text{curl} \Delta_0 \]
\[ \frac{\partial A_0}{\partial t} = \mu \frac{\partial R}{\partial t} + \text{a term which does not act on the electron itself, because it is spherically symmetrical.} \]

\[ \mathbf{R} \times \text{curl } A_0 \text{ disappears by averaging over electron} \]

\[ \mathbf{R} = -\frac{e}{c} \frac{\partial \mathbf{A}'}{\partial t} + \frac{e}{c} \mathbf{A} \times \mathbf{H}' - \nabla U \quad \text{Eq. of motion} \]

Field eqn.

\[ \Box \mathbf{A}' + \Delta \mathbf{A}_0 - \frac{i}{c} \frac{\partial^2 \mathbf{A}_0}{\partial t^2} = -\frac{q}{c} \mathbf{E} \left( \frac{\mathbf{R}}{c} \right) \]

\[ \frac{\partial^2 \mathbf{A}_0}{\partial t^2} \text{ contains } \mathbf{R}^2, \mathbf{R} \mathbf{R}, \text{ and } \mathbf{R} \]

Put \[ \mathbf{A}' = \frac{1}{c} \frac{\partial \mathbf{Z}''}{\partial t} \]

\[ \mathbf{Z}'' = \frac{1}{c} \frac{\partial \mathbf{A}_0}{\partial t} \]

\[ \text{div } \mathbf{Z}'' = 0 \]

\[ \text{curl and } \mathbf{Z}'' = -\frac{1}{c} \frac{\partial^2 \mathbf{Z}''}{\partial t^2} - \frac{1}{c} \frac{\partial \mathbf{A}_0}{\partial t} = -\frac{1}{c} \frac{\partial}{\partial t} (\mathbf{A}' + \mathbf{A}_0) = -\frac{1}{c} \frac{\partial \mathbf{A}}{\partial t} = \mathbf{E} = \mathbf{E}'' + \mathbf{E}_0 \]

Lagrangian

\[ L = \frac{1}{2} m_0 \mathbf{R}^2 + e\mathbf{R} \cdot \mathbf{E} - U(R) + \frac{1}{8\pi} \int \left[ \text{curl } \mathbf{A} \right]^2 - \frac{1}{2} \left( \frac{\partial \mathbf{A}}{\partial t} \right)^2 \right] \frac{dV}{4\pi} \]

Last term necessary to give \[ \text{div } \mathbf{A} = 0 \] the eqn. of motion.

Want to put \[ \mathbf{A} = \mathbf{A}_0 + \mathbf{A}' \]

\[ \mathbf{A}_0 = \mathbf{R} \]

Must introduce supplementary variables

\[ \mathbf{A}' = \mathbf{R} \text{ defined as } \mathbf{P} = \frac{\mathbf{c} \mathbf{A}'}{m_0} \]

New Lagrangian

\[ M = L - V.P - B \cdot \mathbf{P} = M \left( \mathbf{A}, \mathbf{\dot{A}}, \mathbf{R}, \mathbf{P}, \mathbf{\dot{P}} \right) \]

Then can replace \( \mathbf{A} \) by \( \mathbf{A}' + \mathbf{A}_0 \) because now \( \mathbf{A}_0 \) depends only on \( \mathbf{R} \) and \( \mathbf{\dot{R}} \), and longer on velocity, so that...
If this is done, $A'_0$ term introduces $R$ into the lagrangian

$$M = -\frac{1}{2m} \left( \mathbf{p}^2 - \frac{e^2}{\epsilon_0} A'^2 \right) - U(R) - \mathbf{p} R - \frac{i}{2\pi} \int \left( \mathbf{F}'^2 - \epsilon'^2 \right) \, dV$$

$$V = \frac{p - \frac{e}{\epsilon} A'}{m} Z' \text{'defined by curl curl $Z' = E'$}$$

This gives eqns. of motion correctly.

Original

$$H^2 = H^2 + 2 \mathbf{H} \cdot \mathbf{H}_0 + H_0^2$$

will lose disappears $\frac{1}{2} \mu v^2$

$$e^2 = e^2 + 2 E' \cdot E_0 + \epsilon_0 c^2$$

gives last term structure dependent but of higher order

Bad thing: cannot introduce canonical variables

But $H = \mathbf{p} \cdot \mathbf{q} - M$ is constant of motion

$$H = \frac{1}{2m} \left( \mathbf{p}^2 - \frac{e^2}{\epsilon_0} A'^2 \right) + U(R) + \frac{i}{2\pi} \int \left( \mathbf{F}'^2 + \epsilon'^2 \right) \, dt + \frac{e}{\epsilon} \mathbf{V} \cdot \frac{\partial Z'}{\partial R}$$

because $H$ does not contain time derivatives, and $E'$ is a time derivative, ...

but $H = \frac{1}{2m} \left( \mathbf{p}^2 - \frac{e^2}{\epsilon_0} A'^2 \right) + U(R) + \frac{i}{2\pi} \int \left( \mathbf{F}'^2 + \epsilon'^2 \right) \, dt$

Try to get canonical eqns. by assuming $\frac{\partial M}{\partial R}$ small

$$\frac{d}{dt} \left( \frac{\partial M}{\partial R} \right) = \frac{\partial M}{\partial R}$$

If terms of $A_0'$ left out, added to this adds $t'$ eqns. of motion of electron the term

$$-\frac{e}{\epsilon_0} \frac{d}{dt} \frac{\partial}{\partial R} \left( Z \cdot \mathbf{V} \right)$$

which vanishes for dipole radiation

This is now done. Then

canonical conjugate of $R$ is not $-R$ but $-R' = -\left( R + \frac{e}{mc^2} Z' \right)$

$$A' \text{' is } \frac{1}{4\pi} \mathbf{E'} + \frac{e}{mc^2} Z' S (R - \tau)$$
By comparison:

Dirac

\[ \frac{1}{2m} \mathbf{p}^2 - \frac{e}{mc} \mathbf{p} \cdot \mathbf{A} + \frac{e^2}{2mc^2} \mathbf{A}^2 + U(R) + \frac{i}{8\pi} \int (\mathbf{A} \cdot \mathbf{E}) \, d\mathbf{r} \]

Emission and absorption indicated from \( \mathbf{p} \cdot \mathbf{A} \)

Now from \( U \) term which should be written in terms of \( R' \)

\[ U(R) = U(R' - \frac{e}{mc} \mathbf{Z}) = U(R') - \frac{e}{mc} \mathbf{Z} \cdot \nabla U + \frac{i}{m} e^2 \mathbf{Z} \cdot \nabla \cdot \mathbf{E} \]

\( \mathbf{Z} \cdot \nabla U \) term replaces old \( \mathbf{p} \cdot \mathbf{A} \)

\( \nabla U = m \cdot \text{acceleration} = \mathbf{F} = \mathbf{p} \)

\( \mathbf{Z} \cdot \nabla \mathbf{F} \)

A new term \( \mathbf{Z} \cdot \mathbf{k} \) is \( \frac{\nu}{c} \) times old term \( \mathbf{A} \cdot \mathbf{p} \) (\( \nu \), atomic frequency)

\( \therefore \) absorption & emission unchanged

Dispersion theory complicated cancellation. E.g. Compton scattering:

\( \text{Dirac} + \frac{e^2}{mc^2} \mathbf{A}^2 \), Kramers - \( \frac{e^2}{mc^2} A^2 \) but this is compensated by

the fact that \( E' \) is not conjugate to \( A' \). If \( E'' \) is the conjugate, then

\[ E' = E'' + \frac{4\pi e^2}{mc} \mathbf{Z} \cdot \mathbf{E} \]

\[ \frac{i}{8\pi} \int E'^2 \cdot \mathbf{Z}' \, d\mathbf{r} = \frac{i}{8\pi} \int E''^2 \cdot \mathbf{Z}' \, d\mathbf{r} \]

This doubly over-compensates \( A^2 \) term

If we went on to \( e^4 \) term, this would be infinite, i.e. \( \text{post. } \frac{e^4}{8\pi} \mathbf{Z} \cdot \mathbf{E} \)

This is structure dependent
Weisskopf

\[(H_0 + U - E) \gamma_0 + e^2 \frac{1}{H_0 + U - E} H' \Gamma_0 = 0\]

Perturbation theory gives

\[H' \left( \frac{1}{H_0 - E} U \frac{1}{H_0 - E} H' \gamma_0 \right) \]

\[\gamma = \gamma_0 + e \gamma_1\]

\[H \gamma_0 + e^2 H' \gamma_1 = E \gamma_0\]

\[H \gamma_1 + H' \gamma_0 = E \gamma_1\]

\[H = H + H', \quad H = T + V\]

Von Neumann

Space rotational symmetry, quantization desirable

This not possible with classical variables, but known to be possible

for angular momenta

Lorentz group, infinitesimal rotations \(Y_{ij}\), these coordinates \(x_k (x_1, x_2, x_3, x_4 = x, y, z, t)\), called \(Y_{ik}\)

\([Y_{ij}, Y_{ik}] = 0 \quad k \neq i, j\]

\([Y_{ii}, Y_{ij}] = \delta_{ij}\]

1+4 dimensional Lorentz group

Displacement

\(x_i \rightarrow x_i + a_i, \quad x_2, x_3, x_4 \) invariant. This is impossible

Let \(x_i \rightarrow Y_{0i} + a_i\), then \([Y_{0i}, Y_{0j}] \) invariant = \(Y_{ij}\)

\([Y_{ij}, Y_{0i}] \) invariant, \([Y_{ij}, Y_{0j}] \) invariant in contradiction to assumption

\([Y_{ij}, x_k] \) has same symmetry as angular momenta, but is fixed,
Problematics

1. Relative coordinates: Ok, more particles questionable
2. Absolute case: Lorentz invariance, for inhomogeneous Lorentz group.

Oppy

Reversibility of Meson Production

Franklheim \( \sigma(x - y) = \sigma(y - x) \)

Schwinger & Oppy

1) Scattering: \( \sigma \sim a^2 \)
2) Production: Large deflection of protons. Shaking off of mesons on change of direction

\[ N = g^2 \frac{d\sigma}{d\Omega} \quad E_0 \sim g^2 K \quad E \frac{d\sigma}{d\Omega} \]

\[ N \sim E^{2/3} \quad K \sim \frac{1}{a} \]

Couplings: dipole \( \sim \frac{1}{a^3} \), number of mesons increases with energy

Pseudoscalar: Pseudovector coupling \( V_{\mu} \left( \sigma_1 \cdot \sigma_2 - \gamma^5 \frac{1}{2} \gamma_2 \right) \)

\[ d\sigma = d\sigma_0 \frac{g^2 N^2}{E_n} \quad \frac{d\Omega}{E_n} \quad \text{increasing multiplicity} \]

Pseudovector coupling:

(Pseudovector)

\[ \sigma \sim \frac{g^2}{\pi} \frac{N^2}{N!^2} \left( E_1 E_2 \right)^{N-1} \frac{dE_1 dE_2}{E_\text{proton} \left( E_n - E_1 - E_2 \right)^2} \quad \text{finite multiplicity} \]

\[ N = \frac{4 \left( \frac{E_0}{10^9 \text{eV}} \right)^{1/3}}{g^2} \quad \text{for } g^2 = \frac{1}{2} \]

Total cross section assumed unaffected by meson emission.
Why is Schwinger-Oppy wrong?

\[ \sigma = a^2 \] only for emission of single

multiple meson production not

small in strong coupling theory; this will actually occur if enough

energy is available: multiple production in scattering

Present theories require specific couplings

\[(A^- + C^+)^n\]

\[A^- = \text{annihilation of negative particle} \]

\[C^+ = \text{creation of positive} \]

\[A^- n + C^+ n \] would be necessary to avoid multiple processes

produced by mesons; but this is probably not

relativistically invariant (according to Oppy)

Excited nucleon states (hypothesis of Weiskopf)