ESSAYS ON THE IMPACT OF MONETARY POLICY ON REGIONAL AND SECTORAL ECONOMIES IN INDONESIA

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In literature, it is difficult to find the answer on how the monetary policy affects the regional economies and sectoral industries. In the partial equilibrium framework, most of the studies are only focused on some region and particular sector in a region. This is because of the lack of data availability. Moreover, the development of the general equilibrium framework using Computable General Equilibrium (CGE) and Dynamic Stochastic General Equilibrium (DSGE) are concentrated on the national level model. Only a few researchers produce disaggregated model using these methods. It happened because introducing sectoral and regional dimension into general equilibrium framework makes the model much more complicated and hard to solve.

Given this situation, this dissertation attempts to narrow the gap in the literature by building comprehensive tools to analyze the impact of the monetary policy to regional economies and industries that can help policymakers especially central bank as a monetary authority to come out with a policy recommendation. In this study, the scope of monetary policy is limited to the interest rate policy as the primary instrument that is used by the central bank.

This dissertation consists of three essays on different issues and various methods to address the issues. However, the bottom line is similar, namely the impact of the

monetary policy on the regional and sectoral economies in Indonesia. The dissertation introduces the sectoral and regional dimension into the model which can create comprehensive tools for policy analysis as mentioned before.

As a starting point, in the first chapter, the sectoral investment equations are developed using dynamic panel estimator suggested by Arellano and Bond to analyze the determinants of sectoral investment and also the impact of real interest rate to the sectoral investment. Moreover, this chapter also estimates the aggregated national investment as a benchmark and then compares the weighted elasticity parameters of real interest rate between those models to show how the results are different in every sector. It found that the weighted real interest rate coefficients between the aggregated national investment equation and the sectoral investment equation are quite different. According to the sectoral investment model, the most affected sector by the change in real interest rate is Non-Oil and Gas Manufacturing sector instead of Oil and Gas Manufacturing sector as obtained from aggregated national investment model. Thus, the policymakers should use the real interest rate estimate from sectoral investment equations instead of the coefficient from aggregated investment one to avoid the misleading policy recommendation.

The second chapter aims to build a comprehensive model for analyzing the impact of shocks and/or policies not only national economy but also the regional economy in more detail for every sector in the economy. The model is called Financial Computable General Equilibrium (FIRCGE) model. In addition, this paper also analyzes on how central bank as a monetary authority and government as a fiscal authority should coordinate in order to achieve specific objective given the specific shocks. Moreover, this study wants to compare the effectiveness of the combination of interest rate and different fiscal stimulus rule, which are an untargeted fiscal stimulus and a targeted fiscal stimulus.

In this experiment, the FIRCGE model is able to generate an iso-loss curve that might be useful for both central bank, and government in providing the possible combination of fiscal stimulus and interest rate based on the objective or loss function that they choose. It found that in Indonesia, the combination of targeted fiscal stimulus and interest rate is more efficient than the combination of untargeted fiscal stimulus and interest rate in the loss function which has neutral weight on both inflation and GDP stability.

The third chapter aims to construct five region DSGE models, estimates the optimal monetary policy and then analyzes how the central bank should react to the regional shocks. In addition, this chapter tries to address the question of how disaggregation of the national model into the regional model is necessary. This study found that currently, the monetary policy reaction function in Taylor rule is not optimal. The central bank of Indonesia did not react optimally to the regional shocks given the objective function. It also demonstrated that disaggregating national model into regional model would lead to better performance in term of the loss function. Thus, it suggested that the policymakers should use the disaggregated regional DSGE model if they are dealing with the regional shocks in order to get the optimal policy response.

BIOGRAPHICAL SKETCH

Jati Waluyo was born in Jakarta, Indonesia. He received his Bachelor of Computer Science degree from Bandung Institute of Technology (ITB) in Bandung, Indonesia in 2001. After graduating from ITB, he joined Bank Indonesia, the central bank of Indonesia, as a consultant for the economic modeling unit and was fully hired as an economist in 2005. He was responsible for conducting some research projects focusing on developing and maintaining Bank Indonesia's macroeconomic models. He also attained a Master of Art in Applied Economics at University of Michigan, Ann Arbor, U.S.A. in 2012. During his masters, he took Inter-University Consortium for Political and Social Research (ICPSR) Summer Training Program in Quantitative Methods of Social Research at University of Michigan. He was enrolled in doctoral program in Regional Science program at Cornell University, Ithaca, New York, U.S.A. in the fall of 2012, and will receive his Ph.D. degree in May 2017. He is married to Vita Tunjung Sari, and has three sons: Abyaz Altavian Waluyo, Nevan Wistara Waluyo and Keenan Prakasharif Waluyo.

Dedicated to:

My soulmate, Vita Tunjung Sari, my princes, Abyaz, Nevan, and Keenan,

and to my parents and parents-in-law, Samadi, Siti Aisun, Sonny Suharsono (alm.), and Yanu Ismadalia.

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CHAPTER 1

THE IMPACT OF REAL INTEREST RATE ON SECTORAL INVESTMENT IN INDONESIA

1.1 Introduction

Investment is the important catalyst to achieve sustainable economic growth and poverty alleviation, especially in developing countries because it can create entrepreneurship and employment opportunity that improves people's standard of living. Investment is generated from the internal funds such as profits, retained earnings, and stakeholder financing, or from external fund namely private placement and public offerings of shares in the stock markets. In addition, investment also comes from short term financial sector credit, long-term capital raising from the secondary market and foreign direct investment (FDI).

In fact, there is a correlation between investment and the economic growth rate. Suhendra and Anwar (2014) argued that in Indonesia, the faster economic growth rate can be achieved from the higher private and government investment. From this finding, it implies that the government should create a conducive investment climate in the economy that can be done by institutional reforms such as efficient bureaucracy, fiscal incentive, the rule of law, etc.

Based on World Bank (2005), a conducive investment climate creates incentives and opportunities for firms to expand their business, invest in productive machines, and create more jobs. Therefore, it promotes economic growth and poverty reduction. The ingredients of a sound investment climate included macroeconomic stability, the proper level of certainty on government policies, strong contracting and judicial system, reliable social, physical and technology infrastructure, and clear property right, good functioning financial market, and educated healthy individuals. Besides these, the access to international markets is also important because it facilitates the flow of capital, goods, ideas, and technology.

Since investment is a crucial factor in economic growth, it is worthwhile to investigate factors that determines the level of domestic investment in developing countries. This study analyzes the impact of real interest rate in determining the investment in the economy. This research will help the policy maker especially Bank Indonesia to see the tradeoff between fighting inflation by increasing the interest rate and stimulating investment. In addition, based on Dennis et al. (2013), monetary policy has a different impact on the firm's real net sales growth rate in various sectors in the US economy. This finding suggests that we also need to model investment equation for every industry if we want to understand the different impact of real interest rate on the investment in each sector.

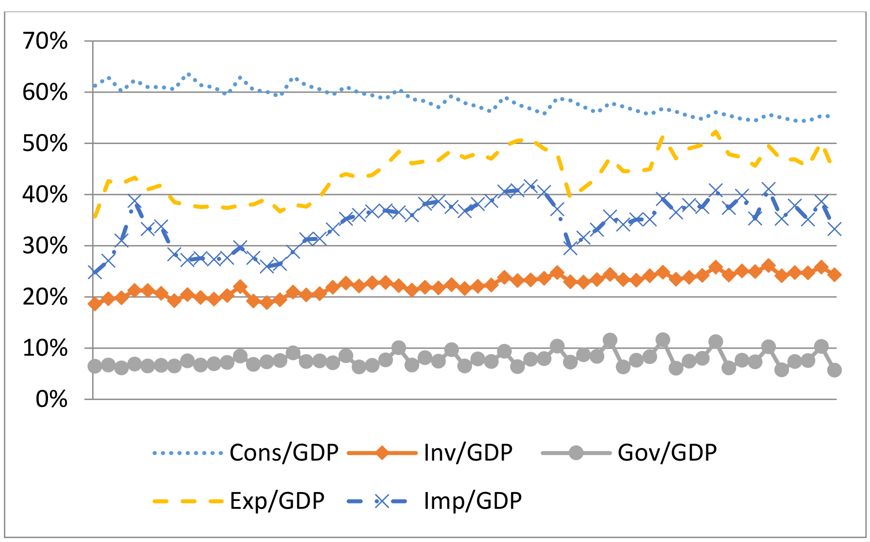

Figure 1.1 The Share of GDP Component from 2000Q1 until 2014Q1

Figure 1.1 above shows the share of GDP component from 2000 quarter 1 (2000Q1) until 2014Q1 in Indonesia. As can be seen, the share of consumption is declining over time with the long-run average of 58%. In contrast, the share of investment is increasing from 18.7% in 2000Q1 to 24.3% in 2014Q1. This indicates that investment has become a critical component for economic development in Indonesia. Hence, central bank as policy maker should understand more about the transmission mechanism of its policies through the investment.

The existing empirical research on the determinants of private investment in developing Asian countries has been dominated by multi-country cross-sectional regression due to the data availability. This regression analysis is based on the assumption of homogeneity across countries. In reality, this assumption is not entirely correct because, in fact, different countries have different economic structures and different institutional aspects. Thus, the analysis of the determinants within a country across regions has more robust results in this case.

Moreover, the current studies about the determinant of investment in Indonesia mainly focus only on a national level. Only a few papers are dealing with more disaggregated level with an emphasis on a particular area in Indonesia. None of them have introduced a comprehensive sectoral dimension in the model. The lack of an empirical study on sectoral investment in Indonesia creates a gap that needs to be filled. This study tries to cover this gap using Indonesian sectoral data.

For the central bank as a monetary authority, the estimation of the sectoral investment equation is more important than that of the regional investment equation because the policymakers in the central bank only focus on the relationship between investment and interest rate, as the main monetary instrument, and the impact of interest rate in the investment in the certain region depends on what sectors are the primary economic driver in that region. In other words, every region has different characteristics based on the industrial or sectoral classification. Therefore, the study on sectoral investment is more relevant in this matter.

The investment consists of expenditures on machinery, capital equipment, inventories, new plants, etc. The investment decision depends on marginal benefits which are the expected rate of businesses' return and marginal cost namely interest rate paid from borrowed fund. Someone will invest if expected rate of return exceeds interest rate, so higher interest rate would reduce investment. Or in other words, there is a negative parameter of interest rate elasticity on investment equation. This parameter indicates the sensitivity of investment in the particular sector to the change in interest rate.

This research aims to analyze the determinants of investment especially the effect of real interest rate on investment in 20 sectors using time-series cross-sectional data across regions/provinces in Indonesia. It also investigates the important of having sectoral investment equation compared to the aggregated national one. This will show how much different in term of elasticity of real interest rate on investment if the central bank still uses the aggregated national investment equation. In addition, this study also tries to identify the most affected sector on the change of real interest rate. This information can help central bank as a policy maker to have a better understanding of how to promote investment in the particular sector.

1.2 Hypothesis

This study tries to investigate the relationship between sectoral investment and its determinants especially real interest rate. The correlation between real interest rate and investment in every sector is expected to be negative. It means that the higher real interest rate will discourage the investment in every sector. Every sector should have different elasticity parameter of real interest rate on investment. This research would expect that more capital intensive industries are more sensitive to the change in interest rate compared to the labor-intensive industries. The capital-intensive industries need more funds to buy or rent more capital, and this fund depends on the interest rate.

The sectoral investment equation is expected to provide more accurate real interest rate elasticity on investment in each sector compared to if the policymakers use the aggregated national investment model.

1.3 Literature Review

The commonly used approaches to explain investment namely the simple accelerator theory, flexible accelerator, Tobin's Q theory associated with Tobin (1969), and neoclassical theory associated with Jorgenson (1963).

The simple accelerator model formulation is based on Clark (1917). It assumes the desired capital-output ratio of the firms is constant.

$$K_t = kY_t$$

Hence, the net investment is also proportional to the change in output:

$$K_t - K_{t-1} = I_t = k\Delta Y_t \tag{1.1}$$

where Y_t is the output, K_t is the capital stock, I_t is the net investment, and k is the desired capital-output ratio. The weakness of this model is due to the fact that the firms' response to the change of demand is always sufficient to keep the actual stock and the desired capital stock equal, which is not necessarily true. This theory ignores the influence on the cost of capital, the expectation of the investors, and profitability.

Due to this problem, the model is reformulated into the flexible accelerator model by Goodwin (1948). This model is flexible because the investment can vary with other variables that related to market imperfection and uncertainty. Another feature of this model is the non-instantaneous adjustment to the desired capital stock. However, the model still lacks a solid theoretical foundation because the investment does not depend on the price of capital.

In his paper, Jorgenson (1963) suggests an alternative to the accelerator model so-called a neoclassical model. It is derived by assuming the firms maximizes their profits subject to the Cobb-Douglas production function. Investment is calculated using the change of desired capital, which is a function of the user cost of capital, the level of output, and the price of output. In turn, this user cost of capital depends on the interest rate, depreciation rate, price of capital goods, and tax structure.

Theory of investment also developed by Tobin (1969) named the Tobin’s Q theory. In this theory, he introduced the Q ratio, which is the ratio of the existing capital and its replacement cost in terms of market value. The firms will increase their capital when the market value of the additional investment exceeds the replacement cost or when $q > 1$. In this condition, the firms gain profits by investing more capital. On the other hand, when $q < 1$, the expected investment are near zero because the profits obtained from the additional investment are less than the cost of capital. The firms will maximize their expected present value to choose the investment level. The shadow price of capital, which is referred to the user cost of capital, is used to obtain the optimal value of capital stock. Capital stock will adjust until no more profit can be made, as the firms maximize their profit.

There are some empirical studies conducted to understand the determinants of investment in both developed and developing countries focusing on different variables. Sakr (1993) examined the determinants of private investment in Pakistan for period 1973/74 – 1991/92 annually. It found a positive correlation of private investment with GDP growth, the growth of credit to private sector and to government investment. If the government investment is divided into infrastructure and non-infrastructure components, private investment has a negative correlated with the non-infrastructure components.

Another study from Dailami and Walton (1992) examined the determinant of private investment in Zimbabwe during 1970 until 1987. They used the lagged dependent variable, which is the lagged of investment, and used the growth of GNP, the real interest rate, the real effective exchange rate, the real wage, the relative price of capital goods, and the real UK government bond yields as the independent variables. The estimation results indicated that the private investment is positively correlated with the lagged of investment, the real interest rate, the growth of GNP, and the real effective exchange rate, and negatively associated with the real wage, the relative price of capital goods, and the real UK government bond yields.

Oshikoya (1994) tried to investigate the determinants of private investment in seven African countries from 1970 to 1988. The equation of private investment was estimated by ordinary least squares using separated pooled data for low-income countries namely Tanzania, Malawi, and Kenya, and for middle-income countries that are Tunisia, Marocco, Mauritius, and Cameroon. The estimation results found a positive impact of GDP growth rate on private investment. Overall, the most affecting variables on private investment in middle-income countries are the domestic inflation, the real exchange rate, the lagged debt service ratio, and the public investment rates. On the other hand, GDP growth, domestic inflation, credit to private sector, and debt service ratio are the largest affecting variables on private investment in low-income countries.

Asante (2000) employed time series and cross-sectional analysis to study the determinants of private investment in Ghana during the period 1970 – 1992. The cross-sectional analysis was used the primary data from the survey of 116 manufacturing firms. The survey was intended to capture certain variables, especially quantitative variables such as political instability, political uncertainty, etc., that may affect private investment but not introduced in the time series analysis. The study found that the growth of real credit to the private sector has a significant and positive effect on private investment. It also showed the complementary between private investment and public investment, so the development of infrastructure from the government is needed to boost the private sector. The others important variables that affect the private investment are macroeconomic instability, political instability, and the trade regime.

Ajide and Lawanson (2012) in their paper tried to model the long-run determinants of domestic private investment in Nigeria from period 1970 to 2010. They employed Auto-Regressive Distributed Lag (ARDL) bounds testing approach. They found that the determinants between short-run and long-run are different. The key determinants in the short-run are real GDP, public investment, and terms of trade, while in the long-run the key determinants are real GDP, public investment, real interest rate, credit to the private sector, exchange rate, external debts, terms of trade, and reform dummy.

Adugna (2013) examined and investigated factors that determine private investment in Ethiopia from 1981 to 2010. The author utilized OLS model on the logarithmic form of the data set. The results from the regression indicate that in the long run, private investment in Ethiopia depends positively and significantly on real GDP per capita, public investment, and external debt, while the lagged of private investment, as a proxy of the investment climate, has a significant negative impact on the private investment. In the short run, the external debt and real GDP per capita have a significant positive effect on the private investment, while the negative effect on private investment comes from inflation.

The recent study by Elbanna (2016) investigated the determinants of private investment in Egypt as a developing country from period 1983 until 2014. The results of running the multiple regression shown that the private investment is affected by three determinants, namely GDP, money supply, and exchange rate. All of these are positively correlated with private investment.

Only a few papers studied about investment in Indonesia. Suhendra and Anwar (2014), for example, their research tried to examine the effect of public investment by government and other macroeconomic variables like GDP growth, interest rate, investment credit, inflation, and exchange rate on the private investment from 1990 to 2011. It also investigated the effect of the private investment on the economic growth during that periods. The results indicated that investment positively associates with the economic growth, public investment, the availability of investment financing, and exchange rate. However, they found that interest rates, and inflation negatively affect the private investment. In the economic growth model, the key determinants of economic growth are private investment, human capital, public investment, and labor. They all positively correlated with the economic growth.

Another research on the determinants of private investment in Indonesia was conducted by Nainggolan et al. (2015). They focus the analysis on North Sumatra Province using data from 1980 to 2011. The results from running Error Correction Model (ECM) method have shown that both in the short-run and long-run, the GDP, investment credit, and exchange rate have a significant and positive impact on private investment. However, the negative effect on private investment comes from the interest rate, government investment, inflation, and economic crisis.

All of these research above mainly estimated the aggregated investment equation at national or regional level. The study on the sectoral investment is limited due to the lack of data availability. Bigsten, et al. (1997) examined the low levels of investment in the manufacturing sector in Africa. They conducted a survey of firms in the manufacturing sectors over the period 1992 to 1995 in four countries, namely Ghana, Cameroon, Zimbabwe, and Kenya. The sample was taken from four sub-sectors in the manufacturing, such as wood and furniture, textile and clothing, machinery and food, and metal working. As determinants of investment, they were focused on the growth of value added, profitability, the size and the age of the firms, and past firms borrowing. The results from the panel data analysis suggested that the large firms are more likely to invest in any year than the small firms. They also argued that the crucial factor adversely affecting investment is the high capital costs facing the firms.

The study about the determinants of agricultural sector investment in Bengkulu province in Indonesia was conducted by Tatiana, et al. (2015). They employed secondary data from Central Bureau of Statistics of Bengkulu from 2010 until 2013 comprises of nine regencies: North Bengkulu, Central Bengkulu, South Bengkulu, Kepahiang, Rejang Lebong, Seluma, Lebong, Mukomuko, Kaur, and one city: Bengkulu city. The panel data regression was conducted. They found that the most dominant factors were GDP per capita, ownership of mining, road infrastructure, and agricultural resources.

This chapter attempts to fill the gap by studying the sectoral investment in Indonesia comprehensively. It estimates all 20 sectoral investment equation in Indonesia. In addition, it also produces the aggregated national investment equation as a benchmark. Thus, this study contributes to the literature on the importance of having disaggregated sectoral investment model by comparing them to the aggregated national investment model, if we want to understand the impact of real interest rate on the sectoral investment.

1.4 The Theoretical Framework

The proposed investment equation is derived using the neo-classical demand for capital. In this framework, the production function is formulated using constant elasticity of substitution (CES) as;

$$F(K_{it}, L_{it}) = TFP_i A_t \left[\beta_i L_{it}^{\frac{\sigma-1}{\sigma}} + \alpha K_{it}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}\nu}, \qquad \alpha_i + \beta_i = 1 \tag{1.2}$$

where σ is the elasticity of substitution between labor and capital, ν denotes returns to scale, and $TFP_i A$ is total factor productivity. The user cost of capital (UC_{it}) equals to the first order condition of firm's optimization problem with respect to capital, and hence leads to the following equality;

$$F_K(L_{it}, K_{it}) = UC_{it} \tag{1.3}$$

The first order conditions of firm profit maximizations are

$$\log K_{it} = \theta \log Y_{it} - \sigma \log UC_{it} + \log H_{it} \tag{1.4}$$

or

$$k_{it} = \theta y_{it} - \sigma uc_{it} + h_{it} \tag{1.5}$$

where $\theta = \left(\sigma + \frac{1-\sigma}{\nu}\right)$ and $h_{it} = \log\left[(TFP_i A_t)^{\frac{\sigma-1}{\nu}} (\nu\alpha_i)^{\sigma}\right]$

$k_{it}$is log of capital stock, $uc_{it}$is the log of user cost of capital, h_{it} is the log of firm specific variables, and y_{it} is the log of output or sales.

In this study, equation (1.5) is estimated using the new specification in terms of Autoregressive Distributed Lag Model (ARDL). The dynamic neoclassical investment model in ARDL(1,0) is defined as follows;

$$k_{it} = \omega_1 k_{i,t-1} + \theta_1 y_{it} - \sigma_1 uc_{it} + \phi_1 h_{it} \tag{1.6}$$

By taking first difference of equation (1.6) and using the approximation $\log K_t - \log K_{t-1} = I_t / K_{t-1} - \delta$, and replacing year-specific productivity growth ($\Delta \log A_t$) by time dummies (λ_t), and also replacing firm-specific effects productivity growth ($\Delta \log TFP_i$) by firm-specific effects (η_i), and adding a random term ε_{it}, yields;

$$\left(\frac{I_{it}}{K_{i,t-1}}\right) = \omega_1 \left(\frac{I_{i,t-1}}{K_{i,t-2}}\right) + \theta_1 \Delta y_{it} - \sigma_1 \Delta uc_{it} + \lambda_t + \eta_i + \varepsilon_{it} \tag{1.7}$$

Based on the neoclassical model, one of the main factors for determining the user cost of capital is the interest rate. The higher the interest rate, the higher the user cost of capital and vice versa. Thus, the monetary policy transmission through the traditional interest rate channel can be explained by the user cost of capital. The most commonly used user cost of capital in the previous studies is based on Jorgenson (1967) approach. The derivation is as follows:

$$q_s = \int_s^\infty e^{-(r+\delta)(t-s)}\, uc_t dt \tag{1.8}$$

where q_s denotes competitive equilibrium price of capital or equals to the series of discounted user cost, and uc_t represents the user cost which is the value of capital service at time *t*. The differentiation of this equation with respect to time s yields the user cost of capital;

$$uc_s = q_s(r + \delta - \pi) \tag{1.9}$$

From this expression, the user cost of capital can be defined as a function of the price of capital, real interest rate, and depreciation. By combining equation (1.7) and (1.9), the investment equation can be formulated as a function of lagged investment, GDP growth, and real interest rate.

1.5 Generalized Methods of Moments (GMM)

The concept of Generalized Methods of Moments (GMM) has been increasingly popular since last two decades. This section will discuss the basic concept of GMM before further discussion the dynamic panel data estimator, particularly the Arellano-Bond Estimator. Most of the dynamic panel data estimators are based on the idea of GMM.

Let define a linear model:

$$y_i = \boldsymbol{z}_i'\boldsymbol{\delta} + \varepsilon_i \qquad (i = 1,2, \dots n) \tag{1.10}$$

where y_i is an independent variable, $\boldsymbol{z}_i$ is and a vector of regressors with L-dimension, $\boldsymbol{\delta}$ is a coefficient of regressors with L-dimension, and ε_i is an error term. Let $\boldsymbol{x}_i$ be a K-dimensional vector of instruments.

The fundamental principle of the method of moments is to find the parameter estimate such that the associated sample moments is equal to zero. Let $E[\boldsymbol{g}(\boldsymbol{\delta})]$ denotes the population moments in the orthogonality conditions. The sample moments then can be obtained by calculating the sample mean of $\boldsymbol{g}(\boldsymbol{\delta})$ evaluated at some hypothetical value $\tilde{\boldsymbol{\delta}}$ of $\boldsymbol{\delta}$, or

$$\boldsymbol{g}_n(\tilde{\boldsymbol{\delta}}) \equiv \frac{1}{n}\sum_{i=1}^{n} \boldsymbol{g}(\tilde{\boldsymbol{\delta}}) \tag{1.11}$$

Using the method of moments to the model actually solves the K number of equations in L unknowns such that $\boldsymbol{g}_n(\tilde{\boldsymbol{\delta}}) = \boldsymbol{0}$. Since the model is linear, $\boldsymbol{g}_n(\tilde{\boldsymbol{\delta}})$ is written as

$$\begin{aligned} \boldsymbol{g}_n(\widetilde{\boldsymbol{\delta}}) &\equiv \frac{1}{n}\sum_{i=1}^{n} \boldsymbol{x}_i(y_i - \mathbf{z}_i'\widetilde{\boldsymbol{\delta}}) \\ &= \frac{1}{n}\sum_{i=1}^{n} \boldsymbol{x}_i y_i - \left(\frac{1}{n}\sum_{i=1}^{n} \boldsymbol{x}_i\,\mathbf{z}_i'\right)\widetilde{\boldsymbol{\delta}} \equiv \boldsymbol{s}_{\boldsymbol{xy}} - \boldsymbol{S}_{\boldsymbol{xz}}\widetilde{\boldsymbol{\delta}} \end{aligned} \tag{1.12}$$

where $\boldsymbol{s}_{\boldsymbol{xy}}$ and $\boldsymbol{S}_{\boldsymbol{xz}}$ are the sample moments of $\boldsymbol{\sigma}_{\boldsymbol{xy}}$ and $\boldsymbol{\Sigma}_{\boldsymbol{xy}}$:

$$\boldsymbol{s}_{\boldsymbol{xy}} \equiv \frac{1}{n}\sum_{i=1}^{n} \boldsymbol{x}_i y_i \quad \text{and} \quad \boldsymbol{S}_{\boldsymbol{xz}} = \frac{1}{n}\sum_{i=1}^{n} \boldsymbol{x}_i\,\mathbf{z}_i'$$

Therefore, the $\boldsymbol{g}_n(\widetilde{\boldsymbol{\delta}}) = \mathbf{0}$ condition can be written as

$$\boldsymbol{S}_{\boldsymbol{xz}}\widetilde{\boldsymbol{\delta}} = \boldsymbol{s}_{\boldsymbol{xy}} \tag{1.13}$$

In the case of $K > L$, the system may not have a solution. The extension of the method of moments known as GMM is used.

In the overidentified equation, when $K > L$, there is no L-dimension of $\widetilde{\boldsymbol{\delta}}$ that satisfy the K-equations in (1.13), so the $\boldsymbol{g}_n(\widetilde{\boldsymbol{\delta}})$ cannot set exactly equal to zero. In this situation, we try to find $\widetilde{\boldsymbol{\delta}}$ such that $\boldsymbol{g}_n(\widetilde{\boldsymbol{\delta}})$ is close to zero by introducing the weighting matrix, $\widehat{\boldsymbol{W}}$, which is a positive definite and symmetric matrix, in the objective function of GMM. The GMM estimator of $\widetilde{\boldsymbol{\delta}}$ is

$$\widetilde{\boldsymbol{\delta}}(\widehat{\boldsymbol{W}}) \equiv \underset{\widetilde{\boldsymbol{\delta}}}{\operatorname{argmin}}\, J(\widetilde{\boldsymbol{\delta}}, \widehat{\boldsymbol{W}}) \tag{1.14}$$

where

$$J(\widetilde{\boldsymbol{\delta}}, \widehat{\boldsymbol{W}}) \equiv n.\,\boldsymbol{g}_n(\widetilde{\boldsymbol{\delta}})'\widehat{\boldsymbol{W}}\boldsymbol{g}_n(\widetilde{\boldsymbol{\delta}})$$

In the linear model, the objective function is quadratic in $\widetilde{\boldsymbol{\delta}}$:

$$J(\widetilde{\boldsymbol{\delta}}, \widehat{\boldsymbol{W}}) = n.\left(\boldsymbol{s_{xy}} - \boldsymbol{S_{xz}}\widetilde{\boldsymbol{\delta}}\right)' \widehat{\boldsymbol{W}}\left(\boldsymbol{s_{xy}} - \boldsymbol{S_{xz}}\widetilde{\boldsymbol{\delta}}\right) \tag{1.15}$$

The first order condition for the minimization problem with respect to $\widetilde{\boldsymbol{\delta}}$ can be written as follows:

$$\boldsymbol{S}'_{\boldsymbol{xz}}\widehat{\boldsymbol{W}}\boldsymbol{s_{xy}} = \boldsymbol{S}'_{\boldsymbol{xz}}\widehat{\boldsymbol{W}}\boldsymbol{S_{xz}}\widetilde{\boldsymbol{\delta}} \tag{1.16}$$

Finally, the GMM estimation can be obtained by solving $\widetilde{\boldsymbol{\delta}}$ as follows:

$$\widetilde{\boldsymbol{\delta}}(\widehat{\boldsymbol{W}}) = \left(\boldsymbol{S}'_{\boldsymbol{xz}}\widehat{\boldsymbol{W}}\boldsymbol{S_{xz}}\right)^{-1}\boldsymbol{S}'_{\boldsymbol{xz}}\widehat{\boldsymbol{W}}\boldsymbol{s_{xy}} \tag{1.17}$$

This estimator will reduce to the IV-estimator when $K = L$.

1.5.1 The Arellano-Bond Estimator

The panel GMM estimator suggested in Arellano and Bond (1991) has become more popular for estimating dynamic data panels. It is because the Arellano-Bond estimator gives consistent estimates in panels with short time series observations per individual or small T, and relies on minimal assumptions, but it needs large samples of the cross-section dimension, or large N. This estimator is similar to the Anderson and Hsiao estimator[1]. However, it utilizes additional moment restrictions, that enhance the instrument set.

The model specification in level is

$$y_{it} = \rho y_{i,t-1} + \boldsymbol{x}'_{it}\boldsymbol{\beta} + \alpha_i + \varepsilon_{it} \tag{1.18}$$

The Arellano and Bond (1991) estimator uses the differenced form of the original equation

[1] See Anderson and Hsiao (1982)

$$y_{it} - y_{i,t-1} = \rho\left(y_{i,t-1} - y_{i,t-2}\right) + \left(\boldsymbol{x}'_{it} - \boldsymbol{x}'_{i,t-1}\right)\boldsymbol{\beta} + \varepsilon_{it} - \varepsilon_{i,t-1} \qquad (1.19)$$

This will cancel the individual fixed effects that assumed to correlate with the exogenous variable. In the matrix notation, we can write both equations above as follows:

$$\boldsymbol{y} = \boldsymbol{y}_{-1}\rho + \boldsymbol{X\beta} + \boldsymbol{D\alpha} + \boldsymbol{\varepsilon} \qquad (1.20)$$

$$\boldsymbol{Fy} = \boldsymbol{Fy}_{-1}\rho + \boldsymbol{FX\beta} + \boldsymbol{F\varepsilon} \qquad (1.21)$$

where

$$\boldsymbol{F} = \boldsymbol{I}_N \otimes \boldsymbol{F}_T \text{ and } \boldsymbol{F}_T = \begin{pmatrix} -1 & 1 & 0 & \dots & 0 & 0 \\ 0 & -1 & 1 & \dots & 0 & 0 \\ \vdots & & & & & \\ 0 & 0 & 0 & \dots & -1 & 1 \end{pmatrix} \text{ with } (T-1)\times T \text{ dimension}$$

The individual fixed effects cancel out because $\boldsymbol{FD} = \boldsymbol{0}$. However, the error term, $\varepsilon_{it} - \varepsilon_{i,t-1}$, clearly correlated with the lagged of endogenous variable, $E\left(dy_{i,t-1}d\varepsilon_{it}\right) \neq 0$. Thus, the estimator will be biased.

Anderson and Hsiao argued that employing the lags 2 of level instruments or lags 2 of difference as an instrument or the lags of difference endogenous regressor can address this problem. This is because these instruments are expected to be uncorrelated with the differenced error term:

$$E\left(y_{i,t-2}d\varepsilon_{it}\right) = 0 \text{ and } E\left(dy_{i,t-2}d\varepsilon_{it}\right) = 0$$

However, Arellano (1989) found that using level instruments is beneficial than using difference instruments since there are no points of singularities and the variance is small. In addition, the advantage of using level instruments is having one year less of losing data. This is relevant in practical use when using few years with a large number of individuals.

For $t = 3$, the equation (1.19) is written as

$$y_{i3} - y_{i2} = \rho(y_{i2} - y_{i1}) + (\boldsymbol{x}'_{i3} - \boldsymbol{x}'_{i2})\boldsymbol{\beta} + \varepsilon_{i3} - \varepsilon_{i2} \tag{1.22}$$

In this example, the available instruments are $y_{i1}, \boldsymbol{x}'_{i2}$, and $\boldsymbol{x}'_{i1}$.

At final period *T*, the equation becomes

$$y_{iT} - y_{i,T-1} = \rho\left(y_{i,T-1} - y_{i,T-2}\right) + \left(\boldsymbol{x}'_{iT} - \boldsymbol{x}'_{i,T-1}\right)\boldsymbol{\beta} + \varepsilon_{iT} - \varepsilon_{i,T-1} \tag{1.23}$$

the available instruments are $y_{i,1}, y_{i,2}, \dots, y_{i,T-2}, \boldsymbol{x}'_{i1}, \boldsymbol{x}'_{i2}, \dots \boldsymbol{x}'_{i,T-1}$.

The instrumented equation in matrix notation is

$$\boldsymbol{W}'\boldsymbol{F}\boldsymbol{y} = \boldsymbol{W}'\boldsymbol{F}\boldsymbol{X}\boldsymbol{\gamma} + \boldsymbol{W}'\boldsymbol{F}\boldsymbol{\varepsilon} \tag{1.24}$$

where

$$\boldsymbol{X}_i = \begin{bmatrix} y_{i2} - y_{i1} & \boldsymbol{x}'_{i3} - \boldsymbol{x}'_{i2} \\ y_{i3} - y_{i2} & \boldsymbol{x}'_{i4} - \boldsymbol{x}'_{i3} \\ \vdots & \vdots \\ y_{i,T-1} - y_{i,T-2} & \boldsymbol{x}'_{iT} - \boldsymbol{x}'_{i,T-1} \end{bmatrix}$$

$$\boldsymbol{X} = (\boldsymbol{y}_{-1}, \boldsymbol{X}), \quad \boldsymbol{\gamma}' = (\rho, \boldsymbol{\beta}'), \quad \boldsymbol{W} = (\boldsymbol{W}'_1, \boldsymbol{W}'_2, \dots, \boldsymbol{W}'_N)'$$

$$\boldsymbol{W}_i = \begin{bmatrix} [y_{i1}, x'_{i1}, x'_{i2}] & 0 & \dots & 0 \\ 0 & [y_{i1}, y_{i2}, x'_{i1}, x'_{i2}, x'_{i3}] & \dots & 0 \\ 0 & 0 & \ddots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & [y_{i1}, y_{i2}, \dots, y_{i,T-2}, x'_{i1}, x'_{i2}, \dots, x'_{i,T-1}] \end{bmatrix}$$

The maximal number of parameters estimated in the *K*-explanatory variable is $T - 2 + K(T - 1)$. The estimation procedure consists of a two-step estimation as in the simple instrumental variable estimation. The first step of estimation utilizes a covariance matrix to take into account the first order autocorrelation in the error term which is introduced by the differencing operation.

$$\boldsymbol{V} = \boldsymbol{W}'\boldsymbol{G}\boldsymbol{W} = \sum_{i=1}^{N} \boldsymbol{W}_i'\boldsymbol{G}_T\boldsymbol{W}_i \tag{1.25}$$

where $\boldsymbol{G} = (\boldsymbol{I}_N \otimes \boldsymbol{G}_T')$ and $\boldsymbol{G}_T = \boldsymbol{F}_T\boldsymbol{F}_T' = \begin{bmatrix} 2 & -1 & & 0 \\ -1 & 2 & \ddots & \\ & \ddots & \ddots & -1 \\ 0 & & -1 & 2 \end{bmatrix}$

The transformation from the original observations into differences is obtained by multiplying the matrix $\boldsymbol{F}$. The covariance matrix $\boldsymbol{V} = \boldsymbol{F}\boldsymbol{F}'$ is used as the approximation of the covariance matrix in the first-step, because the variance of error term in difference equation is $Var(\boldsymbol{F}\boldsymbol{u}) = \boldsymbol{F}\sigma^2\boldsymbol{F}'$.

The second step of GMM estimation utilizes the residuals of the first-step estimation in order to estimate the covariance matrix:

$$\widehat{\boldsymbol{V}} = \sum_{i=1}^{N} \boldsymbol{W}_i'\boldsymbol{F}_T\hat{\boldsymbol{\varepsilon}}_i\hat{\boldsymbol{\varepsilon}}_i'\boldsymbol{F}_T'\boldsymbol{W}_i \tag{1.26}$$

Finally, the estimator is

$$\widehat{\boldsymbol{\gamma}}^{GMM} = \left(\boldsymbol{X}'\boldsymbol{W}\widehat{\boldsymbol{V}}^{-1}\boldsymbol{W}'\boldsymbol{X}\right)^{-1}\boldsymbol{X}'\boldsymbol{W}\widehat{\boldsymbol{V}}^{-1}\boldsymbol{W}'\boldsymbol{y} \tag{1.27}$$

1.5.2 Instrument Validation

The validity of instrument must be checked after the estimation parameters are conducted. There are two types of instrument validation:

- The validity of instrument subsets is verified by the Sargan test (Sargan 1958). The null hypothesis of this test is “the instruments as a group are exogenous or uncorrelated with the residuals." The validity of instruments is obtained when this hypothesis in not rejected. Thus, the higher p-value of the Sargan statistic the better. In the two-step robust estimation, the Hansen J-statistic is used instead of the Sargan statistic. The null hypothesis of the Hansen J-statistic is the same as in the Sargan.

- To investigate the serial correlation in the disturbance, the Arellano and Bond (1991) suggests a test. The validity of instruments can be checked from the existence of serial correlation in the disturbances. When ε_{it} are serially correlated of order 1, the $y_{i,t-2}$ is endogenous or correlated to $\Delta\varepsilon_{it}$ because $\varepsilon_{i,t-1}$ presents in the difference. Thus, we cannot use $y_{i,t-2}$ as a valid instrument.

  Arellano and Bond tested the serial correlation of disturbance using difference, $\Delta\varepsilon_{it}$, instead of level ε_{it}. The serial correlation of order 2 in difference must be tested to check the serial correlation of order 1 in levels. The validation of instrumental variables is achieved when the null hypothesis of this test (no serial correlation) is not rejected.

1.6 Data and Methodology

Most of the data are gathered from CEIC Indonesian premium database. The model used GDP by industry and GDP by expenditure for every province to extract investment by industry for every province. This data is annually from 2000 to 2013. Thus, the panel dataset consists of larger province dimension (*N*=33) and shorter time dimension (*T*=14).

The investment for every sector is estimated using dynamic panel data across provinces since the period of the dataset is too short. The variables used as regressor in the model are private domestic investment (in logarithmic) $INV_{i,t}$; real GDP growth, $GDPG_{i,t}$; and real interest rate, r_t. The index *i* indicates the sectoral variables. There are four exogenous instrument variables: $LOAN_{i,t}$, that is loan given by commercial and rural banks; $M2GDP_t$, which is broad money in the percentage of GDP; $CREDSUP_t$, which is domestic credit provided by financial sector in the percentage of GDP; and $FINNOV_t$, that is ratio of M3 to M1 as a proxy of financial innovation. Meanwhile, the pre-determined instruments are: the lagged of investment; GDP level, denoted by $GDPL_{i,t-j}$; deflator of investment, denoted by $PINV_{i,t-j}$, as a proxy of capital price; and the lagged of real interest rate. These pre-determined instruments are all in level instead of in difference. However, every sector is estimated using different combination of exogenous and pre-determined instruments that are mentioned above.

The model is formulated as follow:

$$INV_{i,t} = \alpha_i + \rho INV_{i,t-1} + \beta_1 GDPG_{i,t} + \beta_2 r_t + \varepsilon_{i,t} \tag{1.28}$$

The Arellano – Bond GMM estimator is used because some econometric problems may arise when estimating the equation above, namely:

1) There might be causality between $GDPG_{i,t}$ and $INV_{i,t}$ from both directions since $GDPG_{i,t}$ is assumed to be endogenous. These regressors might be correlated with error term.
2) The explanatory variables might be correlated with the time-invariant province characteristics (fixed effects), such as demographics and geography.

3) The lagged dependent variable, $INV_{i,t-1}$, that was introduced in the model might give rise to the correlation of this variable with error term.

4) The dataset of the panel has a short time dimension (T =14) and a larger region dimension (N =33).

The first problem can be solved by introducing not only the exogenous instruments but also the lagged levels of the endogenous regressors. Therefore, the endogenous variables are pre-determined and not correlated with the error term.

The second problem can be addressed using the difference GMM by transforming equation (1.28) into the first difference equation:

$$\Delta INV_{i,t} = \rho \Delta INV_{i,t-1} + \beta_1 \Delta GDPG_{i,t} + \beta_2 \Delta r_t + \Delta \varepsilon_{i,t} \quad (1.29)$$

The first difference transformation removes the fixed province-specific effect because it is constant over time.

The correlation of first-differenced lagged dependent variable $\Delta INV_{i,t-1}$ with the error term in problem 3 can also be coped by introducing the instruments of its past levels, i.e. $INV_{i,t-3}, INV_{i,t-4}$, etc.

The fourth problem is solved using the Arellano and Bond estimator because it was designed to deal with the small-T and large-N panels.

All these problems above can be addressed by implementing the Arellano and Bond with difference GMM estimator. Moreover, the two-step robust estimation process is also used to make the resulting standard errors are consistent with panel-specific autocorrelation and heterogeneity in one-step estimation.

1.7 Estimation Results

Due to the limited number of provinces in the panel dataset, it is necessary to keep the number of instruments less than or equal to the number of provinces or groups. The second lag or deeper lags are required because they are not correlated with the current error term, but the first lag is. To find a good instrument, one can experiment with second or deeper lags. However, they will reduce the sample size using deeper lags. As long as the number of instruments less than or equal to the number of groups, one can use all available lags (second and deeper lags) as instruments.

As mentioned before, every sectoral equation is estimated using a different combination of exogenous instruments. Table 1.1 below, shows the combination of exogenous and predetermined instruments that are used for estimation at sectoral and national level including the total number of instruments. As shown, the total number of instruments including predetermined and exogenous is less than or equal to the maximum number of group or province, which is 33.

Before we discuss the estimation results, it is better to check the validity of instruments so that the parameters are not biased. As mentioned in the instrument validation section above, we have to make sure that the estimation results passed those two types of validation. First, the Hansen test should tell that the null hypothesis is not rejected because in this paper we used the two-step robust estimation, so the Sargan test is not relevant in this case. This condition indicates the instruments are exogenous or uncorrelated with the residuals. Second, the Arellano-Bond test for AR(2) in first differences should also reject the null hypothesis to show that there is no serial correlation of order 1 in levels.

Table 1.1 List of Instrument Variables

Sector	Descriptions	List of Instruments: Predetermined	List of Instruments: Exogenous	Total Number of Instru-ments
fcrops	Food Crops	$INV_{fcrops,t-4}, r_{t-4}$	$LOAN_{i,t}, M2GDP_t, FINNOV_t$	23
nfcrop	Non-Food Crops	$INV_{nfcrops,t-4}, GDPL_{nfcrops,t-4}, r_{t-4}$	$LOAN_{i,t}, CREDSUP_t$	32
lvstck	Livestock	$INV_{lvstck,t-3}, GDPL_{lvstck,t-3}$	$LOAN_{i,t}, FINNOV_t$	24
forest	Forestry	$INV_{forest,t-5}, GDPL_{forest,t-5}$	$LOAN_{i,t}, M2GDP_t, FINNOV_t$	30
fish	Fishery	$INV_{fish,t-4}, GDPL_{fish,t-4}, r_{t-4}$	$M2GDP_t, FINNOV_t$	32
oilmq	Oil, Gas and Geothermal Mining	$GDPL_{oilmq,t-7}, GDPL_{oilmq,t-8}$	$LOAN_{i,t}, CREDSUP_t$	15
noilmq	Non-Oil and Gas and Other Mining	$INV_{noilmq,t-7}, r_{t-7}, INV_{noilmq,t-8}, r_{t-8}$	$LOAN_{i,t}, M2GDP_t, CREDSUP_t$	29
oilmf	Oil and Gas Manufacturing	$INV_{oilmf,t-8}$	$LOAN_{i,t}, CREDSUP_t$	8
noilmf	Non-Oil and Gas Manufacturing	$INV_{noilmf,t-4}, GDPL_{noilmf,t-4}$	$LOAN_{i,t}, M2GDP_t, FINNOV_t$	23
utili	Electricity, Gas and Drinking Water	$INV_{utili,t-4}, GDPL_{utili,t-4}$	$LOAN_{i,t}, FINNOV_t$	22
const	Construction	$INV_{const,t-4}, INV_{const,t-5}, INV_{const,t-6}$	$LOAN_{i,t}, M2GDP_t, CREDSUP_t, FINNOV_t$	31
trade	Wholesale and Retail Trade	$GDPL_{trade,t-4}, r_{t-4}$	$LOAN_{i,t}, M2GDP_t, CREDSUP_t, FINNOV_t$	24
hotel	Hotel and Restaurant	$INV_{hotel,t-5}, GDPL_{hotel,t-5}$	$LOAN_{i,t}, M2GDP_t, CREDSUP_t, FINNOV_t$	22
lndtrn	Land Transportation	$GDPL_{lndtrn,t-7}, GDPL_{lndtrn,t-8}$	$LOAN_{i,t}, M2GDP_t, CREDSUP_t$	16

Sector	Descriptions	List of Instruments		Total Number of Instru-ments
		Predetermined	Exogenous	
wtrtrn	Water Transportation	$INV_{wtrtrn,t-9}$, $GDPL_{wtrtrn,t-9}$, r_{t-9}, $INV_{wtrtrn,t-10}$, $GDPL_{wtrtrn,t-10}$, r_{t-10}	$M2GDP_t, CREDSUP_t$	29
airtrn	Air Transportation	$INV_{airtrn,t-4}$, $GDPL_{airtrn,t-4}$, r_{t-4}	$LOAN_{i,t}, CREDSUP_t$, $FINNOV_t$	33
commu	Communications	$INV_{commu,t-4}$, $INV_{commu,t-5}$, $INV_{commu,t-6}$	$LOAN_{i,t}, M2GDP_t$, $CREDSUP_t, FINNOV_t$	31
finan	Financial, Ownership and Business	$INV_{finan,t-7}$, $GDPL_{finan,t-7}$, $INV_{finan,t-8}$, $GDPL_{finan,t-8}$,	$LOAN_{i,t}, M2GDP_t$, $CREDSUP_t$	29
pubsrv	Public Services	$GDPL_{pubsrv,t-4}$	$LOAN_{i,t}, M2GDP_t$, $CREDSUP_t, FINNOV_t$	24
othsrv	Other Services	$INV_{othsrv,t-4}$, $GDPL_{othsrv,t-4}$, r_{t-4}	$CREDSUP_t, FINNOV_t$	32
nat	Aggregated National Investment	$GDPL_{nat,t-5}$, $PINV_{t-5}$, r_{t-5}	$LOAN_{i,t}, M2GDP_t$, $CREDSUP_t, FINNOV_t$	31

Table 1.2 shows the estimation result of aggregated national investment equation using dynamic panel data. The overidentified restrictions test using Hansen and difference in Hansen test of exogeneity are satisfactory, which means the instruments are uncorrelated with the error process. In addition, Arellano-Bond test for second-order autoregression, written as AR(2), in first-differences indicated the null hypothesis could not be rejected. In other words, there is no serial correlation of order 1 in levels.

Table 1.2 Estimation Results of Aggregated National Investment Equation

Variables	National	
$INV_{i,t-1}$	1.004	***
$GDPG_{i,t}$	1.17	*
r_t	-0.35	*
Arellano-Bond test for AR(1) in 1st differences	-1.88	*
Arellano-Bond test for AR(2) in 1st differences	-1.25	
Hansen test of overid restrictions	30.39	
Difference-in-Hansen test of Exogeneity	1.46	

*,**,*** indicate the significance levels of 10%, 5%, and 1% respectively.

As shown, the determinants of aggregated investment at national level are the lagged of investment, GDP growth and real interest rate because they are significantly correlated with aggregated national investment. The coefficient of lagged investment is positive and significant at 1% with a coefficient of 1.004. The coefficient of GDP growth is also positive at 1.17 as expected and significant at 10%. In contrast, the real interest rate elasticity of investment is -0.35, which is significant and negative as expected. The negative coefficient of real interest rate on aggregated national investment means that the higher real interest rate will discourage the national investment.

To make the real interest rate coefficient comparable, the sectoral investment equations are estimated using the disaggregated investment data and utilizing the same model and method of estimation as in aggregated national investment equation. Table 1.3 below show the estimation results of sectoral investment equation for every sector. In term of the validity of the instruments, the Hansen test of overidentified restriction and the difference in Hansen test of exogeneity indicate that there is no correlation between instruments and error term. The Arellano-Bond test for AR(2) in first differences also proves that there is no serial correlation of order 1 in levels. Therefore, the estimation process in every sector has used valid instruments.

Table 1.3 Estimation Results of Sectoral Investment Equation

Variables	Food Crop	Non-Food Crop	Livestock	Forestry	Fishery
$INV_{i,t-1}$	1.03 ***	1.01 ***	0.99 ***	0.92 ***	1.02 ***
$GDPG_{i,t}$	1.36 *	1.35 **	2.10 **	1.32 ***	1.32 ***
r_t	-0.24 **	-0.39 **	-0.32 ***	-0.51 **	-0.30 **
Arellano-Bond test for AR(1) in 1st differences	-2.14 **	-2.22 **	-2.22 **	-2.34 **	-2.45 **
Arellano-Bond test for AR(2) in 1st differences	-1.30	-1.25	-1.19	-1.20	-1.38
Hansen test of overid restrictions	19.61	27.09	20.98	30.01	23.39
Difference-in-Hansen test of Exogeneity	2.98	1.33	1.31	4.68	1.33

Variables	Oil Mining	Non-Oil Mining	Oil Manuf.	Non-Oil Manuf.	Utility
$INV_{i,t-1}$	0.97 ***	0.99 ***	0.87 ***	1.0005 ***	1.02 ***
$GDPG_{i,t}$	0.76 ***	0.97 ***	0.65 *	1.18 ***	0.87 ***
r_t	-0.31 **	-0.27 **	-0.66 **	-0.35 **	-0.38 **
Arellano-Bond test for AR(1) in 1st differences	-1.10	-2.12 **	-1.51	-2.13 **	-2.25 **
Arellano-Bond test for AR(2) in 1st differences	-1.16	-1.40	-1.00	-1.32	-1.39
Hansen test of overid restrictions	14.37	22.55	5.53	26.51	26.51
Difference-in-Hansen test of Exogeneity	1.25	0.73	4.49	4.22	0.36

Variables	Cons-truction	Trade	Hotel	Land Transp.	Water Transp.
$INV_{i,t-1}$	1.002 ***	1.003 ***	0.99 ***	0.98 ***	0.94 ***
$GDPG_{i,t}$	1.17 **	1.55 *	1.58 ***	2.42 **	0.79 ***
r_t	-0.49 **	-0.22 *	-0.28 **	-0.32 **	-0.35 **
Arellano-Bond test for AR(1) in 1st differences	-2.17 **	-2.10 **	-2.11 **	-2.15 **	-2.49 **
Arellano-Bond test for AR(2) in 1st differences	-1.29	-1.37	-1.41	-1.19	-1.63
Hansen test of overid restrictions	29.43	20.66	21.08	13.99	29.08
Difference-in-Hansen test of Exogeneity	5.03	2.55	1.69	4.03	2.14

Variables	Air Transp.	Communi-cation	Finance	Public Service	Other Service
$INV_{i,t-1}$	1.006 ***	1.01 ***	1.01 ***	1.02 ***	1.0001 ***
$GDPG_{i,t}$	0.95 ***	1.67 ***	1.22 **	1.11 **	1.51 **
r_t	-0.32 *	-0.56 **	-0.26 **	-0.23 **	-0.38 **
Arellano-Bond test for AR(1) in 1st differences	-2.14 **	-2.17 **	-2.03 **	-2.15 **	-2.43 **
Arellano-Bond test for AR(2) in 1st differences	-1.37	-1.28	-1.26	-1.39	-1.34
Hansen test of overid restrictions	28.54	31.81	27.66	19.99	30.35
Difference-in-Hansen test of Exogeneity	1.04	1.22	0.09	1.27	2.54

*,**,*** indicate the significance levels of 10%, 5%, and 1% respectively.

As can be seen, the lagged of investment, the sectoral real GDP growth and the real interest rate are the important factors that determine the investment in every sector. The same as in the aggregated investment estimation, the first two factors are positively and significantly correlated with sectoral investment. The elasticity parameter of lagged investment and sectoral real GDP growth show that today investment decisions strongly depend on the last year investment and the current sectoral economic condition which is proxied by sectoral real GDP growth. On the other hand, the last factor has a significant negative impact on the investment in each sector.

From Table 1.3, we can argue that the most affected sector by the change in real interest rate is oil and gas manufacturing with the elasticity of real interest rate on the investment around -0.66. Moreover, the least affected sector is wholesale and retail trade sector with the coefficient of real interest rate on the investment about -0.22.

In order to compare the elasticity of real interest rate on investment for each sector between aggregated investment model and sectoral investment model, we need to use the weighted elasticities by multiplying the real interest rate elasticities with the contribution of sectoral investment on the total investment as described in Table 1.4 below.

In the aggregated investment model, the only way to determine the most affected sector to the change of real interest rate is by looking at the weighted elasticity of real interest rate on investment. According to this approach, Non-Oil and Gas Manufacturing, and Trade sectors are the two sectors that contribute the most to the total investment. Therefore, these two sectors are highly sensitive to the change in real interest rate. However, this approach is biased and does not tell the real story on how the actual real interest rate affects the sectoral investment. It is because the actual results from the sectoral investment model showed that the most affected sector to the change in real interest rate is Oil and Gas Manufacturing.

Table 1.4 Contributions of Sectoral Investment on Total Investment as of 2013

No	Sector	Contribution
1	Non-Oil and Gas Manufacturing	19.93%
2	Trade	17.80%
3	Finance	11.08%
4	Construction	6.93%
5	Food Crop	5.35%
6	Communications	5.29%
7	Other Services	4.95%
8	Public Services	4.86%
9	Hotel and Restaurant	3.72%
10	Oil, Gas and Geothermal Mining	3.67%
11	Non-Oil and Gas and Other Mining	3.29%
12	Non-Food Crop	2.94%
13	Land Transportation	2.48%
14	Fishery	1.74%
15	Livestock	1.48%
16	Oil and Gas Manufacturing	1.40%
17	Electricity, Gas and Drinking Water	0.98%
18	Air Transportation	0.80%
19	Water Transportation	0.67%
20	Forestry	0.63%
	Total	**100%**

Source: author compilation from CEIC data

The comparison between those two models on how the change of real interest rate affects the sectoral investment is described in Table 1.5 below. As seen in that table, the weighted real interest rate elasticities between aggregated nation investment model and sectoral investment model are very different. Some elasticities in the sectoral investment model are higher, some others are lower compared to those in the aggregated national investment model.

This implies the different implications of using aggregated data and using disaggregated data. In this exercise, it is clear that the information that we get from the aggregated investment estimation is misleading. Thus, the policymakers should not rely on the aggregated investment estimation if they want to understand how their policy affects the investment in each sector.

Table 1.5 The comparison of the coefficient of real interest rate on sectoral investment

No	Sector	The Real Interest Rate Elasticity from Sectoral Model	The Weighted Real Interest Rate Elasticity from Sectoral Investment Model	The Weighted Real Interest Rate Elasticity from Aggregated National Model
1	Oil and Gas Manufacturing	-0.658	-0.00923	-0.00491
2	Communications	-0.563	-0.02980	-0.01852
3	Forestry	-0.508	-0.00320	-0.00221
4	Construction	-0.487	-0.03375	-0.02424
5	Non Food Crop	-0.386	-0.01137	-0.01031
6	Other Services	-0.382	-0.01890	-0.01732
7	Electricity, Gas and Drinking Water	-0.380	-0.00374	-0.00344
8	Non-Oil and Gas Manufacturing	-0.352	-0.07015	-0.06976
9	Water Transportation	-0.346	-0.00232	-0.00235
10	Land Transportation	-0.325	-0.00805	-0.00868
11	Air Transportation	-0.316	-0.00252	-0.00279
12	Livestock	-0.315	-0.00468	-0.00519
13	Oil, Gas and Geothermal Mining	-0.308	-0.01129	-0.01284
14	Fishery	-0.295	-0.00514	-0.00609
15	Hotel and Restaurant	-0.281	-0.01043	-0.01301
16	Non-Oil and Gas and Other Mining	-0.274	-0.00904	-0.01153
17	Financial, Ownership and Business	-0.259	-0.02867	-0.03878
18	Food Crop	-0.242	-0.01296	-0.01872
19	Public Services	-0.231	-0.01120	-0.01701
20	Wholesale and Retail Trade	-0.217	-0.03863	-0.06230

Since the central bank's policy instrument is the nominal interest rate, not the real interest rate, in the end, the impact of the policy on the sectoral investment depends on how a central bank controls the inflation level so that the real interest rate can be managed at desired level for the investment.

1.8 Conclusion

The current studies about the determinant of investment in Indonesia are limited to the national level, regional specific or sectoral specific in a particular region. There are no comprehensive studies on sectoral investment in Indonesia. This research attempts to fill this gap by estimating an individual investment equation of 20 sectors in Indonesia and then compare them with the aggregated national investment equation as a benchmark model in term of weighted elasticities.

This chapter successfully estimates both national aggregated and sectoral investment equations using dynamic panel estimator suggested by Arellano and Bond. This estimator argues to be most suitable method given the characteristics of the panel data set and the investment equation as discussed in the previous section.

The estimation results showed that both in the national aggregated and sectoral investment equations, lagged investment, GDP growth, and real interest rate significantly affect the investment in Indonesia. As expected, lagged investment and GDP growth positively affect investment since they promote a conducive investment environment. On the other hand, the real interest rate has a negative impact on investment since the cost of capital increases along with the rise of real interest rate.

Given the current available national aggregated investment equation, the easiest method to determine the most affected sector to the change of real interest rate is by multiplying the estimated parameters with the sectoral contribution to the national economy. From this approach, we found that the most affected sector by the change in real interest rate is Non-Oil and Gas Manufacturing sector. However, if we compare those results with the results from the sectoral investment equations, the most affected sector on the change of real interest rate is Oil and Gas Manufacturing sector. This finding is very different from the previous method.

In terms of the weighted real interest rate coefficient, the results from the aggregated national investment equation and the sectoral investment equation are very different. These differences are quite large, so it is important to have sectoral investment model to understand the impact of real interest rate on investment.

In summary, running the aggregated national investment and then using the sectoral contribution to obtain the weighted sectoral elasticity of real interest rate on the sectoral investment can give the policymakers wrong answers. It is better to use the coefficient of real interest rate on sectoral investment taken from the sectoral investment estimation. This discrepancy can be problematic when it goes to the policymakers, especially central bank as a monetary authority because the policy implication is different in both magnitude of the interest rate policy and the target sector.

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CHAPTER 2
THE COORDINATION OF MONETARY AND FISCAL POLICY IN REGIONAL ECONOMY: THE CASE OF INDONESIA

2.1 Background

Indonesia which is located in the South-East Asian is the largest archipelago country in the world comprising approximately 17,508 islands. It spans over 5,000 km from west to east (equals to the United States) and about 1,900 km from north to south. This geographic condition creates many problems for Indonesia. Two of them are the uneven distribution of growth across the regions and rising inequality concern.

In common situation, the Indonesian policymakers such as a central bank as a monetary authority and government as a fiscal authority usually only take into account the national macro variables as an indicator to see the impact of their policy such as national GDP growth, national inflation, national income inequality and etc. They have limitations to analyze the impact of their policy to the regional economy since the national macroeconomic model is the only option to do the analysis. Even though they have a regional macroeconomic model, the common model to do the analysis is using an econometric method which cannot be used to analyze the impact of the policy comprehensively because the model is categorized as partial equilibrium model. In partial equilibrium model, we always assume that all variables outside the model are exogenous, including and in particular inter-regional trade. The exogeneity assumptions might be inappropriate if for instance we are interested in the interactions between as well as within the Indonesian regions.

In the simple textbook, monetary policy has the same effect on the national economy. However, this is not true in reality. A country like Indonesia with many regions that are linked might have different respond to the changing of economic circumstances. For example, the crude oil price hike in July 2008 affected energy-producing regions very differently from energy-consuming regions. Thus, my argument here that monetary policy has differential effects because of structural differences across regions.

Based on Carlino and Defina (1995), at least there are three reasons from economic theory that support that idea. First, the differences in interest-sensitive industries across regions. Every industry will act differently to the change in interest rate. The capital-intensive industries are likely to be more interest-sensitive than labor-intensive industries since they need more fund to buy or rent capital and this fund depends on the level of interest rate. Second, the different ability of the bank to alter their balance sheets across regions. The ability of banks to make the loan as an effect of the change in interest rate are varied. Small banks have limited funding option than large banks. Therefore, the regions in which the large share of bank loans are made by small banks might be more sensitive to the monetary policy than regions in which the large share of bank loan is made by large banks. Third, the mix of small and large borrower across regions. The difference between a number of small borrowers and large borrower and the source of credit could be the reason for the different response of monetary policy. Some borrowers might have difficulties in obtaining credit from banks as the only source of funds. On the other hand, large borrowers usually have many alternative sources of funds from non-banks. As a result, regions with high concentration of large borrowers should be less sensitive to the level of interest rate than those with a high concentration of small borrowers. In literature, we can interpret borrowers' size as firms' size.

Ridhwan et al. (2014) using a VAR model found that monetary policy between 1990 and 2007 had considerably different effects across the 26 Indonesian provinces. The most affected province by an unanticipated one percent point increase in monetary policy is West Java, the largest manufacturing-based province and the least affected province is Bali. In general, Java Island which is dominated by manufacturing industry is more sensitive to the change in monetary policy than Sulawesi and Eastern Indonesia which are highly dependent on the agriculture sector. Their study also supports the economic theory regarding the importance of cross-regional industrial composition (proxied by the share of manufacturing), bank size and firm size in explaining the differential responses to monetary policy.

In the context of fiscal policy, the government has more flexibility to give a stimulus to particular regions compared to the central bank through its interest rate policy. The stimulus can be given to the local government in the particular region that is affected by the shocks. However, the interest rate policy can only be implemented nationally. When negative shocks hit the economy, both the central bank and government must work together to address the problem. In Indonesia, the central bank and government at national and regional level have been collaborating through the existing coordination forums such as inflation target stipulation forum, Bank Indonesia Governor council coordination meeting with the government, national inflation controlling team, and regional inflation controlling team. In these forums, the tool that can capture the local government stimulus in the particular region and the differential effect of monetary policy on each region is needed, so that they can simulate their policies given the current shocks to come up with the right combination of policies.

For these reasons, it is very important to build the general equilibrium model for the regional economy so the policy makers can analyze the implication of proposed policy recommendation to the regional economy. The model is the extension of Computable General Equilibrium (CGE) Model by employing Inter-Regional Social Accounting Matrix (IRSAM) that has regional dimension combined with Financial Social Accounting Matrix (FSAM) which has a financial sector. The inclusion of this financial sector will give more realistic condition to the economy.

The objective of this study is to build the comprehensive tool for analyzing the impact of shocks and/or policies not only in national economy but also in the regional economy in more detail including real and financial sector in the economy, namely Financial Inter-Regional Computable General Equilibrium Model (FIRCGE) model. In addition, this paper also analyzes on how central bank as a monetary authority and government as a fiscal authority should coordinate in order to achieve specific objective given the specific shocks. Moreover, this study wants to compare the effectiveness of the combination of interest rate and different fiscal stimulus rule, which are a untargeted fiscal stimulus and targeted fiscal stimulus. The detailed explanation of this regulation will be discussed in analysis and simulation section. We will use the backdrop of the world oil price reduction in 2015 and 2016 to demonstrate how the FIRCGE model addresses these questions above.

2.2 Hypothesis

The impact of world oil price reduction should affect each region differently depending on the characteristics of each region. The region with high oil production will suffer more from this shock than the others. Moreover, FIRCGE can generate the iso-loss curve that can be helpful for central bank and government to coordinate and give a picture on the tradeoff between interest rate and fiscal stimulus. Lastly, the combination of monetary policy and targeted fiscal stimulus should be more effective than the combination with the untargeted fiscal stimulus.

2.3 Literature Review

The basic idea behind the CGE model is the interconnection between all markets in the economy that was proposed by Leon Walras (1834-1910). In the classic general equilibrium theory of Arrow and Hahn (1971), and Debreu (1959), the formal statement of the Walrasian economy was provided. The early work of CGE model was developed by Johansen (1960) who is interested in uneven growth between production sector in Norway.

In Indonesia, the first generation of CGE model was built by BPS, ISS and CWFS (1986), Behrman, Lewis, and Lotfi (1989), Ezaki (1989), and Thorbecke (1991). Their works were initiated by the close collaboration with the Indonesian National Planning and Development Agency (Bappenas), the Ministry of Finance, and the Statistics Indonesia (BPS or Badan Pusat Statistik). All of these works were mainly to analyses the government program in response to the decline in the oil price in the early 1980s. In their paper, Behrman, Lewis, and Lotfi (1989), and Ezaki (1989) were utilized

Indonesian Input-Output tables as the primary dataset. Thus, the social behaviors are not completely modeled. A complete model using Social Accounting Matrix (SAM) were developed by BPS, ISS, and CWFS (1986), and Thorbecke (1991).

In the 2000s, the development of CGE models was separated into two categories. First, CGE models that are built using GEMPACK software such as INDORANI CGE model by Abimanyu (2000) joins with the Centre of Policy Studies (CPS), WAYANG model by Warr (2005), and the Indonesia-E3 by Yusuf and Resosudarmo (2008). Wayang has more disaggregated households over INDORANI, since it uses Indonesia SAM. The Indonesia-E3 has more households' classification compared to INDORANI and WAYANG with 100 urban and rural households.

Second, CGE models that used GAMS software. In this category, Azis (2000) developed a new dynamic financial CGE model for Indonesia and tried to analyze the impact of Asian financial crisis in 1997-1998 on the social variables. In his paper, he found that rural households suffered less than urban households through the role of real wage. Mansury (2002) in his dissertation, also utilized financial CGE to establish the link between macroeconomic variables and income distribution and try to analyze the impact of the Asian financial crisis on different household groups. The financial framework of his model follows Bourguignon et al., while the structural model of the real sector is based on Azis (1997), which allows free flows of foreign capital. He showed that the hardest-hits of the financial crisis are urban-high, followed by rural-low and small-farmers. In her dissertation, Min (2014) developed dynamic Financial Computable General Equilibrium (FCGE) Model which explicitly incorporates bank-led capital inflow and dynamic behavior of the banking sector. She found two main findings. First, ignoring the role of exchange rate changes in banks' behavior (i.e., the dynamic component of credit channel) can yield an inaccurate picture of the impact of increased bank-led flows. Secondly, boom and bust cycle can be detrimental to the economy.

All those CGE models above are still limited to the national level analysis. The development of Inter-Regional CGE (IRCGE) for Indonesia was started by Wuryanto (Resosudarmo et al., 1999). In this model, the production side is divided into Java and Non-Java, while the household categories include Sumatra, Java, Kalimantan, Sulawesi, and the rest of Indonesia. It was developed using GAMS, based on Inter-Regional Social Accounting Matrix (IRSAM) dataset and bottom-up approach. In the GEMPACK environment, Pambudi and Parewangi (2004) in collaboration with the CPS at Monash University have developed a provincial level of CGE by utilizing Indonesia Input-Output table. The most recent IRCGE model was built by Resosudarmo, et.al. (2009c) called IRSA-INDONESIA5. This is a dynamic Inter-Regional CGE model for five regions in Indonesia, namely Sumatra, Java and Bali, Kalimantan, Sulawesi, and Eastern Indonesia. However, it does not have a channel from interest rate policy to the real sector. Thus, it cannot be used for monetary policy analysis.

2.4 Data

FIRSAM 2005 is a combination between IRSAM 2005 and FSAM 2005. It consists of five regions namely Sumatera, Java and Bali, Kalimantan, Sulawesi, and Eastern Indonesia and each region comprises of 20 sectors, 18 factors of production, 4 institutions, regional tax, regional subsidy, and local inventory. In addition, there is also another account that contains 5 capital accounts, 20 imports of commodities, 17 financial instruments, and rest of the world. (see Appendix for details). Thus, the size of FIRSAM 2005 is a 339x339 matrix.

According to Resosudarmo et al. (2009b), the IRSAM 2005 for Indonesia is mainly based on the Inter-Regional Input-Output (IRIO) data provided in 2005. In order to fill the data for other accounts in the matrix, it also included (i) National Socio-Economic Survey (SUSENAS), (ii) National and Regional Balance of Payments, (iii) Current Account, (iv) Population Census, (v) National Labor Force Survey (SAKERNAS), (vi) Special Survey on Household Investment and Savings (SKTIR), (vii) Propinsi dalam angka, (viii) Indonesia Statistics, and (ix) Statistik Kesejahteraan Rakyat. Most of these data are taken from Statistics Indonesia / Badan Pusat Statistik (BPS).

In Indonesia, IRIO considered as complicated and costly since Indonesia is a big country with many islands and heterogeneous population, so in order to renew IRIO takes at least 10 (ten) years using survey method by Statistics Indonesia. Other supporting data such as Population Census is also available every ten years, SKTIR is available every two years, SUSENAS and SAKERNAS are available every year, and others are mostly available every year. Even though it has been more than a decade now, currently the latest available data is only IRSAM 2005. Therefore, in this study, IRSAM 2005 is deemed to be sufficiently valid.

Moreover, FSAM 2005 is national-level dataset based on integration between Social Accounting Matrix (SAM), which represents real sector transaction, and Flow of

Fund (FoF), which represents financial sector transaction. In order to connect two data systems in such a way that SAM and FoF are consistent and integrated, the capital account that consists of investment and saving is needed. The dataset to construct SAM comes from the 2005 Indonesian Input-Output (I-O) table with additional supplementary data sources related to the account information of institutions. Meanwhile, FoF is a data system that reflects financial transaction between institutions. It has a set of sources and uses of funds which show the buying and selling of financial instruments, such as money, time deposits, bonds, etc.

Table 2.1 The Structure of FIRSAM 2005

Classification			**Java and Bali**				…	**Eastern Indonesia**									
FIRSAM			Factors	Institutions	Sectors	Regional Tax and Subsidy	…	Factors	Institutions	Sectors	Regional Tax and Subsidy	Capital Account	Financial Instruments	National Government	National Tax and Subsidy	Commodity Import	Rest of the World
			1	2	3	4	…	5	6	7	8	9	10	11	12	13	14
Java and Bali	Factors	1			A1,3		…										A1,14
	Institutions	2	A2,1	A2,2		A2,4	…	A2,5	A2,6					A2,11			A2,14
	Sectors	3		A3,2	A3,3		…		A3,6	A3,7		A3,9		A3,11			A3,14
	Regional Tax and Subsidy	4			A4,3		…										
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Eastern Indonesia	Factors	5					…			A5,7							A5,14
	Institutions	6	A6,1	A6,2			…	A6,5	A6,6		A6,8			A6,11			A6,14
	Sectors	7		A7,2	A7,3		…		A7,6	A7,7		A7,9		A7,11			A7,14
	Regional Tax and Subsidy	8					…			A8,7							
Capital Account		9		A9,2			…		A9,6				A9,10	A9,11			A9,14
Financial Instrument		10					…					A10,9					A10,14
National Government		11	A11,1	A11,2			…	A11,5	A11,6					A11,11	A11,12	A11,13	A11,14
National Tax and Subsidy		12			A12,3		…			A12,7							
Commodity Import		13		A13,2	A13,3		…		A13,6	A13,7		A13,9		A13,11			
Rest of the World		14	A14,1	A14,2			…	A14,5	A14,6				A14,10			A14,13	

Descriptions:

A1,3   : Factor demand in Java and Bali

A1,14 : Income Inflow from RoW to Java and Bali

A2,1 and A2,5 : Income distribution in Java and Bali

A2,2 : Intra-institutional transfer from Java and Bali to Java and Bali

A2,4 : Local government income from tax in Java and Bali

A2,6 : Intra-institutional transfer for Eastern Indonesia to Java and Bali

A2,11 : Transfer from National Government to institutions in Java and Bali

A2,14 : Inflow transfer from RoW to institutions in Java and Bali

A3,2 : Final Demand of goods from Java and Bali to institutions in Java and Bali

A3,3 : Intermediate input from sectors in Java and Bali to sectors in Java and Bali

A3,6 : Final Demand of goods from Java and Bali to institutions in Eastern Indonesia

A3,7 : Intermediate input from sectors in Java and Bali to sectors in Eastern Indonesia

A3,9 : Sectoral Investment in Java and Bali

A3,11 : Final Demand of goods from Java and Bali to national government

A3,14 : Export from Java and Bali

A4,3 : Local indirect tax in Java and Bali

A5,7 : Factor demand in Eastern Indonesia

A5,14 : Income inflow from RoW to Eastern Indonesia

A6,1 and A6,5 : Income distribution in Eastern Indonesia

A6,2 : Intra-institutional transfer from Java and Bali to Eastern Indonesia

A6,6 : Intra-institutional transfer from Eastern Indonesia to Eastern Indonesia

A6,8 : Local government income from tax in Eastern Indonesia

A6,11 : Transfer from National Government to institutions in Eastern Indonesia

A6,14 : Inflow transfer from RoW to institutions in Eastern Indonesia

A7,2 : Final Demand of goods from Eastern Indonesia to Institutions in Java and Bali

A7,3 : Intermediate input from sectors in Eastern Indonesia to sectors in Java and Bali

A7,6 : Final Demand of goods from Eastern Indonesia to Institutions in Eastern Indonesia

A7,7 : Intermediate input from sectors in Eastern Indonesia to sectors in Eastern Indonesia

A7,9 : Sectoral Investment in Eastern Indonesia

A7,11 : Final Demand of goods from Eastern Indonesia to national government

A7,14 : Export from Eastern Indonesia

A8,7 : Local indirect tax in Eastern Indonesia

A9,2 : Saving from institutions in Java and Bali

A9,6 : Saving from institutions in Eastern Indonesia

A9,10 : Financial liabilities

A9,11 : National government saving

A9,14 : Foreign saving

A10,9 : Financial assets

A10,14: RoW financial liabilities

A11,1 : Factor income from Java and Bali to national government

A11,2 : Transfer from institutions in Java and Bali to national government

A11,5 : Factor income from Eastern Indonesia to national government

A11,6 : Transfer from institutions in Eastern Indonesia to national government

A11,11: Intra-national government transfer

A11,12: Government income from tax

A11,13: Import Tariff

A11,14: Transfer from RoW to national government

A12,3 : National indirect tax in Java and Bali

A12,7 : National indirect tax in Eastern Indonesia

A13,2 : Import of Final Goods in Java and Bali

A13,3 : Imported intermediate input of sectors in Java and Bali

A13,6 : Import of Final Goods in Eastern Indonesia

A13,7 : Imported intermediate input of sectors in Eastern Indonesia

A13,9 : Investment Demand of imported goods

A13,11 : Imported national government consumption

A14,1 : Income outflow from Java and Bali to RoW

A14,2 : Outflow transfer from institutions in Java and Bali to RoW

A14,5 : Income outflow from Eastern Indonesia to RoW

A14,6 : Outflow transfer from institutions in Eastern Indonesia to RoW

A14,10: RoW financial assets

A14,13: Total Import

Since there is no FoF dataset at the regional level, the national FoF dataset is used to add financial sector transaction into the IRSAM 2005. The integration of FoF into the IRSAM 2005 needs some adjustment process because both IRSAM 2005 and SAM 2005 are developed by different institutions, and they are both have different elements even though they are using the same 2005 figures. In the adjustment process, the IRSAM 2005 is modified such that the aggregated IRSAM 2005 are matched with the SAM 2005. This process is complicated since the balanced between total rows and columns must be maintained. The final structure of FIRSAM 2005 dataset is illustrated in Table 2.1 above.

2.5 Methodology

The simplest way to do the analysis using FIRSAM 2005 is by creating FIRSAM 2005 multiplier. This method is simple, but there are many limitations such that in this model the prices are constant, demand driven, excess capacity and etc. Due to this limitation, Financial Inter-Regional Computable General Equilibrium Model (FIRCGE Model) will be constructed based on FIRSAM 2005. The FIRCGE model will give us a comprehensive analytical tool in which the equilibrium outcomes of endogenous variables are determined simultaneously from a system of many interrelated markets in the economy. It incorporates the behavior of each agent and its relation to the other agents such that all agents behave optimally in the economy.

The financial sector in FIRCGE allows us to analyze the impact of monetary policy through balance sheet channel in addition to interest rates channel as in the model without financial sector, IRCGE. The interest rates channel works through investment expenditures as the interest rate changes. Higher interest rates induce higher cost of capital so lower investment expenditures. However, the balance sheet channel reflects the impact of monetary policy on institutions' balance sheet. Monetary policy causes a change in financial wealth, which indicated an increase or a decrease in the value of assets or liabilities in balance sheets.

2.5.1 Basic Structure of the Model

The structure of FIRCGE model from the viewpoint of the flow of goods and factors can be illustrated as in Figure 2.1 below. The flows are explained from bottom to top as follows:

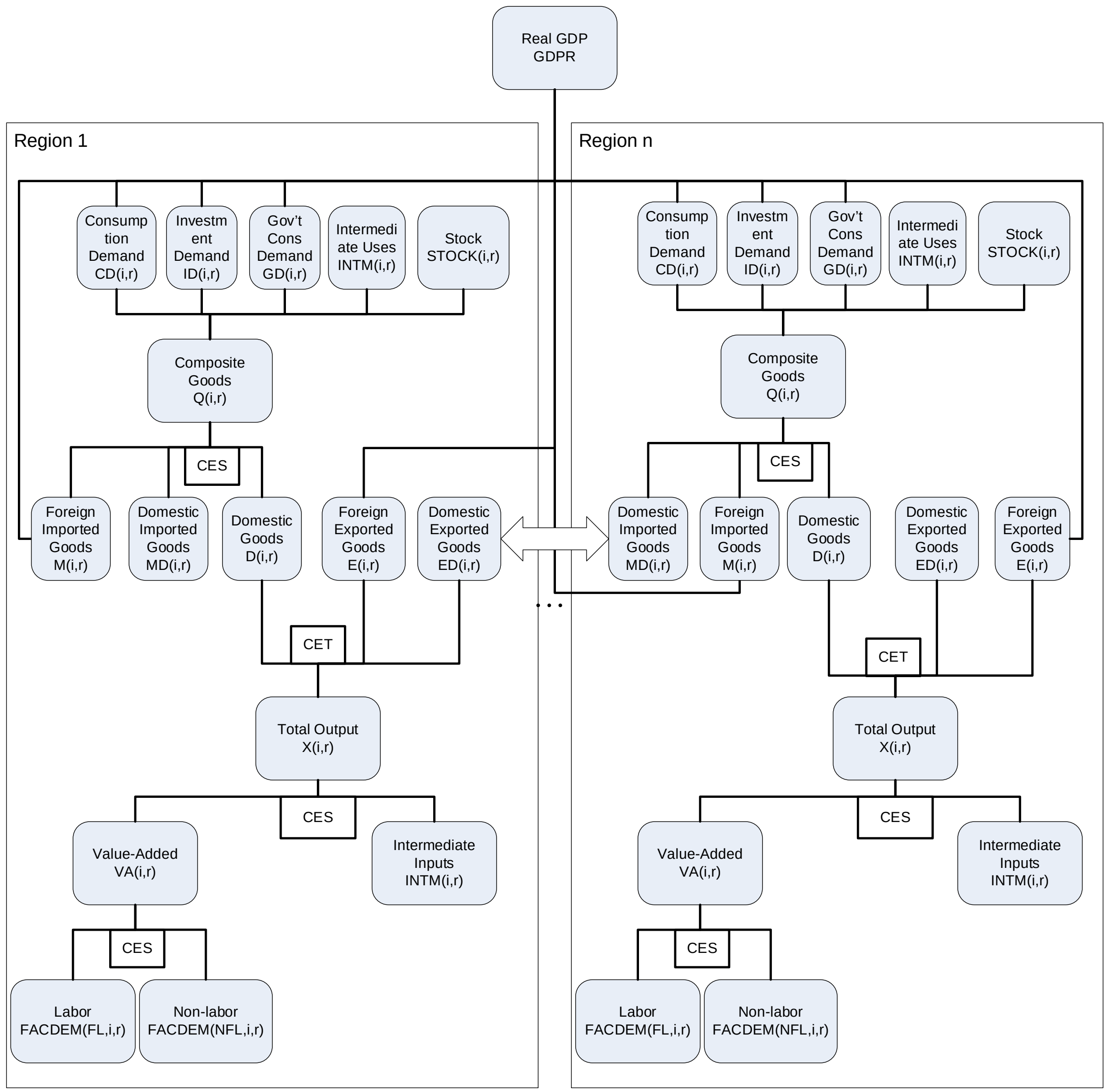

Figure 2.1 Structure of IRCGE Model from the viewpoint of goods and factors

For every region, in the first stage of the production function, Factor Demand of Labor and Non-labor are aggregated using Constant Elasticity of Substitution (CES) into Value-Added (VA). In the second stage of the production function for each region, Value-Added and Intermediate Inputs are combined using CES to produce total output (X). Then, some of the total output (X) are sold domestically, to other regions, and the rest are sold in the foreign country as an export. In this case, the Constant Elasticity of Transformation (CET) is used to disaggregate the total output into domestic goods (D), domestic exported goods (ED) and foreign exported goods (E). Next, domestic goods (D) is combined with domestic imported goods (MD), and foreign imported goods (M) using CES production function to produce composite goods (Q). Then, the composite goods Q is distributed into Consumption Demand (CD), Investment Demand (ID), Government Consumption Demand (GD), Intermediate Uses (INTM) and stock (STOCK). Real GDP (GDPR) then calculated by summing up consumption demand, investment demand, government consumption demand, and net export, which is total export minus import, in every region. Lastly, to make sure the market for intra-national trade is clear, the total domestic export must equal to the total domestic import.

Figure 2.2 below shows the structure of FIRCGE model for income, expenditure, transfer and financial blocks. As can be seen, the total income comprises of the income from government, non-government and foreign institution. The government receives income from indirect tax, import tariff, factor income and transfers. The non-government institutions namely households and companies get income from factor income and transfers. Moreover, foreign institutions’ income is from selling goods as an import, factor income and transfers.

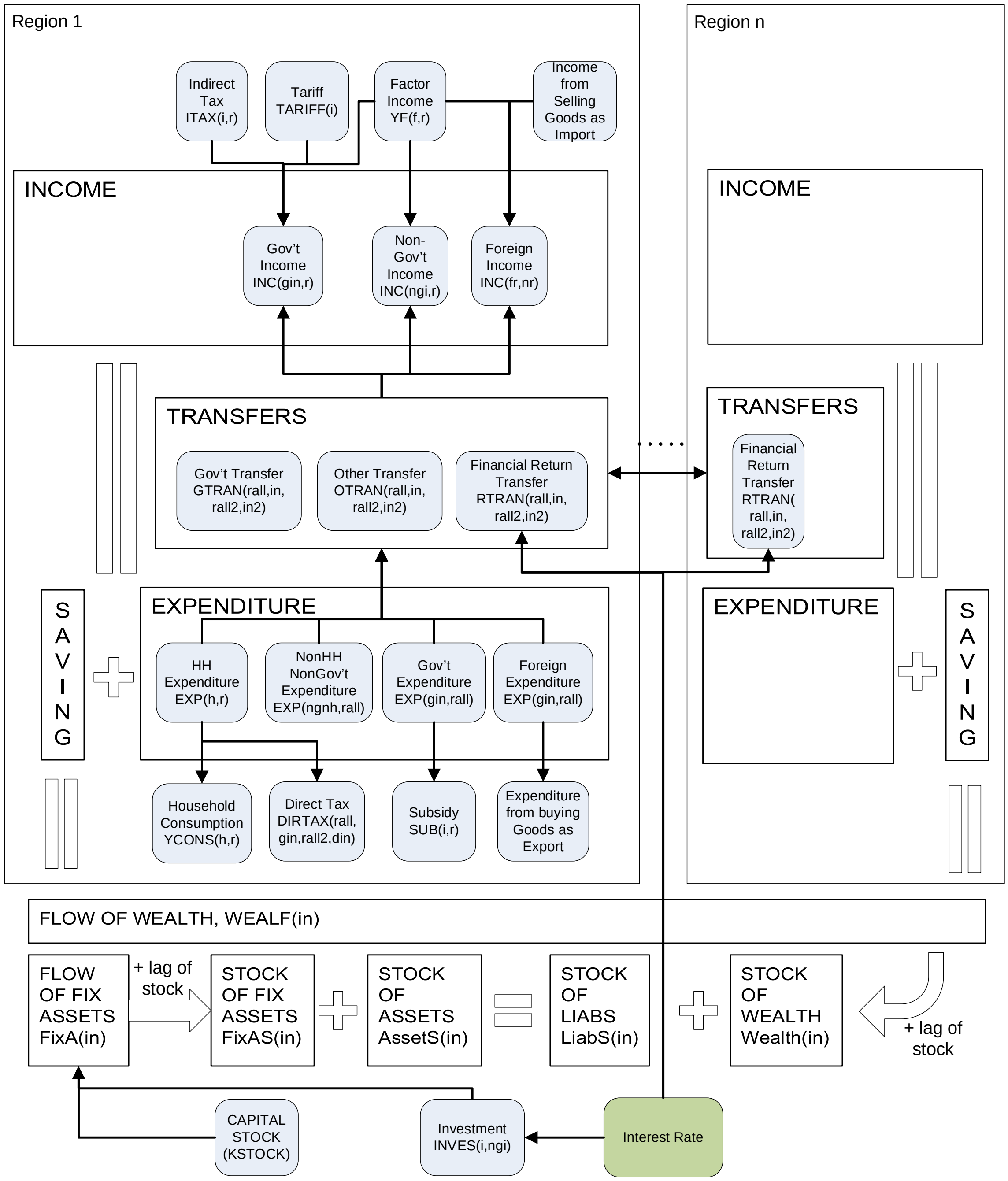

Figure 2.2 Structure of IRCGE Model for Income, Expenditure, and Transfer Block

Some of the total income then is used for expenditure, and the rest is saved as saving. In the expenditure side, households spend part of their income for consumption, direct tax, and transfers. The companies spend part of their money only for transfers.

The government spends part of its income for subsidy and transfers. Lastly, foreign institutions spend their money for transfers and to buy goods as export. Moreover, the transfers not only happen between institutions in the same region but also occur between institution in different regions. The interlink between regions is captured through this kind of transfers.

In the financial block, saving must be equal to flow of wealth. The flow of wealth becomes stock of wealth by adding the lag of saving stock. The stock of wealth and the stock of liabilities are located on the liability side in the balance sheet. The total stock of wealth and stock of liabilities must be equal to the total assets which consist of stock of assets and stock of fix assets. This stock of fix assets is the lag of fix assets stock plus the flow of fix assets from the addition of capital stock and investment. In the model without a financial block, total saving is just equal to the sum of investment in the real sector.

In this model, the investment is determined endogenously mainly by interest rate and the elasticity parameters are estimated using econometric model. With this investment equation, interest rate as a monetary policy can enter the model through the cost of capital in the standard interest rate channel. In addition to the cost of capital, as can be seen in Figure 2.2 above, the interest rate will also affect the model though the institutional transfers as a return from holding assets. The change in interest rate also have implications on the financial wealth of each institution.

2.6 Model

The model is built based on the assumption of perfect competition with Constant-Return-to-Scale (CRS) for simplicity. This section will explain only the important equations in the model. The other equation will be listed in the Appendix.

2.6.1 Output Production

In the first stage of production, the firms choose input factor $FACDEM_{f,i,r}$with price of $p_{f,r}^{F}$ to produce composite factor (or Value Added, $VA_{i,r}$). Their minimizing the cost of production subject to CES technology.

$$\min_{FACDEM_{f,i,r}} \sum_{f} p_{f,r}^{F} FACDEM_{f,i,r}$$

subject to

$$VA_{i,r} = \gamma_{i,r}^{VA} \left(\sum_{f} \theta_{f,i,r}^{F} \left(FACDEM_{f,i,r} \right)^{\eta_i^{VA}} \right)^{\frac{1}{\eta_i^{VA}}}$$

where $\gamma_{i,r}^{VA}$ is scaling coefficient in the Value Added production function

$\theta_{f,i,r}^{F}$is input share coefficient $\left(0 \leq \theta_{f,i,r}^{F} \leq 1\right)$

η_i^{VA} is parameter defined by elasticity of substitution $\left(\eta_i^{VA} = \frac{\sigma_i^{VA}-1}{\sigma_i^{VA}},\ \eta_i^{VA} \leq 1\right)$

$\sigma_i^{VA}$is the elasticity of substitution in Value Added production function

The first order condition for the problem above imply the demand function

$$FACDEM_{f,i,r} = \left(\frac{p_{i,r}^{V} \left(\gamma_{i,r}^{VA}\right)^{\frac{\sigma_i^{VA}-1}{\sigma_i^{VA}}} \theta_{f,i,r}^{F}}{p_{f,r}^{F}} \right)^{\sigma_i^{VA}} VA_{i,r} \tag{2.1}$$

Substituting demand function into production function and then simplifying it yields unit cost function

$$p_{i,r}^{V} = \frac{1}{\gamma_{r,s}^{VA}} \left[\sum_{f} \left(\theta_{f,i,r}^{F}\right)^{\sigma_i^{VA}} \left(p_{f,r}^{F}\right)^{1-\sigma_i^{VA}} \right]^{\frac{1}{1-\sigma_i^{VA}}} \qquad (2.2)$$

In the second stage of production, value added is combined with the intermediate inputs using CES production function to produce total output, $X_{i,r}$. The firms choose Value Added, $VA_{i,r}$, and Intermediate Inputs, $INTM_{i,r}$, with corresponding price $p_{i,r}^{V}$ and $p_{i,r}^{Q}$, respectively. The minimization problem is

$$\min_{VA_{i,r}, INTM_{i,r}} p_{i,r}^{VA} VA_{i,r} + p_{i,r}^{Q}\, INTM_{i,r}$$

subject to

$$X_{i,r} = \gamma_{i,r}^{X} \left(\theta_{i,r}^{VA}\, VA_{i,r}^{\eta_i^X} + \theta_{i,r}^{I}\, INTM_{i,r}^{\eta_i^X} \right)^{\frac{1}{\eta_i^X}}$$

where $\gamma_{i,r}^{X}$ is scaling coefficient in the 2nd stage of output production

$\theta_{i,r}^{VA}$ and $\theta_{i,r}^{I}$ are input share coefficient $\left(0 \le \theta_{i,r}^{VA}, \le 1,\ 0 \le \theta_{i,r}^{I}, \le 1\right)$

η_i^X is parameter defined by elasticity of substitution $\left(\eta_i^X = \frac{\sigma_i^X - 1}{\sigma_i^X},\ \eta_i^X \le 1\right)$

σ_i^X is the elasticity of substitution in 2nd stage of output production

The demand function then can be derived as follows:

$$VA_{i,r} = \left[\frac{\left(\gamma_{i,r}^{X}\right)^{\frac{\sigma_i^X - 1}{\sigma_i^X}} \theta_{i,r}^{VA}\, p_{i,r}^{X}}{p_{i,r}^{V}} \right]^{\sigma_i^X} X_{i,r} \qquad (2.3)$$

$$INTM_{i,r} = \left[\frac{\left(\gamma_{i,r}^{X}\right)^{\frac{\sigma_i^X - 1}{\sigma_i^X}} \theta_{i,r}^{I}\, p_{i,r}^{X}}{p_{i,r}^{Q}}\right]^{\sigma_i^X} X_{i,r} \tag{2.4}$$

and the unit cost function becomes

$$p_{i,r}^{X} = \frac{1}{\gamma_{i,r}^{X}}\left[\left(\theta_{i,r}^{VA}\right)^{\sigma_i^X}\left(p_{i,r}^{V}\right)^{1-\sigma_i^X} + \left(\theta_{i,r}^{I}\right)^{\sigma_i^X}\left(p_{i,r}^{Q}\right)^{1-\sigma_i^X}\right]^{\frac{1}{1-\sigma_i^X}} \tag{2.5}$$

2.6.2 Transformation between Domestic Goods and Export Goods

Total output will be sold domestically as Domestic Goods, $D_{i,r}$, with price $p_{i,r}^{D}$, will be sold abroad as an Export Goods, $E_{i,r}$, with price p_i^E, and will be sold to other regions as a domestic Export, $ED_{i,r}$, with price p_i^N, which is the price of intra-national trade. The firms are assumed to transform total output into good sold in domestic market and in international market using Constant Elasticity of Transformation (CET) as a transformation process. In this case, the firms maximize profit subject to technological constraint.

$$\max_{X_{i,r}, D_{i,r}, E_{i,r}} \pi_{i,r}^{X} = \left((1 - \tau_{i,r}^{DOM} + p_{i,r}^{SUB}) p_{i,r}^{D} D_{i,r} + p_i^E\, E_{i,r} + p_i^N\, ED_{i,r}\right) - p_{i,r}^{X} X_{i,r}$$

subject to

$$X_{i,r} = \gamma_{i,r}^{T}\left(\delta_{i,r}^{D}\, D_{i,r}^{\eta_i^T} + \delta_{i,r}^{EF}\, E_{i,r}^{\eta_i^T} + \delta_{i,r}^{ED}\, ED_{i,r}^{\eta_i^T}\right)^{\frac{1}{\eta_i^T}}$$

where $\tau_{i,r}^{DOM}$ is indirect domestic tax rates

$p_{i,r}^{SUB}$ is production subsidy share

$\gamma_{i,r}^{T}$ is scaling coefficient in transformation

$\delta_{i,r}^{D}$, $\delta_{i,r}^{EF}$, and $\delta_{i,r}^{ED}$ are input share coefficient $\begin{pmatrix} 0 \leq \delta_{i,r}^{D} \leq 1,\ 0 \leq \delta_{i,r}^{EF} \leq 1, \\ 0 \leq \delta_{i,r}^{ED} \leq 1 \end{pmatrix}$

η_i^T is parameter defined by elasticity of transformation $\left(\eta_i^T = \frac{\sigma_i^T + 1}{\sigma_i^T},\ \sigma_i^T \geq 1\right)$

σ_i^T is the elasticity of transformation between domestic goods and export goods

By solving the maximization problem, the supply function of domestic goods and export goods are

$$D_{i,r} = \left[\frac{\left(\gamma_{i,r}^T\right)^{\frac{\sigma_i^T + 1}{\sigma_i^T}} \delta_{i,r}^{D}\, p_{i,r}^{X}}{(1 - \tau_{i,r}^{DOM} + p_{i,r}^{SUB})p_{i,r}^{D}}\right]^{-\sigma_i^T} X_{i,r} \tag{2.6}$$

$$E_{i,r} = \left[\frac{\left(\gamma_{i,r}^T\right)^{\frac{\sigma_i^T + 1}{\sigma_i^T}} \delta_{i,r}^{EF}\, p_{i,r}^{X}}{p_i^E}\right]^{-\sigma_i^T} X_{i,r} \tag{2.7}$$

$$ED_{i,r} = \left[\frac{\left(\gamma_{i,r}^T\right)^{\frac{\sigma_i^T + 1}{\sigma_i^T}} \delta_{i,r}^{ED}\, p_{i,r}^{X}}{p_i^N}\right]^{-\sigma_i^T} X_{i,r} \tag{2.8}$$

After some algebra, the unit cost function is

$$\begin{aligned} p_{i,r}^{X} = \frac{1}{\gamma_{i,r}^T}\Big[&\left(\delta_{i,r}^{D}\right)^{-\sigma_i^T}\left((1 - \tau_{i,r}^{DOM} + p_{i,r}^{SUB})p_{i,r}^{D}\right)^{1+\sigma_i^T} \\ &+ \left(\delta_{i,r}^{EF}\right)^{-\sigma_i^T}(p_i^E)^{1+\sigma_i^T} + \left(\delta_{i,r}^{ED}\right)^{-\sigma_i^T}(p_i^N)^{1+\sigma_i^T}\Big]^{\frac{1}{1+\sigma_i^T}} \end{aligned} \tag{2.9}$$

2.6.3 Substitution between Domestic Goods and Import Goods

Domestic and import goods are not directly consumed by households, firms, and government. However, they are combined into goods called Armington Composite Goods. The firms are assumed to maximize their profit by choosing a suitable combination of domestic goods, imported goods from abroad, $M_{i,r}$ and imported goods from other regions, $MD_{i,r}$ subject to technological constraint that are assumed to follow CES function.

$$\max_{Q_{i,r}, D_{i,r}, M_{i,r}} \pi_{i,r}^{Q} = p_{i,r}^{Q}\, Q_{i,r} - \left(p_{i,r}^{D} D_{i,r} + (1+\tau_i^{M}) p_i^{M} M_{i,r} + p_i^{N} MD_{i,r}\right)$$

subject to

$$Q_{i,r} = \gamma_{i,r}^{A} \left(\theta_{i,r}^{D}\, D_{i,r}^{\eta_i^A} + \theta_{i,r}^{MF}\, M_{i,r}^{\eta_i^A} + \theta_{i,r}^{MD}\, MD_{i,r}^{\eta_i^A}\right)^{\frac{1}{\eta_i^A}}$$

where $\gamma_{i,r}^{A}$ is scaling coefficient in Armington production

$\theta_{i,r}^{D}$, $\theta_{i,r}^{MF}$ and $\theta_{i,r}^{MD}$ are input share coefficient $\begin{pmatrix} 0 \le \theta_{i,r}^{D} \le 1, 0 \le \theta_{i,r}^{MF} \le 1, \\ 0 \le \theta_{i,r}^{MD} \le 1 \end{pmatrix}$

η_i^A is parameter defined by elasticity of substitution $\left(\eta_i^A = \frac{\sigma_i^A - 1}{\sigma_i^A},\ \eta_i^A \le 1\right)$

σ_i^A is the elasticity of substitution between domestic goods and import goods

The optimal demand function for domestic goods, import goods from abroad, and imported goods from other regions are

$$D_{i,r} = \left[\frac{\left(\gamma_{i,r}^{A}\right)^{\frac{\sigma_i^A - 1}{\sigma_i^A}} \theta_{i,r}^{D}\, p_{i,r}^{Q}}{p_{i,r}^{D}}\right]^{\sigma_i^A} Q_{i,r} \qquad (2.10)$$

$$M_{i,r} = \left[\frac{\left(\gamma_{i,r}^{A}\right)^{\frac{\sigma_i^A - 1}{\sigma_i^A}} \theta_{i,r}^{MF} \, p_{i,r}^{Q}}{(1 + \tau_i^M) p_i^M} \right]^{\sigma_i^A} Q_{i,r} \tag{2.11}$$

$$MD_{i,r} = \left[\frac{\left(\gamma_{i,r}^{A}\right)^{\frac{\sigma_i^A - 1}{\sigma_i^A}} \theta_{i,r}^{MD} \, p_{i,r}^{Q}}{p_i^N} \right]^{\sigma_i^A} Q_{i,r} \tag{2.12}$$

The unit cost function is

$$\begin{aligned} p_{i,r}^{Q} = \frac{1}{\gamma_{i,r}^{A}} \Big[\left(\theta_{i,r}^{D}\right)^{\sigma_i^A} \left(p_{i,r}^{D}\right)^{1-\sigma_i^A} + \left(\theta_{i,r}^{MF}\right)^{\sigma_i^A} ((1 + \tau_i^M) p_i^M)^{1-\sigma_i^A} \\ + \left(\theta_{i,r}^{MD}\right)^{\sigma_i^A} (p_i^N)^{1-\sigma_i^A} \Big]^{\frac{1}{1-\sigma_i^A}} \end{aligned} \tag{2.13}$$

2.6.4 Income Block

As shown in Figure 2.2, income block consists of government income, non-government income and foreign income. They are formulated as follows:

<u>Government Income</u>

$$\begin{aligned} INC_{GIN,rall} = \sum_{r2} \sum_{f} factoin_{rall,GIN,r2,f} YF_{f,r2} \\ + \sum_{rall2} \sum_{in2} ITRAN_{rall,GIN,rall2,in2} \\ + gishr_{GIN,rall} \sum_{i} \left(\sum_{r2} \left(ITAX_{i,r2} \right) + TARIFF_i \right) \\ - gsshr_{GIN,rall} \sum_{r} \sum_{i} SUB_{i,r} \end{aligned} \tag{2.14}$$

Non-Government Income

$$INC_{NGI,r} = \sum_{r2}\sum_{f} factoin_{r,NGI,r2,f} YF_{f,r2} + \sum_{rall2}\sum_{in2} ITRAN_{r,NGI,rall2,in2} \tag{2.15}$$

Foreign Income

$$INC_{FR,nr} = \sum_{r2}\sum_{f} factoin_{nr,FR,r2,f} YF_{f,r2} + mrshr_{FR,nr}\sum_{r}\sum_{i} P_i^M M_{i,r} + \sum_{rall}\sum_{in2} ITRAN_{nr,FR,rall,in2} \tag{2.16}$$

where $factoin_{rall,in,r2,f}, gishr_{GIN,rall}, gsshr_{GIN,rall}, mrshr_{FR,nr}$ are share coefficients, $YF_{f,r2}$ is factor income, $ITAX_{i,r}$ is indirect tax, $TARIFF_i$ is import tariff, $SUB_{i,r}$ is government subsidy, and $ITRAN_{rall,in,rall2,in2}$ is transfer between institution. In the income side, the sign of government subsidy is negative because government subsidy is considered as spending.

2.6.5 Expenditure Block

In this block, there are four types of expenditure namely, households expenditure, government expenditure, non-households non-government / company expenditure, and foreign expenditure. Households consume some of their disposable income indicated by one minus marginal propensity to save (MPS). The expenditure equations are described as follows:

Households expenditure

$$EXP_{H,r} = YCONS_{H,r} + \sum_{rall}\sum_{GIN} DIRTAX_{rall.GIN,r,H} + \sum_{r2}\sum_{NGI} ITRAN_{r2.NGI,r,H} + \sum_{nr}\sum_{FR} ITRAN_{nr.FR,r,H} \tag{2.17}$$

Government expenditure

$$EXP_{GIN,rall} = ggshr_{GIN,rall} \sum_{r}\sum_{i} P_{i,r}^{Q} GD_{i,r} + \sum_{rall2}\sum_{in2} ITRAN_{rall2,in2,rall,GIN} \tag{2.18}$$

Non-household and non-government expenditure

$$EXP_{NGNH,r} = \sum_{rall}\sum_{in} ITRAN_{rall.in,r,NGNH} \tag{2.19}$$

Foreign expenditure

$$EXP_{FR,nr} = ershr_{FR,nr} \sum_{r}\sum_{i} p_{i}^{E} E_{i,r} + \sum_{r}\sum_{f} YFROW_{r.f,nr,fr} + \sum_{rall}\sum_{in} ITRAN_{rall.in,nr,fr} \tag{2.20}$$

where $ggshr_{GIN,rall}$ and $ershr_{FR,nr}$ are share coefficient, $YCONS_{H,r}$ is consumption by households, $GD_{i,r}$ is government goods demand, and $DIRTAX_{rall.GIN,r,DIN}$ is direct tax. $YFROW_{r.f,nr,fr}$ is domestic's earning abroad.

2.6.6 Market Clearing

After describing the behavior of each agent above, it is necessary to make sure that supply meets demand in all market. In the market for intra-national trade, the total domestic export must be equal to the total domestic import, or

$$\sum_{r} ED_{i,r} = \sum_{r} MD_{i,r} \tag{2.21}$$

This relationship decides the price of the intra-national trade, which is the same across regions. In the final goods market, the supply of Armington goods, $Q_{i,r}$, equals to summation of final goods demand by institutions, intermediate supply for production, $INTQ_{i,r}$, and also stock $RSTKD_{i,r}$.

$$Q_{i,r} = CD_{i,r} + ID_{i,r} + GD_{i,r} + INTQ_{i,r} + RSTKD_{i,r} \tag{2.22}$$

2.6.7 Investment Equation

In the common CGE model, investment is treated as an exogenous variable which is defined by a value from outside the model. In order to make investment endogenous, we need to specify the investment equation. In this research, the investment equation is defined as a function of value added and interest rate. The elasticity parameters of value added and interest rate to the investment for this function is taken from the estimation of the investment using econometric model.[2]

2.6.8 Financial Block

In general, financial block described the relationship between assets, liabilities, and interest rate. Assets and liabilities can be a stock variable in current and previous

[2] The elasticity parameter of value added is taken from Min (2014), while the elasticity of interest rate is obtained from previous chapter of this dissertation with adjustment.

period namely $AssetS_{as,in}$, $LiabS_{in,as}$, $AssetSLag_{as,in}$, and $LiabSLag_{in,as}$, or can be a flow variable such as $Asset_{as,in}$, $Liab_{in,as}$. The lag of stock variables is not available in FIRSAM 2005 dataset, so additional data from other sources is needed. This stock dataset is useful to calculate the return on assets as additional income from investing in financial sector. This income then goes to the income block through transfer from institution who sell the assets to the institution who own or buy the assets.

For assets or liabilities other than currency and demand deposit, the decision of assets of liabilities allocation depends on the change of corresponding interest rate from the beginning period and the elasticity parameter which shows how much the assets or liabilities will change as the interest rate moving. In general, the assets or liabilities allocation can be formulated as follows

$$\begin{aligned} AssetS_{as,in} &= \theta1_{as,in}\left(\frac{rn_{as}}{rn0_{as}}\right)^{\sigma1_{as,in}} \ or \\ LiabS_{in,as} &= \theta2_{in,as}\left(\frac{rn_{as}}{rn0_{as}}\right)^{\sigma2_{in,as}} \end{aligned} \tag{2.23}$$

where rn_{as} and $rn0_{as}$ are interest rate in current and beginning period, respectively. $\theta1_{as,in}$ and $\theta2_{in,as}$ are elasticity parameters.

As for currency and demand deposit, both assets depend not only on the change in interest rate and elasticity parameter but also from the income of institution and the scale parameter. This relationship is described by money demand equation as follows

$$MD_{in} = \alpha1_{in}\left(\sum_{rall} INC_{in,rall}\right)^{\alpha2_{in}} (rnv1_{in})^{-\alpha3_{in}} \tag{2.24}$$

where $rnv1_{in}$ is average interest rate of currency and demand deposits, $\alpha 1_{in}$ is the scale parameter, and $\alpha 3_{in}$ and $\alpha 3_{in}$are elasticity parameters. All of those elasticity parameters in financial block are taken from previous study by Min (2014).

The final step in this financial block is maintaining the balance of the financial account for each institution.

$$\sum_{as} AssetS_{as,in} + FixAS_{in} = \sum_{as} LiabS_{in,as} + Wealth_{in} \tag{2.25}$$

where $FixAS_{in}$ is the stock value of fix assets, and $Wealth_{in}$is the stock value of wealth or similar to equity. The stock of fix assets reflects the investment accumulation in real sector, while the stock of wealth is equal to the stock of saving in real sector. From the view point of balance sheet for individual institution, assets in the left side, should be equal to liabilities plus equity in the right side. In this equation, assets from investing in financial assets, $\sum_{as} AssetS_{as,in}$, and from investing in real sector, $FixAS_{in}$, in the left hand side should be equal to liabilities, $\sum_{as} LiabS_{in,as}$, plus equity, $Wealth_{in}$.

2.6.9 Closures

Closures are the assumptions of the variables that are exogenous to the model. These assumptions are necessary to make sure the number of equations is the same as the number of endogenous variables, so the model can be solved. The variables that are set to be exogenous in the model are:

a. World import and export prices
b. Exchange rate
c. Price of factors for non-labor

d. Factor income of foreign institutions
e. Marginal Propensity to Save (MPS)
f. Government expenditure
g. Government and other transfers
h. Government and foreign institutions' investment
i. Capital stock and inventory
j. Labor Supply
k. All the lag of stock in financial block
l. All interest rate except interest rate associated with government bond and foreign exchange reserves
m. Stock of foreign exchange reserves held by institution both in asset and liability side
n. Stock of government bond and other financial instruments held by institution as an asset

2.7 Analysis and Simulation

This section will focus on conducting an experiment using FIRCGE model on the current economic shock that is happening in Indonesia and then figuring out the right policies that can support monetary and fiscal authorities to achieve their objectives. In this experiment, the tradeoff between monetary policy and fiscal policy will be discussed in details.

The current economic shock in this experiment is a sharp decline in world crude oil price that happened lately as shown in Figure 2.3 below. The major reason for this significant drop in oil price is the refusal to curb over-production of oil from Saudi Arabia. Some economists said Saudis are trying to price their competitors out of the oil market. As the world's largest oil exporting country, Saudi Arabia can fulfill oil demand at the prevailing market price, while other countries mostly cannot. The other oil exporting countries, such as Russia who is the primary supplier of oil for European countries cannot afford to cut oil production to drive prices up because Russian Federation does not want to lose partners to a competitor especially Saudi Arabia.

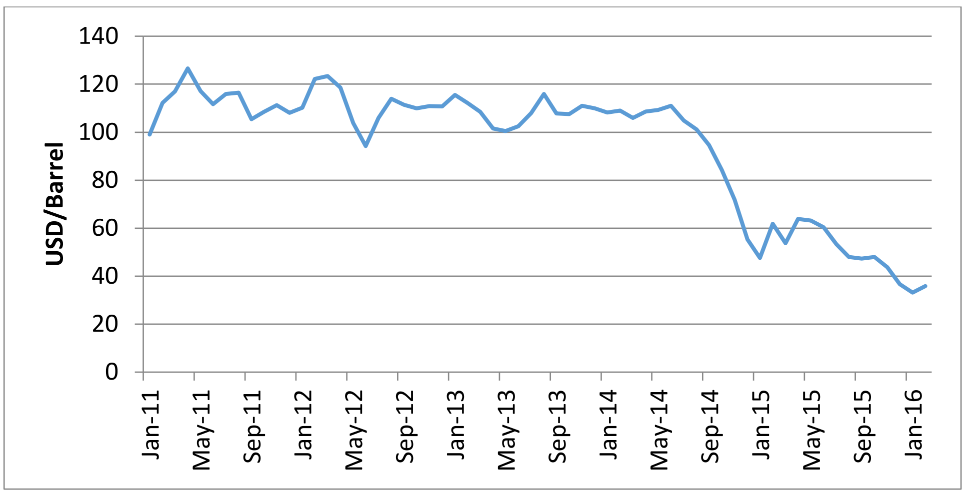

Figure 2.3 Crude Oil Price (Brent) 2011-2016

Such behaviors have led to a global low oil price equilibrium that may have a negative impact on the country such as Indonesia even though Indonesia is now a net importer of oil. This is because the exports value of oil is not small and the high backward and forward linkage of oil sector with other sectors. On the one hand, the backward linkage is defined as the growth of a sector, in this case, oil sector, that leads to the growth of the sector that supplies input to the oil sector. On the other hand, the forward linkage can be described as the growth of oil sector that leads to growth the other sector that uses oil sector output as input. The results of the 10% decline of world oil price to macro variables both at national level and regional level are shown in Table 2.2 and Table 2.3 below.

The impact of 10% world oil price decline is reflected in "Scenario 1" column. As can be seen, GDP decreases by 0.716% from the baseline because of the high backward and forward linkage of oil sector with other sectors as discussed earlier. However, as oil price declines, the inflation falls as well. The oil price reduction leads to lower cost of production and transportation. As a result, the price of final goods is decreasing. Consumption, investment, and export decline as GDP contracted, but import increases as oil import price drops. The unemployment rate is higher than baseline because of lower GDP. In contrast, the gap between rural and urban incomes narrows in comparison to the baseline which means the income of households that live in an urban area has been affected negatively by this kind of shock. Income distribution between rural and urban is defined by dividing the income of rural households by the income urban households. Thus, a higher number indicates a narrowing gap between rural and urban incomes and hence a more equal urban-rural distribution of income at the national level.

Table 2.2 Summary Results of the 10% World Oil Price Decline at National Level

Variable	**Scenario 1**
GDP	-0.716%
Inflation	-0.098%
Consumption	-0.449%
Investment	-0.477%
Export	-0.772%
Import	0.447%
UnemplRate	3.327%
IncDistRuralUrban	0.020%

At the regional level, the most negatively affected region from oil price decline in term of GDP is Sumatra, followed by Jawa and Bali because both regions are the two largest oil and gas exporter region in Indonesia, which is why the exports from those regions significantly drop. Consequently, the output is also decreasing significantly in those regions. Lower output leads to lower investment and lower consumption. As for inflation, all regions are experiencing deflation. In terms of income inequality, all regions except only Jawa and Bali, and Eastern Indonesia have benefited from this oil price shock.

Table 2.3 Results of the 10% World Oil Price Decline at Regional Level

Variable	Region	Scenario 1
GDP	Sumatra	-2.186%
GDP	Java and Bali	-0.444%
GDP	Kalimantan	0.141%
GDP	Sulawesi	-0.123%
GDP	Eastern Indonesia	0.087%
Inflation	Sumatra	-0.122%
Inflation	Java and Bali	-0.083%
Inflation	Kalimantan	-0.172%
Inflation	Sulawesi	-0.131%
Inflation	Eastern Indonesia	-0.125%
Consumption	Sumatra	-1.495%
Consumption	Java and Bali	-0.241%
Consumption	Kalimantan	0.244%
Consumption	Sulawesi	-0.166%
Consumption	Eastern Indonesia	-0.083%
Investment	Sumatra	-0.485%
Investment	Java and Bali	-0.477%
Investment	Kalimantan	-0.382%
Investment	Sulawesi	-0.511%
Investment	Eastern Indonesia	-0.558%
Export	Sumatra	-4.047%
Export	Java and Bali	-0.048%
Export	Kalimantan	0.449%
Export	Sulawesi	-0.052%
Export	Eastern Indonesia	0.335%

Variable	Region	Scenario 1
Import	Sumatra	-0.239%
Import	Java and Bali	0.578%
Import	Kalimantan	0.914%
Import	Sulawesi	-0.683%
Import	Eastern Indonesia	-0.482%
IncDistRuralUrban	Sumatra	0.118%
IncDistRuralUrban	Java and Bali	-0.007%
IncDistRuralUrban	Kalimantan	0.079%
IncDistRuralUrban	Sulawesi	0.025%
IncDistRuralUrban	Eastern Indonesia	-0.067%
Output	Sumatra	-1.830%
Output	Java and Bali	-0.254%
Output	Kalimantan	0.204%
Output	Sulawesi	-0.335%
Output	Eastern Indonesia	-0.191%

Policymakers' objectives must be defined in order to find the right policies to reduce the negative impact of the world oil price reduction shock. The central bank as monetary authority has an objective to maintain price stability. In this paper, the central bank uses interest rate as a primary policy instrument to drive up and down the economy. On the other hand, the government as a fiscal authority has an objective to manage the GDP level back to normal as negative shock hit the economy. For policy simulation, the fiscal policy will be focused on using government spending. Therefore, the objectives of both central bank and government can be defined as achieving stability of price and achieving high economic growth. Since the model has the ability to simulate the impact of the policy at the regional level, the objective must also be obtained at the regional level. This objectives are formulated into loss function type A as follows:

$$LossFunction\ A = 100 \sum_{r} \left[0.5 \left(\frac{p1_r - p0_r}{p0_r} \right)^2 + \boldsymbol{i}\ 0.5 \left(\frac{GDP1_r - GDP0_r}{GDP0_r} \right)^2 \right] \quad (2.26)$$

where $p1_r$ and $p0_r$ are the price level after policy and the price level at the baseline, respectively. $GDP1_r$ is the GDP level after policy was imposed and $GDP0_r$ is the GDP level at the baseline. $\boldsymbol{i}$ is a variable which indicates 1 if the $GDP1_r <= GDP0_r$ and -1 if $GDP1_r > GDP0_r$. This variable is needed to make sure the loss function will increase when GDP level after the shock is less than or equal to the baseline level and vice versa.

This Loss function will maintain the price and GDP level in every region close to the baseline level before oil price shock was imposed. For neutrality between monetary and fiscal authority objective, both regional price and GDP are weighted equally. The right policies can be obtained by simulating possible policies that can make the loss function minimum.

Besides these combined objectives, the simulation also includes individual government's objective to achieve high economic growth. If there is a negative shock, the government wants to minimize the gap between the national GDP after policy and the baseline national GDP. The stability of price is not relevant with the oil price decline shock because this shock has a positive impact on inflation. Thus, there is no simulation using central bank's objective. The stability of GDP can be represented as loss function type B:

$$LossFunction\ B = \left(\frac{NationalGDP1 - NationalGDP0}{NationalGDP0}\right) \tag{2.27}$$

On the one hand, there are two types of government fiscal stimulus. First, the government distributes fiscal stimulus to each region in proportion to the baseline GDP level. This is called untargeted fiscal stimulus. Second, the government distributes fiscal stimulus to each region in proportion to the shortfall of regional GDP level caused by oil price shock, in short, it is called targeted fiscal stimulus. On the other hand, the central bank has only one national policy; it does not have a regional policy. Therefore, in every loss function, there will be two scenarios. The first scenario is a combination of untargeted fiscal stimulus with monetary policy. The second scenario is combination between targeted fiscal policy and monetary policy.

In order to see the tradeoff between fiscal stimulus and monetary policy, the iso-loss curves are drawn. The iso-loss curves are the possible combination of fiscal stimulus and monetary policy such that the certain value of loss function is achieved. For example, in the loss function type A, 90% of Total Loss represents the possible combination of fiscal stimulus and monetary policy if policymakers want to reduce the total loss function by only 10%. In loss function type B, 80% of GDP Decline represents the possible combination of policies such that the GDP shortfall is reduced by 20%. The iso-loss curve is drawn by simulating the combination between interest rate, which ranges from 0 to 100 basis points (bps) with 1 bps increment, and fiscal stimulus, which ranges from 0 to 3 trillion rupiahs with increment of 0.01 trillion rupiah, and then finding the combination of policies that achieved certain value of loss function.

2.7.1 The Combination of Untargeted Fiscal Stimulus and Interest Rate

Figure 2.4 below shows the iso-loss curve of combination between untargeted fiscal stimulus and interest rate using loss function type A. As shown below, the iso-curve of the 90% and 85% of total loss can be approximated by a linear function. The higher the policymakers want to reduce the total loss, the higher the fiscal stimulus is needed. This is shown by the outward shift of iso-loss curve from the origin as the reduction of total loss is getting higher. The slope of the iso-loss curve indicates the tradeoff between interest rate and fiscal stimulus. For example, the slope of -0.0204 means that the government could reduce the total fiscal stimulus by 0.0204 trillion rupiahs as the central bank increase its interest rate by 1 bps. From the graph, the iso-loss curve is getting steeper as the reduction of total loss is getting higher.

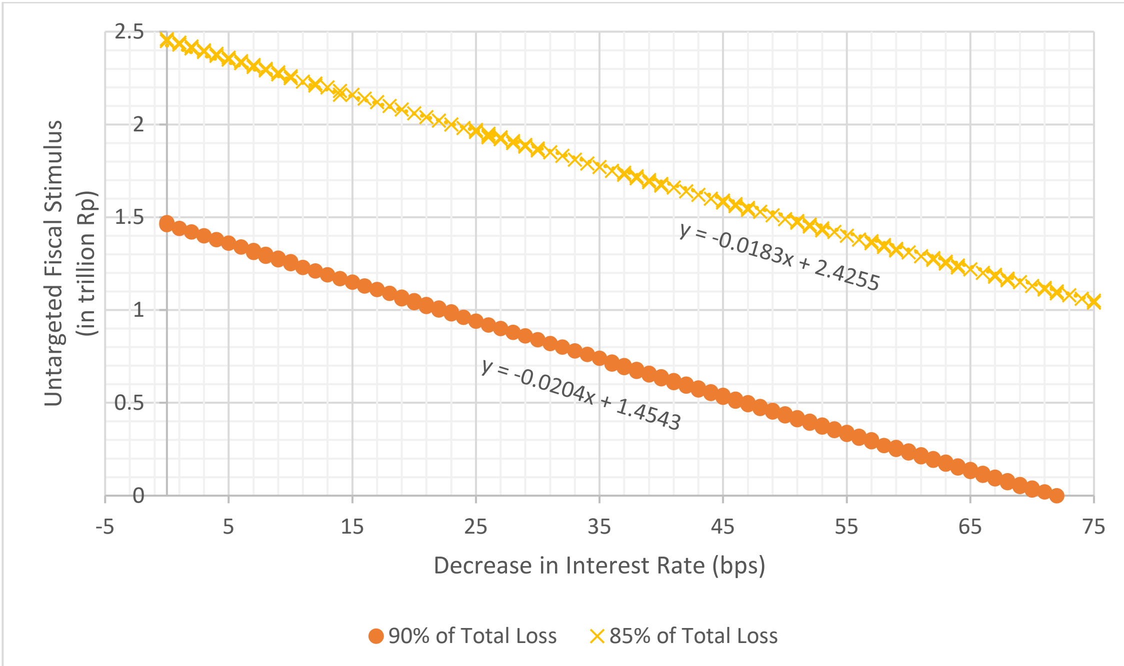

Figure 2.4 Iso Loss Curves for Untargeted Fiscal Stimulus using Loss Function type A

The iso-loss curves using loss function type B are illustrated in Figure 2.5 below. As can be seen in the graph, the slopes between each iso-curve are almost the same, about -0.027. This number may become a guidance for policymakers, especially on how the central bank should coordinate with the government if the objective function is just reducing the GDP shortfall because of oil price shock.

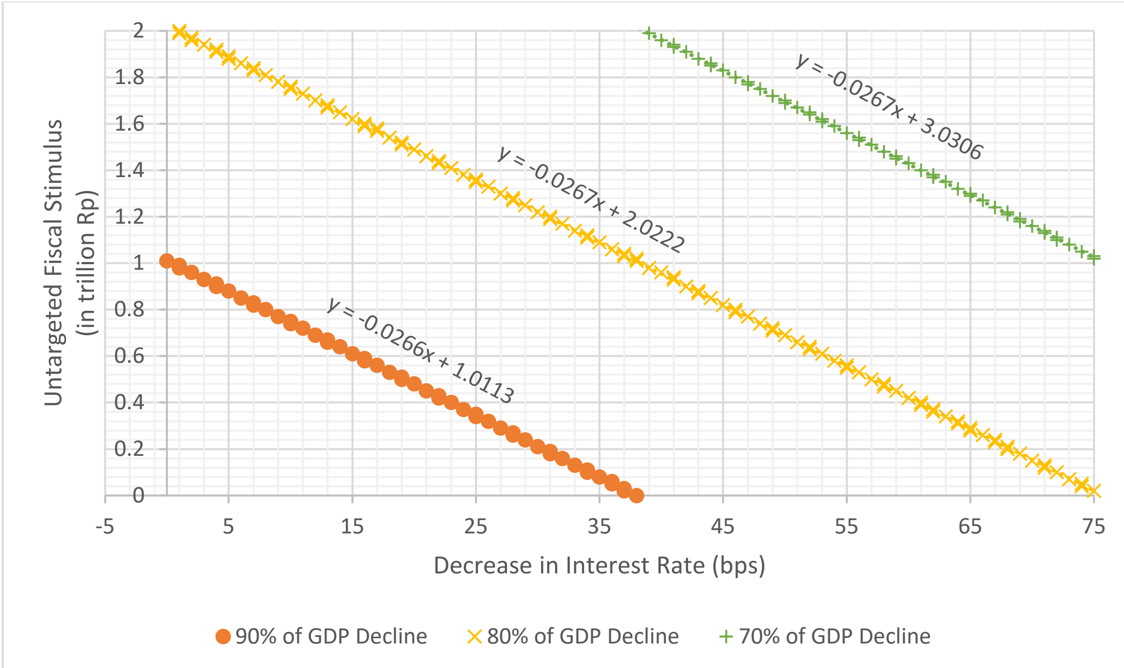

Figure 2.5 Iso Loss Curves for Untargeted Fiscal Stimulus using Loss Function type B

2.7.2 The Combination of Targeted Fiscal Stimulus and Interest Rate

In this scenario, the fiscal stimulus is injected in the proportion of the GDP shortfall because of the oil price decline shock. Figure 2.6 and Figure 2.7 show the iso-curves of combination between targeted fiscal stimulus and interest rate using loss function type A, and the iso-curves using loss function type B, respectively. As seen in Figure 2.6 and Figure 2.7, all the iso-curve can be approximated using the linear function.

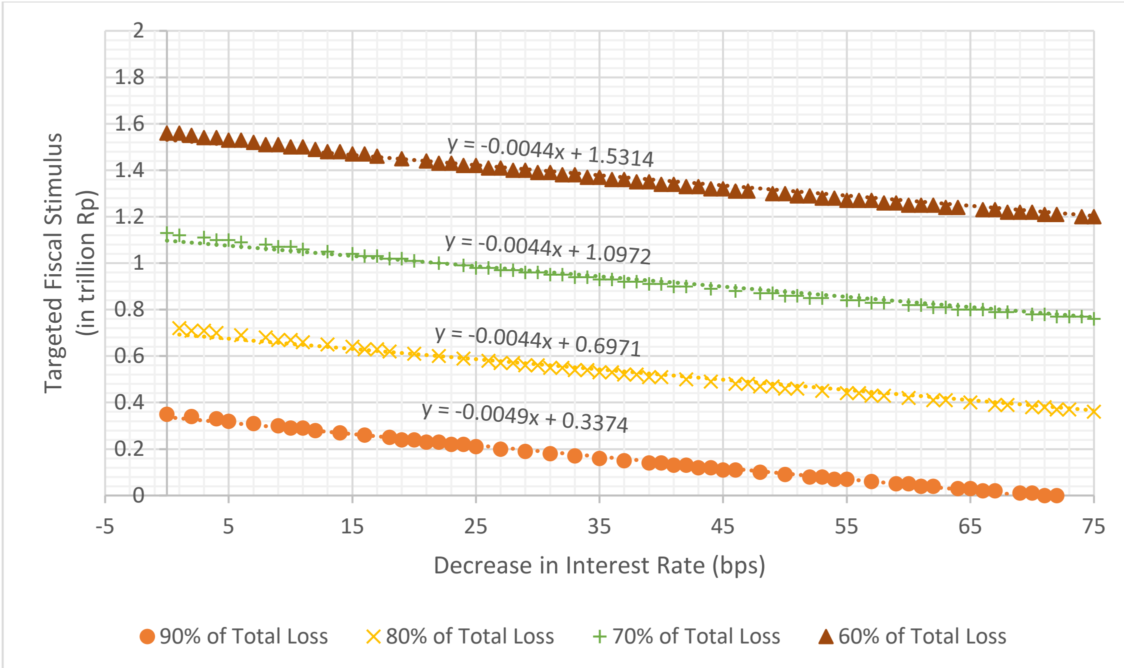

Figure 2.6 Iso Loss Curves for Targeted Fiscal Stimulus using Loss Function type A

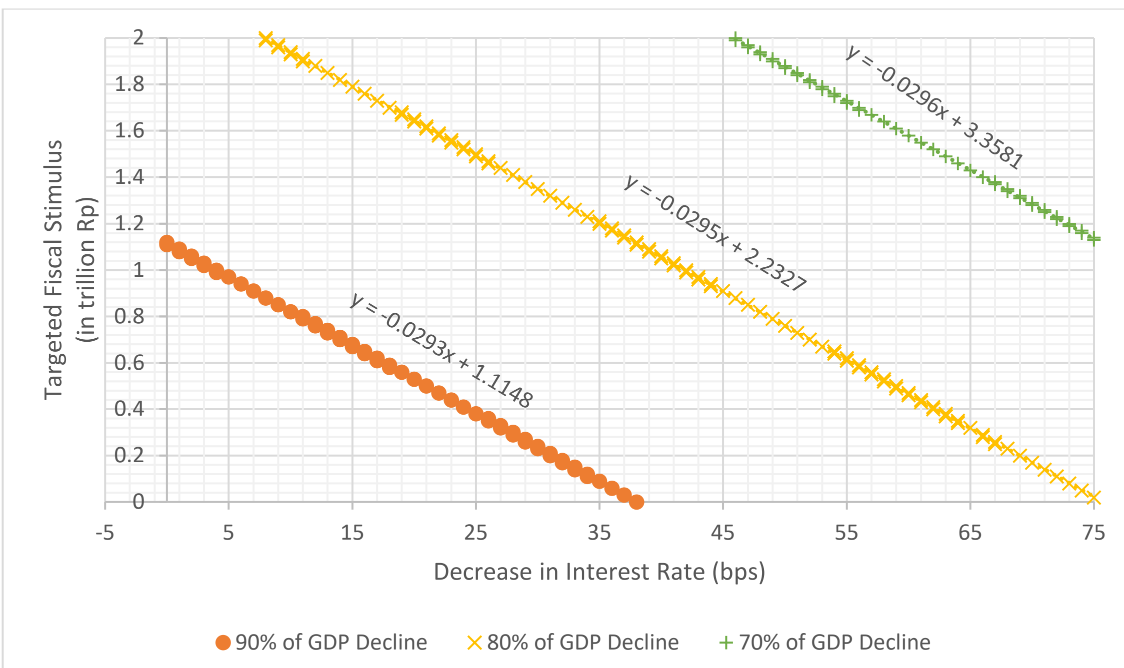

Figure 2.7 Iso Loss Curves for Targeted Fiscal Stimulus using Loss Function type B

2.7.3 Comparison between Untargeted and Targeted Fiscal Stimulus

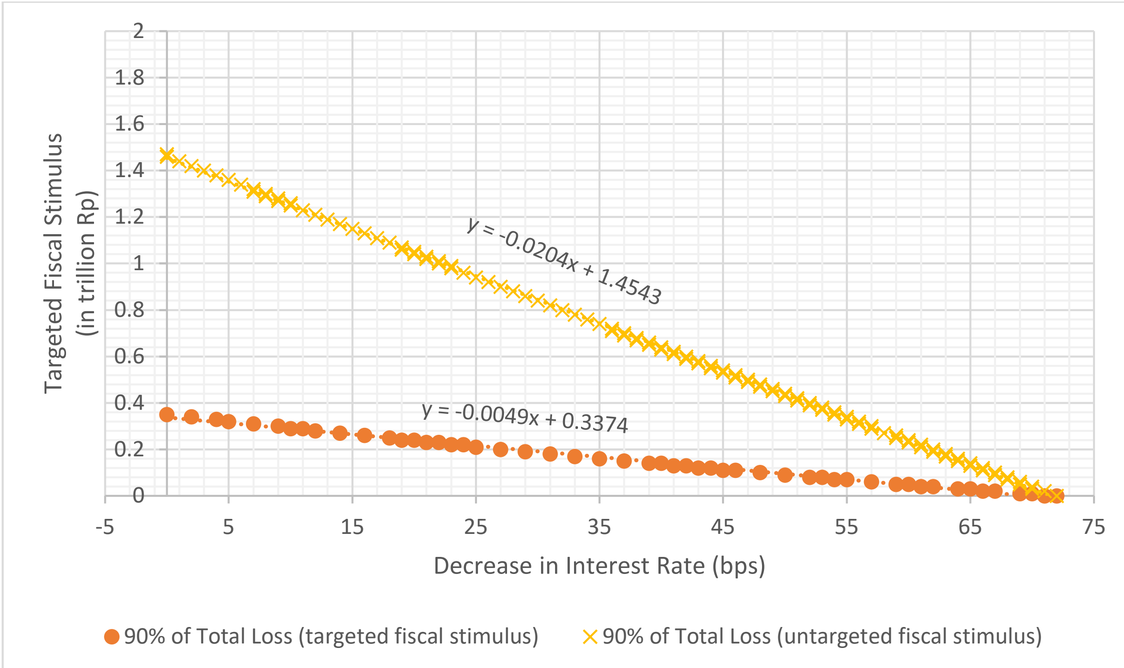

Figure 2.8 Comparison between Untargeted and Targeted Fiscal Stimulus

Figure 2.8 above shows comparison between untargeted and targeted fiscal stimulus at the same value of loss function, which is 90% of the total loss. This comparison is focused on the loss function type A since it considers both central bank's and government's objectives. At the 90% of the total loss function, we can see that targeted fiscal stimulus is more efficient than untargeted one because targeted fiscal stimulus requires less money than untargeted fiscal stimulus. This is shown by the inward tilt of targeted fiscal stimulus iso-curve to the origin.

The right policy can be chosen from those iso-loss curves based on the government budget and the central bank's constraints on how far the interest rate can be reduced. In other words, the iso-loss curves give the options for central bank and government on how they should coordinate in order to achieve the desired target.

The model will be run with the particular combination of policies to see the detailed results of the simulation. In this simulation assume that both central bank and government want to reduce the loss function type A by 10%. The government is using targeted fiscal stimulus and has a limited budget for fiscal stimulus, say 0.21 trillion rupiahs. In this situation, the central bank needs to reduce the interest rate by 25bps.

Table 2.4 Simulation Results at National Level

Variable	Unit	Without Policies	90% of Total Loss
GDP	%	-0.716%	-0.654%
Inflation	%	-0.098%	-0.081%
Consumption	%	-0.449%	-0.392%
Investment	%	-0.477%	-0.314%
Export	%	-0.772%	-0.757%
Import	%	0.447%	0.547%
UnemplRate	%	3.327%	2.799%
IncDistRuralUrban	%	0.020%	0.029%
Loss		0.025505	0.022792
FiscalSpending	million Rp		210,000

Table 2.5 Simulation Results at Regional Level

Variable	Region	Unit	Without Policies	90% of Total Loss
GDP	Sumatra	%	-2.186%	-2.083%
GDP	Java and Bali	%	-0.444%	-0.383%
GDP	Kalimantan	%	0.141%	0.114%
GDP	Sulawesi	%	-0.123%	-0.016%
GDP	Eastern Indonesia	%	0.087%	0.055%
Inflation	Sumatra	%	-0.122%	-0.104%
Inflation	Java and Bali	%	-0.083%	-0.067%
Inflation	Kalimantan	%	-0.172%	-0.141%
Inflation	Sulawesi	%	-0.131%	-0.109%
Inflation	Eastern Indonesia	%	-0.125%	-0.103%
Consumption	Sumatra	%	-1.495%	-1.409%
Consumption	Java and Bali	%	-0.241%	-0.189%

Variable	Region	Unit	Without Policies	90% of Total Loss
Consumption	Kalimantan	%	0.244%	0.232%
Consumption	Sulawesi	%	-0.166%	-0.055%
Consumption	Eastern Indonesia	%	-0.083%	-0.060%
Investment	Sumatra	%	-0.485%	-0.321%
Investment	Java and Bali	%	-0.477%	-0.314%
Investment	Kalimantan	%	-0.382%	-0.233%
Investment	Sulawesi	%	-0.511%	-0.337%
Investment	Eastern Indonesia	%	-0.558%	-0.375%
Export	Sumatra	%	-4.047%	-4.012%
Export	Java and Bali	%	-0.048%	-0.030%
Export	Kalimantan	%	0.449%	0.413%
Export	Sulawesi	%	-0.052%	0.070%
Export	Eastern Indonesia	%	0.335%	0.262%
Import	Sumatra	%	-0.239%	-0.087%
Import	Java and Bali	%	0.578%	0.661%
Import	Kalimantan	%	0.914%	1.036%
Import	Sulawesi	%	-0.683%	-0.448%
Import	Eastern Indonesia	%	-0.482%	-0.246%
IncDistRuralUrban	Sumatra	%	0.118%	0.132%
IncDistRuralUrban	Java and Bali	%	-0.007%	0.001%
IncDistRuralUrban	Kalimantan	%	0.079%	0.078%
IncDistRuralUrban	Sulawesi	%	0.025%	0.030%
IncDistRuralUrban	Eastern Indonesia	%	-0.067%	-0.043%
Output	Sumatra	%	-1.830%	-1.740%
Output	Java and Bali	%	-0.254%	-0.190%
Output	Kalimantan	%	0.204%	0.217%
Output	Sulawesi	%	-0.335%	-0.177%
Output	Eastern Indonesia	%	-0.191%	-0.140%
DomGood	Sumatra	%	-1.265%	-1.135%
DomGood	Java and Bali	%	-0.266%	-0.200%
DomGood	Kalimantan	%	-0.003%	0.033%
DomGood	Sulawesi	%	-0.385%	-0.221%
DomGood	Eastern Indonesia	%	-0.440%	-0.333%
CompGood	Sumatra	%	-1.101%	-0.971%
CompGood	Java and Bali	%	-0.266%	-0.195%

Variable	Region	Unit	Without Policies	90% of Total Loss
CompGood	Kalimantan	%	-0.107%	-0.039%
CompGood	Sulawesi	%	-0.479%	-0.289%
CompGood	Eastern Indonesia	%	-0.522%	-0.371%
RegLoss	Sumatra		0.023962	0.021745
RegLoss	Java and Bali		0.001020	0.000755
RegLoss	Kalimantan		0.000246	0.000164
RegLoss	Sulawesi		0.000161	0.000060
RegLoss	Eastern Indonesia		0.000116	0.000068
FiscalStimulus	Sumatra	million Rp		181780.4
FiscalStimulus	Java and Bali	million Rp		36930.99
FiscalStimulus	Kalimantan	million Rp		-11707.9
FiscalStimulus	Sulawesi	million Rp		10200.53
FiscalStimulus	Eastern Indonesia	million Rp		-7204.04

Table 2.4 and Table 2.5 show the simulation results of 10 % oil decline without and with policies at national level and regional level, respectively. As can be seen, if the central bank coordinates with the government to reduce the loss function by 90%. The national GDP level only decreases by 0.654% compared to the GDP decline before policies, 0.716%. However, the inflation is slightly higher, at -0.081% compared to the price without policies, at -0.098%. This is because of the expansionary fiscal policy and loose monetary policy. The national GDP components like consumption, investment, export, and import are better relative to those without policies. The unemployment rate has an improvement compared to the one without policies, from 3.327% to 2.799%. Moreover, the income distribution between rural and urban are also getting better compared to the one without policies, from 0.020% to 0.029%.

At regional level, every region has improvement in term of GDP level, but some regions such as Sumatra, Java, and Bali, and Sulawesi still have GDP decline at -2.083%, -0.383%, and -0.016% from the baseline, respectively. However, all regions are experiencing lower price compared to baseline. The regional GDP components, in general, have increased compared to those of without policies. Only regional income distribution in Kalimantan are worsening after policies were imposed even though at national level, the income distribution is improving. Lastly, the regional loss function in every region is lower after policies were imposed.

2.8 Conclusion

In the country with many regions like Indonesia, having the comprehensive tool like FIRCGE that can do the analysis of the impact of specific shock to the regional economy with general equilibrium framework is a must for policy makers. The common practice of using partial equilibrium model might be misleading because the ceteris paribus assumption might not correct. The FIRCGE gives you better understanding of not only the impact of the shock to the macro variables but also to micro variables for every sector and every region of the economy. In addition, it also covers both real sector and financial sector.

As shown in the results above, the reduction of world oil price has different impact to the region Indonesia. For example, the most affected region due to the reduction of world oil price is Sumatra, because it is the largest oil and gas production region in Indonesia. The characteristics of each region play an important role that makes the impact of every shock different.

In reality, when the shock hit the economy, and it has negative impacts on the economy, the central bank as a monetary authority and government as a fiscal authority have difficulty on how should they coordinate and how they should react. This is even harder when those questions should also consider the implication on a regional dimension. With FIRCGE, the combination of monetary and fiscal policy can be simulated such that the policymakers' objectives are achieved. The iso-loss curves give the policy makers a better picture on how they should coordinate, and how they should react given their constraints. It gives them the possibilities frontiers which give them flexibilities to choose. Hence, they can come up with the right policies.

Based on the simulation results, the untargeted policy is less efficient than the targeted policy when both price stability and GDP stability are considered. Or in other words, fiscal stimulus should be injected based on priority starting from the most affected region to the less affected region by the shock. This will help the government to save money or reduce the fiscal deficit through a more efficient allocation of government resources.

2.8.1 Further Research

This model only suggests two rules on how fiscal stimulus should be distributed across the regions and shows that targeted fiscal policy rule is better than the untargeted fiscal policy rules in terms of efficiency. There must be any other kind of rules that may give an even better result that the targeted fiscal policy rule. The room to improve the rule is still wide open.

In term of the loss function, the policy makers might also consider any other options like quadratic GDP gap, or different weight of inflation stability and GDP stability on loss function type A. The decision of the objective or loss function is important since different objective or loss function would give us different results.

Lastly, the FIRCGE model in this paper is not dynamic. The movement from static FIRCGE to dynamic FIRCGE might provide us with more comprehensive and better understanding of the impact of the shocks and/or policies to the economy because the change of the economy in one period should have an effect on the next period and so on.

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APPENDIX

FIRSAM Components

No.	Abbreviation	Category	Description
1	LAGRPR	Factor of Production	Formal Rural Agricultural Labor
2	LAGRPU	Factor of Production	Formal Urban Agricultural Labor
3	LAGRUR	Factor of Production	Informal Rural Agricultural Labor
4	LAGRUU	Factor of Production	Informal Urban Agricultural Labor
5	LOPRPR	Factor of Production	Formal Rural Manual Labor
6	LOPRPU	Factor of Production	Formal Urban Manual Labor
7	LOPRUR	Factor of Production	Informal Rural Manual Labor
8	LOPRUU	Factor of Production	Informal Urban Manual Labor
9	LCLEPR	Factor of Production	Formal Rural Clerical Labor
10	LCLEPU	Factor of Production	Formal Urban Clerical Labor
11	LCLEUR	Factor of Production	Informal Rural Clerical Labor
12	LCLEUU	Factor of Production	Informal Urban Clerical Labor
13	LPROPR	Factor of Production	Formal Rural Professional Labor
14	LPROPU	Factor of Production	Formal Urban Professional Labor
15	LPROUR	Factor of Production	Informal Rural Professional Labor
16	LPROUU	Factor of Production	Informal Urban Professional Labor
17	KCAP	Factor of Production	Capital
18	KLAND	Factor of Production	Land
19	HHR	Institution	Rural Households
20	HHU	Institution	Urban Households
21	GOV	Institution	Government
22	COMP	Institution	Companies
23	FCROPS	Production Sector	Food crops
24	NFCROP	Production Sector	Non-food crops
25	LVSTCK	Production Sector	Livestock
26	FOREST	Production Sector	Forestry
27	FISH	Production Sector	Fishery
28	OILGAS	Production Sector	Oil, Gas and Geothermal Mining
29	NOILGS	Production Sector	Coal and Other Mining
30	OILMF	Production Sector	Refinery and Palm oil
31	NOILMF	Production Sector	Fish Processing, Food and Drink Processing, Textiles, Foot and Leather, Wood Processing, Pulp and Paper, Rubber Processing, Petrochemical, Cement, Basic Metal, Metal Processing, Transport Equipment Electricity Machinery, Other Industries
32	UTILI	Production Sector	Electricity, Gas and Drinking Water

No.	Abbreviation	Category	Description
33	CONST	Production Sector	Construction
34	TRADE	Production Sector	Trade
35	HOTEL	Production Sector	Hotel and Restaurant
36	LNDTRN	Production Sector	Land Transportation
37	WTRTRN	Production Sector	Water Transportation
38	AIRTRN	Production Sector	Air Transportation
39	COMMU	Production Sector	Communications
40	FINAN	Production Sector	Finance
41	PUBSRV	Production Sector	Public Services
42	OTHSRV	Production Sector	Other Services
43	INDTAX	Tax	Indirect Tax
44	SUBS	Subsidy	Subsidy
45	STOCK	Inventory	Inventory
46	KAHHR	Capital Account	Rural Households Capital Account
47	KAHHU	Capital Account	Urban Households Capital Account
48	KAGOV	Capital Account	Government Capital Account
49	KACOMP	Capital Account	Companies Capital Account
50	KAROW	Capital Account	Foreign institution Capital Account
51	MFCROPS	Import Commodity	Food crops
52	MNFCROP	Import Commodity	Non-food crops
53	MLVSTCK	Import Commodity	Livestock
54	MFOREST	Import Commodity	Forestry
55	MFISH	Import Commodity	Fishery
56	MOILGAS	Import Commodity	Oil, Gas and Geothermal Mining
57	MNOILGS	Import Commodity	Coal and Other Mining
58	MOILMF	Import Commodity	Refinery and Palm oil
59	MNOILMF	Import Commodity	Fish Processing, Food and Drink Processing, Textiles, Foot and Leather, Wood Processing, Pulp and Paper, Rubber Processing, Petrochemical, Cement, Basic Metal, Metal Processing, Transport Equipment Electricity Machinery, Other Industries
60	MUTILI	Import Commodity	Electricity, Gas and Drinking Water
61	MCONST	Import Commodity	Construction
62	MTRADE	Import Commodity	Trade
63	MHOTEL	Import Commodity	Hotel and Restaurant
64	MLNDTRN	Import Commodity	Land Transportation
65	MWTRTRN	Import Commodity	Water Transportation
66	MAIRTRN	Import Commodity	Air Transportation
67	MCOMMU	Import Commodity	Communications
68	MFINAN	Import Commodity	Finance
69	MPUBSRV	Import Commodity	Public Services

No.	Abbreviation	Category	Description
70	MOTHSRV	Import Commodity	Other Services
71	ROW	Rest of the world	Rest of the world
72	FXRES	Financial Instrument	Government's Forex Reserves
73	MONEY	Financial Instrument	Currency
74	CHKACC	Financial Instrument	Demand Deposit
75	SAVACC	Financial Instrument	Saving Deposit
76	TIMEDEP	Financial Instrument	Time Deposit
77	BICERT	Financial Instrument	Central Bank Certificate (SBI)
78	GBOND	Financial Instrument	Government Bonds
79	LONGSEC	Financial Instrument	Other Long-Term Securities
80	SHRTSEC	Financial Instrument	Short-term Securities
81	WCAPCR	Financial Instrument	Working Capital Credit
82	INVCR	Financial Instrument	Investment Credit
83	CONSCR	Financial Instrument	Consumption Credit
84	NBANKCR	Financial Instrument	Non-Bank Credit
85	TRDCR	Financial Instrument	Trade Credit
86	SHARE	Financial Instrument	Equity & Share
87	INSPRES	Financial Instrument	Insurance, Pension Fund Reserves
88	OTHFIN	Financial Instrument	Others

List of Equations

1. Import Price

$$p_i^M = pwm_i EXR$$

2. Export Price

$$p_i^E = pwe_i EXR$$

3. Value Added Price

$$p_{i,r}^V = \frac{1}{\gamma_{r,s}^{VA}} \left[\sum_f \left(\theta_{f,i,r}^F\right)^{\sigma_i^{VA}} \left(p_{f,r}^F\right)^{1-\sigma_i^{VA}} \right]^{\frac{1}{1-\sigma_i^{VA}}}$$

4. Price of Total Output

$$p_{i,r}^X = \frac{1}{\gamma_{i,r}^X} \left[\left(\theta_{i,r}^{VA}\right)^{\sigma_i^X} \left(p_{i,r}^V\right)^{1-\sigma_i^X} + \left(\theta_{i,r}^I\right)^{\sigma_i^X} \left(p_{i,r}^Q\right)^{1-\sigma_i^X} \right]^{\frac{1}{1-\sigma_i^X}}$$

5. Domestic Good Price

$$p_{i,r}^X = \frac{1}{\gamma_{i,r}^T} \left[\left(\delta_{i,r}^D\right)^{-\sigma_i^T} \left((1 - \tau_{i,r}^{DOM} + p_{i,r}^{SUB}) p_{i,r}^D\right)^{1+\sigma_i^T} + \left(\delta_{i,r}^{EF}\right)^{-\sigma_i^T} \left(p_i^E\right)^{1+\sigma_i^T} \right.$$

$$\left. + \left(\delta_{i,r}^{ED}\right)^{-\sigma_i^T} \left(p_i^N\right)^{1+\sigma_i^T} \right]^{\frac{1}{1+\sigma_i^T}}$$

6. Composite Good Price

$$p_{i,r}^{Q} = \frac{1}{\gamma_{i,r}^{A}}\Big[\left(\theta_{i,r}^{D}\right)^{\sigma_i^A}\left(p_{i,r}^{D}\right)^{1-\sigma_i^A} + \left(\theta_{i,r}^{MF}\right)^{\sigma_i^A}\left((1+\tau_i^M)p_i^M\right)^{1-\sigma_i^A}$$

$$+\left(\theta_{i,r}^{MD}\right)^{\sigma_i^A}\left(p_i^N\right)^{1-\sigma_i^A}\Big]^{\frac{1}{1-\sigma_i^A}}$$

7. Wages

$$WAGES_{i,r} = PINDEX^{vp_i}\left(\frac{PV_{i,r}}{PV0_{i,r}}\right)^{1-vp_i}\left(\frac{X_{i,r}/X0_{i,r}}{\sum_{fl} FACDEM_{fl,i,r}/\sum_{fl} FACDEM0_{fl,i,r}}\right)^{\phi_i}$$

8. Price of Labor

$$p_{fl,r}^{F} = p0_{fl,r}^{F}\sum_{i} WAGES_{i,r}wlshare_{fl,i,r}$$

9. Factor Demand

$$FACDEM_{f,i,r} = \left(\frac{p_{i,r}^{V}\left(\gamma_{i,r}^{VA}\right)^{\frac{\sigma_i^{VA}-1}{\sigma_i^{VA}}}\theta_{f,i,r}^{VA}}{p_{f,r}^{F}}\right)^{\sigma_i^{VA}} VA_{i,r}$$

10. Demand for Value Added Good

$$VA_{i,r} = \left[\frac{\left(\gamma_{i,r}^{X}\right)^{\frac{\sigma_i^{X}-1}{\sigma_i^{X}}}\theta_{i,r}^{VA}\,p_{i,r}^{X}}{p_{i,r}^{V}}\right]^{\sigma_i^X} X_{i,r}$$

11. Demand for Intermediate Inputs

$$INTM_{i,r} = \left[\frac{\left(\gamma_{i,r}^{X}\right)^{\frac{\sigma_i^{X}-1}{\sigma_i^{X}}}\theta_{i,r}^{I}\,p_{i,r}^{X}}{p_{i,r}^{Q}}\right]^{\sigma_i^X} X_{i,r}$$

12. Total Output

$$D_{i,r} = \left[\frac{\left(\gamma_{i,r}^{T}\right)^{\frac{\sigma_i^{T}+1}{\sigma_i^{T}}}\delta_{i,r}^{D}\,p_{i,r}^{X}}{(1-\tau_{i,r}^{DOM}+p_{i,r}^{SUB})p_{i,r}^{D}}\right]^{-\sigma_i^T} X_{i,r}$$

13. Demand for International Exported Good

$$E_{i,r} = \left[\frac{\left(\gamma_{i,r}^{T}\right)^{\frac{\sigma_i^T+1}{\sigma_i^T}} \delta_{i,r}^{E}\, p_{i,r}^{X}}{p_i^E}\right]^{-\sigma_i^T} X_{i,r}$$

14. Demand for Domestic Exported Good

$$ED_{i,r} = \left[\frac{\left(\gamma_{i,r}^{T}\right)^{\frac{\sigma_i^T+1}{\sigma_i^T}} \delta_{i,r}^{ED}\, p_{i,r}^{X}}{p_i^N}\right]^{-\sigma_i^T} X_{i,r}$$

15. Demand for Domestic Good

$$D_{i,r} = \left[\frac{\left(\gamma_{i,r}^{A}\right)^{\frac{\sigma_i^A-1}{\sigma_i^A}} \theta_{i,r}^{D}\, p_{i,r}^{Q}}{p_{i,r}^D}\right]^{\sigma_i^A} Q_{i,r}$$

16. Demand for International Imported Good

$$M_{i,r} = \left[\frac{\left(\gamma_{i,r}^{A}\right)^{\frac{\sigma_i^A-1}{\sigma_i^A}} \theta_{i,r}^{M}\, p_{i,r}^{Q}}{(1+\tau_i^M)p_i^M}\right]^{\sigma_i^A} Q_{i,r}$$

17. Demand for Domestic Imported Good

$$MD_{i,r} = \left[\frac{\left(\gamma_{i,r}^{A}\right)^{\frac{\sigma_i^A-1}{\sigma_i^A}} \theta_{i,r}^{MD}\, p_{i,r}^{Q}}{p_i^N}\right]^{\sigma_i^A} Q_{i,r}$$

18. Direct Tax

$$DIRTAX_{rall,GIN,rall2.DIN} = dtax_{rall,GIN,rall2,DIN} INC_{DIN,rall2}$$

19. Indirect Tax

$$ITAX_{i,r} = tdom_{i,r} p_{i,r}^D D_{i,r}$$

20. Subsidy

$$SUB_{i,r} = psub_{i,r} p_{i,r}^D D_{i,r}$$

21. Import Tariff

$$TARIFF_i = tm_i p_i^M \sum_r M_{i,r}$$

22. Factor Income

$$YF_{f,r} = \sum_i p_{f,r}^F FACDEM_{f,i,r} + \sum_{nr} \sum_{fr} YFROW_{r,f,nr,fr}$$

23. Government Income

$$\begin{aligned} INC_{GIN,rall} = & \sum_{r2} \sum_f factoin_{rall,GIN,r2,f} YF_{f,r2} + \sum_{rall2} \sum_{in2} ITRAN_{rall,GIN,rall2,in2} \\ & + gishr_{GIN,rall} \sum_i \left(\sum_{r2} \left(ITAX_{i,r2} \right) + TARIFF_i \right) \\ & - gsshr_{GIN,rall} \sum_r \sum_i SUB_{i,r} \end{aligned}$$

24. Non-Government Income

$$INC_{NGI,r} = \sum_{r2} \sum_f factoin_{r,NGI,r2,f} YF_{f,r2} + \sum_{rall2} \sum_{in2} ITRAN_{r,NGI,rall2,in2}$$

25. Foreign Income

$$\begin{aligned} INC_{FR,nr} = & \sum_{r2} \sum_f factoin_{nr,FR,r2,f} YF_{f,r2} + mrshr_{FR,nr} \sum_r \sum_i P_i^M M_{i,r} \\ & + \sum_{rall} \sum_{in2} ITRAN_{nr,FR,rall,in2} \end{aligned}$$

26. Disposable Income

$$\begin{aligned} YCONS_{H,r} = & \left(1 - mps_{H,r}\right) \left(INC_{H,r} - \sum_{rall} \sum_{GIN} DIRTAX_{rall,GIN,r,H} \right. \\ & \left. - \sum_{r2} \sum_{NGI} ITRAN_{r2,NGI,r,H} - \sum_{nr} \sum_{fr} ITRAN_{nr,fr,r,H} \right) \end{aligned}$$

27. Households expenditure

$$\begin{aligned} EXP_{H,r} = & YCONS_{H,r} + \sum_{rall} \sum_{GIN} DIRTAX_{rall.GIN,r,H} + \sum_{r2} \sum_{NGI} ITRAN_{r2.NGI,r,H} \\ & + \sum_{nr} \sum_{FR} ITRAN_{nr.FR,r,H} \end{aligned}$$

28. Government expenditure

$$EXP_{GIN,rall} = ggshr_{GIN,rall} \sum_{r} \sum_{i} P_{i,r}^{Q} GD_{i,r} + \sum_{rall2} \sum_{in2} ITRAN_{rall2,in2,rall,GIN}$$

29. Non-household and non-government expenditure

$$EXP_{NGNH,r} = \sum_{rall} \sum_{in} ITRAN_{rall.in,r,NGNH}$$

30. Foreign expenditure

$$EXP_{FR,nr} = ershr_{FR,nr} \sum_{r} \sum_{i} p_{i}^{E} E_{i,r} + \sum_{r} \sum_{f} YFROW_{r.f,nr,fr} + \sum_{rall} \sum_{in} ITRAN_{rall.in,nr,fr}$$

31. Saving

$$SAV_{in,rall} = INC_{in,rall} - EXP_{in,rall}$$

32. Total Saving

$$SAVING = \sum_{rall} \sum_{in} SAV_{in,rall}$$

33. Foreign Saving

$$FSAV.EXR = \sum_{nr} \sum_{fr} SAV_{fr,nr}$$

34. Institutional Transfer

$$ITRAN_{rall,in,rall2,in2} = GTRAN_{rall,in,rall2,in2} + otranshare_{rall,in,rall2,in2} RTRANTOT_{in,in2} + OTRAN_{rall,in,rall2,in2}$$

35. Government Transfer

$$GTRAN_{rall,GIN,rall2,DIN} = DIRTAX_{rall,GIN,rall2,DIN}$$

36. Financial Return Transfer

$$RTRANTOT_{h,in2} = RNshare2_{h,in2} \sum_{asrn} RN_{asrn} LiabSLag_{in2,asrn}$$

37. Consumption Demand

$$CD_{i,r} = \left(\sum_{h} \alpha_{i,r,h}^{q} YCONS_{h,r} \right) \Big/ p_{i,r}^{Q}$$

38. Investment Demand

$$p^{Q}_{i,r} ID_{i,r} = idrshr_{i,r} \sum_{in} INVES_{i,in}$$

39. Total Investment

$$INVEST = \sum_{i} \sum_{in} INVES_{i,in} + KSTOCK$$

40. Domestic Investment Supply

$$INVES_{i,NGI} = \lambda 0_{i,NGI} \left(\sum_{r} X_{i,r} \right)^{\lambda 1_i} (1 + avgRN)^{\lambda 2_i}$$

41. Average Interest Rate

$$avgRN = \left(\sum_{as} \sum_{in} RN_{as} AssetSLag_{as,in} \right) \Big/ \sum_{as} \sum_{in} AssetSLag_{as,in}$$

42. Intermediate Good Demand

$$INTQ_{i,r} = \sum_{r2} \sum_{j} aad_{r,i,r2,j} dintshr_{j,r2} INTM_{j,r2} + \sum_{j} aaf_{i,r,j} fintshr_{j,r} INTM_{j,r}$$

43. Market Clearing for Intra-National Trade

$$\sum_{r} ED_{i,r} = \sum_{r} MD_{i,r}$$

44. Market Clearing Condition

$$Q_{i,r} = CD_{i,r} + ID_{i,r} + GD_{i,r} + INTQ_{i,r} + RSTKD_{i,r}$$

45. Factor Supply

$$FS_{f,r} = \sum_{i} FACDEM_{f,i,r}$$

46. Labor Supply

$$LSUP = (1 + UEMPR) \sum_{fl} \sum_{r} FS_{fl,r}$$

47. Regional Real GDP

$$GDPINDR_{r} = \sum_{i} CD_{i,r} + ID_{i,r} + GD_{i,r} + E_{i,r} - M_{i,r}$$

48. Regional Nominal GDP

$$GDPINDN_r = \sum_i p_{i,r}^Q CD_{i,r} + p_{i,r}^Q ID_{i,r} + p_{i,r}^Q GD_{i,r} + p_i^E E_{i,r} - p_i^M M_{i,r}$$

49. Price Index

$$PINDEX = \frac{\sum_r GDPINDN_r}{\sum_r GDPINDR_r}$$

50. Stock of Assets

$$AssetS_{as,in} = AssetSLag_{as,in} + Asset_{as,in}$$

51. Stock of Liabilities

$$LiabS_{in,as} = LiabSLag_{in,as} + Liab_{in,as}$$

52. Stock of Fix Assets

$$FixAS_{in} = FixASLag_{in} + FixA_{in}$$

53. Stock of Wealth

$$Wealth_{in} = WealthLag_{in} + WEALF_{in}$$

54. Flow of Fix Assets

$$FixA_{in} = \sum_i INVES_{i,in} + kstkshr_{in} KSTOCK$$

55. Flow of Wealth

$$WEALF_{in} = \sum_{rall} SAV_{in,rall}$$

56. Demand of Assets

$$AssetS_{as,in} = \theta 1_{as,in} \left(\frac{rn_{as}}{rn0_{as}}\right)^{\sigma 1_{as,in}}$$

57. Demand of Liabilities

$$LiabS_{in,as} = \theta 2_{in,as} \left(\frac{rn_{as}}{rn0_{as}}\right)^{\sigma 2_{in,as}}$$

58. Money Demand

$$MD_{in} = \alpha 1_{in} \left(\sum_{rall} INC_{in,rall}\right)^{\alpha 2_{in}} (rnv1_{in})^{-\alpha 3_{in}}$$

59. Financial Account Balance

$$\sum_{as} AssetS_{as,in} + FixAS_{in} = \sum_{as} LiabS_{in,as} + Wealth_{in}$$

CHAPTER 3
THE OPTIMAL MONETARY POLICY IN REGIONAL ECONOMY: THE CASE OF INDONESIA

3.1 Background

Dynamic Stochastic General Equilibrium Model (DSGE model) is more consistent with economic theory compared to the traditional econometric model because it is a micro-founded model constructed based on the behavior optimization of every agent that included in the model. That is why DSGE model is resistant to Lucas Critique, which predicts that the effect of economic policy cannot entirely depend on historical data. In DSGE, it is assumed that in the long-run when there are no more shocks, the economy goes to the steady state level. DSGE model generates a unique equilibrium path as the shocks are imposed to the model. Every point in this path represents the equilibrium condition after every agent in the model behaves optimally and interacts with other agents.

If we compare with Computable General Equilibrium model (CGE model), DSGE model is more advanced since it is dynamic and stochastic, unlike the CGE which is static and deterministic. However, both models are micro-founded on assumptions about preferences, technology, and budget constraints. CGE model more focuses on the long-run relationships because the data used are just a snapshot of the particular period usually yearly and this data will define the structural of the economy. On the other hand, DSGE focus on the dynamic of the economy over time and the structure of the economy depends on the series of data and the expert judgment.

In common macro-econometric and CGE model, monetary policy is assumed to be exogenous or is defined outside the model. In this case, there is no feedback effect of the monetary policy in next period. In contrast, standard DSGE model does not assume this. Monetary policy is an endogenously model based on Taylor rule, such that monetary policy will respond to some economic conditions after shock was imposed. In the case of Taylor rule, they are inflation gap and output gap. Given this feature, we can generate the optimal policy responds given the shock.

The economist in central bank mostly used DSGE model to analyze and evaluate macroeconomic policy or even to forecast some key macroeconomic variables for more than two years ahead. This model usually produces analysis for the national level. At national level, it assumed that the effect of a policy is uniform across regions in a country. This is not true if the economic structure of each region is different. Therefore, the regional DSGE model is more appropriate for a country like Indonesia that has many regions with different economic structure.

In DSGE modeling, disaggregating national level into regional level would be very complicated because the system of equations will grow about n times, where n is the number of regions that are introduced into the model. As the system of equations is getting bigger, it is hard to get the stability of the system since the model is dynamic and the equations can contain forward and backward looking variables. That is why the two-region DSGE model is constructed for each region, wherein every model there are one particular region and the rest of the region. For example, in Sumatra DSGE model, there are Sumatra region, and the region outside Sumatra, which contains Java and Bali, Kalimantan, Sulawesi, and Eastern Indonesia.

This study aims to construct five regional DSGE models and tries to estimate the optimal monetary policy of each regional DSGE model and then analyze how central bank as a monetary authority should react to the regional shocks. Moreover, it also wants to address the question of how the disaggregation of the national model into the regional model is necessary by comparing the results between those two models. Besides the monetary policy, the model also provides the optimal local government expenditure in the region. Thus, the central bank optimal policy in this model is the optimal response given the government optimal fiscal policy.

According to Tamegawa (2012) in term of economic geography namely integration, agglomeration and etc., this DSGE model is comparable to the model called New Economic Geography (NEG) models, which are critical for analyzing economic geography. However, the DSGE model has at least one advantage namely agent's forward-looking behavior. This feature might be not interesting for regional analysis, but this is needed for a discussion of dynamic regional model based on DSGE at least to obtain robustness in policy analysis. Their model incorporates several extensions to standard DSGE model because the effect of fiscal policy on the consumption is negative which is not supported by many empirical studies. These extensions include non-Ricardian households (Gali et al., 2007) and "deep habit" (Ravn et al., 2006), a utility that strengthens the complementary between labor and consumption.

The proposed model is built based on a standard small open economy framework by Gali and Monacelli (2008). They created DSGE model for the European Monetary Union (EMU) to analyze the interaction between fiscal and monetary policy in a currency union and also to study its implications for the optimal design of such policies. In their model, they incorporate nominal rigidities that have been used recently for monetary policy analysis. Their model also contains a fiscal policy sector by allowing for a country-specific level of public consumption, and lastly, it comprises many small open economies linked by trade and financial flows. Their model can be implemented for the purpose of analysis because Indonesia as a country with a single currency has many regions that are analogous to the countries in the EMU, where each region has its own region-specific fiscal policy and are linked by trade and financial flows. However, there is only one monetary policy that will affect those regions simultaneously.

As mentioned earlier, separated two-region DSGE model is constructed for each region. Each model emphasized on the regional economy inside Indonesia; it is assumed that the rest of world is constant. Thus, each region is considered as a small open economy to the rest of the region nationally, but it is closed economy to the rest of the world.

The original Gali Monacelli (2008) model is modified such that it can match the reality better. This modification includes backward looking Phillip Curve, backward looking of Taylor rule, and introducing monetary aggregate through utility function. The first one can be done by adding indexation mechanism which nominal prices are indexed to past inflation in the aggregate price level. This feature can accommodate the probability of firms who did not re-optimize their prices when they set the prices. The second one is also important to provide the behavior of central bank as it tends to smooth its interest rate. The last one is necessary for policy makers to manage the dynamics of a monetary aggregate. In addition, the Taylor rule not only depends on inflation gap, the difference between current inflation to its target, but also depends on output gap, the difference between GDP and its potential. This feature reflects the tradeoff of the policy makers between inflation and GDP.

The main advantage of this model is the estimation of the parameters using Bayesian estimation procedure. It allows heterogeneity across the region in terms of economic structure because each region has its own set of parameters that match the reality. This procedure not only estimates the parameters but also calculates the standard error of the shocks. After the estimation process has been conducted, the model is simulated to check the impulse response of each shock. Then, the optimum Taylor rule can be calculated by minimizing objective/loss function subject to some constraints given the estimated parameters and standard errors. Since this study focuses on what the optimal monetary policy when there are regional shocks, only shocks come from the region are included in the minimization process.

3.2 Hypothesis

The current central bank's monetary policy as a response to a regional shock might be not optimal. It is possible to find the optimal policy which maximizes the central bank's objective function given the optimal government's fiscal policy.

When a regional shock hits the economy, the disaggregation of the national level model into regional level model becomes important. It is because the policy recommendation produced by the national model and the regional model might be different. This difference could hurt some agents and regions in the economy. In addition, the regional model is expected to be more optimal than the national one in terms of loss function.

3.3 Literature Review

The idea of DSGE model was started with a seminal paper by Kydland and Prescott (1982) modeling post-war U.S. economy and introducing Real Business Cycle model (RBC model). Since then, DSGE model has become a popular tool for many fields of economic studies especially international economics and macroeconomics to address the problems in economic growth, business cycle, monetary policy, fiscal policy, etc.

In the broad field of macroeconomic models, there are two types of frameworks namely RBC model, and New Keynesian model (NK model). The former is to study business cycle fluctuations based on neoclassical growth model and assumed perfect competitions. Therefore, the prices are flexible. The latter assumed imperfect competitions in goods and labor market which allows price rigidities that are introduced via sticky prices and wages. In the early 1990s, the economists started to combine those two types of frameworks called new neoclassical synthesis, which essentially combined the dynamic aspects of RBC model with imperfect competition and nominal rigidities of the new Keynesian model.

The research on monetary policy using the new neoclassical synthesis becomes popular at the central bank in the last few years. Some central bank has its own name of the DSGE model, for example Bank Indonesia (ARIMBI / Aggregate Rational Inflation-targeting Model for Bank Indonesia), Norges Bank (NEMO / Norwegian

Economy Model), Bank of Canada (ToTEM / Terms of Trade Economic Model), Bank of England (BEQM / Bank of England Quarterly Model), Central Bank of Chile (MAS / Model of Analysis and Simulation), European Central Bank (NAWN / New Area-Wide Model), Sveriges Riksbank (RAMSES / Riskbanks Aggregated Macro model for Studies of the Economy in Sweden), etc.

In Indonesia, Bank Indonesia has been developing its DSGE model since 2000 named GEMBI (General Equilibrium Model of Bank Indonesia) by Joseph, Dewandaru, and Ari (2000). This model was improving every year until 2007. In 2009, BISMA (Bank Indonesia Structural MAcro model) was built by Waluyo (2009) only for policy simulation because it is very theoretical. In 2010, ARIMBI was developed by Harmanta, Bathaludin and Waluyo (2010) to do both policy simulation and forecast model. Now, ARIMBI is the primary model in Bank Indonesia to simulate and forecast the policy and aggregated macro variables and then collaborate with an economic model named SOFIE (ShOrt-term Forecast Model For Indonesian Economy) to disaggregate those forecast variables into more detail variables. This model includes both real and financial sector. However, these models are still at the national level where all the variables are in national macro-aggregated.

Most of the models above are single-country DSGE model. However, some economists also interested in building multi-country DSGE model to see the interaction between countries, for example, Dieppe, Pandiella and Willman (2011) developed NMCM (New Multi-Country Model) for the Euro Area. Their model is intended for doing scenario analysis and forecast for European Central Bank, and it covers the five largest euro area countries. They show the evidence of heterogeneity across countries and that the effect of the shocks to economies depends on whether the shocks are pre-announced, announced and credible or unannounced and uncredible. However, since the model is estimated with GMM (Generalized Method of Moments), it is assumed that optimizing agents are boundedly rational in knowing only parameters related to their problem and not the rest of the model. This assumption might not reflect the real world.

There are few papers discussed regional economy using DSGE model. Tamegawa (2012) in his study explains how to construct *N* region DSGE model, and he simulates the effects of fiscal policy on the economy by using two-region DSGE model. His model has interaction between small region and rest of the regions through trade. However, this model assumed no monetary policy. He found that the fiscal expansion is largest if the more solvent government implements the policy in a region where social capital is productive, and the number of non-Ricardian households is large. Moreover, he also demonstrated that a sudden rise in the interest rate of government bonds as results of a sudden increase in probability of government default could lead to a decrease in national output.

Okano et. al. (2015) constructed DSGE model to examine Japan's Kansai region to find out the causes of its long-run economic stagnation. Their model has two type of firms namely durable and nondurable producers, and it has a monetary and fiscal policy, which comprises local and central government. However, they model the region using single-country closed economy model and split the government expenditure into local and central government using fix ratio. There is no interdependent relationship with another economy, even with other regions. In other words, it is similar to imposing regional dataset into single country DSGE model. The simulation results showed that the structural problems related to the Kansai region stagnation are stagnant private residential and equipment investments and productivity persistence.

In general, the procedure of solving DSGE model involves the setup of the model with some assumptions, derivation of the first-order conditions from all the optimization problems which produce system of non-linear equations, then the linearization of this system of equations which is approximated in the neighborhood of the non-stochastic steady state since the system of equations usually does not have a closed analytical solution. The approximation around the steady-state using (log-linear) approximation produces a system of linear difference equations in the state-space form. Next, the parameters are calibrated by utilizing actual dataset for share type parameters, borrowing parameters from other models that have the same characteristics, and/or getting from expert's judgments. The more sophisticated way to get the parameters is by estimation process using Bayesian methods. These methods allow both data and judgment to be considered in the estimation process.

The use of Bayesian methods in DSGE models began in the late 1990s. Before that, the classical optimization methods were used by macroeconomists to estimate DSGE model. Macroeconomists prefer to use Bayesian methods even though DSGE model can be estimated using classical optimization methods. One reason is that frequentist econometrics often find challenging to estimate a wide variety of macro models. Another reason is that advances in Bayesian theory are giving an expanding array of tools that researchers can utilize to estimate DSGE model. The availability of a modern computer with high computational power makes Bayesian approach popular because the estimation of medium to large-scale DSGE model using Markov Chain Monte Carlo (MCMC) simulators are feasible.

3.4 Bayesian Estimation

The recent DSGE models usually do not have a closed-form solution. Therefore, researchers try to obtain an approximate solution by applying first-order Taylor approximation or log-linearization procedure on the DSGE model around its non-stochastic steady state. The log-deviations of a variable from its steady state is denoted by "$\frown$" symbol, or defined as $\hat{x} = x_t - \bar{x}$ or $\hat{x} = \frac{x_t - \bar{x}}{\bar{x}}$, where that x_t and $\bar{x}$ are the logarithmic level and the logarithmic steady state of x_t, respectively.

Bayesian estimation of the linear approximate DSGE model uses Kalman filter to construct the likelihood of DSGE model. The state-space representation of DSGE model consists of the system of state equations,

$$\mathbb{S}_t = \mathbb{F}\mathbb{S}_{t-1} + \mathbb{Q}\xi_t, \qquad \xi_t \sim NID(\mathbf{0}, \mathbf{I}_m), \tag{3.1}$$

and the system of observation equations,

$$\mathbb{Y}_t = \mathbb{H}\mathbb{S}_t + \xi_{u,t}, \qquad \xi_{u,t} \sim NID(\mathbf{0}, \Sigma_u) \tag{3.2}$$

where $\mathbb{S}_t$is the vector of states, $\mathbb{F}$ and $\mathbb{Q}$ are function of the matrices, $\mathbb{Y}_t$ is the vector of observables, $\mathbb{H}$ is zero and one matrix relates to the definition of model with the data, ξ_t is the vector of structural innovation, and $\xi_{u,t}$ is a vector of measurement errors. The likelihood of the linearized DSGE model can be formulated from the state-space representation above as follows

$$\mathcal{L}(\mathcal{Y}_T|\Theta) = \prod_{t=1}^{T} \mathcal{L}(\mathbb{Y}_t|\mathcal{Y}_{t-1}, \Theta) \tag{3.3}$$

where $\mathcal{L}(\mathbb{Y}_t|\mathcal{Y}_{t-1},\Theta)$ is the likelihood conditional on past information available up to $t-1$ and $\mathcal{Y}_{t-1} \equiv \{\mathbb{Y}_0, \mathbb{Y}_1, \dots, \mathbb{Y}_{t-1}\}$. Let define $\mathbb{S}_{t|t-1}$ as the conditional expectation of $\mathbb{S}_t$ given $\{\mathbb{S}_1, \dots, \mathbb{S}_{t-1}\}$ and its mean square error is $P_{t|t-1} \equiv \mathbb{E}[(\mathbb{S}_t - \mathbb{S}_{t-1})(\mathbb{S}_t - \mathbb{S}_{t-1})']$. Then, Kalman filter is calculated the likelihood by the following steps:

1. Set the initial value, $\mathbb{S}_{1|0} = 0$, and $P_{1|0} = \mathbb{F}P_{0|0}\mathbb{F}' + Q'$, where $Q' = \mathbb{Q}\mathbb{Q}'$
2. Calculate $\mathbb{Y}_{1|0} = \mathbb{H}'\mathbb{S}_{1|0} = 0$, $\Omega_{1|0} = \mathbb{E}\left[\left(\mathbb{Y}_1 - \mathbb{Y}_{1|0}\right)\left(\mathbb{Y}_1 - \mathbb{Y}_{1|0}\right)'\right] = \mathbb{H}'P_{1|0}\mathbb{H} + \Sigma_u$
3. From step 1 and 2, the prediction of period 1 likelihood is

$$\mathcal{L}(\mathbb{Y}_1|\Theta) = (2\pi)^{-m/2}\left|\Omega_{1|0}^{-1}\right|^{1/2} exp\left[-\frac{1}{2}\left(\mathbb{Y}_1'\Omega_{1|0}^{-1}\mathbb{Y}_1\right)\right]$$

4. Update the period 1 forecasts

$$\mathbb{S}_{1|1} = \mathbb{S}_{1|0} + P_{1|0}\mathbb{H}\Omega_{1|0}^{-1}\left(\mathbb{Y}_1 - \mathbb{Y}_{1|0}\right)$$

$$P_{1|1} = P_{1|0} - P_{1|0}\mathbb{H}\Omega_{1|0}^{-1}\mathbb{H}'P_{1|0}$$

5. Do step 2 until 4 again to get Kalman filter predictions of $\mathbb{S}_t$ and $\mathbb{Y}_t$

$$\mathbb{S}_{t|t-1} = \mathbb{F}\mathbb{S}_{t-1}\mathbb{P}_{t|t-1}$$

$$\mathbb{P}_{t|t-1} = \mathbb{F}\mathbb{P}_{t-1|t-1}\mathbb{F}' + \mathbb{Q}'$$

$$\mathbb{Y}_{t|t-1} = \mathbb{H}'\mathbb{S}_{t|t-1}$$

$$\Omega_{t|t-1} = \mathbb{E}\left[\left(\mathbb{Y}_t - \mathbb{Y}_{t|t-1}\right)\left(\mathbb{Y}_t - \mathbb{Y}_{t|t-1}\right)'\right] = \mathbb{H}'\mathbb{P}_{t|t-1}\mathbb{H} + \Sigma_u$$

the likelihood is calculated by

$$\mathcal{L}(\mathbb{Y}_t|\mathcal{Y}_{t-1},\Theta) = (2\pi)^{-m/2}\left|\Omega_{t|t-1}^{-1}\right|^{1/2} exp\left[-\frac{1}{2}\left(\mathbb{Y}_t - \mathbb{Y}_{t|t-1}\right)'\Omega_{t|t-1}^{-1}\left(\mathbb{Y}_t - \mathbb{Y}_{t|t-1}\right)\right]$$

then the state vector and its mean square error for period $t = 2, \dots, T$ can be updated using

$$\mathbb{S}_{t|t} = \mathbb{S}_{t|t-1} + P_{t|t-1}\mathbb{H}\Omega_{t|t-1}^{-1}\left(\mathbb{Y}_t - \mathbb{Y}_{t|t-1}\right)$$

$$P_{t|t} = P_{t|t-1} - P_{t|t-1}\mathbb{H}\Omega_{t|t-1}^{-1}\mathbb{H}' P_{t|t-1}$$

The likelihood of the linearized DSGE model is computed using the likelihood in step 2 and 5 above, and then in the first stage, the parameters are estimated that maximized the likelihood in equation (3.3).

In order to incorporate priors in the Bayesian estimation procedure, the parameter vector Θ is divided into two parts: the priors, Θ_1, which contains the parameters which are to be estimated; and Θ_2, which consists of the fixed or calibrated parameters. The priors and the calibrated parameters are discussed in more detail in the priors and calibrated parameters section later.

In the second stage, after the parameters are estimated using maximum likelihood estimator, the posterior distribution of the DSGE model parameters in Θ_1 is simulated using the MH-MCMC algorithm. The estimated parameters in the first stage are used as initial parameters for MH-MCMC algorithm. These parameters are passed to Kalman filter procedure to get the estimated $\mathcal{L}(\mathcal{Y}_T|\Theta_1;\Theta_2)$. The MH random walk law of motion updates the initial parameters and put the updated parameters back to Kalman filter to generate a second estimate of the likelihood function. The MH algorithm decides whether the proposed update of Θ_1 and its likelihood go to next step. Next given this decision, the MH algorithm is trying to obtain new proposed update of Θ_1 using MH random walk law of motion and generate an estimate of corresponding likelihood. The new likelihood is compared to the previous likelihood to select the Θ_1 and likelihood for the next MH step. In order to generate the posterior of linear approximate DSGE model, $\mathcal{P}(\Theta_1|\mathcal{Y}_T;\Theta_2)$, this procedure needs to be repeated $\mathcal{H}$ times.

In more details, the MH-MCMC algorithm can be written as follows:

1. Initialize the MH algorithm $\widehat{\Theta}_{1,0}$ using the parameters estimated in the first stage from maximum likelihood estimator.
2. Put these parameters $\widehat{\Theta}_{1,0}$ into Kalman filter procedure to produce an initial estimate of the likelihood of the DSGE model, $\mathcal{L}(\mathcal{Y}_T|\widehat{\Theta}_{1,0};\Theta_2)$
3. Generate the proposed update of $\widehat{\Theta}_{1,0}$ which is $\Theta_{1,1}$ using the random walk law of motion of MH, $\Theta_{1,1} = \widehat{\Theta}_{1,0} + \varpi\vartheta\varepsilon_1$, $\varepsilon_1 \sim NID(\mathbf{0}_d, \boldsymbol{I}_d)$, where ϖ is a scalar which indicates the size of the "jump" of the proposed MH random walk update, ϑ is the Cholesky decomposition of the covariance matrix of Θ_1, and d is the dimension of Θ_1. Then, compute the $\mathcal{L}(\mathcal{Y}_T|\Theta_{1,0};\Theta_2)$ by employing $\Theta_{1,1}$ as input.
4. There are two-stage procedure to decide whether to move to proposed update $\Theta_{1,1}$ or to stay with the initial value $\widehat{\Theta}_{1,0}$. In the first stage calculate

$$\omega_1 = \min\left\{\frac{\mathcal{L}(\mathcal{Y}_T|\Theta_{1,1};\Theta_2)\mathcal{P}(\Theta_{1,1})}{\mathcal{L}(\mathcal{Y}_T|\widehat{\Theta}_{1,0};\Theta_2)\mathcal{P}(\widehat{\Theta}_{1,0})}, 1\right\}$$

   where $\mathcal{P}(\Theta_{1,1})$ is the prior at $\Theta_{1,1}$. In the second stage, draw a uniform random variable $\varphi_1 \sim U(0,1)$ to set $\widehat{\Theta}_{1,1} = \Theta_{1,1}$ and the counter $\mathbb{C} = 1$ if $\varphi_1 \leq \omega_1$, otherwise set $\widehat{\Theta}_{1,1} = \widehat{\Theta}_{1,0}$ and $\mathbb{C} = 0$
5. Repeat step 3 and 4 for $\ell = 2,3,\ldots,\mathcal{H}$ by calculating the MH random walk law of motion

$$\Theta_{1,\ell} = \widehat{\Theta}_{1,\ell-1} + \varpi\vartheta\varepsilon_\ell, \qquad \varepsilon_\ell \sim NID(\mathbf{0}_{d\times 1}, \boldsymbol{I}_d)$$

and draw again $\varphi_\ell \sim U(0,1)$ to test again

$$\omega_\ell = \min\left\{\frac{\mathcal{L}(\mathcal{Y}_T|\Theta_{1,\ell};\Theta_2)\mathcal{P}(\Theta_{1,\ell})}{\mathcal{L}(\mathcal{Y}_T|\widehat{\Theta}_{1,\ell-1};\Theta_2)\mathcal{P}(\widehat{\Theta}_{1,\ell-1})}, 1\right\}$$

to set $\widehat{\Theta}_{1,\ell}$ to either $\Theta_{1,\ell}$ or $\widehat{\Theta}_{1,\ell-1}$. The former implies $\mathbb{C} = \mathbb{C} + 1$, while the latter has $\mathbb{C} = \mathbb{C} + 0$.

These five steps produce the posterior $\mathcal{P}(\widehat{\Theta}_1|\mathcal{Y}_T;\Theta_2)$ by drawing $\{\widehat{\Theta}_{1,\ell}\}_{\ell=1}^{\mathcal{H}}$. However, at least there are five main issues when running MH-MCMC algorithm, namely obtaining initial value $\widehat{\Theta}_{1,0}$ for the MH-MCMC, setting ϑ which is the Cholesky decomposition of the covariance matrix of Θ_1, determining $\mathcal{H}$, setting ϖ to get optimal acceptance rate $\mathbb{C}/\mathcal{H}$ for the proposal $\Theta_{1,\ell}$, and checking the convergence of MH-MCMC simulator.

First, the initial value of the MH-MCMC can be obtained by using classical optimization methods and an MH-MCMC "burn-in" stage. A classical optimizer is repeatedly run to the likelihood of DSGE model with initial conditions found by sampling 100 times from $\mathcal{P}(\Theta_1)$. The initial condition for a "burn-in" stage of MH-MCMC algorithm is taken from the mode of the posterior distribution of Θ_1 in the classical optimization in previous step. This procedure needed to remove the dependence of $\mathcal{P}(\widehat{\Theta}_1|\mathcal{Y}_T;\Theta_2)$ on the initial condition $\widehat{\Theta}_{1,0}$. To complete the "burn-in" stage, 10,000 MH steps are run using $\varpi = 1$ and $\vartheta = \mathbf{I}_d$. These steps produce an empirical estimate of the covariance matrix $\vartheta\vartheta'$ which is used after the Cholesky decomposition for the source of ϑ for the MH law of motion.

Determining $\mathcal{H}$ and ϖ are important because they affect the speed of convergence. The number of step or simulation, $\mathcal{H}$, should be sufficient such that the MH-MCMC simulation is convergence. In this model, 20,000 simulations are enough to allow for convergence. However, for large and more complicated DSGE model, it needs a lot more simulations. As for jump scale, ϖ, Gelman et al (2004) suggest with $\varpi = 2.4/\sqrt{d}$ to drive the acceptance rate $\mathbb{C}/\mathcal{H} \in [0.23, 0.30]$ to get the most efficient MH law of motion.

The convergence of the MH-MCMC simulator can be checked by using $\hat{R}$ statistic from Gelman et al. (2004). The $\hat{R}$ statistic compares the variances of the elements within the sequence of $\{\widehat{\Theta}_{1,\ell}\}_{\boldsymbol{\ell}=1}^{\boldsymbol{\mathcal{H}}}$ to the variances between the sequences produced with different initial conditions that are generated by the same methods above. According to Gelman et al (2004), the rule of thumb for convergence is when $\hat{R} \leq 1.1$, if not, they suggest to increase $\mathcal{H}$ until convergence is achieved.

3.5 The Model

Indonesia is modeled as a system consists of a *continuum* of regions represented by the unit interval. Each region is treated as a small open economy, indexed by $i \in [0,1]$. However, the whole model is considered closed economy since this research is only focus on the national interaction. The model in detail is described as follows:

3.5.1 Households

The model assume that a region is inhabited by an infinitely-lived representative household seeking to maximize

$$\max_{C_t^i, N_t^i, \frac{M_t^i}{P_{c,t}^i}, B_{t+1}^i} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t (1-\chi) \log C_t^i + \chi \log G_t^i - \frac{{N_t^i}^{1+\varphi}}{1+\varphi} + \frac{\left(\frac{M_t^i}{P_{c,t}^i}\right)^{1-\nu}}{1-\nu} \right\} \tag{3.4}$$

subject to a sequence of the budget constraint of the form:

$$\int_0^1 P_t^i(j) C_{i,t}^i(j) dj + \int_0^1 \int_0^1 P_t^f(j) C_{f,t}^i(j) dj\, df + E_t\{Q_{t,t+1} B_{t+1}^i\} + M_t^i \leq B_t^i + M_{t-1}^i + W_t^i N_t^i + T_t^i \tag{3.5}$$

where C_t^i, N_t^i, G_t^i, and $\frac{M_t^i}{P_{c,t}^i}$ are private consumption, hours of work, public consumption, and real money balance, respectively. $P_t^f(j)$ is the price of good *j* produced in region *f*. $B_{t+1}^i$is the nominal payoff in period *t+1* of the portfolio held at the end of period *t*. W_t^i is the nominal wage, and T_t^i is lump-sum taxes. $Q_{t,t+1}$ is the stochastic discount factor for one-period ahead nominal payoffs, which is common across regions. Parameter $\chi \in [0,1]$ measures the weight attached to public consumption. φ is the inverse of the elasticity of work effort with respect to the real wage or the inverse of the Frisch elasticity of labor supply. Finally, ν is the inverse of the elasticity of money holdings with respect to the interest rate.

C_t^i is a composite consumption index defined by

$$C_t^i \equiv \frac{\left(C_{i,t}^i\right)^{1-\alpha} \left(C_{F,t}^i\right)^{\alpha}}{(1-\alpha)^{(1-\alpha)} \alpha^{\alpha}} \tag{3.6}$$

where $C_{i,t}^i$ is an index of region *i*'s consumption of locally produced goods given by CES function

$$C_{i,t}^{i} \equiv \left(\int_0^1 C_{i,t}^{i}(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}} \tag{3.7}$$

where $j \in [0,1]$ denotes the type of good in region i. Variable $C_{F,t}^{i}$is an index of region i's consumption of imported goods from other regions, given by:

$$C_{F,t}^{i} = exp \int_0^1 c_{f,t}^{i} \, df \tag{3.8}$$

where $c_{f,t}^{i} = \log C_{f,t}^{i}$. The consumption of goods by region i's households that are produced by region f is

$$C_{f,t}^{i} \equiv \left(\int_0^1 C_{f,t}^{i}(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}} \tag{3.9}$$

Region i's public consumption index is given by

$$G_{t}^{i} \equiv \left(\int_0^1 G_{t}^{i}(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}} \tag{3.10}$$

Notice that in the specification of preference described above $\alpha \in [0,1]$ is the weight of imported goods in the utility of private consumption. Parameter $\epsilon > 1$ denotes the elasticity of substitution between varieties produced within any given region, independently of the producing region. The optimal allocation of consumption in any given region is

$$C_{i,t}^{i}(j) = \left(\frac{P_t^i(j)}{P_t^i}\right)^{-\epsilon} C_{i,t}^{i}; \qquad C_{f,t}^{i}(j) = \left(\frac{P_t^f(j)}{P_t^f}\right)^{-\epsilon} C_{f,t}^{i} \tag{3.11}$$

The price index for locally produced goods for all $i \in [0,1]$ is given by $P_t^i \equiv \left(\int_0^1 P_t^i(j)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}$, and the price index for bundles of imported goods from region f is $P_t^f \equiv \left(\int_0^1 P_t^f(j)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}$. Therefore, $\int_0^1 P_t^i(j) C_{i,t}^{i}(j) dj = P_t^i C_{i,t}^{i}$ and

$\int_0^1 \int_0^1 P_t^f(j) C_{f,t}^i(j) dj\, df = P_t^f C_{f,t}^i$. Moreover, the optimal allocation of consumption on imported goods by region of origin is:

$$P_t^f C_{f,t}^i = P_t^* C_{F,t}^i \text{ for all } f \in [0,1] \tag{3.12}$$

where $P_t^* = exp \int_0^1 p_t^f df$ is the inter-regional trade price. Lastly, by defining the Consumer Price Index (CPI) for region i as $P_{c,t}^i = \left(P_t^i\right)^{1-\alpha} (P_t^*)^\alpha$, the optimal allocation of consumption between local and imported goods in that region is

$$P_t^i C_{i,t}^i = (1-\alpha) P_{c,t}^i C_t^i; \qquad P_t^* C_{F,t}^i = \alpha P_{c,t}^i C_t^i \tag{3.13}$$

By using all those equations above, the budget constraint in equation (3.5) can be simplified as follows:

$$P_{c,t}^i C_t^i + E_t\left\{Q_{t,t+1} B_{t+1}^i\right\} + M_t^i \leq B_t^i + M_{t-1}^i + W_t^i N_t^i + T_t^i \tag{3.14}$$

The optimality conditions for the maximization problem (3.4) subject to the budget constraint (3.14) are given by

$$\beta E_t \left\{ Q_{t,t+1}^{-1} \frac{P_{c,t}^i}{P_{c,t+1}^i} \frac{{C_{t+1}^i}^{-1}}{{C_t^i}^{-1}} \right\} = 1 \tag{3.15}$$

$$\frac{W_t^i}{P_{c,t}^i} = \frac{{N_t^i}^\varphi}{(1-\chi){C_t^i}^{-1}} \tag{3.16}$$

$$\frac{\left(\frac{M_t^i}{P_{c,t}^i}\right)^{-\nu}}{(1-\chi){C_t^i}^{-1}} = \frac{i_t}{1+i_t} \tag{3.17}$$

The first equation is a conventional Euler equation, where $E_t\left\{Q_{t,t+1}^{-1}\right\} = R_t$, which is the (gross) nominal interest rate. The second and the last equation describe the relation between consumption and labor, consumption and real money balance, respectively.

Those three equations can be written in log-linear form as

$$\hat{c}_t^i = E_t \hat{c}_{t+1}^i - \left(\hat{\imath}_t - \mathrm{E_t} \pi_{c,t+1}^i\right) \tag{3.18}$$

$$\widehat{w}_t^i - \hat{p}_{c,t}^i = \varphi \hat{n}_t^i + \hat{c}_t^i \tag{3.19}$$

$$\widehat{m}_t^i = \frac{1}{\nu} \hat{c}_t^i - \frac{1}{(1+\bar{\imath})\nu} \hat{\imath}_t \tag{3.20}$$

The detailed derivation and linearization can be seen in Appendix.

3.5.2 Terms of Trade

The bilateral term of trade between regions *i* and *f* is defined as

$$S_{f,t}^i \equiv \frac{P_t^f}{P_t^i} \tag{3.21}$$

P_t^f is the price of good produced in region *f*. The effective terms of trade for regions *i* are thus given by

$$S_t^i = \frac{P_t^*}{P_t^i} = exp \int_0^1 \left(\log P_t^f - \log P_t^i\right) df = exp \int_0^1 \log S_{f,t}^i \, df \tag{3.22}$$

From this equation, we know that

$$\Delta s_t^i = \pi_t^* - \pi_t^i \tag{3.23}$$

Notice that the CPI and the domestic price levels are related according to

$$P_{c,t}^i = P_t^i \left(S_t^i\right)^\alpha \tag{3.24}$$

Hence, the CPI inflation are linked according to

$$\pi_{c,t}^i = \pi_t^i + \alpha \Delta s_t^i \tag{3.25}$$

where α represents the degree of openness. Using the definition of the effective terms of trade above, alternatively, CPI inflation can be calculated as the weighted average between locally produced goods' inflation and the rest of regions' inflation, as follows

$$\pi_{c,t}^i = (1-\alpha)\pi_t^i + \alpha \pi_t^* \tag{3.26}$$

3.5.3 National Risk Sharing

By assuming complete market for state-contingent securities across the region, the Euler equation analogous to equation (3.15) for representative households in any other region, say region *f*

$$1 = \beta E_t \left\{ Q_{t,t+1}^{-1} \frac{P_{c,t}^{f}}{P_{c,t+1}^{f}} \frac{{C_{t+1}^{f}}^{-1}}{{C_t^{f}}^{-1}} \right\} \tag{3.27}$$

By combining both Euler equation (3.15) and (3.27), we can derive

$$C_t^i = \vartheta_f C_t^f \left(S_{f,t}^i\right)^{1-\alpha}$$

where ϑ_f is some constant that will generally depend on initial conditions regarding relative net asset positions. Without loss of generality, let assume symmetric initial conditions, i.e. zero net foreign assets holdings and ex-ante identical environment. This implies $\vartheta_f = \vartheta = 1 \; \forall f$. Taking logs and integrating over *f* yields

$$\hat{c}_t^i = \hat{c}_t^* + (1-\alpha)\hat{s}_t^i \tag{3.28}$$

where $\hat{c}_t^* \equiv \int_0^1 \hat{c}_t^f df$ is the rest of region consumption, and $\hat{s}_t^i \equiv \int_0^1 \hat{s}_{f,t}^i df$

3.5.4 Firm

Each region has a continuum of firms represented by the interval [0,1]. For simplicity, each firm produces a differentiated good with a linear technology:

$$Y_t^i(j) = A_t^i N_t^i(j) \tag{3.29}$$

for all $i, j \in [0,1]$, where A_t^i is a region-specific productivity shifter. Let define the aggregate output, $Y_t^i \equiv \left(\int_0^1 Y_t^i(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}$. Thus, the aggregate labor hired is given by

$$N_t^i = \int_0^1 N_t^i(j) dj = \frac{Y_t^i Z_t^i}{A_t^i} \tag{3.30}$$

where $Z_t^i = \int_0^1 \frac{Y_t^i(j)}{Y_t^i} dj$. The equilibrium variations of $z_t^i = \log Z_t^i$ around the steady state as shown in Appendix are of second order approximation. Therefore, the first order approximation of the aggregate output for $i \in [0,1]$ is

$$\hat{y}_t^i = \hat{a}_t^i + \hat{n}_t^i \tag{3.31}$$

The region-specific productivity is assumed to follow AR(1) process (in logs):

$$\hat{a}_t^i = \rho_a \hat{a}_{t-1}^i + \varepsilon_t^{a^i} \tag{3.32}$$

where $a_t^i = \log A_t^i$, $\rho_a \in [0,1]$ and $\varepsilon_t^{a^i}$ is white noise.

3.5.5 Price setting

The firms are assumed to set prices in a staggered price fashion, as in Calvo (1983). Hence, some firms choose not to reoptimize their price, and some others reoptimize their price. The former uses lagged inflation for their next period price and the latter sets an entirely new price $\tilde{P}_t^i$ for next period. $Cost_{t+k}^i(j)\{.\}$ is the cost function as a function of output $Y_t^i(j)$. The objective function of the firms in setting up the price is

$$V_t^i = \mathrm{E}_t \sum_{k=0}^{\infty} \left(\beta\theta^i\right)^k \left[\frac{P_{t+k}^i(j)}{P_{t+k}^i} Y_{t+k}^i(j) - Cost_{t+k}^i(j)\left\{Y_{t+k}^i(j)\right\}\right] \tag{3.33}$$

and the law of motion of the aggregate price is given by

$$P_t = \left[\theta\left(P_{t-1}^i \pi_{t-1}^i\right)^{1-\varepsilon} + (1-\theta)\tilde{P}_t^{i\,1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{3.34}$$

A measure $1-\theta$ of (randomy selected) firms sets new prices each period, with an individual firm's probability of re-optimizing in any given period being independent of time elapsed since it last reset its price. So, on average firms reoptimize their price every $1/(1-\theta)$ periods in a year. As is well known, the inflation equation can be obtained as follows:

$$\hat{\pi}_t^i = \frac{1}{(1+\beta)}\hat{\pi}_{t-1}^i + \frac{\beta}{(1+\beta)}\hat{\pi}_{t+1}^i + \frac{(1-\beta\theta^i)(1-\theta^i)}{\theta^i(1+\beta)}\widehat{mc}_t^i \tag{3.35}$$

where $\widehat{mc}_t^i$ is the real marginal cost.

The real marginal cost in the linear technology production function is given by

$$mc_t^i = w_t^i - p_t^i - a_t^i = (w_t^i - p_{c,t}^i) + (p_{c,t}^i - p_t^i) - a_t^i \tag{3.36}$$

Combining equation (3.19), (3.24), (3.31) and (3.39) we can rewrite equation (3.36) as

$$\widehat{mc}_t^i = \left(\frac{1}{(1-\gamma)} + \varphi\right)\hat{y}_t^i - \frac{\gamma}{(1-\gamma)}\hat{g}_t^i - (1+\varphi)\hat{a}_t^i$$

Thus, the final inflation equation is

$$\begin{aligned}\hat{\pi}_t^i = {} & \frac{1}{(1+\beta)}\hat{\pi}_{t-1}^i + \frac{\beta}{(1+\beta)}\hat{\pi}_{t+1}^i \\ & + \frac{(1-\beta\theta^i)(1-\theta^i)}{\theta^i(1+\beta)}\left\{\left(\frac{1}{(1-\gamma)} + \varphi\right)\hat{y}_t^i - \frac{\gamma}{(1-\gamma)}\hat{g}_t^i - (1+\varphi)\hat{a}_t^i\right\}\end{aligned} \tag{3.37}$$

3.5.6 Market Clearing

The market clearing condition for good *j* produced in region *i* is

$$Y_t^i(j) = C_{i,t}^i(j) + \int_0^1 C_{i,t}^f(j)df + G_t^i(j) \tag{3.38}$$

This equation shows that output of region *i*, $Y_t^i(j)$, should equal the sum of consumption goods of region *i* produced locally, $C_{i,t}^i(j)$, total export goods for other regions produced in region *i*, $\int_0^1 C_{i,t}^f(j)df$, and public goods of region *i*.

Using the definition of region i's aggregate output, $Y_t^i = \left(\int_0^1 Y_t^i(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}$ and equation (3.38), the market clearing condition can be written as

$$Y_t^i = C_t^i \left(S_t^i\right)^{\alpha} + G_t^i$$

Log-linearize around the steady state yields:

$$\hat{y}_t^i = (1-\gamma)\left(\hat{c}_t^i + \alpha \hat{s}_t^i\right) + \gamma \hat{g}_t^i \tag{3.39}$$

By aggregating over all regions and solve for $\hat{c}_t^*$ yields

$$\hat{c}_t^* = \frac{1}{(1-\gamma)} \hat{y}_t^* - \frac{\gamma}{(1-\gamma)} \hat{g}_t^* \tag{3.40}$$

Combining equation (3.18), (3.28) and (3.40) produces the region i's consumption

$$\begin{aligned}\hat{c}_t^i = \frac{1}{(1-\gamma)} E_t \hat{y}_{t+1}^* - \frac{\gamma}{(1-\gamma)} E_t \hat{g}_{t+1}^* + (1-\alpha) E_t \hat{s}_{t+1}^i \\ - \left(\hat{\imath}_t - \mathrm{E_t} \pi_{\mathrm{c,t+1}}^i\right)\end{aligned} \tag{3.41}$$

Using the log of (3.24) and integrating it, equation (3.39) can be written as

$$\hat{y}_t^i = (1-\gamma)\hat{c}_t^i + \gamma \hat{g}_t^i - (1-\gamma)\left(p_t^i - p_t^*\right) \tag{3.42}$$

Integrating it yields

$$\hat{y}_t^* = (1-\gamma)\hat{c}_t^* + \gamma \hat{g}_t^* \tag{3.43}$$

where $\hat{y}_t^* \equiv \int_0^1 \hat{y}_t^i \, di$, and $\hat{g}_t^* \equiv \int_0^1 \hat{g}_t^i \, di$. Finally, integrating equation (3.18) for all regions and combining with (3.43) yields the aggregate dynamic IS equation:

$$\hat{y}_t^* = E_t \hat{y}_{t+1}^* - (1-\gamma)(\hat{\imath}_t - \mathrm{E_t} \pi_{\mathrm{t+1}}^*) - \gamma E_t \Delta \hat{g}_{t+1}^* \tag{3.44}$$

where $\pi_t^* \equiv \int_0^1 \pi_t^i \, di$.

3.5.7 Monetary Policy

To improve social welfare, the central bank faces a trade-off between maintaining price stability and output stability relative to its potential. Based on Philips

Curve, the central bank has a preference to decrease the inflation rate, so the real output drops as well. This condition creates this trade-off as mentioned before.

Since July 2005, Bank Indonesia has been implementing Inflation Targeting Framework (ITF) and the monetary policy transmission through the interest rate channel and expectation channel is believed to be strong in Indonesia. In this case, we will implement Taylor rule interest rate reaction function as a mechanism to set the interest rate in the short-run. Taylor rule is a monetary-policy response from the central bank that considers both inflation target achievement and minimizing social cost. It is assumed that Bank Indonesia as a monetary authority has a higher preference in lowering inflation rate than output stability relative to potential output.

According to this framework, the central bank objective function is to set the short-run nominal interest rate, $\hat{\imath}_t$, such that it minimizes the deviation between inflation and its target, which is inflation gap, and also reduces the difference between output and its potential, which is output gap. The formula of Taylor rule is as follows:

$$\hat{\imath}_t = \rho_i \hat{\imath}_{t-1} + (1-\rho_i)\big(\alpha_\pi (E_t \hat{\pi}_{t+1}) + \alpha_y \hat{y}_t\big) + \varepsilon_t^{\hat{\imath}} \tag{3.45}$$

where $\hat{\imath}_t$ is short-run nominal interest rate gap that represents the gap between policy rate and its long-run trend, $E_t \hat{\pi}_{t+1}$ is the expected CPI inflation gap one period ahead that indicates the gap between inflation and its long-run trend, $\hat{y}_t$ is the output gap (the gap between output and its potential) and $\varepsilon_t^{\hat{\imath}}$ is the error term (exogenous shock). Parameter ρ_i represents the interest rate smoothing with previous period of the interest rate. Moreover, $\alpha_\pi > 1$ and α_y are the preference parameter for inflation gap and output gap, respectively.

3.5.8 Fiscal Policy

Government as a fiscal authority has an objective to maximize the aggregate welfare of households, which is reflected by the aggregate utility function of the households. In Appendix, the aggregate welfare loss function is derived using the second order approximation to the average utility losses of the aggregated households, expressed as fluctuations about the efficient steady state can be written as:

$$\mathbb{W}_t = -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t \int_0^1 \left(\frac{\varepsilon}{\lambda}\left(\pi_t^i\right)^2 + (1+\varphi)\hat{y}_t^{i\,2} + \frac{\chi}{1-\chi}\left(\hat{g}_t^i - \hat{y}_t^i\right)^2\right) di \qquad (3.46)$$

where the last term indicates the fiscal gap. Thus, in another word, the sum of welfare losses is trying to minimize the fluctuation of inflation, output gap, and fiscal gap.

Under the assumption of no commitment or discretion case, the optimal fiscal policy is derived by minimizing the period loss function

$$\int_0^1 \left[\frac{\varepsilon}{\lambda}\left(\pi_t^i\right)^2 + (1+\varphi)\hat{y}_t^{i\,2} + \frac{\chi}{1-\chi}\left(\hat{g}_t^i - \hat{y}_t^i\right)^2\right] di \qquad (3.47)$$

subject to the set of constraints

$$\pi_t^i = \frac{\lambda}{(1+\beta)}\left(\frac{1}{1-\gamma} + \varphi\right)\hat{y}_t^i - \frac{\lambda\gamma}{(1+\beta)(1-\gamma)}\hat{g}_t^i + u_t^i \qquad (3.48)$$

$$\hat{y}_t^i = \gamma\hat{g}_t^i + v_t^i \qquad (3.49)$$

where $u_t^i \equiv \frac{1}{(1+\beta)}\hat{\pi}_{t-1}^i + \frac{\beta}{(1+\beta)}E_t\hat{\pi}_{t+1}^i$ and $v_t^i = E_t\hat{y}_{t+1}^i - (1-\gamma)\left(\hat{\imath}_t - \pi_{t+1}^i\right) - \gamma\hat{g}_{t+1}^i$ are taken as given by the policymaker because of the discretion case. After rearranging the FOCs, the optimal fiscal policy takes the form:

$$\hat{g}_t^i = -\varphi\hat{y}_t^i - \frac{\varepsilon\varphi}{(1+\beta)}\pi_t^i \qquad (3.50)$$

All detailed step for deriving the optimal fiscal policy can be seen in Appendix.

3.6 Data

The observed data that are used in the estimation process are national and regional CPI inflation, national and regional real GDP, national and regional real government expenditure, and interest rate. These data are taken from CEIC, and it is quarterly data ranging from the first quarter of 2002 until the second quarter of 2016.

Since the variables in the model are in deviation from the steady state, the data used in the model is in the gap from its trend. The gap data are obtained by using HP filter procedure. Before using this filter, the data are seasonally adjusted using Census X12 because the raw data is still seasonal.

3.7 Calibration and Priors

Some parameters in the model are calibrated, and some others are estimated. As discussed in the previous section, the estimation process needs priors combined with the actual data. Both priors and calibrated parameters are taken from the inter-regional social accounting matrix (IRSAM), from the empirical results of others research in Indonesia and also research from other countries that have similar economic structure, or even from the expertise judgments.

Table 3.1 Priors and Calibrated Parameters

No	Para-meters	Description	Type	Value	Implication	Source
1	β	Discount Factor	Calibrated	0.99	Annual return about 4% in steady state	Gali Monacelli (2008)
2	ρ_a	Autoregressive process for labor productivity	Calibrated	0.95	Persistent productivity shock	King Rebelo (1999)

No	Para-meters	Description	Type	Value	Implication	Source
3	ρ_a^*	Autoregressive process for labor productivity	Calibrated	0.95	Persistent productivity shock	King Rebelo (1999)
4	α_1	Degree of openness of region 1	Prior	*	-	IRSAM Table
5	θ_1	The probability of firms that are not setting a new price in region 1	Prior	0.75	Price adjustment every quarter	Gali Monacelli (2008)
6	φ_1	The inverse of the elasticity of the effort of work with respect to the real wage in region 1	Prior	3	A labor supply elasticity of 1/3	Gali Monacelli (2008)
7	ϵ_1	The elasticity of substitution between different goods in region 1	Prior	6	A steady state markup $\mu =$ 1.2	Gali Monacelli (2008)
8	ν_1	The inverse of the elasticity of money with respect to the interest rate in region 1	Prior	1	Consumption elasticity in money demand is 1	Gali (2008)
9	θ^*	The probability of firms that are not setting a new price for the rest of the region	Prior	0.75	Price adjustment every quarter	Gali Monacelli (2008)
10	φ^*	The inverse of the elasticity of the effort of work with respect to the real wage in the rest of the region	Prior	3	A labor supply elasticity of 1/3	Gali Monacelli (2008)
11	ϵ^*	The elasticity of substitution between different goods in the rest of the region	Prior	6	A steady state markup of 1.2	Gali Monacelli (2008)

No	Para-meters	Description	Type	Value	Implication	Source
12	α_{π}	Inflation gap parameter in Taylor rule	Prior	1.5	Must be greater than 1 for unique equilibrium	Gali Monacelli (2008)
13	α_{y}	Output gap parameter in Taylor rule	Prior	0.1	-	Tjahyono et.al (2006)
14	ρ_{i}	Smoothing parameter in Taylor rule	Prior	0.5	-	Tjahyono et.al (2006)

3.8 Simulation and Results

Table 3.2 The Estimation Results of The National Model

Parameters	Prior mean	Prior Distribution	Prior Stdev	Indonesia (National)
				Post. mean
θ	0.75	beta	0.1	0.4378
φ	3	gamma	0.1	2.9392
ϵ	6	gamma	0.1	5.9318
α_{π}	1.5	normal	0.1	1.5701
α_{y}	0.1	normal	0.1	0.2704
ρ_{r}	0.5	beta	0.1	0.2149
Stdev of shocks	**Prior mean**	**Prior Distribution**	**Prior Stdev**	**Post. mean**
σ_{y}	0.001	inv.gamma	Inf	0.0133
σ_{π}	0.001	inv.gamma	Inf	0.018
σ_{g}	0.001	inv.gamma	Inf	0.2506
σ_{r}	0.01	inv.gamma	Inf	0.0078

Table 3.3 Estimation Results of The Regional Model

Para-meters	Prior mean	Prior Distri-bution	Prior Stdev	Suma-tra	Jawa and Bali	Kali-mantan	Sula-wesi	Eastern Indone-sia
				Post. mean	**Post. mean**	**Post. mean**	**Post. mean**	**Post. mean**
α_1	*	beta	0.1	0.1738	0.1589	0.1782	0.2653	0.2849
θ_1	0.75	beta	0.1	0.3207	0.3916	0.3911	0.2266	0.6627
φ_1	3	gamma	0.1	2.8899	2.8837	2.8589	2.9097	2.8723
ϵ_1	6	gamma	0.1	5.9215	5.9233	5.9158	5.9299	5.9182
ν_1	1	gamma	0.1	1.0024	0.9935	1.0028	0.9993	0.9976
θ_2	0.75	beta	0.1	0.3501	0.3037	0.4341	0.4239	0.4476
φ_2	3	gamma	0.1	2.8905	2.8926	2.8883	2.8917	2.891
ϵ_2	6	gamma	0.1	5.9215	5.9268	5.9237	5.9212	5.923
α_π	1.5	normal	0.1	1.5619	1.5068	1.5213	1.5377	1.5231
α_y	0.1	normal	0.1	0.3636	0.3676	0.329	0.3119	0.3072
ρ_r	0.5	beta	0.1	0.0937	0.0822	0.1679	0.1516	0.2011
Stdev of shocks	**Prior mean**	**Prior Distribution**	**Prior Stdev**	**Post. mean**	**Post. mean**	**Post. mean**	**Post. mean**	**Post. mean**
σ_{y_1}	0.001	inv. gamma	Inf	0.0277	0.0104	0.0287	0.0297	0.0397
σ_{π_1}	0.001	inv. gamma	Inf	0.0288	0.0189	0.0209	0.0292	0.0208
σ_{g_1}	0.001	inv. gamma	Inf	0.2652	0.2342	0.2113	0.258	0.2523
σ_{y_2}	0.001	inv. gamma	Inf	0.0088	0.0114	0.0118	0.0113	0.0122
σ_{π_2}	0.001	inv. gamma	Inf	0.0193	0.022	0.0175	0.0176	0.0173
σ_{g_2}	0.001	inv. gamma	Inf	0.2316	0.231	0.2287	0.2305	0.2318
σ_r	0.01	inv. gamma	Inf	0.0086	0.0086	0.0082	0.0083	0.0083

* : Sumatra = 0.17, Java and Bali = 0.16, Kalimantan = 0.18, Sulawesi = 0.27, Eastern Indonesia = 0.29

Two models are used for simulation namely, national and regional DSGE model. The first model consists of equation (3.37), (3.44), (3.50), (3.32) and (3.45). The second model has three parts. The first part is the region i's system that consists equation of (3.41), (3.20), (3.39), (3.23), (3.26), (3.37), (3.50) and (3.32). The second part is the rest of the region or the region outside the region i's system that comprises of equation (3.37), (3.44), (3.50), and (3.32). The last part is the national level equations which are (3.45), the aggregate inflation and output gap equations which are the weighted average of the regional inflation and output gap, respectively. These models will be estimated with Bayesian estimation using the actual dataset to get the current condition of the central bank's behavior based on Taylor rule. Then, the Taylor rule parameters will be optimized based on the central bank's objective.

The results of Bayesian estimation and simulation of posterior distribution using Metropolis-Hasting yield the estimated parameters and standard deviation of shocks shown in Table 3.2 and Table 3.3 above.

3.8.1 Optimal Policy

The optimal policy rule is obtained by simulating the model given the estimated parameters and estimated standard error of the regional shocks. Then, calculated using the linear-quadratic minimization problem as follows:

$$\min_{\alpha} E(y_t' W y_t)$$

$$\text{st. } P_1 E_t y_{t+1} + P_2 y_t + P_3 y_{t-1} + C e_t = 0$$

where α denotes the set of parameters to be optimized, in this case the inflation gap and output gap parameters in Taylor Rule equation, or α_π and α_y respectively, y are the endogenous variables in the model, e are the exogenous stochastic shocks, P are the estimated parameter set, C are the constant parameters of the exogenous shocks, and W are the weighted matrix of the objective function.

In this study, the central bank's objective is to minimize a weighted sum of unconditional second moments of the inflation gap and output gap, and it is assumed that both inflation gap and output gap have equal weight. Thus, the objective function above can be simplified into a loss function:

$$LossFunction = \sigma_{\hat{\pi}}^2 + \sigma_{\hat{y}}^2$$

where $\sigma_{\hat{\pi}}^2$ and $\sigma_{\hat{y}}^2$ are the variance of inflation gap and output gap respectively.

Table 3.4 below shows the optimal policy parameters across regions and Indonesia country compared to the estimated parameters. Parameters in Taylor rule equation, α_π and α_y indicate how aggressive central bank should react under region shocks on the inflation gap and output gap, respectively. The higher the value, the more aggressive the central bank should react, and vice versa. The aggressiveness of the central bank can be shown by the higher or lower of the interest rate change compared to the common situation. The estimated parameters indicate the current behavior of the central bank to reduce the inflation and output gap. On the other hand, optimal parameters indicate the optimal behavior of the central bank which minimized the loss function of the central bank given the current economic structure and condition.

From Table 3.4 below, the comparison between estimated and optimal parameters both in the national and regional model indicates that currently, the central bank is not reacting optimally, but reacting less aggressively than it is supposed to. In any model, the inflation gap parameters are higher compared to the output gap parameters. This shows how important the central bank should manage the inflation gap across the regions.

Table 3.4 Optimal Policy Parameters

Region	**Parameter Type**	**Parameters**		**Loss Function Value**	**Improvement**
		α_π	α_y		
Sumatra	Estimated Parameter	1.5619	0.3636	1.503E-04	49%
	Optimal Parameter	1.9741	0.0306	1.007E-04	
Jawa and Bali	Estimated Parameter	1.5068	0.3676	8.459E-04	86%
	Optimal Parameter	1.9931	0.0129	4.552E-04	
Kalimantan	Estimated Parameter	1.5213	0.3290	2.057E-05	46%
	Optimal Parameter	1.6810	0.0611	1.412E-05	
Sulawesi	Estimated Parameter	1.5377	0.3119	8.469E-06	18%
	Optimal Parameter	1.6483	0.1276	7.167E-06	
Eastern Indonesia	Estimated Parameter	1.5231	0.3072	6.182E-06	24%
	Optimal Parameter	1.6192	0.1372	4.999E-06	
Indonesia (National)	Estimated Parameter	1.5701	0.2704	1.418E-03	0.04%
	Optimal Parameter	1.5702	0.2703	1.418E-03	

Before further analysis, it is better to ensure that the Impulse Response Function (IRF) of the model behaves correctly as in macroeconomics theory. In this simulation, four types of shock are going to be imposed on the Eastern Indonesia region, for example. From Figure 3.1 below, 1% increase in output gap ($ygap_1$) in Eastern

Indonesia region would also increase the local inflation (inf_1) about 0.5% which in turn the central bank should increase the interest rate (r) by 2 basis points (bps) and the local government should reduce its expenditure (g_1) by 7%. This interest rate rise can be translated into about 0.4% reduction in real money balances. The shock from Eastern Indonesia region will only slightly affect the rest of the region as shown in the figures above because the domestic export in this region is very small.

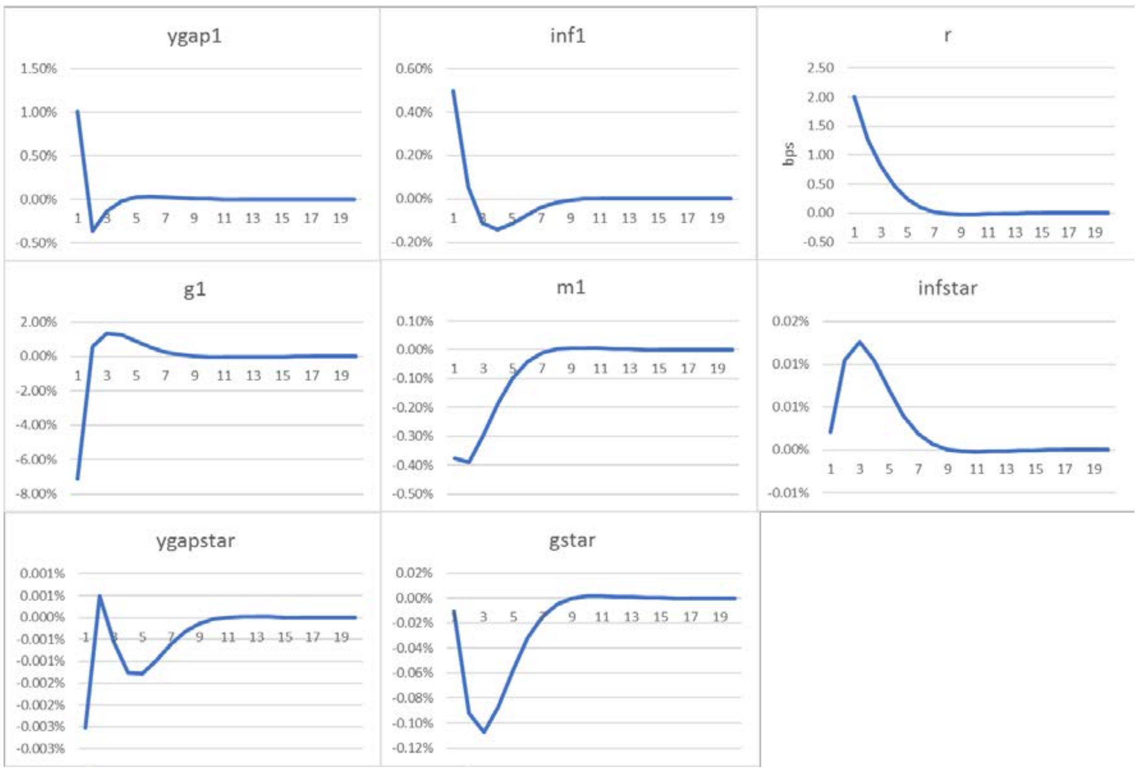

Figure 3.1 The IRF of 1% increase in Output Gap in Eastern Indonesia Region

In the case of 1% increase in inflation in the Eastern Indonesia region, as shown in Figure 3.2 below, the output gap in this region will decrease by 1.5%. In response to this condition, the model suggested the central bank should increase the interest rate by 3.16 bps or equal to -0.7% reduction of real money balance. Also, the government should lower its spending by about 4%. The rest of the region inflation (*infstar*) will slightly increase by 0.02%, but its output gap (ygapstar), and government expenditure (gstar) will slightly drop.

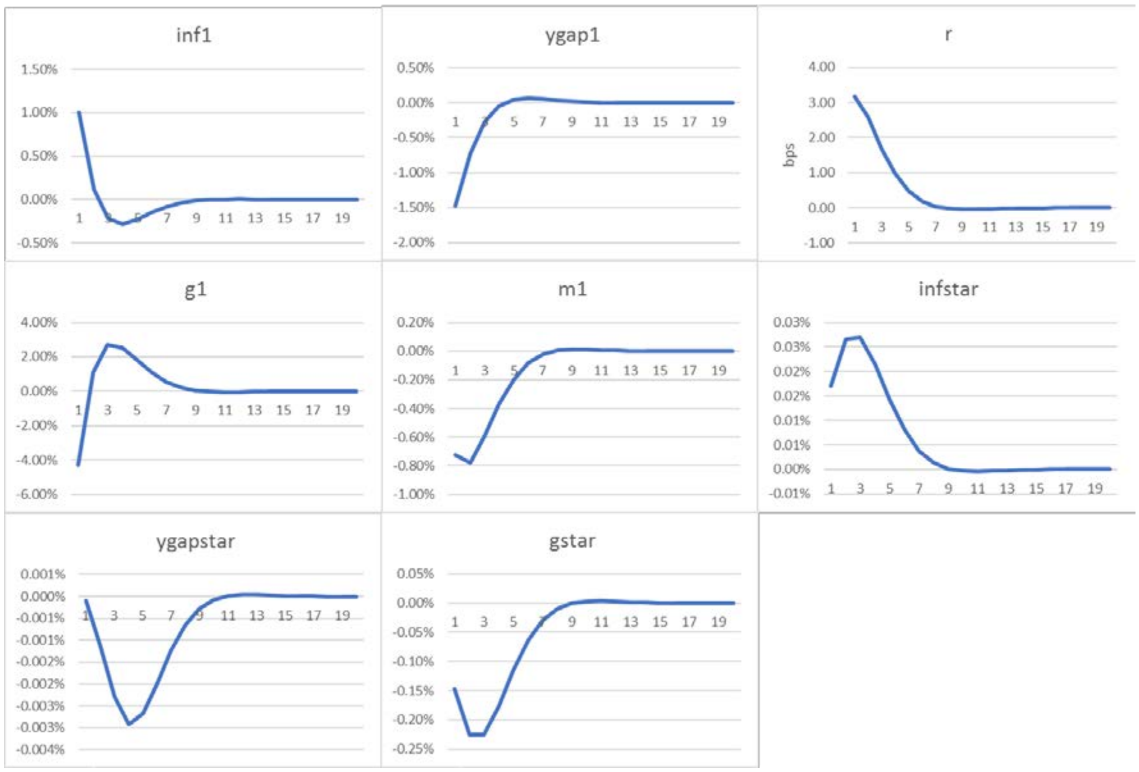

Figure 3.2 The IRF of 1% increase in Inflation in Eastern Indonesia Region

The monetary policy transmission in this model can be shown from the interest rate shock in Figure 3.3 below. The 100 bps rise in interest rate is translated into 3.28% decrease in real money balances. It will reduce the inflation in Eastern Indonesia by 0.7% and also its output gap by 0.8%. In addition, the inflation and output gap in the rest of the region also decline by 1.5% and 0.6%, respectively. As a response, the government in Eastern Indonesia and in the rest of the region should raise its spending by 9% and 15%, respectively.

Lastly, in Figure 3.4 below, suppose there is 1% increase in government expenditure in Eastern Indonesia. It will directly raise the output gap in this region by 0.12% and also inflation by 0.03%. In this scenario, the central bank should slightly increase the interest rate by 0.15 bps or reduce the real money balances by 0.03%. The effects to the rest of the region are tiny, so we can ignore them.

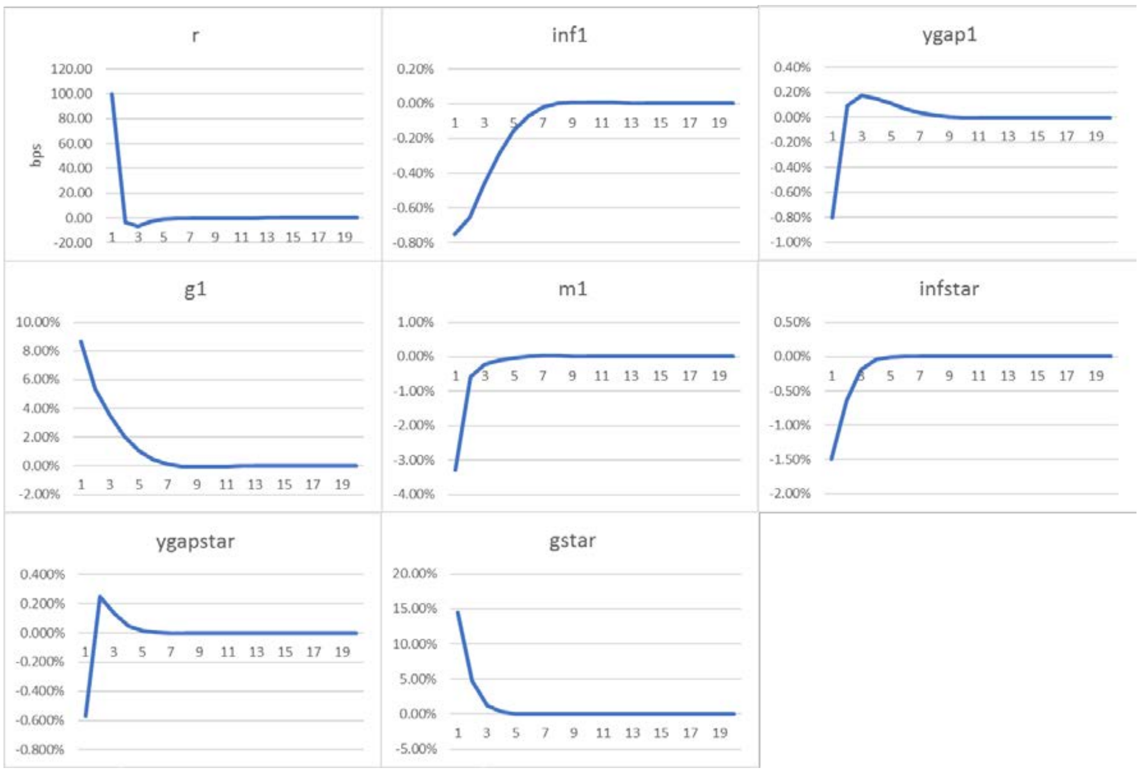

Figure 3.3 The IRF of 100 bps increase in Interest Rate

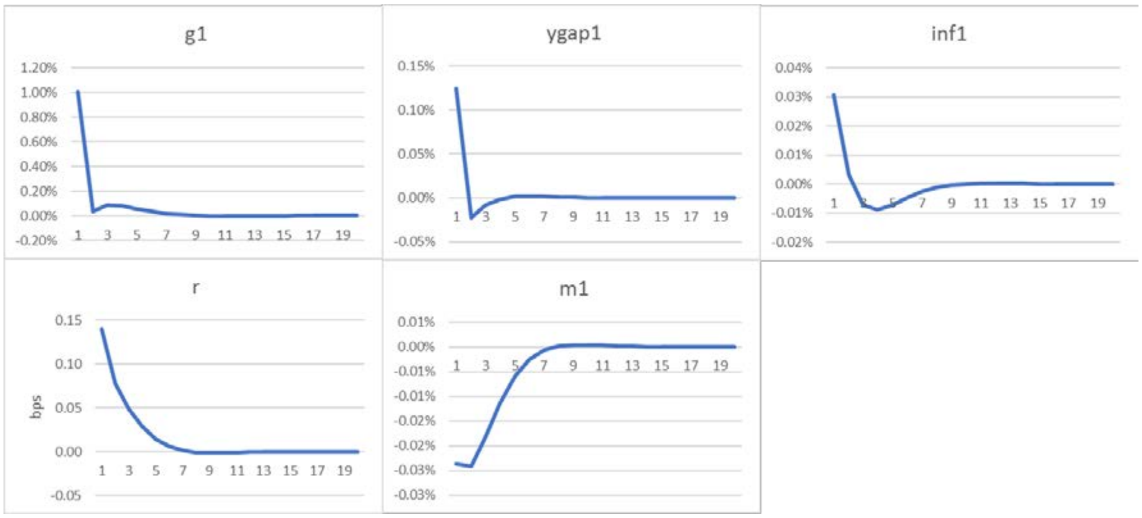

Figure 3.4 The IRF of 1% Increase in Government Expenditure in Eastern Indonesia Region

3.8.2 Does Regional Disaggregation Matter?

Comparing the national model and regional model can address this question. Since those models are different, the comparison cannot be made by just looking at the value of the loss function. One alternative way to make the value of loss function comparable is by plugging the Taylor rule parameters from the national model into a regional model.

Table 3.5 Loss Function Comparison from Different Parameter Types Using Eastern Indonesia Model

Parameter Type	**Parameters**		**Loss Function Value**	**% Diff**
	α_π	α_y		
Optimized Parameter from Eastern Indonesia Model	1.619	0.137	4.999E-06	14%
Optimized Parameter from National Model	1.570	0.270	5.705E-06	

Suppose there is a negative shock from Eastern Indonesia, and we want to see the performance of regional model as compared to the national model. In this case, the regional model that is used for the analysis is Eastern Indonesia model. Table 3.5 above shows the comparison of the loss function value from two different parameter types; first, the value of loss function using the optimized parameter from a regional model which indicates the optimal behavior of central bank given their objectives if they use the regional model; secondly, the loss function value using the optimal parameter from a national model which describe the optimal behavior of central bank if they utilize the national model. As can be seen, the regional model outperformed the national model when using optimized parameter by 14%. If the central bank is still using the national model for doing simulation when there is a regional shock, the results may no longer be optimal. Even though the difference of the loss function value is only 14%, both models might have different results, especially, policy recommendation. Thus, it is clear to say that regional disaggregation is important.

Besides the comparison of the loss function value, the importance of regional disaggregation can be shown by conducting the simulation experiments between national and regional model. There are two types of regional shocks in this experiment; first is negative supply shock which reflects the increase in inflation variables; second is demand shock which illustrates the increase in the output gap.

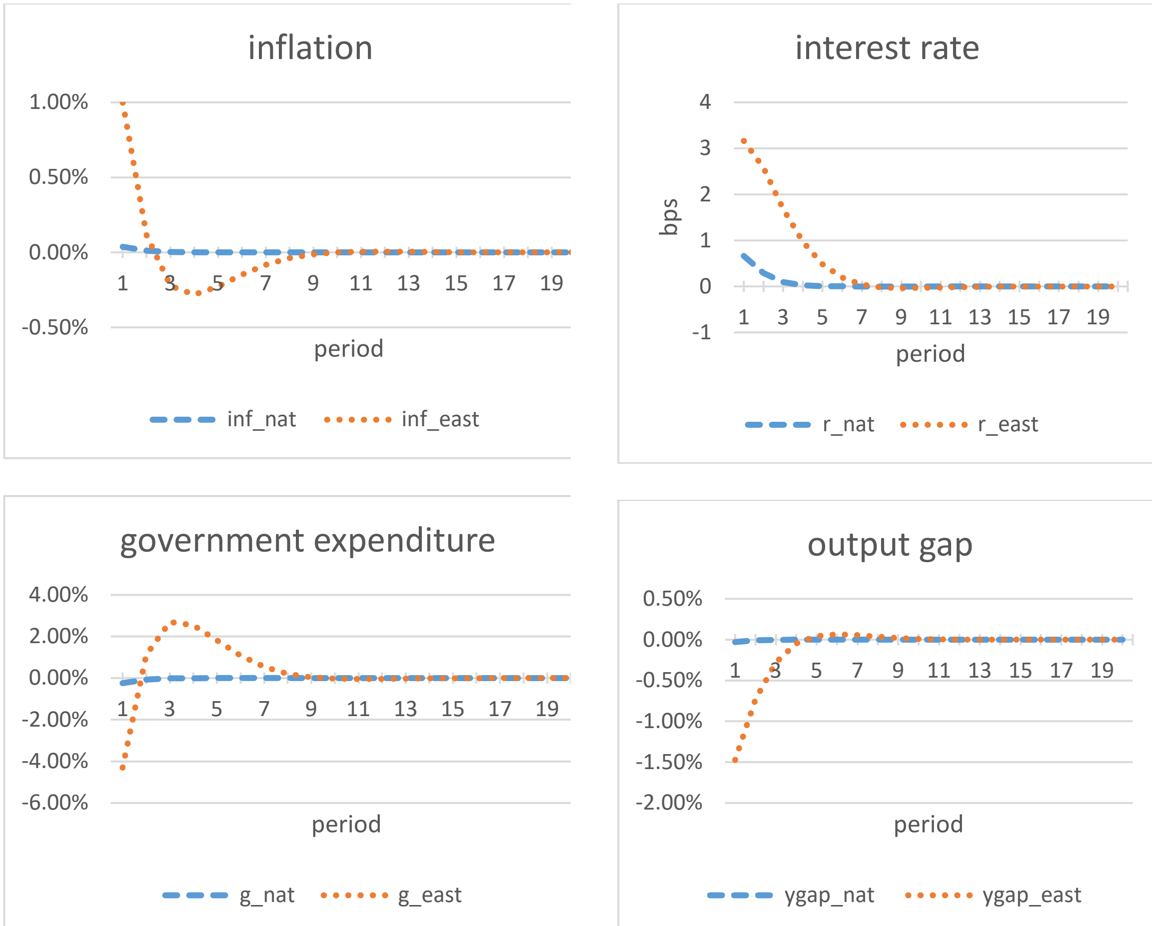

Figure 3.5 The IRF of 1% increase in inflation in Eastern Indonesia

In negative supply shock scenario, there is a sudden increase in inflation in Eastern Indonesia by 1%. In the national model, this shock is translated into 0.037% increase in national inflation because the weight of Eastern Indonesia's inflation to the national inflation is 3.7%. The results of this simulation are illustrated in Figure 3.5 above. According to the regional model, the optimal policy for the central bank as a reaction to the 1% increase in inflation in Eastern Indonesia is increasing the interest rate by 3 bps. In addition, the fiscal authority should also decrease its spending by 4.3%. As consequences, the output gap in Eastern Indonesia drops by 1.47%. However, from the national model, the central bank should only increase the interest rate by 1 bps. It seems that both models have a slightly different policy recommendation, but if the shock is getting bigger, then the different between both models would be also big as well.

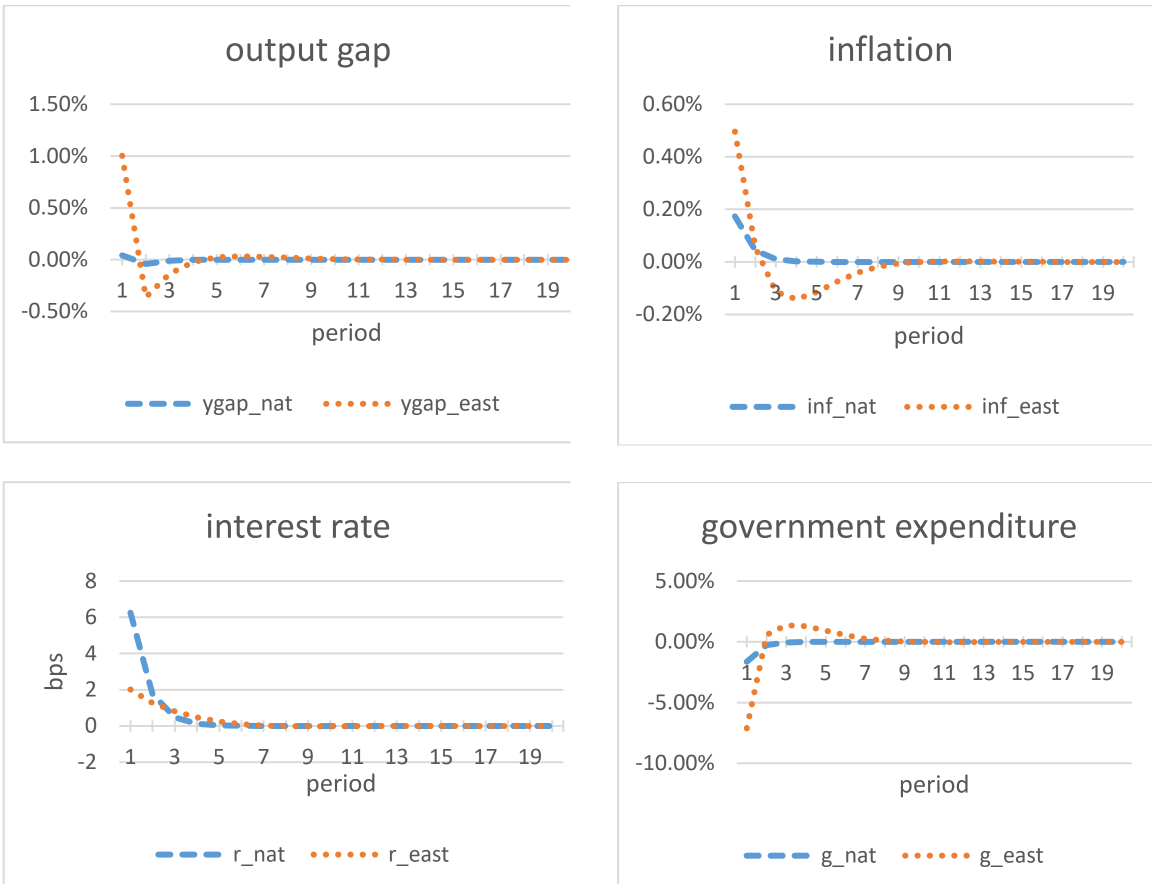

Figure 3.6 The IRF of 1% increase in output gap in Eastern Indonesia

In demand shock scenario, suppose there is a sudden jump in demand in Eastern Indonesia by 1%. This shock equals to 0.0415% increase in output gap in the national model. This number comes from the GDP weight of Eastern Indonesia in national GDP in 2015, which is 4.15%. As can be seen from the regional model in Figure 3.6 above, 1% increase in output gap in Eastern Indonesia would lead to the 0.5% increase in the

Eastern Indonesia's inflation. If the central bank wants to react optimally, it should increase its policy rate (interest rate) by 2 bps. However, the national model gives us different policy recommendation, 6 bps which is 3 times more aggressive than the interest rate in the regional model. This aggressive policy could hurt the local economy which would not be happened if the central bank uses the regional model.

From those two simulations, we can show that when the regional shocks hit the economy, the central bank should use a regional-DSGE model instead of the national-DSGE model because the results from the national-DSGE model could be misleading. In this case, the national model is less aggressive in fighting the negative supply shock in Eastern Indonesia, and more aggressive in dealing with demand shock in Eastern Indonesia if we compared to the regional model. In summary, the regional DSGE model will help the policy makers of the central bank to determine how aggressive they should react when there are specific regional shocks.

3.9 Conclusion

In this chapter, the two-region DSGE model has been successfully developed in order to estimate and simulate the optimal monetary and fiscal policy given the regional shocks. Moreover, it can show how important the regional DSGE model compared to the national DSGE model. This model can give the central bank of Indonesia, Bank Indonesia, a new experience of utilizing DSGE model with a regional dimension and a better understanding of how to react given the specific shocks in particular region.

The results showed that currently, the monetary policy reaction function in Taylor rule is not optimal. The central bank of Indonesia did not react optimally to the regional shocks given the objective function. In this case, the central bank was less aggressive in fighting the inflation gap. Fighting inflation gap should be the primary focus of the central bank since it will lead to the economic stability which is lower output gap.

In this research, we also demonstrated that disaggregating national level into the regional level model would provide us with better performance in terms of the loss function. This means that if the central bank is still using the national level model to simulate the regional shocks, the policy recommendation from the national model is not optimal and could hurt some agents and regions in the economy. In the simulation, the policy response from the national DSGE model seems to be more aggressive than the policy response from the regional model when there is a demand shock in the Eastern Indonesia. On the other hand, when there is a negative supply shock, the national model is less aggressive than the regional model in term of policy response. Therefore, the regional aspect of the model when the regional shocks hit the economy cannot be ignored.

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APPENDIX

Optimal Allocation of Consumption, Money Demand and Labor

The representative household seeking to maximize

$$\max_{C_t^i, N_t^i, \frac{M_t^i}{P_{c,t}^i}, B_{t+1}^i} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t (1-\chi) \log C_t^i + \chi \log G_t^i - \frac{{N_t^i}^{1+\varphi}}{1+\varphi} + \frac{\left(\frac{M_t^i}{P_{c,t}^i}\right)^{1-\nu}}{1-\nu} \right\}$$

subject to

$$P_{c,t}^i C_t^i + E_t\{Q_{t,t+1} B_{t+1}^i\} + M_t^i \leq B_t^i + M_{t-1}^i + W_t^i N_t^i + T_t^i$$

The Lagrange function is

$$\mathcal{L} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t (1-\chi) \log C_t^i + \chi \log G_t^i - \frac{{N_t^i}^{1+\varphi}}{1+\varphi} + \frac{\left(\frac{M_t^i}{P_{c,t}^i}\right)^{1-\nu}}{1-\nu} \right\}$$

$$- \lambda_t^i \left(P_{c,t}^i C_t^i + E_t\{Q_{t,t+1} B_{t+1}^i\} + M_t^i - \left(B_t^i + M_{t-1}^i + W_t^i N_t^i + T_t^i \right) \right)$$

The First Order Conditions (FOCs) are:

$$C_t^i : \beta^t (1-\chi) {C_t^i}^{-1} - \lambda_t^i P_{c,t}^i = 0 \tag{3.51}$$

$$N_t^i : -\beta^t {N_t^i}^{\varphi} + \lambda_t^i W_t^i = 0 \tag{3.52}$$

$$\frac{M_t^i}{P_{c,t}^i} : \beta^t \left(\frac{M_t^i}{P_{c,t}^i}\right)^{-\nu} - \lambda_t^i P_{c,t}^i + E_t \lambda_{t+1} P_{c,t}^i = 0 \tag{3.53}$$

$$B_{t+1}^i : -\lambda_t^i E_t\{Q_{t,t+1}\} + E_t\{\lambda_{t+1}^i\} = 0 \tag{3.54}$$

From (3.54) we have

$$\frac{E_t\{\lambda_{t+1}^i\}}{\lambda_t^i} = E_t\{Q_{t,t+1}\} = \frac{1}{1+i_t} \tag{3.55}$$

From (3.51) and (3.55) yields

$$\lambda_t^i = \frac{\beta^t (1-\chi) {C_t^i}^{-1}}{P_{c,t}^i}$$

$$\Rightarrow E_t\{Q_{t,t+1}\} = \frac{1}{1+i_t} = \frac{\dfrac{\beta^{t+1}(1-\chi) E_t {C_{t+1}^i}^{-1}}{E_t P_{c,t+1}^i}}{\dfrac{\beta^t (1-\chi) {C_t^i}^{-1}}{P_{c,t}^i}} = \beta E_t \left\{ \frac{P_{c,t}^i {C_{t+1}^i}^{-1}}{P_{c,t+1}^i {C_t^i}^{-1}} \right\}$$

$$\Rightarrow \ \beta(1+i_t) E_t \left\{ \frac{P_{c,t}^i}{P_{c,t+1}^i} \frac{{C_{t+1}^i}^{-1}}{{C_t^i}^{-1}} \right\} = 1$$

$$\Rightarrow \ \beta E_t \left\{ Q_{t,t+1}^{-1} \frac{P_{c,t}^i}{P_{c,t+1}^i} \frac{{C_{t+1}^i}^{-1}}{{C_t^i}^{-1}} \right\} = 1 \tag{3.56}$$

From (3.51) and (3.52) yields

$$\frac{\beta^t (1-\chi) {C_t^i}^{-1}}{P_{c,t}^i} = \frac{\beta^t {N_t^i}^{\varphi}}{W_t^i}$$

$$\Rightarrow \frac{W_t^i}{P_{c,t}^i} = \frac{{N_t^i}^{\varphi}}{(1-\chi) {C_t^i}^{-1}} \tag{3.57}$$

From (3.51) and (3.53), we have

$$\frac{\beta^t \left(\dfrac{M_t^i}{P_{c,t}^i}\right)^{-\nu}}{\beta^t (1-\chi) {C_t^i}^{-1}} = \frac{\lambda_t^i P_{c,t}^i - E_t \lambda_{t+1} P_{c,t}^i}{\lambda_t^i P_{c,t}^i} = 1 - \frac{E_t \lambda_{t+1}}{\lambda_t^i} = 1 - E_t\{Q_{t,t+1}\} = 1 - \frac{1}{1+i_t} = \frac{i_t}{1+i_t}$$

$$\Rightarrow \frac{\left(\dfrac{M_t^i}{P_{c,t}^i}\right)^{-\nu}}{(1-\chi) {C_t^i}^{-1}} = \frac{i_t}{1+i_t} \tag{3.58}$$

In log-linearized form the equation (3.56), (3.57), and (3.58) become

$$\Rightarrow \log\beta + \log(1+i_t) - \log\frac{E_t P_{c,t+1}^i}{P_{c,t}^i} - \log E_t C_{t+1}^i + \log C_t^i = \log 1$$

$$\Rightarrow \hat{\imath}_t - \mathrm{E_t}\pi^i_{\mathrm{c,t+1}} - E_t\hat{c}^i_{t+1} + \hat{c}^i_t = 0$$

$$\Rightarrow \hat{c}^i_t = E_t\hat{c}^i_{t+1} - \left(\hat{\imath}_t - \mathrm{E_t}\pi^i_{\mathrm{c,t+1}}\right) \tag{3.59}$$

$$\Rightarrow \log W^i_t - \log P^i_{c,t} = \varphi \log N^i_t + \log C^i_t - \log(1-\chi)$$

$$\Rightarrow \widehat{w}^i_t - \hat{p}^i_{c,t} = \varphi\hat{n}^i_t + \hat{c}^i_t \tag{3.60}$$

$$-\nu \log m^i_t + \log C^i_t - \log(1-\chi) = \log\frac{i_t}{1+i_t}$$

$$\Rightarrow -\nu\frac{dm^i_t}{\overline{m^\iota}} + \frac{dC^i_t}{\overline{C^\iota}} = \frac{1+\bar{\imath}}{\bar{\imath}}\left(\frac{(1+\bar{\imath})-\bar{\imath}}{(1+\bar{\imath})^2}\right)\bar{\imath}\frac{di_t}{\bar{\imath}}$$

$$\Rightarrow -\nu\frac{dm^i_t}{\overline{m^\iota}} + \frac{dC^i_t}{\overline{C^\iota}} = \frac{1}{(1+\bar{\imath})}\frac{di_t}{\bar{\imath}}$$

$$\Rightarrow -\nu\widehat{m}^i_t + \hat{c}^i_t = \frac{1}{(1+\bar{\imath})}\hat{\imath}_t$$

$$\Rightarrow \widehat{m}^i_t = \frac{1}{\nu}\hat{c}^i_t - \frac{1}{(1+\bar{\imath})\nu}\hat{\imath}_t \tag{3.61}$$

<u>Terms of trade and CPI Inflation</u>

Term of trade

$$S^i_t = \frac{P^*_t}{P^i_t} = exp\int_0^1 \left(p^f_t - p^i_t\right)df = exp\int_0^1 s^i_{f,t}df \tag{3.62}$$

where $S^i_{f,t} \equiv \frac{P^f_t}{P^i_t}$ is bilateral term of trade between region i and region f, $P^*_t = exp\int_0^1 p^f_t\,df$ is the price index for domestic imported goods, $s^i_{f,t} \equiv \log S^i_{f,t}$

Let $P_{c,t}^{i} \equiv \left(P_t^i\right)^{1-\alpha}(P_t^*)^{\alpha}$, then

$$P_{c,t}^{i} = P_t^i \left(\frac{P_t^*}{P_t^i}\right)^{\alpha}$$

$$P_{c,t}^{i} = P_t^i \left(S_t^i\right)^{\alpha}$$

In logarithmic:

$$p_{c,t}^{i} = p_t^i + \alpha s_t^i \tag{3.63}$$

CPI inflation becomes:

$$\pi_{c,t}^{i} = \pi_t^i + \alpha \Delta s_t^i \tag{3.64}$$

Under the assumption of complete markets for the securities traded nationally, the first order condition analogous to equation (3.56) must hold for the representative household in any other region, say region f

$$1 = \beta E_t \left\{ Q_{t,t+1}^{-1} \frac{P_{c,t}^{f}}{P_{c,t+1}^{f}} \frac{{C_{t+1}^{f}}^{-1}}{{C_t^{f}}^{-1}} \right\} \tag{3.65}$$

Combine both FOC, equation (3.56) and (3.65) yields

$$1 = \frac{\beta E_t \left\{ Q_{t,t+1}^{-1} \frac{P_{c,t}^{i}}{P_{c,t+1}^{i}} \frac{{C_{t+1}^{i}}^{-1}}{{C_t^{i}}^{-1}} \right\}}{\beta E_t \left\{ Q_{t,t+1}^{-1} \frac{P_{c,t}^{f}}{P_{c,t+1}^{f}} \frac{{C_{t+1}^{f}}^{-1}}{{C_t^{f}}^{-1}} \right\}}$$

$$= \frac{E_t \left\{ \frac{P_{c,t}^{i}}{P_{c,t+1}^{i}} \frac{{C_{t+1}^{i}}^{-1}}{{C_t^{i}}^{-1}} \right\}}{E_t \left\{ \frac{P_{c,t}^{f}}{P_{c,t+1}^{f}} \frac{{C_{t+1}^{f}}^{-1}}{{C_t^{f}}^{-1}} \right\}} = E_t \left\{ \frac{P_{c,t}^{i}}{P_{c,t+1}^{i}} \frac{{C_{t+1}^{i}}^{-1}}{{C_t^{i}}^{-1}} \frac{P_{c,t+1}^{f}}{P_{c,t}^{f}} \frac{C_{t+1}^{f}}{C_t^{f}} \right\} \tag{3.66}$$

Solve for C_t^i:

$$C_t^{i^{-1}} = E_t\left\{\left(\frac{C_{t+1}^f}{C_{t+1}^i}\right) C_t^{f^{-1}} \frac{\frac{P_{c,t+1}^f}{P_{c,t+1}^i}}{\frac{P_{c,t}^f}{P_{c,t}^i}}\right\} = E_t\left\{\left(\frac{C_{t+1}^f}{C_{t+1}^i}\right) C_t^{f^{-1}} \frac{\left(S_{f,t+1}^i\right)^{1-\alpha}}{\left(S_{f,t}^i\right)^{1-\alpha}}\right\}$$

$$C_t^i = E_t\left\{\left(\frac{C_{t+1}^f}{C_{t+1}^i}\right) C_t^{f^{-1}} \frac{\left(S_{f,t+1}^i\right)^{1-\alpha}}{\left(S_{f,t}^i\right)^{1-\alpha}}\right\}^{-1} = E_t\left\{\frac{C_{t+1}^i}{C_{t+1}^f} C_t^f \left(S_{f,t+1}^i\right)^{-(1-\alpha)} \left(S_{f,t}^i\right)^{1-\alpha}\right\}$$

$$= E_t\left\{\frac{C_{t+1}^i}{C_{t+1}^f\left(S_{f,t+1}^i\right)^{1-\alpha}}\right\} C_t^f \left(S_{f,t}^i\right)^{1-\alpha}$$

$$C_t^i = \vartheta_f C_t^f \left(S_{f,t}^i\right)^{1-\alpha} \tag{3.67}$$

where ϑ_f is some constant that will generally depend on initial conditions regarding relative net asset positions. Without loss of generality, let assume symmetric initial conditions, i.e. zero net foreign assets holdings and ex-ante identical environment. This implies $\vartheta_f = \vartheta = 1\ \forall f$.

$$C_t^i = C_t^f \left(S_{f,t}^i\right)^{1-\alpha} \tag{3.68}$$

Taking logs yields:

$$c_t^i = c_t^f + (1-\alpha) s_{f,t}^i \tag{3.69}$$

Noted that the rest of region consumption is given by

$$c_t^* \equiv \int_0^1 c_t^f \, df \tag{3.70}$$

Therefore, integrating (3.69) using (3.70) yields

$$c_t^i = \int_0^1 \left(c_t^f + (1-\alpha) s_{f,t}\right) df = c_t^* + (1-\alpha) s_t^i \tag{3.71}$$

Aggregate Demand

The market clearing for good j produced in country i requires

$$Y_t^i(j) = C_{i,t}^i(j) + \int_0^1 C_{i,t}^f(j)df + G_t^i(j) \tag{3.72}$$

$$\begin{aligned}
Y_t^i(j) &= \left(\frac{P_t^i(j)}{P_t^i}\right)^{-\varepsilon}\left\{(1-\alpha)\left(\frac{P_{c,t}^i}{P_t^i}\right)C_t^i + \alpha\int_0^1\left(\frac{P_{c,t}^f}{P_t^i}\right)C_t^f df + G_t^i\right\} \\
&= \left(\frac{P_t^i(j)}{P_t^i}\right)^{-\varepsilon}\left\{(1-\alpha)\left(\frac{P_{c,t}^i}{P_t^i}\right)C_t^i + \alpha\int_0^1 S_{f,t}^i C_t^f df + G_t^i\right\} \\
&= \left(\frac{P_t^i(j)}{P_t^i}\right)^{-\varepsilon}\left\{(1-\alpha)\left(S_t^i\right)^{\alpha}C_t^i + \alpha S_t^{\alpha}\int_0^1 \left(S_{f,t}^i\right)^{1-\alpha} C_t^f df + G_t^i\right\} \\
&= \left(\frac{P_t^i(j)}{P_t^i}\right)^{-\varepsilon}\left\{(1-\alpha)\left(S_t^i\right)^{\alpha}C_t^i + \alpha\left(S_t^i\right)^{\alpha}C_t^i + G_t^i\right\} \\
&= \left(\frac{P_t^i(j)}{P_t^i}\right)^{-\varepsilon}\left\{C_t^i\left(S_t^i\right)^{\alpha} + G_t^i\right\}
\end{aligned}$$

Thus,

$$Y_t^i = C_t^i\left(S_t^i\right)^{\alpha} + G_t^i \tag{3.73}$$

Log-linearize around the steady state yields:

$$\hat{y}_t^i = (1-\gamma)\left(\hat{c}_t^i + \alpha\hat{s}_t^i\right) + \gamma\hat{g}_t^i \tag{3.74}$$

By aggregating over all regions, a national market clearing condition can be derived as:

$$\hat{y}_t^* = \int_0^1 \hat{y}_t^i di = \int_0^1 \left((1-\gamma)\left(\hat{c}_t^i + \alpha\hat{s}_t^i\right) + \gamma\hat{g}_t^i\right)di = (1-\gamma)\hat{c}_t^* + \gamma\hat{g}_t^* \tag{3.75}$$

Solve for $\hat{c}_t^*$, yields

$$\hat{c}_t^* = \frac{1}{(1-\gamma)}\hat{y}_t^* - \frac{\gamma}{(1-\gamma)}\hat{g}_t^* \tag{3.76}$$

Price Setting

The objective function of the firms in setting up the price is

$$V_t^i = \mathrm{E}_t \sum_{k=0}^{\infty} (\beta\theta^i)^k \left[\frac{P_{t+k}^i(j)}{P_{t+k}^i} Y_{t+k}^i(j) - Cost_{t+k}^i(j)\{Y_{t+k}^i(j)\}\right]$$

$$= \mathrm{E}_t \sum_{k=0}^{\infty} (\beta\theta^i)^k \left[\frac{P_{t+k}^i(j)}{P_{t+k}^i} Y_{t+k}^i \left(\frac{P_{t+k}^i(j)}{P_{t+k}^i}\right)^{-\varepsilon} - Cost_{t+k}^i(j)\left\{Y_{t+k}^i \left(\frac{P_{t+k}^i(j)}{P_{t+k}^i}\right)^{-\varepsilon}\right\}\right]$$

$$= \mathrm{E}_t \sum_{k=0}^{\infty} (\beta\theta^i)^k \left\{Y_{t+k}^i \left(\frac{P_t^i(j)}{P_{t+k}^i}\Phi_{t,k}\right)^{1-\varepsilon} - Cost_{t+k}^i(j)\left\{Y_{t+k}^i \left(\frac{P_t^i(j)}{P_{t+k}^i}\Phi_{t,k}\right)^{-\varepsilon}\right\}\right\}$$

where $\Phi_{t,k} \equiv \pi_t^i \pi_{t+1}^i \ldots \pi_{t+j-1}^i$ and $\Phi_{t,0} \equiv 1$. Note that $Cost_{t+k}^i(j)\{Y_{t+k}^i(j)\}$ is cost as a function of $Y_{t+k}^i(j)$,

The FOC with respect to $P_t^i(j)$ is

$$\frac{\partial V_t^i}{\partial P_t^i(j)} : 0 = \mathrm{E}_t \sum_{k=0}^{\infty} (\beta\theta^i)^k \left[Y_{t+k}^i \frac{\partial \left(\frac{P_t^i(j)}{P_{t+k}^i}\Phi_{t,k}\right)^{1-\varepsilon}}{\partial P_t^i(j)} \right.$$

$$\left. - \frac{\partial Cost_{t+k}^i(j)}{\partial Y_{t+k}^i(j)} \left\{ Y_{t+k}^i \frac{\partial \left(\frac{P_t^i(j)}{P_{t+k}^i}\Phi_{t,k}\right)^{-\varepsilon}}{\partial P_t^i(j)} \right\} \right]$$

$$0 = \mathrm{E}_t \sum_{k=0}^{\infty} (\beta\theta^i)^k \left[Y_{t+k}^i (1-\varepsilon) \left(\frac{P_t^i(j)}{P_{t+k}^i}\Phi_{t,k}\right)^{-\varepsilon} \frac{\Phi_{t,k}}{P_{t+k}^i} \right.$$

$$\left. + MC_{t+k}^i(j) Y_{t+k}^i \varepsilon \left(\frac{P_t^i(j)}{P_{t+k}^i}\Phi_{t,k}\right)^{-\varepsilon-1} \frac{\Phi_{t,k}}{P_{t+k}^i} \right]$$

$$0 = \mathrm{E}_t \sum_{k=0}^{\infty} \left(\beta\theta^i\right)^k Y_{t+k}^i \left(\frac{P_t^i(j)}{P_{t+k}^i}\Phi_{t,k}\right)^{-\varepsilon} \left[\frac{\Phi_{t,k}}{P_{t+k}^i} - \frac{\varepsilon}{\varepsilon-1} MC_{t+k}^i(j)\frac{1}{P_t^i(j)}\right]$$

$$0 = \mathrm{E}_t \sum_{k=0}^{\infty} \left(\beta\theta^i\right)^k Y_{t+k}^i \left(\frac{P_t^i(j)}{P_{t+k}^i}\Phi_{t,k}\right)^{-\varepsilon} \left[\frac{P_t^i(j)}{P_{t+k}^i}\Phi_{t,k} - \frac{\varepsilon}{\varepsilon-1} MC_{t+k}^i(j)\right]$$

where $MC_{t+k}^i(j)$ is the real marginal cost of good *j*. We know that

$$\frac{P_{t+k}^i(j)}{P_{t+k}^i} = \frac{P_t^i(j)\Phi_{t,k}}{P_{t+k}^i} = \frac{P_t^i(j)}{P_t^i}\frac{\pi_t^i\pi_{t+1}^i \dots \pi_{t+k-1}^i}{\pi_{t+k}^i\pi_{t+k-1}^i \dots \pi_{t+1}^i} = \frac{P_t^i(j)}{P_t^i}\frac{\pi_t^i}{\pi_{t+k}^i}$$

Thus,

$$0 = \mathrm{E}_t \sum_{k=0}^{\infty} \left(\beta\theta^i\right)^k Y_{t+k}^i \left(\frac{P_t^i(j)}{P_t^i}\frac{\pi_t^i}{\pi_{t+k}^i}\right)^{-\varepsilon} \left[\frac{P_t^i(j)}{P_t^i}\frac{\pi_t^i}{\pi_{t+k}^i} - \frac{\varepsilon}{\varepsilon-1} MC_{t+k}^i(j)\right]$$

Since all firms make the same decision, we can drop the good index *j* and define the optimum price $\tilde{P}_t^i$. Let also define $\tilde{p}_t^i = \frac{\tilde{P}_t^i}{P_t^i}$, we can rewrite the FOC as

$$0 = \mathrm{E}_t \sum_{k=0}^{\infty} \left(\beta\theta^i\right)^k Y_{t+k}^i \left(\pi_{t+k}^i\right)^{\varepsilon} \left[\tilde{p}_t^i \frac{\pi_t^i}{\pi_{t+k}^i} - \frac{\varepsilon}{\varepsilon-1} MC_{t+k}^i\right] \tag{3.77}$$

The law of motion of the aggregate price is given by

$$P_t = \left[\theta\left(P_{t-1}^i\pi_{t-1}^i\right)^{1-\varepsilon} + (1-\theta)\tilde{P}_t^{i\,1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{3.78}$$

Powered by $1-\varepsilon$ and divided by $\left(P_{t-1}^i\right)^{1-\varepsilon}$ yields

$$\left(\pi_t^i\right)^{1-\varepsilon} = \theta\left(\pi_{t-1}^i\right)^{1-\varepsilon} + (1-\theta)\left(\tilde{p}_t^i\pi_t^i\right)^{1-\varepsilon}$$

$$1 = \theta\left(\frac{\pi_{t-1}^i}{\pi_t^i}\right)^{1-\varepsilon} + (1-\theta)\left(\tilde{p}_t^i\right)^{1-\varepsilon}$$

Taking log and divided each by the steady state value yields:

$$0=\log\left\{\theta\left(\frac{\pi_{t-1}^{i}}{\pi_{t}^{i}}\right)^{1-\varepsilon}+(1-\theta)\left(\tilde{p}_{t}^{i}\right)^{1-\varepsilon}\right\}$$

Take a first order Taylor approximation on both sides around the steady state yields

$$0=0+\theta(1-\varepsilon)\hat{\pi}_{t-1}^{i}-\theta(1-\varepsilon)\hat{\pi}_{t}^{i}+(1-\theta)(1-\varepsilon)\hat{\tilde{p}}_{t}^{i}$$

$$\hat{\tilde{p}}_{t}^{i}=\frac{\theta}{(1-\theta)}\left(\hat{\pi}_{t}^{i}-\hat{\pi}_{t-1}^{i}\right)$$

Rewrite the FOC as

$$\tilde{p}_{t}^{i}\mathrm{E}_{t}\sum_{k=0}^{\infty}\left(\beta\theta^{i}\right)^{k}Y_{t+k}^{i}\left(\pi_{t+k}^{i}\right)^{\varepsilon}\frac{\pi_{t}^{i}}{\pi_{t+k}^{i}}=\frac{\varepsilon}{\varepsilon-1}\mathrm{E}_{t}\sum_{k=0}^{\infty}\left(\beta\theta^{i}\right)^{k}Y_{t+k}^{i}\left(\pi_{t+k}^{i}\right)^{\varepsilon}MC_{t+k}^{i}$$

Now, by defining

$$\Gamma_{t+k}^{i}=Y_{t+k}^{i}\left(\pi_{t+k}^{i}\right)^{\varepsilon},\Omega_{t+k}^{i}=\frac{\pi_{t}^{i}}{\pi_{t+k}^{i}},s_{\varepsilon}=\frac{\varepsilon}{\varepsilon-1}$$

In steady state, we have

$$\Gamma^{i}\sum_{k=0}^{\infty}\left(\beta\theta^{i}\right)^{k}=\frac{\Gamma^{i}}{1-\beta\theta^{i}}=s_{\varepsilon}\frac{\Gamma^{i}}{1-\beta\theta^{i}}MC^{i}=s_{\varepsilon}\Gamma^{i}MC^{i}\sum_{k=0}^{\infty}\left(\beta\theta^{i}\right)^{k}$$

Taking log and a first order Taylor approximation on both sides around the steady state of the FOC yields

$$\log\left(\tilde{p}_{t}^{i}\mathrm{E}_{t}\sum_{k=0}^{\infty}\left(\beta\theta^{i}\right)^{k}\Gamma_{t+k}^{i}\Omega_{t+k}^{i}\right)=\log\left(s_{\varepsilon}\mathrm{E}_{t}\sum_{k=0}^{\infty}\left(\beta\theta^{i}\right)^{k}\Gamma_{t+k}^{i}MC_{t+k}^{i}\right)$$

$$\log\frac{\Gamma^i}{1-\beta\theta^i}+\widehat{\tilde{p}}_t^i+\frac{1-\beta\theta^i}{\Gamma^i}\left(\Gamma_t^i-\Gamma^i\right)+\frac{1-\beta\theta^i}{\Gamma^i}\beta\theta^i\left(\Gamma_{t+1}^i-\Gamma^i\right)+\cdots$$

$$+\frac{1-\beta\theta^i}{\Gamma^i}\Gamma^i\left(\Omega_t^i-\Omega^i\right)+\frac{1-\beta\theta^i}{\Gamma^i}\Gamma^i\beta\theta^i\left(\Omega_{t+1}^i-\Omega^i\right)+\cdots$$

$$=\log\frac{\Gamma^i}{1-\beta\theta^i}+\frac{1-\beta\theta^i}{\Gamma^i}\left(\Gamma_t^i-\Gamma^i\right)+\frac{1-\beta\theta^i}{\Gamma^i}\beta\theta^i\left(\Gamma_{t+1}^i-\Gamma^i\right)+\cdots$$

$$+\frac{1-\beta\theta^i}{\Gamma^i s_\varepsilon MC^i}s_\varepsilon\Gamma^i\left(MC_t^i-MC^i\right)+\frac{1-\beta\theta^i}{\Gamma^i s_\varepsilon MC^i}s_\varepsilon\Gamma^i\beta\theta^i\left(MC_{t+1}^i-MC^i\right)+\cdots$$

$$\widehat{\tilde{p}}_t^i+\left(1-\beta\theta^i\right)\widehat{\Omega}_t^i+\left(1-\beta\theta^i\right)\beta\theta^i\widehat{\Omega}_{t+1}^i+\cdots$$

$$=\left(1-\beta\theta^i\right)\widehat{mc}_t^i++\left(1-\beta\theta^i\right)\beta\theta^i\widehat{mc}_{t+1}^i+\cdots$$

Substitute $\widehat{\Omega}_{t+k}^i=\hat{\pi}_t^i-\hat{\pi}_{t+k}^i$, yields

$$\widehat{\tilde{p}}_t^i+\left(1-\beta\theta^i\right)\beta\theta^i\left(\hat{\pi}_t^i-\hat{\pi}_{t+1}^i\right)+\left(1-\beta\theta^i\right)\left(\beta\theta^i\right)^2\left(\hat{\pi}_t^i-\hat{\pi}_{t+2}^i\right)+\cdots$$

$$=\left(1-\beta\theta^i\right)\widehat{mc}_t^i++\left(1-\beta\theta^i\right)\beta\theta^i\widehat{mc}_{t+1}^i+\cdots$$

Rearrange it to:

$$\widehat{\tilde{p}}_t^i+\hat{\pi}_t^i=\left(1-\beta\theta^i\right)\sum_{k=0}^{\infty}\left(\beta\theta^i\right)^k E_t\left(\widehat{mc}_{t+k}^i+\hat{\pi}_{t+k}^i\right) \tag{3.79}$$

or

$$\begin{aligned}\widehat{\tilde{p}}_t^i+\hat{\pi}_t^i&=\left(1-\beta\theta^i\right)\left(\widehat{mc}_t^i+\hat{\pi}_t^i\right)\\&+\left(\beta\theta^i\right)\left(1-\beta\theta^i\right)\sum_{k=0}^{\infty}\left(\beta\theta^i\right)^k E_t\left(\widehat{mc}_{t+1+k}^i+\hat{\pi}_{t+1+k}^i\right)\end{aligned} \tag{3.80}$$

By taking expectation on equation (3.79) on both sides yields:

$$E_t\left(\widehat{\tilde{p}_{t+1}^i}+\hat{\pi}_{t+1}^i\right)=\left(1-\beta\theta^i\right)\sum_{k=0}^{\infty}\left(\beta\theta^i\right)^k E_t\left(\widehat{mc}_{t+1+k}^i+\hat{\pi}_{t+1+k}^i\right) \tag{3.81}$$

Substitute equation (3.81) into the equation (3.80), yields:

$$\hat{\tilde{p}}_t^i + \hat{\pi}_t^i = (1-\beta\theta^i)(\widehat{mc}_t^i + \hat{\pi}_t^i) + (\beta\theta^i)E_t(\widehat{\tilde{p}_{t+1}^i} + \hat{\pi}_{t+1}^i)$$

Rewrite as:

$$\hat{\tilde{p}}_t^i = (1-\beta\theta^i)\widehat{mc}_t^i + \beta\theta^i E_t(\hat{\pi}_{t+1}^i - \hat{\pi}_t^i) + (\beta\theta^i)E_t(\widehat{\tilde{p}_{t+1}^i})$$

Replace $\hat{\tilde{p}}_t^i = \frac{\theta^i}{(1-\theta^i)}(\hat{\pi}_{t-1}^i - \hat{\pi}_t^i)$, yields

$$\frac{\theta^i}{(1-\theta^i)}(\hat{\pi}_t^i - \hat{\pi}_{t-1}^i)$$
$$= (1-\beta\theta^i)\widehat{mc}_t^i + \beta\theta^i E_t(\hat{\pi}_{t+1}^i - \hat{\pi}_t^i) + (\beta\theta^i)E_t\left(\frac{\theta^i}{(1-\theta^i)}(\hat{\pi}_{t+1}^i - \hat{\pi}_t^i)\right)$$

$$\left(\frac{\theta^i}{(1-\theta^i)} + \frac{\beta\theta^i(1-\theta^i)}{(1-\theta^i)} + \frac{\beta\theta^i\theta^i}{(1-\theta^i)}\right)\hat{\pi}_t^i$$
$$= \frac{\theta^i}{(1-\theta^i)}\hat{\pi}_{t-1}^i + \left(\frac{\beta\theta^i(1-\theta^i)}{(1-\theta^i)} + \frac{\beta\theta^i\theta^i}{(1-\theta^i)}\right)\hat{\pi}_{t+1}^i + (1-\beta\theta^i)\widehat{mc}_t^i$$

$$\left(\frac{\theta^i(1+\beta)}{(1-\theta^i)}\right)\hat{\pi}_t^i = \frac{\theta^i}{(1-\theta^i)}\hat{\pi}_{t-1}^i + \left(\frac{\beta\theta^i}{(1-\theta^i)}\right)\hat{\pi}_{t+1}^i + (1-\beta\theta^i)\widehat{mc}_t^i$$

$$\hat{\pi}_t^i = \frac{1}{(1+\beta)}\hat{\pi}_{t-1}^i + \frac{\beta}{(1+\beta)}\hat{\pi}_{t+1}^i + \frac{(1-\beta\theta^i)(1-\theta^i)}{\theta^i(1+\beta)}\widehat{mc}_t^i \qquad (3.82)$$

Now, we need to derive the marginal cost. Given the assumption of linear technology as shown in equation (3.29), the real marginal cost in logarithmic form is

$$mc_t^i = w_t^i - p_t^i - a_t^i = (w_t^i - p_{c,t}^i) + (p_{c,t}^i - p_t^i) - a_t^i = \hat{c}_t^i + \varphi\hat{n}_t + \alpha s_t^i - a_t^i$$

$$\widehat{mc}_t^i = \hat{c}_t^i + \alpha s_t^i + \varphi\hat{a}_t^i + \varphi\hat{n}_t - (1+\varphi)\hat{a}_t^i$$

$$\widehat{mc}_t^i = \left(\frac{1}{(1-\gamma)} + \varphi\right)\hat{y}_t^i - \frac{\gamma}{(1-\gamma)}\hat{g}_t^i - (1+\varphi)\hat{a}_t^i \qquad (3.83)$$

Finally, the inflation equation is

$$\hat{\pi}_t^i = \frac{1}{(1+\beta)}\hat{\pi}_{t-1}^i + \frac{\beta}{(1+\beta)}\hat{\pi}_{t+1}^i + \frac{(1-\beta\theta^i)(1-\theta^i)}{\theta^i(1+\beta)}\left\{\left(\frac{1}{(1-\gamma)}+\varphi\right)\hat{y}_t^i - \frac{\gamma}{(1-\gamma)}\hat{g}_t^i - (1+\varphi)\hat{a}_t^i\right\} \tag{3.84}$$

Optimal Fiscal Policy

Before we conduct a second order Taylor expansion of consumption, it needed to calculate Taylor expansion of $\log(Y_t^i - G_t^i)$. Let define the government spending share $\gamma \equiv \frac{\bar{G}^i}{\bar{Y}^i}$ and define $\hat{y}_t^i \equiv \log\frac{Y_t^i}{\bar{Y}^i}$ and $\hat{g}_t^i \equiv \log\frac{G_t^i}{\bar{G}^i}$. A second order Taylor expansion of $\log(Y_t^i - G_t^i)$ around the steady state is given by

$$\begin{aligned}
\log(Y_t^i - G_t^i) \simeq{}& \log(\bar{Y}^i - \bar{G}^i) + \frac{\bar{Y}^i}{\bar{Y}^i - \bar{G}^i}\left(\frac{Y_t^i - \bar{Y}^i}{\bar{Y}^i}\right) - \frac{\bar{G}^i}{\bar{Y}^i - \bar{G}^i}\left(\frac{G_t^i - \bar{G}^i}{\bar{G}^i}\right) \\
&- \frac{{\bar{Y}^i}^2}{2(\bar{Y}^i - \bar{G}^i)^2}\left(\frac{Y_t^i - \bar{Y}^i}{\bar{Y}^i}\right)^2 + \frac{2\bar{Y}^i\bar{G}^i}{2(\bar{Y}^i - \bar{G}^i)^2}\left(\frac{Y_t^i - \bar{Y}^i}{\bar{Y}^i}\right)\left(\frac{G_t^i - \bar{G}^i}{\bar{G}^i}\right) \\
&- \frac{{\bar{G}^i}^2}{2(\bar{Y}^i - \bar{G}^i)^2}\left(\frac{G_t^i - \bar{G}^i}{\bar{G}^i}\right)^2 \\
={}& \log\left((1-\gamma)\bar{Y}^i\right) + \frac{\bar{Y}^i}{(1-\gamma)\bar{Y}^i}\left(\frac{Y_t^i - \bar{Y}^i}{\bar{Y}^i}\right) - \frac{\gamma\bar{Y}^i}{(1-\gamma)\bar{Y}^i}\left(\frac{G_t^i - \bar{G}^i}{\bar{G}^i}\right) \\
&- \frac{{\bar{Y}^i}^2}{2\left((1-\gamma)\bar{Y}^i\right)^2}\left(\frac{Y_t^i - \bar{Y}^i}{\bar{Y}^i}\right)^2 + \frac{2\gamma{\bar{Y}^i}^2}{2\left((1-\gamma)\bar{Y}^i\right)^2}\left(\frac{Y_t^i - \bar{Y}^i}{\bar{Y}^i}\right)\left(\frac{G_t^i - \bar{G}^i}{\bar{G}^i}\right) \\
&- \frac{\left(\gamma\bar{Y}^i\right)^2}{2\left((1-\gamma)\bar{Y}^i\right)^2}\left(\frac{G_t^i - \bar{G}^i}{\bar{G}^i}\right)^2
\end{aligned}$$

$$= \log\left((1-\gamma)\bar{Y}^i\right) + \frac{1}{(1-\gamma)}(\hat{y}_t - \gamma\hat{g}_t) + \frac{1}{2(1-\gamma)}(\hat{y}_t^2 - \gamma\hat{g}_t^2)$$

$$- \frac{1}{2(1-\gamma)^2}(\hat{y}_t - \gamma\hat{g}_t)^2$$

$$= \log\left((1-\gamma)\bar{Y}^i\right) + \frac{1}{(1-\gamma)}(\hat{y}_t - \gamma\hat{g}_t) - \frac{\gamma}{2(1-\gamma)^2}(\hat{g}_t - \hat{y}_t)^2$$

Thus, when considering fluctuations around the steady state, we can rewrite

$$\log\left(Y_t^i - G_t^i\right) \simeq \frac{1}{(1-\gamma)}(\hat{y}_t - \gamma\hat{g}_t) - \frac{\gamma}{2(1-\gamma)^2}(\hat{g}_t - \hat{y}_t)^2 + tips \qquad (3.85)$$

where $tips$ denotes terms that are independent of policy.

A Taylor expansion of consumption using equation (3.73), $\int_0^1 s_t^i di = 0$, and the fact that $\gamma = \chi$

$$\int_0^1 \log C_t^i di = \int_0^1 \log\left(Y_t^i - G_t^i\right) - \alpha s_t^i di = \int_0^1 \log\left(Y_t^i - G_t^i\right) di$$

$$\simeq \frac{1}{(1-\chi)} \int_0^1 (\hat{y}_t - \chi\hat{g}_t)\, di \qquad (3.86)$$

$$- \frac{\chi}{2(1-\chi)^2} \int_0^1 (\hat{g}_t - \hat{y}_t)^2\, di$$

Next, a Taylor expansion of logarithmic government expenditure is

$$\log G_t^i \simeq \log \bar{G}^i + \frac{\bar{G}^i}{\bar{G}^i}\left(\frac{G_t^i - \bar{G}^i}{\bar{G}^i}\right) - \frac{{\bar{G}^i}^2}{2{\bar{G}^i}^2}\left(\frac{G_t^i - \bar{G}^i}{\bar{G}^i}\right)^2$$

$$= \log \bar{G}^i + \hat{g}_t^i + \frac{1}{2}{\hat{g}_t^i}^2 - \frac{1}{2}{\hat{g}_t^i}^2$$

$$= \log \bar{G}^i + \hat{g}_t^i$$

Thus, when considering fluctuations around the steady state, we can rewrite

$$\int_0^1 \log G_t^i di = \int_0^1 \hat{g}_t^i \, di \tag{3.87}$$

A second order Taylor expansion of the disutility of labor about a steady state is given by

$$\frac{{N_t^i}^{1+\varphi}}{1+\varphi} \simeq \frac{{\bar{N}^i}^{1+\varphi}}{1+\varphi} + {\bar{N}^i}^{1+\varphi}\left(\frac{N_t^i - \bar{N}^i}{\bar{N}^i}\right) + \frac{\varphi}{2}{\bar{N}^i}^{1+\varphi}\left(\frac{N_t^i - \bar{N}^i}{\bar{N}^i}\right)^2$$

$$= \frac{{\bar{N}^i}^{1+\varphi}}{1+\varphi} + {\bar{N}^i}^{1+\varphi}\left(\hat{n}_t^i + \frac{1}{2}{\hat{n}_t^i}^2\right) + \frac{\varphi}{2}{\bar{N}^i}^{1+\varphi}{\hat{n}_t^i}^2$$

$$= \frac{{\bar{N}^i}^{1+\varphi}}{1+\varphi} + {\bar{N}^i}^{1+\varphi}\hat{n}_t^i + \frac{1}{2}{\bar{N}^i}^{1+\varphi}(1+\varphi){\hat{n}_t^i}^2$$

where $\hat{n}_t^i \equiv \log\frac{N_t^i}{\bar{N}^i}$. Under the optimal policy, the steady state of labor is given by $\bar{N}^i = 1$.

Thus,

$$\frac{{N_t^i}^{1+\varphi}}{1+\varphi} = \hat{n}_t^i + \frac{1}{2}(1+\varphi){\hat{n}_t^i}^2 + tips$$

Using the fact that $N_t^i = \left(\frac{Y_t^i}{A_t^i}\right)\int_0^1 \left(\frac{P_t^i(j)}{P_t^i}\right)^{-\epsilon} dj$, where $N_t^i = \int_0^1 N_t^i(j)dj$, we can derive

$$\hat{n}_t^i = \hat{y}_t^i - a_t^i + z_t^i$$

where $z_t^i \equiv \log\int_0^1 \left(\frac{P_t^i(j)}{P_t^i}\right)^{-\epsilon}$.

In proof of z_t^i below, we know that

$$z_t^i \simeq \frac{\epsilon}{2} var_j\{p_t^i(j)\}$$

Using the fact that a_t^i is independent of policy, we can rewrite the second order approximation to the utility of labor as

$$\frac{{N_t^i}^{1+\varphi}}{1+\varphi} \simeq \hat{y}_t^i + z_t^i + \frac{1}{2}(1+\varphi){\hat{y}_t^i}^2 + tips \tag{3.88}$$

Since the real money balance is also independent of policy, the real money balance is not included in the welfare function. Thus, the aggregate welfare in Indonesia as a whole country in a second order approximation is

$$\mathbb{W}_t \equiv \int_0^1 U\left(C_t^i, G_t^i, N_t^i\right) di$$

$$= (1-\chi)\int_0^1 \log C_t^i \, di + \chi \int_0^1 \log G_t^i \, di - \int_0^1 \frac{{N_t^i}^{1+\varphi}}{1+\varphi} di$$

$$\simeq \int_0^1 \left(\hat{y}_t^i - \chi \hat{g}_t^i\right) di - \frac{\chi}{2(1-\chi)} \int_0^1 \left(\hat{g}_t^i - \hat{y}_t^i\right)^2 di + \chi \int_0^1 \hat{g}_t^i$$

$$- \int_0^1 \left(\hat{y}_t^i + z_t^i + \frac{1}{2}(1+\varphi){\hat{y}_t^i}^2\right) di + tips$$

$$= -\int_0^1 \left(z_t^i + \frac{1}{2}(1+\varphi){\hat{y}_t^i}^2 + \frac{\chi}{2(1-\chi)}\left(\hat{g}_t^i - \hat{y}_t^i\right)^2\right) di + tips$$

The discounted sum of utilities across households is

$$\mathbb{W}_t = -\frac{1}{2}\sum_{t=0}^{\infty} \beta^t \int_0^1 \left(\frac{\varepsilon}{\lambda}\left(\pi_t^i\right)^2 + (1+\varphi){\hat{y}_t^i}^2 + \frac{\chi}{1-\chi}\left(\hat{g}_t^i - \hat{y}_t^i\right)^2\right) di \qquad (3.89)$$

The optimal policy in the present of nominal rigidities under discretion minimizes the loss function

$$\int_0^1 \left[\frac{\varepsilon}{\lambda}\left(\pi_t^i\right)^2 + (1+\varphi){\hat{y}_t^i}^2 + \frac{\chi}{1-\chi}\left(\hat{g}_t^i - \hat{y}_t^i\right)^2\right] di \qquad (3.90)$$

subject to

$$\pi_t^i = \frac{\lambda}{(1+\beta)}\left(\frac{1}{1-\gamma} + \varphi\right)\hat{y}_t^i - \frac{\lambda\gamma}{(1+\beta)(1-\gamma)}\hat{g}_t^i + u_t^i \qquad (3.91)$$

$$\hat{y}_t^i = \gamma \hat{g}_t^i + v_t^i \qquad (3.92)$$

The Lagrange function is

$$\mathcal{L} = \int_0^1 \left[\frac{\varepsilon}{\lambda}\left(\pi_t^i\right)^2 + (1+\varphi)\hat{y}_t^{i\,2} + \frac{\chi}{1-\chi}\left(\hat{g}_t^i - \hat{y}_t^i\right)^2\right] di$$

$$- \mu_t^i \left(\pi_t^i - \frac{\lambda}{(1+\beta)}\left(\frac{1}{1-\gamma} + \varphi\right)\hat{y}_t^i \right. \tag{3.93}$$

$$\left. + \frac{\lambda\gamma}{(1+\beta)(1-\gamma)}\hat{g}_t^i - u_t^i\right) - \omega_t^i\left(\hat{y}_t^i - \gamma\hat{g}_t^i - v_t^i\right)$$

The FOCs are

$$\pi_t^i: \quad \frac{2\varepsilon}{\lambda}\pi_t^i = \mu_t^i \tag{3.94}$$

$$\hat{y}_t^i: \quad 2(1+\varphi)\hat{y}_t^i - \frac{2\chi}{1-\chi}\left(\hat{g}_t^i - \hat{y}_t^i\right) + \mu_t^i\frac{\lambda}{(1+\beta)}\left(\frac{1}{1-\gamma} + \varphi\right) - \omega_t^i = 0 \tag{3.95}$$

$$\hat{g}_t^i: \quad \frac{2\chi}{1-\chi}\left(\hat{g}_t^i - \hat{y}_t^i\right) - \mu_t^i\frac{\lambda\gamma}{(1+\beta)(1-\gamma)} + \omega_t^i\gamma = 0 \tag{3.96}$$

Combining equation (3.94), (3.95), and (3.96) yields

$$2(1+\varphi)\hat{y}_t^i - \frac{2\chi}{1-\chi}\left(\hat{g}_t^i - \hat{y}_t^i\right) + \frac{1}{(1+\beta)}\left(\frac{1}{1-\gamma} + \varphi\right)2\varepsilon\pi_t^i$$

$$= -\frac{2\chi}{\gamma(1-\chi)}\left(\hat{g}_t^i - \hat{y}_t^i\right) + \frac{2\varepsilon}{(1+\beta)(1-\gamma)}\pi_t^i$$

$$(1+\varphi)\hat{y}_t^i - \frac{\chi}{1-\chi}\left(\hat{g}_t^i - \hat{y}_t^i\right) + \frac{\varepsilon\varphi}{(1+\beta)}\pi_t^i = -\frac{\chi}{\gamma(1-\chi)}\left(\hat{g}_t^i - \hat{y}_t^i\right)$$

$$(1+\varphi)\hat{y}_t^i - \frac{\chi\gamma}{\gamma(1-\chi)}\hat{g}_t^i + \frac{\chi\gamma}{\gamma(1-\chi)}\hat{y}_t^i + \frac{\varepsilon\varphi}{(1+\beta)}\pi_t^i = -\frac{\chi}{\gamma(1-\chi)}\hat{g}_t^i + \frac{\chi}{\gamma(1-\chi)}\hat{y}_t^i$$

$$\frac{\chi - \chi\gamma}{\gamma(1-\chi)}\hat{g}_t^i = -\left((1+\varphi) - \frac{\chi - \chi\gamma}{\gamma(1-\chi)}\right)\hat{y}_t^i - \frac{\varepsilon\varphi}{(1+\beta)}\pi_t^i$$

$$\hat{g}_t^i = -\left(\frac{(1+\varphi)\gamma(1-\chi)}{\chi - \chi\gamma} - 1\right)\hat{y}_t^i - \frac{\varepsilon\varphi\gamma(1-\chi)}{(\chi - \chi\gamma)(1+\beta)}\pi_t^i$$

using the fact that $\gamma = \chi$, yields the optimal fiscal policy:

$$\hat{g}_t^i = -\varphi \hat{y}_t^i - \frac{\varepsilon\varphi}{(1+\beta)}\pi_t^i \tag{3.97}$$

Proof of z_t^i

Let define $z_t^i \equiv \log \int_0^1 \left(\frac{P_t^i(\mathrm{j})}{P_t^i}\right)^{-\epsilon}$ and $\hat{p}_t(j) = p_t(j) - p_t$. A second order approximation of $\left(\frac{P_t(j)}{P_t}\right)^{1-\epsilon}$ is

$$\left(\frac{P_t(j)}{P_t}\right)^{1-\epsilon} = exp[(1-\epsilon)\hat{p}_t(j)] \simeq 1 + (1-\epsilon)\hat{p}_t(j) + \frac{(1-\epsilon)^2}{2}\hat{p}_t(j)^2$$

From the definition of P_t, we know that $\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon} di = 1$. Thus,

$$E_j\{\hat{p}_t(j)\} = \frac{\epsilon - 1}{2} E_j\{\hat{p}_t(j)^2\} \tag{3.98}$$

A second order approximation of $\left(\frac{P_t(j)}{P_t}\right)^{1-\epsilon}$ is

$$\left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} \simeq 1 - \epsilon\hat{p}_t(j) + \frac{\epsilon^2}{2}\hat{p}_t(j)^2 \tag{3.99}$$

Combining equation (3.98) and (3.99), yields

$$\int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} dj = 1 + \frac{\epsilon}{2}E_t\{\hat{p}_t(j)^2\} = 1 + \frac{\epsilon}{2}var\{\hat{p}_t(j)\}$$

Therefore,

$$z_t^i \simeq \frac{\epsilon}{2} var_j\{p_t^i(j)\} \tag{3.100}$$