LARGE SCALE CHARGING OF ELECTRIC VEHICLES: TECHNOLOGY AND ECONOMY

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LARGE SCALE CHARGING OF ELECTRIC VEHICLES: TECHNOLOGY AND ECONOMY

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The electrification of the transportation sector through the diffusion of plug-in electric vehicles (EVs), coupled with cleaner electricity generation, is considered a promising pathway to reduce air pollution from on-road vehicles and to strengthen energy security. The annual new EV sales increased 8-fold since 2011. However, the market share of EVs is still less than 1% in the new vehicle market. One of the significant barriers to large-scale adoption of EVs is the charging facilities. Well developed EV charging services are essential for the successful launch of electric vehicles and provide valuable vehicle-to-grid (V2G) services to the smart grid. On the other hand, unplanned charging brings instability, inefficiency and higher cost to the power grid and hinders the rise of clean transportation.

This work focuses on the EV charging, including the methodology, development, and economy. In different contexts, the technologies of EV charging are presented. Both online and off-line approaches are discussed, in decentralized and centralized scenarios. The interaction between the EVs and the power grid is studied, in particular for the V2G services including load shifting, frequency regulation and spinning reserves. The indirect network effect between EVs and EV charging are illustrated. A stylized model for both the EV consumers and the charging facility investor is developed, including the EV price, the coverage of the charging stations, and the price of charging.

BIOGRAPHICAL SKETCH

Zhe Yu received his B.E. degree from Department of Electrical Engineering, Tsinghua University, Beijing, China in 2009, and M.S. from Department of Electrical and Computer Engineering, Carnegie Mellon University, Pittsburgh, PA, USA in 2010. He is currently working toward the Ph.D. degree in the School of Electrical and Computer Engineering, Cornell University. His current research interests are in smart grid, demand response, dynamic programming and optimization.

This document is dedicated to all Cornell graduate students.

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CHAPTER 1

INTRODUCTION

1.1 Electric Vehicle history

The first electric car is considered to be invented by Scotsman Robert Anderson, somewhere in-between 1832 and 1839. Unfortunately, the exact date is unknown and the detail design was lost. In the same time, Sibrandus Stratingh, a Dutch chemistry professor, also carried out a breakthrough in inventing electric horseless carriages.

Figure 1.1: First Electric horseless Carriage, by Sibrandus Stratingh. It was simply a wooden platform carrying a battery and a motor. Now it is on display at the Museum Boerhaave in Netherlands.

The first electric carriages were nothing but some fancy oddity, due to the backward battery technology. The carriage weighted about 6.6 lb and could carry loads of half of its own weight. It was powered by a primeval battery consisting a zinc plate and a copper plate sitting in a jar filled with dilute acid. The battery could provide fifteen minutes driving time before the current was exhausted.

Thanks to the invention of lead-acid rechargeable battery in 1859, the electric vehicles (EVs) became more practical. In 1881, the first electric vehicle powered by rechargeable batteries was demonstrated at the International Exhibition of Electricity in Paris. Using the updated battery technology, the electric vehicle reached a top speed up to around 10 miles per hour and the range of driving was extended to around 25 miles at the dawn of the 20th century.

At that time, the major rivals of the electric vehicles were the traditional horse carriages and the cars powered by steam or internal combustion engines (ICEs). Although the horse-drawn carriages were more dependable and sturdy, the accumulation of horse manure was a serious problem of towns. For a steam power car, the driver needed a long time to light up the loco and build the pressure, comparing to which, charging EV overnight became a minor inconvenience. The gasoline internal combustion engine vehicles, at its birth in 1985, also had it defects including difficult to start, and the need of a complex gearbox which is hard to maintain. On the other hand, electric cars has no emissions nor noise, drives easily and smoothly, and lower cost of upkeep. The limited range of electric cars did not seem to be a great issue since the journey of early mobiles took place in built-up areas, which were limited in 19th century.

In the golden time of electric vehicles from 1900 to 1910, the market share of EVs in the United States achieved almost 40%, comparing to 22% market share of gasoline cars [16]. EVs were running elegantly without noise and pollutants, and became popular with wealthy people. Even the Royalty became a fan and enjoyed the driving. Other urban residents could experience it by taking an electric cab, which had been proved a great success in America. Facing the high demand of electric cars, the manufacturers were working on the biggest drawback of the electric vehicles, the limited range. By 1903, Thomas Edison created a nickel-iron battery for transport applications. The new battery could supply energy for EVs to travel over 45 miles. In the same time, Ferdinand Porsche, the founder of the Porsche, created the first hybrid vehicle, the Semper vivus. The vehicle was powered by a battery and two combustion engines and the maximum speed reached 25 miles per hour.

Figure 1.2: Queen Alexandra's electric car for driving around the grounds of Sandringham House.

However, gasoline vehicles went from strength to strength and turned against electrics. The Kettering electric starter made the internal combustion vehicle as convenient as EVs. Henry Ford and his model T made gasoline cars widely available and affordable. The wide driving range and the cheap price made half of all the cars in the US to be model T by 1918. Meanwhile, better roads and new discovered crude oil reserves helped contribute to the disappearing of EVs in 1935. Over the next 30 years or so, cheap abundant gasoline and continued improvement in the internal combustion engine created little need for alternative fuel vehicles. In 1960s and 1970s, the soaring gas price brought EVs back into the public consciousness temporarily. Sebring Vanguard's CitiCar was one successful example. The wedge-shaped compact car with 50-60 miles driving range made the company the sixth largest U.S. auto maker by 1975. But the limited performance and driving range caused interest in EVs to fade again.

In the 1990s, new federal and state regulations created a renewed interest in EVs. Automakers began modifying popular vehicle models into electric vehicles while improving the performance and range. In 2000, Toyota released the Prius, the first mass-produced hybrid vehicle, worldwide. It became an instant success. The sale of Prius in 2015 achieved 18,000. In 2006, Tesla Motors announced plans of building a luxury electric sports car with a range of 200+ miles. Now three production lines of EVs are available, with a driving range more than 250 miles. In 2010, GM released the Chevy Volt with battery technology developed by the Department of Energy (DoE). In the same year, Nissan released the LEAF, the most popular EV model on the road by 2016.

There are typically three types of electric vehicles, plug-in EVs (PEVs), hybrid EVs and plug-in hybrid EVs (PHEVs). In a plug-in EV, the electric motor is powered by battery packs which are rechargeable by any external source of

Figure 1.3: Tesla model S: powered by a battery pack with capacity 85 k-Wh, driving range 265 miles, 0 to 60 mph in 2.5 seconds.

electricity, such as wall sockets. A hybrid electric vehicle, however, combines internal combustion engines with the electric motor. The ICE is used to charge the battery when the capacity is low or directly delivers power to the wheels when it is needed. Specially, a plug-in hybrid EV (PHEVs) combines the advantages of both EVs, which has internal combustion engine and a battery that can be charged by plugging into the electrical grid.

As of December 2014, the global number of EVs reached over 665,000, which represents 0.08% of total passenger cars. In 2015, total electric drive vehicle sales reaches over 498,000, which is 2.87% of total annual vehicle sales [39]. Norway, Netherlands, Sweden and the US are counturies with most EV adoption. In particular, the market sales shares of EVs reaches over 12% in Norway [68].

The accumulation in EV market also stimulates the investment in EV charging facilities. Through the end of 2014, there were more than 15,000 fast charging

Figure 1.4: Nissan leaf: the most popular EV model by 2016. Powered by a Li-ion battery pack with capacity 24 kWh, driving range 107 miles.

points and 94,000 slow charging points across the world. EV charging station stock more than doubled for slow charging points between the end of 2012 and 2014, and increased eightfold for fast charging points [68]. As of March 2016, there are more than 12,700 electric stations and 31,800 charging outlets deployed in the United States, most are installed in the east and west coast [140].

Despite significant growth, current EV adoption is well below President Obama's call to have 1 million EVs on the road by 2015. Over the long term, however, EVs will likely need to account for a significant share of the vehicle fleet in order for auto makers to achieve the new Corporate Average Fuel Economy standards of 54.5 mpg by 2025, up from current level of 27.5 mpg [1].

1.2 Importance of EV charging.

The anxiety of driving range has been the greatest barrier of wide EV adoption. Thus the convenient access of charging facilities and fast charging technology are essential for success of EVs. Residential are unlikely to purchase EVs until there are sufficient charging facilities on the road. On the other hand, commercial investor will build charging facilities only when there are enough charging demand, *i.e.*, EVs on the road. In this case, both consumers and investors are waiting for the movement of each other. Studies indicate that a positive feedback exists during the initial adoption stage when EV sales and EV charging services exceed a certain threshold; the market share of both EVs and EV charging services will likely grow at a substantially higher rate henceforth [151, 152]. This result justifies that the government needs to spend efforts to establish charging facility network at the launch time of EVs.

The United States, by 2016, has built about 12,700 charging stations with about 31,800 charging outlets, thanks to the direct and indirect investments of federal and local governments. For example, the Department of Energy (DoE) provided $230 million dollars from 2013 to establish 13,000 charging stations. There are also private initiatives in establishing networks of charging facilities. For example, PG&E announced in 2015 a plan to install 25,000 charging stations. Kroger, the largest grocery store owner of the U.S. has installed over 300 charging stations in key markets over the country [56]. Walmart and Kohl's also expanded their charging stations [54, 55].

Sufficient development of EV charging not only helps the successful launch of electric vehicles, but also is believed as a promising solution to the pollution and an essential component of the future smart grid.

The growth in energy demand and greenhouse gas emissions is mainly due to car ownership and economic development [62]. In 2014, 26% of the total greenhouse gas emissions in the USA came from the transportation, which have increased by 17% since 1990 [3]. The majority of greenhouse gas emission from transportation are CO2 emissions, which is due to the combustion of fossil energy in vehicles. The prevailing conjecture is that the electrified transportation with "clean" power generators such as wind turbines or solar panels is an efficient way to reduce this emission. The well managed charging of EV can be helpful to improve the shape of the overall demand, allow the power utilities to operate more efficiently, and maximally make use of the renewable energy. For example, the peak output of wind generators usually happens in the evening when the load is low. Without EVs, the output of wind generators is limited to maintain the stability of the system. However, large scale adoption of EVs makes it possible to fully use the wind energy to charge EVs during the night without any emission. In [44], it has been shown that, EV charging during the off-peak period will contribute to the adoption of renewable energy and largely reduce the emission of CO2 .

Besides the environmental benefits, the adoption of EVs can also provide potential services back to the power grid [80]. Vehicle to grid (V2G) services typically include spinning reserves, frequency regulation and peak power supply. Spinning reserves and frequency regulations are referred as ancillary services, which account for up to 10% of electricity cost, or about $12 billion per year in the U.S. [65].

Spinning reserves refer to additional generating capacity that can quickly response to the requirements of the independent system operator (ISO), typically within 10 minutes. When there is an unplanned event, such as loss of generation, the spinning reserve providers are required to supply power back to the grid. Spinning reserves are paid for by the amount of time the service providers are available. Traditionally, it requires generators to be "spinning" at low or partial speed so that they can act upon the request. For the charging services, however, this only requires having EVs plugged in to the system. When the spinning reserve is required, the EV charging station stops charging EVs or discharges the battery to power the system or simply . The requirements of spinning reserves are called occasionally, limited by the contract to 20 calls per year and 1 hour per call at maximum [82], which requires small amount of energy and makes it suitable for EV charging to participate in.

Frequency regulation accounts for over 80% of the cost of ancillary services. ISOs monitor the load, generation balance, and the frequency deviation of the grid and send out signals to service providers to follow. For example, when the power load is less than the generation and the frequency of the grid increases, ISOs will ask the charging stations, who participate in the regulation market, to consume more power. When the generation is less than the demand, ISO will ask the participants to consume less. The calls for regulation happen more frequently than those to spinning reserve (400 time a day) and require much faster response (less than one minute). The payment is measured based on how fast the response is and how well the demand of the charging station tracks the regulation signal sent by ISO. The frequency regulation market is also suitable for EV charging to participant in. The action of EV charging station is agile due to the fast response of batteries.

The peak power market is established to deal with times of high power consumption, *e.g.,* hot summer afternoons. Since the peak power time during one year lasts about a few hundred hours, it is economic to use some generators with small capacity cost, even with higher marginal cost, such as gas turbines. For EV charging stations, charging EVs during off peak hours and selling energy back to the grid during the peak hour may increase profit.

Given the possibility of the V2G service of EVs, social planners have more motivation to help the successful launch of EVs. Simply by allowing the EV charging stations to participate in the ancillary service and peak power markets, the social planner will help the grid to be cleaner, more reliable, and more efficient. Meanwhile, the option to participate in the power market may attract more investment in charging facilities. The charging cost would be lower not only due to the competition between charging stations, but also due to the extra income from the V2G services. The EV consumers enjoy the cheaper operation cost of EVs and other consumers have cleaner energy. Everyone wins.

1.3 Overview of technology: challenges and limitations.

Impacts and challenges

However, before the win-win situation comes true, multiple technical challenges need to be solved. It has been shown that, with no proper management, the large scale charging of EVs can impact demand peaks, reduce reserve margins, and increase prices. It may also lead to negative consequences on voltage control, power quality (harmonics and subharmonics), phase balance and relay protection [119, 106, 103, 29, 22].

The operation of EVs in a distribution system will be a challenging demand side management problem from the utilities' perspective since EV battery chargers represent sizeable loads. Numerous EV owners tend to arrive home from work within a narrow time period and immediately plug-in their vehicle to a charger during a time of already high peak demand. These un-managed charging activities could significantly change the load profile of the grid, increase system losses, cause inefficient dispatch, increase the real-time localized marginal price, and draw down the voltage level. Studies showed the increase in peak load can be as large as 80% and the drop of the voltage level can be more than 14% [22, 29]. According to the study of Oak Ridge National Laboratory, additional generation capacity is needed to satisfy the added demand when EV charging in the evening in most regions [59]. The authors of [127] show that uncontrolled EV charging may reshape the demand load and create new peak periods. The intensified peak load reduces the spinning margin and makes the system less reliable.

Second, the EV charging introduces extra demand burden to the transformers in the distribution network. A commonly used 25kVA neighborhood transformer serves the load of typically 5-7 households. The level 1 charger accounts for around 1/3 load of a household and the level 2 charger accounts for around 1 more household load. The load during the peak period may cause the transformer to be overloaded, which can decrease a transformer's expected lifespan by 20% [29, 120].

On the other hand, EV chargers draw AC power and convert it to DC, which produces harmonic distortion in the distribution system. The distortion in voltage and harmonics in current in distribution networks cause all kinds of problems, including excessive neutral current and transformer hot spots [104]. The harmonic distortion of different models of batteries varies. However, the impact of EV charging is additive. The charging processes of all EV chargers start at typically the same time in order to use the cheapest energy during the off-peak hour. Massive EV charging will not only affect the magnitude but also the phase angles of individual harmonic components [52].

Besides the negative impact on the grid, there are several commonly-cited barriers to EV and EVCS adoption. First, EVs are more expensive than their conventional gasoline vehicle counterparts. The manufacturer's suggested retail prices (MSRP) for the 2016 model of Nissan Leaf and Chevrolet Volt are $32,450 and $33,170, respectively, while the average price for a comparable conventional vehicle (*e.g.,* Nissan Sentra, Chevrolet Cruze, Ford Focus and Honda Civic) is between $16,000 and $18,000. A major reason behind the cost differential is the cost of the EV battery. As battery technology improves, the cost should come down. In addition, lower operating costs of EVs can significantly offset the high initial purchase costs. A study by EPRI in 2013 compares the lifetime costs (including purchase cost less incentives, maintenance, and operation) of vehicles of different fuel types and finds that under reasonable assumptions, higher capital costs are well balanced by savings in operation costs: EVs are typically within 10% of comparable hybrid and conventional gasoline vehicles [27].

The second notable barrier to EV adoption is the limited driving range. Plug-in electric vehicles have a shorter range per charge than conventional vehicles have per tank of gas, contributing to consumer anxiety of running out of electricity before reaching a charging station. The Nissan LEAF, the most popular battery electric vehicle in the U.S. has an EPA-rated range of 84 miles on a fully charged battery in 2016. The Chevrolet Volt has an all-electric range of 53 miles, beyond which it will operate under gasoline mode. This range is sufficient for daily household trips but may not be enough for longer distance travel.

The third barrier, closely related to the second, is the lack of charging infrastructure. By 2016, there are 12,700 charging stations in the U.S., comparing to 121,446 gas stations [137]. A large network of charging stations can reduce range anxiety and allow PHEVs to operate more under the all-electric mode to save gasoline. The installation of charging stations involves a variety of cost including charging station hardware, other materials, labor and permits. A public Level 2 charging station has 3-4 charging units and costs about $15,000 while a DC fast charging station costs over $50,000. These charging stations can be found at workplace parking lots, shopping centers, grocery stores, restaurants, vehicle dealers and existing gasoline stations. Owners of charging stations are often motivated by a variety of considerations such as boosting their sustainability credentials, attracting customers for their main business, and providing a service for employees. Charging stations are often managed by one of the major national operators such as Blink, ChargePoint, and eVgo.

The fourth barrier is long charging times. It takes much longer to charge EVs than to fill up gasoline vehicles. An EV may not be able to get fully charged overnight if it uses a regular 110 Volt electric plug (*e.g.,* it takes 21 hours for a Nissan LEAF to get fully charged) . To get faster charging, EV drivers either need to install a charging station at home or go to public charging stations. It takes 6-8 hours to fully charge a Nissan Leaf at a Level 2 charging station and only 10-30 minutes at a DC fast charging station. Unlike battery EVs, PHEV batteries can be charged not only by an outside electric power source, but by their own internal combustion engine as well. Having the second source of power in hybrid electric vehicles may alleviate range anxiety but the shorter electric range limits the fuel cost savings from EVs.

1.4 Broad classifications of EV charging methodologies

The great challenge and potential of EV charing problems have attracted plenty of attention to this area. The subject can be probed from different angles. An EV owner wants to minimize the total charging cost with respect to dynamic electricity prices [60]. The objective of a aggregator or a charging station operator is to maximize the total charging profit, facing stochastic EV arrivals, random charging demands, dynamic electricity prices, fluctuating ancillary service requirements and other system constraints [18, 132, 141, 73, 153, 148]. The distribution network operator is concerned about the stability and reliability of the grid and manages the charging of EV taking into account voltage deviation, efficient dispatch, demand deviation, system losses etc. [97, 145, 77, 47, 33].

EV charge scheduling policies can be classified into two categories, static(offline) and dynamic(online) algorithms, by the information available. Static scheduling algorithms require all information about EVs and the power system within the scheduling horizons, *e.g.,* the charging demand, arrival and departure time of all EVs, electricity price and so on. Dynamic scheduling algorithms only know the information about the power system up to the time the scheduling is made and the states of the EVs that have already arrived at the station.

Instead of knowing the future electricity price and charging requirement of EVs, dynamic scheduling algorithms may assume that statistics of this information is available, such as the distribution of the EV arrivals. From the perspective of aggregators or a charging station operator, the centralized control problem may be reasonable. That is, the scheduler has direct access to control the charging of multiple EVs in the charging station. The scheduler can determine when to activate or deactivate the chargers and control the charging rate of each individual EV. As an independent system operator (ISO), it may be more reasonable to consider the decentralized framework. The ISO does not have direct control on the charging of each individual EV. Instead, it can affect the charging behavior of EVs via adjusting the charging price or broadcasting similar signals. The individual EV owner and charging station operator would respond to the signal by modifying their charging profiles.

CHAPTER 2

EV CHARGING MODELS AND ENERGY MANAGEMENT SYSTEMS

In this chapter, we will present frameworks for EV charging in different contexts. We first review the charging technology and behavior of EV batteries in Section 2.1 and 2.2. Then the EV charging frameworks in different levels are presented, including charging at home, in a distribution system, and in a public charging station. In the end, the interaction between the EV charging and the grid is presented in a vehicle-to-grid framework.

2.1 Charging Standards

There are typically three levels of charging stations: Level 1, Level 2 and DC Fast Charge (DCFC) [34] [1]. These levels denote different charging rates of an electric vehicle's battery. The use of higher level charging stations will dramatically reduce charging time, but the building cost is also significantly higher. The characteristics of three charging levels are summarized as follows.

Table 2.1: EV charging classification

	Charging time	Voltage/Current	Cost	Availability
Level 1	Up to 20 hours	110V/15A		Residential
Level 2	Up to 7 hours	240V/40A	$2000-$15000	Residential/Public
DC	30 minutes	480V/125A	$50,000-$100,000	Public

Level 1 charging is the most common form of battery recharging to satisfy all

[1] Level 1, 2, and DCFC are the most widely deployed classes of chargers. For the information of other classes, the information can be found at: http://standards.sae.org/j2836/2_201109/

of a driver's need. It provides charging from a standard residential 110 volt AC outlet. Most electric vehicle manufacturers include a Level 1 cord set so that no additional installation is required. On one end of the cord is a standard NEMA connector (for example, a NEMA 5-15, which is a common three-prong household plug) and on the other end is an SAE J1772 standard connector. The SAE J1772 connector plugs into the car's J1772 charge port and the NEMA connector plugs into a standard NEMA wall outlet. The power consumption is approximately equal to that of a toaster, leading to about 4 miles of range per hour of charging. Overnight charging at 110V can replenish about 40 miles of driving range. However, a completely depleted battery could take up to 20-22 hours to completely recharge.

Figure 2.1: Level 1 charger

Level 2 equipment uses the same connector and charging port as level 1. However, level 2 equipment needs a dedicated electrical circuit to improve safety thus requiring professional electrical installation. The charging station can be installed in either residential houses or in public charging stations. The cost of installing a single port in a residential house is a little bit more than $1000, over half of which is the hard ware cost [72]. The cost of installing a level 2 charging station in public stations varies from $2,000 to $15,000 with the number of ports, station features and brands. The power consumption of Level 2 charging is like a clothes dryer in the house. It will supply up to approximately 15 miles of travel for one hour of charging to vehicles with a 3.3kW onboard charger, or 30 miles of travel for one hour of charging for vehicles with a 6.6kW onboard charger. To fully charge an EV, it would take around 7 hours. Now, level 2 is the dominating class of public charging stations.

DC fast charging requires commercial grade 480V AC power circuits whose power consumption equals approximately 15 average size residential central air conditioning units. So it is only available in public charging stations. DCFC transforms the AC power to DC and supplies up to 40 miles of driving range for every 10 minutes of charging, or a full recharge in 30 minutes. Among the DCFCs deployed in the USA, there are three types of DC fast charging systems: SAE J1772 Combo used by Chevrolet and BMW, CHAdeMO used by Nissan, Mitsubishi, Toyota, and Fuji, and Tesla's supercharger. The cost of a DC charging station is between $50,000 to $100,000 due to the expensive hardware and the frequent need to install a new 480V transformer.

2.2 Charging behavior of an EV battery

To study the EV charging behavior or design the battery management system, we need to understand how the batteries of EVs are charged and discharged. There are four kinds of batteries that are commonly used in electric vehicles,

Figure 2.2: Level 2 charger

Lead-Acid, Li-Ion, NiMH and Ni Cd [139]. The typical discharging behavior curve of a battery is presented in Figure 2.4. The terminal voltage of the battery versus the state of charge (SOC) shows nonlinear behavior. When the battery capacity is almost full, the voltage drops quickly to a steady level as the battery discharges. Then the battery enters a linear range. When the capacity is as low as 20%, the battery enters the other nonlinear zone, in which the terminal voltage drops exponentially.

The modeling of battery has been studied extensively. The most important

Figure 2.3: DC charger

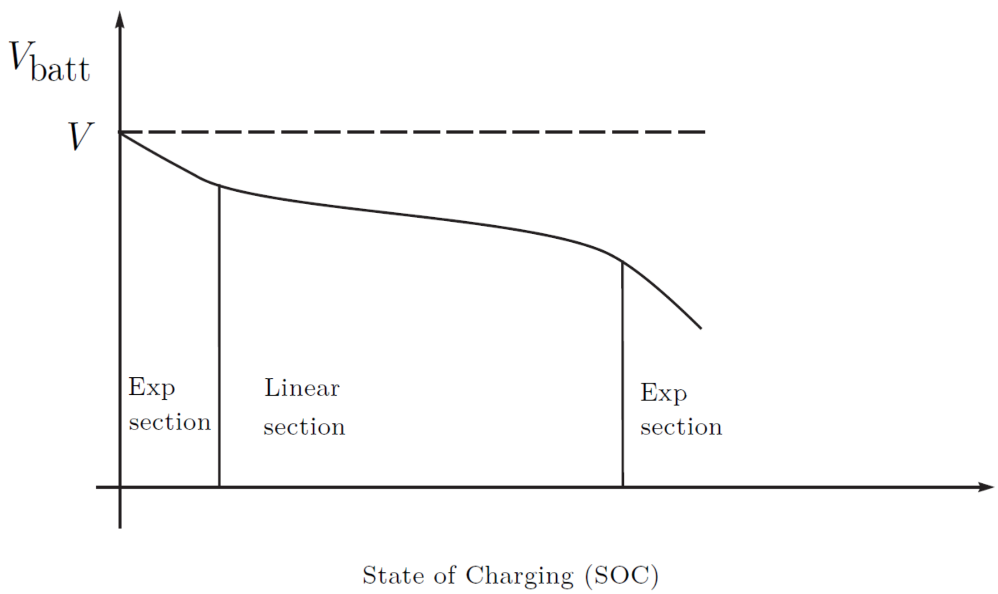


Figure 2.4: Discharge curve

articles in the field date back to 1960s. Shepherd first proposed an equation description of battery discharge and the method to fit the parameters [129]. If all factors except polarization is ignored, then the battery voltage V_{batt} is defined as

$$V_{\mathrm{batt}} = E_0 - K\frac{Q}{Q - it}i$$

where V_{batt} is the battery voltage, E_0 the battery constant voltage, K the polarization resistance, Q the battery capacity, i is the battery current, and $it = \int idt$ the actual battery charge (state of charging). To include the internal resistance, the voltage equation becomes

$$V_{\text{batt}} = E_0 - K\frac{Q}{Q - it}i - R \times i$$

where R is the internal resistance.

The above formula can describe the linear zone well. However, the initial drop at the beginning of a battery discharge is not included. Thus an extra exponential term $A\exp(-B \times it)$ is added to correct for the difference, where A and B are empirical constants. The final equation is as follows.

$$V_{\text{batt}} = E_0 - K\frac{Q}{Q - it}i - R \times i + A\exp(-B \times it).$$

Tremblay & Dessaint extend Shepherd's model and propose detailed models for four different kinds of batteries taking into account the open circuit voltage as a function of SOC [139]. The models are summarized as follows.

- Lead-Acid

  Discharge: $V_{\text{batt}} = E_0 - R \times i - K\frac{Q}{Q-it}(it + i^*) + Exp(t)$

  Charge: $V_{\text{batt}} = E_0 - R \times i - K\frac{Q}{it-0.1Q}i^* - K\frac{Q}{Q-it}it + Exp(t)$

  where V_{batt} is the battery voltage, E_0 the battery constant voltage, K the polarization resistance, Q the battery capacity, $it = \int idt$ actual battery charge (state of charging), i battery current, i^* filtered current, and $Exp(t)$ satisfies

  $$dExp(t)/dt = B|i(t)|[-Exp(t) + Au(t)],$$

  where $u(t)$ is the charge and discharge mode. $u(t) = 1$ is the charge mode and $u(t) = 0$ is the discharge mode.

- Li-Ion

$$\text{Discharge:}\quad V_{\text{batt}} = E_0 - R \times i - K\frac{Q}{Q-it}(it + i^*) + A\exp(-B \times it)$$

$$\text{Charge:}\quad V_{\text{batt}} = E_0 - R \times i - K\frac{Q}{it-0.1Q}i^* - K\frac{Q}{Q-it}it + A\exp(-B \times it)$$

- NiMH and NiCd

$$\text{Discharge:}\quad V_{\text{batt}} = E_0 - R \times i - K\frac{Q}{Q-it}(it + i^*) + Exp(t)$$

$$\text{Charge:}\quad V_{\text{batt}} = E_0 - R \times i - K\frac{Q}{|it|-0.1Q}i^* - K\frac{Q}{Q-it}it + Exp(t)$$

These battery models describe the charging/discharging curves. Using these detailed models of battery charging, one can carefully investigate the charging behavior of an individual EV and the impact on the grid. However, the detailed model of batteries does not help when considering the large scale of charging.

2.2.1 A Simplified EV Charging model

For the purpose of large scale charging, the detailed battery model is usually linearized and simplified. In this setting, the SOC of electricity vehicles is assumed to be linear with the charging time or the consumed energy. This assumption is acceptable since the exponential areas of the battery are relatively small and the charging demand of EVs usually falls into the linear section.

Here we present a simplified EV charging model. The charging request from an EV J_i is specified by the tuple $J_i = (r_i, d_i, j_i)$, where $r_i \in \mathbb{R}^+$ is the time when the EV arrives at the charger and the charging can begin, $d_i \in \mathbb{R}^+$ the deadline by which the charging should be completed, $j_i \in \mathbb{R}^+$ how much the consumer requires to charge.

Nowadays, the control of chargers is often characterized as on/off operations. When the scheduler activates one charger, the EV attached to the charger is charged at a fixed charging rate; when the scheduler deactivate the charger, no power flows through. When the charging rate is constant, the charging demand can be measured by the charging time instead of the energy by dividing the energy by the charging rate times the energy efficiency.

Figure 3.2 gives an example of a linearized EV charging model. The deadline of EV J_i satisfies $d_i \geq r_i + j_i$ at its arrival, where $j_i(t)$ is the remaining charging time to be completed at time t. The leading time $T_i(t)$ and laxity $l_i(t)$ at time t are defined in (2.1).

$$T_i(t) = (d_i - t)^+, l_i(t) = (T_i(t) - j_i(t))^+. \tag{2.1}$$

where $b^+ = \max\{0, b\}$.

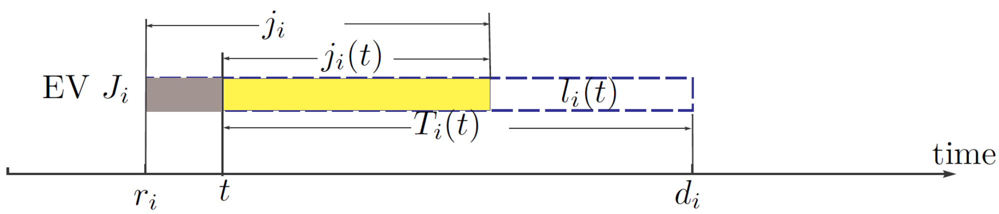


Figure 2.5: EV attributes.

After introducing the charging models of electric vehicles, we present three frameworks of EV charging in different contexts.

2.3 Home charging models and impacts on distribution systems.

The most commonly used charging method would be charging at home using level 1 or level 2 charging outlets. In this section, we present the charging framework of a single EV in a residential house. Figure 2.6 shows the main architec-

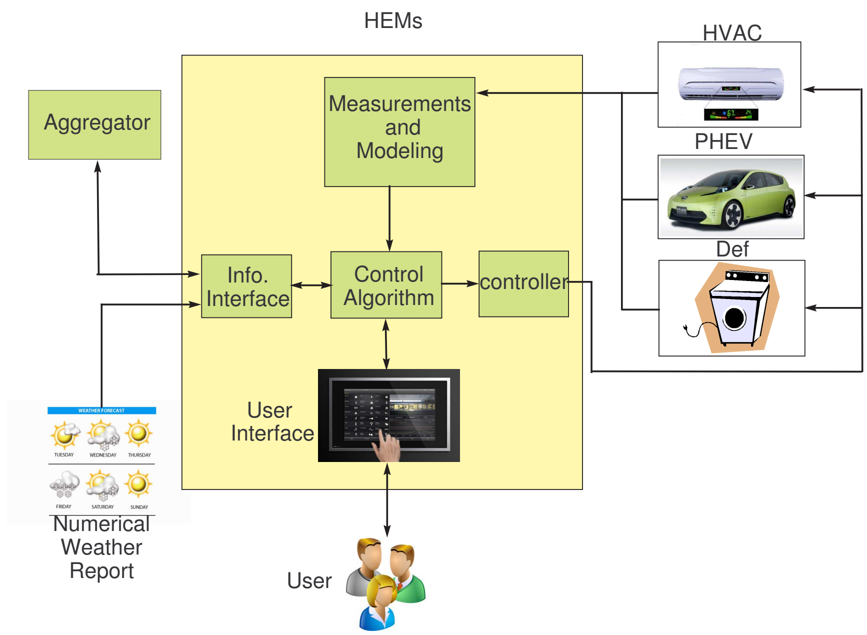


Figure 2.6: Architecture of home energy management system

ture of a home energy management system (HEMs) [150]. A complete home energy management system should be able to optimally control the charging of EVs and other appliances according to the desire of users and the information collected from the electricity aggregator and other sources. The complete HEMs should include the following functions.

- Direct Control: Sends the real time control signal to EVs and other end-use devices or controls the operation status directly.
- Measurements, Modeling and Prediction: Takes measurements of loads and models the behavior of EVs as well as the human activity patterns. Predicts into the future about the behavior of EVs and other information.
- User Interface: Provides customers with statistics on the power usage and allows users to adjust the settings.

- Information Interface: Exchanges price, power profile and weather information with aggregators and numerical weather stations.
- Control Algorithm: Optimally schedule the EV charging and control of other devices to maintain the comfort level of users subject to power limits, budget constraints and physical constraints.

Within this architecture, users upload the charging amount of EVs, the deadline at which EVs leave home, and other daily load profile through HEMs and aggregators determine and release the real-time electric price or direct control signal based on the profile collection. Taking in price signals and numerical weather reports, HEMs schedules the charging of EVs and allocates power to other end devices to maintain the comfort level and finish task required by users.

2.3.1 Charging scheduling of single EV

In this subsection, we present a generic formulation of the scheduling problem of single EV charging. Here we assume the state of charging (SOC) is linear in the power consumptions and discharging is forbidden. Since the real-time electric price is released every 5 minutes or 15 minutes, we assume time is slotted[2] and indexed by t. Assume the scheduler knows the charging cost and the EV is attached to the charger from $t = 0$ to $t = T - 1$. The scheduling problem can be

[2] In this monograph, we use "(t)" to indicate the continuous time and "[t]" to indicate the discrete time

formulated as a optimization problem as follows.

$$\begin{array}{ll} \min & \sum_{t=0}^{T-1} C_t(u[t]) \\ \text{subject to} & \sum_{t=0}^{T} u[t] = j \\ & 0 \le u[t] \le u^{\max}, \forall t. \end{array} \tag{2.2}$$

where $u[t]$ is the charging rate of the EV within time slot t, $C_t(\cdot)$ the charging cost as a function of the charging rate, j the required charging amount, T the plug-in duration (or the lead time on arrival), and $u^{\max}$ the maximum charging rate.

In general, the charging cost function $C_t(\cdot)$ would include the energy cost of purchasing power, the cost offset by the renewable energy, and the regulation price. If the cost function is linear, *i.e.*, $C_t(u[t]) = c[t]u[t]$, the problem (2.2) becomes a linear program, which can be easily solved by "Simplex" method or other commercial solver.

Now we present some characteristics of the optimal solution of (2.2) under the linear cost assumption. In this case, assume there exists an optimal charging profile $\{u[t]\} = \{u[0], \cdots, u[T-1]\}$ and $u[t_1], u[t_2] \in \{u[t]\}$ such that $0 < u[t_1], u[t_2] < u^{\max}$. It follows that

$$\begin{aligned} C_{t_1}(u[t_1]) + C_{t_2}(u[t_2]) &= c[t_1]u[t_1] + c[t_2]u[t_2] \\ &= c[t_1](u[t_1] + u[t_2]) + u[t_2](c[t_2] - c[t_1]) \end{aligned}$$

If the sum of power, $(u[t_1] + u[t_2])$, is kept constant, then this becomes a linear equation with respect to $u[t_2]$. The extreme occur only at both ends of the range of $u[t_2]$. Thus there exists an optimal scheduling such that $u[t]$ should be either 0 or $u^{\max}$[3]. This result implicates that charging control should be an on/off control at the maximum charging rate to minimize the charging cost.

[3]Assume the required charging amount j is a multiple of $u^{\max}$.

In (2.2), the future charging cost function $C_t(\cdot)$ is assumed known by the scheduler. However, the cost is in general stochastic. The charging cost is basically the electricity price offset by the local renewable energy. The electricity price is determined by the operation status of the power grid and the demand. The renewable energy is significantly affected by the weather. The randomness is one of the major difficulties in the scheduling problem.

Different approaches have been proposed to deal with the randomness in charging cost, EV arrival and charging capacity, including dynamic programming [60], Markov decision processes [153], model predictive control [48], and so on. Some of these approaches will be discussed in the following chapters.

2.4 Large Scale Charging in Distribution System

In this section, we discuss the framework of the large-scale EV charging in a distribution system. As shown in Figure 2.7, EVs are attached to residential houses or public chargers distributed in a wide area. A service aggregator, who purchases power from the grid and supply the charging services, connects the power grid and EV customers. EVs communicate the charging scheduling and charging prices with the aggregator through the chargers. The aggregator collects information of multiple chargers and receives the information about the power grid, including the power limit, wholesale energy price, and ancillary service requirement. Based on the information, the aggregator optimally schedules the charging of EVs by directly sending the control signal or affects the charging decision of EVs indirectly by price signals.

For large-scale EV charging, there are two ways of scheduling, centralized (direct) control, and decentralized (indirect) control. The centralized control framework assumes the scheduler has direct access to each EV and can make a direct decision on the charge scheduling. This structure requires more computational power and information communication between EVs and the aggregator. On the other hand, this framework is more likely to achieve optimality. In the centralized framework, assuming the future information about the EVs and the power grid is available, it is shown that the optimal solution satisfies the valley-filling property [98]. A valley-filling aggregate charging schedule schedules more EVs when the total electricity demand (or equivalently, the electricity price) is low to fill the valley and minimize the charging cost.

The decentralized framework shares the same presence and goals of the aggregator/scheduler with the centralized control. The difference is the information and where the charging schedules are calculated. In the decentralized charging framework, the scheduler acts as a coordinator that designs the incentive structure so that the scheduling of EVs minimizes the total cost or achieves other goals of the aggregator. Each EV makes provisional charging decisions and determines its charging schedule according to the aggregator incentives. The decentralized framework distribute the computation burden from the scheduler to individual EV chargers. Each charger solves a much smaller scale problem which relieves the computation cost brought by the large population of EVs. Another benefit is the privacy of the EVs. EVs may be reluctant to upload details of their demand directly to the aggregator. In the decentralized framework, no information but the proposed charging profile of each EV is given out. Despite of these advantages, the stability and optimality of the decentralized charging remain unclear. EVs tend to cluster the charging schedule to the valley of the price which potentially brings up the total cost and leads to unstable scheduling process. One approach to overcome the instability is to impose penalty of changing charging profiles in the incentives. Besides the charging cost, the aggregator penalizes the EVs who dramatically changes their charging plan or deviates from the trend of the total charging profile. Detailed algorithms can be found in [98], [48], and [84]. Under some conditions, the outcome of the decentralized algorithms converges to the optimal solution under the centralized framework.

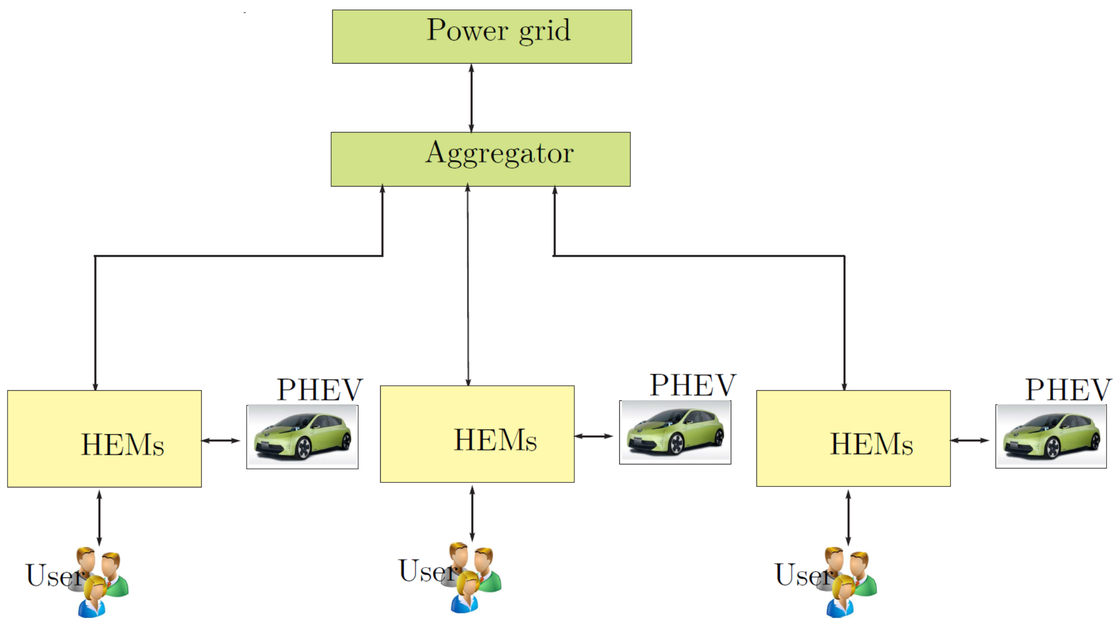


Figure 2.7: Architecture of home energy management system

2.5 Large scale charging at public facilities

In this section, we present the framework of EV charging in public charging facilities. In US, there are several major parties that invest in public EV charging stations. The government spends in charging stations in public facilities such as hospitals and schools due to the environmental consideration. The business runners of grocery stores and shopping malls build charging stations near their business to attract customers for the primary business, provide a bonus service for employees, and boost the sustainability credentials. The major national operators of charging stations, such as ChargePoint and eVgo, invest in charging service targeting the rapid growing EV market. The manufactories of EVs build charging stations to boost the market share of their products and provide after sale services to loyal consumers.

The placement of charging facilities affects the favorability of each station. Factors that influence this may include but are not limited to variations in accessibility and availability of service other than charging, *e.g.,* tyre inflator and mini market. Accessibility refers to how easy it is for consumers to access the charging station and potentially affects the volume of the passenger flow. A similar discussion is presented for gas stations by Salop [125] and Hotelling [66]. For example, a site at a workplace parking lot may be more attractive than a location that is less frequently visited by consumers. Ancillary services refer to other services that a charging station may provide such as vehicle repair and supermarkets. Kroger, the largest grocery store owner of the U.S., has installed over 300 Level 2 and DC fast charging stations in the major markets over the country [56]. Walmart and Kohl's also expanded their charging stations [54, 55]. These public charging station locations are also favored by EV owners since they can charge while shopping.

The intelligent energy management system (iEMS) architecture for public charging stations is illustrated in Figure 2.8. The scheduler here is the owner or the operator of the charging facilities, it is natural to consider centralized con-

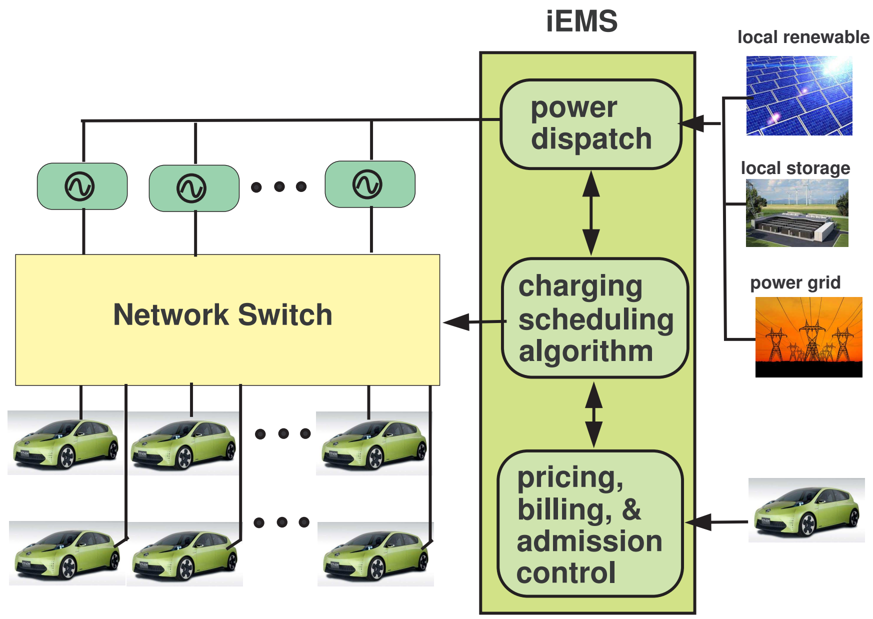


Figure 2.8: Architecture for network switched charging and iEMS.

trol. The hardware system of the proposed iEMS includes a dispatcher that delivers power from a mix of energy sources–local energy (*e.g.*, renewable energy and local storage) and purchased electricity from the grid–to tens or hundreds of chargers. By sending out the control signal, the scheduler activates and deactivates the chargers connected to EVs admitted to the facility to serve more urgent or more profitable requests.

The iEMS is run by the software system that makes engineering and economic decisions. At the core of the software system for the iEMS is the charge scheduling algorithm, which is the focus of this monograph. The scheduler (i) sets the connections of the switch so that a subset of EVs are charged by the available chargers, and (ii) determines the admission of new EVs based on its charging demand and the system operating condition. The software system also has to handle billing, other ancillary services and possibly the forecast of available renewable in the future.

2.6 Interaction between EV charging and the grid

Besides the charging station, EVs are viewed as a possible solution to integrating the renewable energy to the power grid. One of the impact of EVs on renewable energy is load shifting.

The typical daily demand curve is shown in Figure 2.9. The daily peak happens after 6 PM and falls to the valley around 3 AM. The big ramp causes reliability problem to the grid. The existence of power peak makes reserve generation necessary which costs much higher than a regular generation. Another issue is the peak of renewable generation and the peak of demand do not overlap naturally. The peak of wind power usually happens in the evening, and solar energy reaches a peak around noon. Without sufficient storage, the output of renewables will be limited due to the grid consideration.

However, large-scale charging of EVs would play a role in load shifting. At the peak of renewable energy, energy management systems can schedule more charging of EVs and consume the output of wind and solar. When the demand reaches the peak, EVs may supply power back to the electricity grid and relieve the burden of the reserves. In this way, the charging of EVs shifts the load peak to the valley and “shaves” the demand curve. The flat curve would require fewer reserves, consume more green energy and make the grid more reliable.

The other benefit of V2G would be ancillary services provided by EVs. The output of renewables, such as solar and wind, is typically random and uncon-

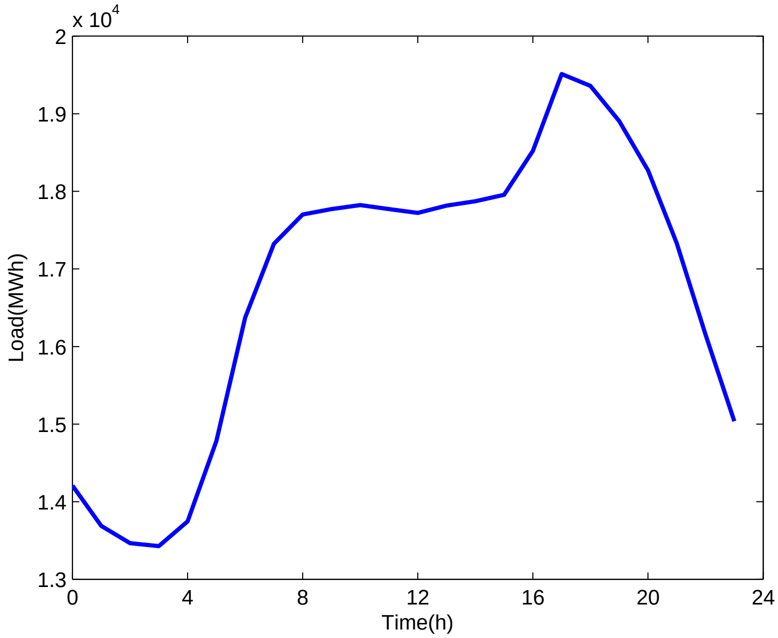


Figure 2.9: Load curve of New York ISO, November 8th, 2016

trollable. As shown in Figure 2.10, the output of renewables changes rapidly and randomly due to the weather. These fluctuation together with demand oscillations cause the electricity frequency deviations and harm the generators, transformers and end use devices. Usually, renewable generators need large storages, expensive devices, to hedge this uncertainty. However, when large-scale EVs take part of the ancillary service market, this oscillation can be dealt by EV batteries. When the renewable output dips, the EV charging station can suspend charging and reduce the power consumption to help to restore the frequency. In this way, the renewable generators do not need to build large capacity storages. The revenue collected from the ancillary services subsidize the EV consumers and makes the charging more economical. Meanwhile, the power grid receives cleaner energy and has a more robust system.

Figure 2.11 illustrates an example of EV charging station participating in the

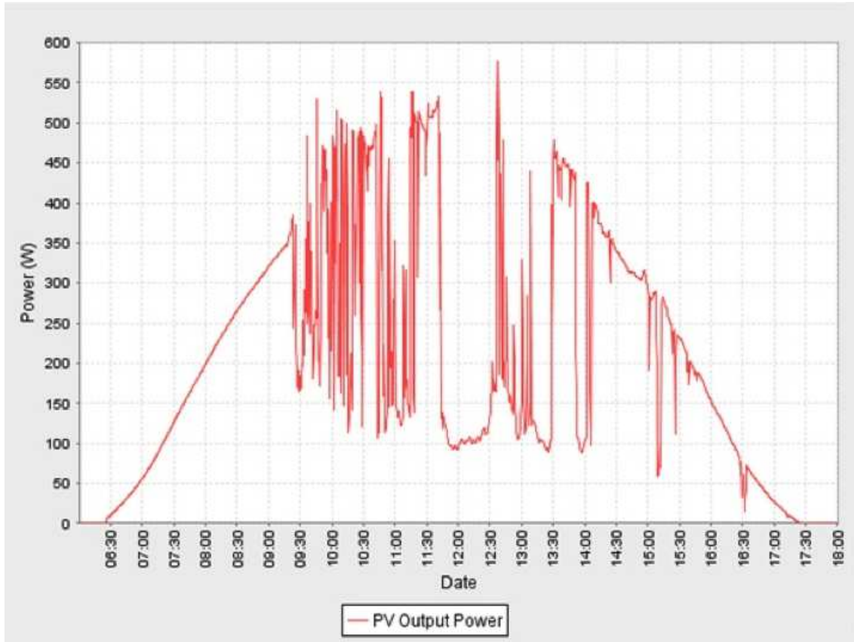


Figure 2.10: Solar power output in a typical day

ancillary service market. In the day-ahead market, the charging station needs to submit the demand bids to the ISO. The bids include an expected demand curve of the next day and a range within which the charging station promises to adjust the power consumption in response to the request of ISO. In the real-time, the ISO releases the electricity price and the regulation signal which the charging station is required to follow. The ancillary service revenue will be paid from the ISO to the charging facility according to the adjustment range and the accuracy and speed of the response.

In the following chapters, we will present the detailed formulation and approaches of EV charging in a distribution system, in a public charging station, and in the vehicle-to-grid service.

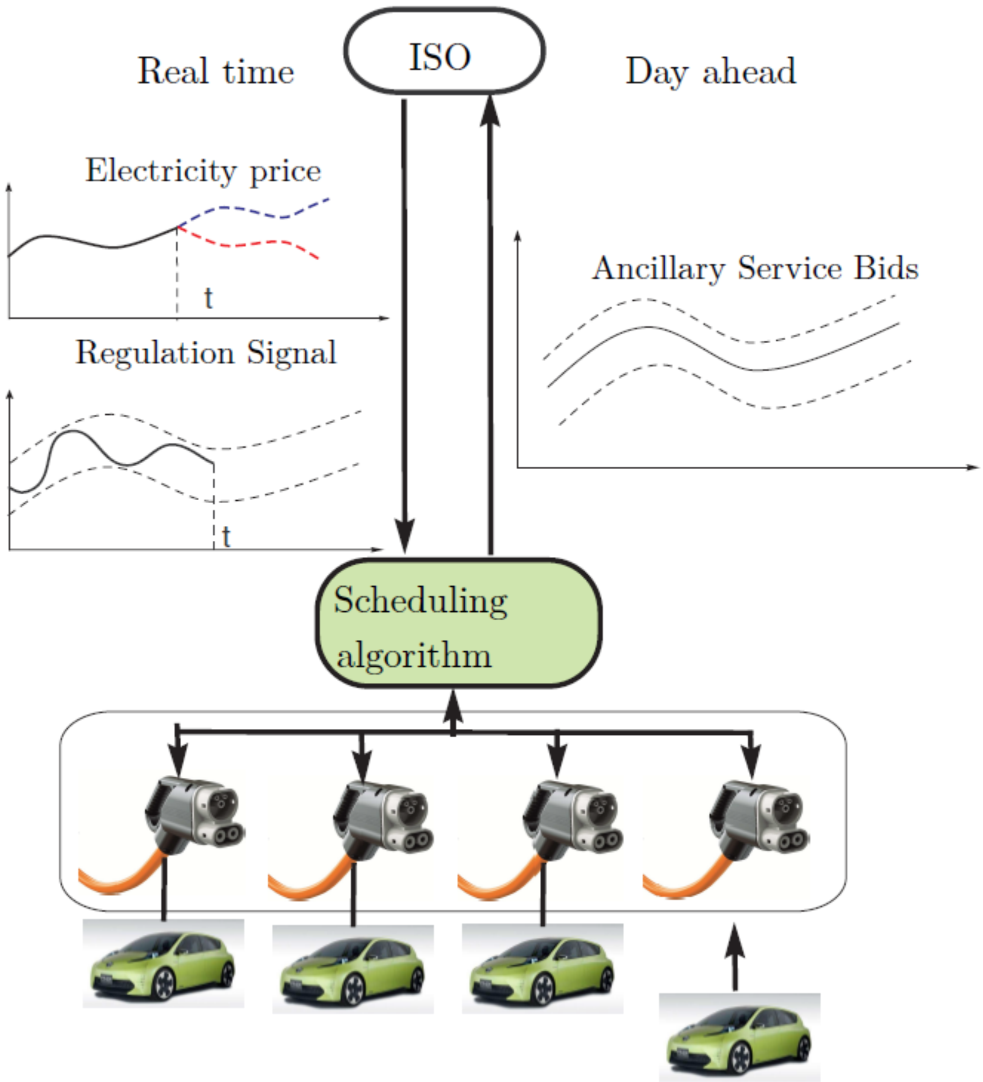


Figure 2.11: Charging station participating demand response.

CHAPTER 3

AN ONLINE CENTRALIZED APPROACH OF LARGE-SCALE EV CHARGING

In the off-line charging problems, the charging algorithm requires the information of the future, *e.g.,* the charging cost function, the arrival and departure time of EVs. However, these kinds of information are hard to obtain, sometimes even impossible. The future electricity price depends on the operation status of generators, the output of the renewables, the state of the transmission network, and the demand of other consumers. The arrivals of EVs and charging demand are determined by individual human behaviors. These kinds of uncertainty make it extremely hard to forecast.

Instead of requiring to know the future exactly, we consider a stochastic scheduling problem, in which, the scheduler is assumed to know only the current information of electricity price and states of EVs that are already in the charging station. The information about future charging cost and EV arrivals, besides some statistics and probability distributions, is not known until the time it is revealed. Under this assumption, the stochastic scheduling problem is to seek a policy that achieves the average optimality overall randomness.

The stochastic scheduling of public charging facilities faces a different set of technical challenges from those associated with the convex optimizations discussed in the previous chapters

First, there is a high level of uncertainty in charging demand. EVs arrive at a charging facility randomly, each with stochastic demand and arbitrary deadlines that are revealed to the scheduler after the arrival. This randomness makes it difficult for the scheduler to meet consumer needs.

Second, the cost (or the profit) of the service provider may be stochastic. For instance, the service provider may participate in the wholesale electricity market and is subject to real-time price fluctuations. Also, the service provider may integrate local renewable energy such as solar with intermittent generation. Here we assume the charging station is a price-taker, which means the charging cost will be stochastic and independent of the total demand at the time of charging.

Third, fast charging techniques are usually used in the public charging stations. The level 2 charger and DC fast charger consume significant power and may have detrimental effects on power system reliability [22, 133] and life time of devices [29, 120]. Thus it is necessary to impose a power limit of the public charging station considering the capacity of transformers and the thermal limit of the transmission line. This power limit translates to a constraint of the number of simultaneous charging EVs. Due to this capacity limit, it may be infeasible to finish charging of all EVs and a penalty may incur due to the economic and reputation loss.

Thus, the objective of the scheduler is to maximize the expected discounted or average charging profit (or equivalently, to minimize the charging cost and incompletion penalty) subject to the power limit. However, the energy management system that schedules EV charging operates in real time. Therefore, the scheduling algorithm must be scalable on the size of the charging facility, which rules out the use of brute-force optimization techniques. In fact, sometimes the optimal solution does not exist. Thus people are interested in some simple heuristics those are in general not optimal, but scalable and perform rea-

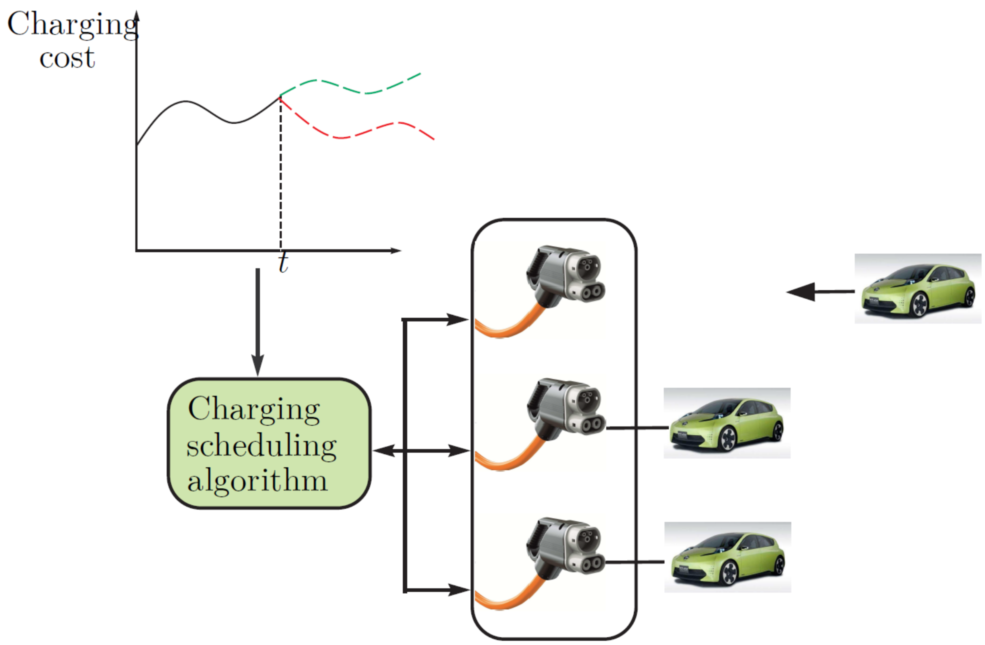


Figure 3.1: Architecture of a charging station

sonably well.

3.1 An MDP Formulation of Stochastic Deadline Scheduling

We now formulate the EV charging problem as a stochastic deadline scheduling problem subject to charging capacity constraints.

Figure 3.1 shows a schematic of an energy management system at an EV charging facility. We assume that the facility has N parking spots, each with a charger. The charging rate of each charger is assumed identical and fixed. This is likely to be a case of practical significance, as charging rates of EVs are constrained by their design specifications and charging station technology.

The electricity price from the ISO is updated every 15 minutes or every hour. So it is reasonable to consider discrete time framework, where the time is slotted and indexed by t. At the beginning of each time slot, new EVs arrive at the facility randomly and the scheduler randomly assign a parking spot to it. The EV owner communicates the charging demand j_i, and the deadline for completion d_i to the scheduler. Since the charging rate is fixed, the charging demand j_i is measured in charging time. The scheduler receives the information and updates the charging cost and the state of chargers in the system. Then the scheduler makes a decision on which chargers to activate or deactivate in the current slot. The power limit translates to a constraint on the number of EVs that are simultaneously being charged. At the end of each time slot, EVs that hit their deadlines leave the charging station regardless whether the charging is completed.

The assumptions of this chapter are summarized as follows; they are approximations of practical operating conditions and are made for tractable analytical developments.

A1. The time is slotted, indexed by t.

A2. The EV arrival process of each charger is independent and identically distributed (i.i.d.) with probability mass function $Q(T, j)$ where T is the lead time defined by the number of time slots before the deadline, and j the charging demand defined by the number of time slots required for completion. Here $Q(0, 0)$ is the probability that no EV arrives. Upon arrival, an EV reveals (T, j) to the scheduler.

A3. At each time, at most M EVs can be simultaneously charged. The activation/deactivation of chargers incurs no cost.

A4. If an EV is charged in time slot t, it generates a unit payment to the charging station and incurs a time varying cost $c[t]$. We assume that $c[t]$ is a homogeneous finite state Markov chain whose evolution is independent of the actions of the scheduler[71, 87].

A5. If the charging demand of an EV is not fulfilled by the deadline, a penalty defined by a convex function of the amount of unfinished charging demand is imposed on the scheduler at the end of the deadline.

Some comments and clarifications on these assumptions are in order. The i.i.d. arrival assumption in A2 is somewhat limiting but necessary for obtaining index policies. This is also consistent with the standard Poisson arrival case when the the arrived EV is randomly assigned to a charger in the charging station. A2 implies that when an EV arrives at a charger that is occupied by an unfinished EV, the newly arrived EV is dropped, which seems unreasonable since the EV could have been assigned to an open charger (if it exists). Asymptotically when $N \to \infty$, there is no loss of performance by imposing these assumptions.

Assumption A4 assumes that the marginal payment from each EV is the same. This can be generalized easily to the case that when the price of completing the EV is a function of the deadline—the so-called service differentiated deadline scheduling problem [12]—by including the initial lead time in the state of the EV. In this case, the charging station will assign different marginal charging price to EVs with different deadlines (or laxity) at their arrival. The initial deadline or laxity is included in the state of the EV and the reward function will be a function of this state. The analysis and results stay the same.

Next, we formulate the constrained MDP by defining the state, the action of the scheduler, the state evolution, the reward, the constraints, and the decision policy.

We now define the constrained MDP by defining the state, the action of the scheduler, reward, the state evolution, constraints, and the decision policy.

3.1.1 State space

Consider first the state of the i-th charger in the station. Let $T_i[t] \triangleq d_i - t$ be the lead time to deadline d_i, and $j_i[t]$ is the remaining charging demand measured in charging time, as illustrated in Figure 3.2.

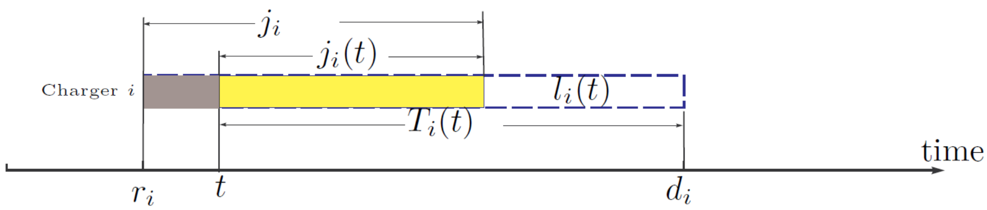


Figure 3.2: An illustration for the charger's state. r_i is the arrival time of an EV at charger i, d_i the deadline for completion, $j_i[t]$ the amount of charging to be completed by d_i, $T_i[t]$ the lead time to deadline.

The state of the i-th charger is defined as

$$S_i[t] \stackrel{\Delta}{=} \begin{cases} (0,0) & \text{if no EV waits at the } i\text{th charger,} \\ (T_i[t], j_i[t]) & \text{otherwise,} \end{cases}$$

The state of the charging cost is denoted by $c[t]$ which is assumed to form a Markov chain with transmission probability $P = [P_{k,k'}]$.

The state of the MDP is defined by the states of all chargers and the charging cost $c[t]$ as $S[t] \stackrel{\Delta}{=} (c[t], S_1[t], \cdots, S_N[t]) \in \mathcal{S}$ and $\mathcal{S}$ the state space.

3.1.2 Action

The action of the scheduler in time slot t is defined by the binary vector $\mathbf{a}[t] = (a_1[t], \cdots, a_N[t]) \in \{0, 1\}^N$ where $a_i[t] = 1$ means that charging at charger i is activated, for which the charger is referred as *active*. The complement, $a_i[t] = 0$, is when charger i is *passive*, *i.e.,* charging is suspended. For convenience, we allow a charger without an EV to be activated, in which case the charger receives no reward and incurs no cost.

3.1.3 State evolution

We assume that the charging cost $c[t]$ evolves as an exogenous finite state Markov chain with transition probability matrix $P = [P_{k,k'}]$. Given the current charging cost $c[t] = c_k$, the next charging cost will be $c[t+1] = c_{k'}$ with probability $P_{k,k'}$. The evolution of the charging cost is independent of the actions taken by the scheduler. The Markov chain of charing cost $c[t]$ can be illustrated in Figure 3.3.

The evolution of chargers' states depend on the scheduling action $\mathbf{a}[t] = \{a_i[t]\}_{i=1}^N$. When $T_i > 1$, the EV is not leaving the system by the end of this time slot and the lead time will decrease by 1. If the EV is not fulfilled, the remaining charging demand will reduce by 1 if the charger is active $a_i = 1$, and remain the same otherwise. If the EV is fulfilled, we assume the charger can also be activated. However, no energy will be delivered and no charging cost will incur. Given $a_i[t]$, the state evolution is stated as follows.

$$(T_i[t+1], j_i[t+1]) = (T_i[t] - 1, (j_i[t] - a_i[t])^+).$$

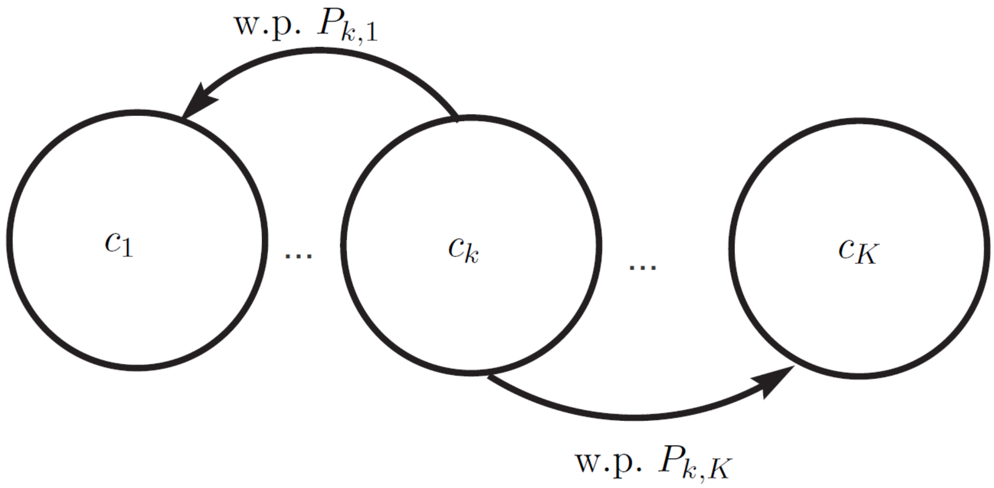


Figure 3.3: An illustration for the charging cost evolution. Given the current state c_k, the next state will be $c_{k'}$ with probability $P_{k,k'}$.

where $b^+ = \max(b, 0)$.

When the deadline is reached, $T_i = 1$, the EV leaves the system by the end of this time interval. The probability mass function (PMF) $Q(\cdot, \cdot)$ governs the initial states of newly arrived EVs, where $Q(0, 0)$ means there is no new arrival in this interval. Formally, the state evolution of charger i with state $S_i[t]$ under action $a_i[t] = 1$ is given by

$$S_i[t+1] = \begin{cases} (T_i[t] - 1, (j_i[t] - a_i[t])^+) & T_i[t] > 1, \\ (T, j) \text{ with prob. } Q(T, j) & T_i[t] \leq 1. \end{cases} \tag{3.1}$$

where $b^+ = \max(b, 0)$.

The entire Markov chain of a charger given action $a = 1$ is summarized in Figure 3.4. The support of the lead time and charging demand is assumed finite since usually consumers charge an EV less than one day.

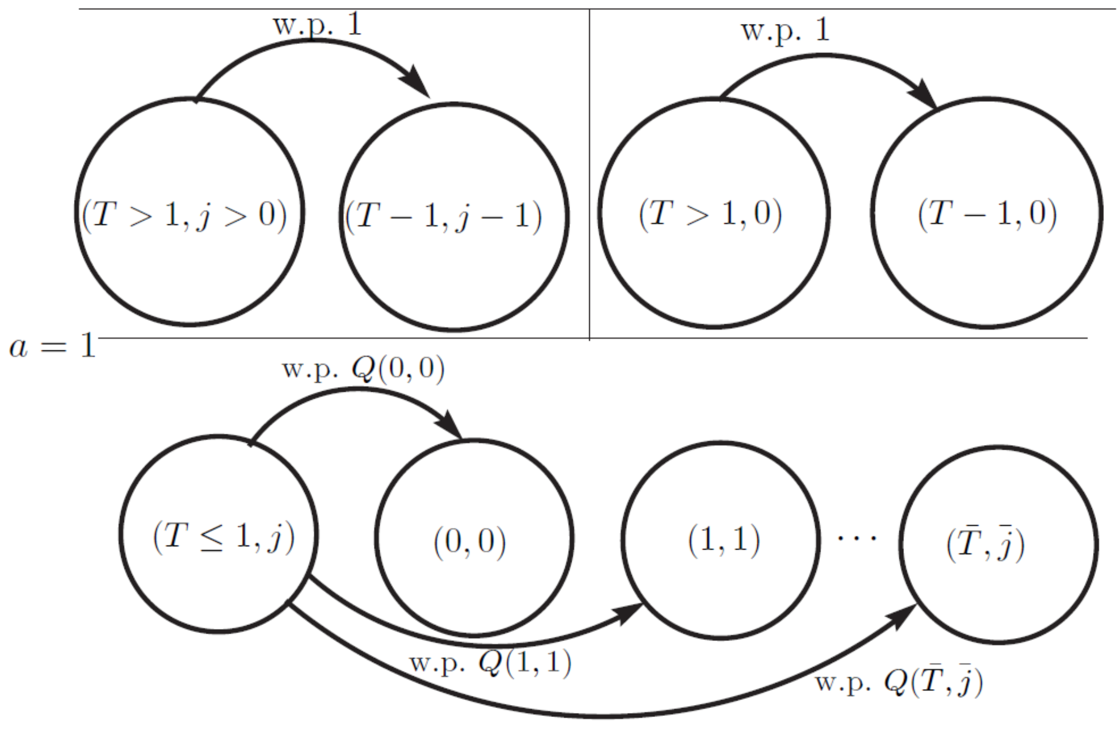


Figure 3.4: An illustration for the charger state evolution given the charger is activated. $\bar{T}$ is the maximum lead time and $\bar{j}$ the maximum charging demand.

3.1.4 Reward

For each EV, the scheduler obtain one unit of reward if the EV is charged for one time slot. At the EV's deadline, *i.e.*, $T_i[t] = 1$, the scheduler pays the penalty for the unfinished work. Let $F(j)$ be the convex penalty function of the amount j of the unfinished charging, and $F(0) = 0$. Denote the cost of charging at time t by $c[t]$. Thus the reward collected from charger i at time t is given by

$$
\begin{aligned}
&R_{a_i[t]}(S_i[t], c[t]) \\
&= \begin{cases} (1 - c[t])a_i[t], & \text{if } j_i[t] > 0,\ T_i[t] > 1, \\ (1 - c[t])a_i[t] - F(j_i[t] - a_i[t]), & \text{if } j_i[t] > 0,\ T_i[t] = 1, \\ 0, & \text{otherwise,} \end{cases}
\end{aligned}
\tag{3.2}
$$

The total reward collected from N chargers given the system state s and the action vector $\mathbf{a}$ is simply the sum.

$$R_{\mathbf{a}}(s) = \sum_i R_{a_i}(s_i, c)$$

3.1.5 Objective Function

Given the initial system state $S[0] = (c[t], S_1[t], \cdots, S_N[t]) = s$ and a policy π that maps each system state $S[t]$ to an action vector $\mathbf{a}[t]$, the expected discounted system reward is defined by

$$G_\pi(s) \triangleq \mathbb{E}_\pi \left(\sum_{t=0}^{\infty} \sum_{i=1}^{N} \beta^t R_{a_i[t]}(S_i[t]) \mid S[0] = s \right), \tag{3.3}$$

where $\mathbb{E}_\pi$ is the conditional expectation over the randomness in costs and EVs arrival under a given scheduling policy π and $0 < \beta < 1$ is the discount factor. The analysis can be extended to the average case [37].

3.1.6 Constrained MDP and Optimal Policies

In practice, there is usually a power limit of the public charging station due to the thermal limit of the transmission line and the capacity constraint of the transformer, which is different in the setting of home charging. Assuming the charging rate of each charger is the same, this power constraint translates to a constraint on the number of simultaneously activated chargers, *i.e.*, $\sum_i^N a_i[t] \leq M$ for all t. This constraint imposes a difficulty on the scheduling problem and sometimes makes the optimality not achievable.

The EV charging scheduling problem can be formulated as a constrained MDP. The maximum expected reward is given by

$$G(s) = \sup_{\{\pi:\sum_i^N a_i^\pi[t]\le M,\forall t\}} G_\pi(s), \tag{3.4}$$

where $a_i^\pi[t]$ is the action generated by policy π. A policy π^* is optimal if $G_{\pi^*}(s) = G(s)$. Without loss of optimality, we will restrict our attention to stationary policies [4].

3.1.7 A Linear Approach

To solve the constrained MDP problem in (3.4), we can use either the linear programming approach (cf. Chap. 3 of [4]) or the dynamic programming approach (cf. Chap. 2 of [117]). Here we introduce the linear programming method.

For each system state $s \in \mathcal{S}$, denote $\mathcal{A}(s)$ the feasible set of actions. Thus $\mathcal{A}(s)$ describes the charging capacity constraint and any action vector $\mathbf{a} \in \mathcal{A}(s)$ satisfies $\sum_i a_i \le M$. For any policy π, we can denote $\rho(s,\mathbf{a})$ the occupation measure of state s and action $\mathbf{a}$ under policy π and initial state s_0. It can be viewed as the probability that state s is visited and action $\mathbf{a}$ is carried out.

It is shown that [4], the measure belongs to the set of measures that satisfy

$$\begin{aligned}
&\rho(s,\mathbf{a}) \ge 0, \forall s, \mathbf{a}\\
&\textstyle\sum_s \sum_{\mathbf{a}} \rho(s,\mathbf{a}) = 1\\
&\textstyle\sum_{\mathbf{a}} \rho(s,\mathbf{a}) = \beta \sum_{s'} \sum_{\mathbf{a}} \rho(s',\mathbf{a}) P^{\mathbf{a}}_{s,s'}, \forall s \ne s_0\\
&\textstyle\sum_{\mathbf{a}} \rho(s_0,\mathbf{a}) = 1 - \beta + \beta \sum_{s'} \sum_{\mathbf{a}} \rho(s',\mathbf{a}) P^{\mathbf{a}}_{s,s'},
\end{aligned}$$

where $P^{\mathbf{a}}_{s,s'}$ is the transition probability from state s to s' given action $\mathbf{a}$. The probability needs to be positive and the sum equals to one. There is also an equality relationship between the measure ruled by the transition matrix.

Thus the constrained MDP in (3.4) is equivalent to the linear programming as follows.

$$\begin{aligned}
\max_{\rho} \quad & \textstyle\sum_{s\in\mathcal{S}}\sum_{\mathbf{a}\in\mathcal{A}(s)} R_{\mathbf{a}}(s)\rho(s,\mathbf{a}) \\
\text{subject to} \quad & \textstyle\sum_{s}\sum_{\mathbf{a}}\rho(s,\mathbf{a}) = 1 \\
& \rho(s,\mathbf{a}) \geq 0, \forall s, \mathbf{a} \\
& \textstyle\sum_{\mathbf{a}}\rho(s,\mathbf{a}) = \beta\sum_{s'}\sum_{\mathbf{a}}\rho(s',\mathbf{a})P^{\mathbf{a}}_{s,s'}, \forall s \neq s_0 \\
& \textstyle\sum_{\mathbf{a}}\rho(s_0,\mathbf{a}) = 1-\beta+\beta\sum_{s'}\sum_{\mathbf{a}}\rho(s',\mathbf{a})P^{\mathbf{a}}_{s,s'},
\end{aligned} \tag{3.5}$$

where the objective is to maximize the expected total reward. The optimizer, $\rho^*(s,\mathbf{a})$, gives the optimal policy. For any state s such that $\sum_{\mathbf{a}}\rho^*(s,\mathbf{a}) > 0$, the optimal policy is to choose action $\mathbf{a}'$ with probability $\rho^*(s,\mathbf{a}')/\sum_{\mathbf{a}}\rho^*(s,\mathbf{a})$.

Note that, the feasible set $\mathcal{A}(s)$ contains all actions that choose less than M chargers out of N. The size of $\mathcal{A}(s)$ is exponentially increasing as N grows. Thus the complexity of the linear programming (3.5) easily explodes and makes the approach computationally unattractive. Thus we want to look for some policies that have linear complexity and perform close to optimality.

3.1.8 A Performance Upper Bound

In (3.4), the power limit must be satisfied for all t. By relaxing this constraint and requiring that the average power usage does not exceed M, we obtain a performance upper bound for (3.4). In particular, a relaxed problem can be stated as

$$\begin{aligned}
\sup_{\pi} \quad & \mathbb{E}_{\pi}\left\{\textstyle\sum_{t=0}^{\infty}\sum_{i=1}^{N}\beta^t R_{a_i[t]}(S_i[t], c[t]) \mid S[0]\right\} \\
\text{subject to} \quad & (1-\beta)\mathbb{E}\textstyle\sum_{t=0}^{\infty}\sum_{i=1}^{N}\beta^t a_i[t] \leq M.
\end{aligned} \tag{3.6}$$

Problem (3.6) is not a practical formulation for the large scale EV charging since the power usage could be far more than M at certain time.

Since the charging cost is the same for all chargers, the relaxed problem (3.6) is equivalent to the following problem (on the scheduling of a single charger i).

$$\begin{array}{ll} \sup_\pi & N\mathbb{E}_\pi \{\sum_{t=0}^{\infty} \beta^t R_{a_i[t]}(S_i[t], c[t]) \mid S_i[0], c[0]\} \\ \text{subjec to} & (1-\beta)\mathbb{E}\sum_{t=0}^{\infty} \beta^t a_i[t] \leq M/N. \end{array} \tag{3.7}$$

Problem (3.7) seeks to maximize the discounted reward from a single charger i with no more than M/N active action (per time period) on average. The optimal solution and the optimal objective of (3.7) are the same as those of (3.6). The optimal objective of (3.7) can be used as a performance upper bound for the original scheduling problem in (3.4).

The constrained MDP problem in (3.7) has a much smaller dimensionality and can be easily solved by linear programming as mentioned in (3.5).

3.2 Deadline Scheduling as An RMAB Problem

The constrained MDP problem stated in the previous sections falls into the category of restless multi-armed bandit (RMAB) problem. In probability theory, the classic multi-armed bandit (MAB) problem is the problem in which, the gambler at a row of N slot machines (sometimes referred as ont-armed bandit) has to decide which M machines to play at each time to maximize the reward. In its original setting, each machine, if played, provides a random reward from a probability distribution specific to that machine and the state of the machine changes according to a Markov chain. Machines that are not played provides no reward and stay frozen.

Gittins first proposed an *index policy* that provides a scalable solution [51]. An index policy computes an indic for each arm according to the state and characteristics (*e.g.,* transitions, rewards) ignoring other arms. Then it ranks all arms by the index and choose to play the top M arms. This kind of policies decouple the complicated MDP problem into N smaller problem and decouple arms, thus are scalable. Clearly, index policies are not optimal in general. However, Gittins showed that, when $M = 1$, the Gittins index policy is in fact optimal.

Whittle extended the classic MAB problem to restless multi-armed bandits (RMAB) [143]. In the classic setting, the arms that are not played (deactivated) provide no reward and stay frozen. In a RMAB, however, deactivated arms change states and generate reward as well, maybe according to different probability distributions from the ones of the activated case. Whittle, inspired by Gittin, proposed the Whittle's index policy. The intuition of Whittle's index is based on the subsidy. For each individual arm, whenever it is not played (idle), a subsidy is provided on the top of the original reward. Whittle's index is the smallest subsidy such that the deactivation is the same attractive as the active action for this arm. In the classic MAB setting, Whittle's index is shown to degenerate to Gittin's index. In the RMAB setting, the Whittle's index is shown to be asymptotically optimal as the scale of the problem grows under some conditions. However, these conditions are usually hard to check.

In this section, we see to obtain an index policy following the principle of Whittle's index. Specifically, the index of charger i is a mapping from its extended state $\tilde{S}_i[t] \triangleq (S_i[t], c[t])$ to a index value.

An RMAB Formulation

We now formulate Problem (3.4) as a restless multi-armed bandit (RMAB) problem. We identify each charger in the station as an arm. To this end, "playing" an arm is equivalent to activate a charger to charge the EV (if there is one) in the station. The resulting multi-armed bandit problem is restless because the state of a charger, in particular, the lead time evolves even if the charger is not activated.

A complication of casting (3.4) as an RMAB problem comes from the inequality constraint on the maximum number of simultaneous activated chargers. This complication can be circumvented by introducing M dummy chargers and requiring that exactly M chargers must be activated in each period. Specifically, each dummy charger always accrues zero reward, and the state of dummy chargers stays at $S_i = (0, 0)$. We let $\{1, \cdots, N\}$ be the set of regular chargers that generate reward (penalty) and $\{N + 1, \cdots, N + M\}$ be the set of dummy chargers. By including dummy chargers, the MDP in (3.4) is equivalent to an RMAB problem where exactly M out of $N + M$ chargers (arms) are active in each time slot.

We define the extended state of each charger as $\tilde{S}_i[t] \triangleq (S_i[t], c[t])$, and denote the extended state space as $\tilde{\mathcal{S}}_i \triangleq \mathcal{S}_i \times \mathcal{S}_c$. The state transition of each arm and the associated rewards are inherited from (3.1- 3.2) of the original MDP.

The corresponding RMAB problem is defined by

$$\begin{aligned} \sup_\pi \quad & \mathbb{E}_\pi \left\{ \sum_{t=0}^{\infty} \sum_{i=1}^{N+M} \beta^t R_{a_i[t]}(\tilde{S}_i[t]) \mid \tilde{S}_i[0] \right\} \\ \text{s.t.} \quad & \sum_{i=1}^{N+M} a_i[t] = M, \quad \forall\, t. \end{aligned} \tag{3.8}$$

In (3.8), the arms are coupled by the charging cost and period index, and are not independent.

3.3 Whittle's Index Policy

To pursue the EV charging problem as an RMAB, we need to establish the indexability of the RMAB and examine the performance.

3.3.1 Indexability

Consider the *ν-subsidized* single arm reward maximization problem [143] that looks for a policy π to activate/deactivate the arm to maximize the discounted accumulative reward:

$$V_i^{\nu}(s) = \sup_{\pi} \mathbb{E}_{\pi}\left(\sum_{t=0}^{\infty} \beta^t R^{\nu}_{a_i[t]}(\tilde{S}_i[t]) \mid \tilde{S}_i[0] = s\right), \tag{3.9}$$

where the subsidized reward is modified single arm reward (3.2) given by

$$R^{\nu}_{a_i[t]}(\tilde{S}_i[t]) = R_{a_i[t]}(\tilde{S}_i[t]) + \nu\mathbb{1}(a_i[t] = 0),$$

where $\mathbb{1}(\cdot)$ is the indicator function. In words, the *ν-subsidized* problem is a modification of the reward such that the scheduler receives a subsidy ν whenever the arm is passive.

Let $\mathcal{L}_a$ be an operator on V_i^{ν} defined by

$$(\mathcal{L}_a V_i^{\nu})(s) \triangleq \mathbb{E}\left(V_i^{\nu}(\tilde{S}_i[t+1]) \middle| \tilde{S}_i[t] = s, a_i[t] = a\right).$$

The maximum discounted reward $V_i^{\nu}(\cdot)$ in (3.9) is determined by the Bellman equation

$$V_i^{\nu}(s) = \max_{a \in \{0,1\}} \left\{ R^{\nu}_a(s) + \beta(\mathcal{L}_a V_i^{\nu})(s) \right\}. \tag{3.10}$$

Let $\mathscr{S}_i$ be the space of extended state of arm i and $\mathscr{S}_i(\nu)$ the set of states under which it is optimal to take the passive action in the ν-subsidy problem. The *indexability of the RMAB* is defined by the monotonicity of $\mathscr{S}_i(\nu)$ as subsidy level ν increases:

Definition 1 (Indexability [143])**.** *Arm i is indexable if the set $\mathscr{S}_i(\nu)$ increases monotonically from $\emptyset$ to $\mathscr{S}_i$ as ν increases from $-\infty$ to $+\infty$. The MAB problem is indexable if all arms are indexable.*

We establish the indexability for the EV charging scheduling problem.

Theorem 1 (Indexability)**.** *Each arm is indexable and the RMAB problem (3.8) is indexable.*

Proof. The proof of Theorem 1 can be found in Appendix A.1. □

3.3.2 Whittle's index policy

Given the definition of indexability, the Whittle's index is defined as follows.

Definition 2 (Whittle's index [143])**.** *If arm i is indexable, its Whittle's index $\nu_i(s)$ of state s is the infimum of the subsidy ν under which the passive action is optimal at state s, i.e.,*

$$\nu_i(s) \triangleq \inf_{\nu}\{\nu : R_0(s) + \nu + \beta(\mathcal{L}_0 V_i^{\nu})(s) \geq R_1(s) + \beta(\mathcal{L}_1 V_i^{\nu})(s)\}.$$

Thus if arm i is indexable, any $\nu < \nu_i(\tilde{s})$ makes activating arm i optimal. Likewise, any $\nu \geq \nu_i(s)$ makes it optimal to deactivate arm i.

To compute the Whittle's index, we need to solve a parametric programming where the subsidy ν forms the constraints. For arm i, the formulation is stated as follows.

$$
\begin{aligned}
\min_{\mu_i(s)} \quad & \textstyle\sum_{s\in\mathcal{S}} p(s)\mu_i(s) \\
\text{s.t.} \quad & \mu_i(s) \geq R_1(s) + \beta \textstyle\sum_{s'\in\mathcal{S}} P^1_{s,s'}\mu_i(s') \\
& \mu_i(s) \geq R_0(s) + \nu + \beta \textstyle\sum_{s'\in\mathcal{S}} P^0_{s,s'}\mu_i(s')
\end{aligned}
$$

where $s = (T, j, c)$ is the extended state of arm i, $p(s)$ the initial-state probability, and $P^a_{s,s'}$ the transition probability from s to s' given action a. For particular value of ν, the optimal solution $\mu_i^*(s)$ equals to the value function $V_i(s)$ and the active constraints give the optimal action. Solving this parametric programming is to find the break point of ν where the optimal action changes. A simplex method to solve the parametric programming is described in [26].

The special structure of the deadline problem, however, allows us to have a closed-form solution when the charging cost is constant.

Theorem 2. *If $c[t] = c_0$ for all t, the Whittle's index of a regular arm $i \in \{1, \cdots, N\}$ is given by*

$$
\nu_i(T, j, c_0) = \begin{cases} 0 & \text{if } j = 0, \\ 1 - c_0 & \text{if } 1 \leq j \leq T - 1, \\ 1 - c_0 + \beta^{T-1}[F(j - T + 1) - F(j - T)] & \text{if } T \leq j. \end{cases} \tag{3.11}
$$

The Whittle's index of a dummy arm is zero.

$$
\nu_i(0, 0, c_0) = 0, \quad i \in \{N + 1, \cdots, N + M\}.
$$

Proof. The proof of closed-form of Whittle's index with constant charging cost can be found in Appendix A.2. □

In (3.11), when it is feasible to finish EV i's request (*i.e.* its lead time is no less than its remaining charging time), charger i's Whittle's index is simply the (per-unit) charging profit $1 - c_0$. When non-completion penalty is inevitable, the index takes into account both the charging profit and the non-completion penalty. We note that the Whittle's index gives higher priority to EVs with less laxity. Here, the laxity of EV i is defined as $L_i[t] \triangleq T_i[t] - j_i[t]$ (cf. Figure 3.2).

Given the definition of Whittle's index, the Whittle's index policy for the deadline scheduling problem is stated as follows.

Definition 3 (Whittle's index policy)**.** *For the RMAB problem defined in (3.8), the Whittle's index policy sorts all arms by their Whittle's indices in a descend order and activates the first M arms.*

Since the states of EVs and charging cost are finite, the Whittle's index can be computed off-line. In real-time scheduling, at the beginning of each time slot, the scheduler looks up the indices for each charger and activates the ones with highest indices. When there is a tie, the scheduler breaks the tie randomly with a uniform distribution.

3.4 Performance of Whittle's Index Policy for Finite-Armed Restless Bandits

In this section, we examine the performance of Whittle's index policy for the stochastic deadline scheduling problem when the power limit (M) is finite. We show that when $M < N$, there does not exist an optimal index policy, hence Whittle's index policy is not optimal. We further derive an upper bound on the gap-to-optimality on the performance of the Whittle's index policy. This result provides the essential ingredient for establishing asymptotic optimality of Whittle's index policy in the next section.

3.4.1 Performance in the Finite Power limit Cases

In general, Whittle's index policy is not optimal except in some special cases [93]. For the deadline scheduling problem, the same conclusion holds. We show in fact that no index policy exists.

Property 1. *An optimal index policy for the RMAB problem formulated in (3.8) does not exist in general when $M < N$. When $M = N$, the Whittle's index policy is optimal.*

Proof. The fact that Whittle's index policy is optimal when $M = N$ is intuitive and a formal proof can be found in Appendix C of [149]. To show that optimal index policy does not exist in general, it suffices to construct a counter example that no index policy can be optimal.

Set the capacity of the queue to be $N = 3$, the power limit $M = 1$, the discounted factor $\beta = 0.4$, the penalty function $F(j) = j^2$, and the charging cost $c[t] = 1$. Assume the arrival is busy ($Q(0,0) = 0$) and the initial laxity is zero ($T = j$ at arrival). For this small scale MDP, a linear programming formulation is used to solve for the optimal policy [117].

Consider two different states,

$$s = ((1,1),(2,2),(2,2))$$
$$s' = ((1,1),(1,1),(2,2))$$

where $s = ((T_1, j_1), (T_2, j_2), (T_3, j_3)) \in \mathcal{S}$ is the state of the system including the states of each arm.

For state s, the optimal action is to charge EV $(2, 2)$. The EV $(2, 2)$ is preferred to $(1, 1)$ in this case. Charging $(2, 2)$ will cause 1 instant penalty, and the state will change to $((T, j), (1, 1), (1, 2))$, where (T, j) is a new arrival. In next stage, a penalty of 2 from the last two EVs will happen. If some policy charges $(1, 1)$ alternately, there will be no penalty in the first stage and the state will change to $((T, j), (1, 2), (1, 2))$. The last two EVs will at lease incur a penalty of 5.

For state s', the optimal action is to charge the EV $(1, 1)$. The EV $(1, 1)$ is preferred to $(2, 2)$ in this case. Charging $(1, 1)$ will cause 1 instant penalty, and the state will change to $((T, j), (T', j'), (1, 2))$, where (T, j) and (T', j') are new arrivals. If some policy charges $(2, 2)$ alternately, there will an instant penalty of 2 from the first two EVs in the first stage and the state will change to $((T, j), (T', j'), (1, 1))$. In this case, a penalty of 1 can be saved by charging $(2, 2)$ in the previous stage. However, due to the discount factor, it is more profitable to charge $(1, 1)$.

An *index policy* assigns each EV an index (that depends only on the EV's current state), and charges the EVs with the highest indices [51]. Therefor, for any "index" policy, the indices of EV $(1, 1)$ and $(2, 2)$ are fixed and the preference of these two EVs should remain the same in these two cases, which is violated by the result here. This counter example shows that no "index" policy that is optimal in general. $\square$

Note that, the Whittle's index policy is an example of index policies, and thus is sub-optimal.

3.4.2 An Upper Bound of the Gap-to-Optimality

In the following lemma, we first establish a result that applies quite generally to the case for a finite queue size N and finite power limit M.

Lemma 1. *Let $G^N(s)$ be the optimal value function defined in (3.4) and $G^N_{RMAB}(s)$ be the value function achieved by the Whittle's index policy, respectively. We have*

$$\begin{aligned} &G^N(s) - G^N_{RMAB}(s) \\ &\quad \le \tfrac{C}{1-\beta}\mathbb{E}[I^N[t] | I^N[t] > M] \Pr(I^N[t] > M), \end{aligned} \tag{3.12}$$

where $I^N[t]$ is the number of EVs admitted in the station with N chargers within time $[t-\bar{T}+1, t]$, $\bar{T}$ is the maximum lead time of EVs, and C is a constant determined by the charging cost and the penalty of non-completion.

The proof can be found in Appendix A.3. The gap-to-optimality is bounded by the tail expectation of the EVs admitted to the system. Note that, the conditional expectation on the right hand side (RHS) of (3.12) is connected to the conditional value at risk (CVaR) [115], which measures the expected losses at a certain risk level and is extremely important in the risk management.

3.4.3 Least Laxity and Longer Processing Time (LLLP) Principle

In this section, we will apply the Less Laxity and Longer remaining Processing time (LLLP) principle (originally proposed in [147]) to improve the Whittle's index policy. The LLLP principle is a priority rule for the scheduling, which is defined as follows.

Definition 4 (The LLLP Principle)**.** *Consider charger (arms) i and i' at time t. We* *say i' dominates i ($i' \geq i$), if i' has less laxity and longer remaining charging time, i.e., $L_{i'}[t] \leq L_i[t]$ and $j_{i'}[t] \geq j_i[t]$, with at least one of the inequalities strictly holds.*

LLLP defines a partial order over the EVs' states such that the EV with less laxity and longer remaining charging demand should be given priority. In [147], the authors applied an interchange argument to show that LLLP could improve the performance of any given policy along every sample path, and further, there exists an optimal stationary policy that follows the LLLP principle under mild conditions.

To apply the LLLP principle, note that the Whittle's index policy for the multi-armed bandit problem is a stationary policy: at each time it orders (the states of) the $M+N$ arms, and activates the first M arms. The proposed heuristic policy re-order every pair of arms that violates the LLLP principle (cf. Algorithm 2). As such, the proposed heuristic policy always gives priority to EVs with less laxity and longer remaining processing time.

3.5 Asymptotic performance of the Whittle's Index Policy

In this section, we establish the asymptotic optimality of the Whittle's index policy when the EV arrival rate μ and the power limit M increase to infinity simultaneously while the system stays stable.

We first consider the case when the aggregated arrival of EVs follows a Poisson distribution. Let $I[t]$ be the total number of EVs arrived in the system within $[t - \bar{T} + 1, t]$, recalling that $\bar{T}$ is the maximum lead time of EVs. Note that $I[t]$ is Poisson distributed.

Algorithm 1: Whittle index with LLLP interchange

1. Calculate the Whittle's indexes of all arms and sort them in a descend order.

2. Suppose the order of arms is $i_1, i_2, \cdots, i_{M+N}$.

for $j = i_1 : i_M$

  for $k = i_{M+1} : i_{M+N}$

    if $k \geq j$ in the sense of Definition 4

      exchange the orders of j and k

    end

  end

end

3. Activate the M arms with highest priority.

When the queue at the service center is finite with N chargers, we assume that each charger receives equally likely $1/N$th of the traffic. Because a newly arrived EV may be rejected when the assigned charger is occupied, the total number of EVs $I^N[t]$ admitted to the system in slot t satisfies $I^N[t] \leq I[t]$. However, as $N \to \infty$, $I^N[t] \to I[t]$ in distribution. Define

$$G(s) - G_{\text{RMAB}}(s) \triangleq \limsup_{N \to \infty} [G^N(s) - G^N_{\text{RMAB}}(s)],$$

then

$$G(s) - G_{\text{RMAB}}(s) \leq \frac{C}{1-\beta} \mathbb{E}[I[t] \mathbb{1}(I[t] > M)]. \tag{3.13}$$

Equation (3.13) characterizes the performance gap for the Whittle's index policy for the asymptotic regime as N increases while the arrival process and power limit stay constant. Now, we check the performance of Whittle's index policy when the charging capacity M increases and the mean of the arrival process $I[t]$ also grows as a function.

Theorem 3. *Suppose that the aggregated arrival $I[t]$ is Poisson with mean μ. The Whittle's index policy is asymptotically optimal as $M \to \infty$ if $\mu < M/e$. In particular,*

$$G(s) - G_{RMAB}(s) = \mathcal{O}(\frac{\mu e^{-\mu}}{\sqrt{M}}). \tag{3.14}$$

The proof of Theorem 3 can be found in Appendix A.4. Beside showing that the Whittle's index is asymptotically optimal, it also shows that the gap-to-optimality decays sub-exponentially when μ grows with M at the constant rate less than $1/e$. When μ grows slower than M, the gap decays to zero but with a slower rate.

In general, suppose that if we don't have the aggregated Poisson arrival, but $I^N[t]$ converges in distribution to $\bar{I}[t] < I[t]$ as $N \to \infty$, we can then show similar results as in Theorem 3.

Theorem 4. *Suppose that $I[t]$ had a light tailed distribution with mean $\tilde{\mu}$, i.e., there exist constant $a \geq 1$ and $b \geq 0$ with*

$$\Pr(I[t] \geq i) \leq a \exp[-ib/\tilde{\mu}], \ \forall i \geq 0. \tag{3.15}$$

the Whittle's index policy is asymptotically optimal as $M \to \infty$ if $\tilde{\mu} = o(M/\ln M)$. In particular,

$$G(s) - G_{RMAB}(s) = \mathcal{O}[\exp(-\frac{Mb}{\tilde{\mu}})(Mb + \tilde{\mu})], \tag{3.16}$$

Under the heavy tail assumption, we have the following theorem.

Theorem 5. *Suppose that $I[t]$ has a heavy tailed distribution with mean $\tilde{\mu}$, i.e., there exist constants $a > 0$ and $b > 2$ with*

$$\Pr(I[t] \geq i) \leq a\tilde{\mu}/i^b, \forall i > 0, \tag{3.17}$$

the Whittle's index policy is asymptotically optimal as $M \to \infty$ if $\tilde{\mu} = o(M^{b-1})$. In particular,

$$G(s) - G_{RMAB}(s) = O(\tilde{\mu}/M^{b-1}). \tag{3.18}$$

The proof of Theorem 4 and 5 can be found in Appendix A.5. Theorem 4 and 5 establish the asymptotic optimality of the Whittle's index policy when both the job arrivals and the processing capacity grow simultaneously while the overall system remains stable in different traffic regime.

3.6 Performance

In this section, results of numerical experiments are presented to compare the performance of the Whittle's index policy with other simple heuristic (index) policies, *i.e.*, EDF (earliest deadline first) [92], LLF (least laxity first) [31], and Whittle's index policy with LLLP interchange (cf. Algorithm 2).

If feasible, EDF charges M EVs with the earliest deadlines, and LLF charges M EVs with the least laxity. Both policies will fully utilize the power limit and activate M EVs as long as there are at least M unfinished EVs in the system. The Whittle's index policy, on the other hand, ranks all arms by the Whittle's index and activates the first M arms, and may put some (regular) arms idle (deactivated) when the charging cost is high.

We first consider a special case of problem (3.8) with a constant charging cost. Since the charging cost is time-invariant, it is optimal to fully utilize the capacity to charge M unfinished EVs.

In Figure 3.5, we fix the traffic of EVs and the number of chargers N and vary the power limit M. All policies besides the EDF scheduling perform well and close to the upper bound of the performance. When $M/N = 1$, all EVs can be finish. Thus all policies achieve optimality. In Figure 3.6, we zoom in the case when $M/N = 0.5$ and vary the total number of chargers N. We observe that the Whittle's index policy with LLLP interchange and LLF achieve similar performance, since both policies roughly follow the least laxity first principle. The performance of these two policies is close to the performance upper bound. The EDF policy perform poorly because it does not take the remaining charging demand into account. The gap between the Whittle's index policy and the Whittle's index policy with LLLP interchange comes from the reordering of EVs with positive laxity (cf. the discussion following Theorem 2).

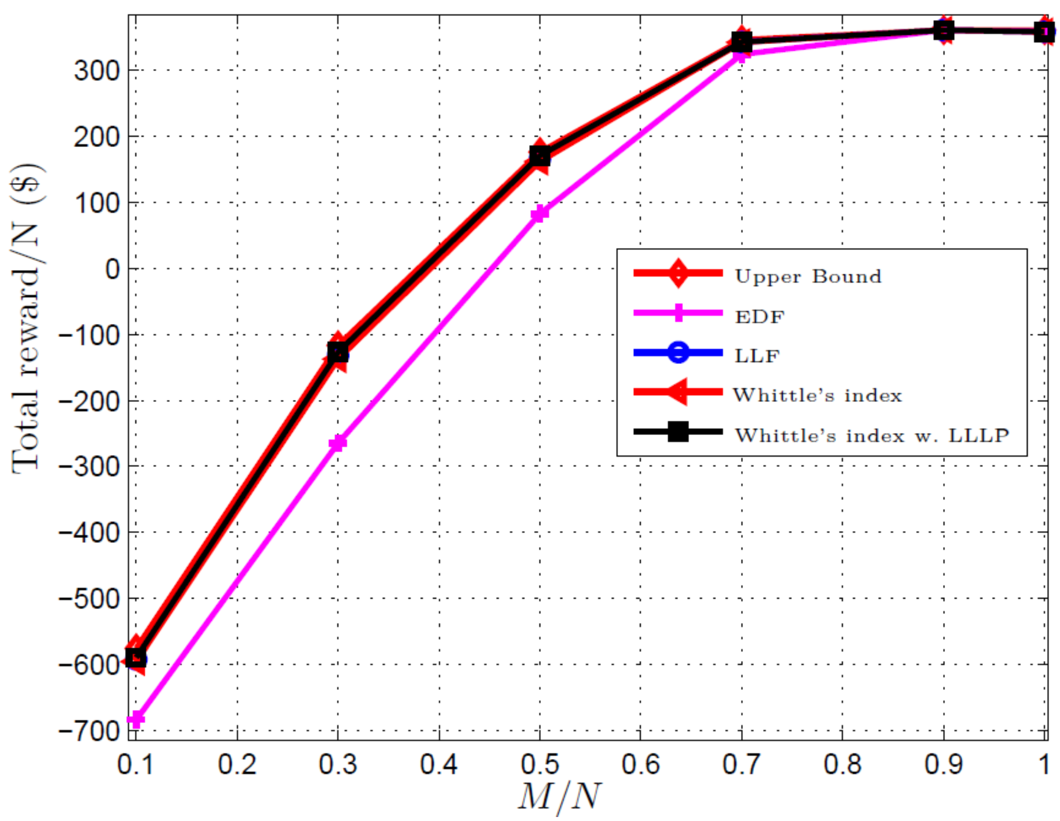


Figure 3.5: Performance comparison: constant charging cost $c[t] = 0.5$, $Q(0, 0) = 0.3$, $\bar{T} = 12$, $\bar{j} = 9$, $\beta = 0.999$, $F(j) = 0.2j^2$, $N = 10$.

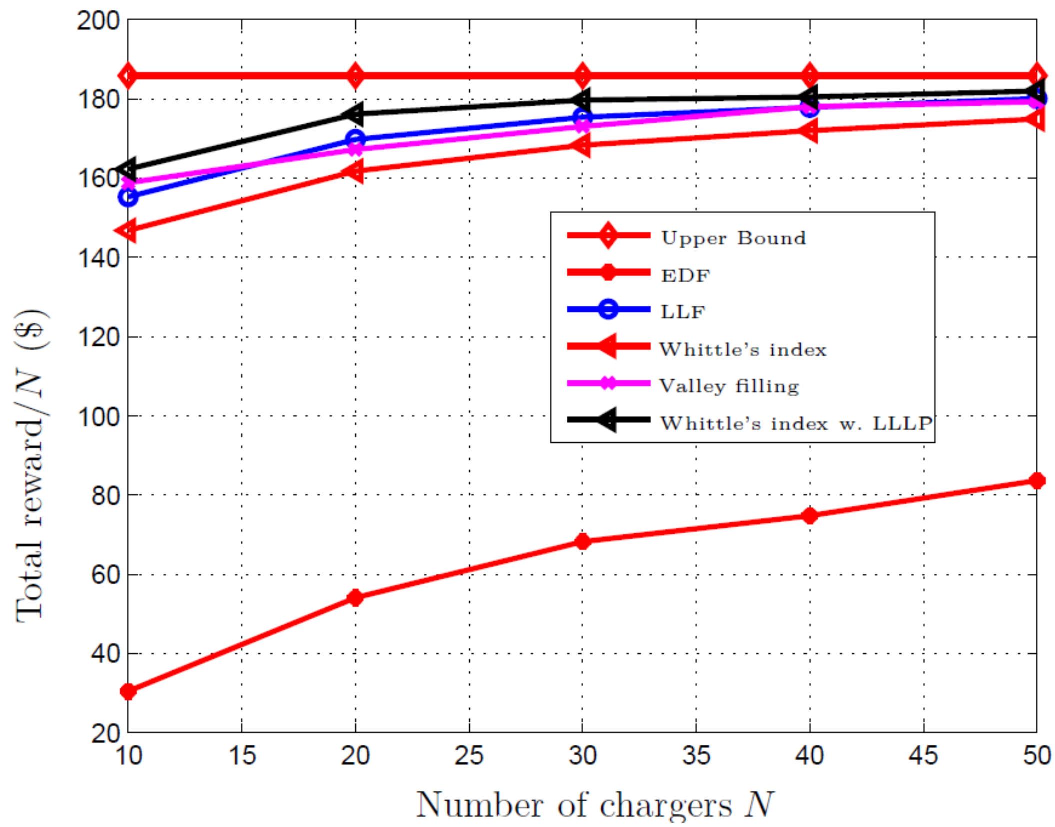


Figure 3.6: Performance comparison: constant charging cost $c[t] = 0.5$, $Q(0, 0) = 0.3$, $\bar{T} = 12$, $\bar{j} = 9$, $\beta = 0.999$, $F(j) = 0.2j^2$, $M/N = 0.5$.

For the dynamic charging cost case, we use the real-time electricity price signal from the California Independent System Operator (CAISO) and train a Markovian model that describes the marginal charging costs (cf. Sections III and V of [86]). Each time slot of the constructed Markov chain (on charging cost) lasts for 1 hour. For each time slot, the real-time price is quantized into discrete price states, and the transition probability (of the Markov chain) is simply the frequency the price change from one state to another.

In Figure 3.7, we fix the EV traffic and the number of chargers $N = 10$ and vary the power limit constraints. When the power limit is low and M/N is s-mall, there is no enough power to finish all EVs and the penalty dominates the

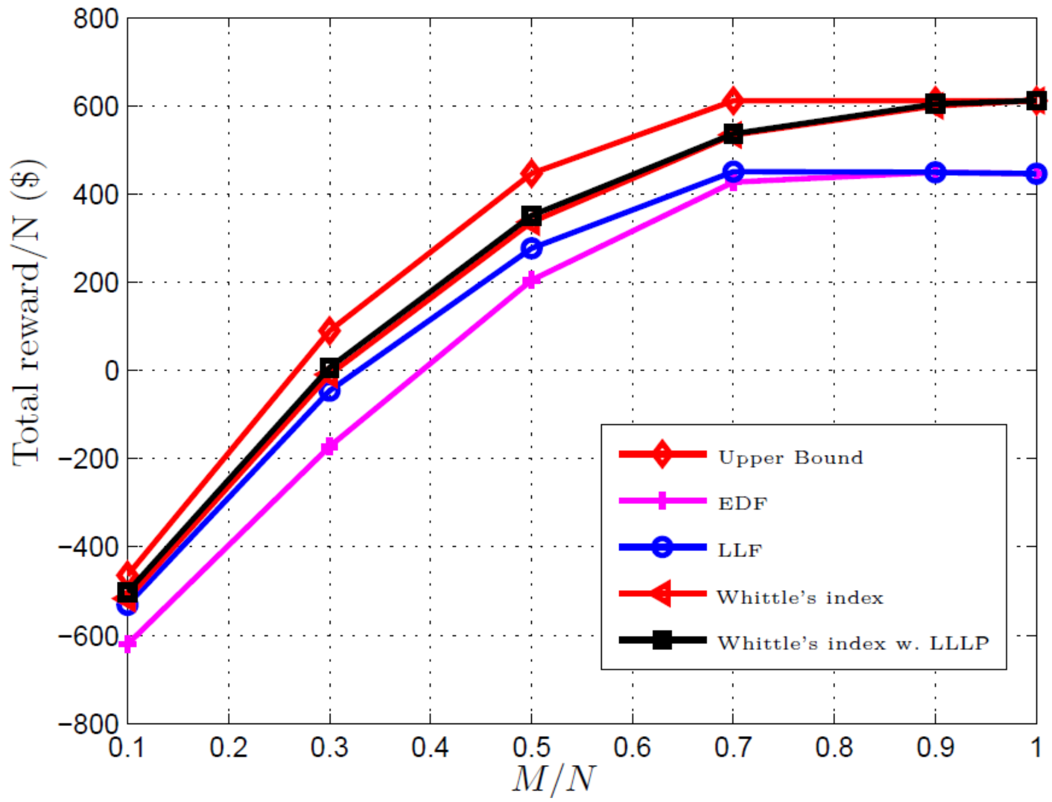


Figure 3.7: Performance comparison: stochastic charging cost, $Q(0, 0) = 0.3$, $\bar{T} = 12$, $\bar{j} = 9$, $\beta = 0.999$, $F(j) = 0.2j^2$.

charging profit. In this case the performance of different policies is close since the limited resource allows not much to do. When the power limit is adequate and $M/N = 1$, all EVs can be finished on time. In this case, the Whittle's index policy solves the problem optimally and achieves the upper bound. The interchange does not happen because the LLLP principle is always satisfied in this case. EDF and LLF do not consider the stochastic charging cost, thus they perform sub-optimal. When the power constraint is neither too tight ($M/N \approx 0$) nor too loose ($M/N \approx 1$), LLLP can reduce the number of unfinished EVs with large unfinished charging demand and therefore reduces the non-completion penalties.

In Figure 3.8, we compare the performance of different policies by fixing the

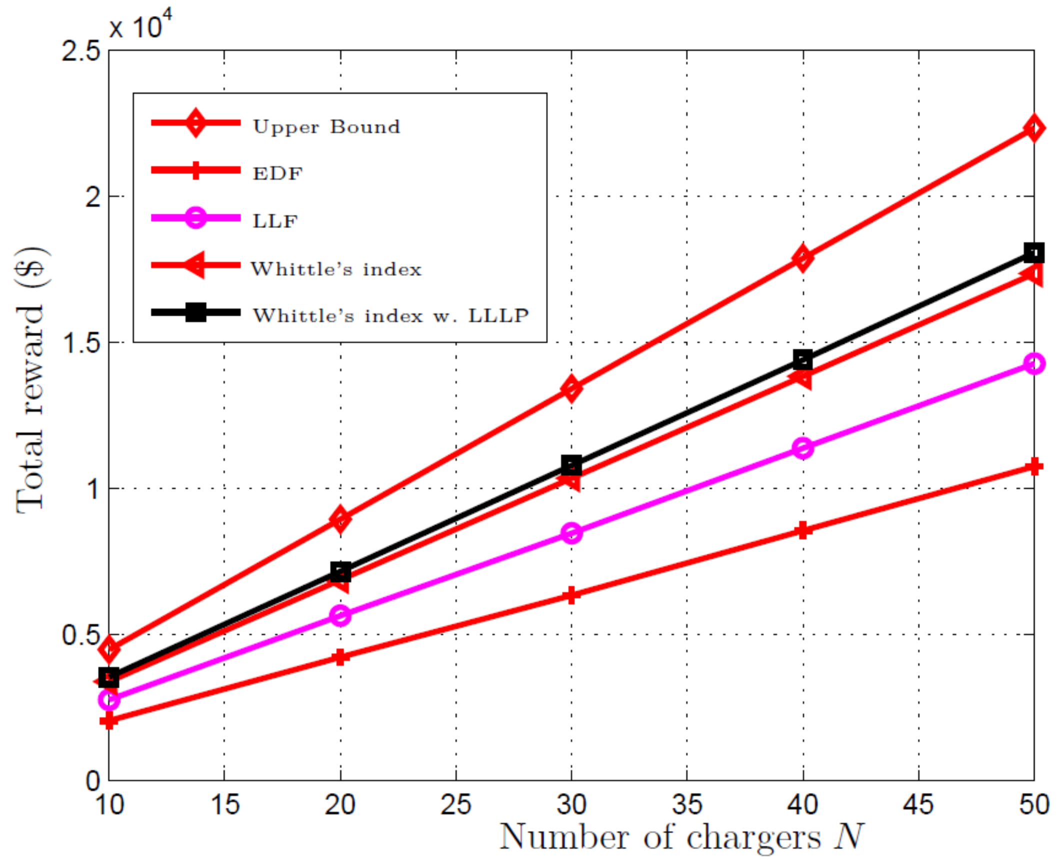


Figure 3.8: Performance comparison: stochastic charging cost, $\rho_i^\tau = 0.7$, $\bar{T} = 12$, $\bar{j} = 9$, $\beta = 0.995$, $F(j) = 0.2j^2$, $M/N = 0.5$.

power limit ratio $M/N = 0.5$ and vary the number of chargers N. Both EDF and LLF seek to activate as many chargers as possible, up to the power constraint M. The Whittle's index policy, on the other hand, take the advantage of the pricing fluctuation and charge more EVs at price valley and keeps chargers idle when charging cost is high. Based on the Whittle's index policy, the LLLP interchange reduces the penalty of unfinished EVs and improves the performance of Whittle's index policy. The total reward achieved by the Whittle's index with LLLP interchange policy is more than 1.7 times of that obtained by EDF; the performance gap between the Whittle's index with LLLP interchange policy and the LLF policy is over 25%. We also note that the LLLP principle improves Whittle's index by around 10%.

3.6.1 Asymptotic optimality

In Figure 3.9, simulation results are presented to compare the performance achieved by various heuristic policies and to validate the theoretic results established in Lemma 1.

In this simulation, the arrival sequence within $\bar{T}$ time slots follows a Poisson process with mean $M^{0.999}$. We fix the number of chargers $N = 1000$ and vary the power limit M as a parameter. The dynamic cost evolves according to a Markovian model that is trained using real-time electricity price signals from CAISO. Each time slot of the constructed Markov chain lasts for 1 hour and the entire simulation horizon lasts for 300 days (with 24×300 time slots).

The EDF and LLF policies do not take into account the dynamics of charging costs, and their gap-to-optimality increases as both the EV arrival rate and power limit grow. On the other hand, the gap between the total rewards achieved by the Whittle's index policy and the optimal policy quickly decreases to zero as the system scales.

3.7 Literature Review

The EV charging model considered in this chapter falls into the category of deadline scheduling problems which is first considered by Liu and Layland [92]. The deadline scheduling problem, in its most generic setting, is the scheduling

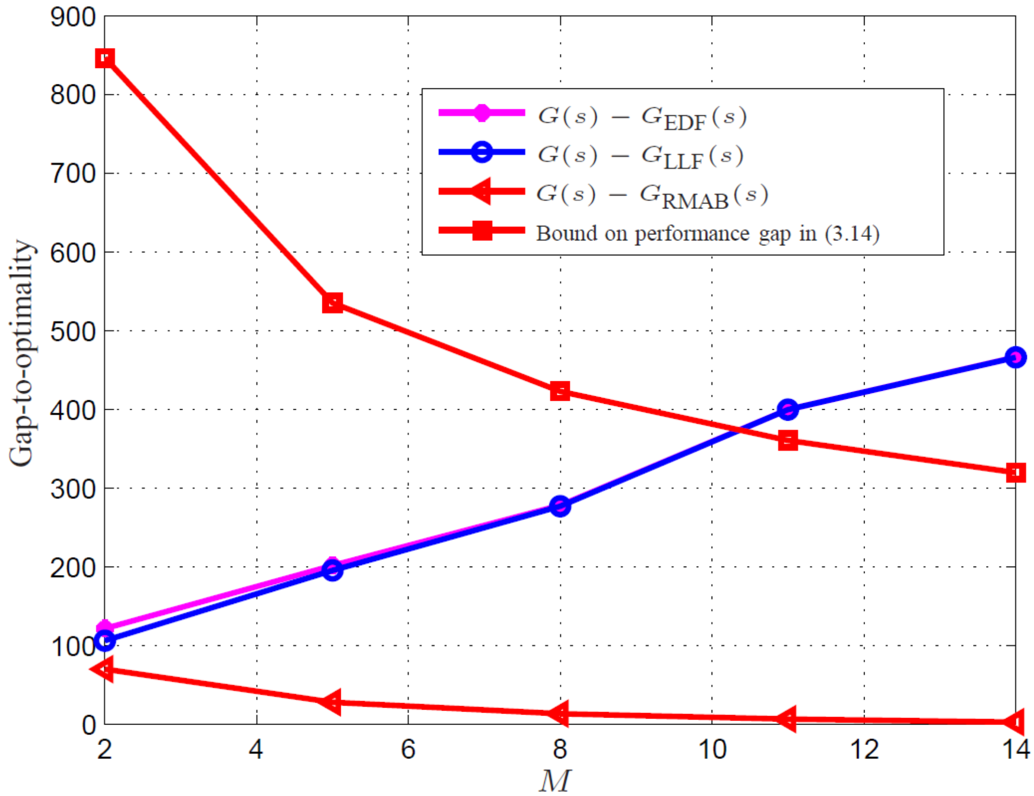


Figure 3.9: Comparison of the total rewards achieved by three different index policies under dynamic charging cost: $Q(0,0) = 0.3$, $\bar{T} = 12$, $\bar{j} = 9$, $\beta = 0.999$, $F(j) = 0.2j^2$, $\theta = 0.999$, $N = 1000$.

of EVs with different workloads and deadlines for completion. Typically, there are not enough processors to satisfy all the demand; the cost of processing may vary with time, and unfinished jobs by their deadlines incur a penalty. In the EV charging framework, each EV can be viewed as a job and the peak power constraint gives the processing capacity.

There are in general two criterions to evaluate a policy, worst case analysis and average case analysis. In the average case analysis, like what we present in this chapter, the objective of a scheduling policy is to achieve a good performance averaged over all possible scenarios of job arrivals, the states of jobs and charging costs. The worst case analysis, on the other hand, focuses on the robustness of the algorithm. An algorithm achieves good performance on average can perform rather bad in some scenarios. The worst case analysis gives an lower bound of the performance of algorithms and looks for algorithms with optimal lower bound in any scenarios. In this section, we present some results in both worst case and average case analysis.

3.7.1 Worst case analysis

Performance measure: competitive ratio

In the worst case analysis, the performance of an online policy π is measured by the competitive ratio against the optimal offline (clairvoyant) scheduler. The offline scheduler has the information of future arrivals while an online policy does not.

Define instance I as a sample path of the realization of the arrival, demand and deadlines of EVs and other randomness. Denote the profit obtained by the online (optimal offline) scheduler by $V_\pi(I)$ ($V^*_{\text{offline}}(I)$) for some instance I. The competitive ratio is defined as below:

Definition 5. *Competitive ratio: An online policy π is α-competitive if* $\inf_I \frac{V_\pi(I)}{V^*_{offline}(I)} \geq \alpha$ *for all instance $I \in \mathcal{I}$ where $\mathcal{I}$ is the collection of all instances with finite number EVs.*

For any instance, the performance of an α-competitive online algorithm is guaranteed to be at least α fraction of the optimal offline algorithm. The objective is to find the online policy that is competitive to the optimal offline schedule across all instances $I \in \mathcal{I}$.

Optimal online policies: $M = 1$

For the single processor case, the results are quite complete. When all EVs can be finished on time, simple index algorithms (with linear complexity) such as the earliest deadline first (EDF) [92, 31] and the least laxity first (LLF) [102] achieve the same performance as the optimal off-line algorithm in the deterministic setting. Recall that, EDF always

To apply the results in the EV charging, we consider a special case of what is descried in Section. 3.1. Assume the parking capacity of the charging station is arbitrarily large ($N = \infty$) but the power capacity can only support charging one EV at any time ($M = 1$). The charging cost is assumed to be constant ($c[t] = c_0 < 1$) and the penalty of unfinished EV is simply linear in unfinished demand $F(j) = j$. We set the discounted factor $\beta = 1$ and consider a finite number of EVs in each instance I so the value function $V_\pi(I)$ is well defined.

In this case, the charging cost c_0 is constant and less than the unit time charging revenue 1. To maximize the charging profit, the scheduler needs to keep charging if there is unfinished EV in the system. The simple online algorithm EDF and LLF can finish charging all EVs if it is possible [31]. That is, both of EDF and LLF achieves the competitive ratio equals to 1.

The proof of the optimality of EDF is intuitive. If there exists some schedule algorithm π that can finish all EVs before the deadlines, one can always transform the policy π into EDF by interchange EVs charging sequence that does not follow the EDF principle while preserving the feasibility of the scheduling. As illustrated in Figure 3.10, suppose policy π charges J_2 before J_1 and $d_2 > d_1$. One can interchange the charging of J_1 and J_2 and keep other charging sequence the same. Denote the policy after the interchange as π'. We can see that π' is always feasible since $d_2 > d_1$. If π can finish charging all EVs before the deadlines, so can π'. One can keep doing the interchange till the charging sequence completely follows the EDF principle.

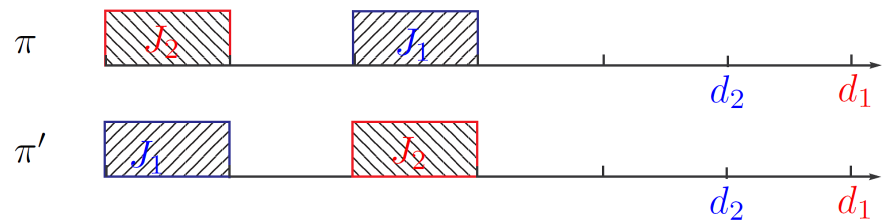


Figure 3.10: Interchange following EDF principle

The proof of the optimality of LLF is similar. As illustrated in Figure3.11, suppose some policy π can fully charge all EVs. Assume at time t, J_1 has less laxity than J_2, $d_1 - j_1 < d_2 - j_2$, and π charges J_2 at time t rather than J_1. We first find t', the first time π charges J_1 after t. We can interchange the charging of J_2 and J_1 at time t and t' and keep all other charging sequence the same as pi and the feasibility is preserved. The interchange can always be made since $d_1 - d_2 < j_1 - j_2 < j_1$. The policy π will not charge J_1 after d_2 for the first time. Otherwise it can not finish charging J_1 on time even if it spends all time from d_2 to d_1 on J_1. The interchange can be carried out till no such EV pair that violates the LLF principle can be found. Thus LLF can also finish all EVs if possible.

There is also a substantial literature on deadline scheduling with multiple processors ($M > 1$) (for a survey, see [28]). It is shown in [32] that optimal online scheduling policies do not exist in general for the worst case performance measure.

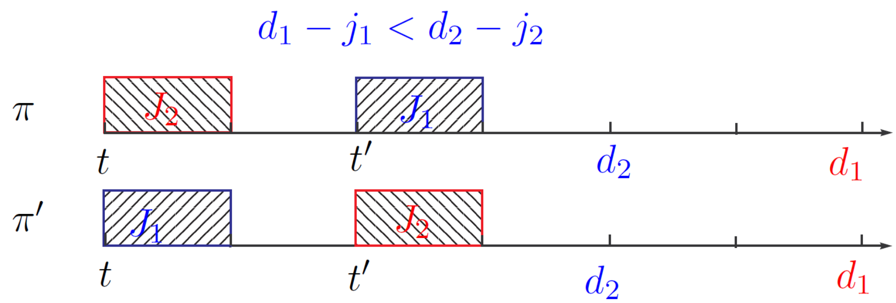


Figure 3.11: Interchange following LLF principle

3.7.2 Average case analysis

The literature on deadline scheduling in the average case analysis is less extensive. Panwar, Towsley and Wolf in [105] and [138] made early contributions in establishing the optimality of EDF in minimizing the unfinished work when there is a single processor ($M = 1$), and the jobs (EVs) are non-preemptive. The performance of EDF is quantified in the heavy traffic regime using a diffusion model in [90, 35, 88].

The multiprocessor ($M > 1$) stochastic deadline scheduling problem is less understood, primarily because the stochastic dynamic programming for such problems are intractable to solve in practice. A particularly relevant class of applications is scheduling in wireless networks where job (packets) arrival is stochastic, and packets sometimes have deadlines for delivery. In [11], the author analyzed the performance of the EDF policy for packets delivery in tree networks. In [112], the packets are assumed to arrive periodically or simultaneously and the Whittle's index policy is applied. Related problems of scheduling packets with deadlines in ad hoc networks are studied in [70].

The work closest to our problem in this chapter is [53] where the deadline scheduling problem with processing capacity is formulated as an RMAB problem and the indexability is established. There are, however, several important differences between [53] and our case. First, in [53], the arrivals are simultaneous. In our case, the arrival is random. Second, the constant processing profit is assumed in [53] while in our case the processing profit (charging cost) is stochastic.

CHAPTER 4

EV CHARGING FOR VEHICLE TO GRID SERVICES

Electric Vehicles can also perform a variety of functions related to the stability and efficient operation of the grid (collectively referred to as Vehicle-to-Grid, or V2G), and to be remunerated for their cooperation. Factoring these cooperation and remuneration structures into the charging problem of the EVs adds a degree of computational complexity, while also Requiring the creation of a new framework for revenue-sharing from the provision of these services given that the EV faces degradation costs and other potential inconvenience. In this chapter, we will first go over the proposed uses of EVs for V2G and studies about their economic viability (§4.1). We discuss incorporating demand-response and load-shifting (§4.2), and frequency-regulation (§4.3) in the scheduling problem.

4.1 Literature Review

Vehicle-to-Grid, or V2G, refers collectively to the provision of energy and ancillary services from an EV to the grid [131]. The former encompasses the use of EVs as distributed storage and to shift load, while the latter encompasses the provision of a range of services, such as frequency regulation and voltage control, to the grid.

4.1.1 EVs as distributed storage and load-shifting agents

Electric vehicles can be used as distributed storage by the grid to store renewable energy and to shift peak demand [79]. Vehicles are only utilized 4% of the time [79], leading to their potential use for grid-scale storage. However, this use is envisioned for large numbers of aggregated vehicles given the amount of storage required to have a significant effect on the grid load, and thus it will have a secondary effect on the uptake of electric vehicles [81]. Furthermore, this additional use of EV batteries leads to degradation of the batteries, and thus the benefit of this additional storage has to be weighed against the cost of this wear-and-tear. In simulation studies, the net annual benefit of this load-shifting versus the cost of battery degradation, even with *a priori* perfect market information, is shown to be \$10 – 120 per EV [108], which pales in comparison to the cost of an EV battery.

4.1.2 EVs as ancillary service providers

The Federal Energy Regulatory Committee (FERC) has defined ancillary services as "those services necessary to support the transmission of electric power . . . to maintain reliable operations of the interconnected transmission system" [24]. FERC has identified 6 types of possible ancillary services [24], but their implementation and availability depends on jurisdictions and ISOs (*e.g.,* [113]). In particular, the charging of fleets of EVs has been envisioned as a potential contributor to unit commitment [123, 122] and loss reduction in the distribution system [134]. However, these effects are only noticeable once there is a large enough coordinated population of EVs. The most well-studied ancillary services to be provided by EVs are those provided on shorter time scales: frequency regulation and spinning reserves [79, 81, 130, 131, 142, 74, 61].

Frequency control encompasses the actions taken to ensure that the grid frequency remains within a range of the nominal frequency, 60 Hz [113]. This process generally involves three levels of control, which are distinguished by their time-frame and where the action is taken [113]. Frequency regulation occurs through a real-time signal sent by the grid operator that calls for either a positive or a negative correction. Regulation typically occurs on an hourly basis and involves dispatch on time-frames of 4 seconds to 1 minute [81]. Typically, market participants contracted for frequency regulation are remunerated based on the amount of capacity available per-hour, with a smaller portion of payment to follow based on actual use of the resources [81]. Providing spinning reserves, however, requires maintaining synchronization with the grid and the ability to provide power immediately. These reserves are called upon during unplanned system events that occur 20-50 times a year [81] and their providers are remunerated based on both their availability and the used capacity. Frequency regulation has been shown to be the most economically beneficial avenue for EVs to participate in V2G markets [79, 81, 146, 142].

However, the total demand for frequency regulation is shown to be fulfilled by the participation of a relatively small number of EVs [6, 81], and thus the profit from participating in this market will decrease with the increasing penetration of EVs. After the large adoption of EVs, participation in load shifting would be another major benefit of EV owners.

4.2 Load Shifting

The lack of cost-effective storage makes the power grid inefficient dealing with the fluctuating demand. The annualy demand peak usually happens in the summer. Without local energy storage, a large number of reserve generators and transmission lines are needed. The peak typically only lasts a few hundred hours, necessitating a large investment for the marginal capacity that can be ameliorated by spreading out delay-tolerant load over time. Single purpose battery storage facilities have not been proven economical except in niche application such as delaying a distribution system upgrade. However, the V2G service of EVs supplies a dual purpose storage resource which has provoked great interest. Numerous studies have shown the ability of EV charging to reshape the demand profile or even move the peak period [59, 127].

One of problems of traditional storage is the huge initial capital cost. However, this problem is naturedly circumvented by the adoption of EVS. The battery of EVs is purchased for driving and the initial capital cost is not assigned to the off-vehicle electricity use. Besides the initial capital cost, the load shaving effect of V2G services also reduces the ramping reserves which usually come at great economic and environment cost.

The authors of [107] perform an economic analysis of a single EV owner participating the load shifting V2G service. The consumer is assumed to know the future electricity price and the battery is assumed to be able to discharge and sell the energy back to the grid. The objective function is to maximize the profit of energy arbitrage taking into account battery degradation. The economic test is carried out using different electricity price scenarios. The result suggests that the economic incentive for EV to participate in the energy arbitrage is small due to the small variation in LMPs and the small battery capacity.

A similar analysis is presented in [43]. The authors shift the charging of a single EV to achieve different objectives such as minimizing the charging cost and maximizing the use of renewable energy. The results show that the benefit strongly depends on the accuracy of predictions of price and renewables.

The load shifting of large scale EVs problem is considered in [135] with network constraints from the view point of an aggregator. The aggregator has direct control of multiple EVs and distributed generators. The objective is to minimize the day-ahead operation cost of the system while satisfying the network constraints, generation limits and the requests of EV charging. A three segments model of EV batteries is considered and the problem is formulated as a mixed-integer linear program (MILP) where the control variables are the output of the generators and the charge/discharge rate of EVs. Case studies are carried out using real network parameters.

4.3 Frequency Regulation

The flexibility inherent to EV charging makes EVs a good candidate to participate in the frequency regulation market. In the frequency regulation market, cleared participants (in the day-ahead ancillary services market) are given regulation signals to follow at the operating time along with payments associated with *up-regulating* or *down-regulating* at the ISOs request. Note that even EV chargers that cannot supply power back to the grid (*e.g.,* due to grid transformers not supporting bi-directional energy flow) can accomplish both actions by changing their consumption patterns. The charger's goal in this setting is to charge EVs up to their desired level while minimizing the expected cost of charging, factoring in possible payments from the ISO for providing frequency regulation services. Given that a participant in the regulation market needs to have the flexibility to accommodate unpredictable regulation signals at charging time, it is reasonable to assume that charging aggregators that operate multiple charger will be participants in the regulation markets, which typically occur a day ahead of charging time, with real time adjustments being made by EVs in response to signals sent by the aggregator that seeks to meet its obligations. We will examine these two problems in turn: we first focus on the problem of the aggregator that has to bid in the day-ahead market, and then focus on the problem of an EV that has to charge in real-time given regulation signals passed on by the aggregator, two problems that can be shown to be somewhat decoupled.

4.3.1 Large Scale EV Charging with Ancillary Service

The participation of public charging stations in the provision of ancillary services has been widley studied [132, 73, 148]. Naturally, the charging station has two related mathematical problems, divided into day-ahead and real-time stages by the time at which the decision has to be finalized.

In the day-ahead market, as all participants, the charging station submits the ancillary service bids to the ISO. The ancillary service bids consist of an expected demand curve for the coming day and an adjustment range within which the charging station promises to be able to adjust the power consumption. The ISO collects all the bids from the participants and decides who are selected to provide ancillary service by clearing the market.

In the real-time market, the ISO distributes the real-time electricity price and the regulation signal for each selected participant. The participants are asked to adjust the power consumption to follow the regulation signal. The revenue of

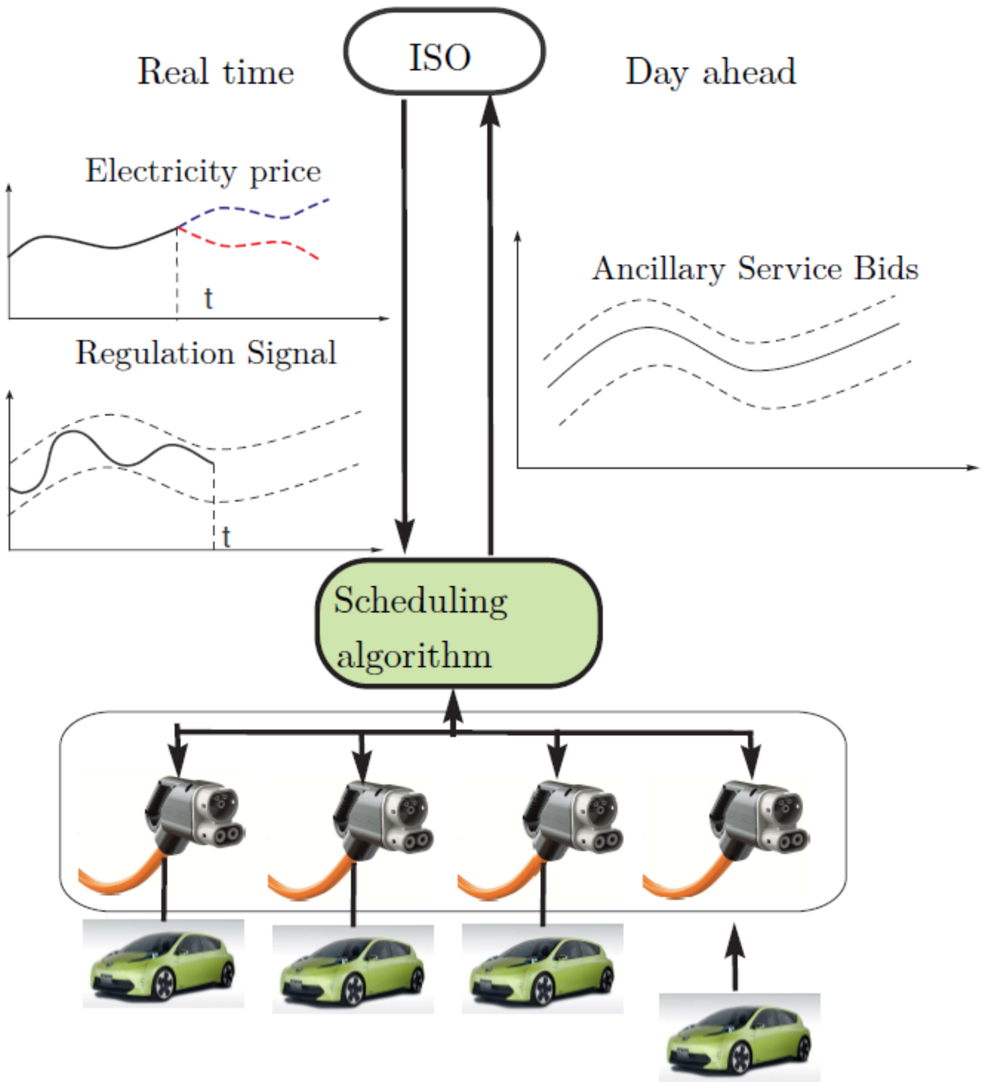


Figure 4.1: Charging station participating demand response.

the ancillary service is computed by the range of the adjustment and the accuracy and the speed of the response. The EV charging problem is more complicated in this case since the charging station needs to deal with the randomness from the EV arrivals, as well as the stochastic regulation signal.

4.3.2 Ancillary Service Bidding

Here we formulate the most general problem of the charging station. In this formulation, the charging station revenue comes from three sources. These sources are charging revenue collected from EV consumers R_{Ch} (usually assumed proportional to the charging energy or time), the revenues from selling regulation and responsive reserves capacity R_{AS}, and the possible revenues from selling energy to the grid R_{En}. The charging cost may include the wholesale cost of energy which is delivered to the EVs and the battery degradation associated with discharging. The bidding and the scheduling problem of the charging station is to maximize the profit subject to physical constraints and consumer requirements which may include but are not limited to battery capacity limits, consumer-defined minimum SOC by departure,and system loading limits.

The realized dispatch of energy in the real-time subject to the ancillary service signal is stochastic and unknown at bidding time. One intuitive way to deal with the randomness is to formulate the bidding problem in the day-ahead market as a linear programming based on the expected regulation signals [132]. The control variables include the preferred charging profile $u_i(t)$ of EV i, the up-regulation capacity $u_i^+(t)$, the down-regulation capacity $u_i^-(t)$, and the reduction in power draw available for spinning reserves $u_i^{\text{SR}}(t)$. The expected power consumption is stated as follows.

$$\mathbb{E}u_i(t) = u_i(t) + u_i^+(t)\mathbb{E}p^+ - u_i^-(t)\mathbb{E}p^- - u_i^{\text{SR}}(t)\mathbb{E}p^{\text{SR}}$$

where $\mathbb{E}p^+$ is the expected percentage of up-regulation capacity dispatched each time, $\mathbb{E}p^-$ the expected percentage of down-regulation capacity, and $\mathbb{E}p^{\text{SR}}$ the expected percentage of responsive reserve capacity.

The optimization problem is formulated as follows.

$$\begin{array}{lll} \max_{u_i(t),u_i^+(t),u_i^-(t),u_i^{\mathrm{SR}}(t)} & R_{\mathrm{Ch}} + R_{\mathrm{AS}} + R_{\mathrm{En}} - \sum_i \sum_t c(t)\mathbb{E}u_i(t) - C_{\mathrm{Deg}} & \\ \text{Subject to} & \text{Charging capacity constraints} & \\ & \text{Customer-defined minimum SOC limits} & \\ & \text{Power constraints} & \end{array} \tag{4.1}$$

where:

- the charging revenue is the charging price times the charging energy $R_{\mathrm{Ch}} = p \sum_i \sum_t \mathbb{E}u_i(t)$,
- the revenue of selling energy back to the grid equals to the energy price times the energy delivered to the grid $R_{\mathrm{En}} = \sum_t p_{\mathrm{En}}(t) \sum_i \mathbb{E}u_i(t)\mathbb{1}(\mathbb{E}u_i(t) < 0)$,
- and C_{Deg} the battery degradation cost.

If the ancillary service revenue is measured linearly in terms of the up and down-regulation range and the spinning reserve capacity, the problem is a linear program. The expected demand curve and the regulation range are thus the sum of expected power consumption $\sum_i \mathbb{E}u_i(t)$ and the sum of regulation ranges of each EV.

This formulation assumes that the charging facility has perfect prediction of electricity price, ancillary service price, and driving pattern of customers. Advanced machine learning algorithms and smart phone applications may help to improve the predictions. However, a study of the impacts of the forecasting errors is lacking.

4.3.3 Real-time Scheduling

When scheduling the real-time EV charging, the charging facility needs to deal with the difference between the prediction made in the day-ahead planning and the realization of the regulation signal and the charging demand. Several approaches are proposed to adjust the planned charging profile. In [132], the amount of power adjustment required by the ISO is distributed to EVs proportional to the adjustment range $u^{+}, u^{-}, u^{\mathrm{SR}}$. In [73], authors investigated the real-time adjustment balancing the deviation from the planned charging trajectories and the error of dispatch signal tracking.

A "priority" policy of large-scale EV charging participating the ancillary service program is proposed in [148]. Following the formulation in Chapter 5, the scheduler does not require the prediction of electricity price, ancillary service dispatch signal, EV charging demand in the real-time scheduling. Under the framework illustrated in Figure 4.1, following assumptions are approximations of the practical operating conditions and made for tractable analytical developments.

- A1. Each charger can be connected to only one EV, and it is removed from the EV at the deadline d_i.
- A2. An EV is charged at a fixed rate normalized to 1 and can not be discharged [60].
- A3. The EV arrivals to the N chargers are independent and identically distributed (i.i.d.).
- A4. The price of charging collected from consumers is proportion to the charging demand, normalized to 1 dollar/hour.

A5. The marginal charging cost $c[t]$ is an exogenous finite state Markov chain whose evolution is independent of the state evolution and actions of charging.

A6. The charging of EVs is preemptive without cost.

A7. The penalty for incomplete charging is a convex function of the incomplete amount at the deadline.

A8. The dispatch signal $M[t]$ is stochastic and independent from the actions of the charging facility.

The state transition and charging reward of each EV are the same as defined in (3.1) and (3.2). The charging profit collected from the EVs is stated as

$$\mathcal{R}_{\text{Ch}}(S[t], \vec{a}[t]) \triangleq \sum_{i=1}^{N} R_{a_i[t]}(S_i[t], c[t]).$$

Based on the new rule of demand response market [110], the frequency regulation credit includes the capacity payment and performance payment. The former one is simply the regulation amount times the regulation price per unit. The performance payment is measured by the tracking accuracy of the regulation signal. The accuracy as defined in PJM ancillary service market is stated as $(1 - |\sum_{i=1}^{N} a_i[t] - M[t]|/M[t])$ [109], where $M[t]$ is the regulation signal and $\sum_{i=1}^{N} a_i[t]$ the extra power usage.

The frequency regulation credit collected from the ISO at time t is stated as follows.

$$\mathcal{R}_{\text{AS}}(S[t], \vec{a}[t]) \triangleq A[t](1 - |\sum_{i=1}^{N} a_i[t] - M[t]|/M[t]) + B[t],$$

where $A[t]$ and $B[t]$ take account of the stochastic demand response price and capacity credit.

The charging rate of each EV is assumed fixed and the action is reduced to binary variables: charging/idle. Thus the dispatch signal $M[t]$ is quantized into the number EVs that are simultaneously being charged.

Given the initial system state $S[0] = s$ and a policy π that determines a sequence of actions $a[t], t = 0, 1, \cdots$, the expected discounted system reward is defined by

$$V_\pi(s) \stackrel{\Delta}{=} \mathbb{E}_\pi \left\{ \sum_{t=0}^{\infty} \beta^t [\mathcal{R}_{\mathrm{Ch}}(S[t], \vec{a}[t]) + \mathcal{R}_{\mathrm{AS}}(S[t], \vec{a}[t])] \middle| S[0] = s \right\} \tag{4.2}$$

where $\mathbb{E}_\pi$ is the conditional expectation for given scheduling policy π and $0 < \beta < 1$ the discount factor.

The random regulation signal and regulation price introduce extra complexity. One intuitive way is to model the tracking of the signal as a constraint on number of active arms and to maximize the charging rewards. The stochastic dynamic programming in () can be viewed as a restless MAB problem that, at each time t, exactly $M[t]$ out of N chargers (arms) are active. The optimization problem is state as follows.

$$\begin{array}{ll} \sup_\pi & \mathbb{E}_\pi \left\{ \sum_{t=0}^{\infty} \sum_{i=1}^{N} \beta^t R_{a_i[t]}(\tilde{S}_i[t]) \mid \tilde{S}_i[0] \right\} \\ \text{subject to} & \sum_{i=1}^{N} a_i[t] = M[t], \quad \forall t. \end{array} \tag{4.3}$$

Similar to Chapter 3, a heuristic policy, the Whittle's index policy with LLLP interchange, is proposed as illustrated in Algorithm 2.

The proposed policy tries the best to follow the regulation signal and takes the advantage of time varying charging cost while balancing the risk of non-completion penalties. In principle, it gives higher priority to EVs with tight deadlines and large remaining demand to avoid potential penalties.

Algorithm 2: Whittle Index with LLLP interchange

1. Calculate the Whittle's index of all EVs and sort them in a descend order.
2. Apply LLLP inter-change to the sorted EVs.
3. Activate the $M[t]$ EVs with highest priority. If there are not enough available EVs in the station, activate as many as possible.

Note that the proposed algorithm does not guarantee the feasibility of (4.3). However, the heuristic itself always works. If there are not enough EVs with positive charging demand in the system, the algorithm simply charges as many EVs as possible to try best tracking the regulation signal.

4.3.4 Participation of EVs in the Frequency Regulation Market

The provision of frequency regulation services is remunerated by the grid based on capacity rather than actual dispatched energy, as the fluctuations required by the grid are evenly distributed around zero [14]. This means that given the fluctuations in the availability of EVs and the need for forward contracts (*e.g.,* commitment to providing frequency regulation service in the day-ahead market), the presence of an aggregator is needed to coordinate the participation of populations of EVs in the frequency regulation market. The aggregator then commits to providing regulation services to the grid for a specific time period through a contract. At operation time, when the aggregator receives a regulation signal, it can call upon available, idle EVs to supply the required power by incentivizing them by passing along a scaled portion of the payment received from the grid for the service.

Since regulation service is only provided by idle EVs (*i.e.,* those not being charged in that time instance), thus the participation of a single EV in frequency regulation can be decoupled from the problem of the aggregator, and its charging schedule can be captured in the same framework as off-line charging given advance knowledge of the regulation price [118, 61].

The decision on whether to participate in the frequency regulation market in each time period will be a function of the state of charge of the EV at that time-frame, as well as its availability in the charging station. Thus, the optimal charging and participation decision can be folded into the same Markov Decision Process framework [118] that can be solved numerically.

Without the frequency regulation decision, the optimal schedule computation laid out in §2.3.1 involved finding the least-costly feasible schedule:

$$\min_{\mathbf{0} \leq \vec{u} \leq \vec{\bar{u}}} \qquad \sum_{t=1}^{T} C_t\left(\sum_{i=1}^{M} u_i[t]\right). \tag{4.4}$$

Notice that the cost of charging in this context depends on the total charging profile at each time. Since energy used to charge by a single EV is negligible compared to the grid, one can assume that in formulating the single EV problem, $C_t(x) = C[t]x$ is a reasonable assumption (*i.e.,* the fluctuations in the price of power are independent of the consumption of a single EV).

Furthermore, the aggregator will provide a financial incentive for the EV to regulate their charging up or down to track the regulation signal given by the grid when they are not being charged. Therefore, there will be an additional term in the objective of each EV that will incorporate the possible pay-off they can achieve from tracking the regulation requirement of the aggregator.

Thus, if $\mathbb{T}_C$ is defined to be the set of times in $\mathbb{T} = \{1, \ldots, T\}$ at which the EV is being charged, the remuneration structure for participation in the regulation market will apply for $\mathbb{T}\backslash\mathbb{T}_C$, so the total remuneration becomes:

$$\begin{aligned} \min_{\mathbf{0}\leq\vec{u}\leq\vec{\bar{u}}} \quad & \textstyle\sum_{t=1}^{T} C(t)u(t) - \sum_{t\in\mathbb{T}\backslash\mathbb{T}_C} C_R(t) \\ & = -\textstyle\sum_{t=1}^{T} C_R(t) + \sum_{t\in\mathbb{T}_C}(C_R(t) + C(t)u(t)). \end{aligned} \tag{4.5}$$

where $C_R(t)$ is the scaled remuneration for provision of regulation service at time $t \in \mathbb{T}\backslash\mathbb{T}_C$. For a single EV, and without strict charging bounds, it is easy to see that an optimal solution exists such that $u(t)$ only takes values $\{0, \bar{u}\}$ for all but at most one $t \in \mathbb{T}_C$ using a replacement argument, as a linear function of a bounded variable is maximized at one of the bounds. The implication, as foreshadowed in §2.3.1, is that for a reasonably large number of time-increments, the optimal schedule will only consist of periods of charging at the maximal rate and periods of idleness. Therefore, the charging action-space has only two values, making the problem suitable to a dynamic programming framework.

This simplified framework, however, does not explicitly consider the charging limits of batteries. In the real world, for example, a fully charged EV cannot provide up-regulation to the grid, as it does not have the means to charge even more, and vice versa. One solution to this problem that has been proposed [61] is to differentiate between the up-regulation and down-regulation signal in the remuneration structure, replacing $C_R(t)$ with $C_{RU}(t)W_{RU}(s(t)) + C_{RD}(t)W_{RD}(s(t))$, where $s(t)$ is the state of charge of the EV at time t, $C_{RU}(t)$ and $C_{RD}(t)$ are the up and down regulation prices offered by the aggregator (potentially equal), and $W_{RU}(\cdot)$ (respectively, $W_{RD}(\cdot)$) is a weight function that is equal to 1 when EV charge is far away from its upper (respectively, lower) limit, and decreases linearly to 0 with EV charge as the EV increases (decreases) to its maximum (minimum) level. This essentially eliminates the benefit of participating in up-regulation services when the EV is fully charged, and participating in down-regulation when the EV has an empty battery, and thus makes the final policy implementable. The final step of the process involves pinning the final state of charge of the EV (at departure time) to the desired amount, and solving the resulting Markov Decision Process, *e.g.*, using backward induction.

The savings from participating in this market have been estimated to be on the order of $600 per EV per year [118] even with uni-directional flows that would not require upgrades to the distribution system.

Numerical Results

In this section, numerical experiments are conducted to compare the performance of different scheduling policies with demand response options in the real-time. One intuitive trajectory tracking algorithm is proposed in [73], which minimizes a trade off between the tracking errors of a predetermined charging trajectory for each vehicle (determined in a day-head optimization like (4.1)) and the deviation from the regulation signal. In the simulation, Algorithm 2 with constant dispatch signal $M[t] = M$ is applied to generate a planned charging trajectory for each charger. After that, the dispatch signal is generated by a uniform distribution with mean M. Since the result of the convex programming in [73] is continuous, the binary charging action is generated by Bernoulli random variables with the results of the convex programming as the probability coefficients.

Figure 4.2 shows an example of regulation signal tracking performance of different policies. When the constraint is feasible, the Whittle's index policy with LLLP interchange always perfectly matches the regulation signal. While, the trajectory tracking policy deviates from the regulation signal slightly due to the difference between the predetermined charging trajectory and the regulation signal.

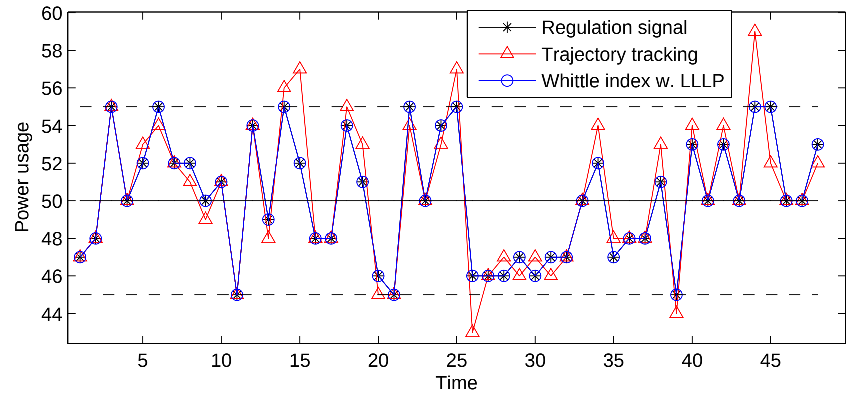


Figure 4.2: Regulation signal tracking: $M = 50$, 10% regulation capacity.

The impact of the regulation range on tracking accuracy and charging rewards is illustrated in Figure 4.3. As the range increases, the tracking error of trajectory tracking policy increases while Whittle Index policy with and without LLLP improvement matches the regulation signal perfectly. For the charging reward, the trajectory tracking algorithm does not consider the state of EVs and the electricity price. The performance in charging reward is worse than Whittle's index. Meanwhile, LLLP improves the Whittle's index policy in the charging reward collected from EV consumers and thus makes more profit.

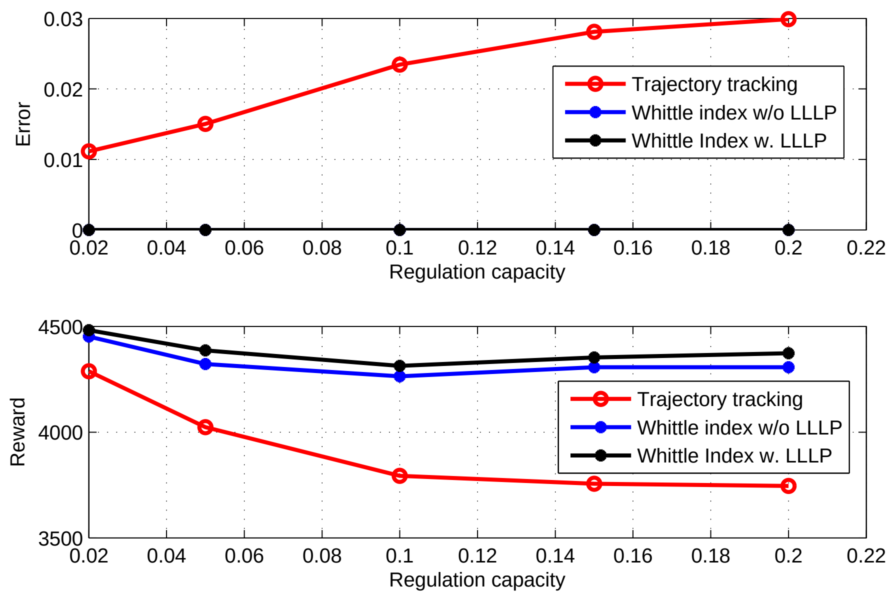
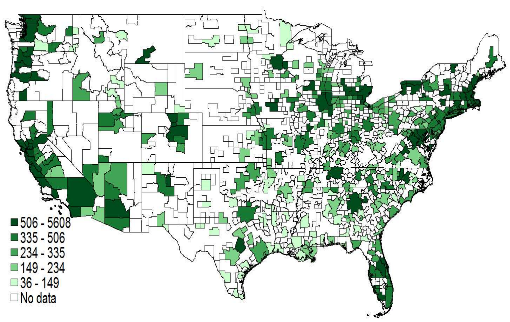


Figure 4.3: Regulation signal tracking accuracy and charging reward Vs. regulation capacity.

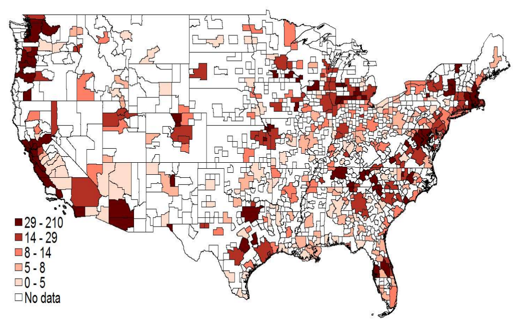
CHAPTER 5

MARKET DYNAMICS AND INDIRECT NETWORK EFFECTS IN ELECTRIC VEHICLE DIFFUSION

As discussed in Chapter 1, both the electric vehicles and the charging stations embrace a rapid development in the past decade. The growth trends of EVs and EVCSs have strong temporal and geographic couplings as shown in Figure 5.1. This is the ”cluster” phenomenon of alternative fuel vehicles discussed in [144], which is the first article to discuss the network effects between the alternative fuel vehicles and the fueling infrastructure. Consumers’ EV adoption in the EV market is affected by the availability of EV charging stations whereas the level of EVCS investment strongly depends on the size of EV stock. Thus areas with a lower cost to adopt charging stations will attract more EVs and more EVs incentivize the investors to build more charging stations.



(a) EV stock per million people.



(b) Public charging stations per million people.

Figure 5.1: EVs and public charging stations in Metropolitan Statistical Areas in 2015 [67].

The reason behind the growth of the EV market, or the lack of it, is multifaceted. The growth is driven partially by the increasing awareness of the environmental impacts of gasoline vehicles, superior designs and performance of some EVs, and, by no small measure, subsidies in the form of tax credits provided by the federal and state governments. However, the EV industry faces stiff skepticism due to the high purchase cost of EVs, the limited driving range, the long charging time, and the lack of public charging facilities.

The manufacturer's suggested retail price (MSRP) of the 2016 Nissan Leaf, Ford Focus Electric, and Chevrolet Volt are $34200, $29170, and $25670, respectively, which are more than $10000 above the purchase price of comparable gasoline vehicles, such as Nissan Sentra, Ford Focus, and Chevrolet Cruze. To make EVs equally attractive as internal-combustion vehicles, federal and state governments provide a tax credit subsidy up to $7500 for each purchase of EVs. Meanwhile, the cost of electric vehicle batteries, which is the major cost of electric vehicles, has been reduced by more than 65% since 2010. It is projected that, by 2020, EVs will become more economic than internal-combustion vehicles even without government subsidies [2]. Besides the quick drop of the battery cost, the low operation cost of EVs also significantly offsets the high capital cost [40].

The limited driving range of EVs, however, restricts the adoption significantly. By 2016, Nissan Leaf has an range of 107 miles after a full charge, comparing with the range of 99 miles of Ford Focus Electric and 53 electric miles of Chevrolet Volt. While the gasoline vehicles can easily achieve a driving range of over 400 miles per tank of gas. The anxiety of running out of electricity makes the EV charging infrastructure and charging time important considerations when consumers choose vehicles.

This chapter provides an analytical study on indirect network effects in the EV market. In particular, we formulate a sequential game for the two-sided market and address analytically and numerically the following questions: how does the EV adoption interact with EVCS investment? How do indirect network effects affect market dynamics? What are the implications of indirect network effects on the public policy?

A perfect and complete sequential game model for the two-sided EV-EVCS market with an investor as the leader and each consumer a follower is introduced. Through profit maximization, the investor decides whether to build EVCSs at sites chosen (optimally) from a list of candidate sites or to defer his investment with earned interest. The candidate sites are heterogeneous; each site may have a different favorability rating and different capital costs. The optimal investment decision also includes the optimal pricing of charging services.

Observing investor's decision which defines the locations of EVCSs and the charging prices, a consumer decides whether to purchase an EV or a gasoline alternative. If the choice is an EV, the consumer also decides the preferred charging service.

We provide a solution to the sequential game that includes the optimal decisions for the consumers and the investor. A random utility maximization (RUM) model [99] with two different distributions of the consumer vehicle preference is considered.

Under the assumption that the consumer vehicle preference has the type I extreme value distribution, we show that the optimal purchasing decision is a threshold policy on the difference of vehicle preferences of the consumer. A closed-form expression for the EV market share η_e is obtained, where η_e is a function of the EV purchasing price, the investor's decisions on the number/locations of charging stations, and the charging prices at those locations. The obtained closed-form solution allows us to examine how the investor's decisions and the cost of EVs affect the overall EV market share.

To obtain the optimal investment decision, we first study the optimal operation decision of the investor by fixing the set of EVCS sites. We show that the optimal pricing for EV charging at these sites is such that profits generated from these sites are equal. We show further that the optimal pricing increases logarithmically with the density of EV charging sites.

The optimal decision on choosing which EVCS sites to build (or deferring investment) is more complicated and is combinatorial in nature. We provide a greedy heuristic and show that the heuristic is asymptotically optimal as the density of EVCS sites increases.

Under the assumption of the uniform vehicle preference, similar results are also obtained. The optimal purchasing decision of a consumer is again a threshold policy on the vehicle preference. There is, however, a *dead zone effect* of the EVCS density. Specifically, when the density of EVCSs is lower than some threshold, the EV market share is zero.

5.1 Literature Review

There is an extensive literature on two-sided markets and indirect network effects for various products; see *e.g.,* the compact disc (CD) player and CD title markets [49], the video game console and video game markets [23, 25, 154], the hardware and software markets [36], the credit card market [7], and the yellow page and advertisement markets [121]. Previous work on indirect network effects dates back to early theoretical studies such as [116, 78, 42]. Rochet and Tirole [114] firstly proposed a restrictive definition to distinguish between one-sided and two-sided markets in the context of charge per usage. Caillaud and Jullien [17] pointed out that, one side of the market always waits for the action from the other side. It is thus critical for players to take the right move during the initial stages of the product diffusion.

Exploratory researches on the diffusion of vehicles with alternative fuels have been conducted. Surveys were carried out to study the consumer attributes on conventional vehicles and plug-in EVs in [85, 111, 89, 30]. The role of consumer environmental awareness and signaling in the market for traditional hybrid vehicles is examined in [75, 76] and [126]. The impacts of government programs both at the federal and state level in promoting the adoption of hybrid vehicles are examined in recent studies, including [9, 46] and [124]. On the other hand, Bunch *et al.* [15] established and estimated a nested multinomial logit model for the clean-fuel vehicles demand. Similar discrete decision model of vehicle is used in [41, 50, 64, 58] to study the preference of hybrid electric vehicle. It is pointed out that charging convenience is a major concern when consumers make the purchase but no analytical result of impact of EV market performance on EVCS investment is presented.

There is a growing literature on the EVCS investment from the operation research and engineering perspectives. For example, the charging station deployment has been formulated as an optimization problem from the social planner's point of view in [128, 45, 63, 21]. A location competition problem of charging stations is considered in [10], where a discrete decision model of charging stations similar to this chapter is used. Efficient design of large scale charging is presented in [20] and the competition of charging operations is considered in [19].

There is a rich literature on externalities in the new technology market [69, 38, 8, 13, 101]. Jaffe and Stavins [69] examine the energy efficient technologies in buildings and suggest that energy paradox could be driven by a host of factors such as the information problem and unobserved costs of new technologies. Economides [38] points out that the network externality is often observed in non-network industries that the value of a unit of good increases with the number of units sold. Arthur [8] examines the outcomes of two competing new technologies under various conditions and shows that the new technology market gradually lock itself into an outcome that is not entirely predictable in advance. Bresnahan and Greenstein [13] characterize the equilibrium and strategic behavior within each segment of the computer industry and explain the dramatic change of the market in 1990s. More closely related to our problem is Meyer and Winebrake [101] which is the first articles to explore the indirect network effects between alternative fuel vehicles and refueling stations. The multinomial logit model of vehicle choice considered in Meyer and Winebrake [101] is similar to the consumer vehicle choice model in this chapter. However, Meyer and Winebrake [101] consider only the consumer vehicle choice without modeling the investment decision on refueling stations and provide simulation results under various scenarios. This chapter focuses on the EV market and provides a theoretical analysis that simultaneously models the two-sides of the market: consumer adoption of EVs and investment decision of charging stations.

The work of Li *et al.* [91] and Yu *et al.* [152] represent the first analysis of the two-sided EV and EVCS market and the related indirect network effects. The work in [91] focuses on an empirical study of indirect network effects whereas [152] focuses on the theoretical analysis. In particular, [152] extends the model in [151] by allowing (unobserved) vehicle preferences to have a type I extreme value distribution consistent with the discrete choice model [100]. Also new in [152] are the comparison between the market solution and the decisions of the social planner and the effects of subsidies for EV purchase and EVCS investments.

This chapter is organized as follows. The structure of the two-sided market and a sequential game model are described in Section 5.2. The solution to the game is obtained by backward induction. In Section 5.3, the consumers' model and the optimal decisions are obtained. The investor's model and optimal strategy are presented in Section 5.4, as well as the social welfare optimization. Discussions about different effects of subsidies and the difference between the private market solution and the socially optimal solution are presented in Section 5.5.

5.2 A Sequential Game Model

In this section, we formulate the two-sided market as a two-player sequential game model with perfect and complete information. We introduce the basic structure of the EV-EVCS market, define the players of the game, and specify the decision process.

5.2.1 Two-sided market structure

A two-sided market typically has a structure as illustrated in Figure 5.2, where we use a generic hardware-software market as an example to describe its basic components. A two-sided market includes a set of platforms, say, Macbook™ and the OS X operating system as a hardware-software platform by Apple Inc. vs. Dell's Inspiron™ and the Windows 8 as an alternative.

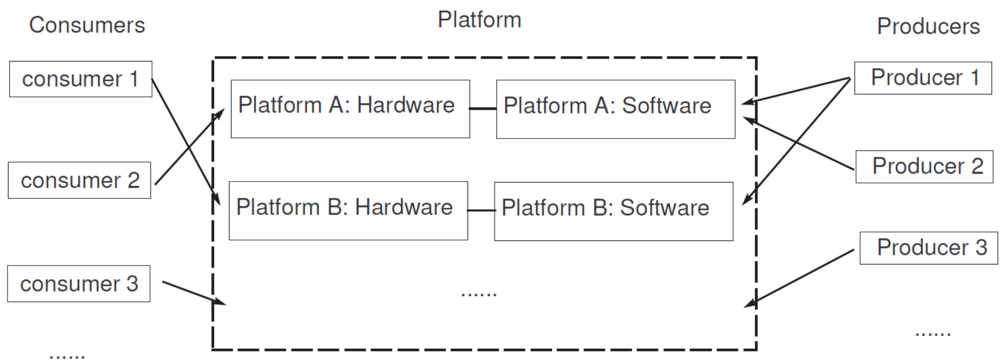


Figure 5.2: The structure of a two-sided market.

On the one side of the platforms is the consumer who makes her purchase decisions based on her platform preferences, the costs of the platforms, and the available softwares for different platforms. On the other side of the platforms are the software developers who invest in developing softwares for one particular platform or multiple platforms. The software developer makes his decision based on, among other factors, the costs of developing softwares and the popularity of platforms.

In Figure. 5.3, we present the two-sided EV market. There are two platforms: one is the EV as the "hardware" and the EVCS as the "software". The other is the conventional gasoline vehicle as the "hardware" and the gas station as the "software". On the one side of the platforms are the consumers who decide which type of vehicles to purchase based on the cost of EVs, the available charging stations, and the cost of charging. On the other side of the platforms is an investor who decides to build and operate charging stations or to defer his investment and earn interest at a fixed rate[1].

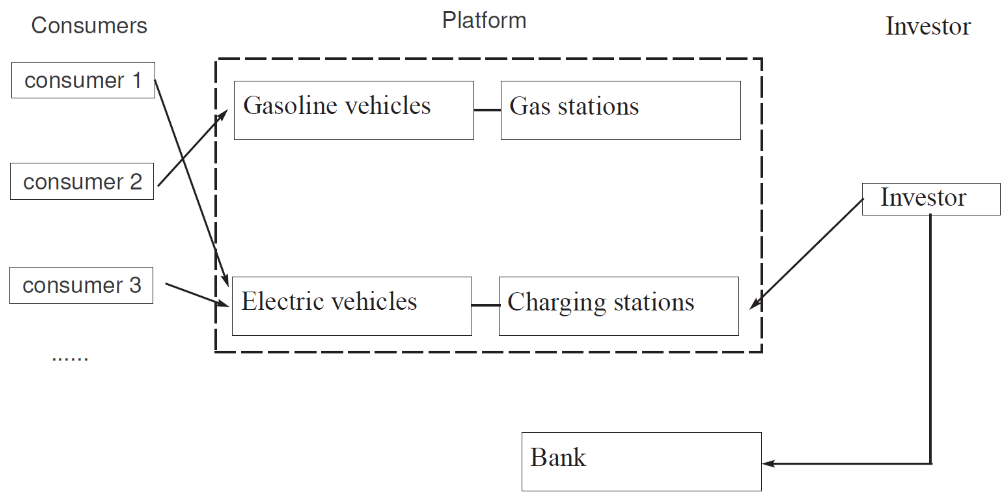


Figure 5.3: The two-sided market model of EVs and EVCSs.

5.2.2 The investor's decision model

We assume that the investor is both the builder and the operator of EVCSs. The monopoly assumption is valid at the launch stage of EVCS market since the investment is mainly conducted by the EV manufactories or the government. The investor's decision has two components: the first is an *investment decision* on whether to build EVCSs from a list of candidate sites or to defer his investment. The second is an *operation decision* on pricing the charging services at those built locations. The type of the charging station to be built is another decision

[1]Because we focus on the early stage of EV diffusion in an environment that gas stations are already well established, the alternative of EVCS investor is not building additional gas stations.

component since the building cost of the charging stations are dramatically different and the cost of charging time is an important concern of consumers when choosing charging service. In this chapter, we assume the charging rate of the EVCSs is the level 2 charging, the dominant charging rate for the public charging stations on the market. The major concern of the DC fast charging is the potential damage to the battery and its impact on the grid.

Let $\mathscr{C} = \{\bar{s}_i = (f_i, \bar{c}_i), i = 1, \cdots, N_L\}$ be the set of candidate sites for charging stations known to the investor. Site $\bar{s}_i = (f_i, \bar{c}_i)$ has two attributes: the favorability rating f_i and the marginal operating cost[2] $\bar{c}_i$. The favorability rating f_i represents the characteristics of charging stations. It may include but not limited to variations in accessibility and availability of ancillary services. Accessibility refers to how easy it is for consumers to access the charging station and potentially affects the volume of the passenger flow. A similar discussion is presented for gas stations by Salop and Hotelling ([125, 66]). For example, a site at workplace parking lots may be more attractive than a location that is less frequently visited by consumers. Ancillary services refers to other services that a charging station may provide such as vehicle repair and super markets. Kroger, the largest grocery store owner of the U.S. has installed over 300 Level 2 and DC fast charging stations in key markets over the country [56]. Walmart and Kohl's also expanded their charging stations [54, 55]. These public charging station locations are also favored by EV owners since they can charge while shopping.

The operating cost $\bar{c}_i$ may vary across stations because of different locational marginal price of wholesale electricity, the labor cost and other cost components.

Given $\mathscr{C}$ and the utility functions of the consumer, the investor's decision is

[2] The marginal cost (\$/mile) here is marginal operating cost (\$/kWh) normalized by EV efficiency (miles/kWh).

denoted by $(C, \vec{\rho}) \in 2^{\mathscr{C}} \times \mathcal{R}^{|C|}$ where $C \subseteq \mathscr{C}$ is the set of locations selected to build charging stations and $\vec{\rho} = (\rho_1, \cdots, \rho_{|C|}) \in \mathcal{R}^{|C|}$ the vector of charging prices at the built stations.

Assuming the consumer maximizes her utility, the investor chooses the investment sites and charging prices to maximize the investment profit within his budget B. The investment optimization is stated as

$$\begin{array}{ll} \max_{C,\vec{\rho}} & \Pi(C, \vec{\rho}) - \sum_{i=1}^{|C|} \mathcal{F}(\bar{s}_i) \\ \text{subject to} & \sum_{i=1}^{|C|} \mathcal{F}(\bar{s}_i) \leq B \end{array} \tag{5.1}$$

where Π is the operational profit and $\mathcal{F}(\bar{s}_i)$ the building cost of station i.

5.2.3 The consumer's decision model

The expected cost of owning and using an EV include the upfront capital cost and the expected future operating cost. The capital costs vary between EVs and gasoline cars reflecting different purchase costs of these two types of vehicles. Our model defines the expected future operating cost to be a function of expected gasoline price for gasoline cars and a function of expected electricity prices for EVs. The gasoline price can be thought of expected average gasoline price in the future. In case that gasoline prices follow a random walk as suggested by Anderson, Kellogg, Sallee [5], the benefit predictor of future gasoline prices would be current gasoline price.

A consumer observes the investor's decision on the location set of charging stations $C = \{s_1, \cdots, s_{N_E}\}$ and the charging price vector $\vec{\rho} = (\rho_1, \cdots, \rho_{N_E})$ where N_E is the number of charging stations. The consumer first chooses the type of vehicle to purchase. If the choice is an EV, the consumer also decides on the location of charging. The action of the consumer is given by $\{v, k\}$ where $v \in \{E, G\}$ is the vehicle choice (either EVs or gasoline vehicles) and $k \in \{0, 1, \cdots, N_E\}$ the preferred charging station. We include $k = 0$ for the home charging option. The consumer chooses $\{v, k\}$ by maximizing the overall vehicle utility that includes the charging utility for the EV purchase.

For the vehicle choice, we assume a widely adopted discrete choice model with random utility functions [10]. The consumer utility model of purchasing a vehicle is assumed as follows.

$$\begin{aligned} V_E &= \beta_1 \mathbb{E}(U_E) - \beta_2 p_E + \Phi + \epsilon_E \\ V_G &= \beta_1 \mathbb{E}(U_G) - \beta_2 p_G + \Phi + \epsilon_G \end{aligned} \tag{5.2}$$

where U_E is the (random) charging utility of consumer's best choice defined in (5.5), $\mathbb{E}(U_E)$ the expected maximum charging utility, p_E the price of an EV, Φ the utility of owning a vehicle, and ϵ_E a random vehicle preference of EV. Variables $\mathbb{E}(U_G)$, p_G, and ϵ_G are similarly defined for the gasoline vehicle.

The coefficient β_1 captures the magnitude of the indirect network effects. More charging stations provide potentially higher charging utility of consumer-s, thus increases the utility of purchasing a electric vehicle. The charging utility U_E captures the operation cost and charging time cost of EVs. The price of electricity is much lower than the price of gasoline which makes the operation cost of an EV much lower. However, the charging time cost of EVs offsets the benefit. In the charging utility model described in (5.4), we explicitly consider the effect of charging price. The charging time cost can be included by offsetting the favorability rating f_i by a constant since we are considering only level 2 charging. The coefficient β_2 is the price elasticity of demand and captures consumer price sensitivity. The preference ϵ_E and ϵ_G represents the consumer preference of EVs and gasoline vehicles, including but not limited to quality and brand loyalty, driving experience, awareness of environment [75, 76] and other demand shocks.

This consumer model is flexible and can be either short-term (EV leasing) or long-term (EV purchasing). In a framework of EV purchasing, the price p_E would be the purchasing cost of electric vehicles. The charging utility is calculated during the life time of an EV. In a framework of EV leasing, the price p_E would be the leasing cost of electric vehicles, for example, for three years, the duration of a typical leasing contract

The consumer's decision is then defined by

$$\max\{V_E, V_G\}. \tag{5.3}$$

The optimization of consumer's vehicle decision also includes optimally choosing charging stations. Specifically, the consumer charging utility at station i is assumed to be random in the following form.

$$U_i = \alpha_1 f_i - \alpha_2 \rho_i + \epsilon_i, i = 0, \ldots, N_E \tag{5.4}$$

where f_i is the favorability rating, ρ_i the charging price determined by the investor, ϵ_i the random preference of charging station i. The coefficient α_2 indicates the price sensitivity of the charging price of consumers.

Given the realization of the charging preference, $\vec{\epsilon} = (\epsilon_0, \ldots, \epsilon_{N_E})$, the EV owner chooses charging station $k \in \{0, 1, \ldots, N_E\}$ to maximize her charging utility, *i.e.*,

$$U_E = \max_{k \in \{0,\ldots,N_E\}} U_k. \tag{5.5}$$

For the option of choosing a gasoline vehicle, the number/locations of gas stations will not change with the investor's decisions. Thus the expected maximum fueling utility, $\mathbb{E}(U_G)$, is a constant.

The indirect network effect between the EV market and the EV charging station market is reflected by the investment decision of the investor and the charging utility U_E. A shock to the system, for example a tax credit, would stimulate the sale of EVs, which will bring more profit to the charging stations and attract the investor to build more charging services. In the other hand, more charging stations make the charging more convenient and result in a greater charging utility U_E. When β_1 is statistically significant, more consumers will choose to purchase an EV. The impact would circle back and forth between the decisions of consumers and the investor and form a positive feedback loop in both sides of the market [91].

5.2.4 The sequential game model

The sequential game structure of the two-sided EV-EVCS market is summarized as follows.

- The investor's decision is defined by the optimization in (5.1). Specifically, given the set of locations, $\mathscr{C}$, the investor decides to invest (build and operate) charging stations at a subset $C \subseteq \mathscr{C}$ and determines the charging price $\vec{\rho}$. When $C = \emptyset$, the investor defers his investment and earns interest at a fixed rate.
- The consumer's decision is defined by (5.3-5.5). Specifically, having observed the investor's decision, $\{C, \vec{\rho}\}$, the consumer chooses $\nu \in \{E, G\}$. If $\nu = E$, the consumer also chooses charging stations to charge by maximizing her charging utility.

The dynamic game is solved by backward induction. In particular, we first consider the consumer's decision by fixing the investor's choice of charging locations and charging prices. The optimal consumer's decision is given in Section 5.3. In Section 5.4, the optimal investor's decision is presented.

5.3 Consumer Decisions

5.3.1 Consumer Decision Model and Assumptions

We first summarize the assumptions of the consumer model given in Section 5.2.3.

A1. Consumers are identical and their decisions are statistically independent. Without loss of generality, we focus on the decision of a single consumer.

A2. The average charging demand is normalized to 1.

A3. The random preference of charging station i, ϵ_i, is independent and identically distributed (IID) and follows the type I extreme value distribution with the probability density function (PDF)

$$f(\epsilon) = e^{-\epsilon} e^{-e^{-\epsilon}}.$$

A4. The random preference of vehicles ϵ_E and ϵ_G are statistically independent.

The type I extreme value distribution is widely used in the discrete choice model. McFadden firstly introduced it in the consumer choice theory and showed it leads to the multinomial logit distribution across choices [100].

5.3.2 Consumer Decisions and EV Market Share

The main result in this section is the structure of the optimal vehicle decision and the characterization of the EV market share as shown in the following theorem.

Theorem 6 (Consumer choice and EV market share)**.** *1. If the vehicle preferences ϵ_E and ϵ_G follow the type I extreme value distribution, the optimal consumer decision is a threshold policy on the difference of the vehicle preferences $\epsilon_E - \epsilon_G$:*

$$\begin{cases} \epsilon_E - \epsilon_G \geq \tau_e & \textit{purchase electric vehicles} \\ \epsilon_E - \epsilon_G < \tau_e & \textit{purchase gasoline vehicles} \end{cases}$$

where

$$\tau_e = \beta_1 \mathbb{E}(U_G) - \beta_2 p_G - \beta_1 \ln[\sum_{i=0}^{N_E} \exp(\alpha_1 f_i - \alpha_2 \rho_i)] + \beta_2 p_E. \tag{5.6}$$

The EV market share is given by

$$\eta_e = \frac{q^{\beta_1}}{q^{\beta_1} + C}, \tag{5.7}$$

where $C = \exp[\beta_1 \mathbb{E}(U_G) - \beta_2 p_G + \beta_2 p_E]$ and $q = \sum_{i=0}^{N_E} \exp(\alpha_1 f_i - \alpha_2 \rho_i)$.

2. *If the vehicle preference of EV is uniformly distributed with $\epsilon_E \sim \mathcal{U}(0,1)$ and $\epsilon_G = 1 - \epsilon_E$, the optimal consumer decision is a threshold policy on the realization of the consumer preference ϵ_E,*

$$\begin{cases} \epsilon_E \geq \tau_u & \textit{purchase electric vehicles} \\ \epsilon_E < \tau_u & \textit{purchase gasoline vehicles} \end{cases}$$

where

$$\tau_u = \Big[[\beta_1 \mathbb{E}(U_G) - \beta_2 p_G - \beta_1 \ln(q) + \beta_2 p_E + 1]/2\Big]_0^1. \tag{5.8}$$

The EV market share is given by

$$\eta_u = 1 - \tau_u. \tag{5.9}$$

3. *Under both assumptions, the charging service market share captured by charging station i is given by*

$$P_i = \frac{\exp(\alpha_1 f_i - \alpha_2 \rho_i)}{\sum_{k=0}^{N_E} \exp(\alpha_1 f_k - \alpha_2 \rho_k)} \triangleq \frac{q_i}{q}. \tag{5.10}$$

Proof. To derive the optimal consumer vehicle decision from (5.2-5.3), we first compute the expected maximum charging utility from (5.5) using the type I extreme value distribution of ϵ_i. Specifically,

$$\begin{aligned} \mathbb{E}(U_E) &= \ln[\textstyle\sum_{k=0}^{N_E} \exp(\alpha_1 f_k - \alpha_2 \rho_k)] \\ &\triangleq \ln(\textstyle\sum_{k=0}^{N_E} q_k) = \ln(q), \end{aligned} \tag{5.11}$$

where $q_k = \exp(\alpha_1 f_k - \alpha_2 \rho_k)$.

Next, by substituting $\mathbb{E}(U_E)$ into (5.2), the consumer's optimal vehicle choice is given by a threshold policy on $\epsilon_E - \epsilon_G$. In particular, the consumer purchases an EV if

$$\epsilon_E - \epsilon_G \geq \beta_1 \mathbb{E}(U_G) - \beta_2 p_G - \beta_1 \ln(q) + \beta_2 p_E. \tag{5.12}$$

Under the assumption of uniform distribution, by substituting $\epsilon_G = 1 - \epsilon_E$ into (5.12), we have (5.8).

From [100, chap. 4], the EV market share and the EVCS market share are given by (5.7), (5.9) and (5.10). □

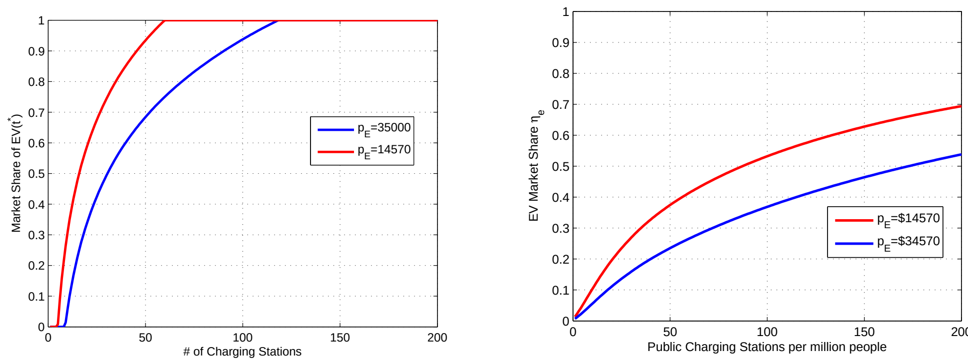
From the expression of the market share, we can explore the effects of operational cost and capital cost of EVs and gasoline vehicles. When the price elasticity β_2 is positive, the EV market share is inversely proportional to the exponential price difference between EVs and gasoline vehicles in the extreme preference case. In the uniform preference case, the market share is linear in the price difference with a negative slope $(-\beta_2/2)$. The effect of charging is reflected in the sum of exponential utilities q. Lower charging cost will increase q thus benefit the EV market share. Meanwhile, limited number of charging station will bring down the exponential utilities and hinder the successful launch of EVs. The threshold on the consumer vehicle preference is a linear combination of the difference in the capital cost and the operational utility (charging utility) between the EVs and gasoline vehicles.

With Theorem 6, we examine the trend of EV market share as a function of the charging station density, the charging price, and the price of EV. In particular, under both preference assumptions, the expressions of EV market share is an increasing and concave function of the number of available charging stations. This means that the marginal return of building additional charging stations reduces with the number of available charging stations.

Figure 5.4(a) and 5.4(b) show numerical evaluations of the market share as the density of charging stations. In addition to the concavity of the market share function, we also observe that the market share accelerates faster with a lower EV purchasing price.

For the uniform preference model, Figure 5.4(a) shows a dead zone effect, as a result of the ceiling operation in (5.8). In particular, the EV market share is zero unless the density of charging stations exceeds a certain level. In addition, we see that lowering EV purchasing cost helps the market share escaping the dead zone. Figure 5.5(a) shows that the critical density of charging stations at which the market share becomes positive grows as a "convex" function of the EV purchasing price. The convexity of this function means that the requirement of initial investment on EVCSs stiffens as the cost of EV purchase increases.

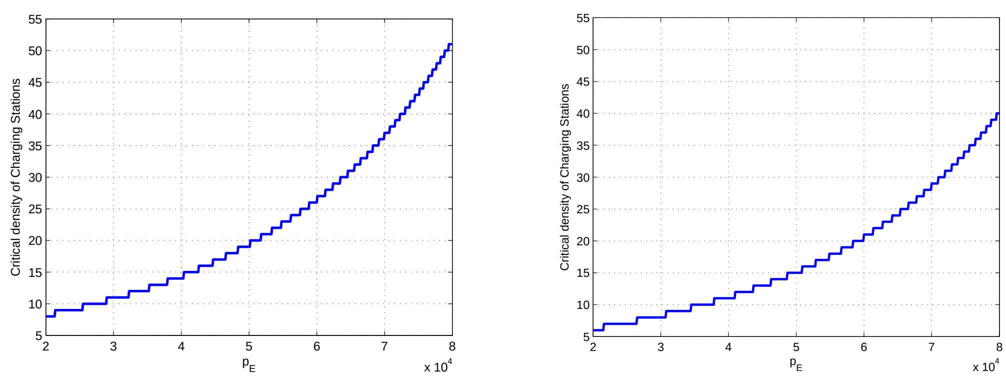
Under the extreme value distribution assumption, there is no dead zone effect. The EV market share η_e is always positive. However, if we treat $\eta_e \leq 5\%$ as a launch failure of EV, there is a critical density below which EV is considered failed. In Figure 5.5(b), the critical density of EVCSs is shown to have a "convex" shape in terms of EV prices.



(a) Uniform preference distribution.

(b) Type I extreme value preference distribution.

Figure 5.4: EV market share vs. density of charging stations. $p_G = \$17450$, $\mathbb{E}(U_G) = 4.5052$, $\rho_i = 0.2\$/\text{kWh}$.



(a) Uniform preference distribution

(b) Type I extreme value preference distribution

Figure 5.5: Critical density of charging stations vs. EV price p_E. $p_G = \$17450$, $\mathbb{E}(U_G) = 4.5052$, $\rho_i = 0.2\$/\text{kWh}$.

The "convex" shape of the critical density of EVCSs in terms of EV prices implies at least two points regarding the successful launch of EVs. From the manufacturers' point of view, the higher the MSRP is, the harder the EV would survive not only due to the prohibitive price but also the gap between the existing number and desired number of EVCSs. Thus lower price would not only attract consumers to purchase EVs but also alleviate the strain of building more charging stations. From the government point of view, it is important to determine the subsidy policy considering that what range the price of EVs lies in. When the price of EV is extremely high, giving more subsidy to EV purchasing would significantly reduce the required number of EVCSs. But as the subsidy increases, the effect on reducing the required number of EVCSs is getting smaller. At some point, it will be less effective to subsidize the EV purchasing than to build more charging stations. This phenomenon is also confirmed in the empirical simulation in [91] that building charging stations may be a more effective way to boost EV sales in the EV launch stage.

5.4 Investor Decisions

After the discussion about the consumer model and her decision, we now focus on the investor decision model that includes the selection of the charging stations locations and the optimal pricing of charging.

5.4.1 Investor Decision Model and Assumptions

We make the following assumptions about the investor model:

B1. We consider a single investor who also operates all charging stations. This implies the monopolistic competition in the charging service market.

B2. We assume that the deferred investment earns interest at a rate of γ.

B3. The investor knows the utility functions of the consumers.

To solve the optimization in (5.1), we proceed with backward induction. In Section 5.4.2, we find the optimal pricing with fixed EVCS locations. In Section 5.4.3 we optimally choose the charging station locations.

5.4.2 Optimal Charging Price

Given the set of charging station locations C, the investor determines the optimal charging price $\vec{\rho}$ to maximize the life time discounted total operation profit. Specifically, the investor has the following optimization.

$$\max_{\vec{\rho}} \Pi = \max_{\vec{\rho}} N_c D \eta(\vec{\rho}) \sum_{i=1}^{N_E} P_i(\vec{\rho})(\rho_i - \bar{c}_i), \tag{5.13}$$

where N_c is the total amount of consumers (householders), D the discounted life time driving mileage of a typical EV, $\eta(\vec{\rho})$ the expected EV market share given in Theorem 6 (here we make the dependency on charging price explicit), $P_i(\vec{\rho})$ the market share of station i, *i.e.*, the fraction of EV owners who charge at station i, and $\bar{c}_i$ the marginal operation cost of station i. The optimal charging price ρ_i^* is given by the following theorem.

Theorem 7 (Charging price)**.** *For fixed set of charging stations* $C = \{(f_i, \bar{c}_i), i = 1, \cdots, N_E\}$*, the optimal charging price* $\rho_i^*, i = 1, \cdots, N_E$ *generates uniform profits across charging stations. In particular,*

1. *Under the type I extreme value vehicle preference distribution assumption,*

$$\rho^*_{i,e} - \bar{c}_i = \frac{1}{\alpha_2\beta_1(1-\eta_e(\vec{\rho}^*_e))(1-P_0(\vec{\rho}^*_e)) + \alpha_2 P_0(\vec{\rho}^*_e)}, \tag{5.14}$$

where η_e *is the market share of EV,* $\vec{\rho}^*_e = (\rho^*_{1,e}, \ldots, \rho^*_{N_E,e})$ *the vector of optimal charging price,* $\rho_{0,e}$ *the cost of charging at home, and* $P_0(\vec{\rho}^*_e) = \frac{\exp(\alpha_1 f_0 - \alpha_2\rho_{0,e})}{\exp(\alpha_1 f_0 - \alpha_2\rho_{0,e}) + \sum_{k=1}^{N_E}\exp(\alpha_1 f_k - \alpha_2\rho^*_{k,e})}$ *the probability that the consumer charges at home.*

2. *Under the uniform vehicle preference distribution assumption,*

$$\rho^*_{i,u} - \bar{c}_i = \frac{1}{\frac{\alpha_2\beta_1(1-P_0(\vec{\rho}^*_u))}{2\eta_u(\vec{\rho}^*_u)} + \alpha_2 P_0(\vec{\rho}^*_u)}, \tag{5.15}$$

where η_u, $\vec{\rho}^*_u$, $\rho_{0,u}$, *and* $P_0(\vec{\rho}^*_u)$ *are similarly defined.*

Proof. This is a direct consequence of the first order optimality condition. □

Note that the right hand sides of equation (5.14) and (5.15) are the same for any charging station $i \in \{1, \ldots, N_E\}$. This means that the profits generated from different charging stations are the same. Equation (5.14) and (5.15) do not have closed-form solutions but the optimal prices can be solved numerically. Since $0 < P_0(\vec{\rho}^*_j) < 1$ and $0 < \eta_j \leq 1$ for either $j \in \{e, u\}$, the revenue is strictly positive. The indirect network effect from the EV market can be found in both charging profit and the total profit. In both uniform and extreme preference cases, if there is a shock in the EV market share, the uniform profit across charging stations will increase and the total profit will also grow, which will encourage the investment in more charging stations.

The profit across different charging stations is the same. However, since the operating cost across charging stations is different, the charging price of stations varies. Intuitively, more attractive charging stations have higher operating cost (labor cost, electricity price, and so on), thus higher charging price. Consumers balance the convenience of the station and the charging cost and make charging decisions. The price of gasoline and gasoline vehicles affects the charging profit indirectly through the EV market share η. The lower the gasoline price and gasoline vehicle price are, the smaller the EV market share is and this brings down the charging profit of charging stations.

As $N_E \to \infty$, the public charging of EV becomes more and more convenient, which not only motivates consumers to purchase EVs but also encourages them to charge outside home. As a result, the EV market share $\eta_j \to 1$ and the fraction of charging at home $P_0(\vec{\rho}_j^*) \to 0$. Based on this trend, we have the convergence of the marginal charging profit shown in the following theorem.

Theorem 8 (Charging price convergence)**.** *Consider a fixed set of charging stations* $C = \{(f_i, \bar{c}_i), i = 1, \cdots, N_E\}$. *Let* $v = \sum_{i=1}^{N_E} \exp(\alpha_1 f_i - \alpha_2 \bar{c}_i)$ *be the sum of exponentials of systematic charging utilities.*

1. *For the extreme distributed vehicle preference, the per-charging station profit* $r_e = \rho_{i,e}^* - \bar{c}_i$ *grows logarithmically with the sum utilities, i.e.,*

$$r_e = \ln v/\alpha_2 + o(\ln v). \tag{5.16}$$

2. *For uniformly distributed vehicle preference, the charging profit* $r_u = \rho_{i,u}^* - \bar{c}_i$ *is strictly increasing with the number of charging facilities and converges to a constant. Specifically,*

$$r_u = 2/\alpha_2\beta_1 + o(1) \;\; (v \to +\infty). \tag{5.17}$$

Proof. The detailed proof can be found in Appendix B.1. □

In Theorem 8, the increasing per-charging station profit is because of the assumption of monopolistic investor. More charging stations motivate more consumers to purchase EVs and bring larger charging demand. The investor will take the advantage and set a higher markup. In both cases, the profit increase corresponding to the increase of number of charging stations N_E (or equivalently, ν,) is decreasing, which is due to the diminishing of the network effect.

The profit is also affected by the consumer sensitivity to the charging price. When consumers are more sensitive to the price (α_2 is large), the optimal charging price is close to the marginal cost across charging stations.

The different convergence comes from different preference assumptions. Under the extremely distributed preference assumption, the EV market share strictly increases as the density of EVCSs increases. Thus the profit grows logarithmically because of the expanding charging demand. While under the uniform distributed preference assumption, the EV market share reaches the upper limit when there are enough EVCSs. So the profit converges to a constant.

5.4.3 Optimal Charging Station Locations

After the discussion about the optimal charging price, we consider the choice of charging station locations. Given the set of location candidates $\mathscr{C} = \{\bar{s}_i = (f_i, \bar{c}_i),\ i = 1, \cdots, N_L\}$, the investor has the following optimization.

$$\begin{array}{ll} \max_{C \subseteq \mathscr{C}} & \Pi(C, \vec{\rho}_j^*(C)) - \sum_{i=1}^{|C|} \mathcal{F}(\bar{s}_i) \\ \text{subject to} & \sum_{i=1}^{|C|} \mathcal{F}(\bar{s}_i) \leq B \end{array} \tag{5.18}$$

where $\mathcal{F}(\bar{s}_i)$ is the building cost of charging station $\bar{s}_i$ and $\Pi(C, \vec{\rho}_j^*(C))$ the operational profit.

In general, the optimal investment decision from (5.18) requires combinatorial search for C, which is computationally inefficient and sometimes not tractable. However, the convergence of the optimal charging prices across charging stations in Theorem 8 makes it possible to separate the price decision and the location choice, which leads to a linear complexity heuristic algorithm.

Algorithm 3: Greedy Investment Algorithm

1. Compute the exponential systematic utility $v_i = \exp(\alpha_1 f_i - \alpha_2 \bar{c}_i)$ and sorted list $\{v_{(i)}\}$.

2. Set $N = 1$.

while $N \leq N_L$ and $\sum_{i=1}^{N} \mathcal{F}(\bar{s}_i) \leq B$ **do**

  Compute $\tilde{P}_N \triangleq \Pi(s_1, \cdots, s_N) - \sum_{i=1}^{N} \mathcal{F}(\bar{s}_i)$.

  if $\tilde{P}_N < \tilde{P}_{N-1}$ or $\sum_{i=1}^{N} \mathcal{F}(\bar{s}_i) > B$ **then**

    STOP;

  else

    $N \leftarrow (N + 1)$.

  end if

end while

The Greedy Investment Algorithm (GIA) given in Algorithm 3 first ranks the charging stations by the exponential systematic part of the charging utility, $v_i = \exp(\alpha_1 f_i - \alpha_2 \bar{c}_i)$. It then adds charging stations to the investment list one at a time in the decreasing order of exponential systematic utility v_i until either the budget is exhausted or the cumulated profit starts to decrease.

By ignoring the dependency of charging locations in the marginal charging profit $(\rho^*_{i,j} - \bar{c}_i)$ in (5.18), the GIA is not optimal in general. As N_E increases, however, the marginal charging profit increases and converges, which makes the algorithm asymptotically optimal.

Theorem 9 (Asymptotic optimality)**.** *Assume the building costs of charging stations are constant, i.e., $\mathcal{F}(\bar{s}_i) = (1+\gamma)\mathcal{F}_0$, where γ is the interest rate. There exists an $M > 0$ such that when $N > M$, the greedy algorithm is optimal under both the type I extreme value distribution and the uniform distribution assumption.*

Proof. The detailed proof can be found in Appendix B.2. □

After obtaining the optimal set of charging stations C^* and the optimal charging price vector $\vec{\rho}^*_j$, the investor makes the investment if the investment profit $[\Pi(C^*, \vec{\rho}^*_j) - \sum_{i=1}^{|C^*|} \mathcal{F}(\bar{s}_i)]$ is positive. Otherwise, the investor will defer his investment and earn interest at rate γ.

To make $[\Pi(C^*, \vec{\rho}^*_j) - \sum_{i=1}^{|C^*|} \mathcal{F}(\bar{s}_i)]$ positive, the EV price and the building costs of charging stations need to be low enough, which implies that the subsidies for EV purchase and charging stations are necessary to the successful launch of EV. Meanwhile the price of gasoline and electricity affect the charging profit Π. The high petroleum price and cheap electricity benefit the EV market.

5.4.4 Social welfare optimization

We now consider the investment problem from the viewpoint of a social planner who makes investment decisions based on social welfare maximization and compare the difference between the solution of the private market defined in Section 5.4.3 and that of a social planner.

Recall the investor utility $S_I(C, \vec{\rho}_j)$ and the consumer utility $S_C(C, \vec{\rho}_j)$ given in (5.1) and (5.3):

$$\begin{aligned} S_I(C, \vec{\rho}_j) &= \Pi(C, \vec{\rho}_j) - \textstyle\sum_{i=1}^{|C|} \mathcal{F}(\bar{s}_i), \\ S_C(C, \vec{\rho}_j) &= \mathbb{E}[\max\{V_E(C, \vec{\rho}_j, \epsilon_E), V_G(\epsilon_G)\}]. \end{aligned}$$

Under the type I extreme value vehicle preference distribution assumption, the consumer utility is stated as

$$\begin{aligned} &S_C(C, \vec{\rho}_e) = \ln\left[\left(\textstyle\sum_{i=0}^{|C|} \exp(\alpha_1 f_i - \alpha_2 \rho_{i,e})\right)^{\beta_1} C_1 + C_2\right], \\ &C_1 = \exp(-\beta_2 p_E + \Phi),\ C_2 = \exp[\beta_1 \mathbb{E}(U_G) - \beta_2 p_G + \Phi]. \end{aligned}$$

Under the uniform vehicle preference distribution assumption, the consumer utility is stated as:

$$S_C(C, \vec{\rho}_u) = \left[\left(\eta_u(C, \vec{\rho}_u)\right)^2 + \beta_1 \mathbb{E}(U_G) - \beta_2 p_G + \Phi - \frac{1}{2}\right].$$

Assume that the social planner does not operate the charging station thus can not determine the charging price or the vehicle price. He only determines the set of charging stations locations to build by regulation. For example, Beijing government recently requires 18% of parking lots in new residential communities have to install charging stations [94].

The social planner's decision is stated as:

$$\begin{aligned} \max_{C \subseteq \mathscr{C}} \quad & \lambda N_c S_C(C, \vec{\rho}_j^*(C)) + S_I(C, \vec{\rho}_j^*(C)) \\ \text{subject to} \quad & \textstyle\sum_{i=1}^{|C|} \mathcal{F}(\bar{s}_i) \leq B \end{aligned}, \tag{5.19}$$

where N_c is the total amount of consumers (householders), $\vec{\rho}_j^*(C)$ the vector of the optimal charging prices determined by the charging station operators given the charging station locations, and $\lambda > 0$ the weight of the consumers surplus.

The greedy investment algorithm in Algorithm 3 can also be applied to solve for the social welfare optimized investment in charging stations. The following theorem characterizes the difference between the social welfare optimal solution and the market solution.

Theorem 10 (Social welfare)**.** *Let C^* be the optimal set of charging stations determined by the investor, and assume $|C^*| \gg 1$. Let C^{**} be the optimal charging locations determined by the social planner. Under both the type I extreme value distribution and the uniform distribution assumptions, $|C^{**}| > |C^*|$.*

Proof. See B.3. □

Theorem 10 implies that the monopolistic market solution tends to under-build charging stations. The under-provision of EVCSs and lower adoption of EVs relative to the socially optimal outcomes are due to two types of market failures: market power and indirect network effects (or externalities). The assumption of monopolistic investor leads to under-provision of EVCSs and a higher charging price than a competitive solution. This will in turn lead to a lower EV adoption. Therefore, introducing competition in EVCS provision will help EV diffusion. While this form of market failure is a result of our model setup and can be relaxed, the second form of market failure is inherent in the EV market as empirically confirmed in [91]. Indirect network effects are externalities which are not accounted for in individual investment and purchase decisions. They will lead to a wedge in socially optimal outcomes and market outcomes, which justifies government interventions. For example, government can provide subsidies to charging station investors as the U.S. DOE does through various funding programs or mandate the provision of charging stations in real estate development as recently implemented in China [94].

5.5 Discussions

5.5.1 Effects of subsidy

We consider here the effects of subsidy, either to EV consumers or to the investor of EV charging stations. The results obtained in Section 5.3 and Section 5.4 provide the basis for numerical results presented here.

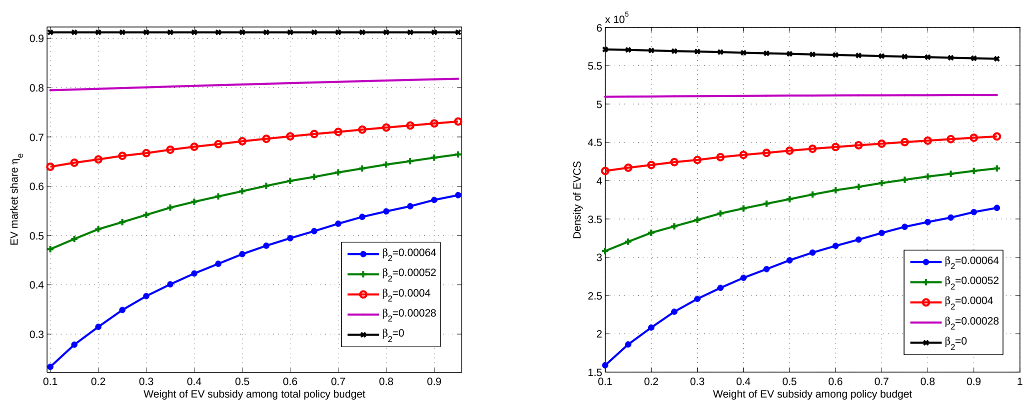
Fixing the total policy budget as 230 million dollars, we vary the weight of subsidy for EV purchase among the total policy budget. A bisection algorithm is applied to search for the subsidy amounts for each EV and EVCS so that the constraints of total budget and budget weight are satisfied.

Figure 5.6(a) shows the EV market share against subsidy weight with different values of β_2. In the utility model (5.2), β_2 represents the consumer sensitivity to the EV price. When β_2 was large, consumers cared more about the EV price than the characteristics of charging facilities. Increasing EV subsidy dramatically boostd up the EV market share. On the other hand, when β_2 was close to 0, consumers mainly concerned about the charging services and the subsidy for EV purchase played a tiny role in the EV market share evolution.

Similar impact exists in the EVCS market. As shown in Figure 5.6(b), when

β_2 was large, the consumer was more sensitive to EV price. In this case, the policy of subsidizing the EV purchase was more effective, not only in stimulating the EV purchasing but also driving the investor to deploy more charging facilities because of the EV popularity. When β_2 was close to 0, consumers were less price sensitive. In this case, putting more weight to EV subsidy was less helpful. While subsidizing the charging station encouraged the investment in charging services which also benefited the EV consumer by making the charging more convenient.

This result may suggest that, at the launch period, more tax credit to EV purchasing may be not efficient in stimulating the market share of EVs. One possible reason is that the early adopters of EVs have a higher wage income comparing to average vehicle consumers. Thus they are less price sensitive and concern more about other characteristics of the vehicle, such as the environmental impact or the driving experience.



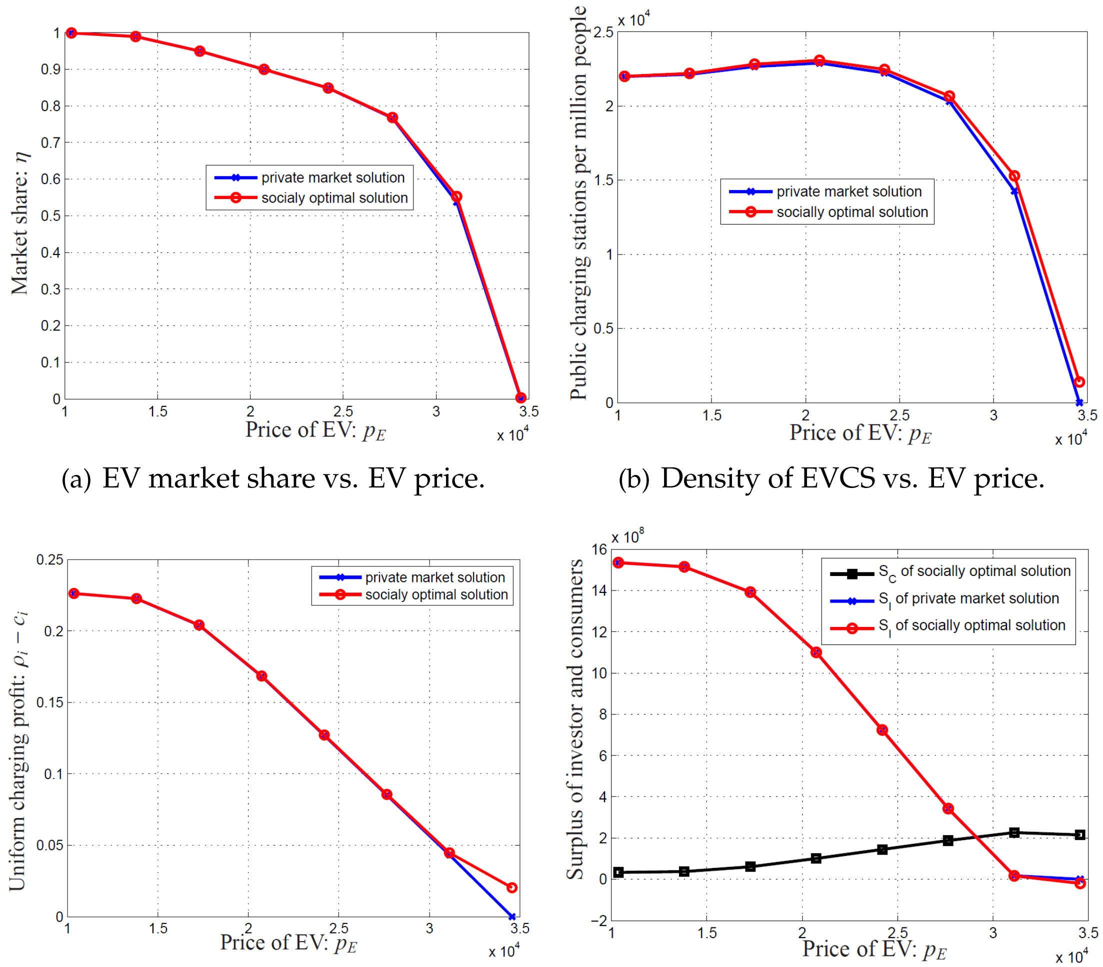
(a) EV market share vs. subsidy weight. (b) Density of EVCSs vs. subsidy weight.

Figure 5.6: Subsidy effect with different coefficients

5.5.2 Socially optimal solution vs. private market solution

In Section 5.4.4, the analytical result shows the socially optimal solution requires to invest in more charging facilities than that from the private market solution. In this section, a numerical result is presented to illustrate this difference in the EVCS market. In the simulation, we assume that all charging stations have the same favorability rate f and building cost F. In the private market setting, the investor simply chooses how many charging stations to invest and determines the optimal charging price to maximize the total profit collected from charging stations defined in (5.18). In the socially optimal solution, the social planner maximizes the social welfare defined in (5.19) by determining the number of charging stations.

When the weight of the consumer was small ($\lambda = 10$), the decision of the social planner was close to the outcome of the private market. As shown in Figure 5.7(a), the market share of the socially optimal solution was slightly greater than the private market solution. We observed that when $p_E = \$34570$, the investor would not invest in any charging stations due to the neglectful market share of EVs. The uniform unit charging profit of the private market solution was also decreasing as the price of EV increased. This could be explained by (5.14) where the profit was decreasing when the market share η shrinked. The result of the socially optimal solution was close to the private market outcome except when the price of EV was high ($p_E = \$34500$). In this case, the investor had no motivation to invest in any charging stations due to the neglectful market share of EVs. However, the social planner would still build some charging stations taking into account of the utilities of EV consumers. As shown in Figure 5.7(d), when the EV price was low, the total operation profit from charging



(a) EV market share vs. EV price.

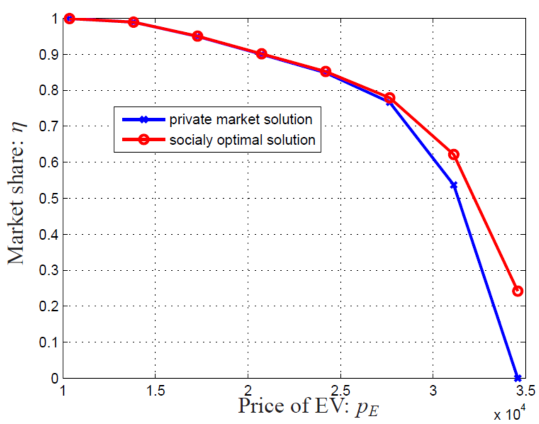
(b) Density of EVCS vs. EV price.

(c) Charging price vs. EV price.

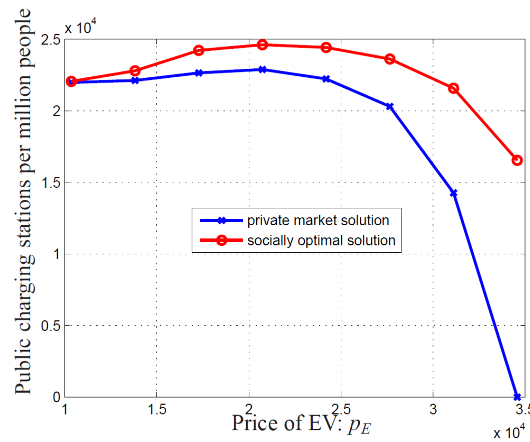
(d) Total profit of building charging stations vs. EV price.

Figure 5.7: Socially optimal solution vs. private market solution. p_G = \$17460, $N_c = 10^6$, $\mathcal{F}$ = \$15000, $f = 0.001$, $c = 0.08\$/\text{mile}$, $\alpha_1 = 10$, $\alpha_2 = 25$, $\beta_1 = 2$, $\beta_2 = 7 \times 10^{-4}$, $\lambda = 10$.

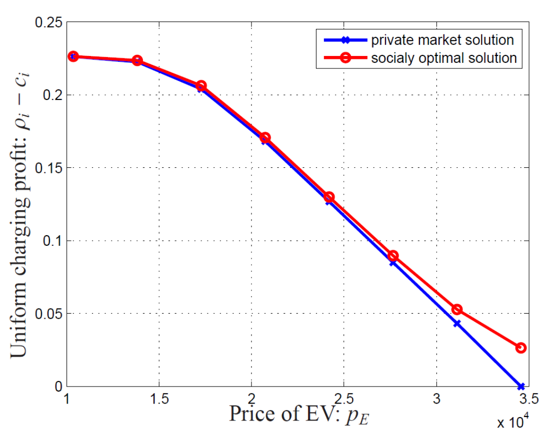
stations dominated the consumer surplus. As the price of EV raised, the profit dropped sharply due to the shrink of the EV market share. However, as the price of EV increased, the charging cost droped, as shown in Figure 5.7(c), and raised the charging utility and the consumer surplus. When the profit dropped below the consumer surplus, the social planner mainly focused on the consumers and built charging stations even when the total charging profit was negative.



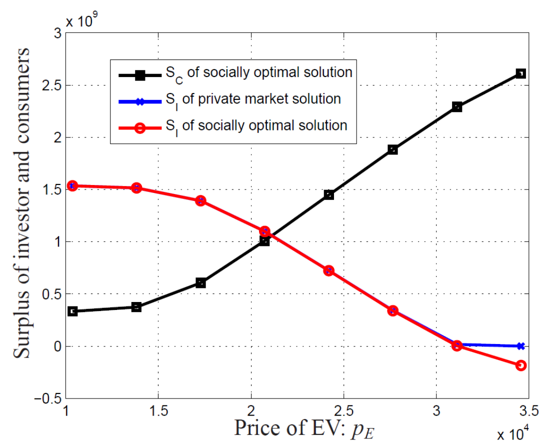
(a) EV market share vs. EV price.



(b) Density of EVCS vs. EV price.



(c) Charging price vs. EV price.



(d) Total profit of building charging stations vs. EV price.

Figure 5.8: Socially optimal solution vs. private market solution. $p_G = \$17460$, $N_c = 10^6$, $\mathcal{F} = \$15000$, $f = 0.001$, $c = 0.08\$/\text{mile}$, $\alpha_1 = 10$, $\alpha_2 = 25$, $\beta_1 = 2$, $\beta_2 = 7 \times 10^{-4}$, $\lambda = 100$.

As shown in Figure 5.8, when the weight of consumer surplus was larger ($\lambda = 100$), the socially optimal solution deviated from the private market outcome significantly, especially when the EV price was high. The market share of EV, the density of charging services and the unit charging profit of the socially optimal solution were larger than the ones of the private market outcome. This could be explained by Figure 5.8(d). When p_E was low, the consumer surplus was comparable to the profit of charging stations and social planner biased to consumers notably. When p_E was high, the profit went to negative and was dominated by the consumer surplus. The social planner's decision deviated from the private market outcome significantly.

The simulation result suggests that, bringing down the cost of EVs makes all parties benefited. The investors of charging station build more charging services and earn more profit with lower EV prices. The consumers face attractive EV price and have more charging options which eliminates the anxiety of driving range. Moreover, the private market outcomes is closer to the socially optimal solution as the price of EV decreases.

When the EV price is not low enough and the private market solution deviates the socially optimal solution, it is reasonable to supply subsidy to the EV purchase. Meanwhile, subsidies to charging stations or other policies can also fill the gap between the private market outcomes and the socially efficient solution.

5.5.3 Other policies

Beyond the subsidy policy, the proposed model can be directly used to study other policies, such as regulations. The government may require a minimal share of EVs in the vehicle fleet [144] such as the Zero Emission Vehicle program in California. This could stimulate the EV market and attract more investment on the EV charging station via the feedback loop between the EV and EVCS market. The impact of this kind of policy can be estimated by adding an external shock to the EV market and simulating the result in the EVCS market. Similarly, the government may mandate the minimal number of EV charging stations in public parking lots as being recently adopted the Beijing municipal government.

Another approach is to allow the EV charging operator to participant in the operation of the power system, not just a price taker. By doing so, the operator of the charging stations may adjust the EV charging profile according to the requirement from the power system operator to make the grid more reliable and efficient. These vehicle to grid (V2G) services typically include spinning reserves, frequency regulation and peak power supply [80, 95, 57, 136].

The option to participate in the power market may attract more investment in charging stations. The charging cost would be lower not only due to the competition between charging stations, but also the income from the V2G services. In this case, the social planner has more motivation to help the successful launch of EVs and support more charging stations, since the adoption of EVs not only generates environmental benefits but also help the grid to be cleaner, more robust and more efficient. The EV owners will enjoy cheaper charging prices and other consumers have cleaner energy. Everybody wins.

To model the impact of the adoption of the V2G services, the decision model of the investor needs to be extended. When making the investment decision, the investor needs to consider not only the charging profit but also the V2G revenue. One possible way is to add one more term in the profit function Π to indicate the V2G income as a function of the EV charging population. When operating the charging stations, the charging price is designed to maximize the total profit from EV charging and V2G services. The density of EV charging stations are expected to increase from the result of the case without V2G options. The profit of investors and consumer surplus are expected to increase as well.

5.5.4 Applications

In this chapter, we considered the two-sided EV market in a homogenous setting. The charging rate of the stations is assumed uniform and the market across areas are indifferent. In this subsection, we will discuss several directions for future research, including the EV market under heterogeneous settings, and how our models can be helpful to understand the market outcome.

One possible direction is to incorporate the choices of different types of charging station technology in the decision model. The charging time of different charging rate varies dramatically, which affects the consumer choice of charging stations, and the capital cost differs greatly, which has an influence on the investment decisions. The deployment of all three levels of charging was considered in Luo et al. ([96]) where a sequential deployment of charging stations with various charging rate was simulated. To consider the option of choosing charging rate in our model, we can add one more variable to indicate the charging level of the charging station in the investor decision model of EV charging stations. In the consumer choice model, the waiting time can be included in the utility function as part of the charging cost. In this case, the attractions and capital cost of EV charging stations with various charging levels are different and the investor's decision is more complicated.

Besides the different charging technologies, we can evaluate different policy impacts in heterogeneous markets. In different locations, the environmental benefit of EVs are different and the policies may also vary. In areas where a large share of electricity is generated from renewables or hydro, the environmental benefits would be higher while in areas with a large share of coal-fired power generation, the environmental impact of EVs could even be negative. To incorporate this heterogeneity, we could include the environmental benefits of EV adoption in the objective function of social planner's decision in equation (19) and solve the social planner's problems in different locations individually. Recognizing this heterogeneity, the government policies could vary across locations in order to achieve the spatially-variant optimal level of EV adoption.

APPENDIX A

APPENDICES FOR CHAPTER 3

A.1 Proof of Theorem 1

In this appendix, we provide an elementary proof of indexability. That is, for any state $\tilde{s}$ of an arm, there is a critical $\nu(\tilde{s})$ such that if and only if $\nu \geq \nu(\tilde{s})$ the first term in the Bellman equation (3.10) is larger than or equal to the the second term in a single arm ν-subsidy problem.

A.1.1 Indexability of dummy arms

The indexability of dummy arms is straightforward. For $i \in \{N+1, \cdots, N+M\}$, there is no EV arrival, and only the charging cost evolve. The Bellman equation of the ν-subsidy problem is given by

$$V_i^\nu(0,0,c_k) = \max\{\beta \textstyle\sum_{k'} P_{k,k'} V_i^\nu(0,0,c_{k'}) + \nu, \beta \textstyle\sum_{k'} P_{k,k'} V_i^\nu(0,0,c_{k'})\}.$$

If and only if $\nu \geq 0$, the first term is larger than the second term and it is optimal to deactivate the dummy arm. Otherwise, the active action is optimal. So a dummy arm is indexable and its Whittle's index is $\nu_i(0,0,c_k) = 0$.

A.1.2 Indexability of regular arms

We now prove the indexability of regular arms by induction. We first show the Whittle's index $\nu_i(t, j, c_k)$ exists for $T \leq 1$ and all j and c_k, and the difference of the value function $g^{\nu}(T, j, c_k) = V_i^{\nu}(T, j+1, c_k) - V_i^{\nu}(T, j, c_k)$ satisfies some special property for $T = 1$. Then assuming the Whittle's index $\nu_i(T, j, c_k)$ exists and the property of $g^{\nu}(T, j, c_k)$ holds for $T = t-1$, we show $\nu_i(T, j, c_k)$ exists and the property of $g^{\nu}(T, j, c_k)$ holds for $T = t$.

Proof. **When** $T = 0$

there is no EV waiting at the charger. The Bellman equation is stated as

$$V_i^{\nu}(0, 0, c_k) = \max\{\nu + \beta W_k^{\nu}, \beta W_k^{\nu}\}.$$

where

$$W_k^{\nu} = Q(0,0)\sum_{k'} P_{k,k'} V_i^{\nu}(0, 0, c_{k'}) + \sum_{T'}\sum_{j}\sum_{k'} Q(T', j) P_{k,k'} V_i^{\nu}(T', j, c_{k'})$$

is the expected reward of possible arrivals. If and only if $\nu \geq 0$, the first term is larger and the passive action is optimal. Thus $\nu_i(0, 0, c_k) = 0$.

When $T = 1$

there are two possible cases.

- If $B = 0$, the Bellman equation is stated as

  $$V_i^{\nu}(1, 0, c_k) = \max\{\nu + \beta W_k^{\nu}, \beta W_k^{\nu}\}.$$

  Thus $\nu_i(1, 0, c_k) = 0$.

- If $j \geq 1$, the Bellman equation is stated as

  $$V_i^{\nu}(1, j, c_k) = \max\{\nu - F(j) + \beta W_k^{\nu}, 1 - c_k - F(j-1) + \beta W_k^{\nu}\}.$$

  If and only if $\nu \geq 1 - c_k + F(j) - F(j-1)$, the passive action is optimal.

Thus the Whittle's index for $T = 1$ exists and the closed-form is stated as follows.

$$\nu_i(1, j, c_k) = \begin{cases} 0 & \text{if } j = 0 \\ 1 - c_k + F(j) - F(j-1) & \text{if } j \geq 1 \end{cases} \tag{A.1}$$

Denote the difference of the value function as

$$g^\nu(T, j, c_k) = V_i^\nu(T, j+1, c_k) - V_i^\nu(T, j, c_k)$$

We note that the difference of the value function is continuous and piecewise linear in ν. Specially, denote $\mathcal{G}$ as a set of functions of ν such that $g(\nu) \in \mathcal{G}$ if and only if $g(\nu)$ is a continuous piecewise linear function in ν, there exist $\underline{\nu}$ and $\bar{\nu}$ such that, $\partial g(\nu)/\partial \nu \geq -1$ when $\nu \in [\underline{\nu}, \bar{\nu}]$, and $\partial g(\nu)/\partial \nu = 0$ when $\nu \notin [\underline{\nu}, \bar{\nu}]$. We show that, when $T = 1$, $g^\nu(T, j, c_k) \in \mathcal{G}$.

- If $j = 0$,

$$g^\nu(1, j, c_k) = V_i^\nu(1, 1, c_k) - V_i^\nu(1, 0, c_k).$$

  - If $\nu_i(1, 1, c_k) > \nu_i(1, 0, c_k) = 0$,

$$g^\nu(1, j, c_k) = \begin{cases} 1 - c_k, & \text{if } \nu < 0; \\ 1 - c_k - \nu, & \text{if } 0 \leq \nu < \nu_i(1, 1, c_k); \\ -F(1), & \text{if } \nu_i(1, 1, c_k) \leq \nu. \end{cases}$$

  - If $\nu_i(1, 1, c_k) \leq \nu_i(1, 0, c_k) = 0$,

$$g^\nu(1, j, c_k) = \begin{cases} 1 - c_k, & \text{if } \nu < \nu_i(1, 1, c_k); \\ \nu - F(1), & \text{if } \nu_i(1, 1, c_k) \leq \nu < 0; \\ -F(1), & \text{if } \nu_i(1, 1, c_k) \leq \nu. \end{cases}$$

- If $j \geq 1$,

$$g^\nu(1, j, c_k) = V_i^\nu(1, j, c_k) - V_i^\nu(1, j, c_k).$$

Since $v_i(1, j+1, c_k) \geq v_i(1, j, c_k)$ by (A.1),

$$g^{v}(1, j, c_k) = \begin{cases} F(j-1) - F(j), & \text{if } v < v_i(1, j, c_k); \\ 1 - c_k - v, & \text{if } v_i(1, j, c_k) \leq v < v_i(1, j+1, c_k); \\ F(j) - F(j+1), & \text{if } v_i(1, j+1, c_k) \leq v. \end{cases}$$

So $g(1, j, c_k)$ is continuous piecewise linear in v and exist $\underline{v}$ and $\bar{v}$ such that $\partial g(1, j, c_k)/\partial v \geq -1$ when $v \in [\underline{v}, \bar{v}]$ and $\partial g(1, j, c_k)/\partial v = 0$ otherwise.

When $T \geq 2$

assuming the Whittle's index $v_i(T, j, c_k)$ exits and $g^{v}(T, j, c_k) \in \mathcal{G}$ for $T = t-1$, we show $v_i(T, j, c_k)$ exits and $g^{v}(T, j, c_k) \in \mathcal{G}$ for the case $T = t$.

First, existence of $v_i(T, j, c_k)$ when $T = t$.

- If $B = 0$, the Bellman equation is stated as follows.

  $$V_i^{v}(t, 0, c_k) = \max\{\beta \sum_{k'} P_{k,k'} V_i^{v}(t-1, 0, c_{k'}) + v, \beta \sum_{k'} P_{k,k'} V_i^{v}(t-1, 0, c_{k'})\}.$$

  If and only if $v \geq 0$, the first term is larger than the second term and the passive action is optimal. Thus $v_i(t, 0, c_k) = 0$.

- If $j \geq 1$, the Bellman equation is stated as follows.

  $$V_i^{v}(t, j, c_k) = \max\{\beta \sum_{k'} P_{k,k'} V_i^{v}(t-1, j, c_{k'}) + v, \beta \sum_{k'} P_{k,k'} V_i^{v}(t-1, j-1, c_{k'}) + 1 - c_k\}. \tag{A.2}$$

  Denote the difference between the two actions as

  $$f^{v}(t, j, c_k) \triangleq v - (1 - c_k) + \beta \sum_{k'} P_{k,k'} g^{v}(t-1, j-1, c_{k'}),$$

  where

  $$g^{v}(t-1, j-1, c_{k'}) = V_i^{v}(t-1, j, c_{k'}) - V_i^{v}(t-1, j-1, c_{k'}).$$

Since $g^{\nu}(t-1, j-1, c_{k'}) \in \mathcal{G}$ by assumption, $f^{\nu}(t, j, c_k)$ is continuous and piece-wise linear in ν. Denote

$$\underline{\nu}(t, j, c_k) \triangleq \min_{k'} \underline{\nu}(t-1, j-1, c_{k'}),$$
$$\bar{\nu}(t, j, c_k) \triangleq \max_{k'} \bar{\nu}(t-1, j-1, c_{k'}).$$

where $\partial g^{\nu}(t-1, j-1, c_{k'})/\partial\nu \geq -1$ if and only if $\nu \in [\underline{\nu}(t-1, j-1, c_{k'}), \bar{\nu}(t-1, j-1, c_{k'})]$. We have

$$\partial f^{\nu}(t, j, c_k)/\partial\nu = \begin{cases} \geq 0, & \text{if } \nu \in [\underline{\nu}(t, j, c_k), \bar{\nu}(t, j, c_k)]; \\ 1, & \text{otherwise.} \end{cases}$$

So $f^{\nu}(t, j, c_k)$ is continuous and non-decreasing in ν. When $\nu = -\infty$, $f^{\nu}(t, j, c_k) = -\infty$. When $\nu = +\infty$, $f^{\nu}(t, j, c_k) = +\infty$. Thus there is a cross point of $f^{\nu}(t, j, c_k)$ and the ν-axis. Define $\nu_i(t, j, c_k) \triangleq \min_{\nu}\{f^{\nu}(t, j, c_k) = 0\}$. If and only if $\nu \geq \nu_i(t, j, c_k)$, the first term in (A.2) is larger or equal to the second term and the passive action is optimal. By definition, $\nu_i(t, j, c_k)$ is the Whittle's index.

The existence of $\nu_i(t, j, c_k)$ is shown. Next we show $g^{\nu}(t, j, c_k) \in \mathcal{G}$.

- If $j = 0$,

$$g^{\nu}(T, j, c_k) = V_i^{\nu}(t, 1, c_k) - V_i^{\nu}(t, 0, c_k).$$

  - If $\nu_i(t, 1, c_k) > \nu_i(t, 0, c_k) = 0$,

$$g^{\nu}(t, 0, c_k) = \begin{cases} 1 - c_k, & \text{if } \nu < 0; \\ 1 - c_k - \nu, & \text{if } 0 \leq \nu < \nu_i(t, 1, c_k); \\ \beta \sum_{k'} P_{k,k'} g^{\nu}(t-1, 0, c_{k'}), & \text{if } \nu_i(t, 1, c_k) \leq \nu. \end{cases}$$

  - If $\nu_i(t, 1, c_k) \leq \nu_i(t, 0, c_k) = 0$,

$$g^{\nu}(t, 0, c_k) = \begin{cases} 1 - c_k, & \text{if } \nu < \nu_i(t, 1, c_k); \\ \nu + \beta \sum_{k'} P_{k,k'} g^{\nu}(t-1, 0, c_{k'}), & \text{if } \nu_i(t, 1, c_k,) \leq \nu < 0; \\ \beta \sum_{k'} P_{k,k'} g^{\nu}(t-1, 0, c_{k'}), & \text{if } 0 \leq \nu. \end{cases}$$

- If $j \geq 1$,

$$g^{\nu}(t, j, c_k) = V_i^{\nu}(t, j+1, c_k) - V_i^{\nu}(t, j, c_k).$$

  - If $\nu_i(t, j+1, c_k) > \nu_i(t, j, c_k)$,

$$g^{\nu}(t, j, c_k) = \begin{cases} \beta \sum_{k'} P_{k,k'} g^{\nu}(t-1, j-1, c_{k'}), & \text{if } \nu < \nu_i(t, j, c_k); \\ 1 - c_k - \nu, & \text{if } \nu_i(t, j, c_k) \leq \nu < \nu_i(t, j+1, c_k); \\ \beta \sum_{k'} P_{k,k'} g^{\nu}(t-1, j, c_{k'}), & \text{if } \nu_i(t, j+1, c_k) \leq \nu. \end{cases}$$

  - If $\nu_i(t, j+1, c_k) \leq \nu_i(t, j, c_k)$,

$$g^{\nu}(t, j, c_k) = \begin{cases} \beta \sum_{k'} P_{k,k'} g^{\nu}(t-1, j-1, c_{k'}), \\ \text{if } \nu < \nu_i(t, j+1, c_k); \\ \nu - (1 - c_k) + \beta \sum_{k'} P_{k,k'} [g^{\nu}(t-1, j, c_{k'}) + g^{\nu}(t-1, j-1, c_{k'})], \\ \text{if } \nu_i(t, B+1, c_k) \leq \nu < \nu_i(t, B, c_k); \\ \beta \sum_{k'} P_{k,k'} g^{\nu}(t-1, j, c_{k'}), \\ \text{if } \nu_i(t, j, c_k) \leq \nu. \end{cases}$$

Clearly, $g^{\nu}(T, j, c_k)$ is a linear combination of $g^{\nu}(T-1, j, c_{k'})$ and $g^{\nu}(T-1, j-1, c_{k'})$. Since $g^{\nu}(T-1, j, c_{k'}) \in \mathcal{G}$ for all j and $c_{k'}$ by assumption, we have $g^{\nu}(T, j, c_k) \in \mathcal{G}$ as well.

Thus, by induction, the Whittle's index $\nu_i(T, j, c_k)$ exists and $g^{\nu}(T-1, j, c_k) \in \mathcal{G}$ for all T, j, and c_k.

□

A.2 Proof of Theorem 2

Proof. Since the charging cost c_0 is constant, we will omit the cost in the state of arms for simplicity. In Appendix A.1.1 we have shown that the Whittle's index of the dummy arms is $\nu_i(0,0) = 0$.

For regular arms, we showed in (A.1) that $\nu_i(1,0) = 0$ and $\nu_i(1,j) = 1 - c_0 + F(j) - F(j-1)$ when $j \geq 1$. Next, we show the closed-form of the Whittle's index for the case of $T \geq 2$ using induction.

A.2.1 When $T = 2$

The discussion is divided into two conditions.

- If $j = 1$,

$$V_i^\nu(2,1) = \max\{\nu + \beta V_i^\nu(1,1), 1 - c_0 + \beta V_i^\nu(1,0)\}.$$

  The difference between active and passive actions

$$\begin{aligned} & f^\nu(2,1) \\ & \quad = \nu - (1 - c_0) + \beta g_1^\nu(1,0)] \\ & \quad = \begin{cases} \nu - (1-\beta)(1-c_0), & \text{if } \nu < 0; \\ (1-\beta)[\nu - (1-c_0)], & \text{if } 0 \leq \nu < 1 - c_0 + F(1); \\ \nu - (1-c_0) - \beta F(1), & \text{if } 1 - c_0 + F(1) \leq \nu; \end{cases} \end{aligned}$$

  equals to 0 when $\nu = 1 - c_0$. Thus $\nu_i(2,1) = 1 - c_0$.

- If $j \geq 2$, the Bellman equation is stated as follows.

$$V_i^\nu(2,j) = \max\{\nu + \beta V_i^\nu(1,j), 1 - c_0 + \beta V_i^\nu(1,j-1)\}.$$

  Denote $\Delta F(j) = F(j) - F(j-1)$. The difference between active and passive actions

$$
\begin{aligned}
&f^{\nu}(2, j)\\
&\quad = \nu - (1 - c_0) + \beta g^{\nu}(1, j-1)\\
&\quad = \begin{cases} \nu - (1 - c_0) - \beta\Delta F(j-1), & \text{if } \nu < 1 - c_0 + \Delta F(j-1);\\ (1-\beta)[\nu - (1 - c_0)], & \text{if } 1 - c_0 + \Delta F(j-1) \leq \nu < 1 - c_0 + \Delta F(j);\\ \nu - (1 - c_0) + \beta\Delta F(j), & \text{if } 1 - c_0 + \Delta F(j) \leq \nu; \end{cases}
\end{aligned}
$$

equals to 0 when $\nu = 1 - c_0 + \beta[F(j-1) - F(j-2)]$. Thus

$$\nu_i(2, j) = 1 - c_0 + \beta[F(j-1) - F(j-2)]$$

when $j \geq 2$.

So (3.11) is true when $T = 2$.

A.2.2 When $T > 2$

Assume equation (3.11) holds when $T = t - 1$, we show that it holds when $T = t$.

- If $j = 1$,

  $$V_i^{\nu}(t, j) = \max\{\nu + \beta V_i^{\nu}(t-1, 1), 1 - c_0 + \beta V_i^{\nu}(t-1, 0)\}.$$

  The difference between actions is

  $$
  \begin{aligned}
  &f^{\nu}(t, 1)\\
  &\quad = \nu - (1 - c_0) + \beta g^{\nu}(t-1, 0)\\
  &\quad = \begin{cases} \nu - (1-\beta)(1 - c_0), & \text{if } \nu < 0;\\ (1-\beta)[\nu - (1 - c_0)], & \text{if } 0 \leq \nu < 1 - c_0;\\ \nu - (1 - c_0) + \beta^2 g^{\nu}(t-2, 0) & \text{if } 1 - c_0 \leq \nu. \end{cases}
  \end{aligned}
  $$

The last case can be rewritten as

$$\begin{aligned}
&\nu - (1-c_0) + \beta^2 g^\nu(t-2,0) \\
&\quad = (1-\beta)[\nu-(1-c_0)] + \beta[\nu-(1-c_0)] \\
&\qquad + \beta^2[V_i^\nu(t-2,1) - V_i^\nu(t-2,0)],
\end{aligned}$$

which equals to 0 when $\nu = 1-c_0$ since by assumption $\nu_i(t-1,1) = 1-c_0$. Thus $\nu_i(t,1) = 1-c_0$.

- If $2 \le j \le t-2$, the difference between actions is stated as follows.

$$\begin{aligned}
&f^\nu(t,j) \\
&\quad = \nu - (1-c_0) + \beta g^\nu(t-1,j-1) \\
&\quad = \begin{cases} \nu-(1-c_0)+\beta^2 g^\nu(t-2,j-2) & \text{if } \nu < 1-c_0; \\ \nu-(1-c_0)+\beta^2 g^\nu(t-2,j-1) & \text{if } 1-c_0 \le \nu. \end{cases}
\end{aligned}$$

  The latter case equals to 0 when $\nu = 1-c_0$ since $\nu_i(t-1,j) = 1-c_0$ when $2 \le j \le t-2$ by assumption. Thus $\nu_i(t,j) = 1-c_0$ when $2 \le j \le t-2$.

- If $j = t-1$,

$$\begin{aligned}
&f^\nu(t,j) \\
&\quad = \nu - (1-c_0) + \beta g_1^\nu(t-1,j-1) \\
&\quad = \begin{cases} \nu-(1-c_0)+\beta^2 g^\nu(t-2,j-2), & \text{if } \nu < 1-c_0; \\ (1-\beta)[\nu-(1-c_0)], & \text{if } 1-c_0 \le \nu < 1-c_0+\beta^{t-2}F(1); \\ \nu-(1-c_0)+\beta^2 g^\nu(t-2,j-1), & \text{if } 1-c_0+\beta^{t-2}F(1) \le \nu; \end{cases}
\end{aligned}$$

  equals to 0 when $\nu = 1-c_0$. So $\nu_i(t,j) = 1-c_0$ when $j = t-1$.

- If $j \ge t$,

$$\begin{aligned}
&f^\nu(t,j) \\
&\quad = \nu - (1-c_0) + \beta g^\nu(t-1,j-1) \\
&\quad = \begin{cases} \nu-(1-c_0)+\beta^2 g^\nu(t-2,j-2) & \text{if } \nu < \nu_i(t-1,j-1); \\ (1-\beta)[\nu-(1-c_0)], & \text{if } \nu_i(t-1,j-1) \le \nu < \nu_i(t-1,j); \\ \nu-(1-c_0)+\beta^2 g^\nu(t-2,j-1), & \text{if } \nu_i(t-1,j) \le \nu. \end{cases}
\end{aligned} \tag{A.3}$$

If $\nu < \nu_i(t-1, j-1)$, according to (3.11)

$$\begin{aligned} \nu &< \nu_i(t-1-T', j-1-T') \\ &\le \nu_i(t-1-T', j-T') \end{aligned}$$

for all $0 \le T' \le t-1$. Thus the first case of (A.3) can be written as,

$$\begin{aligned} &\nu - (1-c_0) + \beta^2 g^\nu(t-2, j-2) \\ &\quad = \nu - (1-c_0) + \beta^3 g^\nu(t-3, j-3) \\ &\quad = \cdots \\ &\quad = \nu - (1-c_0) + \beta^{t-1} g^\nu(1, j-t+1) \\ &\quad = \nu - (1-c_0) + \beta^{t-1}[-F(j-t+1) + F(j-t)]. \end{aligned}$$

So when $\nu = 1 - c_0 + \beta^{t-1}[F(j-t+1) - F(j-t)]$, the first case in equation (A.3) equals to 0 . Thus when $B \ge t$, the closed-form of index is stated as:

$$\nu_i(t, j) = 1 - c_0 + \beta^{t-1}[F(j-t+1) - F(j-t)].$$

So (3.11) holds when $T = t$. By induction, we have (3.11) is true for all T. $\square$

A.3 Proof of Lemma 1

Proof. In the problem of (3.8), given an initial state s, denote the scheduling sequence of the Whittle's index policy given the charging limit M by π_{RMAB}. For the same trajectory, denote the charging sequence given no charging limit (or equivalently, $M = N$) by π_{N}. Recall that when $M = N$, the Whittle's index policy achieves the optimality. Thus the reward of π_{N} serves as an upper bound of the optimal reward for any case when $M \le N$, *i.e.*,

$$G_{\pi_{\text{N}}}(s) \ge G(s) \ge G_{\pi_{\text{RMAB}}}(s),$$

where $G_{\pi_{\mathrm{N}}}(s)$ is the reward collected from π_{N}, $G_{\pi_{\mathrm{RMAB}}}(s)$ the reward collected from the Whittle's index policy π_{RMAB} when $M \leq N$, and $G(s)$ the maximum reward defined in (3.4).

In this appendix, we establish an upper bound of the difference of the value functions of π_{RMAB} and π_{N}, $G_{\pi_{\mathrm{N}}}(s) - G_{\pi_{\mathrm{RMAB}}}(s)$, which serves as an upper bound of the gap-to-optimality of the Whittle's index policy, $G(s) - G_{\pi_{\mathrm{RMAB}}}(s)$. We first quantify $G_{\pi_{\mathrm{N}}}(s) - G_{\pi_{\mathrm{RMAB}}}(s)$ by the number of different actions in the charging sequences resulted by π_{RMAB} and π_{N}. Then we relate the number of different actions to the arrivals of EVs in Lemma 3, which gives the expression in (3.12).

Note that, π_{N} has no capacity limit and activates a regular arm if and only if its Whittle's index is positive. A policy π_{RMAB} activates a regular arm if and only if its indic belongs to the largest M positive ones. Due to the capacity limit M, if facing the same trajectory of charging cost and the same sequence of arrivals, the policy π_{N} and π_{RMAB} generate different charging sequences for identical EVs. As shown in Figure A.1, the charging sequences of an EV J_i with arrival time r and departure time d determined by π_{N} and π_{RMAB} are plotted. Define the action differences as follows.

- Event **A**: π_{N} charges J_i but π_{RMAB} does not.
- Event **B**: π_{RMAB} charges J_i but π_{N} does not.

- If event **A** happens at time t, the instant reward difference between π_{N} and π_{RMAB} is bounded.

$$R_i^{\pi_{\mathrm{N}}}[t] - R_i^{\pi_{\mathrm{RMAB}}}[t] \leq 1 - c_{\min}$$

where $R_i^{\pi}[t]$ is the instant reward collected from J_i by policy π at time t.

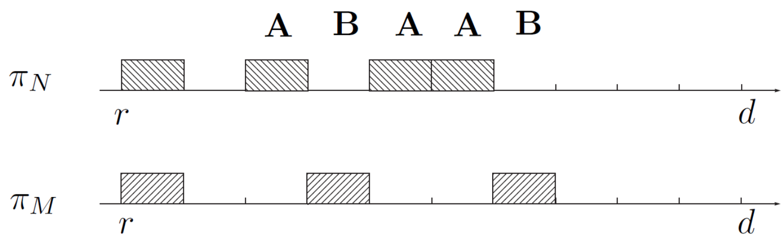


Figure A.1: Action differences between π_N and π_M

- If event **B** happens at time t, the instant reward difference between π_{N} and π_{RMAB} is also bounded.

$$R_i^{\pi_{\text{N}}}[t] - R_i^{\pi_{\text{RMAB}}}[t] \leq |1 - c_{\max}|$$

- At the deadline of J_i, the difference of unfinished EV length resulted by two policies is bounded by the number of event **A**. Thus the penalty difference of two policies is also bounded.

$$\begin{aligned} F_i^{\pi_{\text{N}}}[d] - F_i^{\pi_{\text{RMAB}}}[d] &\leq F(j + \textstyle\sum_{t=r}^{d} \mathbb{1}(\mathbf{A}[t])) - F(j) \\ &\leq F(\bar{j}) \textstyle\sum_{t=r}^{d} \mathbb{1}(\mathbf{A}[t]) \end{aligned}$$

  where $F_i^{\pi}[d]$ is the penalty of J_i resulted by π at deadline d, j the left over charging demand under π_{N} of EV i, $\mathbb{1}(\mathbf{A}[t]) = 1$ if and only if event **A** happens at t, and $F(\bar{j})$ the maximum penalty generated by an EV.

So the reward difference collected from J_i up to time $t < d$ is the sum of the first two cases.

$$\begin{aligned} &\textstyle\sum_{h=r}^{t} \beta^h (R_i^{\pi_{\text{N}}}[h] - R_i^{\pi_{\text{RMAB}}}[h]) \\ &\quad \leq (1 - c_{\min}) \textstyle\sum_{h=r}^{t} \mathbb{1}(\mathbf{A}[h])\beta^h + |1 - c_{\max}| \textstyle\sum_{h=r}^{t} \mathbb{1}(\mathbf{B}[h])\beta^h \end{aligned}$$

The difference up to deadline $t = d$ is the sum of the three cases.

$$\begin{aligned}
&\textstyle\sum_{h=r}^{d} \beta^h (R_i^{\pi_{\mathrm{N}}}[h] - R_i^{\pi_{\mathrm{RMAB}}}[h]) \\
&\quad \le (1 - c_{\min}) \textstyle\sum_{h=r}^{d} \mathbb{1}(\mathbf{A}[h]) \beta^h + \\
&\quad\quad |1 - c_{\max}| \textstyle\sum_{h=r}^{d} \mathbb{1}(\mathbf{B}[h]) \beta^h + \\
&\quad\quad F(\bar{j}) \textstyle\sum_{h=r}^{t} \mathbb{1}(\mathbf{A}[h]) \beta^d
\end{aligned}$$

For each time t, we enlarge the penalty term and get a general bound as follows.

$$\begin{aligned}
&\textstyle\sum_{h=r}^{t} \beta^h (R_i^{\pi_{\mathrm{N}}}[h] - R_i^{\pi_{\mathrm{RMAB}}}[h]) \\
&\quad \le (1 - c_{\min}) \textstyle\sum_{h=r}^{t} \mathbb{1}(\mathbf{A}[h]) \beta^h + \\
&\quad\quad |1 - c_{\max}| \textstyle\sum_{h=r}^{t} \mathbb{1}(\mathbf{B}[h]) \beta^h + \\
&\quad\quad F(\bar{j}) \textstyle\sum_{h=r}^{t} \mathbb{1}(\mathbf{A}[h]) \beta^h
\end{aligned}$$

Note that, the cumulative number of event **A** happened up to any fixed time t is always larger than the number of event **B**. Formally, we state the following lemma to illustrate the relationship between event **A** and **B**. The proof is delayed to Appendix A.3.1.

Lemma 2. *Denote $\mathbb{1}(\boldsymbol{A}[t])$ as whether event $\boldsymbol{A}$ happens at t. Denote $\#\boldsymbol{A}[t]$ as the cumulative number of event $\boldsymbol{A}$ happened from r to time t. Define $\mathbb{1}(\boldsymbol{B}[t])$ and $\#\boldsymbol{B}[t]$ respectively. For any $t \in [r, d]$,*

$$\#\boldsymbol{A}[t] = \textstyle\sum_{h=r}^{t} \mathbb{1}(\boldsymbol{A}[h]) \ge \#\boldsymbol{B}[t] = \sum_{h=r}^{t} \mathbb{1}(\boldsymbol{B}[h])$$

$$\textstyle\sum_{h=r}^{t} \mathbb{1}(\boldsymbol{A}[h]) \beta^h \ge \sum_{h=r}^{t} \mathbb{1}(\boldsymbol{B}[h]) \beta^h$$

So the reward difference is bounded as follows.

$$\begin{aligned}
&\textstyle\sum_{h=r}^{t} \beta^h (R_i^{\pi_{\mathrm{N}}}[h] - R_i^{\pi_{\mathrm{RMAB}}}[h]) \\
&\quad \le (1 - c_{\min} + F(\bar{j}) + |1 - c_{\max}|) \textstyle\sum_{h=r}^{t} \mathbb{1}(\mathbf{A}[h]) \beta^h
\end{aligned}$$

Now we want to quantify the cumulative number of event **A**. Event **A** happens only when there are more than M EVs with positive Whittle's index in the system under π_{RMAB}. This event can only happen when there are at least M EVs in the queue. To bound the number of event **A**, we have the following lemma. The proof is delayed to Appendix A.3.2.

Lemma 3. *Denote $I[t]$ the number of EVs arrived to the system between $[t - \bar{T} + 1, t]$. Then for any t,*

$$\mathbb{1}(A[t]) \leq \mathbb{1}(I[t] > M).$$

Thus for each EV, we have

$$\sum_{h=r}^{t} \beta^h (R_i^{\pi_{\text{N}}}[h] - R_i^{\pi_{\text{RMAB}}}[h]) \leq C \sum_{h=r}^{t} \beta^h \mathbb{1}(I[h] > M), \tag{A.4}$$

for any t, where $C = (1 - c_{\min} + F(\bar{j}) + |1 - c_{\max}|)$.

If we sum arrivals and take expectation, we have the difference of expected value function bounded as follows.

$$\begin{aligned} G_{\pi_{\text{N}}}(s) - G_{\pi_{\text{RMAB}}}(s) &\leq C \textstyle\sum_t \beta^t \mathbb{E}[\mathbb{1}(I[t] > M) I[t]] \\ &= C\mathbb{E}[\mathbb{1}(I[t] > M) I[t]]/(1 - \beta). \end{aligned}$$

Since $G_{\pi_{\text{N}}}(s)$ is an upper bound of $G(s)$, we have

$$G(s) - G_{\pi_{\text{RMAB}}}(s) \leq \frac{C}{1 - \beta} \mathbb{E}[I[t] | I[t] > M] Pr(I[t] > M),$$

which is Lemma 1. $\square$

A.3.1 Proof of Lemma 2

Proof. Denote the remaining charging demand of J_i at time t under policy π_{N} and π_{RMAB} by $j_{\text{N}}[t]$ and $j_{\text{RMAB}}[t]$. Event **B** happens only when $j_{\text{RMAB}}[t] > j_{\text{N}}[t]$. When

$j_{\text{RMAB}}[t] = j_{\text{N}}[t]$, if π_{RMAB} charges J_i, which means the Whittle's index of J_i is positive, π_{N} also charges J_i. Since at the arrival r, $j_{\text{RMAB}}[r] = j_{\text{N}}[r]$, event **B** can only happen when $j_{\text{RMAB}}[t] > j_{\text{N}}[t]$, which means event **A** must have happened before.

This also implies that $j_{\text{RMAB}}[t] \geq j_{\text{N}}[t]$ for all t. □

A.3.2 Proof of Lemma 3

Proof. Recall that, the remaining charging demand under π_{RMAB} is always larger than the one under π_{N}, *i.e.*, $j_{\text{RMAB}}[t] \geq j_{\text{N}}[t]$. Whenever π_{N} charges some EV J_i, the Whittle's index of this EV under π_{N} must be positive. If the Whittle's index is monotonically increasing in j, the indic under policy π_{RMAB} must also be positive, and π_{RMAB} will also charge this EV if the capacity limit allows. Thus event **A** happens must imply that there are more than M EVs with positive Whittle's index, which requires the arrivals larger than M, *i.e.*, $I[t] > M$.

In this subsection, we show that the Whittle's index is increasing in j when the index is positive and the value function is concave when $\nu > 0$ by induction. That is, $\nu_i(T, j+1, c_k) \geq \nu_i(T, j, c_k)$, if $\nu_i(T, j, c_k) > 0$ and $V^{\nu}(T, j, c_k)$ is concave when $\nu > 0$.

When $T = 1$

the Whittle's index is

$$\nu_i(1, j, c_k) = \begin{cases} 0, & \text{if } j = 0; \\ 1 - c_k + F(j) - F(j-1), & \text{if } j \geq 1. \end{cases}$$

If $\nu_i(1, j, c_k) > 0$, $\nu_i(1, j+1, c_k) > \nu_i(1, j, c_k)$ due to the convexity of $F(j)$.

The value function is concave in j when $\nu > 0$.

$$
\begin{aligned}
&V_i^\nu(1, j+2, c_k) - 2V_i^\nu(1, j+1, c_k) + V_i^\nu(1, j, c_k) \\
&= \begin{cases}
-F(j+2) + 2F(j+1) - F(j), & \text{if } \nu_i(1, j, c_k) < \nu, \nu_i(1, j+1, c_k) < \nu, \\
& \quad \text{and } \nu_i(1, j+2, c_k) < \nu; \\
1 - c_k - \nu + F(j+1) - F(j), & \text{if } \nu_i(1, j, c_k) < \nu, \nu_i(1, j+1, c_k) < \nu, \\
& \quad \text{and } \nu \leq \nu_i(1, j+2, c_k); \\
\nu - 1 + c_k + F(j) - F(j+1), & \text{if } \nu_i(1, j, c_k) < \nu \leq \nu_i(1, j+1, c_k); \\
-F(j+1) + 2F(j) - F(j-1), & \text{if } \nu \leq \nu_i(1, j, c_k);
\end{cases} \\
&\leq 0.
\end{aligned}
$$

The first and last cases are negative because of convexity of the penalty. The second and third cases are negative because of the definition of $\nu_i(1, j, c_k)$.

When $T > 1$

assume $\nu_i(T, j+1, c_k) > \nu_i(T, j, c_k)$ when $\nu_i(T, j, c_k) > 0$, and $V_i^\nu(T, j, c_k)$ is concave in j when $\nu > 0$ for $T = t - 1$. We show that these properties are true for $T = t$.

The difference of the activate and deactivate actions at state $(t, j+1, c_k)$ is stated as follows.

$$
\begin{aligned}
& f^\nu(t, j+1, c_k) \\
&= \nu - 1 + c_k + \beta \sum P_{k,k'}[V_i^\nu(t-1, j+1, c_{k'}) - V_i^\nu(t-1, j, c_{k'})] \\
&= \nu - 1 + c_k + \beta \sum P_{k,k'}[V_i^\nu(t-1, j, c_{k'}) - V_i^\nu(t-1, j-1, c_{k'})] \\
&\quad + \beta \sum P_{k,k'}[V_i^\nu(t-1, j+1, c_{k'}) - 2V_i^\nu(t-1, j, c_{k'}) + V_i^\nu(t-1, j-1, c_{k'}] \\
&= f^\nu(t, j, c_k) \\
&\quad + \beta \sum P_{k,k'}[V_i^\nu(t-1, j+1, c_{k'}) - 2V_i^\nu(t-1, j, c_{k'}) + V_i^\nu(t-1, j-1, c_{k'})].
\end{aligned}
$$

When $\nu = \nu_i(t, j, c_k) > 0$, $f^\nu(t, j, c_k) = 0$ according to the definition of $\nu_i(t, j, c_k)$. The second term in the above equation is negative due to the concavity of the value function when $\nu > 0$. Thus $f^{\nu}(t, j+1, c_k) \leq 0$ when $\nu = \nu_i(t, j, c_k) > 0$, which implies $\nu_i(t, j+1, c_k) > \nu_i(t, j, c_k)$.

We have shown the monotonicity of the Whittle's index when $T = t$. Next we show the concavity of the value functions for $T = t$ when $\nu > 0$.

$$
\begin{aligned}
&V_i^{\nu}(t, j+2, c_k) + V_i^{\nu}(t, j, c_k) - 2V_i^{\nu}(t, j+1, c_k) \\
&= \begin{cases}
\beta \sum P_{k,k'} V_i^{\nu}(t-1, j+2, c_{k'}) - 2\beta \sum P_{k,k'} V_i^{\nu}(t-1, j+1, c_{k'}) + \beta \sum P_{k,k'} V_i^{\nu}(t-1, j, c_{k'}), \\
\text{if } \nu_i(t, j, c_k) < \nu, \nu_i(t, j+1, c_k) < \nu, \text{and } \nu_i(t, j+2, c_k) < \nu; \\
1 - c_k - \nu + \beta \sum P_{k,k'} [V_i^{\nu}(t-1, j, c_{k'}) - V_i^{\nu}(t-1, j+1, c_{k'})], \\
\text{if } \nu_i(t, j, c_k) < \nu, \nu_i(t, j+1, c_k) < \nu, \text{and } \nu \leq \nu_i(t, j+2, c_k); \\
\nu - (1 - c_k) + \beta \sum P_{k,k'} [V_i^{\nu}(t-1, j+1, c_{k'}) - V_i^{\nu}(t-1, j, c_{k'})], \\
\text{if } \nu_i(t, j, c_k) < \nu \leq \nu_i(t, j+1, c_k); \\
\beta \sum P_{k,k'} [V_i^{\nu}(t-1, j+1, c_{k'}) - 2V_i^{\nu}(t-1, j, c_{k'}) + V_i^{\nu}(t-1, j-1, c_{k'})], \\
\text{if } \nu \leq \nu_i(t, j, c_k);
\end{cases} \\
&\leq 0.
\end{aligned}
$$

The first and fourth terms are less than zero because by the assumption the value function is concave when $\nu > 0$ for $T - 1$. The second and third terms are negative because of the definition of $\nu_i(t, j+1, c_k)$. So the value function $V_i^{\nu}(t, j, c_k)$ is concave in j when $\nu > 0$.

By induction, we have $\nu_i(T, j+1, c_k) > \nu_i(T, j, c_k)$ when $\nu_i(T, j, c_k) > 0$, and $V_i^{\nu}(T, j, c_k)$ is concave in j when $\nu > 0$ for all T. □

A.4 Proof of Theorem 3

For a Poisson process $I[t]$ with mean μ, we have the expression as follows.

$$\mathbb{E}[I[t]|I[t] > M]Pr(I[t] > M) = \mu Pr(I[t] \geq M)$$

For any $M > \mu - 1$, we have the inequality as follows [83].

$$\begin{aligned}
\mu Pr(I[t] \geq M) \quad & < \mu Pr(I[t] = M)/(1 - \tfrac{\mu}{M+1}) \\
& = \mu^{M+1} e^{-\mu}(M+1)/[(M+1-\mu)M!] \\
& \leq \tfrac{\mu^{M+1} e^{M-\mu}(M+1)}{\sqrt{2\pi} M^{M+1/2}(M+1-\mu)} \\
& = O(\tfrac{\mu e^{-\mu}}{\sqrt{M}})
\end{aligned} \tag{A.5}$$

where the second inequality is because of Stirling formula. When $\mu \leq M/e$, the right hand side decreases to zero, which indicates the asymptotic optimality of the Whittle's index.

A.5 Proof of Theorem 4

Proof. We have

$$\begin{aligned}
G(s) - G_{\text{RMAB}}(s) \quad & \triangleq \limsup_{N\to\infty} \left[G^N(s) - G^N_{\text{RMAB}}(s)\right] \\
& \leq \tfrac{C}{1-\beta} \lim_{N\to\infty} \mathbb{E}[I^N[t]\mathbb{1}(I^N[t] > M)] \\
& = O(\mathbb{E}[I[t]\mathbb{1}(I[t] > M)]).
\end{aligned}$$

We can rewrite the bound as

$$\begin{aligned}
& \mathbb{E}[I[t]\mathbb{1}(I[t] > M)] \\
& \quad = \textstyle\sum_{i>M} i \Pr(I[t] = i) \\
& \quad = (M+1)\Pr(I[t] \geq M+1) + \textstyle\sum_{i=M+2} \Pr(I[t] \geq i).
\end{aligned}$$

- If $\Pr(I[t] \geq i) \leq a \exp[-ib/\lambda]$, we have

$$\begin{aligned}
&(M+1)\Pr(I[t] \geq M+1) + \textstyle\sum_{i=M+2} \Pr(I[t] \geq i) \\
&\quad \leq a(M+1)\exp[-\tfrac{(M+1)b}{\mu}] + a \textstyle\sum_{i=M+2} \exp[-\tfrac{ib}{\mu}] \\
&\quad \leq a(M+1)\exp[-\tfrac{(M+1)b}{\mu}] + a \textstyle\int_{M+1}^{\infty} \exp(-xb/\mu)dx \\
&\quad = a\exp[-\tfrac{(M+1)b}{\mu}][M+1+\mu/b].
\end{aligned}$$

  Thus we have

$$G(s) - G_{\mathrm{RMAB}}(s) = \mathcal{O}[\exp(-\frac{Mb}{\mu})(Mb+\mu)],$$

  which is (3.16) in Theorem 4. When $\mu = o(M/\ln M)$, the bound goes to zero and the Whittle's index policy is asymptotically optimal.

- If $\Pr(I[t] \geq i) \leq a\mu/i^b$, we have

$$\begin{aligned}
&(M+1)\Pr(I[t] \geq M+1) + \textstyle\sum_{i=M+2} \Pr(I[t] \geq i) \\
&\quad \leq a\tfrac{\mu}{(M+1)^{b-1}} + a\mu \textstyle\sum_{i=M+2} i^{-b} \\
&\quad \leq a\tfrac{\mu}{(M+1)^{b-1}} + a\mu \textstyle\int_{M+1}^{\infty} x^{-b}dx \\
&\quad = ab\mu/[(M+1)^{b-1}(b-1)].
\end{aligned}$$

  Thus we have

$$G(s) - G_{\mathrm{RMAB}}(s) = \mathcal{O}(\mu/M^{b-1}),$$

  which is (3.17) in Theorem 5. When $\mu = o(M^{b-1})$, the bound goes to zero and the Whittle's index policy is asymptotically optimal.

Thus we have the bound on the gap-to-optimality decreases to zero as stated in Theorem 4 and 5 . □

APPENDIX B

APPENDICES FOR CHAPTER 4

Here the proofs under the assumption of the type I extreme value vehicle preference distribution are presented. The uniform preference distribution proof can be similarly developed. The subscript j is dropped in this section for convenience.

B.1 Proof of Theorem 8

In Theorem 7, the optimal charging price is shown to generate uniform profits across charging stations. Denote the uniform profit by $r \triangleq \rho_i^* - \bar{c}_i$, the sum of the exponential systematic utility by $v = \sum_{i=1}^{N_E} v_i = \sum_{i=1}^{N_E} \exp(\alpha_1 f_i - \alpha_2 \bar{c}_i)$ and the ratio of the utility and the exponential profit by $\kappa = v/\exp(\alpha_2 r)$. Equation (5.14) can be rewritten as

$$\begin{aligned} g(v,r) &\triangleq \alpha_2\beta_1 r(1-\eta)(1-P_0) + \alpha_2 r P_0 - 1 \\ &= \beta_1 \ln(v/\kappa)\frac{C}{(q_0+\kappa)^{\beta_1}+C}\frac{\kappa}{q_0+\kappa} + \ln(v/\kappa)\frac{q_0}{q_0+\kappa} - 1 \\ &= 0, \end{aligned} \tag{B.1}$$

where $C = e^{(\beta_1 \mathbb{E}(U_G) - \beta_2 p_G + \beta_G p_E)}$ and $q_0 = e^{(\alpha_1 f_0 - \alpha_2 \rho_0)}$.

Note the first term in the left hand side of (B.1), $\alpha_2\beta_1 r(1-\eta)(1-P_0)$, is positive. We can conclude that as $v \to +\infty$, $\kappa \to +\infty$. Otherwise, the second term $\ln(v/\kappa)\frac{q_0}{q_0+\kappa} \to +\infty$ which violates (B.1).

As the sum of exponential systematic utility v increases, the charging service at the EVCSs is more convenient, thus more consumers tend to purchase EVs and prefer to charge at the public charging stations. $\kappa \to +\infty$ gives the convergence of the market share η and P_0, which is stated as follows.

Lemma 4.

$$
\begin{aligned}
\lim_{v\to+\infty} \eta &= \lim_{v\to+\infty} \tfrac{(q_0+\kappa)^{\beta_1}}{(q_0+\kappa)^{\beta_1}+C} &&= 1 \\
\lim_{v\to+\infty} P_0 &= \lim_{v\to+\infty} \tfrac{q_0}{q_0+\kappa} &&= 0.
\end{aligned}
\tag{B.2}
$$

Proof. Since q_0 and C are constant, $\beta_1 > 0$ and $\kappa \to +\infty$, $\eta \to 1$ and $P_0 \to 0$. □

A direct result of Lemma 4 is that the uniform profit $r \to +\infty$ as $v \to +\infty$. Otherwise (B.1) does not hold any more.

By applying the implicit function theorem (IFT) to function $g(v, r)$, we get the derivative of profit r with respect to the exponential systematic utility v,

$$
\frac{\partial r}{\partial v} = -\frac{\partial g(v, r)/\partial v}{\partial g(v, r)/\partial r} = \frac{1}{\alpha_2 v}\frac{1}{1+h}, \tag{B.3}
$$

where

$$
h = \frac{\beta_1(1-\eta)(1-P_0)+P_0}{\alpha_2\beta_1^2 r\eta(1-\eta)(1-P_0)^2 - \alpha_2\beta_1 r(1-\eta)P_0(1-P_0) + \alpha_2 rP_0(1-P_0)}.
$$

The numerator of h, $\beta_1(1-\eta)(1-P_0)+P_0$, converges to 0 as $v \to +\infty$. Denote the denominator of h by H. If $\beta_1 > 1$, H can be rewritten as

$$
\begin{aligned}
H &= \alpha_2\beta_1 r(1-\eta)(1-P_0)(1-P_0) + \alpha_2 rP_0(1-P_0) \\
&\quad +\alpha_2\beta_1 r(1-\eta)(1-P_0)[\beta_1\eta(1-P_0)-1] \\
&= (1-P_0) + \alpha_2\beta_1 r(1-\eta)(1-P_0)[\beta_1\eta(1-P_0)-1],
\end{aligned}
\tag{B.4}
$$

where the last equality is because of (B.1). In (B.4), the first term converges to 1 and the second term is positive as $v \to +\infty$.

If $\beta_1 \le 1$, H can be rewritten as

$$
\begin{aligned}
H &= \alpha_2\beta_1 r(1-\eta)(1-P_0)[\beta_1\eta(1-P_0)-P_0] \\
&\quad +\alpha_2 rP_0[\beta_1\eta(1-P_0)-P_0] + \alpha_2 rP_0[1-\beta_1\eta(1-P_0)] \\
&= [\beta_1\eta(1-P_0)-P_0] + \alpha_2 rP_0[1-\beta_1\eta(1-P_0)],
\end{aligned}
\tag{B.5}
$$

where the last equality is because of (B.1). In (B.5), the first term converges to β_1 as $v \to +\infty$ and the second term is positive.

In both cases, H is bounded below from zero as $v \to +\infty$. Thus $h \to 0$ and $v\frac{\partial r}{\partial v} \to \frac{1}{\alpha_2}$ in (B.3).

By the l'Hôpital's rule, since $r \to +\infty$ as $v \to +\infty$,

$$\lim_{v\to+\infty} \frac{r}{\ln v} = \lim_{v\to+\infty} \frac{\partial r/\partial v}{1/v} = \lim_{v\to+\infty} \frac{\partial^2 r/\partial v^2}{-v^{-2}} = \frac{1}{\alpha_2}, \tag{B.6}$$

which completes the proof.

Note when $v > \bar{v}_1$ for some $\bar{v}_1 > 0$, $h > 0$. $r(v)$ is strictly increasing in v and $v\frac{\partial r}{\partial v} < \frac{1}{\alpha_2}$ when $v > \bar{v}_1$.

B.2 Proof of Theorem 9

First, fixing the number of charging stations to build as N_E, we examine where to build these stations. Denote the sum of exponential systematic utility by $v = \sum_{i=1}^{N_E} v_i$ and the uniform charging profit by $r = \rho_i^* - \bar{c}_i$. The unit operational profit of the investor can be stated as

$$\frac{\Pi(v)}{N_c D} = r(v)\eta(v) \sum_{i=1}^{N_E} P_i(v) = r(v) \frac{[q_0 + \kappa(v)]^{\beta_1}}{[q_0 + \kappa(v)]^{\beta_1} + C} \frac{\kappa(v)}{q_0 + \kappa(v)}, \tag{B.7}$$

where $q_0 = \exp(\alpha_1 f_1 - \alpha_2 \rho_0)$, $\kappa(v) = v\exp(-\alpha_2 r)$, and $C = \exp(\beta_1 \mathbb{E}(U_G) - \beta_2 p_G + \beta_G p_E)$. The derivative of $\kappa(v)$ with respect to v is stated as

$$\frac{\partial \kappa(v)}{\partial v} = \exp(-\alpha_2 r)(1 - \alpha_2 v \frac{\partial r}{\partial v}) \tag{B.8}$$

which is strictly positive when $v > \bar{v}_1$ according to the discussion in Sec. B.1. So the operational profit $\Pi(v)$ is strictly increasing in v when $v > \bar{v}_1$.

The second order derivative of $\Pi(v)$ with respect to v is stated as

$$\begin{aligned}\frac{1}{N_c D}\frac{\partial^2 \Pi}{\partial v^2} = \quad & \frac{\eta}{v^2}\{v^2\frac{\partial^2 r}{\partial v^2}(1-P_0) \\ & +(-2rP_0)(1-P_0)^2 \\ & +\frac{2}{\alpha_2}\frac{1}{1+h}(1-P_0)[(1-\eta)\beta_1(1-P_0)+P_0] \\ & +r(1-\eta)\beta_1(1-P_0)^2[\beta_1(1-2\eta-P_0)+(2\eta+3)P_0-1]\}.\end{aligned} \tag{B.9}$$

As $v \to +\infty$, $v^2\frac{\partial^2 r}{\partial v^2} \to -\frac{1}{\alpha_2}$, $\eta \to +\infty$ and $P_0 \to 0$. So when $v > \bar{v}_2$ for some $\bar{v}_2 > 0$, the first term in (B.9) is negative and bounded above from zero. The second and last term are negative. The third term is positive but converging to zero. The second order derivative of $\Pi(v)$ is negative and we have the concavity of $\Pi(v)$ stated in the following lemma.

Lemma 5. *Asymptotically,* $\Pi(v)$ *is an increasing and concave function of* v*.*

The monotonicity of $\Pi(v)$ implies that, if given two station candidates k and k', fixing the other $(N_E - 1)$ stations, the one with larger $v_i = \exp(\alpha_1 f_i - \alpha_2 \bar{c}_i)$, $i \in \{k, k'\}$ should be built because the building costs are the same. So we have the optimal strategy about where to build stations as follows.

Lemma 6. *Fixing the number of stations to build as* N_E*, the asymptotically optimal strategy of building is to pick* N_E *candidates with largest* $v_i = \exp(\alpha_1 f_i - \alpha_2 \bar{c}_i)$*.*

Next, after we sort the N_L candidate locations by v_i, we can present the cost $\sum_{i=1}^{N_E} \mathcal{F}(\bar{s}_i) = (1+\gamma)\mathcal{F}_0 N_E$ as a function of $v = \sum_{i=1}^{N_E} v_i$. We can treat v as a continuous variable and write $\tilde{F}(v) \triangleq (1+\gamma)\mathcal{F}_0 N_E$ as

$$\tilde{F}(v) = (1+\gamma)\mathcal{F}_0 N + (v - \sum_{i=1}^{N} v_i)(1+\gamma)\mathcal{F}_0 / v_{N+1},$$

if $\sum_{i=1}^{N} v_i < v \le \sum_{i=1}^{N+1} v_i$. Since $v_i \ge v_{i+1}$, the cost $\tilde{F}(v)$ is a piece wise linear convex function of v. The partial derivative $\partial \tilde{F}(v)/\partial v$ is piece wise constant and increasing in v.

The trends of $\Pi(v)$, $\tilde{F}(v)$ and the derivatives are plotted in Figure B.1 and Figure B.2. In Figure B.1, v^* is the optimal point to maximize the profit $(\Pi(v) - \tilde{F}(v))$. Figure B.2 shows the derivative of $\tilde{F}(v)$ is increasing and the marginal profit $\frac{\partial \Pi(v)}{\partial v}$ is decreasing when v is large enough. The last cross point of the derivatives of $\Pi(v)$ and $\tilde{F}(v)$ is the optimal point. Combining Lemma 5 and 6, we have the asymptotic optimality.

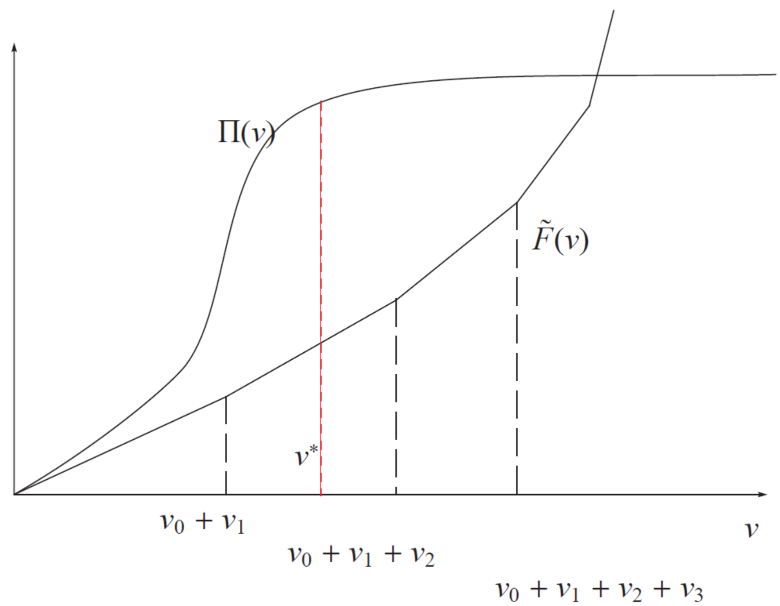


Figure B.1: Profit and cost of charging stations.

B.3 Proof of Theorem 10

Denote the sum of consumer utility and investor's operational profit by $\bar{S}_W(v) = \lambda N_C S_C(v) + \Pi(v)$, the social planner is maximizing $(\bar{S}_W(v) - \tilde{F}(v))$.

The consumer utility, $S_C(v)$, can be rewritten as a function of the total expo-

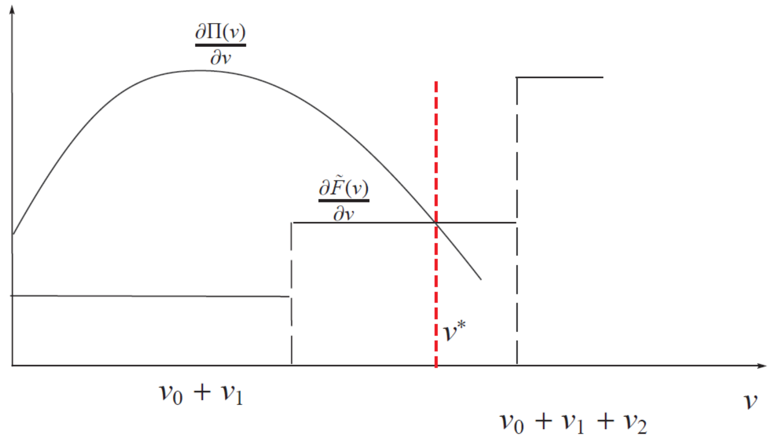


Figure B.2: Profit and cost derivatives of charging stations.

nential systematic utility v as follows.

$$S_C(v) = \ln\left[\left(q_0 + v\exp(-\alpha_2 r)\right)^{\beta_1} C_1 + C_2\right],$$

which is increasing in v. So

$$\frac{\partial \bar{S}_W(v)}{\partial v} = \lambda N_C \frac{\partial S_C(v)}{\partial v} + \frac{\partial \Pi(v)}{\partial v} > \frac{\partial \Pi(v)}{\partial v}.$$

We plot the derivative of the social welfare as well as that of the investor utility in Fig B.3. The optimal social welfare point v^{**} is also the cross point of $\frac{\partial \tilde{F}(v)}{\partial v}$ and $\frac{\partial \bar{S}_W(v)}{\partial v}$. Since $\frac{\partial \bar{S}_W(v)}{\partial v} > \frac{\partial \Pi(v)}{\partial v}$, it is always true that $v^{**} \geq v^*$, which implies the socially optimal solution requires more charging stations than the private market outcomes.

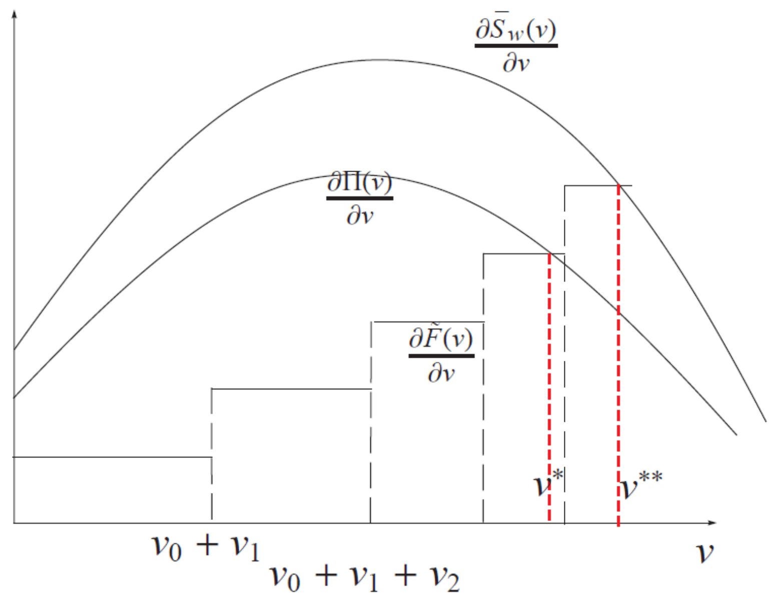


Figure B.3: Derivatives of social welfare and investor utility.

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