Steinkamp's toy can hop 100 times but can't stand up

(appendices only)

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This document contains *only* the appendices and one figure duplicated from the paper.

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For completeness and convenience, here is a figure from the paper that is referred to twice in the appendices:

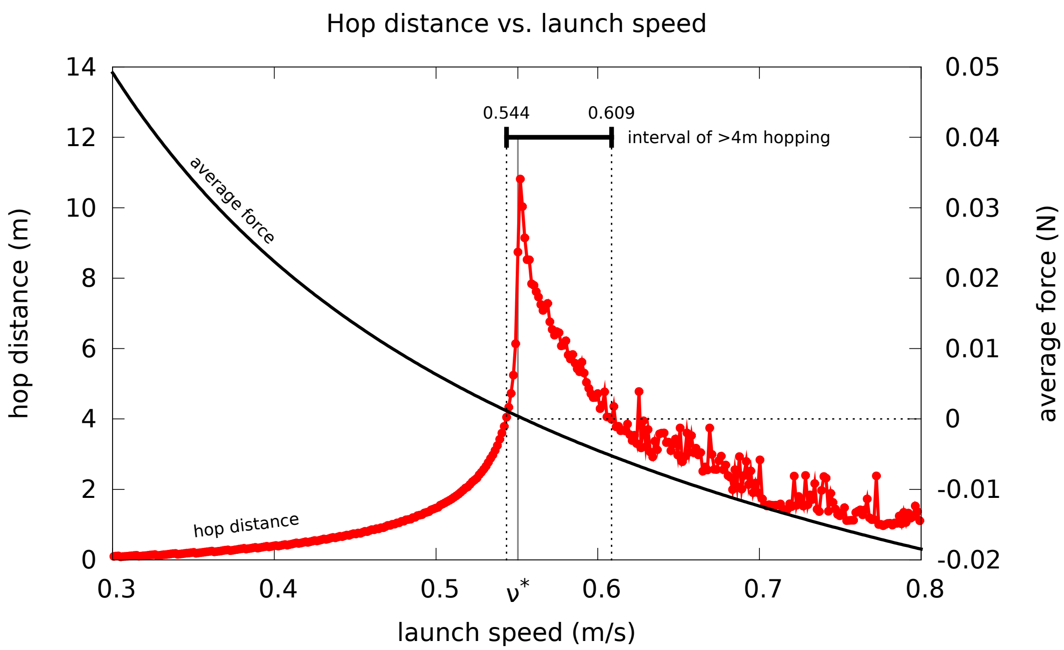

Fig. 14. **Hopping distance as function of launch speed.** The hopper is launched from a stable periodic state of the shock-absorber held system. For each different speed there is a different perturbation from the free periodic motion. Using this launch model, simulations predict a hopping distance greater than 4 meters for a range of launch speeds $0.544 < v_l < 0.609$; the magnitude of this interval is 0.065, somewhat but not substantially larger than the interval for a simulation with a bumpy slope (compare to Figure 17b in Appendix D). The spring and damping coefficients are $k_l = 0.5$ and $c_l = 0.5$. The fixed point of the free hopping map has a speed $\nu^* \approx 0.551$. Note, that even launching at the free-hopper limit-cycle speed, the distance to falling is not infinite because the held hopper has a different fixed point than the free hopper. **Launch force as function of launch speed.** The average launching force is a smooth monotonically decreasing function of the launch speed, and crosses zero near the unstable free hopping map fixed point speed ν^*. The average forces exerted on the device during this type of launch are similar to that of a penny resting on your finger, about 0.03 N. For launch speeds less than ν^*, the instability would cause the robot to decelerate. Therefore a positive force is needed to maintain constant speed. Conversely, at launch speeds greater than ν^*, the instability would cause the robot to accelerate, therefore a negative force is needed.

Appendix A - Simulation Model Parameters

In numerical simulations, we implemented a non-dimensionalized system by setting the leg length ℓ, body mass m_0, and gravitational constant g all to 1. The non-dimensionalized parameter values for the hopper are shown in Figure 15(a).

(a)

$m_0 = 1$	$l = 1$	$EI = 0.417$
$i_0 = 0.610$	$r_{0x} = -0.0270$	$L = 0.260$
$m_1 = 0.0236$	$r_{0y} = 0.0923$	$b = 1.00$
$i_1 = 0.00376$	$r_{1x} = 0.0401$	$c_1 = 0.0619$
$g = 1$	$r_{1y} = 0.391$	$c_2 = 0.838$
$\gamma = 0.0790$	$r_f = 0.0834$	$c_3 = -1.44$
	$\alpha = -0.0761$	

(b)

$m_0 = 1$	$l = 1$	$EI = 0.3102$
$i_0 = 0.21$	$r_{0x} = -0.068$	$L = 0.250$
$m_1 = 0.185$	$r_{0y} = 0.23$	$b = 1$
$i_1 = 0.0103$	$r_{1x} = 0$	$c_1 = 0$
$g = 1$	$r_{1y} = 0.335$	$c_2 = 0$
$\gamma = 0.0790$	$r_f = 0$	$c_3 = 0$
	$\alpha = 0$	

Fig. 15. **(a) Best-fit Steinkamp-model parameters.** The model parameter values that best fit the physical device, non-dimensionalized by body mass, total leg length, and the gravitational constant. **(b) Stable period-2 parameters.** The non-dimensional parameter values of a model significantly different than the physical device and posessing stable period-2 motions are shown. One significant difference from the Steinkamp model is having a leg of much greater mass.

These have been non-dimensionalized by the appropriate combinations of the actual body mass $m_{body} = 0.4025$kg, the actual leg length $l_{leg} = 0.2032$m, and the gravitational constant $g_{earth} = 9.81\text{m/s}^2$.

Figure 15(b) shows the parameter set of a model having stable period-2 motions. One significant difference from the Steinkamp model is a much heavier leg.

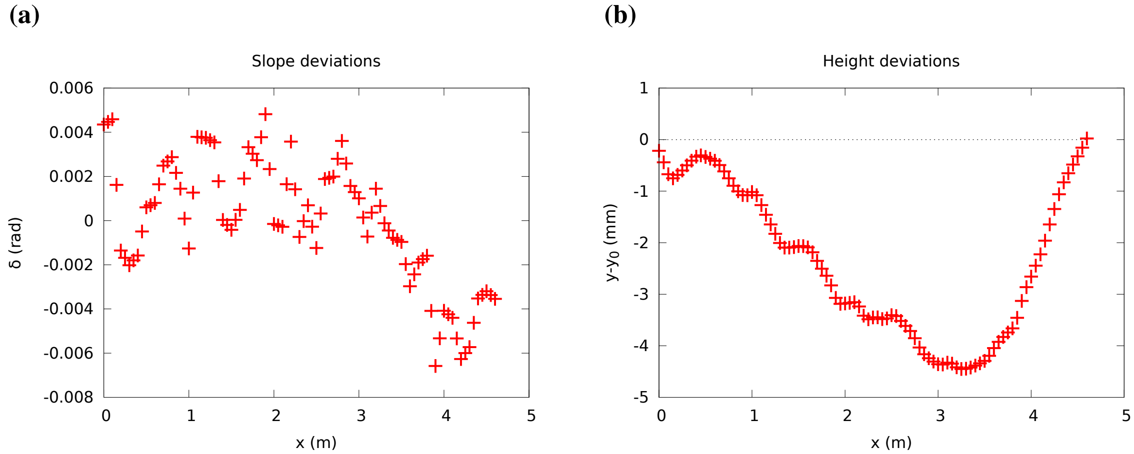

Fig. 16. **Local slope and height deviations of ramp. a)** The angular deviations δ from mean slope vs. position x on the ramp and **b)** the corresponding height deviations $y-y_0$ vs. position x. The height deviations are with respect to a perfectly flat ramp represented by the flat line $y-y_0=0$, and are calculated from the slope deviations as approximately $(y-y_0)(x) = -\int_0^x \delta(x')dx'$.

Appendix B - Contact Models

Our Hybrid simulations have instantaneous changes of state at the intermittent contacts. Further complicating the logical rules determining when to activate/deactivate constraints is the possibility of sliding phases during which there may be forces of sliding friction that may depend on constraint forces.

In the case of Coulomb sliding friction, the sliding friction coefficient μ is completely unknown to us and considered to be a free parameter. All simulations that included Coulomb friction indicated that this parameter has little effect until it is so small that the model cannot reasonably hop anymore (non-dimensional $\mu \approx 0.3$, similar to trying to hop on ice). The case of $\mu = \infty$ does not necessarily mean no sliding occurs; one context in which this limit is studied is described in [1].

We tried four different contact models, three of which have sliding phases. The sliding phases were very short, had little effect, and the motions of interest do not have the floor penetration problem at liftoff [2]. We found that including friction sliding phases tended to increase the maximum eigenvalue by a tiny amount, and overall had a negligible effect on periodic motions. For these reasons and to avoid adding unnecessary complexity to the simulations, for the work in this paper we used the 'simple plastic non-slip' contact model.

Here for only the simple plastic non-slip model, we summarize the rules that apply during transitions between contact and non-contact phases, and during sliding phases if they apply. The vanishing of distance between contact points initiates a collision; a collision impulse is applied, and constraints may be activated. The vanishing of the normal force f_n (defined as the normal force on the foot from the floor) can cause a discontinuation of contact (liftoff). The rules dictating the possible transitions for the simple plastic non-slip model are:

- collision: free $\rightarrow$ non-slip contact
  if the distance between objects becomes zero, apply an impulse that makes the relative velocity of the contact points zero, **then** activate tangential non-slip and normal non-penetration constraints
- liftoff: non-slip contact $\rightarrow$ free
  if the normal force vanishes, **then** relax non-slip contact constraint

Appendix C - The flat ground assumption

One assumption in the model was that the device hops on a tipped, but perfectly flat plane. With close inspection, we found that the ramp used in the lab is measurably not flat. At the time of the experiments, it seemed important to consider any non-ideal effects that may explain differences between theory and experiment. As it turns out, according to simulations the bumpy ramp does not significantly hinder the ability of Steinkamp hopper to hop over 4 meters.

To measure local slope variations, a laser pointer was attached to a 6cm steel block and placed on the ramp aimed at a fixed ruler about 10 meters from the ramp. The laser/block assembly was moved down the ramp at 5cm increments (approximately the same as the distance of one hop); the corresponding positions of the laser light on the ruler were recorded. This data, and simple geometry, yields a map of the deviations of the ramp height from a straight line (Figure 16b). Over the length of the ramp there was a mid-ramp sag of about 4 mm, with slope deviating from flat by up to .006 rad ($\approx 0.4°$). A change in slope affects the hopper at least two ways: 1) the device is gravitationally powered, so an increased average slope must be complimented by an increased speed and/or dissipation; and, 2) the exact timing of the foot impact depends on the

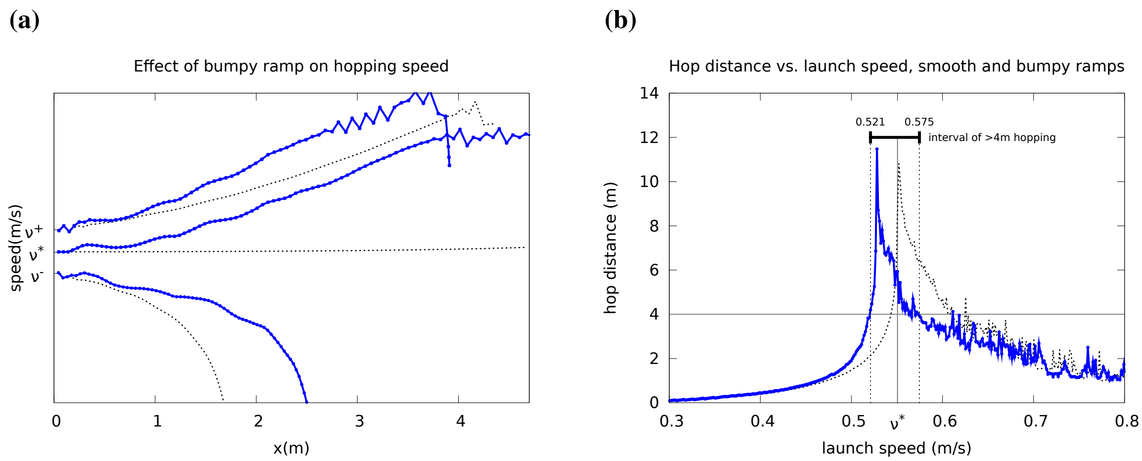

Fig. 17. **Bumpy ramp changes speed. a)** The speed of the simulated hopper with the smooth ramp (dotted line) and bumpy ramp (solid line) for three initial speeds: With too much initial speed (ν^+), the hopper speeds up, becomes erratic and falls forward. A speed on the periodic root ($\nu^* \approx 0.551\text{m/s}$) will theoretically hop forever on a perfect slope; the bumpy slope causes it to fall before hopping 5 meters. Without enough initial speed (ν^-), the hopper slows down and falls backwards. **Small effect on launch-ability. b)** The simulations with smooth slope (dotted line) and simulations with bumpy slope (solid line) both predict a hopping distance greater than 4 meters for a similarly sized interval of launch speeds. The bumpy simulation predicts a hopping distance greater than 4 meters for a range of launch speeds $0.521 < v_l < 0.575$; the magnitude of this interval is 0.054, somewhat but not substantially smaller than the interval for the smooth slope (compare to Figure 14). The stabilizing spring and damping coefficients are $k_l = 0.5$, $c_l = 0.5$; in each case (smooth, bumpy) the hopper is launched from the same fixed point of the held-launch system. The fixed point of the free smooth-sloped system has a speed $\nu^* \approx 0.551$. The bumpy simulation has a distance distribution with slightly higher peak and slightly smaller width than for the smooth simulation.

local surface height. We included these deviations in simulation by starting with initial state a fixed point associated with the average ramp slope, and integrating forwards for many steps with the measured slope as a function of position.

When we included the measured slope deviation in the simulation, the prediction of motion for a particular initial condition is significantly affected. This is expected, as the local slope deviations are up to 8% of the mean slope (see Figure 16a). However, the statistics of the system are relatively unchanged; as shown in Figure 17b the answer to the question "how big can the variation in the launch speed be while still hopping 4 meters?" changes only a little when the bumpy slope is used instead of the smooth one. The bumpy simulation predicts a hopping distance greater than 4 meters for a range of launch speeds $0.521 < v_l < 0.575$, an interval size of 0.054 compared to an interval size of 0.065 for the smooth simulation (Figure 14). Simulations also suggest that the force minimization strategy for launching will still succeed, yielding approximately 6 meters of hopping.

References

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