MEMORY VS MOMENTUM

EXPLORING MOMENTUM STRATEGIES WITH THE HURST EXPONENT

A Thesis

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Qisiyu Miao

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ABSTRACT

Momentum, the strategy of capitalizing on the ongoing trend in the stock price movements, has been one of the most puzzling market anomalies in modern finance. This paper seeks to exploit the momentum profitability from the perspective of the excursion patterns in the stock price movements.

A theoretical framework is developed for momentum strategy analysis and the long memory process in the financial markets. To test the null hypothesis of the Random Walk Hypothesis and the Efficient Market Hypothesis, we employ the Hurst exponent to detect the long-term memory existed in the stock return series. A time series with $0 < H < 0.5$ shows negative correlations between points and a mean-reverting behavior, while a series with $0.5 < H < 1$ indicates positive correlations and a long memory process.

Basic momentum strategies are further applied to the past stock price data, and the back testing results show that there is a U-shaped relationship between the strategy returns and Hurst exponent. This paper also builds on earlier model of a rule-based naïve trading strategy using Hurst exponent as a signal. The strategy generates remarkably higher profitability compared with the benchmark returns. These findings provide new evidence against the random walk assumption and present challenges to a number of rational asset pricing theories.

BIOGRAPHICAL SKETCH

Qisiyu Miao is a second-year master student at Cornell University's Charles H. Dyson School of Applied Economics and Management, where her research concentration is Applied Econometrics and Quantitative Analysis.

Prior to coming to the graduate school, Qisiyu double-majored in International Economics and English Literature at Fudan University, and spent a semester at University of California, Berkeley, as an exchange student.

After graduation, she will be joining the investment bank Morgan Stanley in New York as a sales and trading analyst under the Institutional Equity Division. Excited at the prospect of working on the trading floor, Qisiyu is looking forward to applying the analytical skills she has learned from her studies at Cornell to a real and dynamic business setting.

This thesis is dedicated to my mentor and advisor Professor Calum G. Turvey.

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CHAPTER 1 INTRODUCTION

The phrase "momentum strategy" was first introduced by William F. Hamilton and Michael A. Moses (1973) and made its debut in the setting of corporate financial planning, describing "continuation of present activities in current lines of business", as opposed to "development strategy", which denotes the proposal of changes in the nature or level of the existing business. Narasimhan Jegadeesh and Sheridan Titman (1993) officially coined the term and changed its meaning into what is known to us nowadays — the trading strategies that buy stocks that have performed well in the past and sell stocks that have performed poorly in the past over a 3 – 12 months holding period. In the paper, Jegadeesh and Titman found a surprising fact that the profitability of these strategies are not driven by increasing exposure to systematic risk or the delayed stock price reactions to common factors. Surprise is deepened when similar conclusions are drawn to the contrarian strategies that aim at going in the opposite direction of the prevailing market trends. The evidence on return predictability based on past trends is incompatible with the Efficient Market Hypothesis proposed by Fama (1991).

Despite of a rigorous debate ongoing for years, the interpretations to these market anomalies are ambiguous, and the factors attributing to the momentum profitability stay unclear. In this context, this paper attempts to explore the momentum gains with the long-term memory process existed in the stock price time series, and seeks to find the most salient characteristics that are shared by all of the propositions put forth to explain the momentum profitability by designing an investment strategy using the Hurst exponent as a trading signal.

1.1 Background

1.1.1 Efficient Market Hypothesis And Random Walk

In forecasting the financial time series, the first question that investors encounter is whether a time series data is predicable. If the time series is completely random, there is no hope to predict the prices for the next day. However, if the stock prices exhibit a long memory, this suggests that predictability can be pursued, and speculators could exploit the trend in stocks to increase their portfolio returns.

The efficient market hypothesis (EMH), espoused by Eugene Fama (1970) in his survey paper “Efficient Capital Markets”, states that the expected return of the capital markets is “a fair game”. Prices in an efficient market should fully reflect any information available about the individual stock as well as the stock market as a whole at the moment. Thus, neither fundamental analysis, which feeds on the market short-term mispricing on a company’s financials in order to pick the “undervalued” stocks, nor technical analysis, which studies the stock’s past price movement in an attempt to predict its future trend, would enable the investor to generate a much higher return than those that could be obtained by holding a randomly selected portfolio of individual stocks with comparable risk.

EMH can be broken down into three forms: in the strong form of market efficiency, individual investors or groups have monopolistic access to any information relevant for price formation. In a less restrictive semi-strong form of efficiency, all public information is available; while in the weak form of efficiency, the information available to investors include only the historical prices and return sequences.

The EMH is closely related to the random walk hypothesis (RWH) developed by Malkiel (1970). A random walk is used to characterize a price series where all subsequent price changes are independent of the past price changes. While the market does exhibit some momentum from time to time, it does not occur dependably and there is not enough persistence in stock prices to overwhelm the substantial transactions costs involved in undertaking trend-following strategies. No one has consistently outperformed the placebo of a buy-and-hold strategy. Technical analysis cannot be used to make useful investment strategies. These are the fundamental conclusions of the RWH.

1.1.2 Momentum Gains and Contrarian Profits

However, the empirical results of a number of studies show that the stock price movements do not follow a random walk. Contrary to the RWH, momentum gains and contrarian profits defy the defining property of the RWH, which is the non-existence of correlations of its increments, and suggest that price changes are predictable to some degree. There is an extensive body of literature in the field of momentum strategy that has put forward a large number of explanations to account for the momentum profits.

In the momentum gains analysis (Jegadeesh and Titman 1993), three strategies that buy past winners and sell past losers realize significant greater returns over the time period of 1965 to 1989. For example, in one of the strategies, stocks are selected based on their past 6-month returns and are tested on a holding period of 6 months. A compounded excess annual return of 12.01% is achieved by the strategy. There is strong evidence indicates that the profitability of momentum strategies cannot be attributed to the exposure to systematic risk, nor the lead-lag effects that result from a delayed stock price reactions to common factors. Another example is the earning momentum strategy. Before a firm announces its quarterly earnings, general market or investment banking analysts will form general opinions or estimations towards the actual earnings. After the firm announces earnings, there will be, no matter large or small, a difference from what is estimated and the true value. The return of winners portfolio consists of stocks with positive earning surprises achieved a significantly higher returns than that of the losers portfolio made up of stocks with negative earning surprises.

Comparing with the momentum strategy, a contrarian investing strategy is characterized by purchasing past losers and selling past winners in belief of a negative serial correlation and stock return reversals. Studies (Jegadeesh and Titman 1995, Lo and McKinlay 1990) have found that stock price overreaction to firm-wide information, instead of the lead-lag effects, contributes the most to the contrarian profitability. They also find that there exists a negative serial dependence in individual stock returns while a positive correlation in market indexes, which implies that stock market overreaction might not solely contributes to the contrarian profits, yet the cross effects among different individual stocks account for most of the profit.

Momentum profits suggest that information from past trading data affecting the drift in stock returns in different time length, which is in contrary to the EMH. At the fundamental level, it rejects the simplified view of stock prices evolving over time following a random walk. However, despite of economists' effort to explain the reasons behind the momentum profitability, the evidence of the studies does not allow us to distinguish the positive feedbacks from the negative ones. Henceforth a more fundamental model is needed to explain the observed pattern of returns.

1.1.3 Hurst Exponent

In 1951, the British hydrologist H.E. Hurst was looking for a way to model the levels of the river Nile so that architects could construct an appropriately sized reservoir system. In order to measure the long-term memory of a time series, the Hurst exponent, which relates to the autocorrelations of the time series, and its decreasing speed as lag increases, was introduced by him. Hurst also devised the rescaled range method to calculate the Hurst exponent. As a result, Hurst found that the H value of the Nile River is close to 0.73, which is higher than the $H = 0.5$ that one would expect from independent observations and short autocorrelations.

The application of long memory process, also known as long-range dependency (LCD), has been extended from hydrology, network traffic to the financial markets. The noted mathematician Benoit Mandelbrot has rediscovered and developed Hurst's rescaled range analysis, and applied it to the investigation of the fractal nature of the financial markets. An alternative approach called scaled variance ratio method is explored by Lo (1991) to study the long memory process in the stock returns.

Generally speaking, a stock price series with $H = 0.5$ indicates complete randomness with uncorrelated increments over time. Whereas, in cases of $0 < H < 0.5$ and $0.5 < H < 1$, the time series show long-range dependency. A H value between 0 and 0.5 is indicative of a short-term memory process with a mean-reversion behavior, exhibiting a frequent switching between high and low values. Alternatively, a Hurst value above 0.5 indicates a long memory, and positive autocorrelations between points in the series. In such series, a high value is highly probable to be followed by another high value in the future.

The presence of long-term memory components in stock returns has import implications for making portfolio decisions in the optimal time horizon. Problems also arise in derivative pricing as most employ the martingale methods. If the financial time series shows a long memory, it will not have a normal or log normal distribution. Henceforth traditional tests of the capital asset pricing model and the arbitrage pricing model are no long valid due to the reason that most of the statistical inference do not applying to such time series.

The employment of the Hurst exponent offers a well-established perspective to uncover the anomalous returns of the momentum strategies. In order to test the null of a random walk, we attempt to employ the Hurst exponent as a statistical technique to assess the long memory processes exist in the time series. This information is important as it can be used as a confirming tool to identify the market predictability. Stocks with H values that further deviate from 0.5 would look favorable for applying momentum strategies. Our empirical back testing results confirmed that the notable deviation from a normal distribution is sufficient to sustain a profitable trading strategy. Building on the prior models of a momentum strategy using the Hurst exponent as a trading signal, we construct a moving average Hurst crossover strategy and back test it on the past stock price series. When the H value of the shorter term moving average is smaller then 0.5 and is below the longer term moving average Hurst, it is interpreted as a signal of mean-reverting behavior. Alternatively, when the shorter term Hurst is above that of a longer term moving average, the stock price series is exhibiting a more persistent trend. A buy signal is triggered by a persistent trend combined with a positive return, vice versa. The excess returns the moving average Hurst crossover strategy produces implies the long memory process in the stock market prices is existent, and could be exploitable by the momentum strategies.

1.1.4 Objectives

This paper aims to show that the long memory of the stock price movements is exploitable by the momentum strategies. In addition, we also seek to find the most salient characteristics that are shared by all of the propositions put forth to explain the momentum profitability. We apply the basic momentum strategies to the stocks in different indexes and compare their performance with the Hurst exponent. Furthermore, we design a new momentum strategy by incorporating the use of Hurst exponent as a trading signal, and run the back test. The result will unify what has previously deemed to be mutually exclusive and provides insights to stock market returns.

1.1.5 Organization Of The Thesis

The paper is structured as follows. Chapter 2 presents a literature review on momentum strategies and Hurst exponent. Chapter 3 provides an overview of the theoretical model framework and modification from previous model. Chapter 4 discusses the dataset and methodology used in the study. Chapter 5 presents the back-testing results and limitations, and Chapter 6 concludes and gives remarks on potential future research.

CHAPTER 2 LITERATURE REVIEW

Trading strategies that exploit the momentums in the stock price movements by either buying past winners and selling past losers, or going in the opposite direction of the market trends constitutes a distinct and systematic investment style in the international stock markets. The momentum strategies have long been implemented by professional traders and investors, and predate the scientific evidence that support the ideas.

It has been shown that strategies, which feed either on past stock returns or on earning surprises, could help predict future prices and generate abnormal returns. Study of Lehmann (1990) shows contrarian strategies that select stocks based on their returns in the previous week or month generate significant abnormal returns. In Brock, Lakonishok and LeBaron (1992), two of the simplest and most popular trading rules, moving average crossover divergence (MACD) and trading range breaks, are tested. A simple moving average could help smooth out daily price fluctuations, to determine how far the current price has moved from the trend. Complementary to the moving average and relative strength system, the use of stochastic oscillator (SO) as an indicator to detect if an asset is overbought or oversold, prevails. Debondt and Thaler (1985) document that long-term past losers outperform long-term past winners over the subsequent three to five years. Jagadeesh (1990) and Lehmann (1990) report short-term return reversals. Jagadeesh and Titman (1993) find a twist to the literature by documenting that past winners on average keep outperforming past losers over a time horizon of three to twelve months. Though the process has entailed various indicators and trading systems, there has been a lack of unifying theoretical structure in explaining the momentum.

The momentum strategy, which asserts the predictability of stock prices, contradicts the EMH proposed by Fama (1991). This fuels a heated debate on market efficiency. Fama and French (1996) try to rationalize these "market anomalies" by acknowledging an additional state variable in the CAPM[1]. However, the model fails to explain the profitability of the momentum strategy of buying past winners and selling past losers (Jegadeesh and Titman 1993). Accordingly, there exist a handful of testable hypothesis in regards to the momentum profits while a woeful shortage of solid explanations.

Many papers look at two sources for the predictability of future stock returns. One is the stock past returns. The other relates to the earning releases. Price momentum compares the stock's past (6 month) compound return with ex post returns, while earnings momentum include measures such as standardized unexpected earnings (SUE), cumulative abnormal return around the most recent earnings announcement date, and revisions of analysts' forecasts of earnings. Price momentum relates positively to portfolios' book-to-market and cash-flow-to-price ratios. Earnings performance, abnormal announcement returns, and revisions in analysts' forecasts help to explain price momentum. The results suggest that stock price momentum partially reflects slow adjustment to information about earnings. Stocks with large favorable earnings announcements subsequently tend to outperform those with unfavorable announcements. For instance, the studies of Latane and Jones (1979), Bernard, Thomas and Wahlen (1995) indicate that firms reporting unexpectedly high quarterly earnings outperform

[1] The main difference between ICAPM and CAPM is the additional state variable that accounts for investors hedging against shortfalls in consumption or against changes in the future investment opportunity set.

firms reporting unexpected poor quarterly earning results. The superior performance could persist over a period of about six months after the earning announcement. The portfolio based on standardized unexpected earnings (SUEs) also exhibits excess returns. Givoly and Lakonishok (1979) examine the abnormal returns during the months surrounding the revisions in analysts' forecasts and point out that the sluggish market reaction to the disclosure of analysts' forecasts gives rise to potential abnormal returns to investors who act upon this type of publicly available information. DeLong, Shleifer, Summers and Waldmann (1990) assess the prevalence of positive feedback effect from investors. Positive feedback investors buy stocks when prices go up and sell when prices fall. The type of momentum investment strategy is also referred to as positive feedback trading. An implication of the positive feedback model is that positive-feedback trading causes market overreaction. The price increase in response to good news is greater than the news warrants and would last for a period of time. The price overreaction would eventually lead to subsequent reversals.

In spite of the economists' continuous effort in providing various models to explain the momentum profits, there is a lack of underlying theoretical basis that unifies all these assumptions. Peters (1996) first proposes that fractal market analysis could deliver a robust tool for understanding what the traditional linear models are insufficient in explaining the essence of the capital markets. This tool that employed to assess the fractal structure of the financial markets is Hurst coefficient. The Hurst exponent was first introduced by Hurst (1951) as a technical tool for the quantification of long-term memory in the studies of long-range memory of the Nile River. For independent Gaussian processes the value of the Hurst exponent (H) = 0.5, reflecting the elimination of short-term autocorrelations. Whereas a value of H in range of 0 — 0.5 indicates anti-persistency. If the observation was high in the previous period, it is likely to regress to the mean in the future. The strength of the mean reverting behavior increases as Hurst approaches to zero. When H is above 0.5, the series displays an upward or downward trend. The power of the trend-reinforcing behavior increases as the value of the Hurst increase to 1. When examining the Nile River overflow, Hurst found the H value was around 0.91, which implies the natural system does not follow a random walk.

Mandelbrot (1977) amplifies the theory by developing the R/S analysis, which is then applied to assess the bias in the financial time series. Bias is caused by the non-linear reaction of the series to exogenous factors. If the time series has a fractal distribution it will not have a normal or log-normal distribution, and by definition not follow a true random walk. Based on these, Peters (1996) introduced the Fractal Markets Hypothesis (FMH). He argues that time or memory effects may be investigated by determining whether a system displays a fractal structure. Examination of fractal structures can also be used to determine whether the system displays an underlying trend, and the strength of the trend may be measured by the degree of persistence in the series.

There is an extensive body of literature about on this topic in the financial world. Most of them concluded finding evidence of long memory in financial time series data. For example, while Lo (1991) and Ambrose et al. (1993) deny the presence of long memory in U.S. stock returns, DiSario et al. (2008) support its existence. Lipka and Los (2002) found long memory in the daily returns of eight European stock indices with FTSE displays an ultra-efficient market with abnormally rapid mean-reversion. Corazza and Malliaris (2002) show that the H value of several currency markets including USD, GBP, CAD and JPY is significantly different from 0.5. Cajueiro and Tabak (2004) adopted a rolling sample approach and tested for long memory in stock indices for 11 emerging markets. Their results show that the Asian stock markets is the most inefficient. Kyaw et al. (2006) measures the degree of long-term dependence in Latin American financial markets. All of the studies used the returns series on stock indices, which entails a great deal of aggregation.

CHAPTER 3 THEORETICAL MODELS

We begin with the null hypothesis that stock price series follows a random walk. One important property of the random walk is that the variance of the increments is linear in the observation interval. Contrary to the null, we propose that the fractional Brownian motion can be appropriate for stock prices modeling. In this chapter, we give a basis for our hypotheses and further discuss our choice of theoretical models to assess the long-term memory in the stock price series.

3.1 Null Hypothesis

We begin with the null hypothesis H_0 of a random walk, which states that the increments are iid (identically and independently distributed)). This discrete-time process can be obtained by taking the value at equally spaced intervals of a continuous-time Gaussian process:

$$dx_t = \mu dt + \sigma dZ \qquad (1)$$

Where $dZ = \varepsilon\sqrt{t}$ is a Gauss-Wiener proce. The solution to this stochastic differential equation corresponds to the lognormal diffusion price process.

One important property of the random walk x_t is that there is linearity in the variance at equal intervals. For instance, the variance of $x_t - x_{t-2}$ is twice the variance of $x_t - x_{t-1}$; and the the variance of $x_t - x_{t-2}$ is half of the variance of $x_t - x_{t-4}$. However, to allow for the detection of long memory process in the stock price series, we employ a scaling factor $\frac{1}{\alpha}$ to the variance ratio. Further details would be discussed in the scaled variance ratio method.

3.2 The Classical And Modified R/S Models

The measure to detect the long memory and estimate its characteristic Hurst exponent is essential to our study. A number of methods have been proposed and used in literature, including the classical and modified R/S methods, spectral analysis, and maximum likelihood, etc. Each of the methods has its own advantages and disadvantages. Hurst defined the first method to use a single number, as known as self-similarity coefficient or scaling exponent, to characterize the correlations between points in a time series. In practice, Mandelbrot and Wallis (1969) developed the classical rescaled range (R/S) method by incorporating ordinary least squares regression techniques. The classical rescaled adjusted range statistic measures the standardized range of the partial sum of deviations of a time series from its mean.

$$\frac{R}{S}(n) = \frac{1}{S(n)}[\max_{0\le t\le n}\left(Y(n) - \frac{t}{n}Y(n)\right) - \min_{o\le t\le n}\left(Y(n) - \frac{t}{n}Y(n)\right)], n \ge 1$$

(2)

In practice, the method computes the R/S-statistic in Equation 2 at many different lags n and for a number of different points. The functional relationship over intervals of lengths n is:

$$\mathrm{E}[\frac{\mathrm{R}}{\mathrm{S}}(\mathrm{n})] = \mathrm{c}_1\mathrm{n}^{\mathrm{H}} \text{ as } \mathrm{n} \to \infty \qquad (3)$$

The equation for the best fitting line is determined most simply by using the linear Y on X regression with Y = log(R/S) and X = log(n):

$$\log\left(\frac{R}{S}(n)\right) = \log(c_1) + H\log(n) \qquad (4)$$

By plotting the resulting estimates versus the lags on log, it yields an estimate of the Hurst parameter via the slope of the resulting plot. A straight line indicates there is a self-similar correlation at some scale. A Hurst coefficient larger than 0.5 means positive correlations between points in the time series while a Hurst smaller than 0.5 means negative correlations, or anti-persistence.

However, one often finds that the classical R/S analysis is not that reliable in practice. At its core, the R/S analysis is a linear regression slope that is determined using a series of logarithmic data points. A larger number of data points will generally give a better reading. Data points of shorter and longer intervals at the two ends tend to be scattered below the line, which indicates that the relationship of the R/S statistics and n is concave downward. The slope is artificially too high for short intervals, presumably because observations are taken at unnecessarily short intervals and an overestimated degree of correlation is therefore obtained. This slope is also too low at long intervals because there is a negative correlation between the mean of large segments when near neighbors are positively correlated (Bassingthwaighte and Raymond 1994).

Another shortcoming of the classical R/S method is its sensitivity to the presence of short-range dependency. Unlike the long-range memory, the short-range dependency means that the dependence of two events eventually diminishes as time elapses. Lo (1989) proposed a modified R/S method using $S_q(N)$, a weighted sum of autocovariance, instead of $S\ (N)$, and the sample standard deviation, to compensate for the presence of short-range dependency.

$$S_q(N) = \frac{1}{N}[\sum_{j=1}^{N}(x_j - \overline{x_N})^2 - \frac{2}{N}\sum_{j=1}^{q}\omega_j(q)[\sum_{i=j+1}^{N}(x_j - \overline{x_N})(x_{i-j} - \overline{x_N})^{1/2}]$$

(5)

$$\omega_j(q) = 1 - \frac{j}{q+1}, q < N \qquad (6)$$

Unfortunately, in practice, the modified R/S method may not be that attractive since it is difficult to determine the right choice of q or the range of q-values as truncation lag(s). A small value of q is not likely to account for all the extra short-range dependency while a large q would over-compensate for the long-range dependency existed in the time series data. Trial and error method waiting for the statistics with respect to q to stabilize does not seem to work either as it only becomes stable within the acceptance region for the null hypothesis.

3.3 The Scaled Variance Model

In this paper, we adopt the scaled variance ratio model, which is distinct yet related closely to the Rescaled Range (R/S) model. Lo and MacKinlay (1988) initiated the conventional variance ratio test based on the property that the variance of increments of a Brownian motion is a linear combination of non-overlapping increments. The derivation of the method is explained in details. The stochastic differential equation of the Brownian motion is of the form:

$$dx = \mu x dt + x\sigma dZ \qquad (7)$$

Where $dZ = \varepsilon\sqrt{t}$ is a Gauss-Wiener process, x is the price of the underlying, μ is the expected rate of return, σ is the volatility.

$$dx = x_t - x_{t-1} \qquad (8)$$

$$E[dx] = E[x_t - x_{t-1}] = \mu \qquad (9)$$

In random walk, increments over non-overlapping time intervals $(x_t - x_{t-1}), (x_{t+1} - x_t) \ldots (x_{t+k} - x_t)$ are independent. Each increment has expected value μ. If instead of taking the difference of the prices on a daily basis, we measure the difference over k-day and get:

$$dx = x_{t+k} - x_t \qquad (10)$$

$$E[dx] = E[x_{t+k} - x_t] = E[x_{t+k} - x_{t+k-1} + x_{t+k-1} - \cdots - x_{t+1} + x_{t+1} - x_t]$$

$$= E[x_{t+k} - x_{t+k-1}] + E[x_{t+k-1} - x_{t+k-2}] + \cdots + E[x_{t+1} - x_t] = k\mu \qquad (11)$$

The expected return over k-step is $k\mu$. Then the k-step variance under the no-correlation Markov assumption is:

$$VAR[x_{t+k} - x_t] = E\big[x_{t+k} - x_t - E[x_{t+k} - x_t]\big]^2 = E\big[(x_{t+k} - x_t) - kE[\mu]\big]^2$$

$$= E\big[(x_{t+k} - x_{t+k-1}) + (x_{t+k-1} - x_{t+k-2}) + \cdots + (x_{t+1} - x_t) - kE[\mu]\big]^2$$

$$= k\sigma^2 \qquad (12)$$

If we divide (4) by 1-step variance, we get:

$$\frac{VAR[x_{t+k} - x_t]}{VAR[x_{t+1} - x_t]} = k \qquad (13)$$

In a stationary differenced Gaussian-Markov process, the k-step variance equals to the product of k and 1-step variance. This linearity in the variance of the Brownian motion corresponds to the condition of the SDE: $\mathrm{Var}[dZ] = \mathrm{E}[dZ^2] = \sigma^2 t, t \equiv k, \forall k$.

Mandelbrot was concerned about the leptokurtosis observed in the stock return distribution, and proposed that the shape of the distributions are determined by the scaling properties of a power law[2]. Mandelbrot originally assumed a scaling of $\frac{1}{\alpha}$ and then

[2] A scaling function satisfies certain homogeneity relations of the form $f(s) = \lambda^{\alpha} f(s)$ which encompasses self-similar as well as self-affine scaling behavior (Weissel et al 1994, Feder 1988, Mandelbrot 1968). We are interested in the scaling function $f(T) = \sigma^2 T^{2H}$, where T is a time measure and H is the Hurst exponent.

in Mandelbrot and Van Ness (1969) extended this to the Hurst factor $\alpha = 2$ for a Gaussian process, equivalently, $H = \frac{1}{\alpha} = \frac{1}{2}$.

The SDE of a fractional Brownian motion is written as:

$$dx = \mu x dt + x\sigma dW_H \qquad (14)$$

Where $dW_H = \varepsilon\sqrt{t^{2H}}$.

Plug in Equation 13, we have:

$$\frac{VAR[x_{t+k} - x_t]}{VAR[x_{t+1} - x_t]} = k^{\frac{2}{\alpha}} = k^{2H} \qquad (15)$$

When $H \neq \frac{1}{2}$, the process is a fractional Brownian motion. In the fractional Brownian motion, the covariance over k step is equal to:

$$E(x[t] - x[0])[x(t + \mathrm{k}) - x(t)] = \frac{1}{2}\sigma^2([t + \mathrm{k}]^{2H} - t^{2H} - \mathrm{k}^{2H}) \quad (16)$$

The variance over k is equal to:

$$E[x(t + \mathrm{k}) - x(t)]^2 = \sigma^2(k)^{2H} \qquad (17)$$

When $H = 0.5$, according to Equation 17, the covariance of different time steps is equal to zero, which satisfies for a geometric Brownian motion, and equation reduces to the standard geometric Brownian motion with variance that is linear in time. As $H \to 1$, covariance approaches $\sigma^2 tk > 0$, and variance $\sigma^2 k^2$increases over time. As $H < 0.5$, the covariance decreases as time steps increases.

We take the logarithm of the both sides in Equation 17 and get:

$$H = \frac{1}{2}\ln\left(\frac{VAR[x_{t+k} - x_t]}{VAR[x_{t+1} - x_t]}\right) \qquad (18)$$

The method then obtains an estimated Hurst coefficient.

3.4 Algorithm

The following steps are carried out for different window sizes:

1. Obtain the time series data and suppose x_t is the stock price at time t. Calculate the percentage change in the natural logarithm of prices $ln(x_{t+k}) - ln(x_t)$ for lag k = 1, 2, 3…n, (n equals to the square root of sample size).

2. Calculate the variance of the sub-series $VAR(ln(x_{t+k}) - ln(x_t))$ for each step k using the formula:

$$VAR = \sum_{t=1}^{n}\left(f - f_{average}\right)^2/(n-1) \qquad (19)$$

3. Compute the ${\sigma_{t+k}}^2/{\sigma_t}^2$ over the range of k.

4. Calculate the least squares linear regressions of $\ln\left(\frac{{\sigma_{t+k}}^2}{{\sigma_t}^2}\right)$ versus $\ln k$. The slope of the regression line times ½ is the estimated Hurst coefficient.

It is worth noting that when generating subseries with different lags, we adopt the overlapping prices (k=1, 2, 3…n). The decision to employ overlapping versus contiguous subseries in the computation has been the subject of debate among statisticians. Recent studies (Ellis 2006) have shown that an overlapping subseries is preferred for the R/S analysis as the use of contiguous series with shorter length would yield to more biased exponent estimates. As a general rule, small size of k may not be able to capture the self-similarity attributes of a time series. When the points generated from different subseries are relatively large, the slope of the linear regression line is greater and bias is reduced. Henceforth we expect the same for the calculation of scaled variance series.

We take 1024 observations from 2011/1/1 – 2016/1/1 of each stock to compute its Hurst value. Table 1 in the appendix provides the estimated Hurst value of each stock in the four primary stock indexes. The figures below show the distribution of Hurst exponent in different stock indexes. Give the Hurst coefficients calculated using the scaled variance ratio method, 95% of the values fall into the range of 0.43 – 0.47, which could be seen as consistent with a random walk.

Figure 1 Hurst Exponent Distributions In S&P 500 Stocks

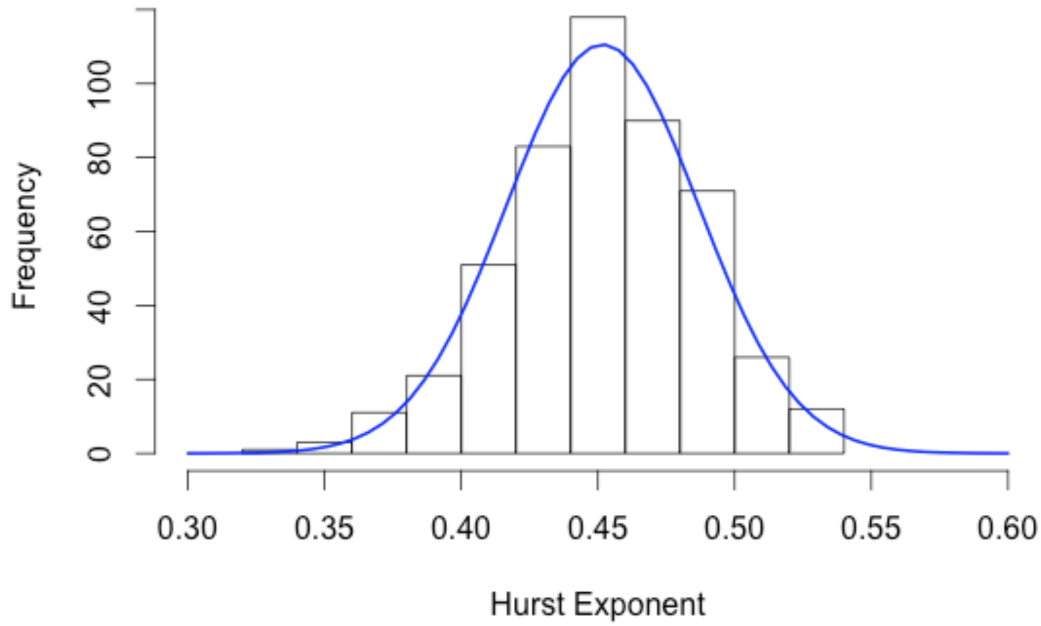


Figure 2 Hurst Exponent Distributions In NASDAQ Stocks

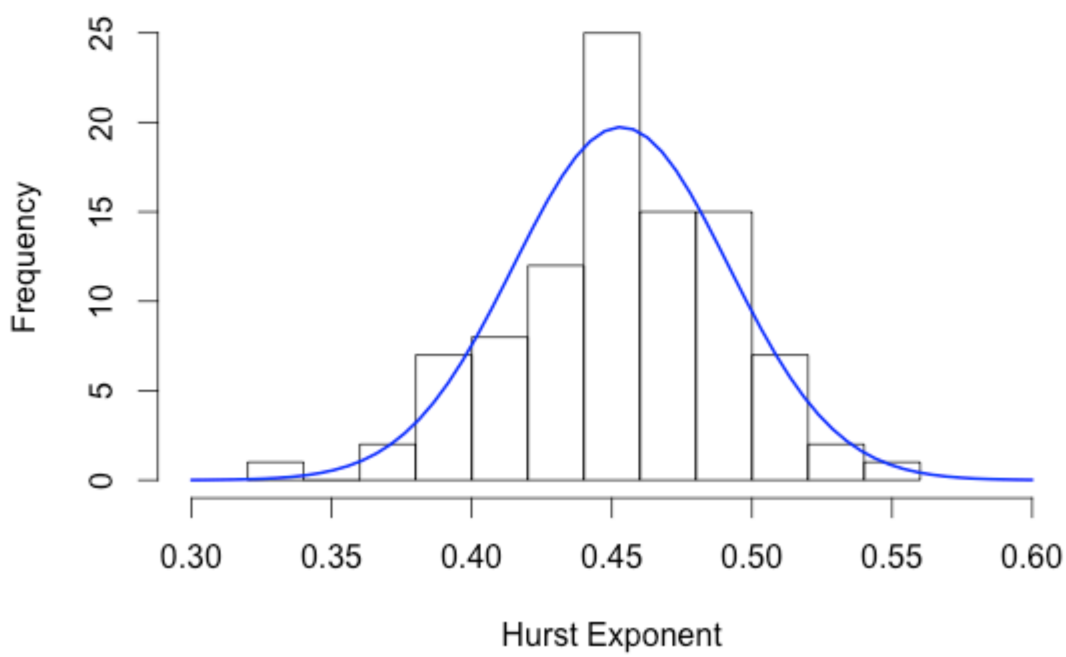


Figure 3 Hurst Exponent Distributions In Dow Jones Stocks

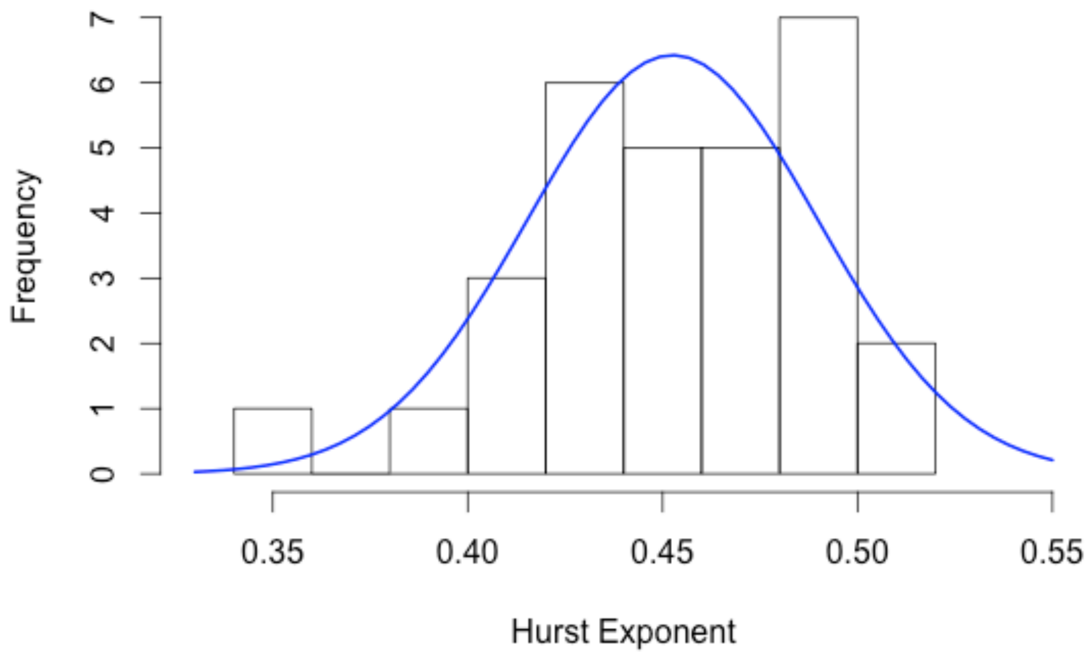
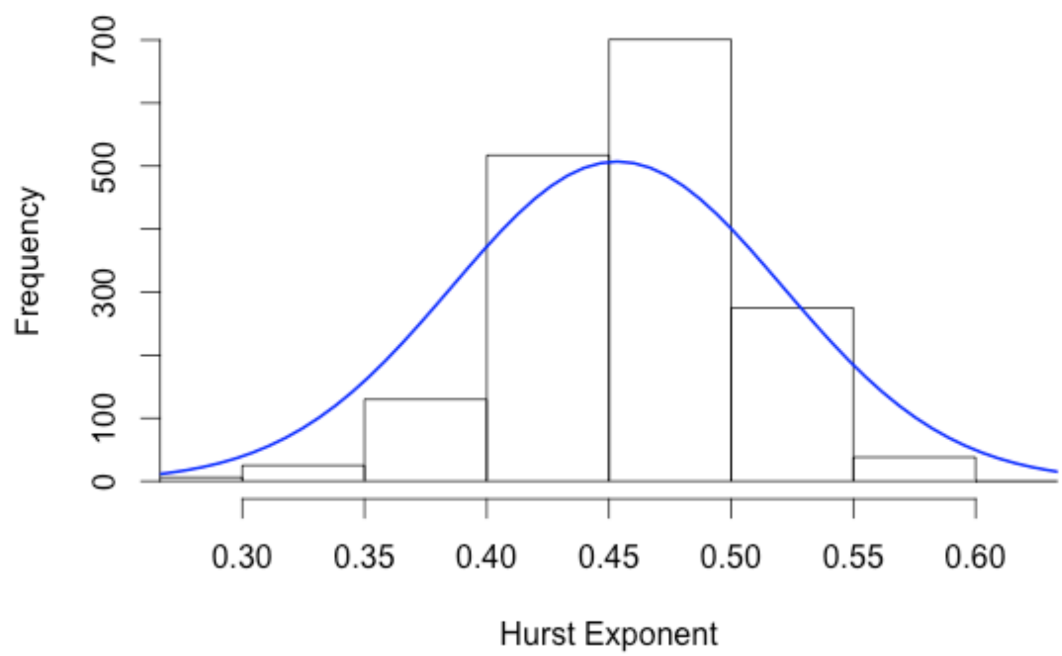


Figure 4 Hurst Exponent Distributions In Russell 2000 Stocks



CHAPTER 4 SAMPLE AND METHODOLOGY

The objectives of this chapter are to: (1) describe the sample data selection, (2) establish the theoretical framework and evaluate the validity of the method, (3) design the testing instrument and apply the procedure to analyzing the data set.

The time series data used in the study includes daily close prices of primary stocks during 2008/1/1 – 2016/1/1. We adopt the scaled variance ratio model[3] to estimate the Hurst exponents of the stocks. Then we apply different momentum trading strategies to groups of stocks in different Hurst ranges and compare their performances. Lastly, we build a new trading algorithm based on the Hurst exponent and justify the model via back testing.

4.1 Sample Data Description

The sample data used in this paper include daily closing prices of the constituent stocks of primary indexes, including S&P 500, NASDAQ, DJIA and RUSSELL2000, covering a period of 2008/1/1 – 2016/1/1.

Since the indexes periodically adjust their constituents to meet the minimal requirements of market-cap, market exposure, liquidity, etc., we adopt the constituents' list as of 2016/1/1, and exclude a few stocks that are not trading on those days.

[3] See Lo (1991) and Cannon et al. (1997) for detailed mathematical proof, and Turvey (2007) for the application of overlapping scaled variance ratio method to agricultural commodity prices.

The daily closing prices of the above were collected from Yahoo Finance, and algorithms are written in R and Python. Back testing was implemented in an interactive development environment of Python called Quantopian.

The database in Quantopian includes all stocks and ETFs that traded since 2008, even ones that are no longer traded. This is very important because it helps avoid survivorship bias in the algorithm. Databases that omit securities that are no longer traded ignore bankruptcies and other important events, and lead to false optimism about an algorithm. For example, "LEH" (Lehman Brothers) is a security in which an algorithm can trade in 2008, even though the company no longer exists in 2014; Lehman's bankruptcy was a major event that affected many algorithms at the time.

4.2 Basic Momentum Trading Strategies

Technical analysis is a common tool for devising market indicators in an effort to forecast the asset prices over short horizons. An extensive body of literature provides evidence that even simple trading rules could generate profits that are both economically and statistically significant. Our study seeks to establish whether the presence of long memory in stock returns is exploitable by the basic momentum strategies. Theoretically speaking, the dispersion of the returns generated by a Gaussian process scales to $\mathrm{H} = 0.5$. When the value of Hurst lies between $0.5 < \mathrm{H} < 1$, it suggests that the returns are positively correlated, and there is a trend following characteristics in the time series. The persistency of the trend increases until Hurst value reaches the ceiling at 1. On the other hand, when the value of Hurst lies within the range of $0 < \mathrm{H} < 0.5$, this denotes that the series reverses itself more than a random walk, and is anti-persistent. Henceforth, when we apply the classical mean-reversion trading strategies, the stocks of the lower Hurst values should generate higher return than that of higher Hurst values. Mean-reversion strategies involve going short the stocks that outperformed in the recent past and going long the underperforming stocks. Three basic mean-reversion strategies were carried out for this study.

4.2.1 Relative Strength Strategy

The relative strength portfolios are constructed as in previous studies (Jegadeesh and Titman 1993, De Bondt and Thaler 1985, 1987). The two-period relative strength strategy is a mean-reversion trading strategy designed to buy or sell securities after a corrective period. The literature has documented that stocks with poor performance over the past 3 to 5 years earned higher average returns than those performed well in the past. The argument is made that characteristics of investor behavior generate a certain inertia or "momentum" in abnormal returns that creates mean-reversion arbitrage opportunity.

A relative strength index (RSI) is calculated using a fairly simple formula as shown per below:

$$RSI = 100 - 100 / (1+ RS) \qquad (20)$$

Where RS* = the mean of 14 days up closes / the mean of 14 days down closes).

The indicator compares the number of days that the stock finishes up with the number of days that it finishes down. The time span is usually between 9 and 15 days. Here we choose a commonly used 14 days time horizon. The relative strength is calculated as the ratio of number of up days by the number of down days. This ratio is then added to one and the result is divided by 100. This number is subtracted from 100 to make the RSI a range between 0 and 100.

We are looking for buying opportunities when two-period RSI moves below 10, which is considered deeply oversold. In other words, the more short-term oversold the security, the greater the subsequent returns on a long position. Conversely, we look for short-selling opportunities when two-period RSI moves above 90. These traditional levels can also be adjusted if necessary to better fit the security. This is consistent with the idea that transactions by short-term momentum traders temporarily move prices away from long-term equilibrium, eventually causing prices to overreact. Once the overreaction is acknowledged, the market will enter a correction.

Table 1 Trading Signals of Relative Strength Strategy

RS Buy Signal	14-day RSI < 10
RS Sell Signal	14-day RSI > 90

4.2.2 Stochastic Oscillator Strategy

The stochastic oscillator is a momentum indicator developed by Dr. George Lane in the late 1950s. This index attempts to predict the price turning point by comparing today's closing price to its price range. The formula for calculating the indicator is as per below:

$$K = (C - L14) / (H14 - L14) *100 \qquad (21)$$

Where C = closing price, L14 = lowest price of the previous 14 days, H 14 = highest price of the previous 14 days.

The lookback horizon for the stochastic oscillator is 14 periods, which can be days, weeks, months or even minutes in an intraday timeframe. A shorter look-back period will produce a choppy oscillator with many overbought and oversold readings. A longer look-back period will provide a smoother oscillator with fewer overbought and oversold readings. We prefer a 14-day timeframe in coherent to the setting of other basic momentum strategies in our study. A 14-day "K" uses the most recent close price, the highest price over the last 14 days and the lowest price over the same period. For example, assume that the highest high equals 110, the lowest low equals 100 and the close equals 108. The high-low range is 10, which is the denominator in the "K" formula. The close less the lowest low equals 8, which is the numerator. 8 divided by 10 equals .80 or 80%. The Stochastic Oscillator is above 50 when the close is in the upper half of the range and below 50 when the close is in the lower half.

Establishing a short-term trading bias with a long-term indicator is a recurring theme for trading strategies. Traders employ the stochastic oscillator to decide the trading bias. The Stochastic Oscillator moves between zero and one hundred, which makes 50 the centerline. Low readings of the index below 20 indicate that price is near its low for the given time period. High readings of the index above 70 indicate that price is near its high for the given time period. The Stochastic Oscillator makes it easy to identify overbought and oversold levels. Closing levels that are consistently at the top of the range indicate a buying pressure, vice versa. We look to sell the stock when the index is above 70 and buy the stock when it goes under 20. Trading at the turning points allows higher chances of making profits.

4.2.3 Moving Average Crossover Strategy

The moving average crossover strategy employs two moving averages of different length of timeframes, or lags, to identify the direction of the price movement. In the simplest form, a moving average crossover rule operates on the assumption that the buy signals emerge when the stock price falls below its moving average, while the sell signals are generated when the stock price rises above its moving average. The strategy is implemented by subtracting the long moving average from the short moving average. In a word, MACD = MA (12) – MA (26). If the number is below zero, and a 5-day moving average return is positive, the stock price is trending downward, and it's time to buy. However, if the number is greater than zero, and a 5-day moving average return is positive, the stock price is trending upward, and it's time to sell.

There are numerous variations of this crossover rule of long moving average and short moving average. For instance, we can impose some form of filters to weed out the false signals. A commonly used one is a fixed percentage band filter to buy only when the buy signal exceeds the moving average by a fixed percentage, for example 2%, and sell only when the sell signal falls below the moving average by a specified percentage. Such filters are employed to confirm that a trend indeed exists before one initiates a trade based on it, and hence minimizing the effects of the noise in the price movement.

The ability of a moving average to reveal an underlying price trend depends on the window length used in computing the moving average. The longer the window period, the smoother the resulting price trend. However, it's worth noting that a long moving average does not respond as quickly to new trend reversals as a short moving average.

4.3 Moving Average Hurst Crossover Strategy

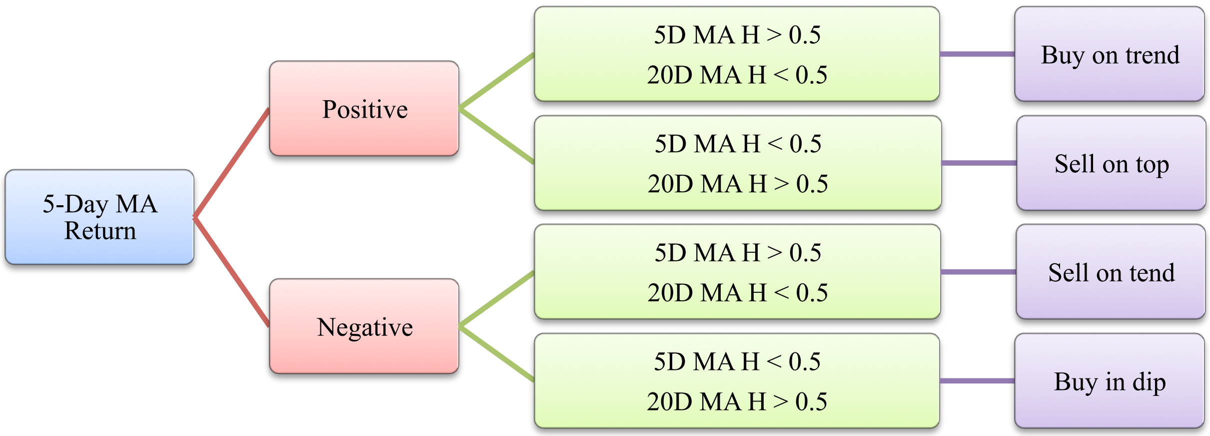
Inspired by the moving average crossover strategy and Paitoon's paper on the fractional momentum and Hurst coefficient (2012), we design a momentum strategy that uses the crossover of long-term and short-term moving average Hurst exponent as a signal.

Similar to the moving average crossover strategy we explain in 4.3, the moving average Hurst crossover strategy subtracts the 5-day moving average Hurst with the 20-day moving average Hurst to decide the excursion pattern of the price movement, then combines the result with a 5-day moving average return. If the difference is above zero, it is signaling a longer and positive excursion in stock prices. However, if the difference is below zero, it indicates a negative excursion and a decline in the existing persistence in the price movement. Likewise, if the 5-day moving average return is positive on a trending pattern, we buy the stock, whereas on a mean-reverting pattern, we sell. If the 5-day moving average return is negative, we proceed with the opposite. Table 2 illustrates the different scenarios of the trend extracted from the moving average Hurst exponent. In order to achieve the best results, we focus on two extreme cases when 5-day moving average Hurst and 20-day moving average Hurst are of different signs.

Table 2 Trading Signals From Moving Average Hurst Exponent

	5D MA Hurst > 0.5	5D MA Hurst < 0.5
30D MA Hurst < 0.5	Trending	
30D MA Hurst > 0.5		Reversal

Figure 5 Diagram Of Moving Average Hurst Crossover Strategy



In order to achieve the best results, we only consider the following four scenarios: a positive 5-day return combined a longer excursion of 5-day moving average H > 0.5 and 20-day moving average H < 0.5; a positive 5-day return combined a longer excursion of 5-day moving average H < 0.5 and 20-day moving average H > 0.5; a negative 5-day return combined a longer excursion of 5-day moving average H > 0.5 and 20-day moving average H < 0.5; and lastly, a negative 5-day return combined a shorter excursion of 5-day moving average H < 0.5 and 20-day moving average H > 0.5.

Figure 2 is a diagram demonstrating the algorithm of the above trading strategy. The four scenarios lead to the results of buying at the bottom, selling at the top, buying on trend and selling on trend, respectively.

We construct the long/short portfolios by investing equal amount of each stock in the portfolios. For the long portfolio, buy one stock at the ask price at the start of the holding period. For the short portfolio, sell one stock at the bid price at the start of the holding period. We close out our position once the trading signal disappears. There is no short selling in one stock in order to prevent severe loss of capital that could lead to margin calls and a false return rate due to high leverages.

Since the choice of the moving average window is arbitrary, we examine a wide range of window periods and find that the pair of 5-day and 20-day moving average Hurst best captures the change of excursion in the price movement. Gencay (1996) mentioned the most popular lookback periods are 5 days, 20 days, 100 days and 200 days. Medium-term momentum can be gauged by looking at moving average that is created with time periods of 100 days or more. For our study, we would like to capture the short-term changes by focusing on the time periods of 20 days or less.

CHAPTER 5 RESULTS AND ANALYSIS

The purpose of our study is to exploit the long memory process existed in the stock price movement. In order to test our assumptions, we explore the basic momentum strategies and compare the performance of stocks in different Hurst range groups. Furthermore, we design a new momentum strategy by incorporating the use of Hurst exponent as a trading signal. In this chapter, the results of the trading strategies discussed in Chapter 4 are presented. We found that Hurst exponent serves as a great categorical indicator in filtering stocks that suitable for mean-reversion strategies, and using Hurst as a trading signal generates considerate returns when compared with the benchmarks.

5.1 Testing Results Of Basic Momentum Strategies

In the each portfolio, the annualized return for each holding period is computed as follows:

Portfolio Return = Long Return + Short Return $= \frac{252}{n} \times \frac{(Short\ value - Long\ value)}{Long\ value}$

(22)

Where n is the holding period between the long and short transactions.

Standard deviation is calculated to measure the volatility of the portfolio and the Sharpe ratio is also computed to indicate the risk-adjusted return:

Sharpe ratio $= \frac{r_p - r_f}{\sigma_p}$ (23)

Where r_p is the mean portfolio return, r_f is the risk-free rate, and σ_p is the standard deviation of portfolio return

Maximum drawdown measures the downward risk over the trading period of time, which is calculated using the following formula:

$$\text{Maximum drawdown} = (\text{Trough Value} - \text{Peak Value}) / \text{Peak Value} \qquad (24)$$

Table 3 shows the cumulative returns generated from each of the three basic momentum trading strategies when applied to the groups of stocks with varying Hurst exponent ranges. From the table, we can see that among the four ranges of Hurst exponent, the stocks within the range of 0.3-0.4 produce the highest returns in all of the basic momentum trading strategies. The strategy return is almost linearly decreasing in $0.3 < H < 0.45$, while gentle increases with $0.45 < H < 0.5$. This indicates that stock price movement becomes easily exploitable by momentum strategies when the H is in the lower range, implying a property of ergodicity of the price process. Whereas the stocks within the range of 0.45-0.5 demonstrate an excursion pattern that is similar to a random walk, henceforth produce lower return, or even negative return on the basic momentum strategies.

Table 3 Testing Period Results Of Basic Momentum Strategies (2008/1/1 - 2016/1/1)

Period Testing Results of Relative Strength Strategy

	Hurst	Alpha	Beta	Volatility	Sharpe Ratio	Max Drawdown	**Total Returns**	Benchmark Returns
SP500	0.3-0.4	0.07	-0.03	0.08	0.81	0.143	72.60%	62.50%
	0.4-0.45	0.03	-0.07	0.07	0.35	0.118	38.70%	62.50%
	0.45-0.5	0.01	-0.10	0.08	-0.01	0.147	17.29%	62.50%
	0.5-0.6	-0.05	-0.19	0.11	-0.55	0.504	-31.40%	62.50%
NASDAQ	0.3-0.4	0.03	0.05	0.11	0.33	0.204	46.80%	127.30%
	0.4-0.45	0.01	-0.06	0.09	0.04	0.136	21.00%	127.30%
	0.45-0.5	0.00	0.02	0.07	-0.03	0.118	16.60%	127.30%
	0.5-0.6	0.01	-0.11	0.12	0.04	0.215	22.30%	127.30%
DIJA	0.3-0.4	0.22	0.17	0.17	1.42	0.215	205.30%	58.30%
	0.4-0.45	0.01	0.04	0.07	-0.07	0.114	13.80%	58.30%
	0.45-0.5	0.03	-0.12	0.10	-0.46	0.236	-9.20%	58.30%
RUSSELL2000	0.3-0.4	0.06	0.05	0.18	0.12	0.483	53.70%	-13.00%
	0.4-0.45	0.02	-0.06	0.11	-0.55	0.182	-10.11%	-13.00%
	0.45-0.5	0.04	-0.08	0.07	-0.54	0.324	-24.40%	-13.00%
	0.5-0.6	0.01	-0.06	0.14	-0.47	0.273	-18.10%	-13.00%

Period Testing Results of Stochastic Oscillator Strategy

	Hurst	Alpha	Beta	Volatility	Sharpe Ratio	Max Drawdown	**Total Returns**	Benchmark Returns
SP500	0.3-0.4	0.07	0.55	0.14	0.71	0.317	97.20%	62.50%
	0.4-0.45	0.03	0.49	0.12	0.49	0.255	66.10%	62.50%
	0.45-0.5	-0.02	0.53	0.13	0.05	0.334	23.00%	62.50%
	0.5-0.6	-0.03	0.56	0.14	0.04	0.394	23.10%	62.50%
NASDAQ	0.3-0.4	0.11	0.41	0.12	1.08	0.249	123.00%	127.30%
	0.4-0.45	0.04	0.45	0.12	0.58	0.272	73.70%	127.30%
	0.45-0.5	-0.17	-1.99	0.61	0.00	2.167	-204.80%	127.30%
	0.5-0.6	-0.03	0.59	0.17	-0.01	0.526	17.39%	127.30%
DIJA	0.3-0.4	0.07	0.48	0.18	0.53	0.435	93.60%	58.30%
	0.4-0.45	0.03	0.44	0.12	0.42	0.156	58.10%	58.30%
	0.45-0.5	0.02	0.53	0.13	0.53	0.328	26.30%	58.30%
RUSSELL2000	0.3-0.4	0.02	0.41	0.12	0.67	0.351	89.10%	-13.00%
	0.4-0.45	0.07	0.48	0.17	0.52	0.348	72.20%	-13.00%
	0.45-0.5	0.03	0.45	0.15	0.38	0.43	49.30%	-13.00%
	0.5-0.6	0.02	0.34	0.25	0.27	0.374	30.70%	-13.00%

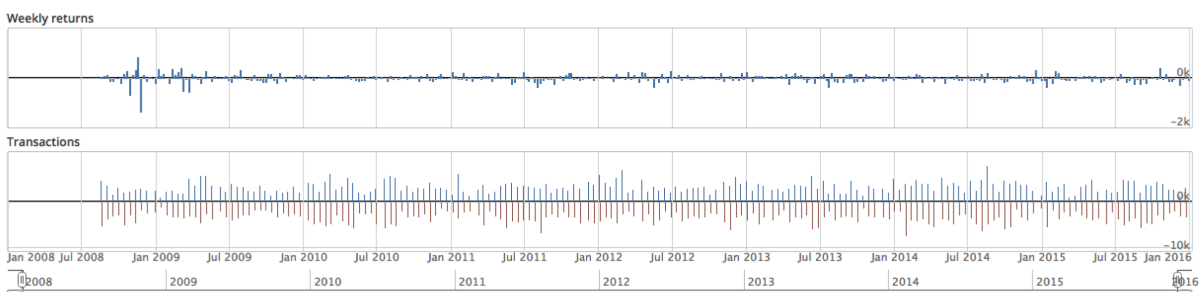
Period Testing Results of Moving Average Crossover Strategy

	Hurst	Alpha	Beta	Volatility	Sharpe Ratio	Max Drawdown	**Total Returns**	Benchmark Returns
SP500	0.3-0.4	-0.01	0.44	0.13	0.09	0.304	27.20%	62.50%
	0.4-0.45	-0.02	0.41	0.12	0.03	0.302	21.70%	62.50%
	0.45-0.5	-0.03	0.44	0.13	-0.03	0.294	15.20%	62.50%
	0.5-0.6	-0.03	0.46	0.14	-0.01	0.319	17.60%	62.50%
NASDAQ	0.3-0.4	0.03	0.47	0.15	0.36	0.388	61.80%	127.30%
	0.4-0.45	0.00	0.37	0.11	0.18	0.305	34.20%	127.30%
	0.45-0.5	-0.02	0.43	0.13	0.01	0.362	18.60%	127.30%
	0.5-0.6	-0.02	0.39	0.13	0.03	0.266	21.20%	127.30%
DIJA	0.3-0.4	0.09	0.43	0.17	0.68	0.257	110.70%	58.30%
	0.4-0.45	0.03	0.35	0.10	-0.03	0.279	15.40%	58.30%
	0.45-0.5	0.03	0.39	0.13	0.08	0.337	25.90%	58.30%
RUSSELL2000	0.3-0.4	0.01	0.35	0.27	0.56	0.306	72.40%	-13.00%
	0.4-0.45	0.18	0.32	0.21	0.16	0.245	56.30%	-13.00%
	0.45-0.5	0.02	0.01	0.24	0.03	0.367	23.90%	-13.00%
	0.5-0.6	0.05	0.33	0.27	0.22	0.306	48.20%	-13.00%

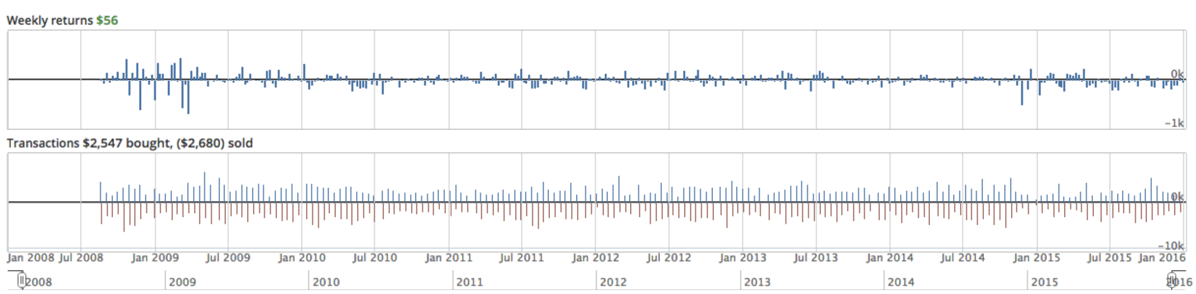
Besides the back-testing results, Quantopian also generates an overview of weekly P&L and the transactions. A sample P&L output of the relative strength strategy on S&P 500 stocks can be seen as per below. Blue columns in the transaction chart demonstrate our weekly long position while red columns show our weekly short positions. As we can see, the level of transactions is more persistent for the stocks with 0.3<H<0.4 as their excursion patterns are better captured by the momentum strategy. Other P&L charts can be found in the Appendix.

Figure 6 Weekly P&L of Relative Strength Strategy on S&P500 Stocks

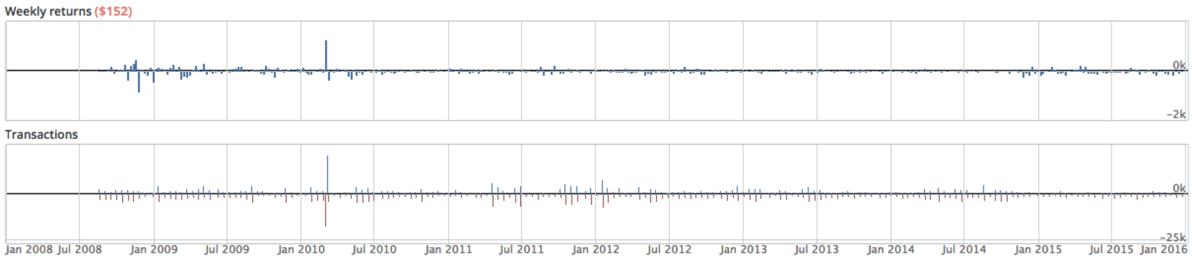
0.3<H<0.4



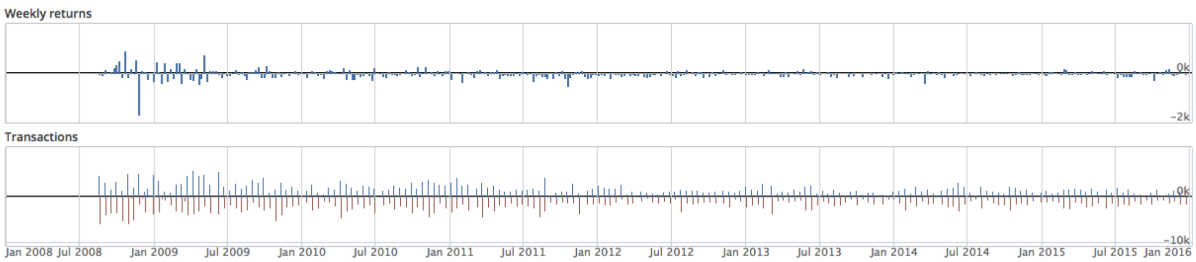
0.4<H<0.45



0.45<H<0.5



0.5<H<0.6



5.2 Testing Results Of Moving Average Hurst Crossover Strategy

5.2.1 Back-testing Results Of S&P 500, NASDAQ, DJIA and RUSSELL 2000 Stocks

Once we start to enter into the more advanced stage of our trading education, it become important to start looking beyond the basic indicators such as the RSI, Stochastic Oscillator and MACD, that are commonly used by a majority of the market. Table 4 summarizes the back-testing results based on the strategy using moving average Hurst crossover as an indicator during the testing period. We apply this strategy to the constituent stocks of large-cap indexes like S&P 500, NASDAQ and DJIA, as well as small-cap index like RUSSELL 2000, and compare the strategy returns with the ETFs that track the performance of these indexes[4].

As shown in the table below, the moving average Hurst crossover strategy produces excessively higher returns than all of the ETFs tracking the performance of the indexes. It triples the return of S&P 500 in eight years, almost doubles the return of NASDAQ, outperforms DJIA by around 30%, and is able to generate a return as high as 79.5% despite of the fact that the RUSSELL 2000 benchmark posts a negative return of -13% over the years. Henceforth, we conclude that an active trading strategy using the Hurst exponent shows competitive advantage over passively investing in the ETFs.

[4] The ETFs used for S&P 500, NASDAQ, DJIA and RUSSELL2000 are: SPDR S&P 500 ETF Trust (Yahoo ticker: SPY), PowerShares QQQ ETF Trust (Yahoo ticker: QQQ), SPDR Dow Jones Industrial AverageSM ETF Trust (Yahoo ticker: DIA), and Russell 2000 High Beta ETF (Yahoo ticker: SHBT), respectively.

Table 4 Testing Period Results Of Moving Average Hurst Crossover Strategy (2008/1/1 - 2016/1/1)

Back-testing Results for Period 2008/1/1-2016/1/1										
	H1Days	H2 Days	MA Days	Alpha	Beta	Volatility	Sharpe Ratio	Max Drawdown	Strategy Return	Benchmark Return
S&P500	5	20	5	0.16	0.39	0.28	0.78	0.283	189.30%	62.50%
NASDAQ	5	20	5	0.21	0.35	0.17	1.56	0.191	224.70%	127.30%
DJIA	5	20	5	0.06	0.41	0.11	0.71	0.128	81.29%	58.30%
RUSSELL2000	5	20	5	0.06	0.47	0.25	0.69	0.2819	79.50%	-13.00%

From a behavioral point of view, an ergotic excursion pattern infers that the price series is in a constant state of mean reversions hence frequent adjustment of long or short positions is required. This suggests that more people are actively trading these stocks and the bid and ask sizes are roughly equal. However, as time evolves, the more profitable security draws more attention from investors, and an imbalance of bid and ask sizes appear. By that time, the stock becomes less profitable and less frequently traded, showing an $H > 0.5$.

5.2.2 Robustness Test

Over-fitting is always a concern when developing a trading strategy. In computing the Hurst exponent of each price process, we adopt the rolling average method by creating different subseries with the length of 500 daily observations. In order to check the robustness of our strategy, we develop a stress testing by changing the inputs such as the look-back window length of the computation of Hurst exponent. Table 5 reports the results of the stress testing.

Table 5 Stress Test Results For Moving Average Hurst Crossover Strategy Using Different Lookback Length

Back-testing Results for Period 2008/1/1-2016/1/1									
	# of Oberservations	Lag	Alpha	Beta	Volatility	Sharpe Ratio	Max Drawdown	Strategy Return	Benchmark Return
S&P500	1000	32	0.04	0.45	0.23	0.46	0.197	71.80%	62.50%
S&P500	700	26	0.04	0.56	0.18	0.37	0.238	73.20%	62.50%
S&P500	500	23	0.16	0.39	0.28	0.78	0.283	189.30%	62.50%
S&P500	200	14	0.28	0.39	0.32	0.42	0.360	138.39%	62.50%
NASDAQ	1000	32	0.28	0.49	0.27	1.08	0.408	190.00%	127.30%
NASDAQ	700	26	0.22	0.53	0.26	1.29	0.389	215.70%	127.30%
NASDAQ	500	23	0.21	0.35	0.17	1.56	0.191	224.70%	127.30%
NASDAQ	200	14	0.55	0.4	0.37	1.4	0.260	221.60%	127.30%
DJIA	1000	32	0.01	0.18	0.06	0.2	0.072	28.19%	58.30%
DJIA	700	26	0.02	0.17	0.06	0.42	0.086	39.80%	58.30%
DJIA	500	23	0.06	0.41	0.11	0.71	0.128	81.29%	58.30%
DJIA	200	14	0.1	0.24	0.1	1.19	0.123	112.00%	58.30%
RUSSELL2000	1000	32	0.15	0.44	0.32	0.62	0.225	64.90%	-13.00%
RUSSELL2000	700	26	0.04	0.14	0.27	0.66	0.215	68.20%	-13.00%
RUSSELL2000	500	23	0.06	0.47	0.25	0.69	0.282	79.50%	-13.00%
RUSSELL2000	200	14	0.18	0.2	0.28	0.42	0.299	59.10%	-13.00%

As we can see, the moving average Hurst crossover strategy still outperforms the benchmarks in all the cases except for RUSSELL 2000, and seems to reach the optimal outcome using 200-500 observations. The results testify to the robustness of our strategy, and imply that the model is less likely to be over-fit, and should be expected to continue to perform well in the future.

5.3 Limitations And Future Improvement

Although the research has achieved its aim, there were some unavoidable limitations that we should be aware of:

First, since most of the stocks we are testing are rather liquid, we ignore the slippage, which often occurs during period of higher volatility. However, there could be a considerable difference between the real-time trade price and the execution price in the case of small stocks in RUSSELL 2000. Future work can be done by including the slippage in the model.

Second, in regards to the algorithm of Hurst trading strategy, the computation of Hurst exponent is relatively time-consuming. For example, if we take 500 observations to compute the 20-day moving average H with a lag of k = 23, the algorithm would need to generate 20 subseries with the length of 500 observations. For each subseries, it will loop through k=1 to 23 and generate 23 subseries in order to calculate the variance for each subseries. In average, an eight-year back testing for the moving average Hurst crossover takes about an hour to complete. The excessive consumption of computing time could be a problem in real-time trading.

Finally, we adopt a simple buy and hold method in the Hurst trading strategy in order to avoid problems such as marginal calls. Future improvement could be made by loosening some of these restrictions and apply short selling to the strategy.

CHAPTER 6 CONCLUSIONS

The paper seeks to test the Random Walk Hypothesis by examining the effect of stock price excursion patterns on momentum trading profitability. The Efficient Market Hypothesis, which is closely related to the Random Walk Hypothesis, states that security prices cannot be predicted based on historical price movement. In order to test the null, we employ the Hurst exponent, to characterize the correlations between of the time series.

Our back testing results show that a naïve trading strategy using the crossover of two different moving averages of the Hurst exponent as a trading signal could generate significantly higher return compared with the benchmark index. It triples the return of S&P 500 in eight years, almost doubles the return of NASDAQ, outperforms DJIA by around 30%, and is able to generate a return as high as 79.5% despite of the fact that the RUSSELL 2000 benchmark posts a negative return of -13% over the years. The robustness test is performed by testing different lengths of look-back period used for calculating the Hurst exponent, and we conclude that the strategy returns are broadly much higher than the benchmarks. These results dispute the hypothesis that the stock market prices follow a weak-form random walk, and suggest that momentum strategies can earn excess returns.

In further analysis, we find that stocks with $0.3 < H < 0.4$ contribute the most to the momentum profit, while those with H in the range of 0.45 – 0.5 earn the least profit. There is a U-shape relationship between the strategy returns and the Hurst exponent. The finding is consistent with the theory that a time series with Hurst exponent less than 0.4

exhibits a more ergotic, mean-reverting excursion property, and is more likely to be exploited by the momentum strategies. Market overreaction could attribute to the profitability in these contrarian strategies. Whereas a time series with H in the range of 0.45 – 0.5 demonstrate an excursion pattern that is close to a random walk, and hence is more unpredictable.

From a behavioral point of view, an ergotic excursion pattern infers that the price series is in a constant state of mean reversions hence frequent adjustment of long or short positions is required. This suggests that more people are actively trading these stocks and the bid and ask sizes are roughly equal. However, as time evolves, the more profitable security draws more attention from investors, and an imbalance of bid and ask sizes appear. By that time, the stock becomes less profitable and less frequently traded, showing an $H > 0.5$.

The existence of a long-term memory in the stock price process provides solid evidence against the Random Walk Hypothesis. Our results present new evidence and challenges to a number of prominent behavioral and rational asset pricing theories. It posts questions on the assumption of log normal distribution in the stock returns, and have an important bearing on pricing of derivatives in the stock markets. Future research on including a long-term memory component in the pricing model would be groundbreaking.

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APPENDIX

Table 6 Hurst Exponents of S&P 500 Stocks for Testing Period 2011/1/ - 2016/1/1

H Value of S&P 500 for Period 2011/1/1-2016/1/1

Ticker	H	Ticker	H	Ticker	H	Ticker	H	Ticker	H
EXPE	0.3381554	CAH	0.4058805	SYK	0.4231330	MTB	0.4324437	QEP	0.4457166
NWL	0.3499727	CNP	0.4060423	PCP	0.4232760	STT	0.4332997	LLL	0.4457874
UNH	0.3557631	FLR	0.4063010	CMCSA	0.4236912	PNC	0.4337264	ROP	0.4458817
CTAS	0.3591012	ZTS	0.4071679	PEP	0.4240020	CERN	0.4337658	JWN	0.4459502
MHK	0.3656426	AZO	0.4078976	BIIB	0.4240903	MON	0.4337892	WAT	0.4459980
NI	0.3671477	MA	0.4084415	SHW	0.4241675	OXY	0.4343471	AON	0.4461479
XL	0.3700534	USB	0.4092551	SJM	0.4243441	CNX	0.4343505	CME	0.4463342
HON	0.3700567	BSX	0.4092654	MJN	0.4246489	LRCX	0.4347317	MHFI	0.4463701
AME	0.3708019	EFX	0.4092899	BRK-B	0.4246904	T	0.4350891	ES	0.4466748
MKC	0.3723009	TWC	0.4104145	CMS	0.4247326	SWK	0.4353919	DO	0.4471778
TJX	0.3729552	PFE	0.4105587	CCI	0.4249040	CTL	0.4354086	MSFT	0.4472290
BLL	0.3750418	CCE	0.4106247	TMK	0.4249383	PRGO	0.4355866	BA	0.4473901
EBAY	0.3778741	YUM	0.4118687	SRE	0.4250614	LM	0.4356516	KMX	0.4474572
L	0.3782142	ETFC	0.4120168	MRK	0.4251208	HIG	0.4357356	NVDA	0.4480559
LEN	0.3782508	AMT	0.4120745	IBM	0.4253154	BHI	0.4368333	ALLE	0.4481950
LLY	0.3805349	PBI	0.4123313	NWSA	0.4253315	AIV	0.4374539	ALTR	0.4483568
WYN	0.3815913	AN	0.4129832	EL	0.4253458	UNP	0.4375393	FTI	0.4483570
HD	0.3816580	AVB	0.4134813	NTRS	0.4253578	BDX	0.4382073	CVX	0.4485860
MAS	0.3822499	ORLY	0.4135683	MCHP	0.4254520	RCL	0.4383335	MAC	0.4486694
GILD	0.3868459	VFC	0.4136080	KEY	0.4258655	APC	0.4386273	JNJ	0.4487341
CELG	0.3873657	ADBE	0.4137670	GWW	0.4260155	XRAY	0.4387545	DHR	0.4490088
DFS	0.3880078	NOC	0.4138645	ISRG	0.4261735	COST	0.4388713	LEG	0.4491236
COF	0.3883140	CHRW	0.4143993	PEG	0.4262794	FLS	0.4391838	PVH	0.4491508
PNR	0.3886222	MOS	0.4153100	PCLN	0.4269021	EOG	0.4394101	WBA	0.4491708
PX	0.3893766	RHI	0.4155781	NSC	0.4272255	BLK	0.4394754	AIZ	0.4494064
TROW	0.3896810	WFC	0.4157581	PBCT	0.4273756	MCD	0.4398652	PGR	0.4495570
LMT	0.3914465	EQR	0.4157700	GPC	0.4274918	LH	0.4402412	UNM	0.4495786
REGN	0.3921261	HBI	0.4164143	JOY	0.4278122	V	0.4405498	PAYX	0.4496689
STZ	0.3928625	PWR	0.4167961	TSS	0.4279048	AAL	0.4408054	K	0.4497441
SNA	0.3935032	AXP	0.4168405	MDT	0.4281599	PKI	0.4409544	TXT	0.4500625
SRCL	0.3938483	CL	0.4168687	DE	0.4282525	BXP	0.4411866	HAS	0.4500811
MMC	0.3957497	CCL	0.4171276	ADI	0.4288192	LOW	0.4415484	VAR	0.4502053
FIS	0.3972739	DAL	0.4179977	TSCO	0.4288774	VMC	0.4417198	IVZ	0.4505582
ACN	0.3978427	HBAN	0.4187339	UHS	0.4289783	EMN	0.4418471	EA	0.4506725
AMG	0.3985231	ORCL	0.4187569	AIG	0.4289846	IFF	0.4419853	PFG	0.4507559
FISV	0.3998325	PSX	0.4190394	AMGN	0.4299900	ADP	0.4431767	NUE	0.4511372
CSX	0.4002790	KSU	0.4195322	CF	0.4302725	EQIX	0.4432681	A	0.4513485
DHI	0.4004008	ECL	0.4196263	RSG	0.4302957	XEL	0.4432761	RAI	0.4519710
VRTX	0.4004087	AES	0.4199472	HUM	0.4305123	MNST	0.4433590	AEP	0.4520820
AET	0.4009437	D	0.4206096	VZ	0.4305240	XYL	0.4437327	ALXN	0.4521615
APH	0.4018249	ACE	0.4213533	XOM	0.4305395	ZBH	0.4439870	FITB	0.4521821
GIS	0.4020053	CB	0.4213533	PPL	0.4305526	CMI	0.4440222	PXD	0.4529836
TAP	0.4031699	HOT	0.4216082	SYY	0.4305938	BMY	0.4441905	UA	0.4531212
MSI	0.4046330	TYC	0.4217796	PM	0.4307984	ICE	0.4444068	MDLZ	0.4531585
KO	0.4051483	XRX	0.4218107	PHM	0.4310021	STI	0.4446908	MUR	0.4534668
APD	0.4051554	TSN	0.4220150	NEE	0.4322281	ROST	0.4447147	ABBV	0.4534838
SBUX	0.4054368	CAM	0.4223326	DLPH	0.4322639	HOG	0.4448964	BF-B	0.4535184
QCOM	0.4055461	MCO	0.4224301	NKE	0.4323779	ABT	0.4454554	DOW	0.4537069

Table 6 Hurst Exponents of S&P 500 Stocks for Testing Period 2011/1/ - 2016/1/1

(Continued)

H Value of S&P 500 for Period 2011/1/1-2016/1/1									
Ticker	H	Ticker	H	Ticker	H	Ticker	H	Ticker	H
VNO	0.4537892	TSO	0.4601253	GNW	0.4705551	DOV	0.4826476	FOSL	0.4958988
MLM	0.4538824	SLB	0.4602324	ETN	0.4717316	PLD	0.4826918	FE	0.4965095
DG	0.4539537	AA	0.4603822	SO	0.4717696	NFLX	0.4828500	SPLS	0.4967883
TMO	0.4539837	BWA	0.4605305	CMA	0.4718118	HAR	0.4828539	KMI	0.4968805
RRC	0.4539955	DTE	0.4606040	TIF	0.4728760	AMZN	0.4829034	KR	0.4971967
MCK	0.4540266	FCX	0.4606436	TEL	0.4729490	NE	0.4831243	WMT	0.4977171
INTU	0.4543675	MU	0.4606967	WU	0.4731911	LB	0.4833340	HCA	0.4980908
SCHW	0.4546000	BCR	0.4606985	DISCA	0.4732763	URI	0.4834840	GMCR	0.4986494
DVA	0.4548412	RL	0.4616295	XLNX	0.4743745	ADSK	0.4840364	TRV	0.4994447
SNI	0.4548424	HRB	0.4618567	AVY	0.4744207	MO	0.4841282	DNB	0.5000321
DRI	0.4548731	CTSH	0.4618729	WM	0.4747228	URBN	0.4845577	M	0.5009959
PH	0.4549805	AFL	0.4624188	SPG	0.4748618	KIM	0.4848321	MRO	0.5011033
SWKS	0.4549890	MMM	0.4624776	AAPL	0.4750167	HAL	0.4848780	SNDK	0.5014586
GOOGL	0.4552027	ADT	0.4625892	BBT	0.4750596	OMC	0.4853582	THC	0.5016707
BRCM	0.4558198	UPS	0.4626447	XEC	0.4751117	COP	0.4854571	TRIP	0.5019912
COH	0.4559237	CTXS	0.4627871	GE	0.4751545	LYB	0.4859142	AKAM	0.5020692
AVGO	0.4559563	ANTM	0.4634944	GRMN	0.4752102	PG	0.4861936	OI	0.5021566
NDAQ	0.4559600	GLW	0.4638373	ENDP	0.4760224	KMB	0.4862341	DUK	0.5021785
LLTC	0.4560356	JEC	0.4640062	KORS	0.4760319	HST	0.4862704	CAT	0.5028860
DGX	0.4560436	STJ	0.4641246	MET	0.4761629	GM	0.4866356	MAT	0.5031972
SLG	0.4560539	SWN	0.4645509	LVLT	0.4765921	WDC	0.4866530	KSS	0.5038679
MAR	0.4562699	TXN	0.4645603	WYNN	0.4766043	PSA	0.4870933	SCG	0.5046102
RF	0.4562853	EQT	0.4647007	NBL	0.4767182	LUV	0.4871883	IRM	0.5048643
SE	0.4562958	BEN	0.4648905	GT	0.4768980	YHOO	0.4875013	VRSN	0.5065031
GD	0.4563908	COL	0.4649905	F	0.4776660	GGP	0.4876953	BBY	0.5066616
ESS	0.4564107	IPG	0.4652478	IR	0.4778710	ARG	0.4879025	CBS	0.5069371
TE	0.4564306	BBBY	0.4652911	EMC	0.4778820	CAG	0.4881324	DD	0.5075349
CLX	0.4567461	WMB	0.4656022	HRS	0.4780392	WEC	0.4882478	ADS	0.5088201
FB	0.4567527	R	0.4656949	EIX	0.4781306	GS	0.4882794	AMAT	0.5120206
RHT	0.4569982	FFIV	0.4657565	CINF	0.4781978	FMC	0.4886468	QRVO	0.5140158
CA	0.4572160	ITW	0.4660585	HCP	0.4783227	UTX	0.4889478	WFM	0.5156689
AEE	0.4573114	ESV	0.4661097	OKE	0.4784836	PCG	0.4890591	O	0.5160405
AMP	0.4574889	ROK	0.4661154	CVS	0.4785508	INTC	0.4891179	CVC	0.5165528
MS	0.4575251	CHK	0.4662270	BAC	0.4788667	SEE	0.4891260	CMG	0.5167121
PDCO	0.4575530	TDC	0.4664907	EXC	0.4793954	CSC	0.4894036	JNPR	0.5189010
CPB	0.4576360	FDX	0.4669839	PRU	0.4794238	JCI	0.4899790	VIAB	0.5206466
ESRX	0.4580138	VLO	0.4671904	RIG	0.4797191	MYL	0.4905904	STX	0.5206618
EXPD	0.4580150	JPM	0.4674352	DVN	0.4798607	PCL	0.4907476	VTR	0.5216980
RTN	0.4580961	HES	0.4675166	HSY	0.4800002	ED	0.4909082	HCN	0.5219606
TWX	0.4582424	CSCO	0.4676854	HRL	0.4804294	LUK	0.4909968	FSLR	0.5231530
GAS	0.4584328	BAX	0.4678738	COG	0.4806126	KLAC	0.4917277	ADM	0.5243012
C	0.4586925	FAST	0.4682428	PPG	0.4809300	CBG	0.4924603	POM	0.5272217
FOXA	0.4587016	WY	0.4683616	NEM	0.4809704	NRG	0.4929607	ABC	0.5296037
IP	0.4590738	ATI	0.4684357	APA	0.4810267	DLTR	0.4932384	GPS	0.5306445
EMR	0.4590790	ALL	0.4688136	DIS	0.4812601	TGNA	0.4937783	GME	0.5350075
HSIC	0.4593366	BK	0.4690761	HP	0.4815614	HPQ	0.4938787	NAVI	0.5358147
WHR	0.4594192	MPC	0.4693780	LNC	0.4816561	PCAR	0.4947318	MNK	0.5370120
NFX	0.4596756	NOV	0.4696268	PNW	0.4820691	NTAP	0.4952859		
DPS	0.4600600	SYMC	0.4696932	CI	0.4823616	ETR	0.4955612		
EW	0.4601238	TGT	0.4704314	FLIR	0.4823803	FTR	0.4958213		

Table 7 Hurst Exponents of NASDAQ Stocks for Testing Period 2011/1/1 - 2016/1/1

H Value of NASDAQ Stocks for Period 2011/1/1-2016/1/1			
Ticker	H	Ticker	H
EXPE	0.3381554	MAR	0.4562699
EBAY	0.3778741	FB	0.4567527
VRSK	0.3784104	CA	0.4572160
GILD	0.3868459	ESRX	0.4580138
CELG	0.3873657	EXPD	0.4580150
CHKP	0.3909999	FOXA	0.4587016
REGN	0.3921261	HSIC	0.4593366
SRCL	0.3938483	MU	0.4606967
CHTR	0.3964582	CTSH	0.4618729
FISV	0.3998325	CTXS	0.4627871
VRTX	0.4004087	ATVI	0.4629480
SBUX	0.4054368	TXN	0.4645603
QCOM	0.4055461	BBBY	0.4652911
NXPI	0.4084262	FFIV	0.4657565
ORLY	0.4135683	CSCO	0.4676854
ADBE	0.4137670	FAST	0.4682428
CHRW	0.4143993	SYMC	0.4696932
GOOG	0.4171765	DISCA	0.4732763
MXIM	0.4203531	XLNX	0.4743745
CMCSA	0.4236912	AAPL	0.4750167
BIIB	0.4240903	GRMN	0.4752102
ISRG	0.4261735	WYNN	0.4766043
PCLN	0.4269021	NFLX	0.4828500
SBAC	0.4281330	AMZN	0.4829034
ADI	0.4288192	ADSK	0.4840364
VOD	0.4288682	BIDU	0.4849751
TSCO	0.4288774	TSLA	0.4862742
AMGN	0.4299900	WDC	0.4866530
CERN	0.4337658	YHOO	0.4875013
COST	0.4388713	INTC	0.4891179
LBTYA	0.4405412	MYL	0.4905904
ADP	0.4431767	KLAC	0.4917277
EQIX	0.4432681	DLTR	0.4932384
MNST	0.4433590	PCAR	0.4947318
LMCA	0.4438995	NTAP	0.4952859
ROST	0.4447147	SPLS	0.4967883
MSFT	0.4472290	GMCR	0.4986494
NVDA	0.4480559	SNDK	0.5014586
ALTR	0.4483424	TRIP	0.5019912
DISH	0.4488350	AKAM	0.5020692
PAYX	0.4496689	MAT	0.5031972
SIRI	0.4515451	AMAT	0.5120206
ALXN	0.4521615	VIP	0.5138055
MDLZ	0.4531585	WFM	0.5156689
INTU	0.4543675	VIAB	0.5206466
BRCM	0.4558198	STX	0.5206618
AVGO	0.4559563	ILMN	0.5552049
LLTC	0.4560356		

Table 8 Hurst Exponents of Dow Jones Stocks for Testing Period 2011/1/1 - 2016/1/1

H Value of NASDAQ Stocks for Period 2011/1/1-2016/1/1	
Ticker	H
UNH	0.3557631
HD	0.3816580
KO	0.4051483
PFE	0.4105587
AXP	0.4168405
MRK	0.4251208
IBM	0.4253154
VZ	0.4305240
XOM	0.4305395
NKE	0.4323779
MCD	0.4398652
V	0.4405498
MSFT	0.4472290
BA	0.4473901
CVX	0.4485860
JNJ	0.4487341
MMM	0.4624776
JPM	0.4674352
CSCO	0.4676854
AAPL	0.4750167
GE	0.4751545
DIS	0.4827770
PG	0.4861936
GS	0.4882794
UTX	0.4889478
INTC	0.4891179
WMT	0.4977171
TRV	0.4994447
CAT	0.5028860
DD	0.5075349

Table 9 Hurst Exponents of RUSSELL 2000 for Testing Period 2011/1/1 - 2016/1/1

H Value of RUSSELL 2000 for Period 2011/1/1-2016/1/1									
Ticker	H	Ticker	H	Ticker	H	Ticker	H	Ticker	H
RRR	0.1149667	SUNS	0.3315937	GBDC	0.3700384	FSCI	0.3871275	SFY	0.3995210
ATX	0.0043828	UPI	0.3324883	SENEA	0.3715955	OSIR	0.3881266	PCBC	0.3999956
GET	0.0069899	YDNT	0.3331022	ELON	0.3718225	PLOW	0.3884644	CLC	0.4004395
VITA	0.0101605	PMI	0.3343875	AMSG	0.3730076	TTGT	0.3890927	COBZ	0.4004649
BKR	0.0112169	MSEX	0.3368189	OCFC	0.3740147	EPHC	0.3896608	SPSC	0.4009323
GGC	0.0351541	HTBK	0.3373870	BKCC	0.3742229	WERN	0.3897108	CBKN	0.4009586
TVL	0.0355725	CNBKA	0.3384232	NDN	0.3743146	DVR	0.3899499	JCOM	0.4013862
GGS	0.0396905	CLP	0.3392220	NPBC	0.3743566	WFD	0.3902078	SNBC	0.4018752
VCI	0.0454719	STBC	0.3408295	SUPX	0.3746017	PPS	0.3906532	STEL	0.4021317
IN	0.0458386	AROW	0.3417663	LBAI	0.3749819	HAYN	0.3907853	FSC	0.4023866
PZG	0.0491612	SHS	0.3420628	EF	0.3752799	PLUS	0.3909056	BRY	0.4028306
FLOW	0.0507992	CTWS	0.3425672	VASC	0.3756903	HITK	0.3910219	SGA	0.4031733
TUC	0.0523935	DGICA	0.3432908	WCBO	0.3757243	ATMI	0.3910884	UFCS	0.4032942
PRM	0.0525340	MSFG	0.3465253	ARTNA	0.3758527	CFNL	0.3916364	NCMI	0.4033238
SFI	0.0532633	WEYS	0.3483050	DPZ	0.3764900	ZIP	0.3917480	EBSB	0.4036427
SNTS	0.0564392	CFNB	0.3486187	UIL	0.3768917	CHDN	0.3918289	NCT	0.4038505
MSW	0.0598469	BMTC	0.3510906	BMI	0.3769396	CVLT	0.3918502	CCBG	0.4038868
CODE	0.0657105	CPF	0.3512047	WTBA	0.3772809	VGZ	0.3922403	TAST	0.4039030
CEC	0.0757591	ORB	0.3514826	BPFH	0.3777591	ALX	0.3934576	SXT	0.4039933
SRX	0.1163046	GAIN	0.3524145	AZZ	0.3780769	KOG	0.3937238	CRWN	0.4044330
SMA	0.1266301	YORW	0.3536351	TISI	0.3782783	BUSE	0.3942067	VSI	0.4045358
SATC	0.1411453	OUTD	0.3539530	AMSWA	0.3784036	FDP	0.3942905	KOPN	0.4046110
CGX	0.1842364	JJSF	0.3545853	RSO	0.3784627	KNX	0.3946393	GBL	0.4046209
ARX	0.2008211	HFWA	0.3548935	GBLI	0.3785453	PHIIK	0.3951014	DRQ	0.4046561
AEA	0.2130441	URRE	0.3549453	ESSA	0.3788274	TNC	0.3953048	UVSP	0.4046750
LSE	0.2166686	CACB	0.3553449	CSFL	0.3800412	NBIX	0.3953488	ELGX	0.4047149
TBL	0.2170341	OB	0.3562315	CTIC	0.3812042	HXL	0.3958740	FRNK	0.4047381
ALC	0.2244112	KCLI	0.3592664	GRIF	0.3827382	UBSH	0.3959361	MKTX	0.4050143
BOX	0.2260772	ORIT	0.3592713	ARJ	0.3831810	GMO	0.3960783	CLMS	0.4050923
LLEN	0.2274168	SFE	0.3595218	FISI	0.3832146	APOG	0.3967853	MSL	0.4053870
MDF	0.2387790	ACO	0.3607625	PSTB	0.3836217	MPR	0.3971950	STRI	0.4056369
SBX	0.2454571	NKSH	0.3607951	TRLG	0.3840641	MLI	0.3973343	GORO	0.4056618
HEV	0.2463027	MBLX	0.3610697	POOL	0.3843249	KFRC	0.3973625	GSOL	0.4056849
SCHS	0.2682363	ISRL	0.3618452	GRC	0.3843355	CNO	0.3974710	HALL	0.4062072
NFBK	0.2799538	ZIGO	0.3623222	AHS	0.3847799	WTFC	0.3975989	DM	0.4062514
CELL	0.2879126	WAVX	0.3624789	FSR	0.3848439	ACLS	0.3978403	MRTN	0.4064781
GCOM	0.2895629	CZNC	0.3627266	ARTC	0.3850625	BDGE	0.3978496	NICK	0.4065607
MPG	0.2976798	FDEF	0.3637182	LMNX	0.3852990	QSFT	0.3979393	LAYN	0.4066119
NMFC	0.2986771	TBNK	0.3637358	SJW	0.3854757	SCS	0.3979999	NWBI	0.4072259
SFN	0.3040292	INCY	0.3646875	BSRR	0.3856545	CSS	0.3981764	CROX	0.4072635
DFG	0.3091101	PROJ	0.3652686	VALU	0.3857845	ICUI	0.3981867	PPC	0.4077563
GLBL	0.3149584	METR	0.3653650	WMK	0.3859036	AINV	0.3983011	FUBC	0.4078800
HERO	0.3185176	CHFN	0.3654790	SBCF	0.3859664	FLIC	0.3983397	SLRC	0.4078815
PCX	0.3199960	INVE	0.3666643	WSO	0.3860711	ARI	0.3983882	MYE	0.4080515
ART	0.3202461	ATLO	0.3667477	OPNT	0.3861921	CSA	0.3983931	SCMR	0.4080629
CXPO	0.3216708	KALU	0.3667672	JMP	0.3865795	CASY	0.3988916	ARDNA	0.4081200
KRNY	0.3240090	AP	0.3691462	WHG	0.3869064	WASH	0.3990298	PPHM	0.4081558
BWINB	0.3285369	CWST	0.3695730	LHCG	0.3869516	PIP	0.3993411	PRXL	0.4083938

Table 9 Hurst Exponents of RUSSELL 2000 for Testing Period 2011/1/1 - 2016/1/1

(Continued)

H Value of RUSSELL 2000 for Period 2011/1/1-2016/1/1									
Ticker	H	Ticker	H	Ticker	H	Ticker	H	Ticker	H
CPSI	0.4084229	AM	0.4148649	BGCP	0.4204245	TCRD	0.4245320	AUMN	0.4293046
MHGC	0.4085041	CRAI	0.4151228	FWRD	0.4205735	BHLB	0.4246494	GLAD	0.4293382
SSNC	0.4085411	HTLD	0.4152322	GNK	0.4206691	ENZN	0.4247541	ISBC	0.4293708
NATR	0.4088821	MWIV	0.4152750	ALGN	0.4207561	CRTX	0.4247625	STEC	0.4294659
NYMX	0.4090221	ELY	0.4154288	CHCO	0.4207759	KNL	0.4247776	BEE	0.4294952
SIMG	0.4090497	NHC	0.4156122	LAD	0.4208058	USPH	0.4248609	CCNE	0.4295225
CSH	0.4091589	PEBO	0.4159588	RAVN	0.4208566	SFL	0.4252150	EME	0.4296898
VPG	0.4093649	PRS	0.4159655	LKFN	0.4209271	FIX	0.4252412	BBRG	0.4298356
STE	0.4093915	NR	0.4159803	HWKN	0.4210391	SPB	0.4252615	MPX	0.4298851
NAUH	0.4096212	VQ	0.4162204	TMP	0.4211283	ROL	0.4252696	FURX	0.4299755
RBN	0.4097271	EBTC	0.4163391	FFBC	0.4214230	ENTG	0.4254268	UFI	0.4300802
MFA	0.4097294	TDY	0.4164515	HURC	0.4216285	KVHI	0.4255473	SBNY	0.4300841
SYNM	0.4098331	CPX	0.4164651	ACUR	0.4216332	TMS	0.4256526	GFIG	0.4301645
CIX	0.4100411	ABAX	0.4166472	CBZ	0.4216699	FBNC	0.4258360	AAON	0.4301681
FC	0.4101042	PLPC	0.4170290	FCBC	0.4217147	CWCO	0.4258419	TTI	0.4303532
LGND	0.4101712	MNR	0.4172108	CVTI	0.4217171	JOUT	0.4259211	UACL	0.4303649
RLOC	0.4102499	JNY	0.4172175	DY	0.4217191	SHOO	0.4259249	EGOV	0.4304209
CAC	0.4103868	KERX	0.4174508	ABG	0.4217796	CATO	0.4259918	CXS	0.4305154
DEL	0.4108979	SAAS	0.4174758	HNR	0.4217826	SPAR	0.4260799	VNDA	0.4308250
NANO	0.4109018	FXCB	0.4175721	SAH	0.4218127	IVAC	0.4261316	RPXC	0.4309573
CEVA	0.4109071	SGS	0.4175903	THFF	0.4219023	WINA	0.4262951	WNC	0.4312690
CSBK	0.4110361	GABC	0.4176076	AYR	0.4221159	THS	0.4263535	CCRN	0.4313975
TMH	0.4110716	NOR	0.4176783	SEB	0.4221380	PGI	0.4263663	MWA	0.4316287
OMI	0.4113175	LYTS	0.4177271	ASGN	0.4221410	NOG	0.4267356	PDLI	0.4317124
ELLI	0.4113767	RNWK	0.4179605	MCF	0.4222396	ELRC	0.4268617	ABCO	0.4318879
HRG	0.4114230	SRCE	0.4180492	MVIS	0.4222906	CTO	0.4270132	ANH	0.4318888
BRKS	0.4114502	MLR	0.4180845	SYBT	0.4223725	ESGR	0.4270628	HBHC	0.4319566
SCHL	0.4115038	HOTT	0.4181618	LMNR	0.4227313	KAMN	0.4271067	ALNC	0.4320546
PNNT	0.4118496	NX	0.4183289	TFM	0.4227521	BELFB	0.4271634	NILE	0.4321408
IBI	0.4119010	PLT	0.4187353	CUB	0.4227872	ATSG	0.4271925	CHTP	0.4322431
PRI	0.4122159	CLH	0.4187662	GSIT	0.4228535	GCO	0.4272290	PAG	0.4322451
STFC	0.4123928	TITN	0.4188548	PRA	0.4229347	MGI	0.4274947	MDSO	0.4322972
NGS	0.4124099	PQ	0.4188611	MIPS	0.4229790	BBOX	0.4275521	BC	0.4323483
BAGL	0.4125033	LAWS	0.4190017	STMP	0.4231863	HURN	0.4277964	CPK	0.4323945
MIDD	0.4125617	MTH	0.4191399	TR	0.4233767	CCC	0.4277987	CAKE	0.4324835
TTMI	0.4126969	HL	0.4191706	GSS	0.4234140	LXRX	0.4278784	ESL	0.4325690
SAFT	0.4127194	SASR	0.4192494	IART	0.4234627	DLGC	0.4279181	DHX	0.4326467
VVI	0.4129918	CMO	0.4192783	TTEK	0.4234773	ANEN	0.4281784	BCPC	0.4328320
DCTH	0.4130657	TICC	0.4193065	ETH	0.4236197	HW	0.4284998	SEAC	0.4328365
NAVG	0.4130688	PWOD	0.4194438	SSD	0.4237419	ESSX	0.4286374	MMS	0.4328492
MDC	0.4131553	EXPO	0.4195166	UNF	0.4237903	EXP	0.4287996	LG	0.4329045
DFZ	0.4131874	USNA	0.4197281	LNCE	0.4238871	MTRX	0.4288095	TINY	0.4329732
NWPX	0.4133449	RUTH	0.4198300	RELL	0.4239173	ACCL	0.4288356	RBCAA	0.4330361
FCEL	0.4138445	PSEC	0.4198767	SBSI	0.4242736	FAF	0.4288608	AGX	0.4330548
KWR	0.4141012	CHG	0.4199224	BKMU	0.4244117	DMRC	0.4289578	CNSL	0.4331038
HEI	0.4143516	CVCO	0.4200159	CLD	0.4244233	KEYN	0.4289839	DLLR	0.4332881
AMWD	0.4144764	VGR	0.4202677	PRK	0.4244383	CWT	0.4290732	ZOLT	0.4333531
BMRC	0.4145463	INFN	0.4203049	BFIN	0.4244920	ISH	0.4290880	PKE	0.4334417

Table 9 Hurst Exponents of RUSSELL 2000 for Testing Period 2011/1/1 - 2016/1/1

(Continued)

H Value of RUSSELL 2000 for Period 2011/1/1-2016/1/1									
Ticker	H	Ticker	H	Ticker	H	Ticker	H	Ticker	H
ASTE	0.4334510	ALNY	0.4370461	NBTB	0.4407991	CLW	0.4445221	BABY	0.4476782
HCSG	0.4335713	SMCI	0.4370791	MNRO	0.4410027	PTX	0.4447387	QADA	0.4477467
RLH	0.4335872	ATRI	0.4371396	ORBC	0.4411290	OMCL	0.4447848	HNH	0.4477492
SNHY	0.4336174	OLP	0.4371707	MTN	0.4412027	INN	0.4448304	BOOM	0.4477641
TOWN	0.4336990	HLS	0.4371801	AGII	0.4412256	AOS	0.4449284	TESO	0.4477831
MCS	0.4339149	BGG	0.4373584	CODI	0.4412500	AIQ	0.4449723	JOSB	0.4479590
MBVT	0.4339555	UAM	0.4373941	ATNI	0.4413025	SKYW	0.4450084	SPRT	0.4480459
PBH	0.4339714	ECHO	0.4374096	GBX	0.4413589	BBG	0.4450277	STNR	0.4482368
FCFS	0.4339964	PEET	0.4374727	TRK	0.4413926	IMMR	0.4450648	FNB	0.4482713
ACET	0.4340614	SUMR	0.4375571	GIFI	0.4414480	DLA	0.4450821	BID	0.4484178
UTL	0.4342091	PERY	0.4375674	TCAP	0.4415739	DLX	0.4451652	COR	0.4484572
MOV	0.4342666	MGLN	0.4376864	NCS	0.4417477	AVAV	0.4451685	MOSY	0.4484987
ONXX	0.4343813	LCUT	0.4377584	MAIN	0.4417827	KRG	0.4451783	GB	0.4485574
SNX	0.4344718	WIFI	0.4379062	DUSA	0.4417839	TRMK	0.4453950	HTGC	0.4485584
EVR	0.4344873	ACTG	0.4379768	MRX	0.4417884	STRA	0.4455624	TXRH	0.4485685
EZPW	0.4345150	CW	0.4379949	STBA	0.4418171	CAVM	0.4455990	VLGEA	0.4485780
GNC	0.4345714	RDEN	0.4380081	PEGA	0.4418227	TRST	0.4456533	HITT	0.4485830
UBNK	0.4346546	CAB	0.4381158	BNCL	0.4418426	KW	0.4457156	FARM	0.4486234
AKRX	0.4348738	WRE	0.4382019	ROIC	0.4419342	MGEE	0.4458680	TBI	0.4487451
CDZI	0.4348903	BLKB	0.4382539	BKI	0.4419628	MTZ	0.4459708	AXAS	0.4488666
DRRX	0.4349800	DXCM	0.4383408	GPK	0.4422519	ZIXI	0.4460686	PMT	0.4488900
MSA	0.4349836	MHO	0.4386534	WSR	0.4422911	LNN	0.4460696	WTS	0.4489050
GRM	0.4350988	JCS	0.4386669	CIR	0.4424511	OFLX	0.4463338	IXYS	0.4489822
OMEX	0.4351129	MGRC	0.4388128	ISCA	0.4424705	MOH	0.4463439	CVGW	0.4490384
IRBT	0.4352321	RNST	0.4389644	APEI	0.4427878	CCOI	0.4464132	CNS	0.4490461
FRME	0.4352612	AZPN	0.4390098	AIT	0.4428484	PWER	0.4464216	INT	0.4490834
STBZ	0.4353932	LDL	0.4391081	ACOR	0.4429282	LFUS	0.4464568	OVTI	0.4492829
BANF	0.4354462	FTEK	0.4391804	EPOC	0.4430155	BOBE	0.4466150	FCF	0.4493615
EFSC	0.4355796	LABL	0.4393591	CKP	0.4430173	IMKTA	0.4466639	GFF	0.4495172
POL	0.4356069	NFP	0.4393823	SGNT	0.4430692	ES	0.4466749	LOGM	0.4495641
LDR	0.4357095	MYRG	0.4393888	CHDX	0.4431457	WSBC	0.4467032	POR	0.4496301
HWCC	0.4357199	NPK	0.4395774	CCF	0.4431759	AVEO	0.4467171	LGF	0.4496429
WWD	0.4357802	TGI	0.4396055	MASI	0.4433512	MDCA	0.4467370	BFS	0.4496477
FNLC	0.4357892	LPX	0.4397870	RRTS	0.4435012	PDC	0.4467447	ISSI	0.4497368
GHDX	0.4358228	JBLU	0.4399128	ALK	0.4436215	KCP	0.4467467	PETS	0.4498015
WNR	0.4358738	MATW	0.4399905	GOOD	0.4436501	NEU	0.4469245	EPAY	0.4498212
DENN	0.4359054	AVA	0.4400548	MKSI	0.4436644	BLDR	0.4469592	WMGI	0.4498738
AGYS	0.4361080	SAFM	0.4401412	EBS	0.4436959	PACW	0.4470324	PRO	0.4499441
AGM	0.4362488	SYKE	0.4401863	MLHR	0.4436968	DCO	0.4470621	ACOM	0.4499577
EXAC	0.4363085	BHE	0.4402098	CHE	0.4438166	ARB	0.4470810	SGI	0.4500384
ASEI	0.4363098	PRMW	0.4402854	LOPE	0.4439384	NAFC	0.4471027	UNIS	0.4500491
THR	0.4364278	KOP	0.4403746	DGI	0.4440487	AMSF	0.4471598	EXR	0.4500521
CALM	0.4365045	ESE	0.4403884	MAA	0.4440535	WBS	0.4472478	BSFT	0.4501852
BSTC	0.4366832	EGY	0.4404844	EPM	0.4442743	HAFC	0.4473013	HIBB	0.4502510
USG	0.4367489	PCBK	0.4405543	PFS	0.4442772	NP	0.4473862	ACIW	0.4503053
ZAGG	0.4367935	SUSS	0.4407196	MVC	0.4444736	ALKS	0.4474450	DAN	0.4504161
UFPI	0.4369786	HTLF	0.4407301	CDI	0.4444871	FEIC	0.4475417	IRDM	0.4504423
SWS	0.4369921	HUBG	0.4407951	MSCC	0.4445132	DMD	0.4475708	VECO	0.4504896

Table 9 Hurst Exponents of RUSSELL 2000 for Testing Period 2011/1/1 - 2016/1/1 (Continued)

H Value of RUSSELL 2000 for Period 2011/1/1-2016/1/1									
Ticker	H	Ticker	H	Ticker	H	Ticker	H	Ticker	H
CASS	0.4505047	ACPW	0.4543399	ONB	0.4568444	VOLC	0.4605040	ACAT	0.4635426
HAE	0.4505960	EMAN	0.4543596	PLXT	0.4569536	NSIT	0.4606931	SQNM	0.4635480
CNC	0.4506672	HVT	0.4543709	OSIS	0.4570593	IMGN	0.4607083	UNS	0.4636692
EGBN	0.4507461	SNCR	0.4545853	ARRS	0.4571367	SWC	0.4607842	EXAM	0.4636891
EXXI	0.4510140	EXEL	0.4545913	ARR	0.4571591	PICO	0.4608097	FELE	0.4637271
DORM	0.4510290	ETM	0.4546350	EFII	0.4573961	FORM	0.4608235	MTRN	0.4637906
WRLD	0.4510350	PRSC	0.4547035	FSYS	0.4576028	SHO	0.4611189	FORR	0.4638520
ALOG	0.4510513	PHX	0.4547193	DWSN	0.4576828	SNTA	0.4611249	CERS	0.4638555
CDXS	0.4511916	GLRE	0.4548001	ABM	0.4576852	FF	0.4611570	UBA	0.4639063
WTI	0.4512128	UHAL	0.4548227	SIX	0.4579096	SKY	0.4611920	LCC	0.4641741
CATY	0.4512592	LMIA	0.4548244	NYT	0.4579209	BTX	0.4611990	DW	0.4642223
LVB	0.4513723	HTWR	0.4548402	TWI	0.4580373	TWO	0.4612308	RIGL	0.4642711
DOLE	0.4515744	IVR	0.4548420	PIR	0.4580767	ISLE	0.4612772	SLTM	0.4643444
TRNO	0.4515954	CASC	0.4548808	MTG	0.4581280	ALG	0.4613205	PRIM	0.4643517
SONC	0.4519179	JKHY	0.4549684	BLC	0.4582051	CTS	0.4614384	VIVO	0.4644212
NAT	0.4522091	AIRM	0.4549748	BODY	0.4582193	ESIO	0.4614398	RUE	0.4644516
EMCI	0.4523151	ERT	0.4550512	NEWS	0.4582643	HI	0.4615057	JBT	0.4646659
GSIG	0.4523885	IO	0.4551127	HMN	0.4583110	CRY	0.4616525	PTIE	0.4647250
AF	0.4524238	PRGS	0.4553086	TCBK	0.4583162	CBNJ	0.4616660	PRTS	0.4647771
BZ	0.4524446	AI	0.4553661	HELE	0.4583940	NNBR	0.4616739	VLTR	0.4647890
ACHN	0.4525954	MOD	0.4553697	FRM	0.4584302	HOS	0.4617074	CPLA	0.4649282
FINL	0.4526250	HNI	0.4553937	GAS	0.4584328	NATL	0.4617461	LNG	0.4650468
LNDC	0.4527079	DHIL	0.4554593	GPX	0.4585504	RUSHA	0.4618141	SJI	0.4650833
GEVO	0.4527698	DRH	0.4554842	COLB	0.4585891	NNI	0.4618229	ICFI	0.4650944
CNL	0.4527834	ININ	0.4555189	SRDX	0.4587040	RLI	0.4618473	MANH	0.4651763
B	0.4528327	SYMM	0.4555652	NWN	0.4587366	GOV	0.4618572	AXL	0.4652431
LYV	0.4528826	SCSC	0.4558287	EDR	0.4587610	CCMP	0.4621190	HIW	0.4652505
FSP	0.4528830	DGII	0.4559735	CPHD	0.4589311	AMCC	0.4621318	CRI	0.4652668
TYL	0.4528965	AEGR	0.4561212	RECN	0.4589650	MNTA	0.4621606	MCC	0.4652800
HUSA	0.4529158	FIBK	0.4561373	CMCO	0.4590121	ELS	0.4621787	POWI	0.4653510
VG	0.4529350	CCG	0.4561601	DX	0.4590297	PHH	0.4622170	KBH	0.4653723
ODFL	0.4529661	FFIC	0.4562019	IHC	0.4590483	HBIO	0.4622790	SNMX	0.4655012
TNK	0.4530273	MDTH	0.4562831	TBBK	0.4590570	LF	0.4623201	IPHI	0.4657339
BOFI	0.4534218	WSFS	0.4563193	INWK	0.4590900	MFB	0.4623876	ALE	0.4657540
ASCMA	0.4535289	WDFC	0.4564382	BCO	0.4591034	JAZZ	0.4624248	DAR	0.4658328
CALD	0.4535296	AYI	0.4565123	GLF	0.4592024	N	0.4624569	INAP	0.4659064
MDVN	0.4535376	NJR	0.4565172	SYNT	0.4592794	EQY	0.4625024	IBOC	0.4659109
INDB	0.4536181	CATM	0.4565945	WAC	0.4593574	CGI	0.4625111	CORE	0.4659110
BECN	0.4537565	NTRI	0.4566063	SSYS	0.4594357	DAKT	0.4625941	TNS	0.4661703
PNM	0.4537659	EE	0.4566454	NL	0.4595455	PULS	0.4627726	ULTI	0.4663990
CHMT	0.4537705	BKE	0.4566828	USAP	0.4597327	WGL	0.4629778	FFG	0.4665388
MBFI	0.4540181	HGR	0.4566894	BIRT	0.4597904	NUTR	0.4630163	TIVO	0.4665509
ECYT	0.4540387	RT	0.4567028	MG	0.4602502	PMC	0.4630700	IIVI	0.4666297
ZUMZ	0.4540715	SANM	0.4567040	CRUS	0.4603053	VVUS	0.4631202	AVD	0.4667485
ESC	0.4540846	CPTS	0.4567696	LZB	0.4603416	COWN	0.4633055	HCKT	0.4667930
RTIX	0.4542254	PRFT	0.4567813	OSUR	0.4603547	MDAS	0.4634141	MF	0.4668949
CPE	0.4543000	ACC	0.4567867	STSA	0.4604072	PVTB	0.4634643	PLAB	0.4669815
CBM	0.4543237	ASYS	0.4568158	FSTR	0.4604549	SYUT	0.4634822	GPOR	0.4670779

Table 9 Hurst Exponents of RUSSELL 2000 for Testing Period 2011/1/1 - 2016/1/1

(Continued)

H Value of RUSSELL 2000 for Period 2011/1/1-2016/1/1									
Ticker	H	Ticker	H	Ticker	H	Ticker	H	Ticker	H
TEN	0.4671823	HSTM	0.4704918	AFSI	0.4735273	CIA	0.4766666	MAPP	0.4802700
HZO	0.4671989	ARNA	0.4705372	AFP	0.4735414	WPP	0.4767035	NWY	0.4805009
LHO	0.4672416	WCG	0.4706299	LANC	0.4736069	SCLN	0.4768203	ALSK	0.4805060
VTG	0.4672837	SONS	0.4707411	HSNI	0.4736867	CMN	0.4769948	NVAX	0.4806875
WSTL	0.4672844	SWHC	0.4709094	PNFP	0.4737041	ZIOP	0.4771519	MED	0.4807442
BPI	0.4673515	QCOR	0.4709359	EEFT	0.4737439	IBKC	0.4773146	KAI	0.4807496
KRC	0.4674065	PLXS	0.4709429	ODC	0.4737786	MTSC	0.4774619	HNRG	0.4807989
LIOX	0.4674402	WGO	0.4709632	GSM	0.4738428	RDN	0.4775453	RAS	0.4808295
SUI	0.4675636	FARO	0.4711131	IPHS	0.4739349	CRVL	0.4776055	HIL	0.4808311
GERN	0.4677706	CNMD	0.4711323	IPXL	0.4741554	JMBA	0.4777551	DFT	0.4809338
HPY	0.4678546	SSS	0.4711452	AMRC	0.4741936	CLNY	0.4777739	BRS	0.4810558
TXI	0.4680848	QTM	0.4711973	HEES	0.4742191	TREX	0.4778067	GMAN	0.4811390
STL	0.4682754	CNK	0.4712619	CBL	0.4742340	PIKE	0.4778245	SLXP	0.4813682
FRPT	0.4682920	RA	0.4712831	CVBF	0.4742374	SWX	0.4778302	TUES	0.4814227
LSCC	0.4684129	CTB	0.4713160	NWLI	0.4742454	PZN	0.4778483	CCIX	0.4814676
COLM	0.4684315	WRC	0.4713563	NCI	0.4742491	IPCC	0.4778588	NEOG	0.4815676
CRZO	0.4685957	SCOR	0.4714649	PSSI	0.4742503	HOME	0.4781392	NVEC	0.4815720
COHR	0.4686705	MOVE	0.4716102	MPWR	0.4743078	FMBI	0.4782008	CALX	0.4815994
ATU	0.4687644	MMR	0.4717263	BRKL	0.4743392	WAL	0.4782346	PVA	0.4817411
HPP	0.4688432	CACC	0.4720248	STAA	0.4743398	UMH	0.4782917	MPAA	0.4818305
ROLL	0.4689210	OKSB	0.4720418	KLIC	0.4744377	FTK	0.4782992	STRL	0.4820744
FR	0.4689218	CRIS	0.4720893	SYX	0.4744433	RAD	0.4783241	UVV	0.4820780
NMRX	0.4690502	RGS	0.4721731	BLT	0.4745097	SFLY	0.4783573	QLIK	0.4822071
CPST	0.4690538	CUZ	0.4721886	XNPT	0.4745127	SHFL	0.4783585	ANGO	0.4822394
KBW	0.4690539	WABC	0.4722905	FNSR	0.4746717	ATRC	0.4783712	QNST	0.4822717
TWIN	0.4690642	NC	0.4724998	HLX	0.4747219	CTRN	0.4783910	SCVL	0.4823143
SKT	0.4690671	CMTL	0.4725229	IOSP	0.4747680	TTEC	0.4786052	LWAY	0.4823168
IIIN	0.4690903	TPC	0.4726362	UTEK	0.4748638	TRR	0.4787293	CBST	0.4823288
GVA	0.4691411	DVAX	0.4726613	URG	0.4748795	MGAM	0.4788410	HGG	0.4824185
MDP	0.4691627	SAM	0.4726752	EBF	0.4748946	BGS	0.4790281	UMPQ	0.4825104
OPY	0.4691806	DIOD	0.4728676	ROG	0.4749814	LUFK	0.4790696	SAIA	0.4825818
PJC	0.4691859	AIN	0.4729591	FCN	0.4750997	NUVA	0.4790926	SGK	0.4826227
SSI	0.4693889	PRAA	0.4729728	PRGX	0.4752381	ARQL	0.4791537	EGP	0.4826624
RCII	0.4695274	KELYA	0.4729999	IRET	0.4753424	CVG	0.4791859	FALC	0.4826662
MSO	0.4696613	BXS	0.4730289	AAWW	0.4755600	MRCY	0.4791866	ATRO	0.4827295
RSTI	0.4696783	DRIV	0.4730466	INSM	0.4756242	COHU	0.4793949	GEOY	0.4828585
SUP	0.4698212	DFR	0.4730957	UBSI	0.4757816	IRWD	0.4795823	ENOC	0.4828978
SCMP	0.4698859	SAVE	0.4731325	MPW	0.4759564	CBOU	0.4797033	MRLN	0.4829787
RNET	0.4699616	GRB	0.4731410	CSOD	0.4759580	IDA	0.4797240	RRGB	0.4830736
BZH	0.4700234	ABMD	0.4731631	TRC	0.4759695	KRA	0.4797433	STNG	0.4830904
CAS	0.4700556	BRC	0.4731772	REXX	0.4760133	FRED	0.4798657	GLT	0.4830909
LUB	0.4700584	PLCE	0.4732299	HF	0.4761061	ZEUS	0.4799392	ADTN	0.4831046
TYPE	0.4701910	TPLM	0.4732440	UTI	0.4761372	EGHT	0.4800711	LB	0.4833339
TWER	0.4702297	FN	0.4733289	PKY	0.4761925	EXLS	0.4800933	RPT	0.4833482
DCOM	0.4702525	RTK	0.4733380	EXPR	0.4763383	FSS	0.4801159	AFFX	0.4833563
SWSH	0.4703984	ULTR	0.4733533	PEB	0.4764084	CTBI	0.4802016	CBU	0.4834296
KFY	0.4704150	STWD	0.4734463	PKD	0.4764335	GLPW	0.4802048	URI	0.4834840
WMAR	0.4704537	TAXI	0.4734857	DSPG	0.4764405	HLIT	0.4802517	HCCI	0.4835036

Table 9 Hurst Exponents of RUSSELL 2000 for Testing Period 2011/1/1 - 2016/1/1

(Continued)

H Value of RUSSELL 2000 for Period 2011/1/1-2016/1/1									
Ticker	H	Ticker	H	Ticker	H	Ticker	H	Ticker	H
IRIS	0.4835198	SEM	0.4863369	GHM	0.4906957	NXTM	0.4943983	MENT	0.4981829
TOWR	0.4835696	SXI	0.4864179	AHT	0.4908930	TC	0.4944089	CMLS	0.4981908
AREX	0.4836220	AXE	0.4866266	GPI	0.4909165	XXIA	0.4944101	MHLD	0.4986188
MLNK	0.4836436	KEYW	0.4866323	RLJ	0.4909519	CORT	0.4944202	ADC	0.4987870
CYS	0.4838696	PGNX	0.4867746	OREX	0.4910001	IMMU	0.4945925	ALTE	0.4989172
PSB	0.4839048	SWI	0.4870272	SEH	0.4910654	UNTD	0.4947739	SYA	0.4989988
CLNE	0.4839131	TTWO	0.4870528	VHC	0.4910692	CVGI	0.4948670	POWR	0.4990319
UDRL	0.4840272	KEM	0.4872458	GST	0.4912395	ACW	0.4951295	HWAY	0.4991765
ABCD	0.4841401	AAT	0.4874279	CBB	0.4913749	ZZ	0.4951298	PCH	0.4992187
CSGS	0.4841524	SFNC	0.4874900	EXAS	0.4914385	ERII	0.4951653	MMSI	0.4992630
GIII	0.4841684	ALJ	0.4875019	FPO	0.4915076	SIGI	0.4954051	MSTR	0.4992840
UMBF	0.4842146	EPR	0.4875418	PBY	0.4915183	VRTU	0.4954281	IRC	0.4993016
GNRC	0.4842286	CHSP	0.4876053	SF	0.4916776	OXM	0.4955239	HTH	0.4994700
CYMI	0.4843679	AMED	0.4876527	MCHX	0.4917925	DEPO	0.4957312	TPCG	0.4995374
ECOL	0.4845069	RGR	0.4876873	VPHM	0.4918037	EXTR	0.4959160	SKS	0.4995767
JRN	0.4846061	PEI	0.4877321	MCRI	0.4918729	TNDM	0.4959259	ACTV	0.4996665
SQI	0.4846929	AVNW	0.4878175	AOI	0.4919248	WWW	0.4959727	AVID	0.4997965
SHEN	0.4847191	MDCO	0.4878768	MINI	0.4921422	KKD	0.4960406	BMR	0.4998123
SIVB	0.4847246	ARRY	0.4878936	HTS	0.4921823	OTTR	0.4960557	GTS	0.4998193
MGPI	0.4847387	NTGR	0.4879676	NKTR	0.4921964	CGNX	0.4960592	NRCI	0.4998660
JAKK	0.4847608	NTCT	0.4879961	NTLS	0.4922807	EDE	0.4961427	TGH	0.5002031
ALIM	0.4847765	NWE	0.4881080	ANDE	0.4923241	GSBC	0.4961980	SGMO	0.5002358
FOR	0.4848484	NEWP	0.4883013	ALTH	0.4923266	EPL	0.4963943	RMBS	0.5002643
ALCO	0.4848686	CENTA	0.4883342	LLNW	0.4923673	MFLX	0.4964047	HNSN	0.5002702
ASI	0.4848764	CHFC	0.4884765	IDCC	0.4924351	AOSL	0.4964074	CSU	0.5005475
ATEC	0.4849182	FUR	0.4885095	DUF	0.4924424	ZN	0.4964946	OAS	0.5006464
ENSG	0.4850106	FUL	0.4885325	CTCT	0.4924563	TSYS	0.4965497	OME	0.5010355
TSRA	0.4850686	CSGP	0.4885672	RP	0.4924618	CETV	0.4965889	AFAM	0.5010438
FNGN	0.4851244	SGY	0.4885853	HOMB	0.4925418	MIND	0.4966376	BGMD	0.5010687
CFX	0.4851526	CLDX	0.4886358	FXEN	0.4926214	USEG	0.4967670	LL	0.5010918
RJET	0.4852066	STAG	0.4888371	RAIL	0.4926570	PACR	0.4967732	DRL	0.5013394
LPSN	0.4853413	PCRX	0.4888721	AKR	0.4926951	CSWC	0.4968088	KND	0.5014411
ZINC	0.4853723	RDNT	0.4888835	ABCB	0.4927046	GTLS	0.4968401	SPNC	0.5014554
TCBI	0.4854161	TRGP	0.4890822	ENV	0.4927366	IMN	0.4969378	GCAP	0.5015047
HHS	0.4854678	EVC	0.4891191	PACB	0.4927450	GNCMA	0.4969597	ROMA	0.5016389
DCT	0.4855340	DYAX	0.4892139	EBIX	0.4927922	TRS	0.4969762	PMTI	0.5018113
EPIQ	0.4855812	UEIC	0.4892314	EGLE	0.4927928	NPTN	0.4970033	CHRS	0.5021536
DTSI	0.4856024	BEAT	0.4893537	DIN	0.4929173	UCTT	0.4970198	SYNA	0.5022199
FIRE	0.4856744	FCH	0.4893616	CDR	0.4929303	MANT	0.4970337	SMP	0.5022868
AMRS	0.4859521	AEIS	0.4894909	CHKE	0.4930782	CQB	0.4970819	SPTN	0.5022912
KTOS	0.4859609	REN	0.4895031	QDEL	0.4932547	BJRI	0.4973104	PCCC	0.5023008
CYNO	0.4860322	ECPG	0.4897440	RBCN	0.4937808	EXAR	0.4973455	UHT	0.5024456
HOV	0.4860397	BBSI	0.4897531	INTX	0.4939158	SGEN	0.4975909	DXPE	0.5026021
FICO	0.4861115	KS	0.4897710	HT	0.4940776	LORL	0.4977686	AMRI	0.5027398
JDAS	0.4861613	BDE	0.4898952	GWR	0.4941520	SCL	0.4977730	ORA	0.5028451
STC	0.4862056	AWR	0.4900762	GLUU	0.4942034	SCSS	0.4978097	PLFE	0.5035946
WST	0.4862195	SWFT	0.4902210	SMTC	0.4943787	AMPE	0.4980566	LXU	0.5036155
MAXY	0.4862465	BKH	0.4903374	BLX	0.4943825	OFIX	0.4981290	QUAD	0.5036662

Table 9 Hurst Exponents of RUSSELL 2000 for Testing Period 2011/1/1 - 2016/1/1

(Continued)

H Value of RUSSELL 2000 for Period 2011/1/1-2016/1/1									
Ticker	H	Ticker	H	Ticker	H	Ticker	H	Ticker	H
NCIT	0.5037775	RST	0.5083819	OLN	0.5147644	IDTI	0.5214834	REX	0.5322854
BH	0.5039129	POZN	0.5085391	WOR	0.5150928	ARC	0.5216505	ARIA	0.5323193
FMER	0.5039606	UEC	0.5086767	TZOO	0.5152823	PODD	0.5216869	LTS	0.5336296
SUBK	0.5040403	PSUN	0.5086894	FSII	0.5156443	VICL	0.5217465	SEMG	0.5337063
GLCH	0.5041722	JACK	0.5086904	AIR	0.5157467	EHTH	0.5217573	STEI	0.5342492
BEBE	0.5043254	SIGM	0.5090630	SBGI	0.5160647	ENZ	0.5224164	ASCA	0.5343031
LXP	0.5044626	UNFI	0.5092756	ASNA	0.5161766	HAIN	0.5228969	GTXI	0.5344143
LBY	0.5045269	CRMT	0.5093418	LTC	0.5161884	WIRE	0.5230614	TNAV	0.5345453
ISIS	0.5048773	OPEN	0.5093717	DK	0.5162069	NUS	0.5231749	TQNT	0.5348866
POWL	0.5049397	MTOR	0.5097159	CWEI	0.5163229	CONN	0.5232311	HSII	0.5349180
VOCS	0.5050581	OPTR	0.5098737	ODP	0.5163476	CSII	0.5233182	SHLM	0.5349420
GEO	0.5051696	MXL	0.5101280	BCRX	0.5167053	QSII	0.5234067	INTL	0.5354899
NNN	0.5053139	SMBL	0.5101820	ARO	0.5167538	AEL	0.5235003	UVE	0.5361858
FHCO	0.5054700	ZLC	0.5102756	MEAS	0.5168298	IDIX	0.5242262	CADX	0.5367853
FOLD	0.5054862	SSP	0.5104501	CDE	0.5169019	AEPI	0.5244451	PRX	0.5368518
SREV	0.5055012	RENT	0.5105097	UIS	0.5169587	PNX	0.5244653	TAYC	0.5370869
BWEN	0.5055077	CAR	0.5105431	ZGNX	0.5170519	NPSP	0.5246089	AHC	0.5372997
OFG	0.5055629	EMKR	0.5106533	ORRF	0.5170865	VRA	0.5246548	FXCM	0.5375911
DEST	0.5056731	FRP	0.5108037	VICR	0.5170924	AUXL	0.5248748	VCBI	0.5378509
MSPD	0.5056816	KDN	0.5109456	LQDT	0.5171201	EIG	0.5251291	FBC	0.5379523
ARII	0.5056984	NPO	0.5110042	ONTY	0.5171491	NHI	0.5255135	MTX	0.5385266
SRI	0.5058595	SGMS	0.5110852	LCI	0.5171761	MKTG	0.5258550	ITMN	0.5387755
IPAR	0.5059268	CLDT	0.5111181	VSAT	0.5173655	OCLR	0.5264031	FLWS	0.5387817
LIZ	0.5059550	TAL	0.5112172	SNSS	0.5176164	FRF	0.5268125	AFFY	0.5389784
HMSY	0.5059775	THRX	0.5113177	GLDD	0.5178616	BKS	0.5268755	SBRA	0.5403005
RTEC	0.5059927	USLM	0.5114666	REV	0.5179269	PNK	0.5272687	MXWL	0.5409664
AVNR	0.5060422	IILG	0.5114826	ARAY	0.5179393	LRN	0.5276548	CBLI	0.5413290
FFIN	0.5062710	RFMD	0.5115914	CBR	0.5179825	PDFS	0.5281537	SMSI	0.5414367
BGFV	0.5065071	VRNT	0.5117852	WIBC	0.5180864	ENS	0.5281608	SMRT	0.5415460
CRAY	0.5065274	MAKO	0.5118014	PNY	0.5183922	GUID	0.5283523	ANAC	0.5421843
CRDN	0.5065934	CVO	0.5118997	CBK	0.5188210	RSYS	0.5284456	HA	0.5422364
FOE	0.5066797	STXS	0.5120039	OMN	0.5188288	APAGF	0.5284601	HR	0.5425889
KCAP	0.5067965	MDCI	0.5122912	ACXM	0.5189248	KNXA	0.5284644	TASR	0.5433176
GBCI	0.5068005	HALO	0.5123859	MEI	0.5192393	RMTI	0.5286750	CBEY	0.5433381
AIMC	0.5068884	OZRK	0.5124221	SKX	0.5196612	CVI	0.5294385	OCN	0.5437547
PSMT	0.5070901	MNKD	0.5125338	ONE	0.5196789	VRTS	0.5297153	DYN	0.5437738
FMD	0.5072540	OGXI	0.5125975	NXST	0.5197232	WD	0.5301141	CYTX	0.5438112
BYD	0.5073034	ATHN	0.5128032	BBW	0.5199421	MW	0.5301218	TWGP	0.5439326
GTY	0.5074543	GDP	0.5128184	EXM	0.5200443	IVC	0.5301340	BAS	0.5439683
ORN	0.5075603	SZYM	0.5134520	FIZZ	0.5200638	OMX	0.5302449	XRTX	0.5441381
CRK	0.5076324	VSEC	0.5138069	TSRX	0.5201507	WLB	0.5305066	GPRE	0.5441740
GLNG	0.5076512	KEG	0.5141393	BWLD	0.5204109	ICON	0.5312298	NRF	0.5448712
IDT	0.5078121	CENX	0.5142573	AXTI	0.5205481	CLUB	0.5315602	IL	0.5453638
ROCK	0.5079453	SPPI	0.5143225	LEAP	0.5208917	CACI	0.5316059	KIRK	0.5454901
CBRL	0.5079946	SHOR	0.5144686	ALGT	0.5211701	DRCO	0.5316307	PTRY	0.5455294
OHI	0.5080864	DGIT	0.5144818	WG	0.5211905	NTSP	0.5316584	TG	0.5457155
SHLO	0.5081173	AMSC	0.5146464	SWM	0.5213541	VOXX	0.5318714	WWE	0.5457189
RWT	0.5082161	ELNK	0.5146917	PZZA	0.5213602	FVE	0.5319485	SRZ	0.5459443

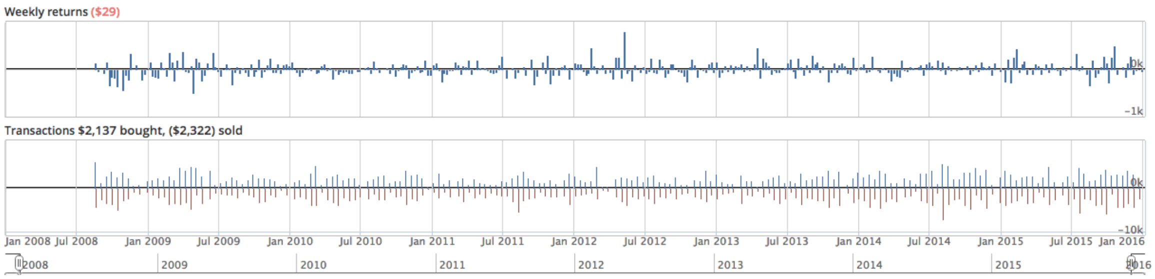
Table 9 Hurst Exponents of RUSSELL 2000 for Testing Period 2011/1/1 - 2016/1/1

(Continued)

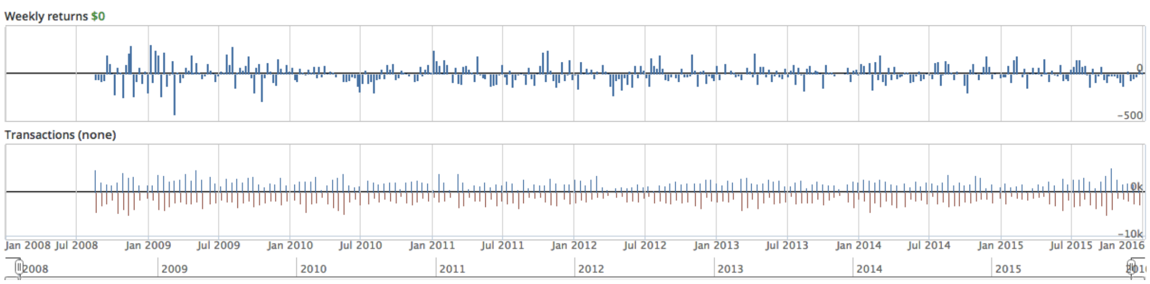
H Value of RUSSELL 2000 for Period 2011/1/1-2016/1/1									
Ticker	H	Ticker	H	Ticker	H	Ticker	H	Ticker	H
SAPE	0.5459600	RLD	0.5501654	FRO	0.5579715	BDC	0.5649062	AT	0.5778821
PKOH	0.5463519	OPK	0.5503331	VDSI	0.5581060	DHT	0.5657079	GTIV	0.5797584
AMAG	0.5464369	NSP	0.5503724	KCG	0.5581745	FFCH	0.5667159	BMTI	0.5841846
OSTK	0.5464970	OMPI	0.5506194	HMPR	0.5582704	INFI	0.5686834	PANL	0.5850261
COKE	0.5465329	WBSN	0.5520177	XRM	0.5588791	ANTH	0.5707367	BONT	0.5864920
FLDM	0.5487263	MNI	0.5532809	RPTP	0.5594312	GNOM	0.5745753	ITG	0.5881678
WRES	0.5490913	CNQR	0.5541992	ENTR	0.5597338	PSS	0.5747807	KSWS	0.5883573
BIOS	0.5501106	MUSA	0.5560617	ANAD	0.5607678	DMND	0.5764076	AMKR	0.5894763
DDD	0.5501630	GSAT	0.5573157	GTN	0.5644309	LINC	0.5776794	SUNH	0.5959816

Figure 7 Weekly P&L of Relative Strength on NASDAQ Stocks

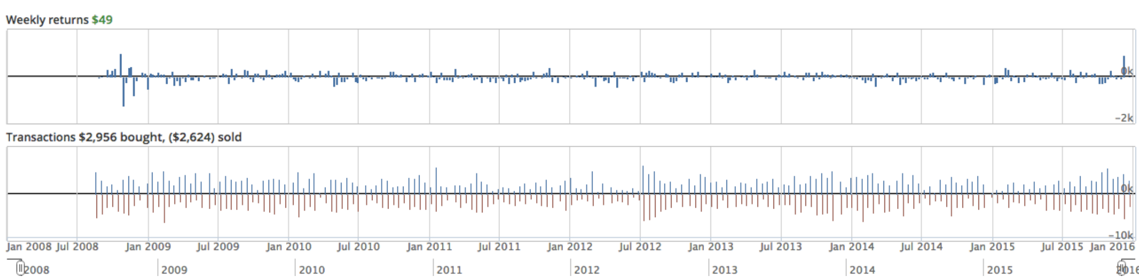
$0.3<H<0.4$



$0.4<H<0.45$



$0.45<H<0.5$



$0.5<H<0.6$

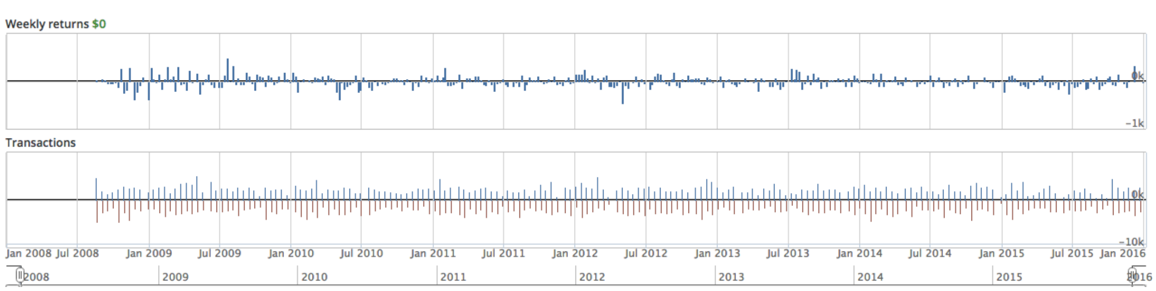
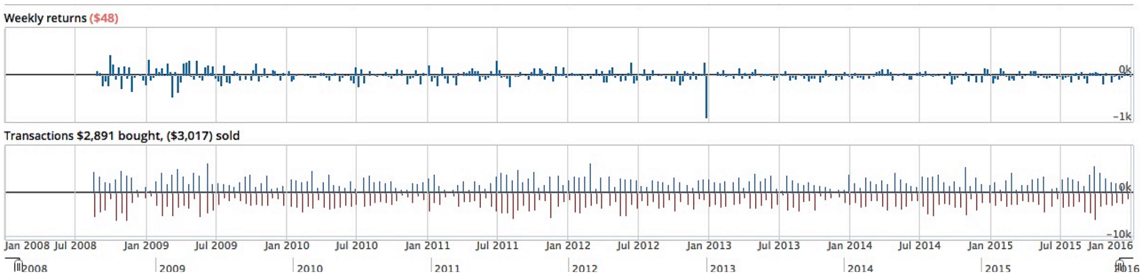
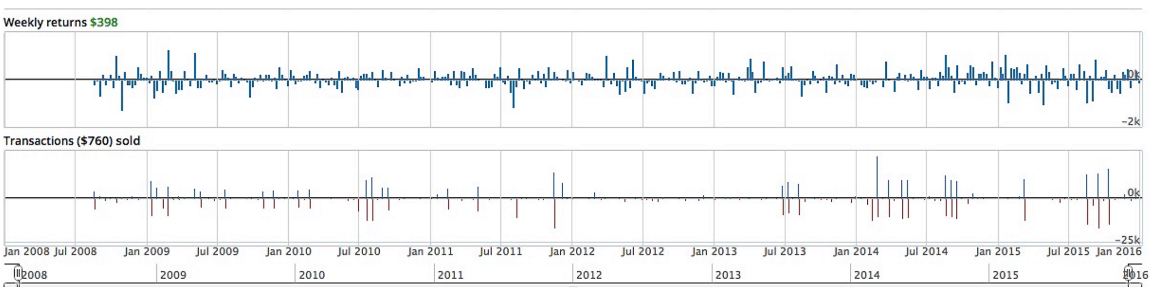


Figure 8 Weekly P&L of Relative Strength on Dow Jones Stocks

0.3<H<0.4



0.4<H<0.45



0.45<H<0.5

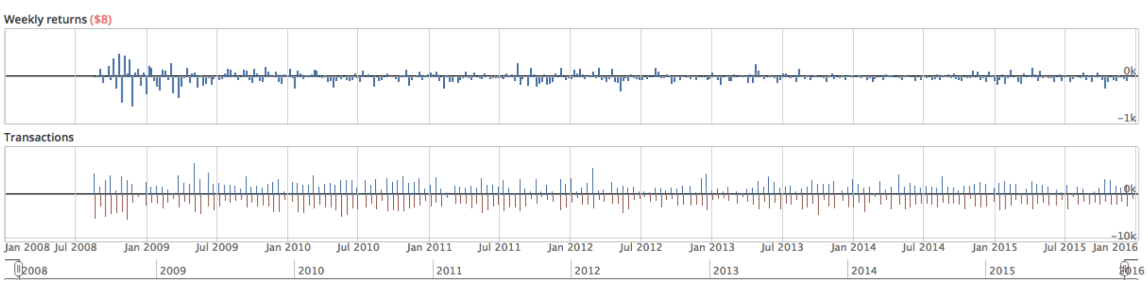
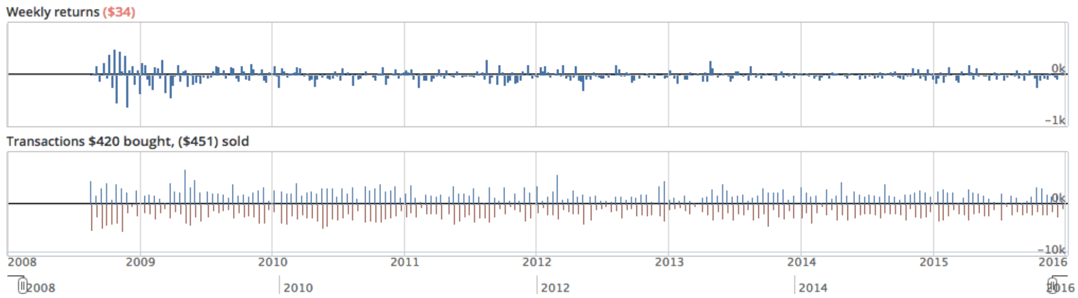
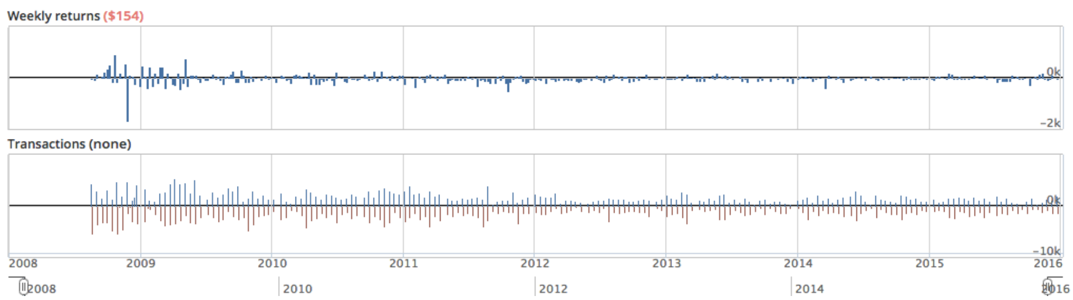


Figure 9 Weekly P&L of Relative Strength on RUSSELL2000 Stocks

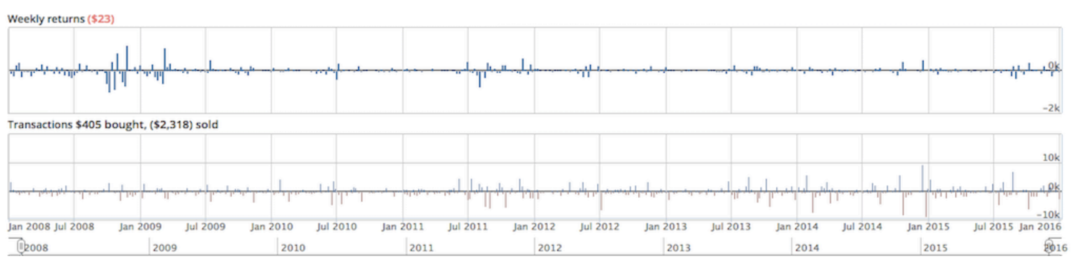
0.3<H<0.4



0.4<H<0.45



0.45<H<0.5



0.5<H<0.6

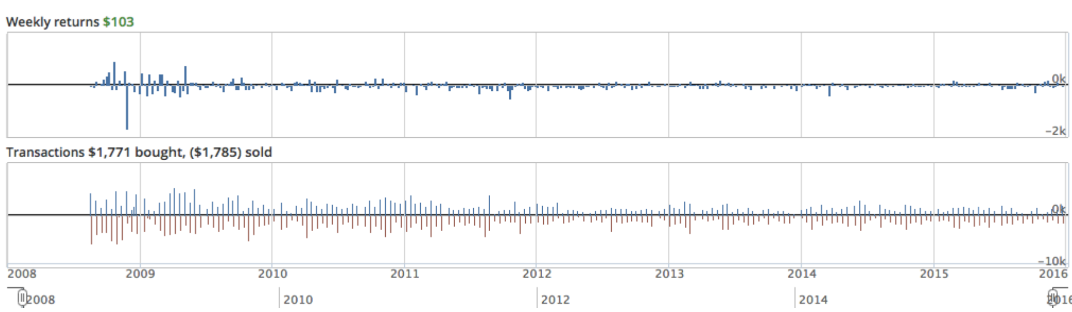
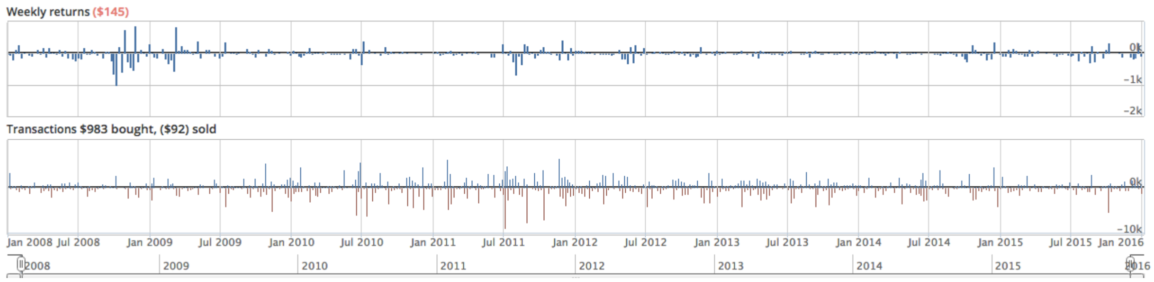
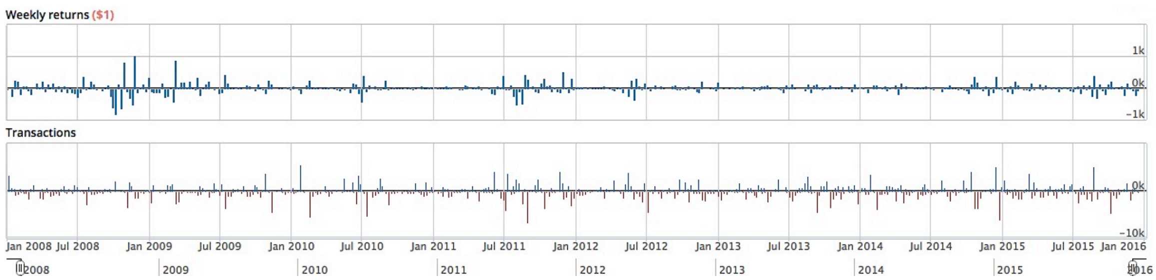


Figure 10 Weekly P&L of Stochastic Oscillator on S&P500 Stocks

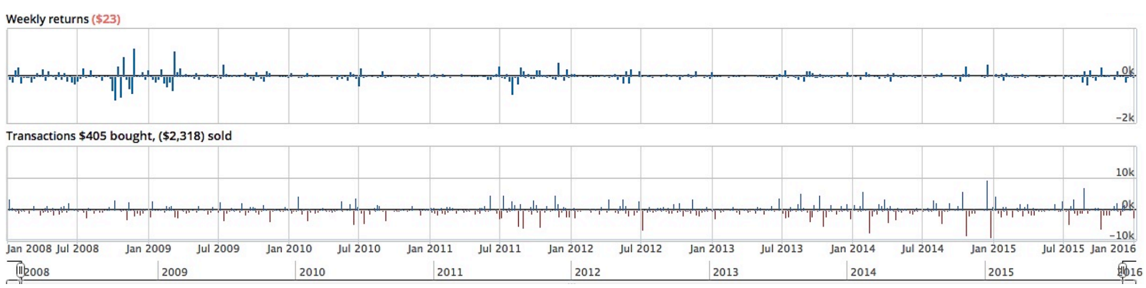
$0.3<H<0.4$



$0.4<H<0.45$



$0.45<H<0.5$



$0.5<H<0.6$

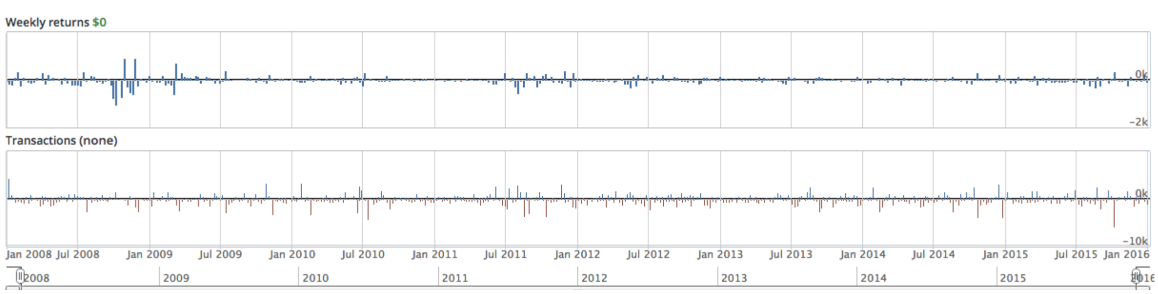
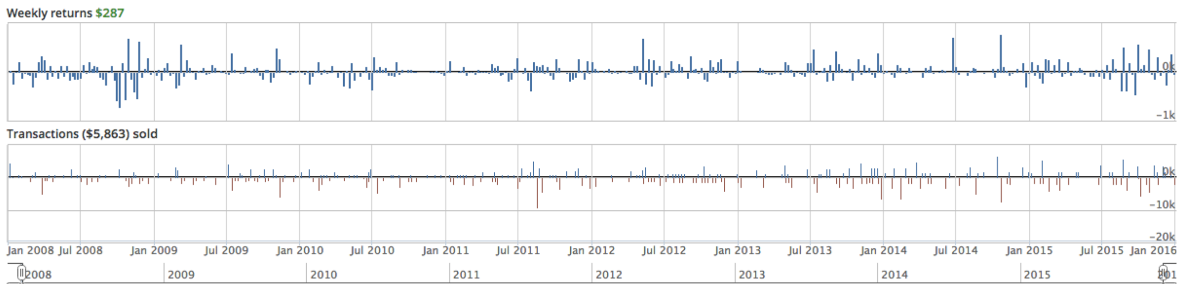
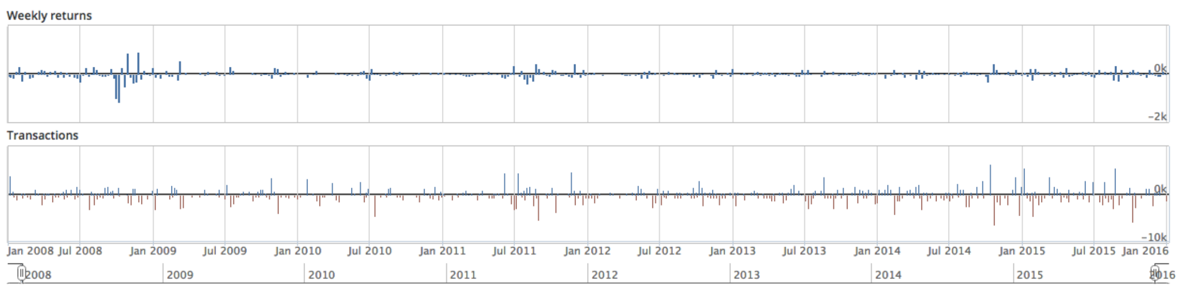


Figure 11 Weekly P&L of Stochastic Oscillator on NASDAQ Stocks

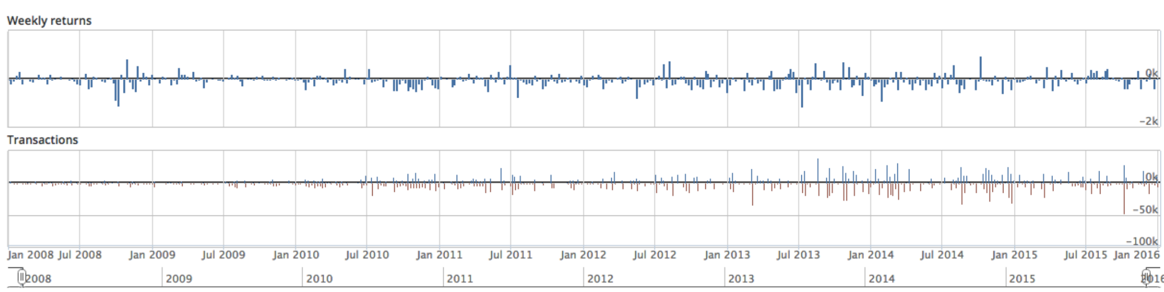
0.3<H<0.4



0.4<H<0.45



0.45<H<0.5



0.5<H<0.6

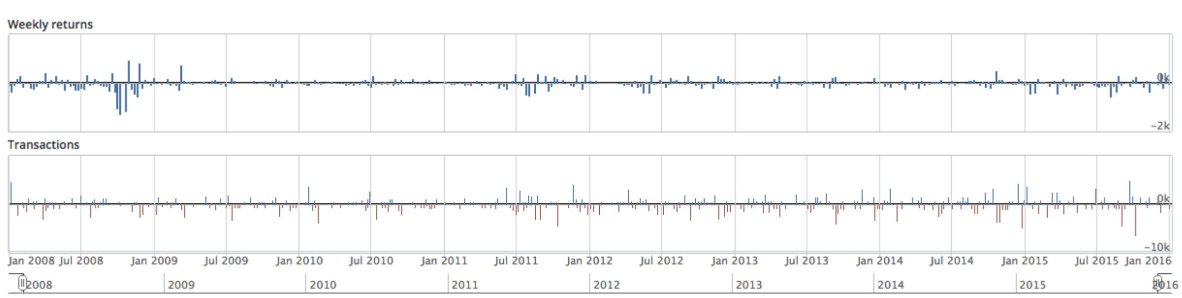
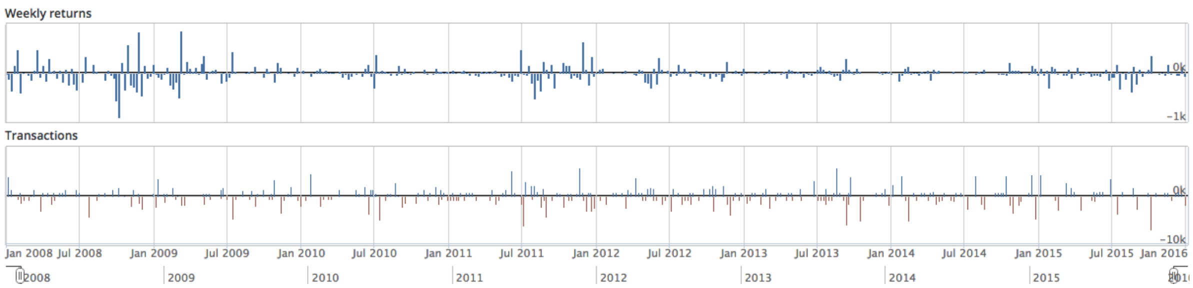
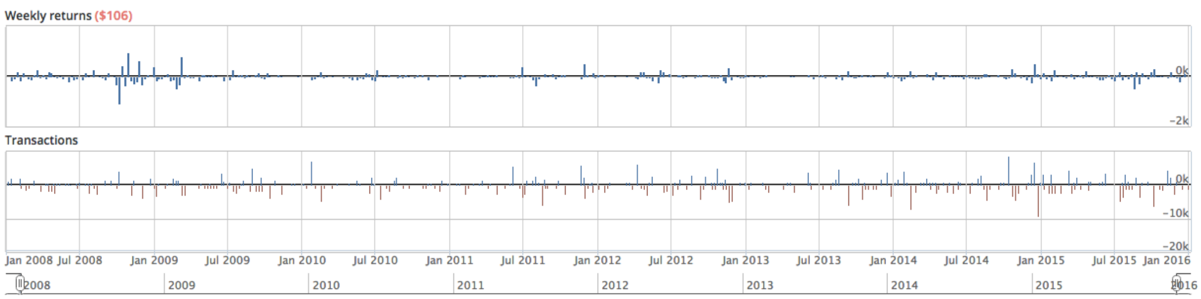


Figure 12 Weekly P&L of Stochastic Oscillator on Dow Jones Stocks

0.3<H<0.4



0.4<H<0.45



0.45<H<0.5

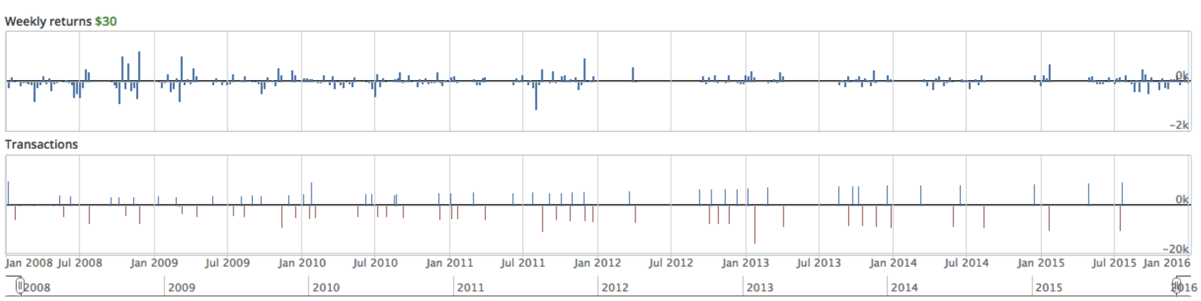
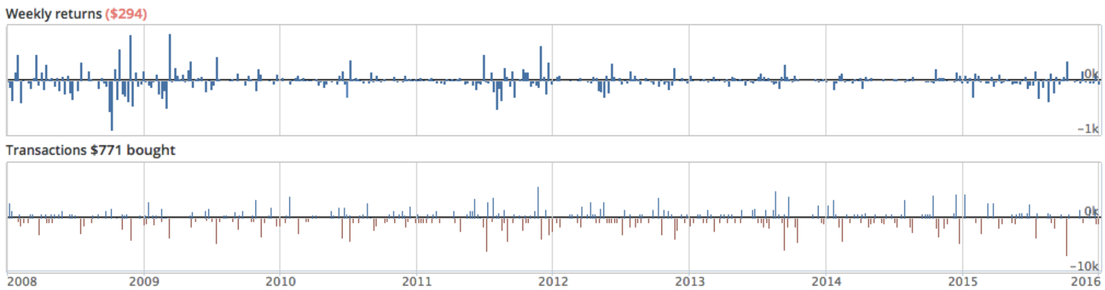
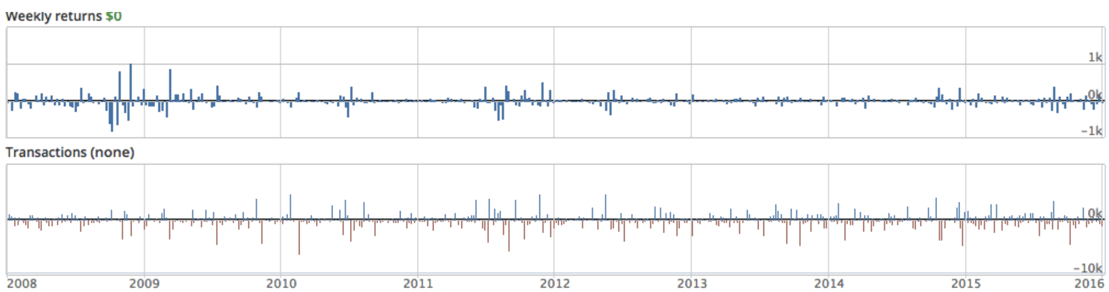


Figure 13 Weekly P&L of Stochastic Oscillator on RUSSELL2000 Stocks

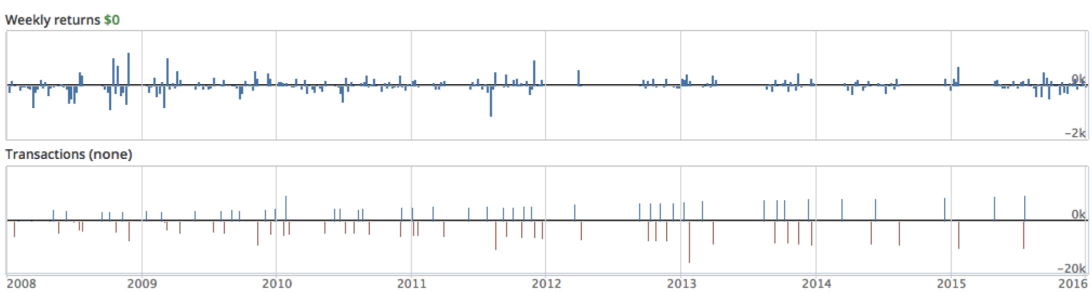
0.3<H<0.4



0.4<H<0.45



0.45<H<0.5



0.5<H<0.6

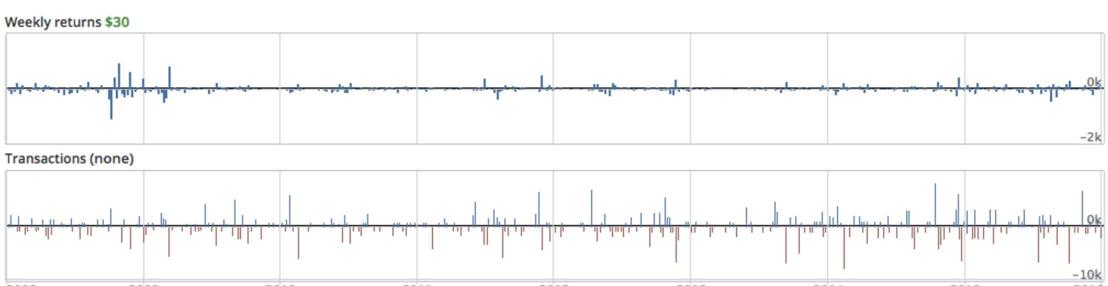
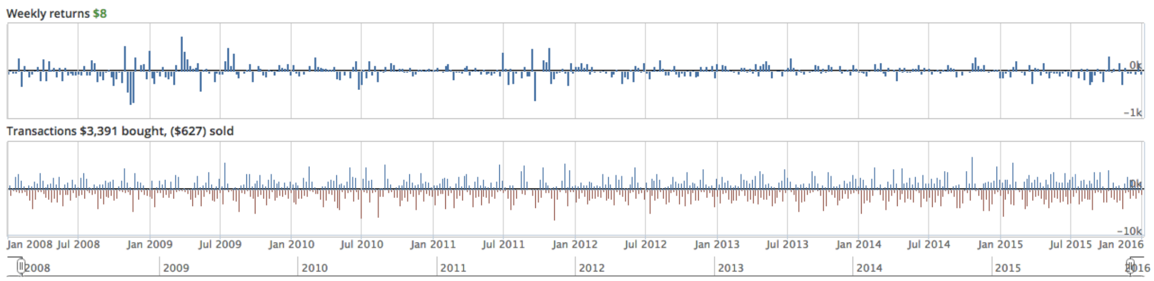
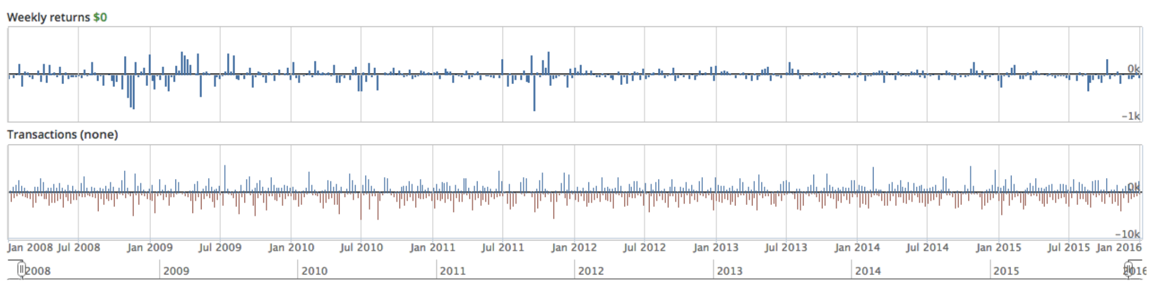


Figure 14 Weekly P&L of Moving Average Crossover on S&P500 Stocks

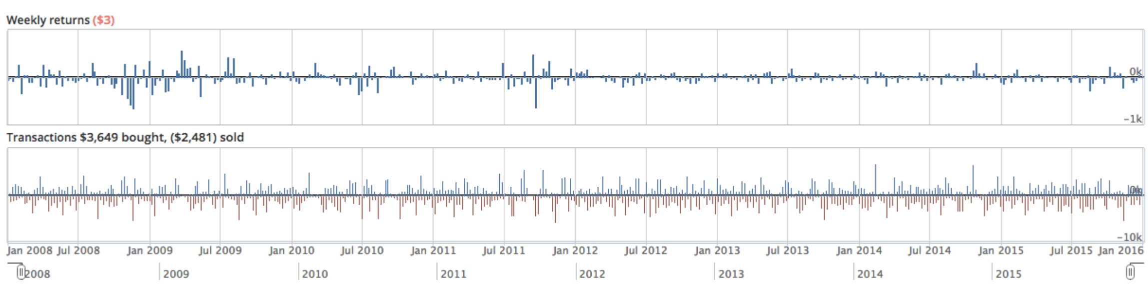
0.3<H<0.4



0.4<H<0.45



0.45<H<0.5



0.5<H<0.6

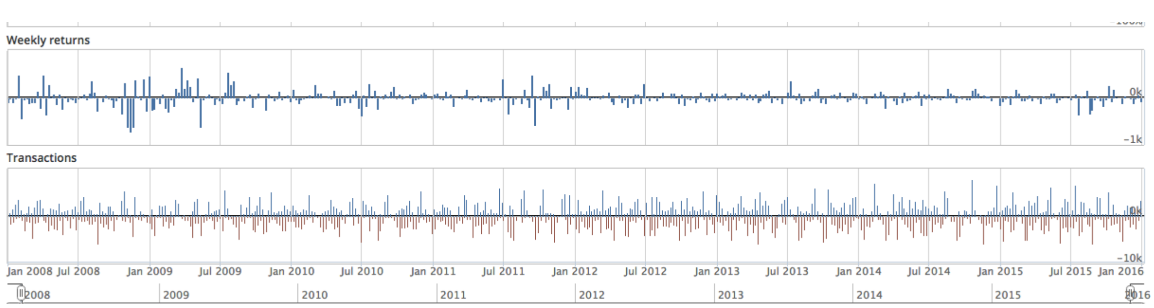
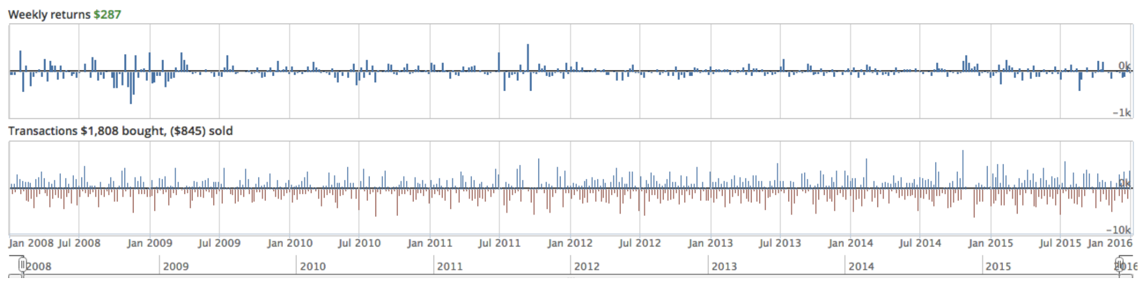
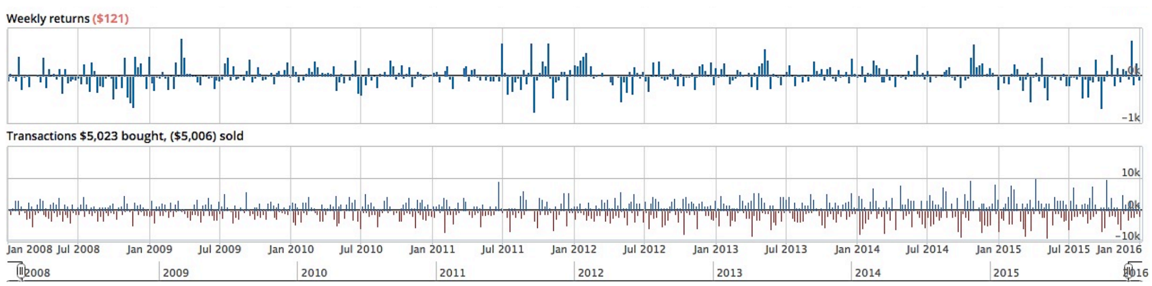


Figure 15 Weekly P&L of Moving Average Crossover on NASDAQ Stocks

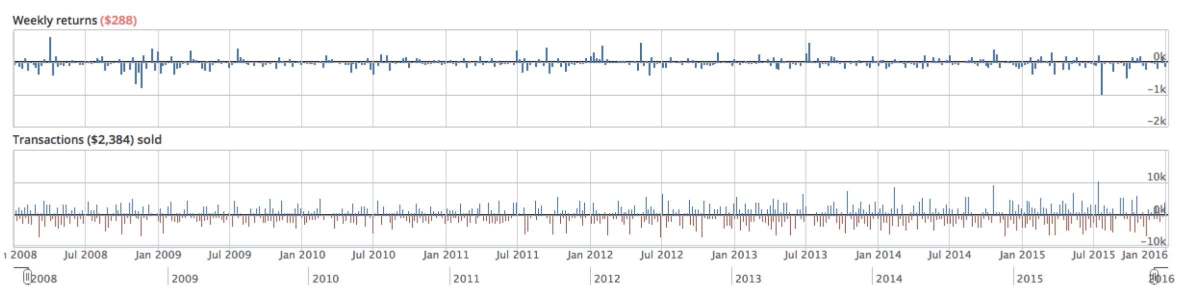
0.3<H<0.4



0.4<H<0.45



0.45<H<0.5



0.5<H<0.6

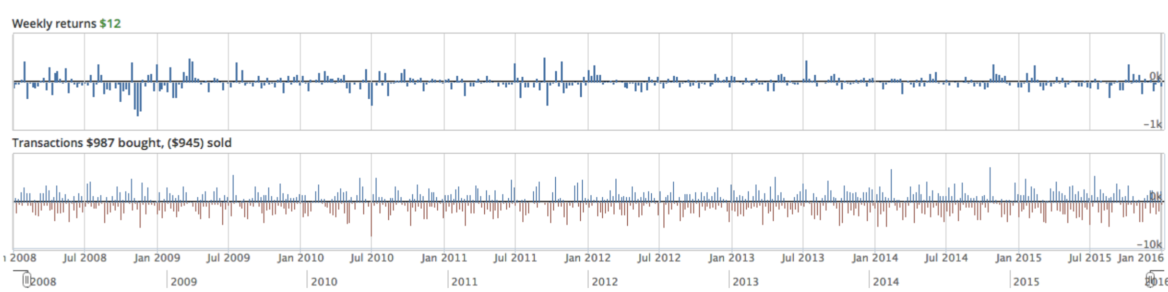
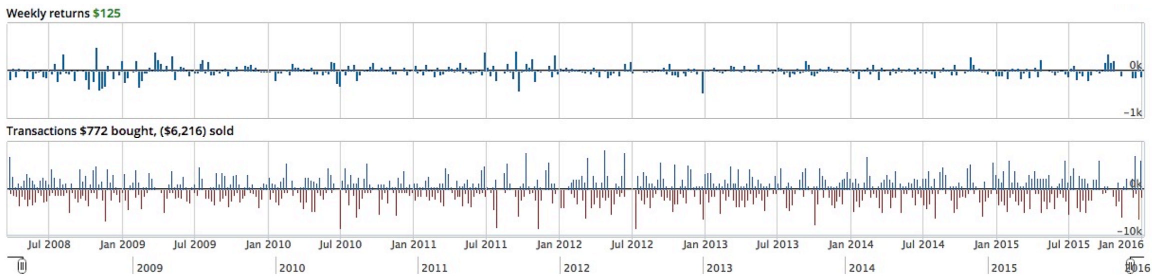
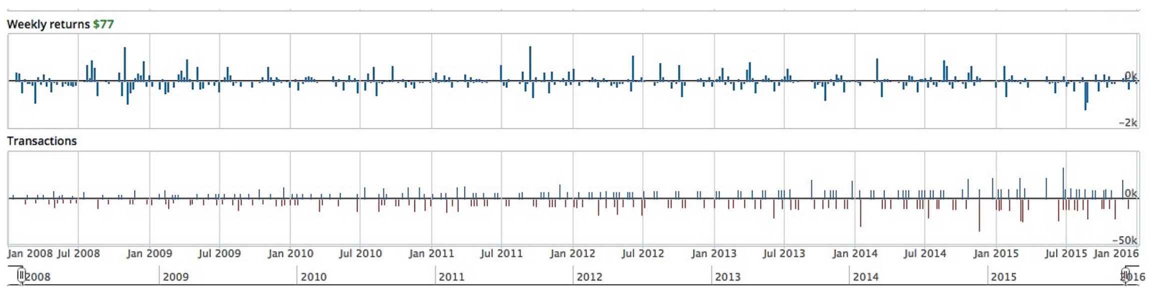


Figure 16 Weekly P&L of Moving Average Crossover on Dow Jones Stocks

0.3<H<0.4



0.4<H<0.45



0.45<H<0.5

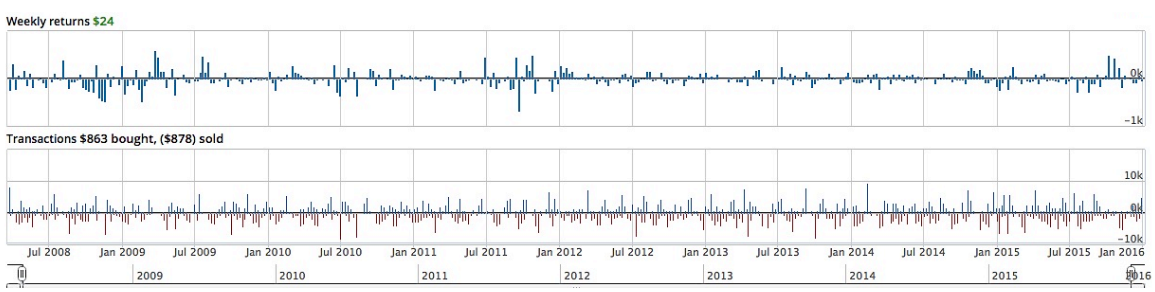
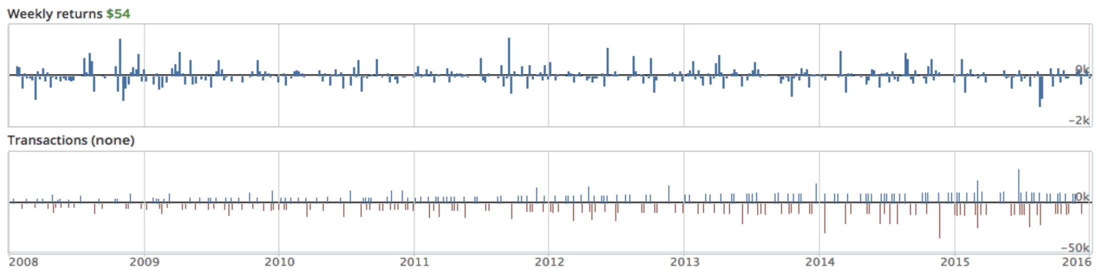
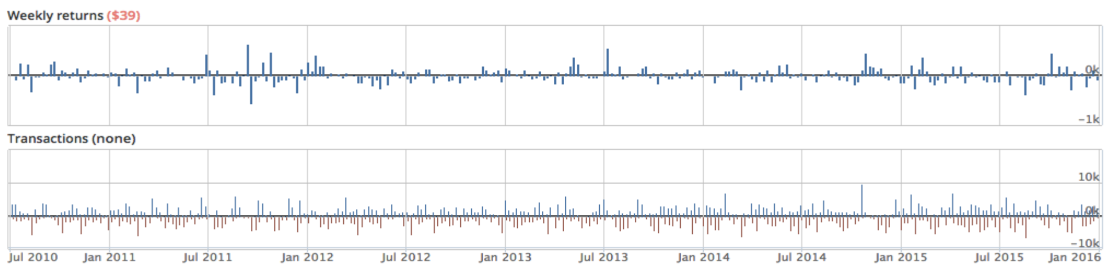


Figure 17 Weekly P&L of Moving Average Crossover on RUSSELL2000 Stocks

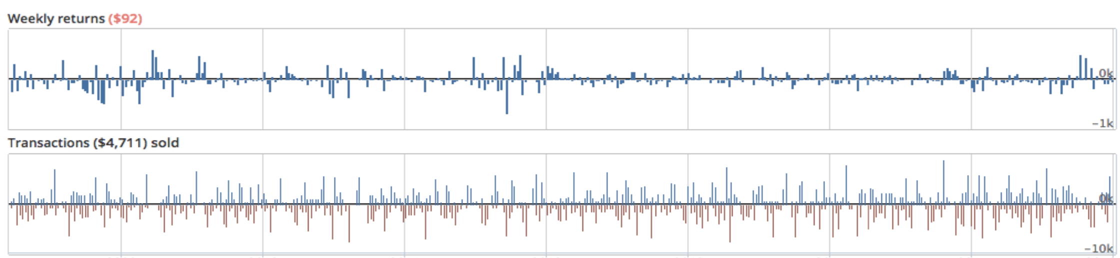
0.3<H<0.4



0.4<H<0.45



0.45<H<0.5



0.5<H<0.6

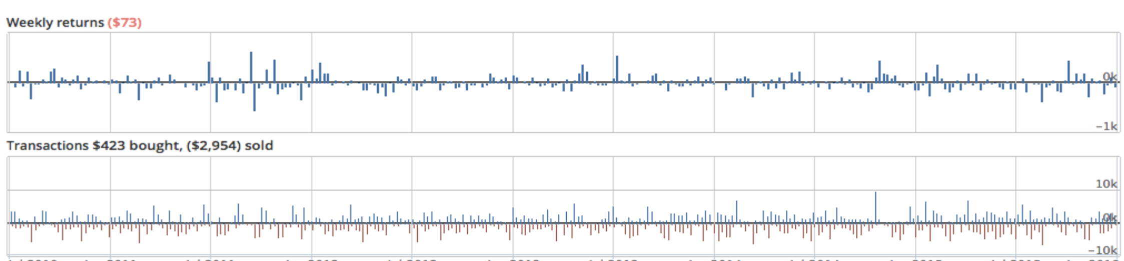


Figure 18 Weekly P&L of Moving Average Hurst Crossover on S&P500 Stocks

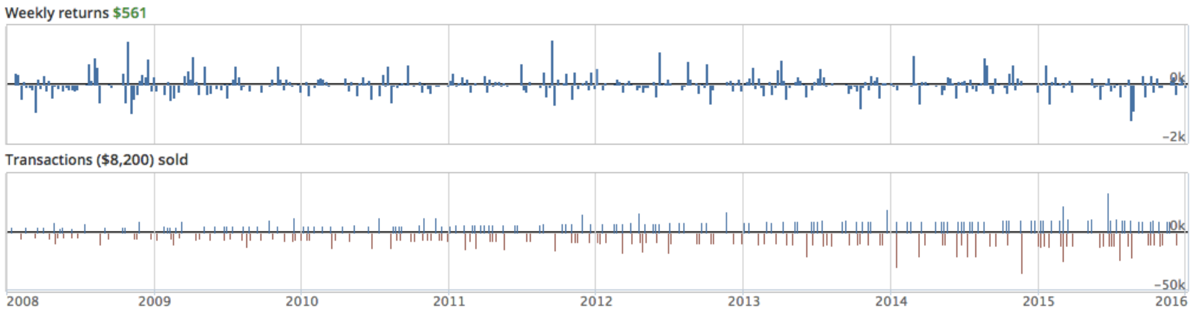


Figure 19 Weekly P&L of Moving Average Hurst Crossover on NASDAQ Stocks

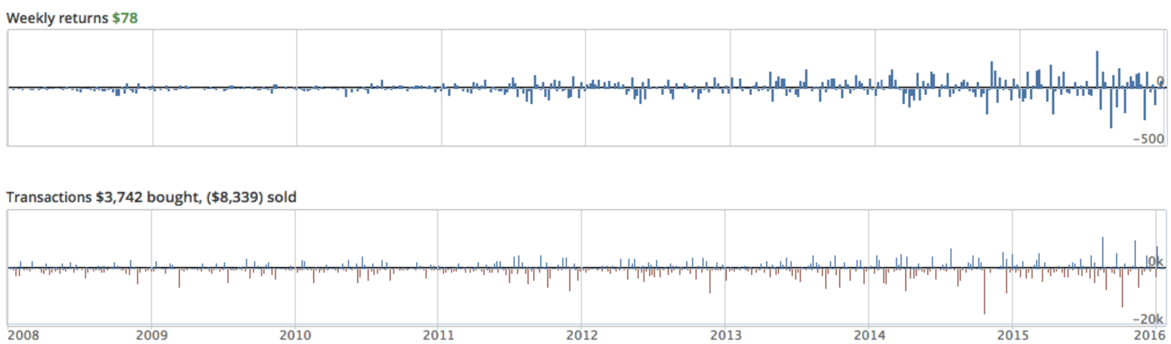


Figure 20 Weekly P&L of Moving Average Hurst Crossover on Dow Jones Stocks

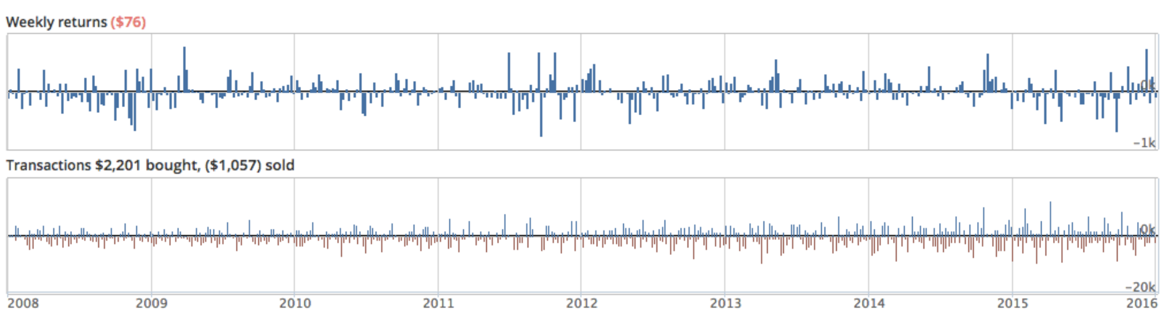
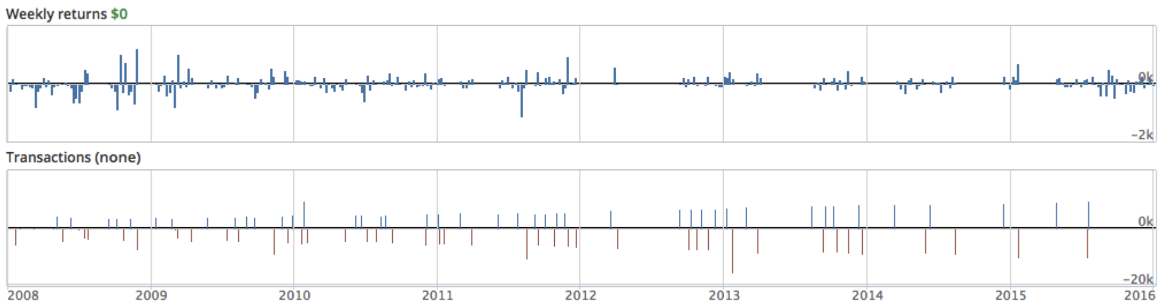


Figure 21 Weekly P&L of Moving Average Hurst Crossover on RUSSELL2000 Stocks



Python Code for Hurst Exponent Calculation Using Scaled Variance Ratio Method

```
'''
Created Jan 2016
@author: Zoe Miao
@summary: Python script to calculate the Hurst exponent of stock price series.
Note: Using 1008 observations, k  = 30
'''

#Enabling csv reading and writing
import pandas as pd
import os

# Import the Time Series library
import statsmodels.tsa.stattools as ts

# Import Datetime and the Pandas DataReader
from datetime import datetime
from pandas.io.data import DataReader

# Import from Numpy
from numpy import cumsum, log, polyfit, sqrt, std, subtract
from numpy.random import randn
```

```
# Hurst Exponent Calculation
def hurst(ts):
    """Returns the Hurst Exponent of the time series vector ts"""
    # Create the range of lag values
    lags = range(2, 30)

    # Calculate the array of the variances of the lagged differences
    tau = [sqrt(std(subtract(ts[lag:], ts[:-lag]))) for lag in lags]

    # Use a linear fit to estimate the Hurst Exponent
    poly = polyfit(log(lags), log(tau), 1)

    # Return the Hurst exponent from the polyfit output
    return poly[0]*2.0

# Download the stock data. Using AAPL as an example here.
ticker = "AAPL"
stock = DataReader(ticker, "yahoo", datetime(2011,1,1), datetime(2016,1,1))
print "Hurst of %s:  %s" % (ticker, hurst(stock['Adj Close']))

# Create a Gometric Brownian Motion, Mean-Reverting and Trending Series
gbm = log(cumsum(randn(100000))+1000)
mr = log(randn(100000)+1000)
```

```
tr = log(cumsum(randn(100000)+1)+1000)


# Output the Hurst Exponent for each of the above series
# and the price of stock (the Adjusted Close price) for
# the ADF test given above in the article
print "Hurst(GBM):   %s" % hurst(gbm)
print "Hurst(Mean Reversion):    %s" % hurst(mr)
print "Hurst(Trending):    %s" % hurst(tr)
```

Quantopian Code for Moving Average Hurst Crossover Strategy

```
import numpy as np
import pandas


def initialize(context):
#     https://dl.dropboxusercontent.com/u/332288829/SP500.csv
#     https://dl.dropboxusercontent.com/u/332288829/NASDAQ.csv
#     https://dl.dropboxusercontent.com/u/332288829/DIJA.csv
#     https://dl.dropboxusercontent.com/u/332288829/RUSSELL2000.csv

    fetch_csv("https://dl.dropboxusercontent.com/u/332288829/SP500.csv",
date_column='date', universe_func=my_universe, date_format = '%y-%m-%d')


def my_universe(context, fetcher_data):
    # fetcher_data is the data resulting from the CSV file from fetcher.
    # set my_stocks to be every security in the fetcher_data
    my_stocks = set(fetcher_data['sid'])

    # # log the size of the universe for debugging
    # context.count = len(my_stocks)
    # print 'total universe size: {c}'.format(c=context.count)

    # return the securities we identified earlier
```

```
    return my_stocks


def handle_data(context, data):
    #get the price data and compute the hurst value
    prices_1 = history(505, '1d', 'price')
    prices_2 = history(520, '1d', 'price')

     #-day MA return
    return_prices = history(6, '1d', 'price')
    returns = (return_prices[1:6].mean()-
return_prices[0:5].mean())/(return_prices[0:5].mean())

    for stock in data:
        H1_list = []
        H2_list = []
        for i in range(5):
            H1_list.append(Hurst(prices_1[stock][0:500+i]))
        H1 = sum(H1_list)/float(len(H1_list))

        for k in range(20):
            H2_list.append(Hurst(prices_2[stock][k:500+k]))
        H2 = sum(H2_list)/float(len(H2_list))
```

```
        if (returns[stock] > 0):
            #RRR, positive return
            if (H1 < H2) and (H1 < 0.5) and (H2 > 0.5):
                order_target(stock,-1)
            #TT, positive return
            elif (H1 > H2) and (H1 > 0.5) and (H2 < 0.5):
                order_target(stock,1)

        elif (returns[stock] < 0):
            #RRR, negative return
            if (H1 < H2) and (H1 < 0.5) and (H2 > 0.5):
                order_target(stock,1)
            #TT, negative return
            elif (H1 > H2) and (H1 > 0.5) and (H2 < 0.5):
                order_target(stock,-1)

'''
    Hurst exponent helps test whether the time series is:
    (1) A Random Walk (H ~ 0.5)
    (2) Trending (H > 0.5)
    (3) Mean reverting (H < 0.5)
'''
```

```
def Hurst( in_list ):
    lagvec = []
    tau    = []
    for lag in range(2,22):
        # Produce price different with lag
        pp = np.subtract(in_list[lag:],in_list[:-lag])
        # Write the different lags into a vector
        lagvec.append(lag)
        # Calculate the variance of the difference
        tau.append((np.std(pp))**2)
    # Linear fit to a double-log graph to get power
    m = np.polyfit(np.log10(lagvec),np.log10(tau),1)
    # Calculate hurst
    hurst = m[0]*0.5
    return hurst
```

Quantopian Code for Basic Momentum Strategies

1. Relative Strength Strategy

```
import numpy as np

import collections

import talib as ta


rsiPeriods   = 14

rsiIndicator = ta.RSI(timeperiod = rsiPeriods)


def initialize(context):

#     https://dl.dropboxusercontent.com/u/332288829/SP500.csv

#     https://dl.dropboxusercontent.com/u/332288829/NASDAQ.csv

#     https://dl.dropboxusercontent.com/u/332288829/DIJA.csv

#     https://dl.dropboxusercontent.com/u/332288829/RUSSELL2000.csv


    fetch_csv("https://dl.dropboxusercontent.com/u/332288829/SP500.csv",

date_column='date', universe_func=my_universe, date_format = '%y-%m-%d')


def my_universe(context, fetcher_data):

    # fetcher_data is the data resulting from the CSV file from fetcher.

    # set my_stocks to be every security in the fetcher_data

    my_stocks = set(fetcher_data['sid'])
```

```
    set_benchmark(  )


    context.dayCount = 0


   # set_commission(commission.PerTrade(cost=1.0))
   # set_slippage(TradeAtTheOpenSlippageModel(.1))


def handle_data(context, data):
    context.dayCount += 1
    if (context.dayCount < rsiPeriods or context.dayCount % 10 != 0):
        return


    ranked = {}
    rsiValues = rsiIndicator(data)
    rsiValuesMean = np.mean(rsiValues)
    if (rsiValuesMean == None or np.isnan(rsiValuesMean)):
        return
    rsiValuesStd  = np.std(rsiValues)
    record(RSIMean = rsiValuesMean)


    for stock in context.stocks:
        if stock not in data:
            continue
```

```
    rsi = rsiValues[stock]
    rsiZScore = (rsi - rsiValuesMean) / rsiValuesStd
    ranked[stock] = rsiZScore

# dictionary sorted by value
ranked = collections.OrderedDict(sorted(ranked.items(), key=lambda t: t[1]))

# Figure out where the long bias percent should end
longBiasPct = rsiValuesMean / 100.0
tradeCount  = len(ranked)
rsiSplit = longBiasPct * tradeCount
shortRsiSplit = tradeCount - rsiSplit

record(LongBias = longBiasPct, RsiSplit = rsiSplit, ShortRsiSplit = shortRsiSplit)
longTotalWeight = (rsiSplit * (rsiSplit + 1)) / 2
shortTotalWeight = (shortRsiSplit * (shortRsiSplit + 1)) / 2
totalLongFraction = 0
totalShortFraction = 0
l = 0
s = 0

# Now buy long those up ranked RSI securities
# and sell short those down ranked RSI securities
```

```
# Split at the boundary of RSI percent
for i in range(0, tradeCount):
    stock = ranked.keys()[i]
    if stock not in data:
        continue

    if (i <= rsiSplit):
        longWeight = rsiSplit - l
        longFraction = (longWeight / longTotalWeight) * longBiasPct
        totalLongFraction += longFraction
        order_target_percent(stock, longFraction)
        #print(" Long Symbol: {0} %: {1}".format(stock.symbol, longFraction))
        l += 1
    else:
        shortWeight = shortRsiSplit - s
        shortFraction = -(shortWeight / shortTotalWeight) * (1.0 - longBiasPct)
        totalShortFraction += shortFraction
        order_target_percent(stock, shortFraction)
        #print("Short symbol: {0} %: {1}".format(stock.symbol, shortFraction))
        s += 1

#print("long %: {0}  short %: {1}".format(totalLongFraction, totalShortFraction))
```

```
#########################################################
class TradeAtTheOpenSlippageModel(slippage.SlippageModel):
    def __init__(self, fractionOfOpenCloseRange):
        self.fractionOfOpenCloseRange = fractionOfOpenCloseRange

    def process_order(self, trade_bar, order):
        openPrice = trade_bar.open_price
        closePrice = trade_bar.price
        ocRange = closePrice - openPrice
        ocRange = ocRange * self.fractionOfOpenCloseRange
        if (ocRange != 0.0):
            targetExecutionPrice = openPrice + ocRange
        else:
            targetExecutionPrice = openPrice

        # Create the transaction using the new price we've calculated.
        return slippage.create_transaction(
            trade_bar,
            order,
            targetExecutionPrice,
            order.amount)
```

2. Stochastic Oscillator Strategy

```
import talib

import numpy as np

import pandas as pd


# Setup our variables

def initialize(context):

#     https://dl.dropboxusercontent.com/u/332288829/SP500.csv

#     https://dl.dropboxusercontent.com/u/332288829/NASDAQ.csv

#     https://dl.dropboxusercontent.com/u/332288829/DIJA.csv

#     https://dl.dropboxusercontent.com/u/332288829/RUSSELL2000.csv


    fetch_csv("https://dl.dropboxusercontent.com/u/332288829/SP500.csv",
date_column='date', universe_func=my_universe, date_format = '%y-%m-%d')


def my_universe(context, fetcher_data):

    # fetcher_data is the data resulting from the CSV file from fetcher.

    # set my_stocks to be every security in the fetcher_data

    my_stocks = set(fetcher_data['sid'])


    set_benchmark(  )
```

```
    # Set the percent of the account to be invested per stock
    context.long_pct_per_stock = 1.0 / len(context.stocks)

    # Create a variable to track the date change
    context.date = None

def handle_data(context, data):
    todays_date = get_datetime().date()

    # Do nothing unless the date has changed
    if todays_date == context.date:
        return
    # Set the new date
    context.date = todays_date

    # Load historical data for the stocks
    high = history(30, '1d', 'high')
    low = history(30, '1d', 'low')
    close = history(30, '1d', 'close_price')

    # Iterate over our list of stocks
    for stock in context.stocks:
        current_position = context.portfolio.positions[stock].amount
```

```
slowk, slowd = talib.STOCH(high[stock],
                           low[stock],
                           close[stock],
                           fastk_period=5,
                           slowk_period=3,
                           slowk_matype=0,
                           slowd_period=3,
                           slowd_matype=0)

# get the most recent value
slowk = slowk[-1]
slowd = slowd[-1]

# If either the slowk or slowd are less than 10, the stock is
# 'oversold,' a long position is opened if there are no shares
# in the portfolio.
if slowk < 10 or slowd < 10 and current_position <= 0:
    order_target_percent(stock, context.long_pct_per_stock)

# If either the slowk or slowd are larger than 90, the stock is
# 'overbought' and the position is closed.
elif slowk > 90 or slowd > 90 and current_position >= 0:
    order_target(stock, 0)
```

2. Moving Average Crossover Strategy

```
import talib

import numpy as np

import pandas as pd


def initialize(context):

#     https://dl.dropboxusercontent.com/u/332288829/SP500.csv

#     https://dl.dropboxusercontent.com/u/332288829/NASDAQ.csv

#     https://dl.dropboxusercontent.com/u/332288829/DIJA.csv

#     https://dl.dropboxusercontent.com/u/332288829/RUSSELL2000.csv


    fetch_csv("https://dl.dropboxusercontent.com/u/332288829/SP500.csv",

date_column='date', universe_func=my_universe, date_format = '%y-%m-%d')


def my_universe(context, fetcher_data):

    # fetcher_data is the data resulting from the CSV file from fetcher.

    # set my_stocks to be every security in the fetcher_data

    my_stocks = set(fetcher_data['sid'])


    set_benchmark(  )



    context.pct_per_stock = 1.0 / len(context.stocks)
```

```
    # Create a variable to track the date change
    context.date = None

def handle_data(context, data):
    todays_date = get_datetime().date()

    # Do nothing unless the date has changed and its a new day.
    if todays_date == context.date:
        return

    # Set the new date
    context.date = todays_date

    # Load historical data for the stocks
    prices = history(40, '1d', 'price')

    # Create the MACD signal and pass in the three parameters: fast period, slow period,
and the signal.
    # This is a series that is indexed by sids.
    macd = prices.apply(MACD, fastperiod=12, slowperiod=26, signalperiod=9)

    # Iterate over the list of stocks
```

```
    for stock in context.stocks:
        current_position = context.portfolio.positions[stock].amount

        # Close position for the stock when the MACD signal is negative and we own shares.
        if macd[stock] < 0 and current_position > 0:
            order_target(stock, 0)

        # Enter the position for the stock when the MACD signal is positive and
        # our portfolio shares are 0.
        elif macd[stock] > 0 and current_position == 0:
            order_target_percent(stock, context.pct_per_stock)


# Define the MACD function
def MACD(prices, fastperiod=12, slowperiod=26, signalperiod=9):
    '''
    Function to return the difference between the most recent
    MACD value and MACD signal. Positive values are long
    position entry signals

    optional args:
        fastperiod = 12
```

```
    slowperiod = 26
    signalperiod = 9

Returns: macd - signal
'''
macd, signal, hist = talib.MACD(prices,
                                fastperiod=fastperiod,
                                slowperiod=slowperiod,
                                signalperiod=signalperiod)
return macd[-1] - signal[-1]
```