ESSAYS IN PUBLIC FINANCE

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Kiel Haynes Albrecht

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ESSAYS IN PUBLIC FINANCE

Kiel Haynes Albrecht, Ph.D.

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In Chapter 2, I develop a basic model of housing prices, in which the level of capitalization depends positively on the absolute value of the elasticity of demand, and negatively on the elasticity of supply and the level of housing demand growth. Using 2001 – 2009 annual public pension asset and liability data from the Boston College Center for Retirement Research, I test the implications of the model. I measure the extent to which unfunded public pension liabilities are capitalized into housing prices, and test whether states with higher housing demand growth experience less capitalization. Instrumenting for unfunded liabilities using initial asset class levels adjusted for average market returns, I find that a one dollar per household increase in unfunded pension liabilities corresponds to a six and a half dollar decrease in average housing prices. In addition, I find that states which experience faster future household growth experience less capitalization.

Chapter 3 outlines a new argument for the use of tax credits for contributions to the public good: that tax credits for contributions to the public good can reduce tax avoidance. Specifically, I develop a simple model with non-labor distorting taxes in which the introduction of tax credits for voluntary contributions to the public good can completely eliminate tax avoidance and allow the social planner to implement the first-best level of the public good. This result is robust to a number of extensions, including the introduction of warm-glow preferences. I

also show that when labor-distorting taxes are used to finance the public good, the introduction of tax credits for contributions to the public good can eliminate tax avoidance and decrease the labor wedge.

Chapter 4 outlines an explanation for the relative generosity of deferred public sector compensation relative to deferred private sector compensation. Using a basic political economy model, I demonstrate that the preferences of the median public sector employee can lead to policies which favor increases in defined benefit pensions over either increases in salary or decreases in employee contributions towards their pensions. In this chapter, I present a simple model in which the current median public sector employee determines how to accept a given increase in the present value of employer pension contributions: through either decreased employee contributions or through increased pension benefits. I demonstrate that if the discount rate is sufficiently small, then the present value of increased future pension benefits exceeds the present value of the decrease in employee pension contributions for the median public employee, and the median public employee will thus prefer a pension benefit increase over a decrease in employee contributions. In addition, I demonstrate that under a wide range of plausible parameters, the median public employee will prefer benefit increases over employee contribution cuts.

BIOGRAPHICAL SKETCH

Kiel Albrecht is a Ph.D. student in the Department of Economics. Originally from Akron, OH, he graduated *Summa cum Laude* from Washington University with a B.A. in Economics and Mathematics. Kiel resides in Gaithersburg, MD with his wife, Katie and daughter, Liliana.

This document is dedicated to my family, for their help and support during my time at Cornell.

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CHAPTER 1

INTRODUCTION

1.1 Overview

This thesis focuses on two distinct areas within public finance: public sector pensions, and the provision of public goods. Specifically, Chapter 3 explores how tax expenditures for contributions to a public good can reduce, and under certain conditions, eliminate tax avoidance. Both Chapters 2 and 4 focus on public sector pensions. Chapter 2 evaluates the effect of public pension liabilities, while Chapter 4 provides an explanation for the generosity of public sector pensions. In Chapter 2, I estimate the level of capitalization of unfunded public pension liabilities on state-level average housing prices. Chapter 4 analyzes the political economy of unions impact on preferences for deferred compensation.

1.2 Capitalization of Unfunded Public Pension Liabilities

Public pensions are a significant and growing liability for state and local governments. In Chapter 2, I develop a basic model of housing prices, in which the level of capitalization depends positively on the absolute value of the elasticity of demand, and negatively on the elasticity of supply and the level of housing demand growth. Using 2001-2009 annual public pension asset and liability data from the Boston College Center for Retirement Research, I test the implications of the model. I measure the extent to which unfunded public pension liabilities are capitalized into housing prices, and test whether states with higher housing demand growth experience less capitalization. I instrument for the level of underfunding by taking the initial assets of the pension system invested in domestic equities, international equities, domestic bonds, international bonds, and real estate investment trusts, and adjust each asset class by average indexed returns. Consistent with the previous literature, I find large negative impacts of underfunding of public pension systems on housing prices. Controlling for state-specific time trends, I find that a one dollar per household increase in unfunded pension liabilities corresponds to a six and a half dollar decrease in average housing prices. In addition, consistent with the model, I find modest evidence that states which experience faster future household growth experience less capitalization.

1.3 Tax Credits and Avoidance

Chapter 3 outlines a new argument for the use of tax credits for contributions to the public good: that tax credits for contributions to the public good can reduce tax avoidance. Specifically, I develop a simple model with non-labor distorting taxes in which the introduction of a non-refundable tax credits for voluntary contributions to the public good can completely eliminate tax avoidance and implement the first-best level of the public good. By setting the non-refundable tax credit sufficiently high, such that the private marginal utility from giving is at least as large as the marginal utility from avoiding taxes, the government can induce each taxpayer to contribute equally to the public good and eliminate tax avoidance. In equilibrium, the government contributes nothing directly towards the public good, and the public good is fully provided by private contributions. This result is robust to a number of extensions, including the introduction of warm-glow preferences.

I also show that when labor-distorting taxes are used to finance the public good, the introduction of a non-refundable tax credit for contributions to the public good can eliminate tax avoidance, and decrease the labor wedge for each tax payer. In addition, I show that for two public goods, the government can implement given levels of the public good by employing an ordinary income tax with a tax credit for contribution to both public goods, and an alternative minimum tax (AMT) with qualifying tax credits for only one of the public goods.

1.4 Pension Increases vs. Contribution Cuts

Chapter 4 outlines an explanation for the relative generosity of deferred public sector compensation relative to deferred private sector compensation. Using a basic political economy model, I demonstrate that the preferences of the median public sector employee can lead to policies which favor increases in defined benefit pensions over either increases in salary or decreases in employee contributions towards their pensions. Economists including Inman have argued that short-term political concerns will lead politicians to rely on deferred compensation for public employees over increased current compensation. This has been presented as an explanation for relatively generous public pensions. However, deferred compensation could take its form as increased pension benefits or decreased employee contributions to the pension system. Short-term political concerns do not explain why public sector pension benefit increases are so common even when public employees are making positive contributions to the system. This chapter presents a simple model in which the current median public sector employee determines how to accept a given increase in the present value of employer pension contributions: through either decreased employee contributions or through increased pension benefits. I demonstrate that if the discount rate is sufficiently small, then the present value of increased future pension benefits exceeds the present value of the decrease in employee pension contributions for the median public employee, and the median public employee will thus prefer a pension benefit increase over a decrease in employee contributions. I also demonstrate that under a wide range of plausible parameters, the median public employee will prefer benefit increases over employee contribution cuts.

CHAPTER 2

CAPITALIZATION OF UNFUNDED PUBLIC PENSION LIABILITIES

2.1 Introduction

Public pensions are a significant and growing liability for state and local governments.[1] In 2009, the level of assets held by US public sector pensions was 1.94 trillion dollars, and the level of liabilities has been estimated between 2.76 trillion and 6.36 trillion dollars. Thus, depending on the discount rate and other actuarial cost assumptions, the level of unfunded liabilities has been estimated between 0.82 trillion and 3.6 trillion dollars (Novy-Marx and Rauh, 2010).[2] Future unfunded liabilities are a significant fiscal burden for state and local governments. Based on Novy-Marx and Rauh's estimates, the average per household unfunded liability ranges between approximately 7,000 and 30,000 dollars. The high level of unfunded liabilities could potentially have a very significant impact on the housing market. If there is not full capitalization of unfunded liabilities into housing prices, then unfunded liabilities could distort potential residents' locational decisions and distort labor markets.

[1]Based on data from the Public Plans Database at the Boston College Center for Retirement Research, actuarial liabilities of defined benefit public pensions increased 58% between 2001 and 2009.

[2]Novy-Marx and Rauh calculate the level of public pension liabilities as of June 30, 2009 under a variety of assumptions. Consider the extreme set of actuarial assumptions. If liabilities are calculated using the accumulated benefit obligation (ABO), and future liabilities are discounted using state's self-determined expected rate of return, on average eight percent, then Novy-Marx and Rauh find that the level of liabilities are 2.76 trillion dollars. If liabilities are calculated using the projected value of benefits (PVB), and liabilities are discounted using the interest rate on treasury bonds, then Novy-Marx and Rauh calculate the level of liabilities to be 6.36 trillion dollars.

Following the financial crisis, the underfunding of public sector pensions has increasingly gained public notice. Between 2001 and 2010, 28 states increased employee contributions, and 36 decreased pension benefits. In 2010 alone, 15 states cut employee pension benefits and nine increased employee contributions (Pew, 2010). In 2011, the Wisconsin legislature at the encouragement of Governor Scott Walker passed legislation which substantially curtailed public sector workers' bargaining rights and placed constraints on employee compensation, including requiring public sector workers to pay one half of the total pension contribution. Despite the relatively strong level of funding in the Wisconsin state pension system, one of the main rationales for passing these restrictions was growth in pension liabilities.

Within the past few years, pension liabilities have pushed several states and municipalities to make drastic changes to their pension systems. In 2012, severe underfunding of pensions led San Jose residents to vote to severely cut back the generosity of the city's public sector pension for current workers. Certain pension systems have even significantly cut the benefits of current retirees. In 2011, due to the severity of its unfunded pension liabilities in its three pension systems, New Jersey suspended cost of living adjustments (COLA) for all retirees, until the asset-to-liability ratio has achieved a more robust level of funding.

Municipalities and states are not always able to reduce their pension benefits, or increase employee contributions in response to large pension liabilities.[3] The li-

[3]Municipalities are not even necessarily able to reduce their total employee costs by increasing employee contributions. To the extent that employees bargain over salary and benefits, higher

abilities from public sector pensions have contributed to significant fiscal changes. In 2011, partially in response to Illinois' high level of unfunded pension liabilities, the Illinois legislature increased the state income tax rate from three percent to five percent and increased the corporate tax rate from 4.8 percent to 7.0 percent. In the past few years, there have been a number of municipal bankruptcies. In July 2012, Stockton, CA filed for Chapter 9 bankruptcy protection. A major contributing factor towards Stockton's entry into Chapter 9 bankruptcy was the city's 125 million dollars in pension obligation bonds (POB) that the city incurred in order to cover its required contributions towards the California Public Employee Retirement System (CALPERS). One year later in July 2013, Detroit, MI entered Chapter 9 bankruptcy. With 9 billion dollars in debt and 9 billion dollars in unfunded liabilities, it was the largest municipal bankruptcy in US history.

If pension liabilities are not fully capitalized into housing prices, state and local governments have an incentive to pay for current services using future unfunded pension benefits, pushing costs onto future residents and away from current residents. If there is less than full capitalization, then this would explain the current high level of unfunded pension liabilities. Less than full capitalization of underfunding of liabilities into housing prices could encourage inefficiently large levels of current government spending, and lead to overly high future marginal tax rates. In addition to distributional concerns, incomplete capitalization could increase the total cost of living in a given state and distort the locational decisions

employee contributions may be offset by higher salaries, which may then crowd out employer contributions.

of future residents, leading to distortions in the labor market.

The main objective of this chapter is to determine the extent to which state and local governments are limited in their ability to push the costs of current services onto future residents by determining how much unfunded liabilities are capitalized into housing prices. In addition, states with increasing demands for housing are likely to experience less capitalization of unfunded liabilities into their home prices than states with stagnant populations. This chapter also seeks to determine if states with increasing demands for housing do experience less capitalization of unfunded liabilities than states with slower growing or stagnant populations. This follows from two factors: one is from the additional taxpayers who will contribute towards covering unfunded liabilities, and the second is from the lower elasticity of housing supply that corresponds to having an increasing housing stock. Using 2001 – 2009 annual panel data from the Boston College Center for Retirement Research's Defined Benefit Pension Plan, I estimate the impact of underfunding of public sector pensions on state level average home prices. In order to address the potential correlation of unfunded liabilities with other economic and political factors which may also affect housing prices, I instrument for the level of underfunding by taking the initial assets of the pension system invested in domestic equities, international equities, domestic bonds, international bonds, and real estate investment trusts, and adjust each asset class by average market returns.

I find very large point estimates of the effect of underfunding of public sector pensions on state level average housing prices. Controlling for state specific time-trends and employing 2-SLS, I find that a 1.0 dollar increase in underfunding corresponds to more than a 6.5 dollar decrease in housing prices. While these estimates are significantly larger than one, they are consistent with prior estimates. In addition, I find only very weak evidence that states which experience faster growth also experience less capitalization. Section 2 outlines the current state of the literature, section 3 introduces the model, sections 4 and 5 discuss the identification strategy and the data sources, and section 6 presents the results and the discussion.

2.2 Literature Review

Starting with Oates (1969), there is an extensive literature measuring the extent to which differences in tax rates and amenities are capitalized into housing prices at the municipal and school district level. These studies estimate the level of capitalization of tax rates and amenities into housing prices by assuming that current tax rates and public service levels are reasonable proxies for future tax rates and public services respectively (Yinger et. al, 1988; Palmon 1998; Palmon and Smith, 1998). Most studies of local property tax capitalization find levels of capitalization anywhere between 0.15 and 1.20.[4] While this approach captures current conditions of taxes and public goods, it often ignores debt levels and underfunding of

[4] Assumes a real discount rate of 3 percent. A capitalization level of C implies that a one dollar increase in the present value of future taxes corresponds to a drop in current housing prices C.

liabilities.[5] This omission is problematic if tax rates are correlated with debt levels or unfunded liabilities. Thus studies based on differences in current tax rates and amenities do not necessarily capture the impact of future tax rates and public services on housing prices.

There is a very limited literature on capitalization of unfunded pension liabilities on housing prices. Epple and Schipper (1980), Inman(1980) and Leeds (1985) all develop basic political economy models in which the level of capitalization is driven by the level of myopia towards future liabilities. This chapter differs from this tradition by employing a basic model in which the level of capitalization does not depend on myopia, but is driven by the elasticity of supply and demand, and by housing demand growth for a particular location. Epple and Schipper (1980) and Leeds (1985) attempt to estimate the level of capitalization of unfunded liabilities using cross-sectional data. Using 1972 Census of Local Governments census tract data in the Chicago MSA, Leeds (1985) estimates the effect of public pension underfunding on housing prices, by using the ratio of pension contributions to pension assets as a proxy for the level of underfunding. Leeds finds no effect of the level of underfunding on housing prices. Epple and Schipper estimate the level of capitalization of unfunded liabilities using 1976 and 1978 cross-sectional census tract level data. Taking advantage of Pennsylvania's pension reporting requirements, they use actual unfunded liabilities reported by pension systems to estimate the level of capitalization. Epple and Schipper find very large estimates

[5] Although most municipalities are required to have a balanced budget, most can carry debt for capital projects, and thus current taxes are not an accurate indicator of future taxes.

for the level of capitalization; finding that a one dollar increase in unfunded liabilities per housing unit corresponds to a six and half dollar decrease in housing prices. Using cross-sectional data, it is quite likely that current levels of unfunded public pensions do not capture the full present value of future growth of unfunded liabilities, and will lead to biased over-estimates.

MacKay (2011) is the only prior empirical paper that does not rely on cross-sectional data. MacKay employs a differences-in-difference strategy to estimate the effect of underfunding of public pensions on housing prices. Following a 2004 revelation that San Diego had understated its pension by approximately one billion dollars, MacKay compares the change in San Diego housing prices relative to the prices of similar houses in adjacent municipalities. MacKay finds that housing prices dropped 2.50 dollars relative to similar homes for every one dollar per housing unit in newly revealed pension liabilities. One concern with MacKay's identification strategy is that the revelation of the additional one billion dollars in pension underfunding was accompanied by a credit rating downgrade for San Diego. If the credit downgrade was a result of not only the greater level of underfunding, but also the negative signal from San Diego concealing liabilities, then MacKay's estimates may be biased over-estimates. Furthermore, if current and future unfunded liabilities are capitalized into housing prices, then the estimates are likely to be over-estimates. If current and future residents are not fully aware of the exact level of future unfunded liabilities and residents use current estimates as an indicator for future liability growth, then the revelation of a billion dollars in current liabilities would imply a significantly larger increase in the present value of unfunded liabilities. In addition, if reported actuarial liabilities understate true liabilities of the pension system, then the revelation of an additional one billion dollars of actuarial liabilities may imply a far greater actual underreported liability, leading to biased over-estimates.

I attempt to address several major issues in the estimation of the effect of unfunded pension liabilities on housing prices: omitted variable bias, actuarial liability reporting's underestimation of actual liabilities, and the current capitalization of unrealized future expected unfunded liabilities. I address both the omitted variable bias and actuarial liability reporting's underestimation of actual liabilities by employing a novel instrument for unfunded liabilities: initial assets indexed by average market performance. Instrumenting for the level of pension underfunding using the predicted assets instrument, I am able to identify changes in unfunded liabilities off of changes in average market performance. This facilitates the estimation of the effect of underfunding of pensions on housing prices even when actuarial liabilities underestimate the full current pension liability. Furthermore, using state-specific time trends, I am able to estimate the effect of unfunded liabilities off of deviations from underlying trends. Thus, state-specific time trends provide estimates that are not biased due to current capitalization of future liability growth. This work differs from the previous literature in several other ways. In contrast to the previous literature's focus on capitalization on the local level, I estimate the effect of underfunding of pensions on average state housing prices.[6]

[6]Demand for state level housing should be less elastic than demand for local housing and thus should be associated with a lower level of capitalization.

Additionally, this is the first attempt to measure the extent that capitalization depends on demand conditions of the locale.

2.3 Basic Model

In this section, I derive a basic formula for the level of capitalization that depends on the elasticity of demand, the elasticity of supply, and the rate of household growth. Consider a simple framework in which residents of a given state incur unfunded liabilities in period 0. In period 1 the unfunded liabilities are repaid lump-sum. For simplicity I assume that there is no depreciation of the housing stock. There are two possibilities for the level of household growth in this community: either there is zero growth in the number of households or there is a positive level of household growth. U is the total level of unfunded liabilities, Q_0 is the level of housing in period 0, P is the price of housing in period 1, and $C(\Delta Q) = C(Q - Q_0)$ is the marginal cost of building a house. $Q_D(P + \frac{U}{Q}) = \theta\Omega(P + \frac{U}{Q})$ is the demand function for housing in the given state, where θ is a measure of the preference for the state. Q represents the level of housing in period 1. P_f represents the full cost of housing, and equals both the purchase price of housing and the unfunded liabilities that must be repaid: $P + \frac{U}{Q}$. If there is zero growth in the number of households, then demand for housing must equal the supply of housing from period 0:

$$Q_D(P + \frac{U}{Q_0}) = Q_0. \tag{2.1}$$

Totally differentiating both sides yields:

$$(\frac{dP}{dU} + \frac{1}{Q_0})Q'_D(P + \frac{U}{Q}) = 0, \tag{2.2}$$

which implies

$$\frac{dP}{dU} = -\frac{1}{Q_0}. \tag{2.3}$$

When there is no construction of new housing, and the elasticity of supply is perfectly inelastic, then there is full capitalization.

In the case in which there is a positive level of household growth and thus a positive level of construction growth, the price of housing is equal to the marginal cost of housing,

$$P = C(Q_D(P + \frac{U}{Q}) - Q_0). \tag{2.4}$$

In addition, in equilibrium the level of housing demand must equal the quantity of housing:

$$Q = Q_D(P + \frac{U}{Q}). \tag{2.5}$$

Plugging (2.4) into (2.5) yields:

$$Q = Q_D(C(Q - Q_0) + \frac{U}{Q}). \tag{2.6}$$

By differentiating the above with respect to U, the change in the level of housing is given by

$$\frac{dQ}{dU} = Q'_D(C(Q-Q_0) + \frac{U}{Q})[C'(Q-Q_0)\frac{dQ}{dU} - \frac{U}{Q^2}\frac{dQ}{dU} + \frac{1}{Q}]. \tag{2.7}$$

Rearranging the above yields:

$$\frac{dQ}{dU} = \frac{\frac{Q'_D(C(Q-Q_0)+\frac{U}{Q})}{Q}}{1 - C'(Q-Q_0)Q'_D(C(Q-Q_0) + \frac{U}{Q}) + Q'_D(C(Q-Q_0) + \frac{U}{Q})\frac{U}{Q^2}}. \tag{2.8}$$

ξ_D is the elasticity of demand with respect to the full price of housing, and ξ_S is the elasticity of supply of housing. Thus, the elasticities can be re-written as

$$C'(Q-Q_0) = \frac{1}{\xi_S}\frac{P}{Q} \tag{2.9}$$

and

$$Q'_D(P + \frac{U}{Q}) = (\xi_D\frac{Q}{P_f}). \tag{2.10}$$

Plugging $\frac{1}{\xi_S}\frac{P}{Q}$ and $(\xi_D\frac{Q}{P_f})$ in for $C'(Q-Q_0)$ and $Q'(P+\frac{U}{Q})$ respectively and rearranging yields:

$$\frac{dQ}{dU} = \frac{\xi_S\xi_D Q}{\xi_S P - \xi_D P + (\xi_S + \xi_S\xi_D)\frac{U}{Q}}. \tag{2.11}$$

Since the relationship between prices and quantities is given by

$$P = C(Q-Q_0), \tag{2.12}$$

the level of capitalization can be written in terms of the change in the level of housing with respect to unfunded liabilities:

$$\frac{dP}{dU} = C'(Q - Q_0)\frac{dQ}{dU}. \tag{2.13}$$

Plugging (2.11) into (2.13), and simplifying yields:

$$\frac{dP}{dU} = \frac{\xi_D}{\xi_S Q - \xi_D Q + \frac{(\xi_S + \xi_S \xi_D)U}{P}}. \tag{2.14}$$

The above gives the basic level of capitalization of total unfunded liabilities on housing prices. Multiplying by Q_0 yields the level of capitalization of unfunded liabilities per base number of households on housing prices:

$$\frac{dP}{d\frac{U}{Q_0}} = \frac{Q_0 \xi_D}{\xi_S Q - \xi_D Q + \frac{(\xi_S + \xi_S \xi_D)U}{P}}. \tag{2.15}$$

The above capitalization equation gives the effect of unfunded liabilities per period 0 households on housing prices. Alternatively, the above capitalization equation can be interpreted as the level of incidence of unfunded liabilities for a seller. When U is zero, capitalization is bounded between 0 and 1. The table below provides changes in housing prices as a function of the elasticity of demand for housing in a given state, elasticity of supply for housing in a given state, and the level of unfunded liabilities as a percentage of total housing prices. For low levels of $\frac{U}{PQ}$, the level of capitalization depends positively on the absolute value of the elasticity

Level of Capitalization: $Q = 1.1Q_0$

	$\frac{U}{QP} = 0$	$\frac{U}{QP} = .05$	$\frac{U}{QP} = .1$	$\frac{U}{QP} = .4$
$\xi_S = 0$	-0.91	-0.91	-0.91	-0.91
$\xi_S = 2, \xi_D = -12$	-0.779	-0.846	-0.924	-2.098
$\xi_S = 2, \xi_D = -8$	-0.73	-0.78	-0.83	-1.48
$\xi_S = 2, \xi_D = -5$	-0.65	-0.67	-0.69	-.87
$\xi_S = 2, \xi_D = -1$	-0.30	-0.30	-0.30	-0.30
$\xi_S = 2, \xi_D = -.2$	-0.083	-0.080	-0.078	-0.065
$\xi_S = 5, \xi_D = -8$	-0.560	-0.645	-0.765	-7.273
$\xi_S = 5, \xi_D = -5$	-0.45	-0.50	-0.56	-1.67
$\xi_S = 5, \xi_D = -1$	-0.15	-0.15	-0.15	-0.15
$\xi_S = 5, \xi_D = -.2$	-0.083	-0.080	-0.078	-0.065
$\xi_S = \infty$	0	0	0	0

Each entry in the table is the corresponding level of capitalization for a given ξ_S, ξ_D, and $\frac{U}{QP}$ calculated directly from equation (15).

of demand and negatively on the elasticity of supply.[7] Consider the extreme cases for both elasticity of demand and elasticity of supply. In the case of supply, when supply is perfectly elastic and the absolute value of the elasticity of demand is greater than one, then the level of capitalization is zero. However, when the elasticity of supply is zero, then the level of capitalization is $\frac{Q_0}{Q}$.[8] These general results are quite intuitive and analysis is analogous to the incidence of a tax on buyers and sellers. If supply is perfectly inelastic, then the full level of underfunding will be fully capitalized into housing prices, and the full burden of the unfunded liabilities will fall on current residents. If the supply of housing is perfectly elastic, then unfunded liabilities will have no effect on housing prices, and the full burden will fall on future residents. For the extreme cases for the elasticity of demand, if

[7]A level of capitalization of z, implies that for a one dollar increase in unfunded liabilities per Q_0, average housing prices decrease by z.

[8]Since an elasticity of supply of zero implies that there is no growth in the level of households, then $Q_0 = Q$, and the level of capitalization is 1.

the elasticity of demand is zero, then the level of capitalization is zero. If demand is perfectly elastic, then the level of capitalization is $\frac{Q_0}{-Q+\frac{\xi_S U}{P}}$. However, the level of capitalization can well exceed one, if the level of unfunded liabilities is significant and the elasticity of demand for housing well exceeds one. If an increase in unfunded liabilities produces a large enough decrease in the number of households, then the unfunded liabilities per household could increase significantly and the level of capitalization could well exceed one. I estimate the basic capitalization level above by estimating the effect of unfunded liabilities on average housing prices.

Alternatively, I can rewrite the level of capitalization of unfunded liabilities per period zero households as the following:

$$\frac{dP}{d\frac{U}{Q_0}} = \frac{\xi_D}{\xi_S(1+\frac{U}{PQ})+\frac{\xi_D\xi_S U}{QP}-\xi_D}(\frac{Q_0}{Q}). \tag{2.16}$$

The above equation can also be rewritten as:

$$\frac{dP}{d\frac{U}{Q_0}} = \frac{\xi_D}{\xi_S(1+\frac{U}{PQ})+\frac{\xi_D\xi_S U}{QP}-\xi_D} - \frac{\xi_D}{\xi_S(1+\frac{U}{PQ})+\frac{\xi_D\xi_S U}{QP}-\xi_D}(\frac{Q-Q_0}{Q}). \tag{2.17}$$

The above expression gives the amount of the unfunded liabilities that are capitalized into total house values of period 1 residents. The level of capitalization is generally decreasing with respect to equilibrium housing growth. If the elasticity of demand for housing in a given state and the elasticity of supply for housing in a given state are constant, then the change in the level of capitalization with respect to changes in housing demand θ is given by the following:

$$\frac{\partial^2 P}{\partial\theta\partial\frac{U}{Q_0}} = \frac{-Q_0\xi_D(\xi_S - \xi_D - \frac{(1+\xi_D)U}{P^2}\frac{P}{Q})}{(\xi_S Q - \xi_D Q + \frac{(\xi_S+\xi_S\xi_D)U}{P})^2}\frac{Q}{\theta[1 - \frac{\xi_D}{\xi_S}\frac{P}{P+\frac{U}{Q}} + \frac{\xi_D U}{PQ+U}]}. \tag{2.18}$$

If $\frac{\xi_S - \xi_D}{1-\xi_D} > \frac{U}{PQ}$ then $\frac{\partial^2 P}{\partial\theta\partial\frac{U}{Q_0}} > 0$.[9] Thus if the level of unfunded liabilities is below a very high threshold, then the level of capitalization is decreasing with respect to household growth. I test this implication by determining whether states that experience higher levels of housing demand growth experience less capitalization. I test this implication of the model by estimating the effect of the interaction of household growth and unfunded liabilities on housing prices. If the coefficient on the interaction is negative then that affirms this implication of the model.

2.4 Empirical Implementation

In order to estimate the basic relationship between housing prices and unfunded liabilities, I regress housing prices on unfunded liabilities per household while controlling for state and year fixed effects. Specifically, I estimate the following equation:

$$P_{i,t} = \alpha\frac{U_{it}}{Q_0} + \beta X + \xi_T T + \delta_I I + \epsilon_{it}, \tag{2.19}$$

[9] Alternatively if $-\xi_D > \frac{U-PQ\xi_s}{PQ+U}$ then $\frac{\partial^2 P}{\partial\theta\partial\frac{U}{Q_0}} > 0$.

where $P_{i,t}$ is the average home value and X is a set of controls, including state per capita income and the unemployment rate.[10] $\frac{U_{it}}{Q_0}$ is unfunded liability per base year occupied households. T and I are indicator vectors, and capture year and state fixed effects. In the above equation, unfunded liabilities are divided by the number of base year household instead of dividing unfunded liabilities by the number of current-year households. While unfunded liabilities per current-year households is a more intuitive measure, the number of households and the price of housing are endogenously determined. If unfunded liabilities per current household are used in the estimation, demand shocks for housing will lead to biased over-estimates of the capitalization coefficient. This will occur since a positive demand shock will increase both equilibrium prices and quantity of housing, which will cause fewer unfunded liabilities per household, and will thus artificially magnify the coefficient on unfunded liabilities per household.[11]

A major concern regarding panel data estimates for the level of capitalization is that unfunded liabilities are consistently growing over time. If current and potential homeowners fully expect future growth in unfunded liabilities, then this future growth in unfunded liabilities will already be capitalized into housing prices. Thus, the panel data estimate above might fail to capture the relationship between unfunded liabilities and housing prices. This will lead the capitalization coefficient in equation 2.19 to be significantly biased towards zero. In order to help ad-

[10] I estimate the models both with and without controls for household growth.

[11] Supply shocks could bias the results towards finding a positive effect of unfunded liabilities on housing prices.

dress this issue, I also estimate the effect of unfunded liabilities on housing prices while controlling for state-specific time trends. The state-specific time trend allows us to estimate the level of capitalization by estimating the effect of unfunded liabilities on deviations from the trend in housing prices,

$$P_{i,t} = \alpha \frac{U_{it}}{Q_0} + \beta X + \xi_T T + \delta_I I + \gamma_I t + \epsilon_{it}. \tag{2.20}$$

Even controlling for state-specific time trends, OLS estimates of the effect of unfunded liabilities on housing prices are still likely to be biased. Changes in unfunded liabilities are likely to be correlated with other economic or political variables which are also correlated with housing prices. Specifically, unfunded liabilities may be correlated with both general economic conditions and other changes in governance structure or fiscal conditions.[12] For example, states which experience positive shocks to governance are likely to experience lower unfunded liabilities, as well as lower debt and other positive governmental characteristics, which will also be correlated with higher housing prices. Alternatively, positive growth shocks may be correlated with both increased unfunded liabilities through increased public sector hiring, and higher housing prices. Another issue with capitalization estimates is the endogeneity of the actuarial assumptions that states and local governments use to calculate their liabilities.

[12]Consider the election of Illinois Governor Rod Blagojevich. The election of Rod Blagojevich may have led to an increase in unfunded pension liabilities. However, the election of Rod Blagojevich may have also led to other poor policies which also adversely affect housing prices including a doubling of the amount of outstanding bonds.

The pension liability data utilized in this chapter is calculated using pension system specific actuarial standards. Most pension systems follow the Governmental Accounting Standards Board (GASB) pension liability reporting standards, which give states discretion in choosing the expected inflation rate, expected public sector wage growth, discount rate and other actuarial assumptions. For example, the GASB standards advise pension systems to discount their future liabilities using the expected rate of return on their investment assets. Thus, changes in unfunded liabilities according to the data may reflect actual changes in unfunded liabilities or changes in actuarial assumptions. If changes in actuarial assumptions are correlated with housing prices, then this will bias estimates of the capitalization coefficient. In order to address the endogeneity of the level of underfunding with outside economic and political factors, I instrument for the level of underfunding with a predicted assets variable. The predicted assets instrument is constructed by adjusting the initial asset allocation of each state's pension system in the base year, and adjusting each asset class by average market performance of that asset class. Specifically the instrument equals

$$A_{i,t} = \sum_{j} S_{i,j,2001} I_{j,t}. \tag{2.21}$$

where $S_{i,j,2001}$ is the total dollar amount of state $i's$ pension fund allocation in asset class j in year 2001 and $I_{j,t}$ is the index value of asset class j in year t, normalizing $I_{j,b}$ as 1.

The identification strategy for the 2-SLS specification in the absence of state time trends assumes that the share of initial assets, the amount of initial assets, and the performance of initial assets are not correlated with changes in housing prices except through the instrument. Additionally, in the 2-SLS specification without state-specific time trends, the identification strategy assumes that the predicted asset instrument affects unfunded liabilities solely through the asset level. Thus, the identification strategy assumes that the asset price instrument is not correlated with the total level of liabilities. For the 2-SLS specification without time trends, the asset price instrument may be correlated with total liabilities. The 2-SLS specification without state-specific time trends may not fully address the issue that future unfunded liabilities are already capitalized into housing prices. If initial asset levels are correlated with the future growth of actuarial liabilities, then estimates may be biased.[13] In addition, although the instrument addresses the correlation between many economic variables and actuarial assumptions, predicted asset levels may be positively correlated with more conservative actuarial assumptions. Additionally, the predicted assets instrument may be negatively correlated with contributions to the pension system and positively correlated with actual liabilities.

The identification strategy could also be problematic to the extent that states which experience increasing quality of governance over this period are also invested disproportionately in asset classes that experience greater levels of return. This could hold if states that are more competently invested are also experiencing

[13]Specifically, if initial asset levels are positively correlated with linear future growth of actuarial liabilities, then this will lead to biased first-stage under-estimates, and thus biased over-estimates of the effect of unfunded liabilities on housing prices.

increases in their governance quality, which would also affect housing prices. Another possible problem with the identification strategy could occur if states with higher initial levels of assets experience greater housing price growth. This could occur if states which are experiencing increases in the quality of governance have greater assets.[14]

The identification strategy for the model with state-specific time trends is similar to the model without time trends. The identification strategy assumes that the predicted assets instrument affects unfunded liabilities solely through the asset level, and is not correlated with total liabilities. In addition, the identification strategy assumes that the size of initial pension assets, the distribution of those assets, and the average market returns to asset classes are not correlated with housing price growth deviating from general price trends except through the instrument. The inclusion of state-specific time trends addresses the concern that any initial asset allocations are correlated with linear future growth.[15]

In order to test the main hypothesis that faster growing states experience less capitalization, I regress the interaction of household growth and unfunded liabilities on housing prices. Specifically, I estimate the following equations:

$$P_{i,t} = \alpha \frac{U_{it}}{Q_0} + \phi \frac{U_{it}}{Q_0} \frac{(Q_{t+1} - Q_t)}{Q_t} + \rho \frac{(Q_{t+1} - Q_t)}{Q_t} + \beta X + \xi_T T + \delta_I I + \epsilon_{it}, \tag{2.22}$$

[14]However, it is possible that the level of assets is negatively correlated with housing price growth. This is quite plausible since states in the Midwest and Northeast, which have more generous pensions have been experiencing less population growth relative to the West and South.

[15]However, I will test whether the predicted assets instrument with time trends is correlated with total liabilities.

and

$$P_{i,t} = \alpha\frac{U_{it}}{Q_0} + \phi\frac{U_{it}}{Q_0}\frac{(Q_{t+1} - Q_t)}{Q_t} + \rho\frac{(Q_{t+1} - Q_t)}{Q_t} + \beta X + \xi_T T + \delta_I I + \gamma_I t + \epsilon_{it}. \tag{2.23}$$

I will once again instrument for the level of unfunded liabilities by using the predicted asset instrument. In addition, I also estimate the above equations with percentage change in expected employment growth in the place of the change in the equilibrium level of households (Bartik, 1991). The Bartik decomposition used in this chapter is given by :

$$G_{it} = \Sigma_{j=1}^{J} e_{i,j,b}(\frac{e'_{i,j,t} - e'_{i,j,t-1}}{e'_{i,j,b}}), \tag{2.24}$$

where $e_{i,j,t}$ is employment in state i in industry j in year t, $e'_{i,j,t}$ is national employment in industry j outside of state i in year t, and b represents base year. G_{it} is predicted change in employment for state in year t. G_{it} equals the base share of employment by industry interacted with the change in national employment, excluding state i.

$$E_{it} = \Sigma_{j=1}^{J}(e_{i,j,b}(\frac{e'_{i,j,t} - e'_{i,j,b}}{e'_{i,j,b}}) + e_{i,j,b}) = \Sigma_{j=1}^{J}(e_{i,j,b}\frac{e'_{i,j,t} - e'_{i,j,b}}{e'_{i,j,b}}) + e_{i,b} \tag{2.25}$$

E_{it} is predicted total employment using initial employment interacted with national change in employment. Alternatively E_{it} equals $\Sigma_{\tau=b}^{t} G_{it}$.

2.5 Data Sources

2.5.1 Housing prices

In order to capture the effect of unfunded liabilities, I employ an index of housing prices denominated in dollars. The home value index used is an index of average home prices by state. It is constructed by taking decennial average home prices, and adjusting them by the Federal Housing Finance Administration (FHFA) quarterly repeat transactions index. The Home Value Index was created and compiled by Morris A. Davis and Jonathan Heathcoate, and is publicly available from the Lincoln Institute of Land Policy in conjunction with Wisconsin School of Business.[16]

2.5.2 Unfunded Pension liabilities

2001 – 2009 annual unfunded pension liabilities come from the Public Plans Data Base, Center for Retirement Research at Boston College and Center for State and

[16]Since unfunded liabilities are only available annually, I convert the quarterly index to an annual index by calculating the annual geometric average.

Local Excellence. The data is available for 107 State retirement systems, covering approximately 90 percent of retirement systems, and 86 percent of assets.[17] The data reports the current level of assets, the level of liabilities, and various actuarial assumptions including discount rate, and cost method for calculating liabilities for each pension system for each year. I construct the unfunded liabilities figure by adding up the total actuarial liabilities of each pension plan for each state for each year and subtracting the market assets of each pension system for each state for each year.[18]

2.5.3 Predicted Assets Instrument

The predicted asset instrument is constructed by adjusting the initial asset allocation by the average market performance for each asset class. Asset allocations for each pension system are broken down by the level of assets invested in domestic equities, international equities, domestic bonds, international bonds, real estate investment trusts, cash, and others. I index initial domestic equities by the Standard & Poor's 500, initial international equities by the MSCI ex US, initial domestic bonds by Barclays Capital Aggregate Bond Index, initial international

[17]Most states require their pension systems to report assets and liabilities in compliance with the Governmental Accounting Standards Board reporting standards. These standards give the public sector pensions wide flexibility in choosing their actuarial assumptions.

[18]Most public pension plans in the data set calculate actuarial liabilities using entry age normal (EAN). EAN assumes that pension liabilities are accumulated evenly over an employees career, and that pension accruals are a set percentage of salaries. Thus EAN does capture future liability growth. In fact it underestimates the current level of liabilities if public employee compensation is consistent with implicit contracts.

bonds by Barclays Bank of America Merrill Lynch Emerging Markets Corporate Plus Index, and initial real estate holdings by DOW Real Estate Investment Trust (REIT). The predicted assets instrument is

$$A_{i,t} = \sum_{j} S_{i,j,2001} I_{j,t}. \tag{2.26}$$

The predicted assets instrument represents total initial assets adjusted for average market returns for each asset class. $S_{i,j,2001}$ is the total dollar amount of state $i's$ pension benefits in asset class j in year 2001 and $I_{j,t}$ is the index value of asset class j in year t, normalizing $I_{j,b}$ as 1. Each index used is a total return index, giving the total gross return from both changes in values of the asset class and assuming that any dividend or coupon payments are reinvested in the fund. The international indices are dollar denominated hedged indices.[19]

2.5.4 Other Variables

State level annual per capita income come from the Bureau of Economic Analysis (BEA). Annual state level unemployment rates are from the Bureau of Labor Statistics. Annual state level households is the occupied household units from the American Community Survey (ACS).

[19] Hedged indices are indices which are adjusted for changes in the exchange rate, and are chosen to control for fluctuations in currencies. Pension systems do spend a non-trivial level of assets on currency futures to minimize their risks from currency fluctuations. This allows pension systems to diversify their portfolios in international investments, while minimizing risk of currency fluctuations, since all liabilities are denominated in US dollars.

2.6 Results

Table 1 reports the basic descriptive statistics. Between 2001 and 2009, while actuarial liabilities increased over 58%, market assets increased only 6.2%. This stagnation in pension assets relative to liabilities led unfunded liabilities per household to increase from $1,419$ to 10486 dollars. While actuarial liabilities experienced significant growth and unfunded liabilities increased rapidly over this period, housing prices experienced positive but modest growth: increasing 39.7%. Since both actuarial liabilities and housing prices were increasing over this period, it is important to incorporate time trends into the analysis.

Using OLS, I find very modest estimates of the effect of unfunded liabilities on average housing prices. Table 3 reports the coefficients of the regression of average housing prices on unfunded liabilities per number of 2001 households. Using panel data and controlling for year and state fixed effects, I find modest estimates of the coefficient on unfunded liabilities. The bivariate regression yields non-distinguishable estimates from zero. When adding controls for per capita income and the unemployment rate, the estimate for the capitalization coefficient is -0.39 and is statistically significant at the 10 percent level. However, when I add controls for state-specific time trends, the estimate of the coefficient of the effect of unfunded liabilities on housing prices is positive, although not statistically different from zero.

Table 4 reports the first stage estimates of effect of predicted assets on unfunded liabilities. Without state-specific time trends, a one dollar increase in predicted assets is associated with a 20 cent decrease in unfunded liabilities. However, the first stage without time trends is quite weak; the coefficient on the effect of the asset instrument on unfunded liabilities is only significant at the 10 percent level, and the F statistic is less than 4. The First-stage is relatively strong when state-specific time trends are included; the first-stage F-statistic exceeds 13. With state-specific time trends, a one dollar increase in predicted assets causes a 66 cent decrease in unfunded liabilities. Since the first stage in the estimation without time trends is quite weak, it is quite likely that the asset instrument is also positively correlated with unfunded liabilities.

Table 5 reports the main results in the chapter. This table reports the 2-SLS estimates of the effect of unfunded liabilities on average housing prices. In the estimation without state-specific time trends, the estimates of the effect of unfunded liabilities are very large, and statistically insignificant. Without state-specific time trends the capitalization coefficients exceeds -20. Although statistically insignificant, these results are consistent with a story in which the asset instrument is correlated with future liability growth. With state-specific time trends, both of the 2-SLS estimates of the effect of unfunded liabilities on housing are -6.60 and -6.75 and statistically significant at the 5 percent level. These coefficients are quite large, and imply that a one dollar increase in unfunded liabilities per household causes a six and half dollar decrease in unfunded liabilities. However the standard errors for both are still quite large at 3.32 and 3.38 respectively.

A major concern regarding the predicted assets instrument is its relationship with total unfunded liabilities. If the two are positively correlated, and the changes in liabilities are being driven by changes in actuarial assumptions, then the 2-SLS estimates will be biased over-estimates. Furthermore, if the predicted assets instrument is positively correlated with total unfunded liabilities, and actuarial liabilities are driven by increases in pension liabilities without any increase in services, then this will contribute to over biased 2-SLS estimates. Table 6 reports the effect of the asset instrument on total liabilities. There is a strong positive relationship between the asset instrument and total liabilities. For every one dollar in predicted assets there is almost fifty cents in increased liabilities. If these increased liabilities have no positive impact on housing prices[20], then the effect of predicted assets on total liabilities accounts for two thirds of the capitalization coefficient in the estimation without state-specific time trends. However, there is no relationship between predicted assets and total liabilities when one controls for state-specific time trend. The coefficient on the effect of predicted assets on total liabilities is 0.007, with relatively large standard errors.

The −6.60 and −6.75 point estimates of the effect of underfunding of public sector pensions on state level average housing prices result are very large. The point estimates are consistent with the model only if there are very high levels of unfunded liabilities and if the average elasticity of demand for a given state is very high. If the elasticity of demand is sufficiently high, an increase in unfunded

[20] i.e. The increased liabilities are not associated with increased services that capitalized into housing prices

liabilities could decrease the equilibrium level of households, thus increase the amount of unfunded liabilities that must be repaid by significantly more than a dollar, leading to levels of capitalization greater than one. Additionally, the distortionary effects of taxation could help account for a level of capitalization greater than one. However on its own, it is quite unlikely that the marginal cost of public funds is high enough to account for a level of capitalization greater than 6.50.[21] Another, factor that could be driving up these results is the salience of public sector pensions. If individuals over estimate the costs of the market losses to pension systems, then there could be larger than theoretically anticipated level of capitalization. Additionally, there are a few econometric concerns. If the asset instrument is negatively correlated with debt levels, or positively correlated with government spending which is positively capitalized into housing prices, then the estimates will be biased over estimates.[22]

Table 7 reports the 2-SLS estimates of the effect of household growth on the level of capitalization. I instrument for underfunding using the predicted assets instrument, and I instrument for unfunded liabilities multiplied by one-period future percent housing growth using predicted assets multiplied by one-period future housing growth. Without controlling for state-specific time trends, the coefficient on the effect of underfunding on average housing prices is -17.31, and the coefficient on the interaction of household growth and unfunded liabilities is 1.21.

[21] Furthermore, since higher income individuals are more mobile within the US, the marginal resident of a given state likely faces a higher marginal tax rate, and thus is likely to face a higher marginal cost of governmental revenue.

[22] Additionally, if the econometric model is misspecified, then the specification with state-specific time trends may be identifying the effect off changes from the wrong underlying trend.

Both estimates of the coefficients are not statistically significantly different from zero. Although not statistically significant, the coefficient on the interaction term is consistent with faster housing growth, leading to less capitalization. Specifically, the coefficient implies that for each increased dollar in unfunded liabilities, a one percent increase in housing growth causes there to be one less dollar of capitalization. The results are similar when controlling for state-specific time trends, but are statistically significantly different than zero. When controlling for time trends, the coefficient on the unfunded liabilities is extraordinarily large: −32.76 and is significant at the 10 percent level, and the coefficient on the interaction is 1.62 and statistically significant at the 5 percent level.

Table 8 reports the 2-SLS estimates of the effect of Bartik predicted growth on the level of capitalization. Once again, I instrument for unfunded liabilities using the predicted assets instrument, but I instrument for unfunded liabilities multiplied by the one-period future growth in the predicted Bartik variable with the asset instrument multiplied by the one-period future growth in the predicted Bartik variable. However, the instruments are extraordinarily weak.[23] Overall, the evidence for decreasing levels of capitalization with increasing household growth is quite modest. Even using time trends, only one of the two specifications had marginally statistically significant results.

[23]The F-statistics for the first stages are .89 for the specification without state-specific time trends and .51 for the specification with state-specific time trends. In the absence of state-specific time trends, the capitalization coefficient is −19.75 and the coefficient on the interaction term is quite large: 6.24. Although the capitalization coefficient is not statistically significantly different than zero, the interaction term is statistically significant at the 10 percent level. If state-specific time trends are included, the capitalization coefficient is −15.30 and the interaction coefficient is very large: 9.91. Neither estimate is statistically significant.

2.7 Conclusion

Public pension liabilities are a significant fiscal burden on state and local governments. In this chapter, I estimate the effect of unfunded liabilities on housing prices. In order to address the endogeneity of the level of underfunding with outside economic and political factors, I construct a unique instrument for the level of pension underfunding, a predicted assets variable. The predicted assets instrument is constructed by adjusting the initial asset allocation of each state's pension system in the base year, and adjusting each asset class by average market performance of that asset class. I instrument for the level of underfunding by taking the initial assets of the pension system invested in domestic equities, international equities, domestic bonds, international bonds, and real estate investment trusts, and adjust each asset class by average market returns. Instrumenting for unfunded liabilities using initial asset class levels adjusted for average market returns, I find very large point estimates of the effect of underfunding of public sector pensions on state level average housing prices. I find that a one dollar increase in unfunded liabilities per household corresponds to a greater than 6.5 dollar decrease in average housing prices. These large estimates are consistent with the early literature, and match the results of Epple and Schipper (1981). These point estimates are consistent with a high level of capitalization, which implies a very high marginal cost of public funds or very elastic demand for housing on the state level.

CHAPTER 3

TAX CREDITS AND AVOIDANCE

3.1 Introduction

Tax avoidance is used in the literature to describe any legal action that is intended to decrease one's tax obligation. Generally, tax avoidance can take the form of postponement of taxes, tax arbitrage across individuals or time, or tax arbitrage across income streams (Stiglitz, 1986). Avoidance activities can constitute significant behavioral responses, and have a substantial impact on tax revenue. For example, following the passage of the Tax Reform Act of 1986 and just prior to the increase in the capital gains tax rate, the realization of capital gains in December 1986 increased seven fold relative to the realization of capital gains in December 1985 (Burman, Clausing, and O'Hare 1994). While specific estimates of the extent of tax avoidance do not exist, estimates of the magnitude of evasion and compliance costs are quite substantial. In the United States, the difference between the total tax liability owed and the amount collected by the IRS, known as the tax gap, is estimated at 14.5 percent of total tax liability.[1] The compliance costs of the US income tax system are estimated to be approximately 10 percent of revenue collected (Slemrod 1996).

Economists including Roberts (1986) and Feldstein (1980) justify subsidies for

[1] In 2006, the IRS estimated the net tax gap at 14.5 percent. The Gross Tax gap, the gap between total tax liabilities and the level of taxes paid in voluntary compliance, was estimated to be even higher at 17 percent.

contributions to the public good since subsidies can provide a given level of the public good with less revenue than direct government provision, which can lead to lower gross tax receipts and thus a lower excess burden. This welfare benefit is contingent upon the government relying on labor-distorting taxes. Furthermore, even if the government cannot rely on non-distortionary taxes, the welfare gains from the subsidy are quite limited since the subsidy required to implement the optimal level of the public good is quite large. If the government relies on private provision of the public good, then for every dollar given to the public good, the optimal tax refund to the individual taxpayer is very close to one. The resulting reduction in gross tax receipts necessary to implement the Samuelson level of the public good is quite modest. This corresponds to only a very modest welfare gain from the subsidy.

This chapter outlines a new argument for the use of tax expenditures for contributions to the public good: that the introduction of tax credits for contributions to the public good can decrease and under certain conditions even eliminate tax avoidance. Specifically, I develop a simple model with non-labor distorting taxes in which the introduction of a non-refundable tax credit for contributions towards the public good can completely eliminate tax avoidance and facilitate the implementation of the Samuelson level of the public good. More generally, I demonstrate that for any level of the public good, non-refundable tax credits can implement the given level of the public good and eliminate tax avoidance. This result is robust to a number of extensions, including the introduction of warm-glow preferences.

In addition, I demonstrate that in the case of two public goods, for any given levels of the public goods an altered piece-meal tax credit function can eliminate tax avoidance and provide the given levels of the public good. I also show that when labor-distorting taxes are used to finance the public good, the introduction of tax credits for contributions to the public good can eliminate tax avoidance and decrease the labor wedge.

In section 2, I review the related literature. In sections 3, 4, and 5, I introduce the basic model and demonstrate how the introduction of a non-refundable tax credit can implement the Samuelson level of the public good. In section 6, I introduce heterogeneity in income and preferences. In section 7, I expand the model to incorporate multiple public goods, and in section 8, I incorporate warm-glow preferences into the model. Section 9 expands the model to incorporate labor-distorting taxes.

3.2 Literature Review

Feldstein (1980), Roberts (1987), and Roberts (1992) demonstrate that by employing tax expenditures for contributions to the public good, the government can finance a public good with less tax revenue than if it relied on direct financing. All three papers study a model in which individuals derive utility from a tax-favored public good, a non-tax favored private consumption good, and leisure. Specifically, all three papers find that for a given level of the public good, fully funding the public good with a refundable tax credit will require a lower tax rate than direct government provision. Feldstein calculates the welfare loss from providing an additional unit of the tax-favored good using both direct financing and subsidies for private contributions. He finds that if the private sector can produce the good at least as efficiently as the government, then regardless of the amount of good already being provided, the government should rely on subsidies. Roberts (1986) finds that if there is one-for-one crowd-out of private contributions for each dollar of public financing, then for any level of direct governmental provision of the public good direct subsidization can provide the same level of the public good at a lower gross cost and thus lower tax rate.[2]

A practical shortcoming of Robert's result is that it relies on a level of subsidization that is very close to one.[3] Subsidization rates approaching one result in gross tax revenues that are very close to the gross revenue requirements under direct provision. The resulting reduction in deadweight loss is very minor. If lump-sum taxation is allowed then the optimal subsidy is $\frac{N-1}{N}$. Specifically, a one dollar donation to the public good should be matched with a $\frac{N-1}{N}$ dollar reduction in the individuals tax bill. For a given level of the public good, full funding via subsidies requires the government to raise $\frac{N-1}{N}$ proportion of the funding required under direct provision. Even under labor-distorting taxes, the welfare gains from using subsidies relative to direct financing are relatively modest. Roberts (1992)

[2]The main underlying assumption for both Robert's and Feldstein's results as well as the results in this chapter is that individuals take the government provision of the public good as given.

[3]A subsidization rate of one implies that a one dollar contribution to the public good would lead to a one dollar decrease in an individual's tax liability.

demonstrates that the per capita tax burden reduction from introducing a tax deduction for contributions to the public good is bounded above by $\frac{(1+MCPF)}{N}$.[4] In this chapter, I take the basic Robert's model and introduce a simple tax avoidance technology based on Slemrod (2001). In this model, the introduction of a sufficiently tax credits can lead to non-trivial welfare gains from the elimination of tax avoidance. In contrast to Robert's model, the welfare gains per person under this model do not approach zero as the number of individuals increase.

This chapter also deviates from the previous literature, in its introduction of non-refundable tax credits. Feldstein (1980), Roberts (1986), and Roberts (1992) all analyze refundable tax credits. While, for a given level of tax credit generosity, refundable tax credits may elicit greater contributions than non-refundable tax credits, they do not necessarily lead to a Pareto-improvement. In contrast, I demonstrate that even with heterogeneous preferences, the introduction of a non-refundable tax credit can provide a given level of the public good while decreasing the labor-wedge and increasing the welfare of each taxpayer.

To my knowledge, Diamond (2006) is the only other paper to find potentially significant welfare gains from funding a public good with tax expenditures. Diamond (2006) introduces an additional rationale for tax expenditures for private contributions to public goods: tax expenditures for high income individuals' contributions to the public good are part of the optimal tax system. Diamond demonstrates that subsidization of contributions to the public good by high ability indi-

[4]The marginal cost of public funds (MCPF) is the marginal change in the sum of Equivalent Variation to consumers from raising one dollar in additional tax revenue.

viduals can ease the incentive constraints on these high ability individuals. Thus tax deductibility can help lessen the resource constraint on government but also lessen the incentive constraint. Thus for a given level of the public good, the introduction of subsidies for high income individuals can increase the level of redistribution possible.

This chapter is also closely related to the literature on reducing tax evasion and avoidance. Most of the literature focuses on optimal audit strategies (Reinganum and Wilde 1985, Sanchez and Sobel 1993); or optimal taxation and evasion (Cremer and Ghavari 1993). Within this literature, this paper is most similar to Liu (2012), which develops a model in which increased reliance on environmental taxes which are difficult to evade, may decrease the overall level of evasion. This work and Liu (2012) both explore seemingly unrelated policies which reduce tax avoidance or evasion. Liu develops a model in which increased reliance on environmental taxes which are difficult to evade, may decrease the overall level of evasion.

3.3 Model

There are N identical agents who derive utility from a private consumption good, c_i and a public good, x. Each individual i has preferences represented by the strictly concave function, $u(c_i, x)$. The public good is funded directly by both government tax receipts and direct voluntary contributions by individuals. Individual $i's$ contribution to the public good is represented by x_i. For the purpose of this chapter, I abstract from the specific provision of the public good and focus exclusively on the financing of the public good. Regardless of who funds the public good, I assume that the resulting provision will be equally efficient, and that individuals are indifferent between private and government financing of the public good. Thus total provision of the public good is the sum of individual contributions and government financing:

$$x = x_G + \sum_{i=1}^{N} x_i. \tag{3.1}$$

For simplicity, I normalize the public good and assume that one unit of the consumption good can be converted without cost into one unit of the public good. The government sets the gross tax liability per individual at T. However, each individual taxpayer i has access to some technology that allows him to avoid amount e_i in taxes, and face a net tax liability of $T - e_i$. The cost of avoiding amount e_i in taxes is given by the increasing and convex function $\Phi(e_i)$.[5] The corresponding level of taxes paid by taxpayer i is t_i. The government must maintain a balanced budget:

$$x_G = \sum_{i=1}^{N} t_i. \tag{3.2}$$

In the absence of tax credits the budget constraint for the individual is $c_i + t_i + x_i +$

[5]More generally, $\Phi(e_i)$ is the equivalent variation of sheltering e_i dollars of tax liability. $\Phi(e_i)$ captures both the direct dollar costs required to reduce tax liability and the inefficiencies of altering consumption income timing and form. For example, increasing contributions to a 401(K) may decrease the present value of one's life time tax liability, but may lead to liquidity constraints or less flexibility and less implicit insurance from savings.

$\Phi(T - t_i) = Y$. Now consider a tax credit for contributions to the public good. The tax credit for individual i is given by $\Lambda(x_i)$. The budget constraint for individual i is given by the following expression:

$$c_i + t_i + x_i + \Phi(T - \Lambda(x_i) - t_i) = Y. \tag{3.3}$$

The timing of the model is as follows. The government sets the gross tax liability, T; the tax credit policy, $\Lambda(x_i)$; and sets the level of direct government financing of the public good, x_G. Next, the individual decides the amount she will contribute towards the public good, the amount she will consume, and the amount she will avoid. Each individual i solves the following maximization problem taking as given T, $\Lambda(x_i)$, $x_j \forall j \neq i$, and x_G:

$$\begin{aligned} \max_{c_i, x_i, t_i} \quad & u(c_i, x_G + \sum_{j \neq i}^{N} x_j + x_i) \\ \text{s.t.} \quad & c_i + t_i + x_i + \Phi(T - \Lambda(x_i) - t_i) = Y. \end{aligned} \tag{3.4}$$

3.4 Analysis

In this section, I outline the Samuelson condition for a Pareto-efficient level of the public good, and I characterize the utilitarian social-welfare maximizing allocation of the public good.

First, conditional on resource constraints, I characterize the level of the public good for a Pareto-efficient allocation:

$$\begin{aligned} \max_{x,[c_i]_{i=1}^N} \quad & u(c_k, x) \\ \text{s.t.} \quad & u(c_i, x) \geq \bar{u}_i, \forall i \neq k \\ & x + \sum_i^N c_i \leq \sum_i^N Y_i = NY. \end{aligned} \tag{3.5}$$

The F.O.C. for the above simplify to the following:

$$\sum_i^N \frac{u_2(c_i, x)}{u_1(c_i, x)} = 1. \tag{3.6}$$

Equation (3.6) characterizes the Samuelson level of the public good. Specifically, looking at the unique Pareto-efficient allocation with equal treatment for each consumer yields:

$$u_1(Y - \frac{x}{N}, x) = N u_2(Y - \frac{x}{N}, x), \forall i \in I. \tag{3.7}$$

Equation (6) also corresponds to the optimal utilitarian social welfare maximizing allocation.[6] In oder to see this, consider the first-best social welfare maximizing

[6] It also corresponds to the first-best social welfare maximizing for any concave individual-specific social welfare function.

allocation of the public good:

$$\begin{aligned}
&\max_{c_i \geq 0, x_i \geq 0, t_i \geq 0} && \sum_{i=1}^{N} u(c_i, x_G + \sum_{j \neq i}^{N} x_j + x_i) \\
&\text{s.t.} && \sum_{i=1}^{N} c_i \leq \sum_{i=1}^{N} Y_i = NY.
\end{aligned} \tag{3.8}$$

The F.O.C. of the above are the following:

$$u_1(c_i, x) = u_1(c_j, x), \forall i, j \in I \tag{3.9}$$

and

$$u_1(c_j, x) = \sum_{j=1}^{N} u_2(c_i, x), \forall i \in I. \tag{3.10}$$

Define (x^*, c^*) to be the unique allocation of consumption and government services s.t $c^* = Y - \frac{x^*}{N}$. x^* is the unique value s.t. $u_1(Y - \frac{x^*}{N}, x^*) = Nu_2(Y - \frac{x^*}{N}, x^*)$. $x = x^*$ and $c_i = c^*, \forall i \in N$ is the unique allocation which maximizes utilitarian social welfare.

3.5 Results

If a taxpayer's marginal cost of reducing his taxable income is less than one dollar, then the government is not able to implement the first-best solution, even if non-labor distorting taxes are feasible. In this section, I show that in the absence of tax credits there will be a positive level of tax avoidance. I then show that by introducing tax credits for contributions to the public good, the social planner can actually implement the first-best level of the public good and eliminate tax avoidance. If the marginal cost of tax avoidance is low for low levels of avoidance, then the optimal tax credit will actually be greater than one dollar for each one dollar contributed to the public good. More precisely, if $\frac{N-1}{N}$ exceeds the marginal cost of avoiding the first dollar, then the optimal tax credit is greater than one for one. If $\Phi'(0) < 1$, then in the absence of tax credits the government can implement the first-best level of the public good but there will exist a positive level of tax avoidance, and the second best provision of the public good.[7]

Proposition 1

If $\Phi'(0) < 1$ and $T > 0$, then there exists a positive level of tax avoidance.

Proof:

In the absence of a tax credit for contributions to the public good, the individual solves the following maximization problem:

$$\begin{aligned} \max_{c_i \geq 0, x_i \geq 0, t_i \geq 0} \quad & u(c_i, x_G + \sum_{j \neq i}^{N} x_j + x_i) \\ \text{s.t.} \quad & c_i + t_i + x_i + \Phi(T - t_i) \leq Y. \end{aligned} \tag{3.11}$$

[7] A positive level of tax avoidance implies that real resources are wasted and thus the allocation is not Pareto-efficient.

Regardless of the level of the x_i, since the individual takes x_G as given, the individual chooses t and minimizes the following:

$$\min_{t_i \geq 0} \quad t_i + \Phi(T - t_i). \tag{3.12}$$

Taking the first order condition:

$$1 = \Phi'(T - t_i). \tag{3.13}$$

Since $\Phi'(0) < 1$, then the level of tax avoidance must be positive.■

The above result demonstrates that there will be a strictly positive level of tax avoidance as long as the gross tax liability is positive and the marginal cost of avoiding the first dollar of taxes is below one. If on the other hand, gross tax liabilities are set to zero, then the voluntary contributions equilibrium will lead to an equilibrium level of the public good below the first-best.[8] Thus if $\Phi'(0) < 1$, then even with lump-sum taxation, it is not possible to implement the first-best solution.[9] The introduction of tax credits for contributions to the public good can eliminate tax avoidance, and implement the first-best solution. However the level of tax credits required to eliminate tax avoidance is quite high. Unless the initial marginal cost of employing the tax avoidance technology is very close to one, the subsidy required to eliminate tax avoidance exceeds 1. More specifically, tax

[8]Even in the absence of tax credits, if the marginal cost of avoiding the first dollar is greater than one, then there will not be any tax avoidance.

[9]If $\Phi'(0) > 1$, then the taxpayer will never employ the tax avoidance technology.

credits can eliminate tax avoidance yet still be less than one if $\frac{N-1}{N} < \Phi'(0) < 1$.

Proposition 2 demonstrates that by setting $\Lambda(x_i) = ((N-1)/(N\Phi'(0)))x_i$ and $T = [\frac{N-1}{N\Phi'(0)}]\frac{x^*}{N}$, the government can induce each tax payer to contribute $\frac{x^*}{N}$ to the public good and fully eliminate her tax liability. In equilibrium, the Samuelson level of the public good is fully funded by direct contributions by tax payers: the government does not collect any revenue and provides zero units of the public good.

Proposition 2

If $\Phi'(0) < 1$, then tax avoidance can be eliminated and the Pareto-efficient level of the public good with equal treatment of individuals can be implemented by setting $\Lambda(x_i) = ((N-1)/(N\Phi'(0)))x_i$.

Proof:

Individual i specifically solves the following maximization problem:

$$\begin{aligned} \max_{c_i \geq 0, x_i \geq 0, t_i \geq 0} \quad & u(c_i, x_G + \sum_{j \neq i}^{N} x_j + x_i) \\ \text{s.t.} \quad & c_i + t_i + x_i + \Phi(T - t_i - \Lambda(x_i)) \leq Y. \end{aligned} \tag{3.14}$$

The government can implement the Samuelson level of the public good, while eliminating tax avoidance by setting $\Lambda(x_i) = ((N-1)/(N\Phi'(0)))x_i$, $x_G = 0$, and $T = [\frac{N-1}{N\Phi'(0)}]\frac{x^*}{N}$. If $x_G + \sum_{j \neq i}^{N} x_j = \frac{(N-1)x^*}{N}$, then the F.O.C.s for an interior solution (with a positive level of tax avoidance and a positive contribution to the public good) can be written as

$$u_2(c_i, \frac{(N-1)x^*}{N} + x_i) = u_1(c_i, \frac{(N-1)x^*}{N} + x_i)[1 - \Lambda'(x_i)\Phi'(T - \Lambda(x_i) - t_i)], \tag{3.15}$$

which simplifies to

$$u_2(c_i, \frac{(N-1)x^*}{N} + x_i) = u_1(c_i, \frac{(N-1)x^*}{N} + x_i)[1 - (N-1)/(N\Phi'(0))\Phi'(T - \Lambda(x_i) - t_i)]. \tag{3.16}$$

Individual i will avoid zero in taxes. In order to demonstrate this, assume the contrary. If individual i engages in a strictly positive level of tax avoidance, then

$$u_2(c_i, \frac{(N-1)x^*}{N} + x_i) \leq u_1(c_i, \frac{(N-1)x^*}{N} + x_i)[1 - (N-1)/(N\Phi'(0))\Phi'(T - \Lambda(x_i) - t_i)] < \frac{1}{N}u_1(c_i, \frac{(N-1)x^*}{N} + x_i). \tag{3.17}$$

which implies that individual i's contribution to the public good is greater than one Nth of the Samuelson level, $x_i > \frac{x^*}{N}$. However, if individual i contributes more than the Samuelson level of the public good, then individual i will owe zero in net tax liability, and thus will not engage in any tax avoidance activities. Since individual i will not utilize the tax avoidance technology, the F.O.C. simplifies to

$$u_2(c_i, \frac{(N-1)x^*}{N} + x_i) = \frac{1}{N}u_1(c_i, \frac{(N-1)x^*}{N} + x_i). \tag{3.18}$$

Thus individual i will choose (x_i, c_i) s.t. $x_i = \frac{x^*}{N}$, and $c_i = c^*$, where $c^* = Y - \frac{x^*}{N}$ and x^* is the unique value s.t. $u_1(Y - \frac{x^*}{N}, x^*) = Nu_2(Y - \frac{x^*}{N}, x^*)$. ■

The intuition behind the previous result is quite straightforward. The level of tax credit for contributions to the public good must be sufficiently large, such that the private marginal utility from giving is at least as large as the marginal utility from avoiding taxes. Specifically, the individual marginal utility divided by the subsidy for the public good must be greater than or equal to one minus the marginal cost of avoiding multiplied by the marginal utility of consumption. This condition implies that the private marginal benefit of giving a dollar to the public good equals the marginal utility of consumption multiplied by $[1 - \frac{N-1}{N\Phi'(0)}]$. A subsidy of $\frac{N-1}{N\Phi'(0)}$ will lead each individual to contribute until the Samuelson level of the public good is provided.

3.6 Heterogeneity

In the previous section, I demonstrated how tax credits for contributions to the public good can induce identical individuals to contribute up to the Samuelson level of the public good and abstain from engaging in tax avoidance. In this section, I relax the assumption that all individuals are identical, and I demonstrate that the basic result still holds with heterogeneity of preferences and multiple public goods. I also demonstrate that in the presence of income inequality, tax credits for contributions to the public good can both eliminate tax avoidance and fully fund the public good with contributions coming from only the most affluent taxpayers.

3.6.1 Heterogeneity in Preferences

Even if there is a large variance in the preference for the public good, the government can set the tax credit sufficiently high in order to induce the taxpayer with the weakest positive preference for the public good to contribute an equal share to the public good. Consider the scenario in which individuals have varying levels of intensity of preference over the value of the public good. The preferences of each individual i are represented by the strictly concave function, $u(c_i, \theta_i(x_G + \sum_{j \neq i}^{N} x_j + x_i))$. Define k to be the individual with the lowest MRS of the public good for the consumption good:

$$k = \arg\min_{i \in I} \{ \frac{u_{i2}(Y - \frac{\bar{x}}{N}, \bar{x})}{u_{i1}(Y - \frac{\bar{x}}{N}, \bar{x})} \}. \tag{3.19}$$

Since the tax credit is non-refundable, it is sufficient to set everyone's tax credit equal to the value of the tax credit necessary to induce the individual with the lowest valuation of the public good to contribute equally to its provision. Proposition 3 demonstrates that for any level of public good $\bar{x}$, sufficiently large s.t. $\forall i \in I$, $u_{i1}(Y - \frac{\bar{x}}{N}, \bar{x}) > u_{i2}(Y - \frac{\bar{x}}{N}, \bar{x})$, the introduction of a tax credit for contributions to the public good can implement $\bar{x}$ and eliminate tax avoidance.

Proposition 3

If $\Phi'(0) < 1$, then for a given level of the public good $\bar{x}$, tax avoidance can be eliminated and public good level $\bar{x}$ implemented by setting $\Lambda(x_i) = \frac{u_{k1}(Y-\frac{\bar{x}}{N},\bar{x})-u_{k2}(Y-\frac{\bar{x}}{N},\bar{x})}{u_{k1}(Y-\frac{\bar{x}}{N},\bar{x})\Phi'(0)}x_i$.

Proof:

Individual i specifically solves the following maximization problem:

$$\begin{aligned} \max_{c_i \geq 0, x_i \geq 0, t_i \geq 0} \quad & u(c_i, x_G + \sum_{j \neq i}^{N} x_j + x_i) \\ \text{s.t.} \quad & c_i + t_i + x_i + \Phi(T - t_i - \Lambda(x_i)) \leq Y. \end{aligned} \tag{3.20}$$

The government can implement $\bar{x}$, while eliminating tax avoidance by setting $\Lambda(x_i) = \frac{u_{k1}(Y-\frac{\bar{x}}{N},\bar{x})-u_{k2}(Y-\frac{\bar{x}}{N},\bar{x})}{u_{k1}(Y-\frac{\bar{x}}{N},\bar{x})\Phi'(0)}x_i$, $x_G = 0$, and $T = [\frac{u_{k1}(Y-\frac{\bar{x}}{N},\bar{x})-u_{k2}(Y-\frac{\bar{x}}{N},\bar{x})}{u_{k1}(Y-\frac{\bar{x}}{N},\bar{x})\Phi'(0)}]\frac{\bar{x}}{N}$. At an equilibrium, a taxpayer will refrain from engaging in tax avoidance if

$$u_2(c_i, x) \geq u_1(c_i, x)[1 - \Lambda'(x_i)\Phi'(0)], \tag{3.21}$$

which simplifies to

$$\Lambda'(x_i) \geq \frac{1 - \frac{u_2(c_i,x)}{u_1(c_i,x)}}{\Phi'(0)}. \tag{3.22}$$

If $x_G + \sum_{j \neq i}^{N} x_j = \frac{(N-1)\bar{x}}{N}$, and the equilibrium level of x is $\bar{x}$, then

$$\Lambda(x_i) = \frac{u_{k1}(Y - \frac{\bar{x}}{N}, \bar{x}) - u_{k2}(Y - \frac{\bar{x}}{N}, \bar{x})}{u_{k1}(Y - \frac{\bar{x}}{N}, \bar{x})\Phi'(0)}x_i \tag{3.23}$$

satisfies the above inequality. Thus if taxpayer i faces $\Lambda(x_i) = \frac{u_{k1}(Y-\frac{\bar{x}}{N},\bar{x})-u_{k2}(Y-\frac{\bar{x}}{N},\bar{x})}{u_{k1}(Y-\frac{\bar{x}}{N},\bar{x})\Phi'(0)}x_i$, $x_G = 0$, $T = \Lambda(x_i) = \frac{u_{k1}(Y-\frac{\bar{x}}{N},\bar{x})-u_{k2}(Y-\frac{\bar{x}}{N},\bar{x})}{u_{k1}(Y-\frac{\bar{x}}{N},\bar{x})\Phi'(0)}\frac{\bar{x}}{N}$, and $x_G + \sum_{j\neq i}^{N} x_j = \frac{(N-1)\bar{x}}{N}$, then she will contribute $x_i = \frac{\bar{x}}{N}$ to the public good, receive a tax credit equal to T, pay $t_i = 0$ in taxes, and consume $c_i = Y - \frac{\bar{x}}{N}$. See appendix for alternative proof. ■

The above result demonstrates that the introduction of a sufficiently large tax credit can implement any public good level $\bar{x}$ and lead each individual to consume $Y-\bar{x}$. Thus, the tax credit can be utilized to implement a Pareto-efficient allocation. Once again, define x^* such that it satisfies the Samuelson condition:

$$\sum_{i=1}^{N} \frac{u_2(Y - \frac{x^*}{N}, x^*)}{u_1(Y - \frac{x^*}{N}, x^*)} = 1. \tag{3.24}$$

Thus by setting the tax credit equal to $\Lambda(x_i) = \frac{u_{k1}(Y-\frac{x^*}{N},x^*)-u_{k2}(Y-\frac{x^*}{N},x^*)}{u_{k1}(Y-\frac{x^*}{N},x^*)\Phi'(0)}x_i$, $x_G = 0$, and $T = \frac{u_{k1}(Y-\frac{x^*}{N},x^*)-u_{k2}(Y-\frac{x^*}{N},x^*)}{u_{k1}(Y-\frac{x^*}{N},\bar{x})\Phi'(0)}\frac{x^*}{N}$, the government can eliminate tax avoidance and implement a Pareto-efficient allocation.

3.6.2 Heterogeneity in Income

Consider the simple situation in which there are various incomes that are exogenously endowed to the population. I assume that income is strictly exogenous, and there is no income distortion caused by taxation. Each individual is endowed with income Y_i. Index each individual s.t. $Y_n > Y_m$ if and only if $n > m$. Assume that for each level of income Y_i there is an exogenously given level of contributions z_i that are expected to be either paid to the government in taxes or contributed directly to the public good. Let $\bar{x} = \sum_{i=1}^{N} z_i$ be the corresponding level of the public good. Define o s.t.

$$o \in \arg\min_{i \in I}\{\frac{u_2(Y_i - z_i, \bar{x})}{u_1(Y_i - z_i, \bar{x})}\}. \tag{3.25}$$

Proposition 4

If there is heterogeneity in income and $\Phi'(0) < 1$, then tax avoidance can be eliminated and the public good level $\bar{x}$ implemented with contributions $(z_i)_{i=1}^{N}$ by setting $\Lambda(x_i) = \frac{u_1(Y_o - z_o, \bar{x}) - u_2(Y_o - z_o, \bar{x})}{u_1(Y_o - z_o, \bar{x})\Phi'(0)} x_i$.

Proof:

Individual i specifically solves the following maximization problem:

$$\begin{aligned} \max_{c_i \geq 0, x_i \geq 0, t_i \geq 0} \quad & u(c_i, x_G + \sum_{j \neq i}^{N} x_j + x_i) \\ \text{s.t.} \quad & c_i + t_i + x_i + \Phi(T - t_i - \Lambda(x_i)) \leq Y. \end{aligned} \tag{3.26}$$

The government can implement the first-best level of the public good, while eliminating tax avoidance by setting $\Lambda(x_i) = \frac{u_1(Y_o - z_o, \bar{x}) - u_2(Y_o - z_o, \bar{x})}{u_1(Y_o - z_o, \bar{x})\Phi'(0)} x_i$, $x_G = 0$, and $\forall i \in I$ $T_i = \frac{u_1(Y_o - z_o, \bar{x}) - u_2(Y_o - z_o, \bar{x})}{u_1(Y_o - z_o, \bar{x})\Phi'(0)} z_i$. If $x_G + \sum_{j \neq i}^{N} x_j = \bar{x} - z_i$, then at an equilibrium a taxpayer i will refrain from engaging in tax avoidance if

$$u_2(c_i, x) \geq u_1(c_i, x)[1 - \Lambda'(x_i)\Phi'(0)], \tag{3.27}$$

which simplifies to

$$\Lambda'(x_i) \geq \frac{1 - \frac{u_2(c_i,x)}{u_1(c_i,x)}}{\Phi'(0)}. \tag{3.28}$$

If $x_G + \sum_{j \neq i}^{N} x_j = \bar{x} - z_i$, and the equilibrium level of x is $\bar{x}$, then

$$\Lambda(x_i) = \frac{u_1(Y_o - z_o, \bar{x}) - u_2(Y_o - z_o, \bar{x})}{u_1(Y_o - z_o, \bar{x})\Phi'(0)} x_i \tag{3.29}$$

satisfies the above inequality. Thus if taxpayer i faces $\Lambda(x_i) = \frac{u_1(Y_o - z_o, \bar{x}) - u_2(Y_o - z_o, \bar{x})}{u_1(Y_o - z_o, \bar{x})\Phi'(0)} x_i$, $x_G = 0$, $T_i = \frac{u_1(Y_o - z_o, \bar{x}) - u_2(Y_o - z_o, \bar{x})}{u_1(Y_o - z_o, \bar{x})\Phi'(0)} z_i$, and $x_G + \sum_{j \neq i}^{N} x_j = \bar{x} - z_i$, then she will contribute $x_i = z_i$ to the public good, receive a tax credit equal to $T_i = \frac{u_1(Y_o - z_o, \bar{x}) - u_2(Y_o - z_o, \bar{x})}{u_1(Y_o - z_o, \bar{x})\Phi'(0)} z_i$, pay $t = 0$ in taxes, and consume $c_i = Y_i - z_i$. See Appendix for alternative proof. ■

If I relax the assumption that net contributions to the public good are imposed externally, then a specific set of $(z_i)_{i=1}^{N}$ can implement a Samuelson level of the public good, and finance it exclusively by the $N - q$ highest income individuals using the following set of net contributions:

$$(z_i^*)_{i=1}^{N} = argmax\{q \mid \sum_{i=1}^{N} \frac{u_2(Y_i - z_i, \sum_{i>q} z_i)}{u_1(Y_i - z_i, \sum_{i>q} z_i)} = 1, \text{and } Y_i - T_i = Y_j - z_j \geq Y_q \ \forall i > q \text{ and } \forall j > q\}. \tag{3.30}$$

If there is heterogeneity in income, then even with the introduction of a linear tax credit it is not possible to equalize after-tax incomes and implement the utilitarian maximum. However, by choosing $(z_i^*)_{i=1}^{N}$, the government is able to satisfy the Samuelson condition, and implement a Pareto-efficient level of the public good while financing the public good exclusively from higher income taxpayers.

3.6.3 Heterogeneity in Preferences and Income

In this sub-section, I generalize the previous two results. Regardless of the source of the variation in the marginal rate of substitution, the government can fully fund the public good by setting the tax credit sufficiently high in order to induce the taxpayer with the lowest marginal rate of substitution of the public good for the consumption good (MRS_{gc}) to fully contribute to the public good. For any given set of exogenous contributions $(z_i)_{i=1}^N$ and corresponding level of the public good $\bar{x} = \sum_{i=1}^N z_i$, the government can implement $\bar{x}$ with voluntary contributions $(z_i)_{i=1}^N$ using taxes and a tax credit. Define l to be the individual with the lowest MRS of the public good for the consumption good:

$$l = \arg\min_{i \in I} \{\frac{u_{i2}(Y_i - z_i, x^*)}{u_{i1}(Y_i - z_i, x^*)}\}. \tag{3.31}$$

Proposition 5

If there is heterogeneity in income and preferences, and $\Phi'(0) < 1$, then tax avoidance can be eliminated and public good level $\bar{x}$ implemented with contributions $(z_i)_{i=1}^N$ by setting $\Lambda(x_i) = \frac{u_{l1}(Y_l - z_l, \bar{x}) - u_{l2}(Y_l - z_l, \bar{x})}{u_{l1}(Y_l - z_l, \bar{x})\Phi'(0)} x_i$.

Proof:

Individual i specifically solves the following maximization problem:

$$\max_{c_i \geq 0, x_i \geq 0, t_i \geq 0} \quad u_i(c_i, x_G + \sum_{j \neq i}^{N} x_j + x_i)$$
$$\text{s.t.} \qquad c_i + t_i + x_i + \Phi(T - t_i - \Lambda(x_i)) \leq Y. \tag{3.32}$$

The government can implement the first-best level of the public good, while eliminating tax avoidance by setting $\Lambda(x_i) = \frac{u_{l1}(Y_l - z_l, \bar{x}) - u_2(Y_l - z_l, \bar{x})}{u_{l1}(Y_l - z_l, \bar{x})\Phi'(0)} x_i$, $x_G = 0$, and $\forall i \in I$ $T_i = \frac{u_{l1}(Y_l - z_l, \bar{x}) - u_{2l}(Y_l - z_l, \bar{x})}{u_{l1}(Y_l - z_l, \bar{x})\Phi'(0)} z_i$. If $x_G + \sum_{j \neq i}^{N} x_j = \bar{x} - z_i$, then at an equilibrium a taxpayer i will refrain from engaging in tax avoidance if

$$u_{i2}(c_i, x) \geq u_{i1}(c_i, x)[1 - \Lambda'(x_i)\Phi'(0)], \tag{3.33}$$

which simplifies to

$$\Lambda'(x_i) \geq \frac{1 - \frac{u_{i2}(c_i, x)}{u_{i1}(c_i, x)}}{\Phi'(0)}. \tag{3.34}$$

If $x_G + \sum_{j \neq i}^{N} x_j = \bar{x} - z_i$, and the equilibrium level of x is $\bar{x}$, then

$$\Lambda(x_i) = \frac{u_{l1}(Y_l - z_l, \bar{x}) - u_{l2}(Y_l - z_l, \bar{x})}{u_{l1}(Y_l - z_l, \bar{x})\Phi'(0)} x_i \tag{3.35}$$

satisfies the above inequality. Thus if taxpayer i faces $\Lambda(x_i) = \frac{u_{l1}(Y_l - z_l, \bar{x}) - u_{l2}(Y_l - z_l, \bar{x})}{u_{l1}(Y_k - z_k, \bar{x})\Phi'(0)} x_i$, $x_G = 0$, $T_i = \frac{u_{l1}(Y_l - z_l, \bar{x}) - u_{l2}(Y_l - z_l, \bar{x})}{u_{l1}(Y_k - z_k, \bar{x})\Phi'(0)} z_i$, and $x_G + \sum_{j \neq i}^{N} x_j = \bar{x} - z_i$, then she will contribute $x_i = z_i$ to the public good, receive a tax credit equal to $T_i = \frac{u_{l1}(Y_l - z_l, \bar{x}) - u_{l2}(Y_l - z_l, \bar{x})}{u_{l1}(Y_l - z_l, \bar{x})\Phi'(0)} z_i$, pay $t = 0$ in taxes, and consume $c_i = Y_i - z_i$. See Appendix for alternative proof. ■

If I relax the assumption that net contributions to the public good are imposed externally, then a specific set of $(z_i)_{i=1}^N$ can implement a Pareto-efficient level of the public good, and finance it exclusively by the $N-k$ highest income individuals using the following set of net contributions:

$$(z_i^*)_{i=1}^N \in argmax\{k \mid \sum_{i=1}^N \frac{u_{i2}(Y_i - z_i, \sum_{i>k} T_i)}{u_{i1}(Y_i - z_i, \sum_{i>k} z_i)} = 1, \text{and } Y_i - z_i = Y_j - z_j \geq Y_k \; \forall i > k \text{ and } \forall j > k\}. \tag{3.36}$$

3.7 Multiple Public Goods

In the previous sections, the government could ensure the provision of a given level of the public good by setting the tax credit sufficiently high in order to induce taxpayers to contribute towards the public good. The presence of multiple public goods presents additional challenges. However, for any given levels of the public goods, the introduction of tax credits can eliminate tax avoidance and implement the desired levels of the public goods. The government can even implement a Pareto-efficient allocation of the public goods, while maintaining equal levels of consumption for the two types of individuals.

Consider the introduction of a second public good y. Assume that there are M individuals of type 1 who derive positive utility from public good x, and that there are N individuals of type 2 who derive positive utility from public good y. The respective set of individuals are indexed by I and J. The preferences of individuals of type 1 are identical to the preferences already presented. The preferences of individuals of type 2 are analogous; the only change being that their public good preferences are derived from good y instead of good x. Without loss of generalizability, assume that $\frac{\bar{x}}{M} > \frac{\bar{y}}{N}$. Analogously to the previous sections, define $\Lambda(x_i, y_i)$ to be the generic tax credit function.

The individual maximization problem for an individual of type 1 is

$$\begin{aligned} \max_{c_i, x_i, y_i, t_i} \quad & u(c_i, x_G + \sum_{j \neq i}^{M} x_j + \sum_{k=1}^{N} x_k + x_i) \\ \text{s.t.} \quad & c_i + t_i + x_i + y_i + \Phi(T - \Lambda(x_i, y_i) - t_i) \leq Y, \end{aligned} \tag{3.37}$$

and the maximization problem for an individual of type 2 is

$$\begin{aligned} \max_{c_i, x_i, y_i, t_i} \quad & u(c_i, x_G + \sum_{j \neq i}^{M} x_j + \sum_{k=1}^{N} x_k + x_i) \\ \text{s.t.} \quad & c_i + t_i + x_i + y_i + \Phi(T - \Lambda(x_i, y_i) - t_i) \leq Y. \end{aligned} \tag{3.38}$$

Define e' st. $\Phi'(e') = 1$. For any $\bar{x}$ and $\bar{y}$ define $T_0(\bar{x}, \bar{y})$ s.t. $T_0(\bar{x}, \bar{y}) = \frac{N\bar{x} - M\bar{y}}{MN + N^2}$. Define $s_x(\bar{x}, \bar{y})$ to equal $\frac{1}{\Phi'(0)}$.

Define $T(\bar{x}, \bar{y})$ such that $T(\bar{x}, \bar{y}) = s_x(\bar{x}, \bar{y})\frac{\bar{x}+\bar{y}}{M+N} = \frac{1}{\Phi'(0)}\frac{\bar{x}+\bar{y}}{M+N}$. Define $s_y(\bar{x}, \bar{y})$ such that $s_y(\bar{x}, \bar{y}) = \frac{T(\bar{x}, \bar{y}) - T_0(\bar{x}, \bar{y})}{\frac{\bar{y}}{N}} = \frac{s_x(\bar{x}, \bar{y})\frac{\bar{x}+\bar{y}}{M+N} - \frac{N\bar{x} - M\bar{y}}{MN+N^2}}{\frac{\bar{y}}{N}}$.

Define $\alpha_{\bar{x},\bar{y}}(y_i)$ s.t.

$$\alpha_{\bar{x},\bar{y}}(y_i) = \begin{cases} s_y(\bar{x},\bar{y})y_i & \text{if } t_i \geq T_0(\bar{x},\bar{y}) \text{ and } y_i \leq \frac{T(\bar{x},\bar{y})-T_0(\bar{x},\bar{y})}{s_y(\bar{x},\bar{y})} \\ T(\bar{x},\bar{y}) - T_0(\bar{x},\bar{y}) & \text{if } t_i \geq T_0(\bar{x},\bar{y}) \text{ and } y_i > \frac{T(\bar{x},\bar{y})-T_0(\bar{x},\bar{y})}{s_y(\bar{x},\bar{y})} \\ 0 & \text{if } t_i < T_0(\bar{x},\bar{y}). \end{cases}$$

Define $\Omega_{\bar{x},\bar{y}}(x_i)$ s.t.

$$\Omega_{\bar{x},\bar{y}}(x_i) = \begin{cases} s_x(\bar{x},\bar{y})x_i & \text{if } x_i \leq \frac{T(\bar{x},\bar{y})}{s_x(\bar{x},\bar{y})} \\ T(\bar{x},\bar{y}) & \text{if } x_i > \frac{T(\bar{x},\bar{y})}{s_x(\bar{x},\bar{y})}. \end{cases}$$

The combined tax credit function can be re-written as $\Theta_{\bar{x},\bar{y}}(x_i, y_i)$ s.t.

$$\Theta_{\bar{x},\bar{y}}(x_i, y_i) = \begin{cases} s_x(\bar{x},\bar{y})x_i + s_y(\bar{x},\bar{y})y_i & \text{if } t_i \geq T_0(\bar{x},\bar{y}) \text{ and } s_x(\bar{x},\bar{y})x_i + s_y(\bar{x},\bar{y})y_i \leq T(\bar{x},\bar{y}) - T_0(\bar{x},\bar{y}) \\ T(\bar{x},\bar{y}) - T_0(\bar{x},\bar{y}) & \text{if } t_i \geq T_0(\bar{x},\bar{y}) \text{ and } s_x(\bar{x},\bar{y})x_i + s_y(\bar{x},\bar{y})y_i > T(\bar{x},\bar{y}) - T_0(\bar{x},\bar{y}) \\ s_x(\bar{x},\bar{y})x_i & \text{if } t_i < T_0(\bar{x},\bar{y}) \text{ and } x_i \leq \frac{T(\bar{x},\bar{y})}{s_x(\bar{x},\bar{y})}. \end{cases}$$

While $\Omega_{\bar{x},\bar{y}}(x_i)$ is a non-refundable standard linear tax credit, $\alpha_{\bar{x},\bar{y}}(y_i)$ is a piece-meal tax credit. $\alpha(y_i)$ is a linear tax credit only available to a taxpayer i if i remits a level of tax payment greater than or equal to $T_0(\bar{x},\bar{y})$. The tax credit provides a reduction in tax liability of $s_y(\bar{x},\bar{y})$ for each dollar contributed to public good y up to a total contribution of $\frac{T(\bar{x},\bar{y})-T_0(\bar{x},\bar{y})}{s_y(\bar{x},\bar{y})}$ towards public good y and a total tax liability reduction of $T(\bar{x},\bar{y}) - T_0(\bar{x},\bar{y})$.[10] Proposition 6 demonstrates that for any given levels of the public goods, by employing tax credits $\Omega(x_i)$ and $\alpha(y_i)$, it is possible to implement a system such that individuals of type 1 and type 2 contribute the same sum of net taxes and contributions to the public goods. If $\frac{\bar{x}}{M} > \frac{\bar{y}}{N}$, then individuals of type 2 contribute less per capita towards public good 2 than individuals of type 1 contribute towards public good 1 but subsidize public good 1 via tax revenue.

If $s_x(\bar{x},\bar{y}) > 1$, then it must be that $s_y(\bar{x},\bar{y}) > s_x(\bar{x},\bar{y})$ in order for x and y to contribute the same sum total amount of taxes and direct contributions to the public good. A policy setting $\Lambda(x_i,y_i) = \Theta_{\bar{x},\bar{y}}(x_i,y_i) = \Omega_{\bar{x},\bar{y}}(x_i) + \alpha_{\bar{x},\bar{y}}(y_i)$ and $T = T(\bar{x},\bar{y}) = \frac{1}{\phi'(0)}\frac{\bar{x}+\bar{y}}{M+N}$ is equivalent to a form of the alternative minimum tax (AMT) which allows for tax credits for contributions to x but disallows tax credits for contributions towards y. Specifically, a policy setting $\Lambda(x_i,y_i) = \Theta_{\bar{x},\bar{y}}(x_i,y_i) = \Omega_{\bar{x},\bar{y}}(x_i) + \alpha_{\bar{x},\bar{y}}(y_i)$ and $T(\bar{x},\bar{y}) = \frac{1}{\phi'(0)}\frac{\bar{x}+\bar{y}}{M+N}$ is equivalent to facing non-refundable tax credits $s_x(\bar{x},\bar{y})$ and $s_y(\bar{x},\bar{y})$, combined with an AMT of $T_0(\bar{x},\bar{y})$, which qualifies for a tax credit equal to $s_x(\bar{x},\bar{y})$ for contributions to x in excess of $\frac{T(\bar{x},\bar{y})-T_0(\bar{x},\bar{y})}{s_x(\bar{x},\bar{y}}$.[11]

Proposition 6

For any $\bar{x}$ and $\bar{y}$, the introduction of tax $T(\bar{x},\bar{y}) = \frac{1}{\phi'(0)}\frac{\bar{x}+\bar{y}}{M+N}$ and tax credit function $\Lambda(x_i,y_i) = \Omega_{\bar{x},\bar{y}}(x_i) + \alpha_{\bar{x},\bar{y}}(y_i)$ can implement $\bar{x}$ and $\bar{y}$, eliminate tax avoidance, and result in equal consumption for both types.

Proof:

[10] In term of taxable income, this is equivalent to a limited tax credit worth up to $T(\bar{x},\bar{y}) - T_0(\bar{x},\bar{y})$ available to individuals with reported taxable incomes above Y_0.

[11] The structure of $\Theta(y_i)$ is also similar to the use of the standard deduction and itemized deductions in the current US tax code.

An individual of type 1 solves the following maximization problem:

$$\max_{c_i \geq 0, x_i \geq 0, y_i \geq 0, t_i \geq 0} u(c_i, x_G + \sum_{j \neq i}^{M} x_j + \sum_{k=1}^{N} x_k + x_i) \tag{3.39}$$
$$c_i + t_i + x_i + y_i + \Phi(T(\bar{x}, \bar{y}) - \Omega_{\bar{x},\bar{y}}(x_i) - \alpha_{\bar{x},\bar{y}}(y_i) - t_i) \leq Y,$$

At an equilibrium, a taxpayer of type 1 will pay zero in net taxes since she can reduce her tax bill more effectively using the tax credit $\Omega_{\bar{x},\bar{y}}(x_i)$ since

$$\Omega'_{\bar{x},\bar{y}}(x_i) + \frac{u_3(c_i, x)}{u_1(c_i, x)} \geq 1, \ \forall x_i \in [0, \frac{\bar{x} + \bar{y}}{M + N}]. \tag{3.40}$$

She will refrain from tax avoidance since she can decrease her tax liability more effectively with the tax credit $\Omega_{\bar{x},\bar{y}}(x_i)$, then she can with the avoidance technology since

$$\Omega'_{\bar{x},\bar{y}}(x_i) \geq \frac{1 - \frac{u_2(c_i,x)}{u_1(c_i,x)}}{\Phi'(0)}, \ \forall x_i \in [0, \frac{\bar{x} + \bar{y}}{M + N}]. \tag{3.41}$$

Since $\Omega_{\bar{x},\bar{y}}(x_i)$ and $\alpha_{\bar{x},\bar{y}}(y_i)$ are both convex functions, and since for any $s \in [0, \frac{\bar{x}+\bar{y}}{M+N}]$

$$\Omega_{\bar{x},\bar{y}}(s) \geq \alpha_{\bar{x},\bar{y}}(s), \tag{3.42}$$

then a taxpayer of type 1 will choose to rely solely on the tax credit $\Omega_{\bar{x},\bar{y}}(x_i)$. Thus, if $T(\bar{x}, \bar{y}) = \frac{1}{\Phi'(0)} \frac{\bar{x}+\bar{y}}{M+N}$, $\Lambda(x_i, y_i) = \Omega_{\bar{x},\bar{y}}(x_i) + \alpha_{\bar{x},\bar{y}}(y_i)$, and $x_G + \sum_{j \neq i}^{M} x_j + \sum_{k=1}^{N} x_k = \frac{(M+N-1)(\bar{x}+\bar{y})}{M+N}$, then an individual of type 1 will choose $c_i = Y - \frac{\bar{x}+\bar{y}}{M+N}$, $x_i = \frac{\bar{x}+\bar{y}}{M+N}$, and $t_i = 0$.

An individual of type 2 solves the maximization problem

$$\max_{c_i \geq 0, x_i \geq 0, y_i \geq 0, t_i \geq 0} u(c_i, y_G + \sum_{j \neq i}^{N} y_j + \sum_{k=1}^{M} y_k + y_i) \tag{3.43}$$
$$c_i + t_i + x_i + y_i + \Phi(T(\bar{x}, \bar{y}) - \Omega_{\bar{x},\bar{y}}(x_i) - \alpha_{\bar{x},\bar{y}}(y_i) - t_i) \leq Y.$$

At an equilibrium, a taxpayer of type 2 will pay zero in net taxes since she could reduce her tax bill more effectively using tax credit $\Omega_{\bar{x},\bar{y}}(x_i)$ since

$$\Omega'_{\bar{x},\bar{y}}(x_i) + \frac{u_3(c_i, y)}{u_1(c_i, y)} \geq 1, \ \forall x_i \in [0, \frac{\bar{x} + \bar{y}}{M + N}]. \tag{3.44}$$

She will refrain from tax avoidance since she can decrease her tax liability more cost effectively with the tax credit $\Omega_{\bar{x},\bar{y}}(x_i)$, then she can with the avoidance technology since

$$\Omega'_{\bar{x},\bar{y}}(x_i) \geq \frac{1 - \frac{u_2(c_i,x)}{u_1(c_i,x)}}{\Phi'(0)}, \ \forall x_i \in [0, \frac{\bar{x} + \bar{y}}{M + N}]. \tag{3.45}$$

If a taxpayer of type 2 did not have access to $\alpha_{\bar{x},\bar{y}}(y_i)$, then she would pay zero in net taxes, refrain from tax avoidance, and fully utilize tax credit by $\Omega_{\bar{x},\bar{y}}(x_i)$ by contributing $x_i = \frac{\bar{x}+\bar{y}}{M+N}$ towards good x. However, since since she has access to $\alpha_{\bar{x},\bar{y}}(y_i)$; $\Omega_{\bar{x},\bar{y}}(x_i)$ and $\alpha_{\bar{x},\bar{y}}(y_i)$ are both convex functions; and since

$$\Omega_{\bar{x},\bar{y}}(s) \geq \alpha_{\bar{x},\bar{y}}(s), \tag{3.46}$$

then a taxpayer of type 2 will choose to rely solely on the tax credit $\alpha_{\bar{x},\bar{y}}(y_i)$. Thus, if $T(\bar{x},\bar{y}) = \frac{1}{\Phi'(0)}\frac{\bar{x}+\bar{y}}{M+N}$, $\Lambda(x_i, y_i) = \Omega_{\bar{x},\bar{y}}(x_i) + \alpha_{\bar{x},\bar{y}}(y_i)$, and $y_G + \sum_{j\neq i}^{N} y_j + \sum_{k=1}^{M} y_k$, then an individual of type 2 will choose $c_i = Y - \frac{\bar{x}+\bar{y}}{M+N}$, $y_i = \frac{\bar{y}}{N}$, and $t_i = T_0(\bar{x},\bar{y}) = \frac{N\bar{x}-M\bar{y}}{MN+N^2}$. ■

3.8 Warm-Glow Preferences

The above analysis assumes that individuals only have preferences over the total supply of a pure public good. However, if individuals derive utility from the amount that the individual directly contributes towards the public good in addition to the total amount of the public good supplied, then this will affect the subsidy necessary to induce individuals to contribute to the public good. In this section, I assume that individuals derive utility from both the level of the public good, and their contribution to the public good. I adopt the basic warm-glow preferences from Andreoni (1990). Individual preferences are represented by $u(c_i, x, x_i)$, which is assumed to be concave. If individuals experience warm-glow from giving to the public good, then the pure public good will exhibit properties of a mixed public good.[12] In fact, for any given level of the public good, warm-glow preferences will necessitate a smaller subsidy than in the case of their absence.

Proposition 7

If warm-glow preferences are included in social welfare and $\Phi'(0) < 1$, then for a given level of the public good $\bar{x}$, tax avoidance can be eliminated and $\bar{x}$ units of the public good provided with equal consumption for each taxpayer by setting

[12] The results in this section generalize to all mixed public goods.

$\Lambda(x_i) = \frac{N-1}{N\Phi'(0)}[1 - \frac{u_3(Y-\frac{\bar{x}}{N},\bar{x},\frac{\bar{x}}{N})}{u_1(Y-\frac{\bar{x}}{N},\bar{x},\frac{\bar{x}}{N})}]x_i$.

Proof:

Let the government set $\Lambda(x_i) = \frac{N-1}{N\Phi'(0)}[1 - \frac{Y-\frac{\bar{x}}{N},\bar{x},\frac{\bar{x}}{N})}{u_1(Y-\frac{\bar{x}}{N},\bar{x},\frac{\bar{x}}{N})}]x_i$, and $T = \frac{N-1}{N\Phi'(0)}[1 - \frac{u_3(Y-\frac{\bar{x}}{N},\bar{x},\frac{\bar{x}}{N})}{u_1(Y-\frac{\bar{x}}{N},\bar{x},\frac{\bar{x}}{N})}]\frac{\bar{x}}{N}$. The individual specifically solves the following maximization problem:

$$\begin{aligned} \max_{c_i \geq 0, x_i \geq 0, t_i \geq 0} \quad & u(c_i, x_G + \sum_{j \neq i}^{N} x_j + x_i, x_i) \\ \text{s.t.} \quad & c_i + t_i + x_i + \Phi(T - \Lambda(x_i) - t_i) \leq Y. \end{aligned} \tag{3.47}$$

From the first-order conditions, at the equilibrium, a taxpayer will rely exclusively on tax credits, and pay zero in net taxes and will not utilize the tax avoidance technology if

$$\Lambda'(x_i) + \frac{u_2(c_i, x, x_i) + u_3(c_i, x, x_i)}{u_1(c_i, x, x_i)} \geq 1 \tag{3.48}$$

and

$$\Lambda'(x_i) + \frac{u_2(c_i, x, x_i) + u_3(c_i, x, x_i)}{u_1(c_i, x, x_i)} \geq 1. \tag{3.49}$$

If, at the equilibrium $\Lambda(x_i) = \frac{N-1}{N\Phi'(0)}[1 - \frac{Y-\frac{\bar{x}}{N},\bar{x},\frac{\bar{x}}{N})}{u_1(Y-\frac{\bar{x}}{N},\bar{x},\frac{\bar{x}}{N})}]x_i$ satisfies the above inequalities, and the taxpayer fully utilizes the tax credit, pays zero in net taxes, and does not utilize the tax avoidance technology. If $x_G + \sum_{j \neq i}^{N} x_j = \frac{(N-1)\bar{x}}{N}$, $\Lambda(x_i) = [\frac{N-1}{N\Phi'(0)} - \frac{u_3(Y-\frac{\bar{x}}{N},\bar{x},\frac{\bar{x}}{N})}{u_1(Y-\frac{\bar{x}}{N},\bar{x},\frac{\bar{x}}{N})\Phi'(0)}]x_i$, and $T = [\frac{N-1}{N\Phi'(0)} - \frac{u_3(Y-\frac{\bar{x}}{N},\bar{x},\frac{\bar{x}}{N})}{u_1(Y-\frac{\bar{x}}{N},\bar{x},\frac{\bar{x}}{N})\Phi'(0)}]\frac{\bar{x}}{N}$, then the solution to the individual problem is $(c_i, x_i, t_i) = (Y - \frac{\bar{x}}{N}, \frac{\bar{x}}{N}, 0)$. ■

There are two possibilities for welfare analysis with warm-glow preferences: include warm-glow in social welfare or exclude exclude warm-glow from social welfare. In Proposition 8, I include warm-glow in the welfare calculation. In Proposition 9, I follow the recommendation of Diamond (2006) for the analysis, and exclude warm-glow from welfare considerations. In the case without warm-glow, I assume separable preferences between warm-glow and the other two inputs in the utility function. This allows for welfare analysis while excluding any preferences represented by warm-glow. The basic results hold up in the presence of warm-glow preferences. The social planner is still able to implement a Pareto-efficient level of the public good and eliminate tax avoidance. For both cases, the tax credit required to implement the first-best level of the public good is smaller than if individuals' preferences do not exhibit warm-glow. The level of the tax credit required in the case in which warm-glow is omitted from social welfare is smaller than the optimal in the case that warm-glow is included in social welfare. The formal result is given in Proposition 8.

When warm-glow is included in the welfare analysis, the social planner takes into account the warm-glow each individual experiences when giving to the public good.[13] Thus the Pareto-efficient level of the public good is higher than the Pareto-efficient level of the public good in the absence of warm-glow preferences. The subsidy required to implement the optimum in the presence of warm-glow preferences is smaller than the subsidy required to implement the optimum in the absence of warm-glow preferences. Although, in this case the social planner takes into account the warm-glow the giver experiences, there is no need to provide a

[13] I also implicitly assume that individuals do not experience any warm-glow via the taxes that they pay.

subsidy for the individual since warm-glow is only providing a private benefit for the giver. When warm-glow is included in the welfare analysis, the optimal subsidy is the original subsidy multiplied by one minus the MRS between private consumption and warm-glow. Thus, the optimal subsidy is reduced by the relative weight of warm-glow to take account that it provides no positive externality to other individuals. The formal result is given in Proposition 8.

Necessary for Proposition 8, define c^{**} and x^{**} to be the Pareto-efficient allocation with equal consumption for all individuals. Specifically, c^{**} and x^{**} satisfy $c^{**} = Y - \frac{x^{**}}{N}$. x^{**} is the unique value s.t. $u_1(Y - \frac{x^{**}}{N}, x^{**}, \frac{x^{**}}{N}) - u_3(Y - \frac{x^{**}}{N}, x^{**}, \frac{x^{**}}{N}) = Nu_2(Y - \frac{x^{**}}{N}, x^{**}, \frac{x^{**}}{N})$.[14]

Proposition 8

If warm-glow preferences are included in social welfare and $\Phi'(0) < 1$, then tax avoidance can be eliminated and the Pareto-efficient level of the public good with equal consumption for each taxpayer implemented by setting $\Lambda(x_i) = \frac{N-1}{N\Phi'(0)}[1 - \frac{u_3(c^{**},x^{**},\frac{x^{**}}{N})}{u_1(c^{**},x^{**},\frac{x^{**}}{N})\Phi'(0)}]x_i$.

Proof:

Let the government set $\Lambda(x_i) = \frac{N-1}{N\Phi'(0)}[1 - \frac{u_3(c^{**},x^{**},\frac{x^{**}}{N})}{u_1(c^{**},x^{**},\frac{x^{**}}{N})}]x_i$, and $T = \frac{N-1}{N\Phi'(0)}[1 - \frac{u_3(c_i,x^{**},\frac{x^{**}}{N})}{u_1(c^{**},x^{**},\frac{x^{**}}{N})}]\frac{x^{**}}{N}$, where x^{**} is the Samuelson level of the public good. More specifically, consider, x^{**} and c^{**}, such that $c^{**} = Y - \frac{x^{**}}{N}$. x^{**} is the unique value s.t. $u_1(Y - \frac{x^{**}}{N}, x^{**}, \frac{x^{**}}{N}) - u_3(Y - \frac{x^{**}}{N}, x^{**}, \frac{x^{**}}{N}) = Nu_2(Y - \frac{x^{**}}{N}, x^{**}, \frac{x^{**}}{N})$. $\forall i, (c_i, x_i) = (Y - \frac{x^{**}}{N}, \frac{x^{**}}{N})$ is the unique set which is Pareto-efficient with equal consumption for all individuals.

[14]See the appendix for proof.

The individual specifically solves the following maximization problem:

$$\begin{aligned} \max_{c_i \geq 0, x_i \geq 0, t_i \geq 0} \quad & u(c_i, x_G + \sum_{j \neq i}^{N} x_j + x_i, x_i) \\ \text{s.t.} \quad & c_i + t_i + x_i + \Phi(T - \Lambda(x_i) - t_i) \leq Y. \end{aligned} \tag{3.50}$$

From the first-order conditions, at the equilibrium, a taxpayer will rely exclusively on tax credits, and pay zero in net taxes and will not utilize the tax avoidance technology if

$$\Lambda'(x_i) + \frac{u_2(c_i, x, x_i) + u_3(c_i, x, x_i)}{u_1(c_i, x, x_i)} \geq 1 \tag{3.51}$$

and

$$\Lambda'(x_i) + \frac{u_2(c_i, x, x_i) + u_3(c_i, x, x_i)}{u_1(c_i, x, x_i)} \geq 1. \tag{3.52}$$

At the equilibrium, if $x = x^*$, then $\Lambda(x_i) = \frac{N-1}{N\Phi'(0)}[1 - \frac{u_3(c^{**}, x^{**}, \frac{x^{**}}{N})}{u_1(c^{**}, x^{**}, \frac{x^{**}}{N})}]x_i$ satisfies the above inequalities and the taxpayer fully utilizes the tax credit, pays zero in net taxes, and does not utilize the tax avoidance technology. If $x_G + \sum_{j \neq i}^{N} x_j = \frac{(N-1)x^{**}}{N}$, $\Lambda(x_i) = [\frac{N-1}{N\Phi'(0)} - \frac{u_3(c^{**}, x^{**}, \frac{x^*}{N})}{u_1(c_i^{**}, x^{**}, \frac{x^{**}}{N})\Phi'(0)}]x_i$, and $T = [\frac{N-1}{N\Phi'(0)} - \frac{u_3(c^{**}, x^{**}, \frac{x^*}{N})}{u_1(c^{**}, x^{**}, \frac{x^*}{N})\Phi'(0)}]\frac{x^{**}}{N}$, then the solution to the individual problem is $(c_i, x_i, t_i) = (Y - \frac{x^{**}}{N}, \frac{x^{**}}{N}, 0)$. ■

For simplicity, in the case in which warm-glow preferences are excluded from welfare analysis, I assume that warm-glow preferences are separable from total public good provision and private good consumption. More specifically, assume that they take the form $u(c_i, x, x_i) = v(c_i, x) + w(x_i)$. The first-best level of the public good when warm-glow preferences exist but are excluded from welfare considerations is equal to the first-best level of the public good in the case without any warm-glow preferences, but less than the first-best level of the public good when warm-glow preferences are included in welfare analysis. Thus, the subsidy required to implement the first-best level of the public good when warm-glow preferences exist but are excluded from welfare considerations is smaller than the subsidy required to implement the first-best when warm-glow exits and is included in welfare analysis. In the previous case, the subsidy was simply altered so it did not subsidize the donor for the warm-glow feel that others were not experiencing. In this case the subsidy must counteract the fact that the donor is experiencing warm-glow.

Proposition 9

If warm-glow preferences exist but are excluded from social welfare and $\Phi'(0) < 1$, then tax avoidance can be eliminated and the Pareto-efficient level of the public good with equal consumption for each taxpayer implemented by setting $\Lambda(x_i) = [\frac{N-1}{N\Phi'(0)} - \frac{u_3(c^*,x^*,\frac{x^*}{N})}{u_1(c^*,x^*,\frac{x^*}{N})\Phi'(0)}]x_i$.

Proof:

Let the government set $\Lambda(x_i) = [\frac{N-1}{N\Phi'(0)} - \frac{u_3(c^*,x^*,\frac{x^*}{N})}{u_1(c^*,x^*,\frac{x^*}{N})\Phi'(0)}]x_i$, and $T = \Lambda(x_i) = [\frac{N-1}{N\Phi'(0)} - \frac{u_3(c^*,x^*,\frac{x^*}{N})}{u_1(c^*,x^*,\frac{x^*}{N})\Phi'(0)}]\frac{x^*}{N}$, where x^* is the Samuelson level of the public good. Recall that x^* and c^* are the Pareto-efficient allocation with equal consumption, Recall that $c^* = Y - \frac{x^*}{N}$, and that x^* is the unique value s.t. $u_1(Y - \frac{x^*}{N}, x^*) = Nu_2(Y - \frac{x^*}{N}, x^*)$.[15]

[15] $x = x^*$ and $c_i = c^*, \forall i \in N$ is the unique allocation that maximizes utilitarian social welfare or any concave social welfare function.

The individual specifically solves the following maximization problem:

$$\max_{c_i, x_i, t_i} \quad u(c_i, x_G + \sum_{j \neq i}^{N} x_j + x_i) + v(x_i)$$
$$\text{s.t.} \quad c_i + t_i + x_i + \Phi(T - \Lambda(x_i) - t_i) \leq Y \tag{3.53}$$

From the first-order conditions, at the equilibrium, a taxpayer will rely exclusively on tax credits, and pay zero in net taxes and will not utilize the tax avoidance technology if

$$\Lambda'(x_i) + \frac{u_2(c_i, x, x_i) + u_3(c_i, x, x_i)}{u_1(c_i, x, x_i)} \geq 1. \tag{3.54}$$

and

$$\Lambda'(x_i) + \frac{u_2(c_i, x, x_i) + u_3(c_i, x, x_i)}{u_1(c_i, x, x_i)} \geq 1. \tag{3.55}$$

If, at the equilibrium $x = x^*$, then $\Lambda(x_i) = [\frac{N-1}{N\Phi'(0)} - \frac{u_3(c_i^*, x^*, \frac{x^*}{N})}{u_1(c_i^*, x^*, \frac{x^*}{N})\Phi'(0)}]x_i$ satisfies the above inequalities, and the taxpayer fully utilizes the tax credit, pays zero in net taxes, and does not utilize the tax avoidance technology. If $x_G + \sum_{j \neq i}^{N} x_j = \frac{(N-1)x^*}{N}$, $\Lambda(x_i) = [\frac{N-1}{N\Phi'(0)} - \frac{u_3(c_i^*, x^*, \frac{x^*}{N})}{u_1(c_i^*, x^*, \frac{x^*}{N})\Phi'(0)}]x_i$, and $T = [\frac{N-1}{N\Phi'(0)} - \frac{u_3(c_i, x, x_i)}{u_1(c_i, x, x_i)\Phi'(0)}]\frac{x^*}{N}$, then the solution to the individual problem is $(c_i, x_i, t_i) = (Y - \frac{x^*}{N}, \frac{x^*}{N}, 0)$. ■

3.9 Labor Distorting Taxes

In the basic model, if taxes are not labor-distorting and $\Phi'(0) < \frac{N-1}{N}$, then the tax credit required to implement the Samuelson level of the public good and eliminate tax avoidance exceeds one. However, if the tax credit required to eliminate tax avoidance exceeds one, then the gross tax liabilities required to implement a given level of the public good with tax credits may exceed the gross tax liabilities required to fund the same level of the public good with direct government financing. If lump-sum taxes are employed, increasing the gross tax liability has no efficiency costs, however it would seem intuitive that this may not be the case with labor distorting taxes.

If the government must rely on labor distorting taxes, then economic intuition may lead one to believe that the optimal policy may involve balancing higher marginal tax rates against less tax avoidance. It would seem that taxpayers would have to endure greater labor-distortions through higher marginal tax rates in exchange for higher tax credits and lower tax avoidance. It would follow that if the government must rely on labor distorting taxes, then the optimal policy may not involve fully eliminating tax avoidance. If the tax credit required to eliminate tax avoidance exceeds one, then (it would seem that) there is a tension between reducing avoidance, and reducing the tax rate. It would follow that the presence of tax avoidance would pull the optimal solution towards larger tax credits, and the presence of labor distorting taxes would push the optimal solution towards tax credits smaller than one. However, as I demonstrate in this section, this is not the case.

It is unsurprising that for low levels of the tax credit rate, a slight increase in the level of the tax credit decreases both the magnitude of labor distorting taxes and decreases tax avoidance. However, as demonstrated in this section, the introduction of tax credits for contributions to the public good can eliminate tax avoidance and lower the labor-distortion wedge. Specifically, I demonstrate that for a given tax rate and a given level of public good provision, there exists a tax credit function and an alternative tax rate such that tax avoidance is eliminated and the labor-distortion wedge is decreased.

While the level of tax avoidance depends primarily on the size of the subsidy and size of the tax burden, at a corner solution in which there is zero tax avoidance, the level of labor distortion depends primarily on the magnitude of the ratio of the tax rate to the subsidy rate.[16] The introduction of a subsidy sufficiently large enough to eliminate tax avoidance would also require a substantially increased tax rate. Although the introduction of a tax credit greater than one would necessitate a higher tax rate, the effective labor wedge, $\frac{\tau}{s}(1 - \frac{u_3(c_i,l_i,\bar{x})}{u_1(c_i,l_i,\bar{x})})$, may not necessarily exceed the original tax rate.

However, in this section I demonstrate that the introduction of a tax credit greater than one can both eliminate tax avoidance and reduce the labor wedge. In fact, if the initial costs of tax avoidance are sufficiently high, then the optimal subsidy in an environment without avoidance is the identical subsidy in an environment with avoidance. Thus, under this case a subsidy can implement the traditional second best optimum.

Consider the simple model with a single public good from section 3. I take the

[16]Specifically, as I demonstrate below, the labor-distortion wedge equals $\frac{\tau}{s}(1 - \frac{u_3(c_i,l_i,\bar{x})}{u_1(c_i,l_i,\bar{x})}$.

basic model, and endogenize income by incorporating disutility from labor. For simplicity, I assume that the public good is funded using a flat income tax with rate $\tau(\bar{x})$, but the tax credit function can take any form. The maximization problem for each individual i, taking as given $w, \tau, x_j\ \forall j \neq i$, and x_G is the following:

$$\begin{aligned} \max_{c_i \geq 0, l_i \geq 0, x_i \geq 0, t_i \geq 0} \quad & u(c_i, l_i, x) \\ \text{s.t.} \quad & c_i + t_i + x_i + \Phi(\tau w l_i - \Lambda(x_i) - t_i) \leq w l_i \end{aligned} \tag{3.56}$$

Assume that $\Lambda(x_i) = s x_i$. For a tax credit strictly less than one, consider an increase in the level of the tax credit. There are three possibilities: that an individual does not engage in any tax avoidance, that an individual engages in a positive level of tax avoidance but also pays a positive amount in taxes, or that an individual avoids all of her taxes. In the absence of a tax credit, if $\Phi'(0) < 1$, then a taxpayer will avoid a positive amount of her tax liability.

Avoidance, Tax Payments, and Contributions to the Public Good ($\Phi'(0) < 1$)

		Avoidance	Tax Payments	Public Good Contributions
$1 - \frac{u_3(c_i,l_i,\bar{x})}{u_1(c_i,l_i,\bar{x})} < s\Phi'(0)$	$\Phi'(\bar{T}) \in \mathbb{R}_+$	0	0	+
$s\Phi'(0) < 1 - \frac{u_3(c_i,l_i,\bar{x})}{u_1(c_i,l_i,\bar{x})} < s\Phi'(\bar{T})$	$\Phi'(\bar{T}) < 1$	+	0	+
$s\Phi'(\bar{T}) < 1 - \frac{u_3(c_i,l_i,\bar{x})}{u_1(c_i,l_i,\bar{x})}$	$\Phi'(\bar{T}) < 1$	+	0	0
$s\Phi'(0) < 1 - \frac{u_3(c_i,l_i,\bar{x})}{u_1(c_i,l_i,\bar{x})} < s$	$\Phi'(\bar{T}) > 1$	+	0	+
$s\Phi'(0) < 1 - \frac{u_3(c_i,l_i,\bar{x})}{u_1(c_i,l_i,\bar{x})} = s$	$\Phi'(\bar{T}) > 1$	+	+	+
$s < 1 - \frac{u_3(c_i,l_i,\bar{x})}{u_1(c_i,l_i,\bar{x})}$	$\Phi'(\bar{T}) > 1$	+	+	0

The table above indicates the presence of tax avoidance, payments, and contributions to the public good as a function of an individual's first-order conditions. For a given level of the public good $\bar{x}$, total tax liability $\bar{T}$, and tax credit s, if $s\Phi'(0) > 1 - \frac{u_3(c_i,l_i,\bar{x}))}{u_1(c_i,l_i,\bar{x})}$ then there will be zero avoidance. In contrast, for a given level of the public good $\bar{x}$, total tax liability $\bar{T}$, and tax credit s, if $s < 1 - \frac{u_3(c_i,l_i,\bar{x})}{u_1(c_i,l_i,\bar{x})}$, then there will be zero contributions to the public good, and the tax credit will have no effect on the level of tax avoidance. Holding the tax rate constant, if there is already a positive level of tax avoidance and a positive contribution to the public good, then an increase in the tax credit will lead to a decrease in tax avoidance and an increase in the provision of the public good.

For a given level of the public good $\bar{x}$, I demonstrate that the introduction of a tax credit can reduce tax avoidance while reducing the tax rate required to fund the public good. Specifically, Proposition 10 demonstrates that the introduction of a dollar-for-dollar non-refundable tax credit, can reduce the labor wedge and decrease tax avoidance. For all cases, assume that $\Phi'(0) < 1$. Also, for all the of the remaining proofs in this chapter, define e' s.t. $\Phi'(e') = 1$.

Proposition 10

If $\Phi'(0) < 1$, then the introduction of the tax credit $s = 1$ can lead to the provision of $\bar{x}$ with a strictly lower tax rate and strictly less tax avoidance.

Proof:

The first-order conditions for an individual who engages in a positive level of tax avoidance and contributes a positive amount towards the public good are

$$s\Phi'(\tau w l_i - s x_i - t_i) = 1 - \frac{u_3(c_i, l_i, x)}{u_1(c_i, l_i, x)} \tag{3.57}$$

and

$$u_2(c_i, l_i, x) = w(1 - \tau\Phi'(\tau w l_i - s x_i - t_i))u_1(c_i, l_i, x). \tag{3.58}$$

Facing tax credit $s = 1$ and tax rate τ, each individual's optimal choice is characterized by the first-order conditions:

$$\frac{\tau}{s} w l_i = \frac{x}{N} + e_i, \tag{3.59}$$

$$u_2(c_i, l_i, x) = w(1 - \frac{\tau}{s}(1 - \frac{u_3(c_i, l_i, x)}{u_1(c_i, l_i, \bar{x})}))u_1(c_i, l_i, x), \tag{3.60}$$

$$c_i = (1 - \frac{\tau}{s}) w l_i + e_i - \Phi'(e_i), \tag{3.61}$$

and

$$\Phi'(e_i) = 1 - \frac{u_3(c_i, l_i, x)}{u_1(c_i, l_i, x)}. \tag{3.62}$$

Generally, let (τ, s, a, ω) denote the policy and economic environment consisting of tax rate τ, non-refundable tax credit s, presence of tax avoidance technology a, and wealth endowment ω. Access to the avoidance technology is indicated by a: $a = a_1$ indicates access to the avoidance technology and $a = a_0$ indicates a lack of access to the avoidance technology. Furthermore, conditional on s, a, and ω, let tax rate $\tau(\bar{x}, s, a, \omega)$ characterize the smallest tax rate such that $\bar{x}$ is provided:

$\frac{\bar{x}}{N} = \tau(\bar{x}, s, a, \omega)wl(\tau(\bar{x}, s, a, \omega)))$. Consider the introduction of $s = 1$. For simplicity define $\tau^*(\bar{x})$ such that $\tau^*(\bar{x}) = \tau(\bar{x}, s, a_1, 0)$. For a given level of $\bar{x}$, and corresponding $\tau^*(\bar{x})$ and must satisfy the following set of equations:

$$u_2(c_i, l_i, \bar{x}) = w(1 - \tau^*(\bar{x})(1 - \frac{u_3(c_i, l_i, \bar{x})}{u_1(c_i, l_i, \bar{x})}))u_1(c_i, l_i, x), \tag{3.63}$$

$$l_i = \frac{\frac{\bar{x}}{N} + e_i^*(\bar{x})}{\tau^*(\bar{x}}, \tag{3.64}$$

$$c_i = \frac{\frac{\bar{x}}{N} + e_i^*(\bar{x})}{\tau^*(\bar{x})} - \frac{\bar{x}}{N} - \Phi'(e_i^*(\bar{x})), \tag{3.65}$$

and

$$\Phi'(e_i^*(\bar{x})) = 1 - \frac{u_3(c_i, l_i, x)}{u_1(c_i, l_i, x)}. \tag{3.66}$$

Formally

$$\tau^*(\bar{x}) = \min_{(c_i, l_i, \tau) \in \mathbb{R}_+^3} \Big\{ \tau \; |(c_i, l_i, 0, \frac{\bar{x}}{N}, \tau) \in U(\tau, s^*) \Big\}, \tag{3.67}$$

such that

$$U(\tau, s^*) = \underset{(c_i, l_i, t_i, x_i) \in \mathbb{R}_+^4}{\arg\max} \quad u(c_i, l_i, x)$$
$$\text{s.t.} \quad c_i + t_i + x_i + \Phi(\tau w l_i - s^* - x_i - t_i) \leq (1 - \tau) w l_i. \tag{3.68}$$

Since the first order conditions are sufficient and necessary for an optimum,

$$\begin{aligned}\tau^*(\bar{x}) = \min_{(c_i,l_i,e_i,\tau)\in\mathbb{R}_+^{3N+1}} \{\tau \mid u_2(c_i,l_i,\bar{x}) = w(1-\tau(1-\frac{u_3(c_i,l_i,\bar{x})}{u_1(c_i,l_i,\bar{x})}))u_1(c_i,l_i,\bar{x}),\\ l_i = \frac{\frac{\bar{x}}{N}+e_i(\bar{x})}{\tau},\\ c_i = \frac{\frac{\bar{x}}{N}+e_i(\bar{x})}{\tau} - \frac{\bar{x}}{N} - \Phi'(e_i(\bar{x})),\\ \Phi'(e_i(\bar{x})) = 1 - \frac{u_3(c_i,l_i,x)}{u_1(c_i,l_i,x)}\}.\end{aligned} \tag{3.69}$$

Under the above system of equations, the taxpayer faces a labor wedge equal to $\tau(1-\frac{u_3(c_i,l_i,\bar{x})}{u_1(c_i,l_i,\bar{x})})$. Define $e_i^*(\bar{x})$ to the e_i which solves the above set of equations In an environment in which the taxpayer does not have access to the tax avoidance technology, define $\tilde{\tau}(\bar{x})$ such that $\tilde{\tau}(\bar{x}) = \tau(\bar{x}+Ne_i^*(\bar{x}), 0, a_0, e_i^*(\bar{x}) - \Phi'(e_i^*(\bar{x})))$: the minimum tax rate required to provide public good level $\bar{x}$ in the absence of a tax credit. Thus,

$$\begin{aligned}\tilde{\tau}(\bar{x}) = \min_{(c_i,l_i,\tau)\in\mathbb{R}_+^{2N+1}} \{\tau \mid u_2(c_i,l_i,\bar{x}) = w(1-\tau)u_1(c_i,l_i,\bar{x}),\\ l_i = \frac{\frac{\bar{x}}{N}+e_i^*(\bar{x})}{\tau},\\ c_i = \frac{\frac{\bar{x}}{N}+e_i^*(\bar{x})}{\tau} - (\frac{\bar{x}}{N}+e_i^*(\bar{x})) + e_i^*(\bar{x}) - \Phi'(e_i^*(\bar{x})),\\ \Phi'(e_i(\bar{x})) = 1 - \frac{u_3(c_i,l_i,x)}{u_1(c_i,l_i,x)}\}.\end{aligned} \tag{3.70}$$

Rearranging yields

$$\tilde{\tau}(\bar{x}) = \min_{(c_i,l_i,\tau)\in\mathbb{R}_+^{2N+1}} \{\tau \mid u_2(c_i, l_i, \bar{x}) = w(1-\tau)u_1(c_i, l_i, \bar{x}),$$
$$l_i = \frac{\frac{\bar{x}}{N} + e_i^*(\bar{x})}{\tau}, \tag{3.71}$$
$$c_i = \frac{\frac{\bar{x}}{N} + e_i^*(\bar{x})}{\tau} - \frac{\bar{x}}{N} - \Phi'(e_i^*(\bar{x}))\}.$$

It follows that $\tau^*(\bar{x}) < \tilde{\tau}(\bar{x})$. The system of equations determining $\tau^*(\bar{x}) = \tau(\bar{x}, 1, a_1, 0)$ differs from the system of equations determining $\tilde{\tau}(\bar{x}) = \tau(\bar{x} + Ne_i^*(\bar{x}), 0, a_0, e_i^*(\bar{x}) - \Phi'(e_i^*(\bar{x})))$, only in the addition of the factor $(1 - \frac{u_3(c_i,l_i,\bar{x})}{u_1(c_i,l_i,\bar{x})})$ in the expression for the first-order condition determining the trade-off between labor and consumption.[17] Since the set of equations determining c_i and l_i under both policy $(\tau^*(\bar{x}), s^*, a_1, 0)$ and policy $(\frac{\tilde{\tau}(\bar{x})}{s^*}, 0, a_0, 0)$ are identical except for the addition of the factor $(1 - \frac{u_3(c_i,l_i,\bar{x})}{u_1(c_i,l_i,\bar{x})})$, conditional on a given τ, the labor wedge in the latter will exceed the labor wedge in the former.[18] Thus, in order to provide a given level of the public good $\bar{x}$, $\tau^*(\bar{x}) < \tilde{\tau}(\bar{x})$.[19] [20]

If the government funds public good level $\bar{x}$ without tax credits, the taxpayer's

[17]Under both $(\tau^*(\bar{x}), 1, a_1, 0)$ and $(\frac{\tilde{\tau}(\bar{x})}{s^*}, 0, a_0, 0)$, there is no tax avoidance.

[18]Thus, for a given τ, the taxpayer, who does have access to the tax avoidance technology, and faces tax credit s^* has a lower labor wedge than a taxpayer who does not have access to the tax avoidance technology nor access to a tax credit.

[19]$\tau^*(\bar{x}) < \tilde{\tau}(\bar{x})$ is identical to the basic result from Roberts (1987). In the absence of a tax avoidance technology, the introduction of a tax expenditure for contributions to the public good can provide a given level of the public good with a lower gross tax rate.

[20]Choosing the smallest τ which satisfies the above system of equations determining $\tilde{\tau}(\bar{x})$ ensures that at the equilibrium the elasticity of gross income with respect to the net wage is less than $\frac{1-\tau^*(\bar{x})}{\tau^*(\bar{x})}$.

first-order condition at the equilibrium is

$$u_2(c_i, l_i, x) = w(1 - \bar{\tau}(\bar{x}))u_1(c_i, l_i, x). \tag{3.72}$$

In the absence of tax credits and facing tax rate $\tau(\bar{x}, 0, a_1, 0) = \bar{\tau}(\bar{x})$, the equilibrium must satisfy

$$u_2(c_i, l_i, \bar{x}) = w(1 - \bar{\tau}(\bar{x}))u_1(c_i, l_i, \bar{x}). \tag{3.73}$$

$$c_i + \phi(e') + \frac{\bar{x}}{N} = wl_i \tag{3.74}$$

and

$$e' + \frac{\bar{x}}{N} = \bar{\tau}(\bar{x})wl_i. \tag{3.75}$$

Rearranging, the equilibrium must satisfy

$$u_2(c_i, l_i, \bar{x}) = w(1 - \bar{\tau}(\bar{x}))u_1(c_i, l_i, \bar{x}), \tag{3.76}$$

$$c_i = \frac{e' + \frac{\bar{x}}{N}}{\bar{\tau}(\bar{x})} - (e' + \frac{\bar{x}}{N}) + (e' - \phi(e')), \tag{3.77}$$

and

$$l_i = \frac{e' + \frac{\bar{x}}{N}}{\bar{\tau}(\bar{x})w}. \tag{3.78}$$

The corresponding tax rate equals

$$\bar{\tau}(\bar{x}) = \min_{(c_i,l_i,\tau)\in\mathbb{R}^3_+} \{\tau \mid u_2(c_i, l_i, \bar{x}) = (1 - \bar{\tau}(\bar{x}))u_1(c_i, l_i, \bar{x}), \quad l_i = \frac{e' + \frac{\bar{x}}{N}}{\bar{\tau}(\bar{x})w}, \quad c_i = \frac{e' + \frac{\bar{x}}{N}}{\bar{\tau}(\bar{x})} - (e' + \frac{\bar{x}}{N}) + (e' - \phi(e'))\}. \tag{3.79}$$

The above set of equations determining $\bar{\tau}(\bar{x}) = \tau(\bar{x}, 0, a_1, 0)$ is equivalent to the set of equations determining $\tau(\bar{x} + Ne', 0, a_0, e' - \phi(e')$ so that the two tax rates are equal. Since the set of equations for $\bar{\tau}(\bar{x} + Ne', 0, a_0, e' - \phi(e')$ differs from the set of equations $\tilde{\tau}(\bar{x}) = \tau(\bar{x} + Ne^*_i(\bar{x}), 0, a_0, e^*_i(\bar{x}) - \Phi'(e^*_i(\bar{x})))$ only in the addition of an additional revenue requirement and an additional endowment effect, it follows that $\tilde{\tau}(\bar{x}) = \tau(\bar{x}+Ne^*_i(\bar{x}), 0, a_0, e^*_i(\bar{x})-\Phi'(e^*_i(\bar{x}))) < \bar{\tau}(\bar{x}+Ne', 0, a_0, e'-\phi(e') = \bar{\tau}(\bar{x}, 0, a_1, 0)$. Since, as previously shown $\tau(\bar{x}, 1, a_1, 0) < \bar{\tau}(\bar{x} + Ne', 0, a_0, e' - \phi(e')$, it follows that $\bar{\tau}(\bar{x}) = \tau(\bar{x}, 1, a_1, 0) < \tau(\bar{x}, 0, a_1, 0) = \tau^*(\bar{x})$. ■

In the traditional models developed by Feldstein and Roberts, the introduction of a tax credit for contributions to the public good leads to minimal welfare gains: since the optimal subsidies approach one as the number of taxpayers grows, the reduction in labor-distortion is very close to zero. In contrast, in the presence of tax avoidance the introduction of a tax credit can lead to a non-trivial welfare gain. Specifically, Proposition 11 demonstrates that the introduction of a tax credit of $s = 1$ leads to welfare gains at least as great as the reduction in the costs from tax avoidance. This results stems from the fact that the tax credit serves two functions: it both reduces tax avoidance and decreases the level of labor-distortion.

Proposition 11

If $\Phi'(0) < 1$, for each individual, the compensating variation for a policy change from $(\tau(\bar{x}), 0)$ to $(\tau(\bar{x}), 1)$ is greater than or equal to $\Phi(e') - \Phi(e'')$.

Proof:

Facing policy $(\tau(\bar{x}), 1)$, an individual could choose to work l', avoid e'' in taxes, and contribute $\tau''(l'' - l')$ to the public good. If the taxpayer chose labor l', avoidance level e'', and contribution to the public good $\tau''(l'' - l')$, then the taxpayer's compensating variation from a change in policy from $(\tau(\bar{x}), 0)$ to $(\tau(\bar{x}), 1)$ would equal $\Phi(e') - \Phi(e'')$. Since an individual facing $(\tau(\bar{x}), 1)$ might choose a more preferred policy, then compensating variation from a change in policy from $(\tau(\bar{x}), 0)$ to $(\tau(\bar{x}), 1)$ would be greater than or equal to $\Phi(e') - \Phi(e'')$ ■

Proposition 11 demonstrates that for any level of the public good, if $\Phi'(0) < 1$, then for each individual, the compensating variation for a policy change from $(\tau(\bar{x}), 0)$ to $(\tau(\bar{x}), 1)$ is greater than or equal to $\Phi(e') - \Phi(e'')$. Proposition 11 provides a lower bound on the welfare gain from the introduction of a tax credit; however, the welfare gains may in fact be higher than $\Phi(e') - \Phi(e'')$. Proposition 12 captures the main insight of this chapter: that the introduction of a tax credit can both eliminate tax avoidance and decrease the labor wedge. If the tax credit is sufficiently large such that it induces the taxpayer from abstaining from tax avoidance, then at equilibrium public good level $\bar{x}$, the first-order condition capturing the leisure-consumption trade-off for the taxpayer is

$$u_2(c_i, l_i, x) = w(1 - \frac{\tau}{s}(1 - \frac{u_3(c_i, l_i, \bar{x})}{u_1(c_i, l_i, \bar{x})}))u_1(c_i, l_i, x), \tag{3.80}$$

and her labor wedge equals $\frac{\tau}{s}(1 - \frac{u_3(c_i,l_i,\bar{x})}{u_1(c_i,l_i,\bar{x})})$.[21] At the equilibrium the taxpayer fully contributes to the public good amount $\bar{x}$, and fully eliminates her tax liability. [22]

The labor wedge depends on the ratio of the tax rate to the tax credit, and one minus the MRS. In order to eliminate tax avoidance and decrease the labor wedge, the government can choose an $s^*(\bar{x})$ and $\tau^*(\bar{x})$ sufficiently large such that tax avoidance is eliminated and such that the ratio, $\frac{\tau^*(\bar{x})}{s^*(\bar{x})}$ is smaller than the original tax rate. Thus, if there is zero tax avoidance, then the level of labor distortion is not determined by the marginal income tax rate but by the ratio of the marginal income tax rate to the subsidy rate. $\tau^*(\bar{x})$ may exceed the original tax rate, however if $\frac{\tau^*(\bar{x})}{s^*(\bar{x})}$ is smaller than the original tax rate, then there will be a smaller labor wedge.

Proposition 12

For a given level of the public good $\bar{x}$ and corresponding tax rate $\bar{\tau}(\bar{x})$, there exists an alternative tax rate $\tau^*(\bar{x})$ and tax credit s^* such that the introduction of tax credit s^* and tax rate $\tau^*(\bar{x})$ eliminates tax avoidance and leads to a strictly lower level labor-distortion than if $\bar{x}$ was exclusively financed directly by the government.

Proof:

[21] If the taxpayer remits a positive amount of tax payments to the government, then $\frac{\tau}{s}(1 - \frac{u_3(c_i,l_i,\bar{x})}{u_1(c_i,l_i,\bar{x})})$ is also the labor wedge. If an individual engages in a strictly positive level of tax avoidance, then her labor wedge equals $\tau\Phi'(\tau w l_i - s x_i - t_i)$.

[22] Although she pays zero in net tax liability, she still faces a positive but diminished tax liability.

The first-order conditions for an individual who engages in a positive level of tax avoidance and contributes a positive amount towards the public good are

$$s\Phi'(\tau w l_i - s x_i - t_i) = 1 - \frac{u_3(c_i, l_i, x)}{u_1(c_i, l_i, x)} \tag{3.81}$$

and

$$u_2(c_i, l_i, x) = w(1 - \tau\Phi'(\tau w l_i - s x_i - t_i))u_1(c_i, l_i, x). \tag{3.82}$$

In order for there to be a corner solution in which there is no tax avoidance, evaluated at the equilibrium it must be that for each i

$$s\Phi'(0) \geq 1 - \frac{u_3(c_i, l_i, x)}{u_1(c_i, l_i, x)}. \tag{3.83}$$

The tax credit s must be sufficiently large in order for tax avoidance to be eliminated:

$$s \geq \frac{1 - \frac{u_3(c_i, l_i, x)}{u_1(c_i, l_i, x)}}{\Phi'(0)}. \tag{3.84}$$

Choose any s which satisfies the above equation. For simplicity choose $s^* = \frac{1}{\Phi'(0)}$. Combining the first-order conditions for the leisure-labor tradeoff and the tax avoidance-private contribution to public good tradeoff yields an equation which captures the labor-leisure trade-off for a taxpayer who does not engage in tax avoidance and eliminates her tax liability exclusively using the tax credit:

$$u_2(c_i, l_i, x) = w(1 - \frac{\tau}{s}(1 - \frac{u_3(c_i, l_i, \bar{x})}{u_1(c_i, l_i, \bar{x})}))u_1(c_i, l_i, x). \tag{3.85}$$

The marginal tax rate τ is not sufficient to calculate the distortion to labor-leisure trade-off. The labor wedge equals

$$\frac{\tau}{s}(1-\frac{u_3(c_i,l_i,x)}{u_1(c_i,l_i,x)}). \tag{3.86}$$

Facing tax credit s^* and tax rate τ, each individual's optimal choice is characterized by the first-order conditions:

$$\frac{\tau}{s^*}wl_i=\frac{x}{N}, \tag{3.87}$$

$$u_2(c_i,l_i,x)=w(1-\frac{\tau(\bar{x})}{s^*}(1-\frac{u_3(c_i,l_i,x)}{u_1(c_i,l_i,\bar{x})}))u_1(c_i,l_i,x), \tag{3.88}$$

and

$$c_i=(1-\frac{\tau}{s^*})wl_i. \tag{3.89}$$

Thus, for a given level of $\bar{x}$ and corresponding $\tau^*(\bar{x})$ must satisfy the following set of equations:

$$u_2(c_i,l_i,\bar{x})=w(1-\frac{\tau^*(\bar{x})}{s^*}(1-\frac{u_3(c_i,l_i,\bar{x})}{u_1(c_i,l_i,\bar{x})}))u_1(c_i,l_i,\bar{x}), \tag{3.90}$$

$$l_i=\frac{\bar{x}}{\frac{\tau^*(\bar{x})}{s^*}wN}, \tag{3.91}$$

and

$$c_i=\frac{\bar{x}}{\frac{\tau^*(\bar{x})}{s^*}N}-\frac{\bar{x}}{N}. \tag{3.92}$$

Generally, let (τ,s,a,ω) denote the policy and economic environment consisting of tax rate τ, non-refundable tax credit s, presence of tax avoidance technology a, and wealth endowment ω. Access to the avoidance technology is indicated by a: $a = a_1$ indicates access to the avoidance technology and $a = a_0$ indicates a lack of access to the avoidance technology. Furthermore, conditional on s, a, and ω, let tax rate $\tau(\bar{x}, s, a, \omega)$ characterize the smallest tax rate such that $\bar{x}$ is provided.

Consider the introduction of s^*. For simplicity define $\tau^*(\bar{x})$ such that $\tau^*(\bar{x}) = \tau(\bar{x}, s, a_1, 0)$. Formally,

$$\tau^*(\bar{x}) = \min_{(c_i, l_i, \tau) \in \mathbb{R}^3_+} \left\{ \tau \mid (c_i, l_i, 0, \frac{\bar{x}}{N}, \tau) \in U(\tau, s^*, a_1, 0) \right\}, \tag{3.93}$$

such that $U(\tau, s, a, 0)$ is the maximal utility as a function of τ, s, a, and ω, thus

$$\begin{aligned} U(\tau, s^*, a_1, 0) = \underset{(c_i, l_i, t_i, x_i) \in \mathbb{R}^4_+}{\arg\max} \quad & u(c_i, l_i, x) \\ \text{s.t.} \quad & c_i + t_i + x_i + \Phi(\tau w l_i - s^* x_i - t_i) \leq (1 - \tau) w l_i. \end{aligned} \tag{3.94}$$

Since the F.O.C. are a sufficient and necessary condition for an optimum, then

$$\begin{aligned} \tau^*(\bar{x}) = \min_{(c_i, l_i, \tau) \in \mathbb{R}^3_+} \{ \tau \mid u_2(c_i, l_i, \bar{x}) = (1 - \frac{\tau}{s^*}(1 - \frac{u_3(c_i, l_i, \bar{x})}{u_1(c_i, l_i, \bar{x})}))u_1(c_i, l_i, \bar{x}), \\ l_i = \frac{\bar{x}}{\frac{\tau}{s^*} w N}, \\ c_i = \frac{\bar{x}}{\frac{\tau}{s^*} N} - \frac{\bar{x}}{N} \}. \end{aligned} \tag{3.95}$$

Under the above system of equations, the taxpayer does not engage in tax avoidance, and faces a labor wedge equal to $\frac{\tau}{s^*}(1 - \frac{u_3(c_i,l_i,\bar{x})}{u_1(c_i,l_i,\bar{x})})$. In an environment in which the taxpayer does not have access to the tax avoidance technology, define $\frac{\tilde{\tau}(\bar{x})}{s^*}$ such that $\frac{\tilde{\tau}(\bar{x})}{s^*} = \tau(\bar{x}, 0, a_1, 0)$: the minimum tax rate required to provide public good level $\bar{x}$ in the absence of a tax credit. Thus,

$$\begin{aligned} \tilde{\tau}(\bar{x}) = \min_{(c_i,l_i,\tau)\in\mathbb{R}^3_+} \{\tau \mid u_2(c_i, l_i, \bar{x}) = (1 - \frac{\tau}{s^*})u_1(c_i, l_i, \bar{x}), \\ l_i = \frac{\bar{x}}{\frac{\tau}{s^*)} wN}, \\ c_i = \frac{\bar{x}}{\frac{\tau}{s^*}N} - \frac{\bar{x}}{N}\}. \end{aligned} \tag{3.96}$$

It follows that $\tau^*(\bar{x}) < \tilde{\tau}(\bar{x})$. The system of equations determining $\tau^*(\bar{x}) = \tau(\bar{x}, s^*, a_1, 0)$ differs from the system of equations determining $\frac{\tilde{\tau}(\bar{x})}{s^*} = \tau(\bar{x}, 0, a_0, 0)$, only in the addition of the factor $(1 - \frac{u_3(c_i,l_i,\bar{x})}{u_1(c_i,l_i,\bar{x})}))$ in the expression for the first-order condition determining the trade-off between labor and consumption.[23] Thus, for a given τ, the taxpayer, who does have access to the tax avoidance technology, and faces tax credit s^* has a lower labor wedge than a taxpayer who does not have access to the tax avoidance technology nor access to a tax credit. Thus, in order to provide a given level of the public good $\bar{x}$, $\tau^*(\bar{x}) < \tilde{\tau}(\bar{x})$.[24]

If the government funds public good level $\bar{x}$ without tax credits, the taxpayer's

[23]Under both $(\tau^*(\bar{x}), s^*, a_1, 0)$ and $(\frac{\tilde{\tau}(\bar{x})}{s^*}, 0, a_0, 0)$, there is no tax avoidance.

[24]$\tau^*(\bar{x}) < \tilde{\tau}(\bar{x})$ is identical to the basic result from Roberts (1987). In the absence of a tax avoidance technology, the introduction of a tax expenditure for contributions to the public good can provide a given level of the public good with a lower gross tax rate.

first-order condition at the equilibrium is

$$u_2(c_i, l_i, x) = w(1 - \bar{\tau}(\bar{x}))u_1(c_i, l_i, x). \tag{3.97}$$

In the absence of tax credits and facing tax rate $\tau(\bar{x}, 0, a_1, 0) = \bar{\tau}(\bar{x})$, the equilibrium must satisfy

$$u_2(c_i, l_i, \bar{x}) = w(1 - \bar{\tau}(\bar{x}))u_1(c_i, l_i, \bar{x}). \tag{3.98}$$

$$c_i + \phi(e') + \frac{\bar{x}}{N} = wl_i \tag{3.99}$$

and

$$e' + \frac{\bar{x}}{N} = \bar{\tau}(\bar{x})wl_i. \tag{3.100}$$

Rearranging, the equilibrium must satisfy

$$u_2(c_i, l_i, \bar{x}) = w(1 - \bar{\tau}(\bar{x}))u_1(c_i, l_i, \bar{x}), \tag{3.101}$$

$$c_i = \frac{e' + \frac{\bar{x}}{N}}{\bar{\tau}(\bar{x})} - (e' + \frac{\bar{x}}{N}) + (e' - \phi(e')), \tag{3.102}$$

and

$$l_i = \frac{e' + \frac{\bar{x}}{N}}{\bar{\tau}(\bar{x})w}. \tag{3.103}$$

The corresponding tax rate equals

$$\bar{\tau}(\bar{x}) = \min_{(c_i,l_i,\tau)\in\mathbb{R}^3_+} \{\tau \mid u_2(c_i, l_i, \bar{x}) = (1 - \bar{\tau}(\bar{x}))u_1(c_i, l_i, \bar{x}), \quad l_i = \frac{e' + \frac{\bar{x}}{N}}{\bar{\tau}(\bar{x})w}, \quad c_i = \frac{e' + \frac{\bar{x}}{N}}{\bar{\tau}(\bar{x})} - (e' + \frac{\bar{x}}{N}) + (e' - \phi(e'))\}. \tag{3.104}$$

The above set of equations determining $\bar{\tau}(\bar{x}) = \tau(\bar{x}, 0, a_1, 0)$ is equivalent to the set of equations determining $\tau(\bar{x} + Ne', 0, a_0, e' - \phi(e'))$ so that the two tax rates are equal. Since the set of equations for $\tau(\bar{x} + Ne', 0, a_0, e' - \phi(e'))$ differs from the set of equations determining $\tau(\bar{x}, 0, a_0, 0)$ only in the addition of an additional revenue requirement and the addition of an endowment effect, it follows that $\bar{\tau}(\bar{x}, 0, a_0, 0) < \tau(\bar{x} + Ne', 0, a_0, e' - \phi(e')) = \tau(\bar{x}, 0, a_1, 0) = \bar{\tau}(\bar{x})$. Since, as previously shown $\tau^*(\bar{x}) = \tau(\bar{x}, s^*, a_1, 0) < \bar{\tau}(\bar{x}, 0, a_0, 0)$, it follows that $\tau^*(\bar{x}) = \tau(\bar{x}, s^*, a_1, 0) < \bar{\tau}(\bar{x}, 0, a_1, 0) = \bar{\tau}(\bar{x})$. Thus, the introduction of $\bar{\tau}(\bar{x})$ and s^* will eliminate tax avoidance and lead to a lower level of labor-distortion. ■

3.9.1 Labor taxes, heterogeneous income, and preferences

The previous section derives results under the relatively strict assumption that individuals are homogenous in preferences and economic capabilities. In this section, I relax the assumption that there is homogeneity of preferences, wage rates, and avoidance technology. The setup of the model is the same as in the previous section except for allowing heterogeneity in preferences, wage rates, and avoidance technology: Individual i is assumed to have preferences represented by the strictly concave function, $u_i(c_i, l_i, x)$, earn wage rate $w_i \in \mathbb{R}_{++}$, and face avoidance costs $\Phi_i'(e_i)$. Allowing for heterogeneity of preferences, wage rates, and avoidance technology, the basic result from proposition 12 still holds. Proposition 13 is a simple generalization of Proposition 12: For a given level of the public good $\bar{x}$ and corresponding tax rate $\tau(\bar{x})$, there exists a tax credit and an alternative tax rate, which can eliminate tax avoidance and lead to a lower labor wedge for each taxpayer.[25]

The government must set the tax credit sufficiently high in order to induce each taxpayer to contribute to the public good a sufficient amount such that her net tax liability is zero and she engages in zero tax avoidance. Specifically, the government must set the tax credit such that for each taxpayer, $s \geq \frac{1-MRS^{i}_{gc}}{\Phi_i'(0)}$. By setting the tax credit equal to $\frac{1}{\Phi_k'(0)}$, the government can induce each taxpayer to contribute enough towards the public good to reduce her tax liability to zero.[26]

Proposition 13

Assuming heterogenous preferences and income, then for a given level of the public good $\bar{x}$ and corresponding tax rate $\bar{\tau}(\bar{x})$, there exists a tax credit s^* and alternative tax rate $\tau^*(\bar{x})$ such that tax avoidance is eliminated and there is strictly less labor-distortion than if $\bar{x}$ was exclusively financed directly by the government.

[25]The result also relies on strong monotonicity of preferences over consumption and the public good.

[26]I assume that each taxpayer's preferences are strictly increasing in the public good.

Proof:

The first-order conditions for an individual i who engages in a positive level of tax avoidance and contributes a positive amount towards the public good are

$$s\Phi_i'(\tau w_i l_i - sx_i - t_i) = 1 - \frac{u_{i3}(c_i, l_i, x)}{u_{i1}(c_i, l_i, x)} \tag{3.105}$$

and

$$u_{i2}(c_i, l_i, x) = (1 - \tau\Phi_i'(\tau w_i l_i - sx_i - t_i))u_{i1}(c_i, l_i, x). \tag{3.106}$$

In order for there to be a corner solution in which there is no tax avoidance at the individual optimum, it must be that $\forall i \in I$

$$s\Phi'(0) \geq 1 - \frac{u_{i3}(c_i, l_i, x)}{u_{i1}(c_i, l_i, x)}. \tag{3.107}$$

At the equilibrium, define k such that

$$k = \arg\max_{i\in I}\{\frac{1 - \frac{u_{i3}(c_i,l_i,\bar{x})}{u_{i1}(c_i,l_i,\bar{x})}}{\Phi_i'(0)}\}. \tag{3.108}$$

At the equilibrium, the tax credit s must be sufficiently large in order for tax avoidance to be eliminated:

$$s \geq \frac{1 - \frac{u_{k3}(c_k,l_k,x)}{u_{k1}(c_k,l_k,\bar{x})}}{\Phi_k'(0)}. \tag{3.109}$$

Choose any s which satisfies the above equation. For simplicity choose $s^* = \frac{1}{\Phi'(0)}$ Combining the first-order conditions for the leisure-labor tradeoff and the tax avoidance- private contribution to public good tradeoff yields an equation which captures the labor-leisure trade-off for a taxpayer who is not engaging in tax avoidance and eliminating her tax liability exclusively using the tax credit:

$$u_{i2}(c_i, l_i, x) = (1 - \frac{\tau}{s}(1 - \frac{u_{i3}(c_i, l_i, \bar{x})}{u_{i1}(c_i, l_i, \bar{x})}))u_{i1}(c_i, l_i, x). \tag{3.110}$$

The marginal tax rate τ is not sufficient to calculate the distortion to labor-leisure trade-off. The labor wedge equals

$$\frac{\tau}{s}(1 - \frac{u_{i3}(c_i, l_i, x)}{u_{i1}(c_i, l_i, x)}). \tag{3.111}$$

Facing tax credit s^* and tax rate τ, each individual's optimal choice is characterized by the first-order conditions:

$$\frac{\tau}{s^*} w l_i = x_i, \tag{3.112}$$

$$\forall i \in I, u_{i2}(c_i, l_i, \bar{x}) = (1 - \frac{\tau}{s^*}(1 - \frac{u_{i3}(c_i, l_i, x)}{u_{i1}(c_i, l_i, x)}))w_i u_{i1}(c_i, l_i, x), \tag{3.113}$$

and

$$\forall i \in I, c_i = (1 - \tau) w_i l_i. \tag{3.114}$$

Let $\tau(x, s, a, 0, \omega)$ denote the minimum tax rate required to provide quantity x of the public good, given refundable tax credit s, access to the tax avoidance technology, and the vector of wealth endowments. For simplicity define $\tau^*(\bar{x}) = \tau(\bar{x}, s, a_1, 0)$

Thus, chose $\tau^*(\bar{x})$ such that:

$$\tau^*(\bar{x}) = \min_{([c_i,l_i]_{i=1}^N,\tau)\in\mathbb{R}_+^{2I+1}} \{\tau \mid \sum_{i=1}^{N} \frac{\tau}{s^*(\bar{x})} w l_i = \bar{x};$$
$$\forall i \in I, u_{i2}(c_i, l_i, \bar{x}) = (1 - \frac{\tau}{s^*(\bar{x})}(1 - \frac{u_{i3}(c_i, l_i, \bar{x})}{u_{i1}(c_i, l_i, \bar{x})}))u_{i1}(c_i, l_i, \bar{x}); \tag{3.115}$$
$$\forall i \in I, c_i = (1 - \frac{\tau}{s^*(\bar{x})}) w_i l_i\}.$$

Under the above system of equations, the taxpayer does not engage in tax avoidance, and faces a labor wedge equal to $\frac{\tau}{s^*(\bar{x})}(1 - \frac{u_{i3}(c_i,l_i,\bar{x})}{u_{i1}(c_i,l_i,\bar{x})})$. In an environment in which the taxpayer does not have access to the tax avoidance technology, define $\frac{\tilde{\tau}(\bar{x})}{s^*(\bar{x})} = \tau(\bar{x}, 0, a_1, 0)$ to be the minimum tax rate required to provide public good level $\bar{x}$ in the absence of a tax credit:

$$\tilde{\tau}(\bar{x}) = \min_{([c_i,l_i]_{i=1}^N,\tau)\in\mathbb{R}_+^{2I+1}} \{\tau \mid \sum_{i=1}^{N} \frac{\tau}{s^*(\bar{x})} w l_i = \sum_{i=1}^{N} \bar{x}_i;$$
$$\forall i \in I, u_{i2}(c_i, l_i, \bar{x}) = (1 - \frac{\tau(\bar{x})}{s^*(\bar{x})})u_{i1}(c_i, l_i, \bar{x}); \tag{3.116}$$
$$\forall i \in I, c_i = (1 - \frac{\tau}{s^*(\bar{x})}) w_i l_i\}.$$

It follows that $\tau^*(\bar{x}) < \tilde{\tau}(\bar{x})$. The system of equations determining $\tau^* = \tau(\bar{x}, s^*, a_1, 0)$ differs from the system of equations determining $\frac{\tilde{\tau}(\bar{x})}{s^*} = \tau(\bar{x}, s^*, a_0, 0)$, only in the addition of the factor $(1 - \frac{u_{i3}(c_i,l_i,\bar{x})}{u_{i1}(c_i,l_i,\bar{x})}))$ in the expression for the first-order condition determining the trade-off between labor and consumption. Thus, for a given τ, the taxpayer, who does have access to the tax avoidance technology, and faces tax credit s^* has a lower labor wedge than a taxpayer who does not have access to the tax avoidance technology nor access to a tax credit. Thus, in order to provide a given level of the public good $\bar{x}$, $\tau^*(\bar{x}) < \tilde{\tau}(\bar{x})$. The first-order condition which captures the leisure-labor trade off:

$$u_{i2}(c_i, l_i, x) = (1 - \tau\Phi_i'(\tau w l_i - s x_i - t_i))u_{i1}(c_i, l_i, x). \tag{3.117}$$

If the government funds public good level x without any tax credits, the taxpayer's first-order condition at the equilibrium is

$$u_{i2}(c_i, l_i, x) = (1 - \tau)u_{i1}(c_i, l_i, x). \tag{3.118}$$

In the absence of tax credits and facing tax rate $\bar{\tau}(\bar{x}) = \tau(\bar{x}, 0, a_1, 0)$, the equilibrium must satisfy[27]

$$\sum_{i=1}^{N} \bar{\tau} w_i l_i = \bar{x} + \sum_{i=1}^{N} e_i', \tag{3.119}$$

$$\forall i \in I, u_{i2}(c_i, l_i, \bar{x}) = (1 - \bar{\tau})w_i u_{i1}(c_i, l_i, \bar{x}), \tag{3.120}$$

and

$$\forall i \in I, c_i = (1 - \bar{\tau})w_i l_i + e_i' - \Phi(e_i'). \tag{3.121}$$

[27] Define e_i' such that $\Phi_i'(e_i') = 1$. If $\tau w_i \bar{l}_i \geq e_i'$ then $\bar{e}_i = e_i'$.

Rearranging, the equilibrium must satisfy

$$\forall i \in I, u_{i2}(c_i, l_i, \bar{x}) = w_i(1 - \bar{\tau}(\bar{x}))u_{i1}(c_i, l_i, \bar{x}), \tag{3.122}$$

$$\forall i \in I, c_{i1} = \frac{e_i' + x_i}{\bar{\tau}(\bar{x})} - (e_i' + x_i) + (e_i' - \phi(e_i')), \tag{3.123}$$

and

$$\forall i \in I, l_i = \frac{e_i' + x_i}{\bar{\tau}(\bar{x}) w_i}. \tag{3.124}$$

The corresponding tax rate equals

$$\begin{aligned}\bar{\tau}(\bar{x}) = \min_{(c_i, l_i, \tau) \in \mathbb{R}_+^3} \{\tau \mid \forall i \in I, u_{i2}(c_i, l_i, \bar{x}) = (1 - \bar{\tau}(\bar{x}))u_{i1}(c_i, l_i, \bar{x}), \\ \forall i \in I, l_i = \frac{e_i' + x_i}{\bar{\tau}(\bar{x}) w_i}, \\ \forall i \in I, c_i = \frac{e_i' + x_i}{\bar{\tau}(\bar{x})} - (e_i' + x_i) + (e_i' - \phi(e_i'))\}.\end{aligned} \tag{3.125}$$

The above set of equations determining $\bar{\tau}(\bar{x}) = \tau(\bar{x}, 0, a_1, 0)$ is equivalent to the set of equations determining $\tau(\bar{x} + Ne', 0, a_0, e' - \phi(e'))$ so that the two tax rates are equal. Since the set of equations determining $\bar{\tau}(\bar{x} + Ne', 0, a_0, e' - \phi(e')$ differs from the set of equations determining $\tau(\bar{x}, 0, a_0)$ only in the addition of an additional revenue requirement and the addition of an endowment effect, it follows that $\tilde{\tau}(\bar{x}) = \tau(\bar{x}, 0, a_0, 0) < \tau(\bar{x} + Ne', 0, a_0, e' - \phi(e')) = \tau(\bar{x}, 0, a_1, 0) = \bar{\tau}(\bar{x})$. Since, as previously shown $\tau^*(\bar{x}) = \tau(\bar{x}, s^*, a_1, 0) < \bar{\tau}(\bar{x} + Ne', 0, a_0, e' - \phi(e')) = \tilde{\tau}(\bar{x})$, it follows that $\tau^*(\bar{x}) = \tau(\bar{x}, s^*, a_1, 0) < \tau(\bar{x}, 0, a_1, 0) = \bar{\tau}(\bar{x})$. ■

Proposition 13 demonstrates that the introduction of a non-refundable tax credit can be Pareto-enhancing. In contrast to refundable tax credits evaluated by Roberts (1987), the introduction of non-refundable tax credits can lower the labor-wedge for each individual relative to direct funding via tax revenue. For a refundable tax credit, in order for the tax credit to be fully financed by tax revenue, the tax credit must be strictly less than one. For a given level of the public good, the introduction of a refundable tax credit will lead to greater contributions towards the public good from individuals who highly value the public good on the margin. Since the tax credit is less than one, individuals who fund a disproportionately large portion of the contributions may be worse off than they would have been under direct provision.

In contrast, for any level of total income, the non-refundable tax credit can provide a given level of the public good with the effective marginal tax rate: In contrast with the introduction of a non-refundable tax credit in order to fund a given level of the public good $\bar{x}$, for each dollar of income earned the marginal contribution to the public good equals $\frac{\tau(\bar{x})}{s^*}$, which is less than tax rate under direct provision.

3.9.2 Multiple Public Goods

In this section, I introduce a second public good when taxes are labor-distorting. While a single refundable tax credit is sufficient to fund a given level of a single public good, a single tax credit is not sufficient to fund particular levels of multiple public goods: multiple tax credits are generally necessary. In the case of a single public good, a single non-refundable tax credit can be Pareto-enhancing. With multiple public goods, multiple linear non-refundable tax credits do not necessarily constitute a Pareto-improvement. Proposition 14 demonstrates that linear tax credits constitute a Pareto improvement under relatively restrictive assumptions. Proposition 15 demonstrates that the introduction of an alternative minimum tax (AMT) can fund any given levels of the public goods $\bar{x}$ and $\bar{y}$ with strictly lower labor wedges for each type of taxpayer.

Once again, for simplicity assume that there are M individuals of type 1 who derive positive utility from public good x, and that there are N individuals of type 2 who derive positive utility from public good y. The respective set of individuals are indexed by I and J. The preferences of individuals of type 1 are identical to the preferences already presented. The preferences of individuals of type 2 are analogous; the only change being that their public good preferences are derived from good y instead of good x.

Proposition 14

Assuming heterogenous preferences and income: For a given level of the public good $\bar{x}$, a given level of the public good $\bar{y}$, and corresponding tax rate $\bar{\tau}(\bar{x}, \bar{y})$, s.t. $\bar{x} < \sum_{i=1}^{M} \bar{\tau}(\bar{x}, \bar{y}) w_i \bar{l}_i$ and $\bar{y} < \sum_{j=1}^{N} \bar{\tau}(\bar{x}, \bar{y}) w_j \bar{l}_j$, then there exists linear tax credits $s_x(\bar{x}, \bar{y})$ and $s_y(\bar{x}, \bar{y})$, and alternative tax rate $\tau^{**}(\bar{x}, \bar{y})$ such that tax avoidance is eliminated and there is a strictly smaller labor-wedge than if $\bar{x}$ and $\bar{y}$ were exclusively financed directly by the government.

Proof:

In order for there to be a corner solution in which there is no tax avoidance, at the optimum (for each individual of type 1) it must be that $\forall i \in M$

$$s_x \geq \frac{1 - \frac{u_{i3}(c_i,l_i,x)}{u_{i1}(c_i,l_i,\bar{x})}}{\Phi_i'(0)}. \tag{3.126}$$

Analogously (for each individual of type 2), in order for there to be no tax avoidance, it must be that $\forall j \in N$

$$s_y \geq \frac{1 - \frac{u_{j3}(c_j,l_j,\bar{y})}{u_{j1}(c_j,l_j,\bar{y})}}{\Phi_j'(0)}. \tag{3.127}$$

Define k s.t.:

$$k = \arg\max_{l \in M \cup N}\{\frac{1}{\Phi_l'(0)}\}. \tag{3.128}$$

For an individual of type 1, combining the first-order conditions for the leisure-labor tradeoff and the tradeoff between tax avoidance and private contributions to the public good yields an equation which captures the labor-leisure trade-off for a taxpayer who is not engaging in tax avoidance and eliminating her tax liability exclusively using the tax credit:

$$u_{i2}(c_i, l_i, x) = (1 - \frac{\tau}{s}(1 - \frac{u_{i3}(c_i, l_i, x)}{u_{i1}(c_i, l_i, x)}))u_{i1}(c_i, l_i, x). \tag{3.129}$$

The marginal tax rate τ is not sufficient to calculate the distortion to labor-leisure trade-off. The shadow tax rate which determines the level of labor-distortion is:

$$\frac{\tau}{s}(1 - \frac{u_{i3}(c_i, l_i, x)}{u_{i1}(c_i, l_i, x)}). \tag{3.130}$$

Choose $\tau^{**}(\bar{x}, \bar{y})$ s.t. $\tau^{**}(\bar{x}, \bar{y})$, c_i^{**}, l_i^{**} satisfy the following set of equations:

$$\sum_{i=1}^{M} \frac{\tau}{s_x} w l_i = \bar{x}, \tag{3.131}$$

$$\sum_{j=1}^{N} \frac{\tau}{s_y} w l_j = \bar{y}, \tag{3.132}$$

$$\forall i \in M, u_{i2}(c_i, l_i, \bar{x}) = (1 - \frac{\tau(\bar{x})}{s_x}(1 - \frac{u_{i3}(c_i, l_i, \bar{x})}{u_1(c_i, l_i, \bar{x})}))w_i u_1(c_i, l_i, \bar{x}), \tag{3.133}$$

$$\forall j \in N, u_{j2}(c_i, l_i, \bar{y}) = (1 - \frac{\tau(\bar{x})}{s_y}(1 - \frac{u_{j3}(c_j, l_j, \bar{y})}{u_1(c_i, l_i, \bar{x})}))w_j u_1(c_j, l_j, \bar{y}), \tag{3.134}$$

$$\forall i \in M, c_i = (1 - \frac{\tau}{s_x}) w_i l_i, \tag{3.135}$$

and

$$\forall j \in N, c_j = (1 - \frac{\tau}{s_y}) w_j l_j. \tag{3.136}$$

For notational purposes, define $U^l(\tau, s_x, s_y, a, 0)$ to be the maximal utility for individual i as a function of τ, s_x, s_y, a, and ω: for an individual i of type 1,

$$U^i(\tau, s_x, s_y, a, \omega) = \underset{(c_i, l_i, t_i, x_i, y_i) \in \mathbb{R}^5_+}{\arg\max} \quad u(c_i, l_i, x)$$
$$\text{s.t.} \quad c_i + t_i + x_i + y_i + \Phi(\tau w l_i - s_x x_i - s_y y_i - t_i) \leq (1-\tau) w l_i, \tag{3.137}$$

and for an individual j of type 2,

$$U^j(\tau, s_x, s_y, a, \omega) = \underset{(c_j, l_j, t_j, x_j, y_j) \in \mathbb{R}^5_+}{\arg\max} \quad u(c_i, l_i, y)$$
$$\text{s.t.} \quad c_j + t_j + x_j + y_j + \Phi(\tau w l_j - s_x x_j - s_y y_j - t_j) \leq (1-\tau) w l_j. \tag{3.138}$$

Define $\tau^{**}(\bar{x}, \bar{y})$ to be the minimal tax credit which funds $\bar{x}$ and $\bar{y}$ and such that $s_x \geq \frac{1}{\Phi_k i'(0)}$ and $s_y \geq \frac{1}{\Phi'_k(0)}$. Thus, chose $\tau^{**}(\bar{x}, \bar{y})$ such that:

$$\begin{aligned} \tau^{**}(\bar{x}, \bar{y}) = & \\ \min_{(\tau, s_x, s_y, c, l) \in \mathbb{R}^{2(M+N)+3}_+} \{\tau \mid \sum_{i=1}^{M} \frac{\tau}{s_x} w l_i = \bar{x}; & \\ \sum_{j=1}^{N} \frac{\tau}{s_y} w l_i = \bar{y}; & \\ \forall i \in M, (c_i, l_i, t_i, x_i, y_i) \in U^i(\tau, s_x, s_y, a, \omega); & \\ \forall j \in N, (c_j, l_j, t_j, x_j, y_j) \in U^j(\tau, s_x, s_y, a, \omega); & \\ s_x \geq \frac{1}{\Phi'_k(0)}, s_y \geq \frac{1}{\Phi'_k(0)}\}, & \end{aligned} \tag{3.139}$$

and more explicitly

$$\begin{aligned}
&\tau^{**}(\bar{x}, \bar{y}) = \\
&\min_{(\tau, s_x, s_y, c, l) \in \mathbb{R}_+^{2(M+N)+3}} \{\tau \mid \sum_{i=1}^{M} \frac{\tau}{s_x} w l_i = \bar{x}; \\
&\sum_{j=1}^{N} \frac{\tau}{s_y} w l_i = \bar{y}; \\
&\forall i \in M, u_{i2}(c_i, l_i, \bar{x}) = (1 - \frac{\tau}{s_x}(1 - \frac{u_{i3}(c_i, l_i, \bar{x})}{u_1(c_i, l_i, \bar{x})})) w_i u_{i1}(c_i, l_i, \bar{x}); \\
&\forall j \in N, u_{j2}(c_j, l_j, \bar{y}) = (1 - \frac{\tau}{s_y}(1 - \frac{u_{j3}(c_j, l_j, \bar{y})}{u_{j1}(c_j, l_j, \bar{y})})) w_j u_1(c_j, l_i, \bar{y}); \\
&\forall i \in M, c_i = (1 - \frac{\tau}{s_x}) w_i l_i; \\
&\forall j \in N \; c_j = (1 - \frac{\tau}{s_y}) w_j l_j; \\
&s_x \geq \frac{1}{\Phi_k'(0)}, s_y \geq \frac{1}{\Phi_k'(0)}\}.
\end{aligned} \tag{3.140}$$

Define $A(\bar{x}, \bar{y})$ to be the set of $(\tau, s_x, s_y, c, l) \in \mathbb{R}_+^{2(M+N)+3}$ which funds $\bar{x}$ and $\bar{y}$ with the lowest tax rate such that $s_x \geq \frac{1}{\Phi_k i'(0)}$ and $s_y \geq \frac{1}{\Phi_k'(0)}$.[28]

$$
\begin{aligned}
&A(\bar{x}, \bar{y}) = \\
&\underset{(\tau, s_x, s_y, c, l) \in \mathbb{R}_+^{2(M+N)+3}}{\arg\min} \{\tau \mid \sum_{i=1}^{M} \frac{\tau}{s_x} w l_i = \bar{x}; \\
&\sum_{j=1}^{N} \frac{\tau}{s_y} w l_i = \bar{y}; \\
&\forall i \in M, (c_i, l_i, t_i, x_i, y_i) \in U^i(\tau, s_x, s_y, a, \omega); \\
&\forall j \in N, (c_j, l_j, t_j, x_j, y_j) \in U^j(\tau, s_x, s_y, a, \omega); \\
&s_x \geq \frac{1}{\Phi_k'(0)}, s_y \geq \frac{1}{\Phi_k'(0)}\}.
\end{aligned}
\tag{3.141}
$$

[28]Since $s_x \geq \frac{1}{\Phi_k'(0)}$ and $s_y \geq \frac{1}{\Phi_k'(0)}$, minimizing τ also minimizes $\frac{\tau}{s_x}$ and $\frac{\tau}{s_y}$.

or alternatively

$$\begin{aligned}
A(\bar{x},\bar{y}) = \\
\underset{(\tau,s_x,s_y,c,l)\in\mathbb{R}_+^{2(M+N)+3}}{\arg\min} \{\tau \mid \sum_{i=1}^{M} \frac{\tau}{s_x} w l_i = \bar{x}; \\
\sum_{j=1}^{N} \frac{\tau}{s_y} w l_i = \bar{y}; \\
\forall i \in M, u_{i2}(c_i, l_i, \bar{x}) = (1 - \frac{\tau}{s_x}(1 - \frac{u_{i3}(c_i, l_i, \bar{x})}{u_1(c_i, l_i, \bar{x})}))w_i u_{i1}(c_i, l_i, \bar{x}); \\
\forall j \in N, u_{j2}(c_j, l_j, \bar{y}) = (1 - \frac{\tau}{s_y}(1 - \frac{u_{j3}(c_j, l_j, \bar{y})}{u_{j1}(c_j, l_j, \bar{y})}))w_j u_1(c_j, l_i, \bar{y}); \\
\forall i \in M, c_i = (1 - \frac{\tau}{s_x})w_i l_i; \\
\forall j \in N, c_j = (1 - \frac{\tau}{s_y})w_j l_j; \\
s_x \geq \frac{1}{\Phi_k'(0)}, s_y \geq \frac{1}{\Phi_k'(0)}\}.
\end{aligned} \tag{3.142}$$

In an environment in which the taxpayer does not have access to the tax avoidance technology or in which $\Phi_i'(0) > 1 \forall i \in I$, define $\frac{\tilde{\tau}(\bar{x},\bar{y})}{s_x^{**}(\bar{x},\bar{y})}$ and $\frac{\tilde{\tau}(\bar{x},\bar{y})}{s_y^{**}(\bar{x},\bar{y})}$ to be the minimum tax rates required to provide public good levels $\bar{x}$ and $\bar{y}$ in the absence of a tax credit:

$$\begin{aligned}
\tilde{\tau}_x(\bar{x},\bar{y}) = min\{\tau \mid \sum_{i=1}^{M} \frac{\tau}{s_x^{**}(\bar{x},\bar{y})} w l_i = \bar{x}; \\
\forall i \in M, u_{i2}(c_i, l_i, \bar{x}) = (1 - \frac{\tau}{s_x^{**}(\bar{x},\bar{y})})w_i u_{i1}(c_i, l_i, \bar{x}); \\
\forall i \in M, c_i = (1 - \frac{\tau}{s_x^{**}(\bar{x},\bar{y})})w_i l_i\}.
\end{aligned} \tag{3.143}$$

$$\tilde{\tau}_y(\bar{x}, \bar{y}) = min\{\tau \mid \sum_{j=1}^{N} \frac{\tau}{s_y^{**}(\bar{x}, \bar{y})} wl_i = \bar{y};$$
$$\forall j \in N, u_{j2}(c_j, l_j, \bar{y}) = (1 - \frac{\tau}{s_y^{**}(\bar{x}, \bar{y})}) w_j u_1(c_j, l_i, \bar{y}); \quad (3.144)$$
$$\forall j \in N \; c_j = (1 - \frac{\tau}{s_y^{**}(\bar{x}, \bar{y})}) w_j l_j\}.$$

It follows that $\tau^{**}(\bar{x}, \bar{y}) < \tilde{\tau}_x(\bar{x}, \bar{y})$ and $\tau^{**}(\bar{x}, \bar{y}) < \tilde{\tau}_y(\bar{x}, \bar{y})$. Under both $\tau^{**}(\bar{x}, \bar{y})$, $\tilde{\tau}_x(\bar{x}, \bar{y})$ and $\tilde{\tau}_y(\bar{x}, \bar{y})$ there is no tax avoidance. The system of equations determining the equilibrium for $\tau^{**}(\bar{x}, \bar{y})$ differs from the system of equations determining the equilibrium for $\frac{\tilde{\tau}_x(\bar{x}, \bar{y})}{s_x^{**}(\bar{x}, \bar{y})}$ and $\frac{\tilde{\tau}_y(\bar{x}, \bar{y})}{s_y^{**}(\bar{x}, \bar{y})}$, only in the addition of the factor $(1 - \frac{u_{i3}(c_i, l_i, \bar{x})}{u_{i1}(c_i, l_i, \bar{x})})$ and $(1 - \frac{u_{j3}(c_j, l_j, \bar{x})}{u_{j1}(c_j, l_j, \bar{x})})$ in the expressions for the first-order condition determining the trade-off between labor and consumption. Thus, for a given τ, the taxpayer, who does have access to the tax avoidance technology, and faces tax credit $s^*(\bar{x})$ has a lower labor wedge than a taxpayer who does not have access to the tax avoidance technology nor access to a tax credit. However, for given equilibrium levels of the public good $\bar{x}$ and $\bar{y}$, there is no income effect. For a given τ at a given level of $\bar{x}$, the labor supply in the first case will exceed the labor supply of the second case. Thus, in order to provide a given level of the public good $\bar{x}$, $\tau^*(\bar{x}) < \tilde{\tau}(\bar{x})$.

Furthermore, by assumption since $\bar{x} < \sum_{i=1}^{M} \bar{\tau}(\bar{x}, \bar{y}) w_i \bar{l}_i$ and $\bar{y} < \sum_{j=1}^{N} \bar{\tau}(\bar{x}, \bar{y}) w_j \bar{l}_j$, then $\tilde{\tau}_x(\bar{x}, \bar{y}) < \bar{\tau}(\bar{x}, \bar{y})$ and $\tilde{\tau}_y(\bar{x}, \bar{y}) < \bar{\tau}(\bar{x}, \bar{y})$. Since $\tau^{**}(\bar{x}, \bar{y}) < \tilde{\tau}_x(\bar{x}, \bar{y})$ and $\tau^{**}(\bar{x}, \bar{y}) < \tilde{\tau}_y(\bar{x}, \bar{y})$, then $\tau^{**}(\bar{x}, \bar{y}) < \bar{\tau}(\bar{x}, \bar{y})$ and $\tau^{**}(\bar{x}, \bar{y}) < \bar{\tau}(\bar{x}, \bar{y})$. Thus the introduction of ta credits $s_x^{**}(\bar{x}, \bar{y})$ and $s_y^{**}(\bar{x}, \bar{y})$. ■

Proposition 14 demonstrates under relatively strict assumptions that linear tax credits can decrease the effect labor wedge for each type of taxpayer. Once again, at the equilibrium individuals pay zero in net taxes and contribute fully to their preferred public good. Proposition 15 demonstrates that a piece-meal tax credits function can fund any levels $\bar{x}$ and $\bar{y}$ of the public goods while eliminating tax avoidance and decreasing the labor wedge for each individual. Without loss of generalizability assume that $\bar{y}$ would require a lower subsidy rate than $\bar{x}$, then the combined tax credit function can be written as $\Theta_{\bar{x},\bar{y}}(x_i, y_i)$ s.t.

$$\Theta_{\bar{x},\bar{y}}(x_i, y_i) = \begin{cases} s(\bar{x},\bar{y})x_i + s(\bar{x},\bar{y})y_i & \text{if } t_i \geq \frac{1-p_y}{s(\bar{x},\bar{y})p_y}y_i \text{ and } s(\bar{x},\bar{y})x_i + s(\bar{x},\bar{y})y_i \leq T_i - \frac{1-p_y}{s(\bar{x},\bar{y})p_y}y_i \\ T(\bar{x},\bar{y}) - T_0(\bar{x},\bar{y}) & \text{if } t_i \geq \frac{1-p_y}{s(\bar{x},\bar{y})p_y}y_i \text{ and } s(\bar{x},\bar{y})x_i + s(\bar{x},\bar{y})y_i > T_i - \frac{1-p_y}{s(\bar{x},\bar{y})p_y}y_i \\ s(\bar{x},\bar{y})x_i & \text{if } t_i < \frac{1-p_y}{s(\bar{x},\bar{y})p_y}y_i \text{ and } x_i \leq \frac{T_i(\bar{x},\bar{y})}{s(\bar{x},\bar{y})}. \end{cases}$$

If $\bar{y}$ requires a lower subsidy rate than $\bar{x}$, then the basic structure of the tax credit function is such that contributions to good x are only limited by one's total gross tax liability. Tax credits for contributions to good y are limited as a proportion of gross tax liability p.[29]

Proposition 15

Assuming heterogenous preferences and income: For a given level of the public

[29] This is similar to an alternative minimum tax. Tax rate τ and tax credit s for contributions to x and y; and AMT of $\frac{(1-p_y)\tau}{s}$
Define $\Omega_{\bar{x},\bar{y}}(x_i)$ s.t.

$$\Omega_{\bar{x},\bar{y}}(x_i) = \begin{cases} s_x(\bar{x},\bar{y})x_i & \text{if } x_i \leq \frac{T(\bar{x},\bar{y})}{s_x(\bar{x},\bar{y})} \\ T(\bar{x},\bar{y}) & \text{if } x_i > \frac{T(\bar{x},\bar{y})}{s_x(\bar{x},\bar{y})}. \end{cases}$$

good $\bar{x}$, a given level of the public good $\bar{y}$, and corresponding tax rate $\bar{\tau}(\bar{x}, \bar{y})$, there exists a tax credit function $s_{\bar{x},\bar{y}}(x_i, y_i)$ and alternative tax rate $\tau^{***}(\bar{x}, \bar{y})$ such that tax avoidance is eliminated and there is strictly less labor-distortion than if $\bar{x}$ and $\bar{y}$ were exclusively financed directly by the government.

Proof:

Define k s.t.:

$$k = \arg\max_{l \in M \cup N} \{\frac{1}{\Phi_l'(0)}\}. \tag{3.145}$$

Set $s^* = \frac{1}{\Phi_k'(0)}$. Under tax credit s^*, if individual contributions are sufficient to fund good $\bar{y}$ but tax revenue is required to fully fund $\bar{x}$, then the following determine the constraints for the government:

$$\frac{\tau}{s^*} = \frac{p_y \tau}{s^*} + (1 - p_y)\tau, \tag{3.146}$$

$$\sum_{j=1}^{N} \frac{p_y \tau}{s^*} w l_j = \bar{y}, \tag{3.147}$$

and

$$\sum_{i=1}^{M} \frac{\tau}{s^*} w l_i + \sum_{j=1}^{N} \frac{(1 - p_y)\tau}{s^*} w l_j = \bar{x}. \tag{3.148}$$

If these constraints are satisfied then the tax system will achieve effective horizontal equity: for a given level of income, each individual of each type will pay the same total in taxes and contributions to the public good.

Combining the first-order conditions for the leisure-labor tradeoff and the tax avoidance versus private contribution to public good tradeoff, yields equations which captures the labor-leisure trade-off for a taxpayer who is not engaging in tax avoidance and eliminating her tax liability exclusively using the tax credit. Under tax credit s^*, if individual contributions are sufficient to fund good $\bar{y}$ but tax revenue is required to fully fund $\bar{x}$, then for a taxpayer of type 1

$$u_{i2}(c_i, l_i, x) = (1 - \frac{\tau}{s^*}(1 - \frac{u_{i3}(c_i, l_i, \bar{x})}{u_{i1}(c_i, l_i, \bar{x})}))u_{i1}(c_i, l_i, x), \tag{3.149}$$

and

$$\forall j \in N \; u_{j2}(c_j, l_j, \bar{y}) = (1 - (1 - p_y)\tau - \frac{p_y \tau}{s^*}(1 - \frac{u_{j3}(c_j, l_j, \bar{x})}{u_{j1}(c_j, l_j, \bar{x})}))w_j u_{j1}(c_j, l_j, \bar{y}). \tag{3.150}$$

The marginal tax rate τ is not sufficient to calculate the distortion to labor-leisure trade-off. The labor wedges for which determines the level of labor-distortion are

$$\frac{\tau}{s^*}(1 - \frac{u_{i3}(c_i, l_i, \bar{x})}{u_{i1}(c_i, l_i, \bar{x})}) \tag{3.151}$$

and

$$(1 - p_y)\tau - \frac{p_y \tau}{s^*}(1 - \frac{u_{j3}(c_j, l_j, \bar{x})}{u_{j1}(c_j, l_j, \bar{x})}). \tag{3.152}$$

Under tax credit s^*, if individual contributions are sufficient to good $\bar{y}$ but tax revenue is required to fully fund $\bar{x}$, then choose $\tau^{***}(\bar{x}, \bar{y})$ s.t. $\tau^{***}(\bar{x}, \bar{y})$, c_i^{***}, l_i^{***} satisfy the following set of equations:

$$\frac{\tau}{s^*} = \frac{p_y \tau}{s^*} + (1 - p_y)\tau, \tag{3.153}$$

$$\sum_{j=1}^{N} \frac{p_y \tau}{s^*} w l_j = \bar{y}, \tag{3.154}$$

$$\sum_{i=1}^{M} \frac{\tau}{s^*} w l_i + \sum_{j=1}^{N} (1 - p_y)\tau w l_j = \bar{x}, \tag{3.155}$$

$$\forall i \in M, u_{i2}(c_i, l_i, \bar{x}) = (1 - \frac{\tau(\bar{x})}{s^*}(1 - \frac{u_{i3}(c_i, l_i, \bar{x})}{u_{i1}(c_i, l_i, \bar{x})}))w_i u_{i1}(c_i, l_i, \bar{x}), \tag{3.156}$$

$$\forall j \in N \; u_{j2}(c_j, l_j, \bar{y}) = (1 - (1 - p_y)\tau - \frac{p_y \tau}{s^*}(1 - \frac{u_{j3}(c_j, l_j, \bar{x})}{u_{j1}(c_j, l_j, \bar{x})}))w_j u_{j1}(c_j, l_j, \bar{y}), \tag{3.157}$$

$$\forall i \in M \; c_i = (1 - \frac{\tau}{s^*})w_i l_i, \tag{3.158}$$

and

$$\forall j \in N \; c_j = (1 - (\frac{p_y \tau}{s^*} + (1 - p_y)\tau))w_j l_j. \tag{3.159}$$

In the case when individual contributions from s^* are sufficient to fund good $\bar{y}$ but tax revenue is required to fully fund $\bar{x}$, define $A(\bar{x}, \bar{y})$ to be the set of $(\tau, p_y, c, l) \in \mathbb{R}_+^{2(M+N)+2}$ with the lowest tax rate, such that tax credit $s^* = \frac{1}{\Phi'(0)}$ and policy parameter p_y provides $\bar{x}$ and $\bar{y}$:

$$
\begin{aligned}
A(\bar{x},\bar{y}) = \underset{(\tau,p_y,c,l)\in\mathbb{R}_+^{2(M+N)+2}}{\arg\min} \{\tau \mid \frac{\tau}{s^*} &= \frac{p_y\tau}{s^*} + (1-p_y)\tau, \\
\sum_{j=1}^{N} \frac{p_y\tau}{s^*} wl_j &= \bar{y}, \\
\sum_{i=1}^{M} \frac{\tau}{s} wl_i + \sum_{j=1}^{N} \frac{(1-p_y)\tau}{s^*} wl_j &= \bar{x}, \\
\forall i \in M\ u_{i2}(c_i,l_i,\bar{x}) = (1 - \frac{\tau}{s^*}(1 - \frac{u_{i3}(c_i,l_i,\bar{x})}{u_1(c_i,l_i,\bar{x})}))w_i u_{i1}(c_i,l_i,\bar{x}), \\
\forall j \in N\ u_{j2}(c_j,l_j,\bar{y}) = (1 - (1-p_y)\tau - \frac{p_y\tau}{s^*}(1 - \frac{u_{j3}(c_j,l_j,\bar{y})}{u_{j1}(c_j,l_j,\bar{y})}))w_j u_{i1}(c_j,l_j,\bar{y}), \\
\forall i \in M\ c_i = (1 - \frac{\tau}{s^*})w_i l_i, \\
\forall j \in N\ c_j = (1 - (\frac{p_y\tau}{s^*} + (1-p_y)\tau))w_j l_j \}.
\end{aligned}
\tag{3.160}
$$

In the case when individual contributions from s^* are sufficient to fund good $\bar{x}$ but tax revenue is required to fully fund $\bar{y}$, define $B(\bar{x},\bar{y})$ to be the set of $(\tau, p_x, c, l) \in \mathbb{R}_+^{2(M+N)+2}$ with the lowest tax rate, such that tax credit $s^* = \frac{1}{\Phi'(0)}$ and policy parameter p_x provides $\bar{x}$ and $\bar{y}$:

$$\begin{aligned}
B(\bar{x},\bar{y}) = \underset{(\tau,p_x,c,l)\in\mathbb{R}_+^{2(M+N)+2}}{\arg\min} \{\tau \mid \frac{\tau}{s^*} = \frac{p_x\tau}{s^*} + (1-p_x)\tau, \\
\sum_{j=1}^{N} \frac{p_x\tau}{s^*} wl_j = \bar{y}, \\
\sum_{i=1}^{M} \frac{\tau}{s^*} wl_i + \sum_{j=1}^{N} \frac{(1-p_y)\tau}{s^*} wl_j = \bar{x}, \\
\forall i \in M \; u_{i2}(c_i,l_i,\bar{x}) = (1-(\frac{p_x\tau}{s^*} + (1-p_x)\tau)(1-\frac{u_{i3}(c_i,l_i,\bar{x})}{u_1(c_i,l_i,\bar{x})}))w_i u_{i1}(c_i,l_i,\bar{x}), \\
\forall j \in N \; u_{j2}(c_j,l_j,\bar{y}) = (1-\frac{\tau}{s^*}(1-\frac{u_{j3}(c_j,l_j,\bar{y})}{u_{j1}(c_j,l_j,\bar{y})}))w_j u_1(c_j,l_i,\bar{y}), \\
\forall i \in M \; c_i = (1-(\frac{p_x\tau}{s^*} + (1-p_x)\tau))w_i l_i, \\
\forall j \in N \; c_j = (1-\frac{\tau}{s^*})w_j l_j\}.
\end{aligned} \tag{3.161}$$

Choose $(\tau^{***}(\bar{x},\bar{y}), p^{***}(\bar{x},\bar{y}), c^{***}(\bar{x},\bar{y}), l^{***}(\bar{x},\bar{y}))$ such that

$$(\tau^{***}(\bar{x},\bar{y}), p^{***}(\bar{x},\bar{y}), c^{***}(\bar{x},\bar{y}), l^{***}(\bar{x},\bar{y})) = \underset{(\tau,p,c,l)\in\mathbb{R}_+^{2(M+N)+3}}{\arg\min} \{\tau \mid (\tau,p,c,l) \in A(\bar{x},\bar{y}) \cup B(\bar{x},\bar{y}), p \in [0,1]\}. \tag{3.162}$$

Let $\tau(x,y,s,p,a,\omega)$ denote the minimum tax rate required to provide quantity x and quantity y of the public good, given refundable tax credit s, and proportion of gross tax p eligible for the both tax credits, access to the tax avoidance technology, and the vector of wealth endowments. Without loss of generality assume that

$$(\tau^{***}(\bar{x},\bar{y}), p^{***}(\bar{x},\bar{y}), c^{***}(\bar{x},\bar{y}), l^{***}(\bar{x},\bar{y})) \in A(\bar{x},\bar{y}). \tag{3.163}$$

Recall that $s^* = \frac{1}{\Phi'(0)}$. Thus,

$$\begin{aligned}
\tau^{***}(\bar{x},\bar{y}) = \min_{(\tau,p_y,c,l)\in\mathbb{R}_+^{2(M+N)+2}} \{\tau \mid \frac{\tau}{s^*} &= \frac{p_y\tau}{s^*} + (1-p_y)\tau, \\
\sum_{j=1}^{N} \frac{p_y\tau}{s^*} wl_j &= \bar{y}, \\
\sum_{i=1}^{M} \frac{\tau}{s^*} wl_i + \sum_{j=1}^{N} (1-p_y)\tau wl_j &= \bar{x}, \\
\forall i \in M \; u_{i2}(c_i,l_i,\bar{x}) = (1 - \frac{\tau}{s^*}(1 - \frac{u_{i3}(c_i,l_i,\bar{x})}{u_{i1}(c_i,l_i,\bar{x})}))w_i u_{i1}(c_i,l_i,\bar{x}), \\
\forall j \in N \; u_{j2}(c_j,l_j,\bar{y}) = (1 - (1-p_y)\tau - \frac{p_y\tau}{s^*}(1 - \frac{u_{j3}(c_j,l_j,\bar{y})}{u_{j1}(c_j,l_j,\bar{y})}))w_j u_{i1}(c_j,l_j,\bar{y}), \\
\forall i \in M \; c_i &= (1 - \frac{\tau}{s^*})w_i l_i, \\
\forall j \in N \; c_j &= (1 - (\frac{p_y\tau}{s^*} + (1-p_y)\tau))w_j l_j\}.
\end{aligned} \tag{3.164}$$

In an environment in which the taxpayer does not have access to the tax credit, define $\frac{\tilde{\tau}(\bar{x},\bar{y})}{s^*} = \tau(\bar{x},\bar{y},0,0,a_0,0)$. Thus,

$$\begin{aligned}
\tilde{\tau}(\bar{x},\bar{y}) = \min_{(\tau,p_y,c,l)\in\mathbb{R}_+^{2(M+N)+2}} \{\tau \mid \frac{\tau}{s^*} &= \frac{p_y\tau}{s^*} + (1-p_y)\tau, \\
\sum_{i=1}^{M} \frac{\tau}{s^*} wl_i + \sum_{j=1}^{N} \frac{\tau}{s^*} wl_j &= \bar{x} + \bar{y}, \\
\forall i \in M \; u_{i2}(c_i,l_i,\bar{x}) &= (1 - \frac{\tau}{s^*})w_i u_{i1}(c_i,l_i,\bar{x}), \\
\forall j \in N \; u_{j2}(c_j,l_j,\bar{y}) &= (1 - \frac{\tau}{s^*})w_j u_{i1}(c_j,l_j,\bar{y}), \\
\forall i \in M \; c_i &= (1 - \frac{\tau}{s^*})w_i l_i, \\
\forall j \in N \; c_j &= (1 - \frac{\tau}{s^*})w_j l_j\},
\end{aligned} \tag{3.165}$$

which is equivalent to

$$
\begin{aligned}
\tilde{\tau}(\bar{x},\bar{y}) = \min_{(\tau,p_y,c,l)\in\mathbb{R}_+^{2(M+N)+2}} \{\tau \mid \frac{\tau}{s^*} &= \frac{p_y\tau}{s^*} + (1-p_y)\tau, \\
\sum_{j=1}^{N} \frac{p_y\tau}{s^*} w l_j &= \bar{y}, \\
\sum_{i=1}^{M} \frac{\tau}{s^*} w l_i + \sum_{j=1}^{N} (1-p_y)\tau w l_j &= \bar{x}, \\
\forall i \in M \; u_{i2}(c_i,l_i,\bar{x}) &= (1-\frac{\tau}{s^*}) w_i u_{i1}(c_i,l_i,\bar{x}), \\
\forall j \in N \; u_{j2}(c_j,l_j,\bar{y}) &= (1-(1-p_y)\tau - \frac{p_y\tau}{s^*}) w_j u_{i1}(c_j,l_j,\bar{y}), \\
\forall i \in M \; c_i &= (1-\frac{\tau}{s^*}) w_i l_i, \\
\forall j \in N \; c_j &= (1-(\frac{p_y\tau}{s^*} + (1-p_y)\tau)) w_j l_i\}.
\end{aligned}
\tag{3.166}
$$

It follows that $\tau^{***}(\bar{x},\bar{y}) < \tilde{\tau}(\bar{x},\bar{y})$. The system of equations determining $\tau^{***}(\bar{x},\bar{y}) = \tau(\bar{x},\bar{y},s^*,p^{***},a_1,0)$ differs from the system of equations determining $\frac{\tilde{\tau}(\bar{x})}{s^*} = \tau(\bar{x},s^*,1,p^{***}a_0,0)$, only in the addition of the factor $(1-\frac{u_{i3}(c_i,l_i,\bar{x})}{u_{i1}(c_i,l_i,\bar{x})})$ and $(1-\frac{u_{j3}(c_j,l_j,\bar{x})}{u_{j1}(c_j,l_j,\bar{x})})$ in the expression for the first-order condition determining the trade-off between labor and consumption. Thus, for a given τ, the taxpayer, who does have access to the tax avoidance technology, and faces tax credit s^* has a lower labor wedge than a taxpayer who does not have access to the tax avoidance technology nor access to a tax credit. For a given τ at a given level of $\bar{x}$, the labor supply in the first case will exceed the labor supply of the second case. Thus, in order to provide given levels of the public goods $\bar{x}$ and $\bar{y}$, $\tau^{***}(\bar{x},\bar{y}) < \tilde{\tau}(\bar{x},\bar{y})$.

In the absence of tax credits and facing tax rate $\bar{\tau}$, the equilibrium $\bar{c}_i$, $\bar{l}_i$, $\bar{x}$, and $\bar{y}$ must satisfy:

$$\sum_{i=1}^{M} \bar{\tau} w_i l_i + \sum_{j=1}^{N} \bar{\tau} w_j l_j = \sum_{i=1}^{M} (\bar{x}_i + e'_i) + \sum_{j=1}^{N} (\bar{y}_j + e'_j), \tag{3.167}$$

$$\forall i \in M, u_{i2}(c_i, l_i, \bar{x}) = (1 - \bar{\tau}) w_i u_{i1}(c_i, l_i, \bar{x}), \tag{3.168}$$

$$\forall j \in N, u_{j2}(c_j, l_j, \bar{y}) = (1 - \bar{\tau}) w_j u_{j1}(c_j, l_j, \bar{y}), \tag{3.169}$$

and

$$\forall i \in I, c_i = (1 - \bar{\tau}) w_i l_i + e'_i - \Phi(e'_i). \tag{3.170}$$

Alternatively,

$$\begin{aligned} \bar{\tau}(\bar{x}, \bar{y}) = min\{\tau \mid \sum_{i=1}^{M} \tau w l_i + \sum_{j=1}^{N} \tau w l_j = \bar{x} + \sum_{i=1}^{M} e'_i + \bar{y} + \sum_{j=1}^{N} e'_j, \\ \forall i \in M \; u_{i2}(c_i, l_i, \bar{x}) = (1 - \tau) w_i u_{i1}(c_i, l_i, \bar{x}), \\ \forall j \in N \; u_{j2}(c_j, l_j, \bar{y}) = (1 - \tau) w_j u_{i1}(c_j, l_j, \bar{y}), \\ \forall i \in M \; c_i = (1 - \tau) w_i l_i + e'_i - \Phi'(e'_i), \\ \forall j \in N \; c_j = (1 - \tau) w_j l_j + e'_j - \Phi'(e'_j)\}. \end{aligned} \tag{3.171}$$

The above set of equations determining $\bar{\tau}(\bar{x}, \bar{y})$ is equivalent to the set of equations determining $\tau(\bar{x} + \sum_{i=1}^{M} e'_i, \bar{y} + \sum_{j=1}^{N} e'_j, 0, 0, a_0, e' - \Phi'(e'))$. Since the set of equations determining $\tau(\bar{x} + \sum_{i=1}^{M} e'_i, \bar{y} + \sum_{j=1}^{N} e'_j, 0, 0, a_0, e' - \Phi'(e'))$ differs from the set of equations determining $\frac{\bar{\tau}(\bar{x}, \bar{y})}{s^*} = \tau(\bar{x}, \bar{y}, 0, 0, a_0, 0)$ only in the addition of an additional revenue requirement and endowment effect, it follows that $\frac{\bar{\tau}(\bar{x},\bar{y})}{s^*} = \tau(\bar{x}, \bar{y}, 0, 0, a_0, 0) < \tau(\bar{x} + \sum_{i=1}^{M} e'_i, \bar{y} + \sum_{j=1}^{N} e'_j, 0, 0, a_0, e' - \Phi'(e')) = \bar{\tau}(\bar{x}, \bar{y})$. Since as previously shown $\tau^{***}(\bar{x}, \bar{y}) < \tilde{\tau}(\bar{x}, \bar{y})$, it follows that $\tau^{***}(\bar{x}, \bar{y}) < \bar{\tau}(\bar{x}, \bar{y})$. ■

3.10 Conclusion

This chapter outlines an additional justification for a tax credit for contributions to public goods; a reduction in tax avoidance. The introduction of tax credits for contributions can increase the opportunity cost of avoiding and thus encourage individuals to contribute to the public good instead of devoting resources towards tax avoidance. In the absence of labor-distorting taxes, the introduction of tax credits can lead to the implementation of a Pareto-efficient level of the public good and fully eliminate tax avoidance. The elimination of tax avoidance and implementation of a Pareto-efficient level of the public good is robust to heterogeneous preferences and varying levels of endowments. Unsurprisingly, under labor-distorting taxes, the social planner can no longer implement a Pareto-efficient allocation. However under labor distorting taxes, the introduction of tax credits for contributions to a public good can eliminate tax avoidance, and fund a given level of the public good with less labor-distortion.

CHAPTER 4

PENSION INCREASES VS. CONTRIBUTION CUTS

4.1 Introduction

Current public sector pensions are large relative to public sector compensation.[1] Furthermore, the relative ratio of retirement benefits to current compensation has tended to increase over the past two decades: During the 1990s, a number of states increased pension benefits while employee contributions remained unchanged. In 1999 five states increased pension benefits and three states increased the tax exemption of pension benefits, while only two states lowered either the employee or employer contribution.

Economists including Inman (1981), and Glaeser and Ponzetto (2013) have argued that the political process may lead politicians to favor future compensation for public employees over current compensation for public employees. This political distortion may explain the high level of public sector pension underfunding. However, given that most public employees make a positive contribution to their pensions from their gross salaries, it can not explain the extent of the generosity

[1]Munnell et al. (2011a) compare the compensation packages of private sector and and public sector employees. They estimate that the average private sector employer costs for defined contribution plans equals 3.0 percent of payroll costs and that the average private sector employer costs for defined benefit plans equals 5.2 percent of payroll costs, while the average public sector employer costs for defined benefit plans equals 7.0 percent of payroll costs. The authors estimate that while only 18 percent of private sector employees are provided with retiree-health benefits, 64 percent of public sector employees are provided such benefits.

of defined benefit pensions relative to current net wages.[2]

Based on Inman's model, an increase in benefits or a decrease in employee contributions should have the same effect on the current/future compensation mix from a politician's perspective. If the union views pension benefits to be overly generous relative to net wages,[3] then there should be room for an efficiency gain by decreasing the employee's contribution rate to the pension, while maintaining the same combination of present employer expenses and future employer expenses. However, this is not the case. In the absence of binding institutional restraints on employee contributions, the widespread presence of positive employee contributions to pension systems indicate that public sector unions and thus at least a sub-set of public sector employees do not view retirement benefits as being overly generous.

In this chapter, I develop a basic political economy model in which the median public sector employee determines the generosity of the defined benefit pension system. Given a set level of current funds devoted towards current worker compensation and the total present value of employer contributions to the pension system, the median public sector employee determines the final wage replacement rate for retirees by deciding what portion of employee wages should be invested towards defined-benefit pensions. More concretely, holding unfunded liabilities and gross wage rates constant, the median public sector employee de-

[2]For 2010, out of 104 public pensions with employee contribution data in Boston College Center for Retirement Research Defined Benefit Pension Plan Dataset, 101 required strictly positive employee contributions.

[3]I use current net wages to refer to wages minus the employee's pension contribution.

termines the employee contribution rate towards the pension system, and thus the pension replacement rate.

In section 2, in a setting with perfect capital markets, I evaluate how the median public employee and thus the public sector union would prefer to receive an increase in the present value of total employer contributions to the pension system: through either an increase in the pension benefit replacement rate or through a decrease in the employee contribution rate. The median public employee will prefer an increase in the pension replacement rate if the present value of increased future pension benefits for the median public employee exceeds the present value of the decrease in employee pension contributions for the median public employee. I find that if the discount rate is sufficiently small, then the median public employee will prefer an increase in the pension benefit replacement rate over a decrease in the employee contribution rate.

If employees do not face borrowing constraints, then the union's political process will affect the distribution of lifetime compensation but will lead to a Pareto-efficient outcome. However, if workers face borrowing constraints, then a relatively large employee contribution rate and relatively large pension replacement rate may lead employees and younger employees in particular to face a binding liquidity constraint. Thus the political process may lead the union to select a level of pension generosity in which the current and future compensation mix is not Pareto-efficient. The median public sector employee will choose an employee contribution rate and pension replacement rate which may maximize her well-being but may not be Pareto-efficient. If the median public employee does not face liquidity constraints then for the selected level of pension benefits and contributions, the present value for the median public employee from a one dollar marginal increase in total benefits will exceed the present value for the median public employee from a one dollar marginal cut in total employee contributions. The process may lead to a Pareto-inefficient mix of pension contributions and benefits: In particular younger workers may be better off with lower pension benefits and lower contribution rates.[4] In order to quantify this efficiency loss, I incorporate borrowing constraints into the model. In section 3, I derive the compensating variation for each cohort of a one dollar increase in the present value of their pension benefits, and the compensating variation for each cohort of a one dollar increase in the present value of decreased employee contributions. At the optimum, the ratio of the median public employee's marginal present values equal the ratio of her compensating variations.

In addition, I demonstrate that under a wide range of parameters, the median public employee prefers a pension increase over an employee contribution cut. In section 4, I develop a continuous-time version of the model without liquidity constraints. I demonstrate that if the median public employee is not borrowing constrained, then under the most reasonable economic parameters, the median public employee would prefer a pension increase over an employee contribution cut if and only if the real interest rate is less than 8.5 percent. I also demonstrate

[4]The median public employee may even choose a pension benefit and contribution scheme such that the median public employee is liquidity constrained.

that even under the least favorable parameters for a pension benefit increase, the median public employee would prefer a pension increase if the real interest rate is less than 4.2 percent.

4.2 Discrete-Time Model

Assume that a politician has agreed to increase public employee compensation by ΔB, where ΔB is the present value of the increase in total public employee compensation for both current and future public employees. For political purposes, the government funds the additional compensation using future tax dollars. I assume that the public employees must receive this additional compensation via either decreased employee contributions to the pension system or increased pension benefits. I assume that the policy must be implemented in this period and be uniform for all current and future hires. In which form should the public employees unions want this compensation? If public employees' unions can decide how to receive the additional compensation then they are likely to prefer it in terms of pension benefits, even if there is no social insurance benefit for preferring pensions. I assume that public employees are identical except for their tenure on the job. Thus, the median public employee is simply the public employee with median tenure on the job. If the public employees' union implements the preferences of the median public employee, then so long as the discount rate is sufficiently low, the median public employee will choose an increase in the defined benefit pension plan over a reduction in the contribution to the pension plan.

This is a simple overlapping generations model with T periods. At any date t, there are R cohorts of public employees currently working, and $T - R$ cohorts of retired public employees. Each cohort consists of mass one. Each public sector employee works for R periods and is retired for $T - R$ periods. The number of years of experience for each worker is given by τ. Each cohort of public sector employees is indexed by h such that h equals the year in which they were first hired. For simplicity, I assume that cross-sectional wages increase by g. Wages for public sector wages are increasing in tenure: Also for simplicity assume that in any given year t, one additional year of experience corresponds to a k increase in salary. The defined benefit pension replacement rate for retired public employees equals p, and is calculated based on the public employee's end of career salary. The retired public employee from cohort h receives pension benefit equal to $pw_{h+R-1,R-1}$. In this section, I assume that public employees can borrow and save at the interest rate r, which is equal to the pure-time discount rate ρ. I assume that each individual experiences no uncertainty over the length of her life span or shocks to her preferences. I assume that an increase in pension benefits or a decrease in contributions is determined in period s and also immediately implemented in period s. At time s, the utility of a public sector employee of cohort h ($s \geq h$):

$$U_s^h(c) = \sum_{t=s\geq h}^{h+T-1} \frac{u(c_t^h)}{(1+r)^{t-s}}. \tag{4.1}$$

Total decreased contributions to the pension system equal

$$\Delta B = \sum_{t=s}^{\infty} \sum_{\tau=0}^{R-1} \frac{\Delta e w_{t,\tau}}{(1+r)^{t-s}}. \tag{4.2}$$

Define $G_{t,s}$ to be indexed growth in cross-sectional public employee salaries from period s: $G_{t,s}$ equals the ratio of wages for new public-employees in period t to the wages of new public-employees in initial period s. Also, define $K_{\tau,t}$ to be the indexed relative wage of public-employees with tenure τ in period t relative to the wages of new public-employees with tenure $\tau = 0$ in period t. If the present value of total compensation for all current and future workers increases by ΔB, then the change in the employee contribution rate Δe must satisfy

$$\sum_{t=s}^{\infty} \sum_{\tau=0}^{R-1} \frac{\Delta e w_{s,0} G_{t,s} K_{\tau,t}}{(1+r)^{t-s}} = \Delta B. \tag{4.3}$$

For simplicity assume that cross-sectional wage growth constantly equals g, and that in any year t, an additional year of experience corresponds to an increase in income by a factor of k, then the sum total of decreased employee contributions equals

$$\sum_{t=s}^{\infty} \sum_{\tau=0}^{R-1} \frac{\Delta e w_{s,0} (1+g)^{t-s} (1+k)^{\tau}}{(1+r)^{t-s}} = \Delta B. \tag{4.4}$$

Simplifying the right hand side of the above equation and simplifying yields

$$\Delta e = \frac{\Delta B (r-g) k}{w_{s,0} (1+r)((1+k)^R - 1)}. \tag{4.5}$$

For a mid-career public sector employee, the personal benefit from a decrease in employee contribution rate Δe equals

$$\sum_{t=s}^{s+\frac{R-1}{2}} \frac{\Delta e w_{t,t+\frac{R-1}{2}-s}}{(1+r)^{t-s}}. \tag{4.6}$$

Assuming that cross-sectional wages grow at rate g, and that in any year t an additional year of experience corresponds to an increase in income by a factor of k. Then, the sum total of decreased employee contributions would equal

$$\Delta e w_{t,0} \sum_{t=s}^{s+\frac{R-1}{2}} \frac{(1+g)^{t-s}(1+k)^{t-s}}{(1+r)^{t-s}}. \tag{4.7}$$

Substituting in the expression for Δe, the present value of decreased contributions equals

$$\frac{\Delta B(r-g)k}{(1+r)((1+k)^R-1)}(1+k)^{\frac{R-1}{2}} \frac{(1+r)^{\frac{R+1}{2}} - (1+g)^{\frac{R+1}{2}}(1+k)^{\frac{R+1}{2}}}{(1+r)-(1+g)(1+k)}. \tag{4.8}$$

The present value of all increased pension benefits must equal ΔB. The present value of total increased pension benefits of a pension replacement rate increase of Δp starting immediately in period s equals

$$\sum_{t=s}^{\infty} \sum_{\tau=R}^{T-1} \frac{\Delta p w_{t-\tau+R-1,R-1}}{(1+r)^{t-s}}. \tag{4.9}$$

Once again, assume that cross-sectional wage growth equals g. Then, the present value of pension benefits equals

$$\sum_{t=s}^{\infty}\sum_{\tau=R}^{T-1}\frac{\Delta p w_{s-T+R,R-1}(1+g)^{t-s+T-1-\tau}}{(1+r)^{t-s}}. \tag{4.10}$$

Evaluating the present value of pension benefits and rearranging to solve for the change in the pension benefit yields

$$\Delta p = \frac{\Delta B}{w_{s,R-1}\frac{1+r}{r-g}[\frac{1}{g}-\frac{(1+g)^{R-T}}{g}]}. \tag{4.11}$$

Once again, assuming that in a given year t one more year of experience corresponds to a k increase in salary, then

$$\Delta p = \frac{\Delta B(r-g)g}{w_{s,0}(1+k)^{R-1}(1+r)[1-(1+g)^{R-T}]}. \tag{4.12}$$

The present value of the increased pension benefit to the median public sector employee equals

$$\sum_{t=s+\frac{R+1}{2}}^{s+T-\frac{R+1}{2}}\frac{\Delta p w_{t+\frac{R-1}{2},R-1}}{(1+r)^{t-s}} \tag{4.13}$$

Since cross sectional wages increase by a constant factor g, and an additional year of experience corresponds to a constant increase in wages of k, the present value of the increased pension benefit to the median public sector employee equals

$$\sum_{t=s+\frac{R+1}{2}}^{s+T-\frac{R+1}{2}} \frac{\Delta p w_{s,0}(1+g)^{\frac{R-1}{2}}(1+k)^{R-1}}{(1+r)^{t-s}}. \tag{4.14}$$

Plugging in the expression for Δp and simplifying yields

$$\frac{\Delta B(r-g)g(1+g)^{T-\frac{R+1}{2}}}{r(1+r)^{T-\frac{R-1}{2}}} \frac{(1+r)^{T-R}-1}{(1+g)^{T-R}-1}. \tag{4.15}$$

The median public sector employee prefers a benefit increase over a contribution cut if and only if the present value of an increase in pension benefits to the median public employee exceeds the present value of a decrease in the contribution rate to the median public employee:

$$\begin{aligned} \frac{\Delta B(r-g)g(1+g)^{T-\frac{R+1}{2}}}{r(1+r)^{T-\frac{R-1}{2}}} \frac{(1+r)^{T-R}-1}{(1+g)^{T-R}-1} \geq \\ \frac{\Delta B(r-g)k}{(1+r)((1+k)^R-1)}(1+k)^{\frac{R-1}{2}} \frac{(1+r)^{\frac{R+1}{2}}-(1+g)^{\frac{R+1}{2}}(1+k)^{\frac{R+1}{2}}}{(1+r)-(1+g)(1+k)}. \end{aligned} \tag{4.16}$$

Simplifying the above equation, it follows that the median public sector employee prefers a benefit increase over a contribution cut if and only if

$$\begin{aligned} \frac{(1+r)^{T-R}-1}{r(1+r)^{T-\frac{R+1}{2}}} \frac{(1+r)-(1+g)(1+k)}{(1+r)^{\frac{R+1}{2}}-(1+g)^{\frac{R+1}{2}}(1+k)^{\frac{R+1}{2}}} \geq \\ \frac{k}{((1+k)^R-1)}(1+k)^{\frac{R-1}{2}} \frac{(1+g)^{T-R}-1}{g(1+g)^{T-\frac{R+1}{2}}}. \end{aligned} \tag{4.17}$$

The derivative of the left-hand side of equation (4.17) with respect to r is strictly positive:

$$\frac{-\frac{1}{r}((1+r)^{T-R}-1)-\frac{R-1}{2}(1+r)^{T-R}+\frac{T-\frac{R+1}{2}}{1+r}}{r(1+r)^{T-\frac{R+1}{2}}}\frac{(1+r)-(1+g)(1+k)}{(1+r)^{\frac{R+1}{2}}-(1+g)^{\frac{R+1}{2}}(1+k)^{\frac{R+1}{2}}}$$

$$+\frac{(1+r)^{\frac{R+1}{2}}-(1+g)^{\frac{R+1}{2}}(1+k)^{\frac{R+1}{2}}-((1+r)-(1+g)(1+k))\frac{R+1}{2}(1+r)^{\frac{R-1}{2}}}{((1+r)^{\frac{R+1}{2}}-(1+g)^{\frac{R+1}{2}}(1+k)^{\frac{R+1}{2}})^2}\frac{(1+r)^{T-R}-1}{r(1+r)^{T-\frac{R+1}{2}}}<0. \tag{4.18}$$

For any given g, k, and R, there exists an $\alpha(g,k,R)$ such that equation (4.17) holds with equality. Since the left-hand side of equation (4.17) is decreasing in r, the median public-employee will choose the increased pension benefit if and only if $r \leq \alpha(g,k,R)$.[5]

4.2.1 Four Period Model

It is difficult to gain intuition directly from equation (4.17). In order to illuminate the intuition behind the median public employee's decision more clearly, I analyze a basic four period life cycle model. I relax the assumption that one year of

[5] There exists an a such that if $r < a$, then the left-hand side of equation (4.17) will exceed the right-hand side of the equation, and the median public-employee will choose the increased pension benefit. Similarly, there exists a b such that if $r > b$, then the left-hand side of equation (4.17) will be less than the right-hand side of the equation, and the median public-employee will choose the increased benefit. Since the left-hand side of equation (4.17) is decreasing in r, it follows that there exists an α' such that the median public-employee will choose the increased pension benefit if and only if $r < \alpha'$.

experience corresponds to a set k increase in wages. The present value of the total decrease in employee contributions must equal ΔB:

$$\sum_{t=s}^{\infty} \frac{\Delta e(1+g)^{t-s}(w_{s,0}+w_{s,1}+w_{s,2})}{(1+r)^{t-s}} = \Delta B, \tag{4.19}$$

Simplifying the above equation, the decrease in the employee contribution rate Δe equals

$$\frac{\Delta B(r-g)}{(1+r)(w_{s,0}+w_{s,1}+w_{s,2})}. \tag{4.20}$$

The discounted present value of the decreased employee contribution to the pension fund for the mid-career public employee equals

$$\Delta e w_{s,1} + \frac{\Delta e w_{s,2}}{(1+r)}. \tag{4.21}$$

Plugging in the expression for Δe yields

$$\frac{\Delta B(r-g)((1+r)w_{s,1}+w_{s,2})}{(1+r)^2(w_{s,0}+w_{s,1}+w_{s,2})}. \tag{4.22}$$

If the government uses ΔB to immediately increase the pension replacement rate, then

$$\sum_{t=s}^{\infty} \frac{\Delta p(1+g)^{t-s}w_{s-1,2}}{(1+r)^{t-s}} = \Delta B. \tag{4.23}$$

Rearranging and simplifying the above equation, the increase in the pension benefit replacement rate Δp equals

$$\frac{\Delta B(r-g)}{(1+r)w_{s-1,2}}. \tag{4.24}$$

The discounted present value of increased pension benefits for the mid-career public employee equals

$$\frac{\Delta p w_{s+1,2}}{(1+r)^2}. \tag{4.25}$$

Plugging in for Δp, the above equation simplifies to

$$\frac{\Delta B(r-g)(1+g)^2}{(1+r)^3}. \tag{4.26}$$

The median public sector employee prefers increased pension benefits if and only if the present value of an increase in the pension benefit replacement rate for the median public employee exceeds the present value of a decrease in the employee contribution rate for the median public employee:

$$\frac{\Delta B(r-g)(1+g)^2}{(1+r)^3} > \frac{\Delta B(r-g)((1+r)w_{s,1}+w_{s,2})}{(1+r)^2(w_{s,0}+w_{s,1}+w_{s,2})}. \tag{4.27}$$

Simplifying the above expression, the median public sector employee prefers an increase in the pension benefit replacement rate over a decrease in the employee contribution rate if and only if

$$(1+g)^2(w_{s,0}+w_{s,1}+w_{s,2}) > (1+r)^2 w_{s,1} + (1+r) w_{s,2}. \tag{4.28}$$

Alternatively, the median public sector employee prefers an increase in the pension benefit replacement rate if and only if

$$(w_{s+2,0}+w_{s+2,1}+w_{s+2,2}) > (1+r)^2 w_{s,1} + (1+r) w_{s,2}. \tag{4.29}$$

For a given $g > 0$, $w_{s,0}$, $w_{s,1}$, and $w_{s,2}$ there exists a $\bar{r} > 0$ s.t. that the median public employee would prefer the increase in the pension benefit replacement rate if and only if $r < \bar{r}$.

More generally, the preference for an increase in the pension benefit replacement rate over a decrease in the employee contribution rate depends positively on g and $w_{s,0}$, and negatively on r. An increase in the growth rate of wages g adversely affects the present value of expected benefits of either an increase in the pension benefit replacement rate or a decrease in the employee contribution rate. Since an increase in the pension benefit replacement rate relatively favors earlier cohorts compared to a decrease in the employee contribution rate, an increase in g, will cause a relatively larger decrease in the share of expected benefit from a decrease in the employee contribution rate. This will increase the relative attractiveness of an increase in the pension benefit replacement rate.

4.3 Liquidity Constraint

In the previous section, I demonstrated that unions may prefer a pension benefit increase over a decrease in the pension contribution rate since the median public employee will likely experience higher expected benefits from the former. If workers utility is linear in consumption or credit markets are complete, then there is no Pareto inefficiency from this process. However, if workers are liquidity constrained, then the political process and the preference for higher pension benefits will lead to a Pareto inefficiency. The median public employee may choose a policy bundle which leaves numerous workers with larger pension benefits and lower current net income than is optimal for a given level of total lifetime compensation for a given worker.

In this section I expand the model by introducing borrowing constraints. Once again, at any date t, there are R cohorts of public employees currently working, and $T-R$ cohorts of retired public employees. Cohorts are indexed by h, and each cohort has mass one. For simplicity, I assume that cross sectional wage growth equals g, and that in period t, one year of additional experience corresponds to a wage increase by a factor of k. At time t, the wage of a public employee with experience τ equals $w_{t,\tau}$. The defined benefit pension replacement rate for retired public employees equals p and is figured on their ending public employee salary. The retired public employee pension benefit for cohort h in period $t > h + R - 1$ equals $pw_{h+R-1,R-1}$. I assume that the interest rate r is equal to ρ. Individuals can save at interest rate $r = \rho$ but are unable to borrow. However, the government can both save and borrow at the interest rate $r = \rho$. Once again, I also assume that individuals do not face any uncertainty over longevity nor experience any shocks to taste. Recall that at time s, the utility of an individual in cohort $h \leq s$:

$$U_s^h(c) = \sum_{t=s}^{s+T} \frac{u(c_t^h)}{(1+r)^{t-s}}. \tag{4.30}$$

Each individual of cohort h in year s maximizes

$$\begin{aligned}
&\max_{\{c_t^h, b_t^h\}_{t=s}^{h+T-1}} && \sum_{t=s}^{h+T-1} \beta^{t-s} u(c_t^h) \\
&\text{s.t.} && \forall s \leq t \leq s + T - \frac{R}{2}, c_t^h + b_t^h = y_t^h + (1+r) b_{t-1}^h \\
& && Y_t^h = (1-e) w_{t,t-h} \text{ if } t \leq h + R - 1 \\
& && Y_t^h = p w_{h+R-1,R-1} \text{ if } t > h + R - 1
\end{aligned} \tag{4.31}$$

The compensating variation for cohort h in period s of a one dollar increase in cohort h's total pension benefits equals

$$CV_s^h(p) = \frac{\sum_{t=h+R}^{h+T-1} \frac{\beta^{t-s} u'(c_t^h)}{\sum_{t=s+\frac{R+1}{2}} \frac{1}{(1+r)^{t-s}}}}{u'(c_s^h)}. \tag{4.32}$$

The compensating variation for cohort h of a one dollar decrease in cohort h's total pension contributions equals

$$CV_s^h(e) = \frac{\sum_{t=s}^{h+R-1} \frac{w_{t,t-h}}{\sum_{t=s}^{s+\frac{R-1}{2}} \frac{w_{t,t-h}}{(1+r)^{t-s}}} \beta^{t-s} u'(c_t^h)}{u'(c_s^h)} \tag{4.33}$$

The CV for the median public employee in period s of a one dollar increase in present value of the median public employee's pension benefits equals[6]

$$CV_s^{s-\frac{R-1}{2}}(p) = \frac{\sum_{t=s+\frac{R+1}{2}}^{s+T-\frac{R+1}{2}} \frac{\beta^{t-s} u'(c_t^{s-\frac{R-1}{2}})}{\sum_{t=s+\frac{R+1}{2}}^{s+T-\frac{R+1}{2}} \frac{1}{(1+r)^{t-s}}}}{u'(c_s^{s-\frac{R-1}{2}})}. \tag{4.35}$$

The CV for the median public employee in period s of a one dollar decrease in the median public employee's total pension contributions equals[7]

$$CV_s^{s-\frac{R-1}{2}}(e) = \frac{\sum_{t=s}^{s+\frac{R-1}{2}} \frac{w_{t,\frac{R-1}{2}-s+t}}{\sum_{t=s}^{s+\frac{R-1}{2}} \frac{w_{t,\frac{R-1}{2}-s+t}}{(1+r)^{t-s}}} \beta^{t-s} u'(c_t^{s-\frac{R-1}{2}})}{u'(c_s^{s-\frac{R-1}{2}})} \tag{4.37}$$

[6]If the retiree is liquidity constrained in all periods then

$$CV_s^{s-\frac{R-1}{2}}(p) = \frac{\sum_{t=s+\frac{R+1}{2}}^{s+T-\frac{R+1}{2}} \frac{\beta^{t-s} u'(pw_{s+\frac{R-1}{2},R-1})}{\sum_{t=s+\frac{R+1}{2}}^{s+T-\frac{R+1}{2}} \frac{1}{(1+r)^{t-s}}}}{u'(pw_{s,\frac{R-1}{2}})}. \tag{4.34}$$

[7]CV of a decrease in pension contributions of one dollar(if the employee is liquidity constrained in all prior periods).

$$CV_s^{s-\frac{R-1}{2}}(e) = \frac{\sum_{t=s}^{s+\frac{R-1}{2}} \frac{w_{t,\frac{R}{2}-s+t}}{\sum_{t=s}^{s+\frac{R-1}{2}} \frac{w_{t,\frac{R-1}{2}-s+t}}{(1+r)^{t-s}}} \beta^{t-s} u'((1-e)w_{t,\frac{R-1}{2}-s+t})}{u'(c_{s,\frac{R}{2}})}. \tag{4.36}$$

Once again, assume that members of the public employee's unions can decide how to accept this compensation. I assume that only current public employees are members of the union but not retirees. However, in this section I assume that the total present value of current and future employer provided pension benefits B is given. Conditional on B, public employees determine contribution rate $e(E)$ and replacement rate $p(E)$ where E is the present value of all current and future employee contributions. such that $P = E + B$.

Since preferences in this case are single peeked, then the median voter determines the outcome, and thus the median career public employee should be pivotal. Since the mid-career public sector employee determines the mix of current and future compensation, then the mix of current and future compensation would be given by

$$\begin{aligned}
&\max_{\{c_t^{s-\frac{R-1}{2}}, b_t^{s-\frac{R-1}{2}}\}_{t=s}^{s+T-\frac{R+1}{2}}} && \sum_{t=s}^{s+T-\frac{R+1}{2}} \beta^{t-s} u(c_t^{s-\frac{R-1}{2}}) \\
&\text{s.t.} && \forall s \leq t \leq s+T-\frac{R}{2}, c_t^{s-\frac{R-1}{2}} + b_t^{s-\frac{R-1}{2}} = y_t^{s-\frac{R-1}{2}} + (1+r) b_{t-1}^{s-\frac{R-1}{2}} \\
& && Y_{t,\frac{R}{2}-s+t} = (1-e(E)) w_{t,\frac{R-1}{2}-s+t} \text{ if } t \leq s + \frac{R-1}{2} \\
& && Y_{t,\frac{R}{2}-s+t} = p(E) w_{s+\frac{R-1}{2},R-1} \text{ if } t > s + \frac{R-1}{2} \\
& && P = E + B
\end{aligned} \tag{4.38}$$

Evaluating at the optimum E yields[8]

$$\sum_{t=s}^{s+\frac{R-1}{2}} w_{t,\frac{R-1}{2}-s+t}\beta^{t-s}u'(c_t^{s-\frac{R-1}{2}})\frac{de}{dE} = \sum_{t=s+\frac{R+1}{2}}^{s+T-\frac{R+1}{2}} w_{s+\frac{R-1}{2},R-1}\beta^{t-s}u'(c_t^{s-\frac{R-1}{2}})\frac{dp}{dE}. \tag{4.40}$$

Define $A_s^{s-\frac{R-1}{2}}$ to be the ratio of the PV of marginal decreased pension contributions for a median public sector employee from reducing e from a one dollar increase in total pension funding relative to the PV of marginal future pension benefits for a median public sector employee from increasing p from a one dollar increase in total pension funding:

$$A_s^{s-\frac{R-1}{2}} = \frac{\sum_{t=s}^{s+\frac{R-1}{2}} \frac{w_{t,t-s+\frac{R-1}{2}}}{(1+r)^{t-s}}\frac{de}{dE}}{\sum_{t=s}^{s+\frac{R-1}{2}} \frac{w_{s+\frac{R-1}{2},R-1}}{(1+r)^{t-s}}\frac{dp}{dP}}. \tag{4.41}$$

Rearranging equation (67) and substituting in $A_s^{s-\frac{R-1}{2}}$, then for the median public-employee, at the optimal pension replacement rate

$$A_s^{s-\frac{R-1}{2}} = \frac{CV_s^{s-\frac{R-1}{2}}(p)}{CV_s^{s-\frac{R-1}{2}}(e)}. \tag{4.42}$$

At the optimum, the ratio of the marginal present values to the median public

[8] There are three possibilities; that median career public employees are net savers regardless of the form of additional compensation, that mid-career public employees are liquidity constrained regardless of the form of additional compensation, or mid-career public employees will be liquidity constrained depending on the form of additional compensation. If the mid-career employee is liquidity constrained in each period then

$$\sum_{t=s}^{s+\frac{R-1}{2}} w_{t,\frac{R-1}{2}-s+t}\beta^{t-s}u'((1-e)w_{t,\frac{R-1}{2}-s+t})\frac{de}{dE} = \sum_{t=s+\frac{R+1}{2}}^{s+T-\frac{R+1}{2}} \beta^{t-s}w_{s+\frac{R}{2},R-1}u'(pw_{s+\frac{R-1}{2},R-1})\frac{dp}{dE}. \tag{4.39}$$

employee equal the ratio of the compensating variations. The above equation captures the efficiency of the marginal pension benefit relative to the efficiency of the marginal decrease in employee contributions. If the median public-employee is liquidity constrained, then $CV_s^{s-\frac{R-1}{2}}(p) < CV_s^{s-\frac{R-1}{2}}(e)$ and at the equilibrium there is a relative efficiency loss for the median public employee.

4.4 Continuous Model

In this section I adapt the model to a continuous overlapping generations framework. Most assumptions remain unchanged from the discrete-time model. At any time t, there are a continuum of public employees and public employee-retirees. Public-employees have tenure on the job over the domain $[0, R]$, retirees have been retired over the domain $[R, T]$. For simplicity, I assume that each cohort is the same size, and thus the distribution of public employees and retirees is uniform; without loss of generality assume density 1. Cohorts are indexed by h and the age of each cohort equals τ. Each cohort of public-employees works for R periods; is retired for $T - R$ periods; and dies with certainty at time T. In any period t, wages for a public employee with experience τ equal $w_{t,\tau}$. Retirement benefits are proportional to wages at time R such that retirement benefits for a member of cohort h equals $pw_{h+R,R}$.

For simplicity, I assume that for any given worker in any time period t with a given level of experience $\tau \in [0, R)$, her wages grow at a constant rate $g + k$, compounded continuously: k is the growth in wages attributed to increases in tenure, and g is the cross sectional increase in wages. Preferences are also analogous to the discrete time case. For a given member of cohort h, her preferences[9] in period s are represented by

$$U_s^h(c) = \int_s^{h+T} e^{-r(t-s)} u(c_t^h) \mathrm{d}t. \tag{4.43}$$

In this section, I assume that individuals can borrow and save at interest rate $r = \tau$. Thus, individuals preferences over income can represented by their discounted present value of income. Once again, I assume that a policy change will take effect immediately in period s, and that the public employees must choose how to accept increased total compensation equal to ΔB. The present value of the benefit from a Δp increase in the pension replacement rate must equal ΔB:

$$\begin{aligned} &\int_s^{s+T-R} \int_R^{R+t-s} \Delta p w_{s,0} e^{kR-r(t-s)+g(t-s-\tau+R)} \mathrm{d}\tau \mathrm{d}t + \\ &\int_{s+T-R}^{\infty} \int_R^{T} \Delta p w_{s,0} e^{kR-r(t-s)+g(t-s-\tau+R)} \mathrm{d}\tau \mathrm{d}t = \Delta B. \end{aligned} \tag{4.44}$$

Evaluating the present value of a benefit increase and rearranging for Δp yields

$$\Delta p = \frac{\Delta B(r-g)r}{w_{s,0}(e^{kR} - e^{kR-r(T-R)})}. \tag{4.45}$$

If increased compensation ΔB is provided in the form of a reduction in the con-

[9]Period h corresponds to the beginning of her public sector career.

tribution rate Δe to the pension system, the present value of decreased pension contributions equals

$$\int_{s}^{\infty}\int_{0}^{R} \Delta e w_{s,0} e^{k\tau-(r-g)(t-s)} \mathrm{d}\tau \mathrm{d}t = \Delta B. \tag{4.46}$$

Evaluating the present value of an employee contribution decrease and rearranging for Δe yields

$$\Delta e = \frac{\Delta B(r-g)k}{w_{s,0}(e^{kR}-1)}. \tag{4.47}$$

The present value of increased pension benefits from an increased pension benefit rate to a mid-career public employee equals

$$\int_{s+\frac{R}{2}}^{s+T-\frac{R}{2}} e^{-r(t-s)} \Delta p w_{s,0} e^{kR+g\frac{R}{2}} \mathrm{d}t. \tag{4.48}$$

Plugging in Δp and evaluating the above equation, the benefit to a mid-career public sector employee from an increased pension benefit rate equals

$$\Delta B(r-g)e^{-(r-g)\frac{R}{2}}. \tag{4.49}$$

The benefit to a mid-career public employee from a reduction in the employee contribution rate equals

$$\int_{s}^{s+\frac{R}{2}} \Delta e w_{s,0} e^{k\frac{R}{2}-(r-g-k)(t-s)} \mathrm{d}t, \tag{4.50}$$

which simplifies to

$$\frac{\Delta B(r-g)k}{r-(k+g)}\frac{1}{(e^{kR}-1)}[e^{k\frac{R}{2}}-e^{kR-(r-g)(\frac{R}{2})}]. \tag{4.51}$$

The present value of increased benefits to a mid-career public employee from a ΔB increase in total benefits will exceed the present value of decreased contributions to a mid-career employee from a ΔB decrease in total contributions if and only if

$$\Delta B(r-g)e^{-(r-g)\frac{R}{2}} \geq \frac{\Delta B(r-g)k}{r-(k+g)}\frac{1}{(e^{kR}-1)}[e^{k\frac{R}{2}}-e^{kR-(r-g)(\frac{R}{2})}]. \tag{4.52}$$

Simplifying the above equation, the median public-employee will choose the increased benefit if and only if

$$1 \geq \frac{k}{e^{kR}-1}\frac{e^{(k+(r-g))\frac{R}{2}}-e^{kR}}{(r-g)-k}. \tag{4.53}$$

The right hand side of the above equation equals the ratio of the present value of decreased contributions from a decrease in the contribution rate for a median public employee to the present value of increased pension benefits from an increase in pension benefit replacement rate to a median public employee. Define the right hand side of the equation to equal $A(k,R)$. Note that equation (4.53) is a function of $r-g$. The derivative of the right hand side of the above equation with respect to $r-g$ is strictly positive:

$$\frac{dA}{d(r-g)} = \frac{ke^{k\frac{R}{2}}}{e^{kR}-1} \frac{[\frac{R}{2}((r-g)-k)e^{(r-g)\frac{R}{2}} + e^{k\frac{R}{2}} - e^{(r-g)\frac{R}{2}}]}{(r-g)-k)^2} > 0. \tag{4.54}$$

There exists an $\alpha(k,R)$ such that equation (4.53) holds with equality. Since the right hand side of equation (4.53) is increasing, the median public-employee will choose the increased pension benefit if and only if $r-g \leq \alpha(k,R)$.[10]

Taking the derivative of A with respect to R yields

$$\frac{dA}{dR} = \frac{ke^{(k+(r-g))R/2}}{e^{kR}-1}[\frac{e^{(k+(r-g))R/2} - ekR}{(r-g)-k}] + \frac{k}{e^{kR}-1}\frac{\frac{(r-g)+k}{2}e^{((r-g)+k)} - ke^{kR}}{(r-g)-k} > 0. \tag{4.55}$$

By definition of $\alpha(R,k)$,

$$A(\alpha(R,k),R,k) = 1. \tag{4.56}$$

Holding k constant and fully differentiating the above yields

$$\frac{\partial A}{\partial \alpha}\frac{d\alpha}{dR} + \frac{\partial A}{\partial R} = 0. \tag{4.57}$$

Since $\frac{\partial A}{\partial \alpha} > 0$ and $\frac{\partial A}{\partial R} >$, then

$$\frac{d\alpha}{dR} < 0. \tag{4.58}$$

[10]There exists an a such that if $r-g > a$, then the right-hand side of equation (4.53) will exceed one and the median public-employee will choose the decreased contribution. Similarly, there exists a b such that if $r-g < b$, then the right-hand side of equation (4.53) will be less than one and the median public-employee will choose the increased benefit. Since the right hand side of equation (4.53) is increasing, it follows that there exists an α' such that the median public-employee will choose the increased pension benefit if and only if $r-g < \alpha'$.

Since α is decreasing in R, this implies that an increase in R corresponds to a lower real interest sufficient for the median public employee to prefer the pension benefit over the contribution cut. Thus, an increase in the retirement age would increase the likelihood that a public-employee union would prefer a contribution cut and a decrease in the retirement age would increase the likelihood that a public-employee union would prefer a pension increase.

$\alpha(k, R)$

	$R = 30$	$R = 35$	$R = 40$	$R = 50$
$k = .01$	.0773	.0653	.0564	.0439
$k = .02$	.0711	.0592	.0504	.0380
$k = .03$	.0653	.0536	.0448	.0327

The table above gives the level of $\alpha(k, R)$ for various levels of k and R. The table implies that a high real interest rate would be required to induce a median employee to prefer a pension contribution decrease over a pension benefit increase. If $k = .02$ and $R = 35$, then $\alpha = .0653$. Thus, if a one year increase in experience is associated with a two percent increase in salary, the median public employee retires after 35 years, and average wage growth equals two percent, then the median public employee would prefer an increase in the pension benefit replacement rate over a decrease in the employee contribution rate if the real interest rate $r \leq .0853$. Furthermore, if average wage growth equals two percent, then for any parameter combination in the table, the median public employee prefers a pension benefit increase over contribution decrease if the real interest rate is less than five percent.

4.5 Conclusion

This chapter presents a basic model demonstrating how the political process within unions can lead to overly generous public-sector pensions. In this chapter, I develop a basic political economy model in which the median public sector employee determines the generosity of the defined benefit pension system. Given a set level of current funds devoted towards current worker compensation and the total present value of employer contributions to the pension system, the median public sector employee determines the final wage replacement rate for retirees by deciding what portion of employee wages should be invested towards defined-benefit pensions. Specifically, I demonstrate that when presented with an increase in the present value of total pension benefits, if the discount rate is sufficiently small, then the present value of increased future pension benefits exceeds the present value of the decrease in employee pension contributions for the median public employee. In the absence of borrowing constraints, if the present value of increased future pension benefits exceeds the present value of the decrease in employee pension contributions for the median public employee, then the median public employee will thus prefer a pension benefit increase over a decrease in employee contributions. In the absence of borrowing constrains, then under a wide range of parameters the public sector employee will prefer a pension benefit increase over an employee contribution increase if the real interest rate is less than five percent.

APPENDIX A

CHAPTER 1 OF APPENDIX

A.1 Primary Tables for Chapter 2

Table 1: Descriptive Statistics

	Mean	SD
2001 Housing Prices	184366	73156
Growth in Housing Prices (2001-2009)	73792	46455
2001 Actuarial Liabilities per Household	19889	10732
Change in Act. Liabilities (2001-2009)	11625	6960
2001 Market Assets Per Household	18471	7985
Change in Market Assets (2001-2009)	1138	2572
2001 Unfunded Liabilities per Household	1419	8038
Change in Unfunded Liabilities (2001-2009)	10486	6840

Source: Public Plans Database. 2001-2009. Center for Retirement Research at Boston College, Center for State and Local Government Excellence, and National Association of State Retirement Administrators. Changes in market assets, actuarial liabilities, and unfunded liabilities are divided by the number of 2001 households. SD is weighted by the number of households

Table 2: 2001 Pension Assets

	Total Assets	Domestic Equities	Domestic Bonds	Inter. Equities	Inter. Bonds	REIT	Other
Level in Billions	1761.3	741.8	574.2	233.0	16.3	100.9	95.8
Percent of Total Assets	-	42.1	32.6	13.2	.9	5.7	5.5
Assets Per Household	16542	6967	5393	2189	153	948	893
(SD)	8282	4519	2344	1659	268	958	848
25^{th} Percentile	9777	3862	3031	925	0	11	127
50^{th} Percentile	13553	6054	4337	1732	0	505	394
75^{th} Percentile	16922	8100	5804	2723	251	931	831
Maximum of States	43303	18111	13069	7148	1966	4087	4105
Minimum of States	4674	0	2049	0	0	0	0

Source: *Public Plans Database*. 2001 – 2009. Center for Retirement Research at Boston College, Center for State and Local Government Excellence, and National Association of State Retirement Administrators. Asset levels are adjusted by the respective index in order to approximate the level of assets as of June 30, 2001. Assets are adjusted by the number of households from the 2001 ACS. Assets per household reflect the mean level of assets per household for the entire nation, and the SD is weighted by the number of households.

Table 3: OLS estimates of the Effect of Unfunded Liabilities on Housing Prices

Dependent Variable: Price of Housing

Unfunded Liabilties	-0.252	-0.392**	0.265
	(0.317)	(0.179)	(0.634)
Per Capita Income	-	11.911	15.375
	-	(3.992)	(8.749)
Unemployment	-	-16196.33	-34871.84
		(7687.775)	(18319.77)
State-Specific Time Trends	No	No	Yes
R-squared	0.912	0.929	0.954
N	459	459	459

Source: *Public Plans Database*. 2001 – 2009. Center for Retirement Research at Boston College, Center for State and Local Government Excellence, and National Association of State Retirement Administrators. Unfunded liabilities are market level assets per 2001 occupied household size minus actuarial liabilities per 2001 occupied household size. Each regression controls for year and state fixed effects, and is weighted by 2001 household size. Standard errors are clustered at the state level. * indicates significance at the 10 percent level, ** indicates significance at the 5 percent level, and *** indicates significance at the 1 percent level.

Table 4: First Stage Estimates of the Effect of Predicted Assets on Unfunded Liabilities

Dependent Variable: Unfunded Liabilities

	(1)	(2)	(3)	(4)
Predicted Assets	-0.212*	-0.659***	-0.202	-0.662***
	(0.115)	(0.177495)	(0.128)	(0.181)
Lagged Household Change	yes	yes	no	no
State-Specific Time Trends	No	Yes	No	Yes
First-Stage F-Statistic	3.39	13.78	2.48	13.40
R-squared	0.768	0.897	0.768	0.897
N	459	459	459	459

Source: *Public Plans Database*. 2001 – 2009. Center for Retirement Research at Boston College, Center for State and Local Government Excellence, and National Association of State Retirement Administrators. Actuarial liabilities are divided by the state level number of households in 2001. The predicted assets instrument is constructed by adjusting initial asset shares by the indexed rate of return of each asset class, and dividing by the state level number of households in 2001. Lagged household change measures the percent change in occupied households between period $t-1$ and period t. Each regression is weighted by 2001 household size and controls for pci, the unemployment rate, and year and state fixed effects. Standard errors are clustered at the state level. * indicates significance at the 10 percent level, ** indicates significance at the 5 percent level, and *** indicates significance at the 1 percent level.

Table 5: 2-SLS Estimates of the Effect of Underfunding on Housing Prices

Dependent Variable: Price of Housing

	(1)	(2)	(3)	(4)
Unfunded	-24.713	-6.601**	-27.307	-6.754**
	(23.000)	(3.319)	(26.954)	(3.381)
Lagged Household Change	yes	yes	no	-no
State-Specific Time Trends	No	Yes	No	Yes
R-squared	-	0.919	-	0.917
N	459	459	459	459

Source: *Public Plans Database*. 2001 – 2009. Center for Retirement Research at Boston College, Center for State and Local Government Excellence, and National Association of State Retirement Administrators. Unfunded liabilities equal market level assets divided by the state level number of households in 2001 minus actuarial liabilities divided by the state level number of households in 2001. The predicted assets instrument is constructed by adjusting initial asset shares by the indexed rate of return of each asset class. Lagged household change measures the percent change in occupied households between period $t-1$ and period t. Each regression is weighted by 2001 household size, and controls for pci, the unemployment rate, and year and state fixed effects. Standard errors are clustered at the state level. * indicates significance at the 10 percent level, ** indicates significance at the 5 percent level, and *** indicates significance at the 1 percent level.

Table 6: OLS Estimates of the Effects of Predicted Assets on Actuarial Liabilities

Dependent Variable: Actuarial Liabilities

	(1)	(2)
Predicted Assets	0.475	0.007
	(0.213)	(0.054)
State-Specific Time Trends	No	Yes
R-squared	0.840	0.932
N	459	459

Source: *Public Plans Database*. 2001 – 2009. Center for Retirement Research at Boston College, Center for State and Local Government Excellence, and National Association of State Retirement Administrators. Unfunded liabilities equal market level assets divided by the state level number of households in 2001 minus actuarial liabilities divided by the state level number of households in 2001. Predicted assets is constructed by adjusting initial asset shares by the indexed rate of return of each asset class, and dividing by 2001 occupied household size. Each regression is weighted by 2001 household size, and controls for pci, the unemployment rate, and year and state fixed effects. Standard errors are clustered at the state level. * indicates significance at the 10 percent level, ** indicates significance at the 5 percent level, and *** indicates significance at the 1 percent level.

Table 7: 2-SLS Estimates of the Effect of Expected Housing Growth on Capitalization

Dependent Variable: Price Housing

	(1)	(2)
Unfunded Liabilities	-17.309	-32.756*
	(15.404)	(18.526)
Unfunded Liabilities x Household Percent Change	1.205	1.618**
	(1.022)	(0.706)
State-Specific Time Trends	No	Yes
First-Stage F-statistic	27.70	9.04
R-squared	0.936	0.942
N	459	459

Source: *Public Plans Database*. 2001 – 2009. Center for Retirement Research at Boston College, Center for State and Local Government Excellence, and National Association of State Retirement Administrators. Unfunded liabilities equal market level assets divided by the number of households in 2001 minus actuarial liabilities divided by the number of households in 2001. I instrument for unfunded liabilities using the predicted assets instrument, and I instrument for unfunded liabilities interacted with the future one period household percentage change using the asset instrument interacted with the future one period household percentage. Household percent change measures the percent change in number of occupied households between year t and year $t+1$. Each regression is weighted by 2001 household size and controls for pci, the unemployment rate, household percent change, and year and state fixed effects. Standard errors are clustered at the state level. * and ** indicate significance at the 10 percent and the 5 percent levels respectively.

Table 8: 2-SLS estimates of the Effect of Bartik Employment Growth on Capitalization

Dependent Variable: Price Housing

	(1)	(2)
Unfunded Liabilities	-19.746	-15.305
	(19.704)	(10.952)
Unfunded Liabilities x Bartik Percent Change	-6.240*	9.907
	(3.481)	(6.154)
State-Specific Time Trends	no	yes
First stage F statistic	. 89	.51
R-squared	0.943	0.942
N	459	459

Source: *Public Plans Database*. 2001 – 2009. Center for Retirement Research at Boston College, Center for State and Local Government Excellence, and National Association of State Retirement Administrators. Unfunded liabilities equal market level assets divided by the number of households in 2001 minus actuarial liabilities divided by the number of households in 2001. I instrument for unfunded liabilities using the predicted assets, and I instrument for unfunded liabilities interacted with one period future Bartik percent growth by instrumenting with the asset instrument interacted with one period future Bartik percent growth. Bartik percent change is the predicted employment growth for period $t + 1$ relative to period t. Each regression is weighted by 2001 household size, and controls for pci, the unemployment rate, Bartik percent change, and year and state fixed effects. Standard errors are clustered at the state level. * indicates significance at the 10 percent level.

A.2 Secondary Tables for Chapter 2

Table A-1: 2001 Pension Assets

	Total Assets	Domestic Equities	Domestic Bonds	Inter. Equities	Inter. Bonds	REIT	Other
Alabama	13206	5,209	5471	955	79	662	829
Alaska	43303	18111	13069	7148	1966	3009	0
Arizona	5475	1907	3365	33	0	115	54
Arkansas	10143	4565	3142	1245	0	359	832
California	28061	10226	8423	5053	473	1979	1908
Colorado	19092	9401	2049	2473	379	1921	2870
Connecticut	13553	6391	5042	1779	0	338	3
Delaware	15750	7653	5267	1960	477	0	394
DC	5596	1641	2201	745	0	0	1009
Florida	15123	8259	3823	1732	0	622	688
Georgia	7352	0	7105	0	0	0	247
Hawaii	22773	10003	6172	2980	1490	1490	638
Idaho	13059	5853	3255	2933	54	616	349

Source: *Public Plans Database*. 2001 – 2009. Center for Retirement Research at Boston College, Center for State and Local Government Excellence, and National Association of State Retirement Administrators. Asset levels are adjusted by the respective index in order to approximate the level of assets as of June 30, 2001. Assets are adjusted by the number of households from the 2001 ACS.

Table A-1: 2001 Pension Assets (Continued)

	Total Assets	Domestic Equities	Domestic Bonds	Inter. Equities	Inter. Bonds	REIT	Other
Illinois	13539	4621	4670	2209	426	890	723
Indiana	6057	2732	2916	314	0	0	95
Iowa	12259	4039	5293	1950	0	836	141
Kansas	9464	4034	2930	1310	0	632	558
Kentucky	14450	7051	4939	720	0	249	1492
Louisiana	10161	4891	2585	1371	545	262	507
Maine	13289	6591	4970	1714	0	0	13
Maryland	13963	6697	2648	2648	97	1872	0
Massachusetts	11833	5056	3011	2609	153	667	338
Michigan	13466	6054	2511	984	0	1188	2730
Minnesota	17152	9198	4337	2449	0	312	856
Mississippi	14266	7490	4779	1954	0	0	43
Missouri	13835	5679	5685	2121	36	67	247

Source: *Public Plans Database*. 2001 – 2009. Center for Retirement Research at Boston College, Center for State and Local Government Excellence, and National Association of State Retirement Administrators. Asset levels are adjusted by the respective index in order to approximate the level of assets as of June 30, 2001. Assets are adjusted by the number of households from the 2001 ACS.

Table A-1: 2001 Pension Assets (Continued)

	Total Assets	Domestic Equities	Domestic Bonds	Inter. Equities	Inter. Bonds	REIT	Other
Montana	14544	7588	4715	1001	0	838	403
Nebraska	6399	3136	2368	896	0	0	0
Nevada	18852	5667	7222	1592	1788	1686	898
New Hampshire	8582	3889	2164	655	290	788	796
New Jersey	23356	11469	7299	3847	0	0	741
New Mexico	21392	10033	7912	3232	0	23	192
New York	33931	16568	10461	3446	93	1843	1519
North Carolina	7574	0	7031	0	0	481	62
North Dakota	9859	3877	2783	1761	491	834	113
Ohio	25346	12436	3052	5108	24	4087	638
Oklahoma	8308	3847	3249	955	60	0	197
Oregon	27327	10028	7587	4278	0	1330	4105
Pennsylvania	14894	6322	3893	2814	212	854	799

Source: *Public Plans Database*. 2001 – 2009. Center for Retirement Research at Boston College, Center for State and Local Government Excellence, and National Association of State Retirement Administrators. Asset levels are adjusted by the respective index in order to approximate the level of assets as of June 30, 2001. Assets are adjusted by the number of households from the 2001 ACS.

Table A-1: 2001 Pension Assets (Continued)

	Total Assets	Domestic Equities	Domestic Bonds	Inter. Equities	Inter. Bonds	REIT	Other
Rhode Island	15269	7942	4319	2873	131	0	4
South Carolina	11741	3021	7010	0	0	0	1709
South Dakota	14821	6906	3922	2797	0	972	224
Texas	12222	3287	5516	973	377	126	1942
Tennessee	8639	1510	6168	587	0	202	171
Utah	14819	6761	3484	2044	784	1107	638
Vermont	9695	3637	2636	1366	1237	819	0
Virginia	4674	254	3526	63	0	505	327
Washington	16691	7754	4329	2838	0	1536	234
West Virginia	5251	2310	2205	735	0	0	0
Wisconsin	22238	16280.54304	5332	0	0	304	322
Wyoming	20880	12213	5922	1605	0	0	1139

Source: *Public Plans Database*. 2001 – 2009. Center for Retirement Research at Boston College, Center for State and Local Government Excellence, and National Association of State Retirement Administrators. Asset levels are adjusted by the respective index in order to approximate the level of assets as of June 30, 2001. Assets are adjusted by the number of households from the 2001 ACS.

Table A-2: First Stage normalized by Current Household Size

Dependent Variable: Unfunded Liabilities

	(1)	(2)
Asset Instrument	0.029	-0.211
	(0.140)	(0.129)
Per Capita Income	0.034	.-0.056
	(0.151)	(0.331)
Unemployment	1472.037***	1128.531
	(315.538)	(334.039)
State-Specific Time Trends	No	Yes
R-squared	0.623	0.623
N	459	459

Source: Public Plans Database. 2001 – 2009. Center for Retirement Research at Boston College, Center for State and Local Government Excellence, and National Association of State Retirement Administrators. Unfunded liabilities equal market level assets divided by the number of current year households minus actuarial liabilities divided by the number of current year households. Asset Instrument is constructed by adjusting initial asset shares by the indexed rate of return of each asset class and dividing by current year asset size. Each regression controls for year and state fixed effects, and is weighted by 2001 household size. Standard errors are clustered at the state level. * indicates significance at the 10 percent level, ** indicates significance at the 5 percent level, and *** indicates significance at the 1 percent level.

Table A-3: 2-SLS Estimates Normalized by Current Household Size

Dependent Variable: Price Housing

	(1)	(2)
Revised Unfunded	-17.314	-5.304
	(17.700)	(3.345)
Per Capita Income	14.108	9.372
	(9.170)	(5.219)
Unemployment	-6508.488	-37623.20
	(16821.39)	(15722.42)
State-Specific Time Trends	No	Yes
R-squared	0.384	0.932
N	459	459

Source: *Public Plans Database*. 2001 – 2009. Center for Retirement Research at Boston College, Center for State and Local Government Excellence, and National Association of State Retirement Administrators. Unfunded liabilities are market level assets per current year occupied household size minus actuarial liabilities per current year occupied household size. Predicted assets is constructed by adjusting initial asset shares by the indexed rate of return of each asset class and dividing by current year asset size. Each regression controls for year and state fixed effects, and is weighted by 2001 household size. Standard Errors are clustered at the state level. * indicates significance at the 10 percent level, ** indicates significance at the 5 percent level, and *** indicates significance at the 1 percent level.

Table A-4: Reduced Form with Quarterly Predicted Assets

Dependent Variable: Price of Housing

	(1)	(2)
Quarterly Predicted Assets	-17.314	-5.304
	(17.700)	(3.345)
Per Capita Income	14.108	9.372
	(9.170)	(5.219)
Unemployment	-6508.488	-37623.20
	(16821.39)	(15722.42)
State-Specific Time Trends	No	Yes
R-squared	0.384	0.932
N	459	459

Source: *Public Plans Database*. 2001 – 2009. Center for Retirement Research at Boston College, Center for State and Local Government Excellence, and National Association of State Retirement Administrators. Unfunded liabilities are market level assets per current year occupied household size minus actuarial liabilities per current year occupied household size. Asset Instrument is constructed by adjusting initial asset shares by the indexed rate of return of each asset class and dividing by current year asset size. Each regression controls for year and state fixed effects, and is weighted by 2001 household size. Standard errors are clustered at the state level. * indicates significance at the 10 percent level, ** indicates significance at the 5 percent level, and *** indicates significance at the 1 percent level.

A.3 Capitalization with Labor Distorting Taxes and Exogenous Spending

In this section, I expand the basic model in Chapter 2 to incorporate income taxes. I relax the assumption that revenue is raised using lump-sum taxes and assume that all revenue is raised using labor-distorting taxes. With the introduction of income taxes, the distortion from unfunded liabilities will come from both the distortion derived from the impact on locational choices and the distortion to the labor-leisure tradeoff. I derive a basic formula for the level of capitalization that depends on the marginal cost of public funds,[1] the elasticity of demand, the elasticity of supply, and the rate of household growth. In this simple extension, government spending per capita is exogenous. Once again, residents of a given state incur unfunded liabilities in period 0. In period 1, the unfunded liabilities are repaid and the government provides per capita public services, g_1. In period 1, both unfunded liabilities and government expenditures are financed using a flat income tax. For simplicity I will assume that there is no depreciation of the housing stock. There are two possibilities for the level of household growth in this community: either there is zero growth in the number of households or there is a positive level of household growth. U is the total level of unfunded liabilities, Q_0 is the level of housing in period 0, and Q is the level of housing in period 1. $Q(P + EV(\frac{U}{Q}))$ represents the demand for housing in period 1. P is the price of

[1] For this thesis, MEB will refer to equivalent variation marginal excess burden derived from the labor-leisure distortion.

housing in period 1, and $C(Q)$ is the marginal cost of housing. P_f represents the full cost of housing, and equals both the purchase price of housing and the unfunded liabilities that must be repaid: $P+EV(\frac{U}{Q})$. $EV(\frac{U}{Q})$ is the equivalent variation for each household associated with raising $\frac{U}{Q}+g_1$ in tax revenue per household. $EV(U)$ is the equivalent variation household of the government having to raise $\frac{U}{Q}$ per household. In order to avoid wealth effects, preferences of period 1 residents residing in state s are represented by

$$c_i + v(l_i, g_1). \tag{A.1}$$

If there is zero growth in the number of households, then demand for housing must equal the supply of housing from period 0:

$$Q(P + EV(\frac{U}{Q_0})) = Q_0. \tag{A.2}$$

Totally differentiating both sides yields:

$$(\frac{\partial P}{\partial U} + \frac{MEB(\frac{U}{Q_0})}{Q_0})Q'(P + EV(\frac{U}{Q_0})) = 0, \tag{A.3}$$

which implies

$$\frac{\partial P}{\partial U} = -\frac{MEB(\frac{U}{Q_0})}{Q_0}. \tag{A.4}$$

When there is no construction of new housing, and the elasticity of supply is perfectly inelastic, then the effect of a one dollar increase in unfunded liabilities per household is just the $1 + MEB$.

In the case in which there is a positive level of household growth and thus a positive level of construction growth, the price of housing is equal to the marginal cost of housing,

$$P = C(Q_D(P + EV(\frac{U}{Q})) - Q_0). \tag{A.5}$$

In addition, in equilibrium the level of housing demand must equal the quantity of housing:

$$Q = Q_D(P + EV(\frac{U}{Q})). \tag{A.6}$$

Plugging (A.5) into (A.6) yields:

$$Q = Q_D(C(Q - Q_0) + EV(\frac{U}{Q})). \tag{A.7}$$

By differentiating the above with respect to U, the change in the level of housing is given by

$$\frac{dQ}{dU} = Q'_D(C(Q - Q_0) + \frac{U}{Q})[C'(Q - Q_0)\frac{dQ}{dU} + (1 + MEB(\frac{U}{Q}))(-\frac{U}{Q^2}\frac{dQ}{dU} + \frac{1}{Q})]. \tag{A.8}$$

Rearranging the above yields:

$$\frac{dQ}{dU} = \frac{\frac{Q'_D(C(Q-Q_0)+EV(\frac{U}{Q}))(1+MEB(\frac{U}{Q}))}{Q}}{1 - C'(Q - Q_0)Q'_D(C(Q - Q_0) + (1 + MEB(\frac{U}{Q}))) + Q'_D(C(Q - Q_0) + (1 + MEB(\frac{U}{Q})))MEB(\frac{U}{Q})\frac{U}{Q^2}}. \tag{A.9}$$

ξ_D is the elasticity of demand with respect to the full price of housing, and ξ_S is the elasticity of supply of housing. Thus, we can rewrite the elasticities as

$$C'(Q) = \frac{1}{\xi_S}\frac{P}{Q} \tag{A.10}$$

and

$$Q'(P + \frac{U}{Q}) = (\xi_D \frac{Q}{P_f}). \tag{A.11}$$

Plugging $\frac{1}{\xi_S}\frac{P}{Q}$ and $(\xi_D \frac{Q}{P_f})$ in for $C'(Q)$ and $Q'(P + \frac{U}{Q})$ respectively and rearranging yields:

$$\frac{dQ}{dU} = \frac{\xi_S \xi_D Q(1 + MEB(\frac{U}{Q}))}{\xi_S P - \xi_D P + (\xi_S + \xi_S \xi_D(1 + MEB(\frac{U}{Q}))\frac{U}{Q}}. \tag{A.12}$$

Since the relationship between prices and quantities is given by

$$P = C(Q), \tag{A.13}$$

the level of capitalization can be written in terms of the change in the level of housing with respect to unfunded liabilities:

$$\frac{dP}{dU} = C'(Q)\frac{dQ}{dU}. \tag{A.14}$$

Plugging (A.12) into (A.14), and simplifying yields:

$$\frac{dP}{dU} = \frac{\xi_D(1 + MEB(\frac{U}{Q}))}{\xi_S Q - \xi_D Q + \frac{(\xi_S + \xi_S \xi_D (1 + MEB(\frac{U}{Q})))U}{P}}. \tag{A.15}$$

The above gives the basic level of capitalization of total unfunded liabilities on housing prices. Multiplying by Q_0 yields the level of capitalization of unfunded liabilities per base number of households on housing prices:

$$\frac{dP}{d\frac{U}{Q_0}} = \frac{\xi_D MEB(\frac{U}{Q})Q_0}{\xi_S Q - \xi_D Q + \frac{(\xi_S + \xi_S \xi_D (1 + MEB(\frac{U}{Q})))U}{P}}. \tag{A.16}$$

A.4 Capitalization with Commuting Costs

In this section, I derive the level of capitalization of unfunded liabilities for a state in which residents reside around a central business district where all employment is concentrated. Development around the central business district is concentric. Each household consists of one resident who is employed in the central business district, and the cost of commuting is constant for each resident at k dollars per unit of distance. Houses at the periphery of the area are located distance d from the central business district. Thus, the total developed area surrounding the central business district equals πd^2. Even with constant construction costs and unlimited land, the level of capitalization is positive. Since each resident has constant commuting costs, the market price of centrally located property must equal:

$$P = A + C + kd. \tag{A.17}$$

The full price of housing including the repayment of unfunded liabilities:

$$P_f = A + C + kd + \frac{U}{Q}, \tag{A.18}$$

and the demand curve for housing equals

$$Q = Q(A + C + kd + \frac{U}{Q}). \tag{A.19}$$

Differentiating the demand equation with respect to U, and re-arranging yields:

$$\frac{dQ}{dU}[1 + Q'(A + C + kd + \frac{U}{Q})\frac{U}{Q^2}] = Q'(A + C + kd + \frac{U}{Q})[k\frac{dd}{dU} + \frac{1}{Q}]. \tag{A.20}$$

The supply of housing is given by

$$Q = \pi d^2. \tag{A.21}$$

Taking the derivative of the supply equation with respect to distance,

$$\frac{dQ}{dd} = 2\pi d. \tag{A.22}$$

Plugging $\frac{dQ}{dd}$ into the equation for $\frac{dQ}{dU}$ yields

$$\frac{dQ}{dU} = \frac{dQ}{dd}\frac{dd}{dU} = \frac{dQ}{dd} = 2\pi d\frac{dd}{dU}. \tag{A.23}$$

Plugging (A.23) into (A.20) yields

$$2\pi d\frac{dd}{dU}[1 + Q'(A + C + kd + \frac{U}{Q})\frac{U}{Q^2}] = Q'(A + C + kd + \frac{U}{Q})[k\frac{dd}{dU} + \frac{1}{Q}]. \tag{A.24}$$

Rearranging yields:

$$\frac{dd}{dU} = \frac{Q'(A + C + kd + \frac{U}{Q})}{Q[2\pi d + Q'(A + C + kd + \frac{U}{Q})(2\pi d\frac{U}{Q^2} - k)]}]. \tag{A.25}$$

Substituting in $\xi_D \frac{Q}{P_f}$ for $Q_D'(P + \frac{U}{Q}A)$, the above equation can be rewritten as

$$\frac{dd}{dU} = \frac{\xi_D Q}{2\pi d(PQ + U) + \xi_D(kQ^2 - 2\pi dU)}]. \tag{A.26}$$

Since $\frac{dQ}{dU} = \frac{dQ}{dd}\frac{dd}{dU} = 2\pi d \frac{dd}{dU}$, it follows that

$$\frac{dQ}{dU} = \frac{2\pi d \xi_D Q}{2\pi d(PQ + U) - \xi_D(kQ^2 - 2\pi dU)}]. \tag{A.27}$$

From the basic arbitrage condition

$$\frac{dP}{dU} = k\frac{dd}{dU}. \tag{A.28}$$

Plugging in $\frac{dd}{dU}$ into the above equation and rearranging yields

$$\frac{dP}{dU} = k\frac{\xi_D}{2\pi dP - \xi_D kQ + (1 - \xi_D)2\pi d\frac{U}{Q})}]. \tag{A.29}$$

It follows that the level of capitalization in terms of unfunded liabilities per period 0 population is

$$\frac{dP}{\partial \frac{U}{Q_0}} = k\frac{\xi_D Q_0}{2\pi dP - \xi_D kQ + (1 - \xi_D)2\pi d\frac{U}{Q})}]. \tag{A.30}$$

For a given $\frac{U}{Q}$, the level of capitalization is decreasing in Q. If $\xi_D < -1$, then the level of capitalization is increasing in U. The average level of capitalization of existing homes will be the same as for the most centrally-located property.

A.5 Alternative Capitalization Derivation

In this section, I derive a basic formula for the level of capitalization that depends on the elasticity of demand, the elasticity of supply, and the rate of household growth. Consider a simple framework in which residents of a given state make from period 0 make expenditure and revenue decisions for period 0. These resulting fiscal decisions result in unfunded liabilities, U_1 at the beginning period 1. After U_1 unfunded liabilities are incurred, residential housing market decisions are made and housing market activity occurs. After the housing market commences, period 1 residents make their fiscal decisions. Next, period 2 begins and unfunded liabilities U_2 are realized. The housing market reopens and housing market decisions are made. Finally period 2 residents repay all remaining unfunded liabilities lump-sum. Q_t is the equilibrium level of housing in period t, and P_t is the equilibrium price of housing in period t. $C(Q_t, Q_{t-1})$ represents the marginal cost of housing in period t. $Q_{D1}(P_1 + \alpha\frac{U_2}{Q_2} - \alpha P_2) = \theta_1\Omega(P_1 + \alpha\frac{U_2}{Q_2} - \alpha P_2)$ is the demand function in period 1. $Q_{D2}(P_2 + \frac{U_2}{Q_2}) = \theta_2\Omega(P_2 + \frac{U_2}{Q_2})$ is the demand for housing in period 2. [2] For simplicity I assume that there is no depreciation of the housing stock.

[2]Alternatively, utility may depend only α outcome.

There are two possibilities for the level of household growth in this community: either there is zero growth in the number of households or there is a positive level of household growth. If there is zero growth in the number of households between $t-1$ and t, then demand for housing must equal the supply of housing from period $t-1$. Also, assume that there is complete turnover between period 1 and period 2. If period 1 individuals sell their homes at the beginning of period 2, consume the proceeds, and die, then the demand for housing in period 1 is

$$Q(P_1 - P_2). \tag{A.31}$$

Period 2 homeowners repay the unfunded liabilities, and thus have demand for housing given by

$$Q(P_2 + \frac{u_1 Q_1}{Q_2}). \tag{A.32}$$

In this section assume that the total cost of construction in period 2 is $C(Q_2, Q_1) = C(Q_2)$ In order to calculate the level of capitalization in period 1, the effect of u_1 on P_2 must be calculated. If there is zero growth in equilibrium level of housing between period 1 and period 2:

$$Q_{D2}(P_2 + \frac{u_1 Q_1}{Q_2}) = Q_1, \tag{A.33}$$

which equals

$$Q_{D2}(P_2 + u_1) = Q_1, \tag{A.34}$$

Taking the derivative with respect to u_1 yields:

$$(\frac{dP_2}{du_1} + 1)Q'_{D2}(P_2 + u_1) = 0, \tag{A.35}$$

which implies

$$\frac{dP_2}{du_1} = -1. \tag{A.36}$$

If there is positive growth in the equilibrium level of housing between period 1 and period 2, then the price of housing equals the marginal cost of production:

$$P_2 = C(Q_{D2}(P_2 + \frac{u_1 Q_1}{Q_2})) \tag{A.37}$$

Substituting the marginal cost of housing into the demand function for housing yields

$$Q_2 = Q_{D2}(C(Q_{D2}(P_2 + \frac{u_1 Q_1}{Q_2})) + \frac{u_1 Q_1}{Q_2}). \tag{A.38}$$

Differentiating the above with respect to u_1:

$$\frac{dQ_2}{du_1} = Q'_{D2}(C(Q_2) + \frac{u_1 Q_1}{Q_2})[C'(Q_2)\frac{dQ_2}{du_1} + \frac{u_1}{Q_2}\frac{dQ_1}{du_1} - \frac{u_1 Q_1}{Q_2^2}\frac{dQ_2}{du_1} + \frac{Q_1}{Q_2}], \tag{A.39}$$

and simplifying yields

$$\frac{dQ_2}{du_1} = \frac{Q'_{D2}(P_f)[\frac{u_1}{Q_2}\frac{dQ_1}{du_1} + \frac{Q_1}{Q_2}]}{[1 - C'(Q_2)Q'_{D2}(P_f) + Q'_{D2}(P_f)\frac{u_1 Q_1}{Q_2^2}]}. \tag{A.40}$$

Substituting in the elasticity of demand in period 2, $\xi_{D2} = Q'_{D2}(P_f)\frac{P}{Q}$ and the elasticity of supply in period 2, $\xi_{S2} = \frac{1}{C'(Q_2)}\frac{P}{Q}$ and simplifying, it follows that

$$\frac{dQ_2}{du_1} = \frac{\xi_{D2}\frac{1}{P_2}[u_1\frac{dQ_1}{du_1} + Q_1]}{[1 - \frac{1}{\xi_{S2}}\xi_{D2} + \xi_{D2}\frac{u_1 Q_1}{P_2 Q_2}]}. \tag{A.41}$$

Since

$$\frac{dP_2}{du_1} = C'(Q_2)\frac{dQ_2}{du_1}, \tag{A.42}$$

then plugging in $\frac{dQ_2}{du_1}$ for the above yields

$$\frac{dP_2}{du_1} = \frac{1}{\xi_{S2}}\frac{P_2}{Q_2}\frac{\xi_{D2}\frac{1}{P_2}[u_1\frac{dQ_1}{du_1} + Q_1]}{[1 - \frac{1}{\xi_{S2}}\xi_{D2} + \xi_{D2}\frac{u_1 Q_1}{P_2 Q_2}]}. \tag{A.43}$$

Tautologically,

$$\frac{dQ_1}{du_1} = \frac{\frac{dP_1}{du_1}}{C'(Q_1)}, \tag{A.44}$$

substituting in $\frac{dQ_2}{du_1} = \frac{1}{C'(Q_1)}\frac{P_1}{Q_1}$ and rearranging:

$$\frac{dQ_1}{du_1} = \frac{\xi_{S1} Q_1 \frac{dP_1}{du_1}}{P_1}. \tag{A.45}$$

Plugging $\frac{dQ_1}{du_1}$ into equation (A.44) and rearranging yiedls

$$\frac{dP_2}{du_1} = \frac{\xi_{D2}\xi_{S1}u_1\frac{Q_1\frac{dP_1}{du_1}}{P_1}}{Q_2\xi_{S2} - Q_2\xi_{D2} + \xi_{D2}\xi_{S2}\frac{u_1 Q_1}{P_2}} + \frac{\xi_{D2}Q_1}{Q_2\xi_{S2} - Q_2\xi_{D2} + \xi_{D2}\xi_{S2}\frac{u_1 Q_1}{P_2}}. \tag{A.46}$$

In equilibrium, the price of housing in period 1 is

$$P_1 = C(Q_1(P_1 - P_2)). \tag{A.47}$$

Differentiating P_1 with respect to u_1 yields

$$\frac{dP_1}{du_1} = C'(Q_1(P_1 - P_2))Q_1'(P_1 - P_2)[\frac{dP_1}{du_1} - \frac{dP_2}{du_1}]. \tag{A.48}$$

Plugging $C'(Q_2, Q_1) = \frac{1}{\xi_{S1}}\frac{P_1}{Q_1}$ and $Q_{D1}'(P_1 - P_2) = \xi_{D1}\frac{Q_1}{P_1}$ into the above equation yields

$$\frac{dP_1}{du_1} = \frac{\xi_{D1}}{\xi_{S1}}[\frac{dP_1}{du_1} - \frac{dP_2}{du_1}]. \tag{A.49}$$

Substituting $\frac{dP_2}{du_1}$ into the above, yields:

$$\frac{dP_1}{du_1} = \frac{\xi_{D1}}{\xi_{S1}}[\frac{dP_1}{du_1} - \frac{\xi_{D2}\xi_{S1}u_1\frac{Q_1\frac{dP_1}{du_1}}{P_1}}{Q_2\xi_{S2} - Q_2\xi_{D2} + \xi_{D2}\xi_{S2}\frac{u_1Q_1}{P_2}} - \frac{Q_1\xi_{D2}}{Q_2\xi_{S2} - Q_2\xi_{D2} + \xi_{D2}\xi_{S2}\frac{u_1Q_1}{P_2}}]. \tag{A.50}$$

Rearranging yields:

$$\frac{dP_1}{du_1} = \frac{-\xi_{D1}\xi_{D2}}{(\xi_{S1} - \xi_{D1})(\frac{Q_2}{Q_1}\xi_{S2} - \frac{Q_2}{Q_1}\xi_{D2} + \xi_{D2}\xi_{S2}\frac{u_1}{P_2}) + \frac{\xi_{D1}\xi_{D2}\xi_{S1}u_1}{P_1}}. \tag{A.51}$$

The level of capitalization is decreasing in housing demand growth: θ_2. If ξ_{S1}, ξ_{S2}, ξ_{D1}, and ξ_{D2} are constant then, the change in the level of capitalization with respect to a change in housing demand in period 2 is

$$\frac{\partial^2 P_1}{\partial u_1 \partial \theta_2} = \frac{-\xi_{D1}\xi_{D2}}{(\xi_{S1} - \xi_{D1})(\frac{Q_2}{Q_1}\xi_{S2} - \frac{Q_2}{Q_1}\xi_{D2} + \xi_{D2}\xi_{S2}\frac{u_1}{P_2}) + \frac{\xi_{D1}\xi_{D2}\xi_{S1}u_1}{P_1}}. \tag{A.52}$$

APPENDIX B

CHAPTER 2 OF APPENDIX

B.1 Alternative Proofs and Welfare Analysis for Chapter 3

B.1.1 Alternative Proof of Proposition 1

In the absence of tax credit for contributions to the public good, the individual specifically solves the following maximization problem:

$$\begin{aligned}
\max_{c_i, x_i, t_i} \quad & u(c_i, x_G + \sum_{j \neq i}^{N} x_j + x_i) \\
\text{s.t.} \quad & c_i + t_i + x_i + \Phi(T - t_i) \leq Y \\
& - c_i \leq 0 \\
& - x_i \leq 0 \\
& - t_i \leq 0 \\
& - (T - t_i) \leq 0
\end{aligned} \tag{B.1}$$

Taking the first order conditions:

$$u_1(c_i, x_G + \sum_{j \neq i}^{N} x_j + x_i) = \lambda_1 - \lambda_2 \tag{B.2}$$

$$u_2(c_i, x_G + \sum_{j \neq i}^{N} x_j + x_i) = \lambda_1 - \lambda_3 \tag{B.3}$$

$$\lambda_1[1 - \Phi'(T - t_i)] - \lambda_4 + \lambda_5 = 0 \tag{B.4}$$

$$\lambda_1[Y - c_i - t_i - x_i - \Phi(T - t_i)] = 0 \tag{B.5}$$

$$\lambda_2[-c_i] = 0 \tag{B.6}$$

$$\lambda_3[-x_i] = 0 \tag{B.7}$$

$$\lambda_4[-t_i] = 0 \tag{B.8}$$

$$\lambda_5[-(T - t_i)] = 0 \tag{B.9}$$

Assume $T = t_i$ and consider equation (B.4). If $T = t$, then (B.4) implies $\lambda_1[1 - \Phi'(0)] = \lambda_4 - \lambda_5$, which would require λ_4 to be positive. However by assumption, t_i is positive and thus by (B.8), λ_4 must be zero. Thus it cannot be the case that $T = t_i$. ■

B.1.2 Alternative Proof of Proposition 2

Proposition 2

If $\Phi'(0) < 1$, then tax avoidance can be eliminated and the Pareto-efficient level of the public good with equal treatment of individuals can be implemented by setting $\Lambda(x_i) = ((N-1)/(N\Phi'(0)))x_i$.

Proof:

Individual i specifically solves the following maximization problem:

$$\begin{aligned} \max_{c_i \geq 0, x_i \geq 0, t_i \geq 0} \quad & u(c_i, x_G + \sum_{j \neq i}^{N} x_j + x_i) \\ \text{s.t.} \quad & c_i + t_i + x_i + \Phi(T - t_i) \leq Y. \end{aligned} \tag{B.10}$$

The government can implement the first-best level of the public good, while eliminating tax avoidance by setting $\Lambda(x_i) = ((N-1)/(N\Phi'(0)))x_i$, $x_G = 0$, and $T = [\frac{N-1}{N\Phi'(0)}]\frac{x^*}{N}$. If $x_G + \sum_{j \neq i}^{N} x_j = \frac{(N-1)x^*}{N}$. At an equilibrium a taxpayer will refrain from engaging in tax avoidance if

$$u_2(c_i, x) \geq u_1(c_i, x)[1 - \Lambda'(x_i)\Phi'(0)], \tag{B.11}$$

which simplifies to

$$\Lambda'(x_i) \geq \frac{1 - \frac{u_2(c_i, x)}{u_1(c_i, x)}}{\Phi'(0)}. \tag{B.12}$$

If $x_G + \sum_{j \neq i}^{N} x_j = \frac{(N-1)x^*}{N}$, and the optimal is x^*, then

$$\Lambda'(x_i) \geq \frac{1 - \frac{1}{N}}{\Phi'(0)}, \tag{B.13}$$

which equals

$$\Lambda'(x_i) \geq \frac{N-1}{N\Phi'(0)}. \tag{B.14}$$

Thus if taxpayer i faces $\Lambda(x_i) = ((N-1)/(N\Phi'(0)))x_i$, $x_G = 0$, $T = [\frac{N-1}{N\Phi'(0)}]\frac{x^*}{N}$, and $x_G + \sum_{j\neq i}^{N} x_j = \frac{(N-1)x^*}{N}$, then she will contribute $x_i = \frac{x^*}{N}$ to the public good, receive a tax credit equal to T, pay $t = 0$ in tax liability, and consume $c_i = Y - \frac{x^*}{N}$. ■

B.1.3 Alternative Proof of Proposition 3

Proposition 3

If $\Phi'(0) < 1$, then for a given level of the public good $\bar{x}$, tax avoidance can be eliminated and public good level $\bar{x}$ implemented by setting $\Lambda(x_i) = \frac{u_{k1}(Y-\frac{\bar{x}}{N},\bar{x}) - u_{k2}(Y-\frac{\bar{x}}{N},\bar{x})}{u_{k1}(Y-\frac{\bar{x}}{N},\bar{x})\Phi'(0)} x_i$.

Proof:

Let the government set $\Lambda(x_i) = \frac{u_{k1}(Y-\frac{\bar{x}}{N},\bar{x}) - u_{k2}(Y-\frac{\bar{x}}{N},\bar{x})}{u_{k1}(Y-\frac{\bar{x}}{N},\bar{x})\Phi'(0)} x_i$, $x_G = 0$, and $T = \frac{u_{k1}(Y-\frac{\bar{x}}{N},\bar{x}) - u_2(Y-\frac{\bar{x}}{N},\bar{x})}{u_{k1}(Y-\frac{\bar{x}}{N},\bar{x})\Phi'(0)} \frac{\bar{x}}{N}$.

Each individual i specifically solves the following maximization problem:

$$\begin{aligned}
\max_{c_i, x_i, t_i} \quad & u_i(c_i, x_G + \sum_{j \neq i}^{N} x_j + x_i) \\
\text{s.t.} \quad & c_i + t_i + x_i + \Phi(T - \Lambda(x_i) - t_i) \leq Y \\
& - c_i \leq 0 \\
& - x_i \leq 0 \\
& - t_i \leq 0.
\end{aligned} \tag{B.15}$$

Taking the first order conditions yield:

$$u_{i1}(c_i, \theta_i(x_G + \sum_{j \neq i}^{N} x_j + x_i)) = \lambda_1 - \lambda_2, \tag{B.16}$$

$$u_{i2}(c_i, x_G + \sum_{j \neq i}^{N} x_j + x_i) = \lambda_1[1 - \Lambda'(x_i)\Phi'(T - \Lambda(x_i) - t_i)] - \lambda_3, \tag{B.17}$$

$$\lambda_1[1 - \Phi'(T - \Lambda(x_i) - t_i)] - \lambda_4 = 0, \tag{B.18}$$

$$\lambda_1[Y - c_i - t_i - x_i - \Phi(T - \Lambda(x_i) - t_i)] = 0, \tag{B.19}$$

$$\lambda_2[-c_i] = 0, \tag{B.20}$$

$$\lambda_3[-x_i] = 0, \tag{B.21}$$

and

$$\lambda_4[-t_i] = 0. \tag{B.22}$$

If $x_G + \sum_{j\neq i}^N x_j = \frac{(N-1)\bar{x}}{N}$, then the solution to each individual i's problem is $(c_i, x_i, t_i) = (Y - \frac{\bar{x}}{N}, \frac{\bar{x}}{N}, 0)$. The corresponding values of the K-T multipliers are $\lambda_1 = u_{i1}(Y - \frac{\bar{x}}{N}, \frac{\bar{x}}{N})$, $\lambda_2 = 0$, $\lambda_3 = 0$, and $\lambda_4 = u_1(Y - \frac{\bar{x}}{N}, \frac{\bar{x}}{N})[1 - \Phi'(0)]$. ■

B.1.4 Alternative Proof of Proposition 4

Proposition 4

If there is heterogeneity in income and $\Phi'(0) < 1$, then tax avoidance can be eliminated and the public good level $\bar{x}$ implemented with contributions $(z_i)_{i=1}^N$ by setting $\Lambda(x_i) = \frac{u_1(Y_o - z_o, \bar{x}) - u_2(Y_o - z_o, \bar{x})}{u_1(Y_o - z_o, \bar{x})\Phi'(0)} x_i$.

Proof:

Let the government set $\Lambda(x_i) = \frac{u_1(Y_o - T_o^*, x^*) - u_2(Y_o - T_o^*, x^*)}{u_1(Y_o - T_o^*, x^*)\Phi'(0)} x_i$, $x_G = 0$, and $T_i = \frac{u_1(Y_o - T_o^*, x^*) - u_2(Y_o - T_o^*, x^*)}{u_1(Y_o - T_o^*, x^*)\Phi'(0)} T_i^*$.

Each individual i specifically solves the following maximization problem:

$$\begin{aligned}
\max_{c_i, x_i, t_i} \quad & u(c_i, x_G + \sum_{j\neq i}^N x_j + x_i) \\
\text{s.t.} \quad & c_i + t_i + x_i + \Phi(T - \Lambda(x_i) - t_i) \leq Y \\
& -c_i \leq 0 \\
& -x_i \leq 0 \\
& -t_i \leq 0.
\end{aligned} \tag{B.23}$$

Taking the first-order conditions yield:

$$u_1(c_i, x_G + \sum_{j \neq i}^{N} x_j + x_i) = \lambda_1 - \lambda_2, \tag{B.24}$$

$$u_2(c_i, x_G + \sum_{j \neq i}^{N} x_j + x_i) = \lambda_1[1 - \Lambda'(x_i)\Phi'(T_i - \Lambda(x_i) - t_i)] - \lambda_3 + \lambda_5\Lambda'(x_i), \tag{B.25}$$

$$\lambda_1[1 - \Phi'(T_i - \Lambda(x_i) - t_i)] - \lambda_4, \tag{B.26}$$

$$\lambda_1[Y - c_i - t_i - x_i - \Phi(T_i - \Lambda(x_i) - t_i)] = 0, \tag{B.27}$$

$$\lambda_2[-c_i] = 0, \tag{B.28}$$

$$\lambda_3[-x_i] = 0, \tag{B.29}$$

and

$$\lambda_4[-t_i] = 0. \tag{B.30}$$

If $x_G + \sum_{j \neq i}^{N} x_j = x^* - T^*$, then the solution to the individual problem is $(c_i, x_i, t_i) = (Y - T^*, T^*, 0)$. The corresponding values of the K-T multipliers are $\lambda_1 = u_1(Y - T^*, T^*)$, $\lambda_2 = 0$, $\lambda_3 = 0$, and $\lambda_4 = u_1(Y - T^*, T^*)[1 - \Phi'(0)]$. ■

B.1.5 Alternative Proof of Proposition 5

Proposition 5

If there is heterogeneity in income and preferences, and $\Phi'(0) < 1$, then tax avoidance can be eliminated and public good level $\bar{x}$ implemented with contributions $(z_i)_{i=1}^N$ by setting $\Lambda(x_i) = \frac{u_{l1}(Y_l - z_l, \bar{x}) - u_{l2}(Y_l - z_l, \bar{x})}{u_{l1}(Y_l - z_l, \bar{x})\Phi'(0)} x_i$.

Proof:

Let the government set $\Lambda(x_i) = \frac{u_{l1}(Y_l - z_l, \bar{x}) - u_{l2}(Y_l - z_l, \bar{x})}{u_{l1}(Y_l - z_l, \bar{x})\Phi'(0)} x_i$, $x_G = 0$, and $T_i = \frac{u_{l1}(Y_l - z_l, \bar{x}) - u_{l2}(Y_l - z_l, \bar{x})}{u_{l1}(Y_l - z_l, \bar{x})\Phi'(0)} z_i$.

Each individual i specifically solves the following maximization problem:

$$\begin{aligned} \max_{c_i, x_i, t_i} \quad & u_i(c_i, x_G + \sum_{j \neq i}^{N} x_j + x_i) \\ \text{s.t.} \quad & c_i + t_i + x_i + \Phi(T - \Lambda(x_i) - t_i) \leq Y \\ & -c_i \leq 0 \\ & -x_i \leq 0 \\ & -t_i \leq 0. \end{aligned} \tag{B.31}$$

Taking the first-order conditions yield:

$$u_{i1}(c_i, x_G + \sum_{j \neq i}^{N} x_j + x_i) = \lambda_1 - \lambda_2, \tag{B.32}$$

$$u_{i2}(c_i, x_G + \sum_{j \neq i}^{N} x_j + x_i) = \lambda_1[1 - \Lambda'(x_i)\Phi'(T_i - \Lambda(x_i) - t_i)] - \lambda_3 + \lambda_5\Lambda'(x_i), \tag{B.33}$$

$$\lambda_1[1 - \Phi'(T_i - \Lambda(x_i) - t_i)] - \lambda_4, \tag{B.34}$$

$$\lambda_1[Y - c_i - t_i - x_i - \Phi(T_i - \Lambda(x_i) - t_i)] = 0, \tag{B.35}$$

$$\lambda_2[-c_i] = 0, \tag{B.36}$$

$$\lambda_3[-x_i] = 0, \tag{B.37}$$

and

$$\lambda_4[-t_i] = 0. \tag{B.38}$$

If $x_G + \sum_{j \neq i}^{N} x_j = \bar{x} - z_i$, then the solution to the individual problem is $(c_i, x_i, t_i) = (Y - z_i, z_i, 0)$. The corresponding values of the K-T multipliers are $\lambda_1 = u_{i1}(Y - z_i, z_i)$, $\lambda_2 = 0$, $\lambda_3 = 0$, and $\lambda_4 = u_{i1}(Y - z_i, z_i)[1 - \Phi'(0)]$. ■

B.1.6 Welfare Analysis with Warm-glow Preferences

I derive sufficient conditions for the utilitarian social optimum in the presence of warm-glow. Consider the following maximization problem:

$$\begin{aligned} \max_{[c_i, x_i]_{i=1}^N} \quad & u(c_k, x_G + \sum_{i \neq k}^N x_i + x_k, x_k) \\ \text{s.t.} \quad & \forall j \neq k,\ u(c_j, x_G + \sum_{i \neq j}^N x_k + x_j, x_j) \geq u_j \\ & \sum_{i=1}^N (c_i + t_i + x_i + \leq NY \end{aligned} \tag{B.39}$$

Let λ_k be the multiplier on the budget constraint, and let λ_j be the multiplier on minimum utility for individual j. The first order conditions are the following:

$$u_1(c_k, x_G + \sum_{i \neq k}^N x_i + x_k, x_k) = \lambda_k \tag{B.40}$$

$$\forall j \neq k,\ -\lambda_j u_1(c_j, x_G + \sum_{i \neq j}^N x_i + x_j, x_j) + \lambda_k = 0 \tag{B.41}$$

$$\begin{aligned} u_2(c_k, x_G + \sum_{i \neq k}^N x_i + x_k, x_k) + u_3(c_k, x_G + \sum_{i \neq k}^N x_i + x_k, x_k) = \\ \sum_{j \neq k}^N -u_2(c_j, x_G + \sum_{i \neq j}^N x_i + x_j, x_j)\lambda_j + \lambda_k \end{aligned} \tag{B.42}$$

$$\forall j \neq k,\ u_2(c_k, x_G + \sum_{i \neq k}^{N} x_i + x_k, x_k) = \sum_{n \neq k}^{N} -(\lambda_n u_2(c_n, x_G + \sum_{i \neq n}^{N} x_i + x_n, x_n)) \\ -\lambda_j u_3(c_j, x_G + \sum_{i \neq j}^{N} x_i + x_j, x_j) + \lambda_k \tag{B.43}$$

Set $\lambda_j = \lambda_k$ for $\forall j$, then combining the first and third first-order conditions equations yields:

$$u_1(c_k, x_G + \sum_{i \neq k}^{N} x_i + x_k, x_k) - u_3(c_k, x_G + \sum_{i \neq k}^{N} x_i + x_k, x_k) = N u_2(c_k, x_G + \sum_{i \neq k}^{N} x_i + x_k, x_k) \tag{B.44}$$

Given that $\lambda_j = 1$ for $\forall j$, then combining the first and second first-order conditions equations yields:

$$\forall j \neq k,\ u_1(c_j, x_G + \sum_{i \neq j}^{N} x_i + x_j, x_j) = u_1(c_k, x_G + \sum_{i \neq k}^{N} x_i + x_k, x_k) \tag{B.45}$$

Combining the third and fourth first and second first-order conditions equations with the above equation yields:

$$\forall j, \text{including } k,\ \sum_{i=1}^{N} u_2(c_j, x_G + \sum_{i \neq j}^{N} x_i + x_j, x_j) \\ = \lambda_k - u_3(c_j, x_G + \sum_{i \neq j}^{N} x_i + x_j, x_j) \tag{B.46}$$

These imply that private consumption and charitable giving must be equal for all consumers. Thus, the optimal x^{**} must satisfy $u_1(Y - \frac{x^{**}}{N}, x^{**}, \frac{x^{**}}{N}) - u_3(Y - \frac{x^{**}}{N}, x^{**}, \frac{x^{*}}{N}) = Nu_2(Y - \frac{x^{**}}{N}, x^{**}, \frac{x^{**}}{N})$. ■

B.1.7 Alternative Proof of Proposition 8

Proposition 8

If warm-glow preferences are included in social welfare and $\Phi'(0) < 1$, then tax avoidance can be eliminated and the Pareto-efficient level of the public good with equal consumption for each taxpayer implemented by setting $\Lambda(x_i) = \frac{N-1}{N\Phi'(0)}[1 - \frac{u_3(c^{**}, x^{**}, \frac{x^{**}}{N})}{u_1(c^{**}, x^{**}, \frac{x^{**}}{N})\Phi'(0)}]x_i$.

Proof:

Let the government set $\Lambda(x_i) = \Lambda(x_i) = \frac{N-1}{N\Phi'(0)}[1 - \frac{u_3(c^{**}, x^{**}, \frac{x^{**}}{N})}{u_1(c^{**}, x^{**}, \frac{x^{**}}{N})\Phi'(0)}]x_i$, and $T = \Lambda(x_i) = \frac{N-1}{N\Phi'(0)}[1 - \frac{u_3(c^{**}, x^{**}, \frac{x^{**}}{N})}{u_1(c^{**}, x^{**}, \frac{x^{**}}{N})\Phi'(0)}]\frac{x^{**}}{N}$, where x^* is the Samuelson level of the public good. More specifically, consider, x^{**} and c^{**}, in which $c^{**} = Y - \frac{x^{**}}{N}$. x^{**} is the unique value s.t. $u_1(Y - \frac{x^{**}}{N}, x^{**}, \frac{x^{**}}{N}) - u_3(Y - \frac{x^{**}}{N}, x^{**}, \frac{x^{**}}{N}) = Nu_2(Y - \frac{x^{**}}{N}, x^{**}, \frac{x^{**}}{N})$. $\forall i, (c_i, x_i) = (Y - \frac{x^{**}}{N}, \frac{x^{**}}{N})$ is the unique set that maximizes utilitarian social welfare, including warm-glow preferences. See the appendix for proof.

The individual specifically solves the following maximization problem:

$$\begin{aligned}
\max_{c_i, x_i, t_i} \quad & u(c_i, x_G + \sum_{j \neq i}^{N} x_j + x_i, x_i) \\
\text{s.t.} \quad & c_i + t_i + x_i + \Phi(T - \Lambda(x_i) - t_i) \leq Y \\
& - c_i \leq 0 \\
& - x_i \leq 0 \\
& - t_i \leq 0
\end{aligned} \tag{B.47}$$

Taking the first-order conditions:

$$u_1(c_i, x_G + \sum_{j \neq i}^{N} x_j + x_i, x_i) = \lambda_1 - \lambda_2, \tag{B.48}$$

(B.48)

$$\begin{aligned}
u_2(c_i, x_G + \sum_{j \neq i}^{N} x_j + x_i, x_i) + u_3(c_i, x_G + \sum_{i \neq j}^{N} x_j + x_i, x_i) \\
= \lambda_1[1 - \Lambda'(x_i)\Phi'(T - \Lambda(x_i) - t_i)] - \lambda_3 +
\end{aligned} \tag{B.49}$$

$$\lambda_1[1 - \Phi'(T - \Lambda(x_i) - t_i)] - \lambda_4 = 0, \tag{B.50}$$

$$\lambda_1[Y - c_i - t_i - x_i - \Phi(T - \Lambda(x_i) - t_i)] = 0, \tag{B.51}$$

$$\lambda_2[-c_i] = 0, \tag{B.52}$$

$$\lambda_3[-x_i] = 0, \tag{B.53}$$

and

$$\lambda_4[-t_i] = 0. \tag{B.54}$$

If $x_G + \sum_{j\neq i}^{N} x_j = \frac{(N-1)x^{**}}{N}$, then the solution to the individual problem is $(c_i, x_i, t_i) = (Y - \frac{x^{**}}{N}, \frac{x^{**}}{N}, 0)$. The corresponding values of the K-T multipliers are $\lambda_1 = u_1(Y - \frac{x^{**}}{N}, x^{**}, \frac{x^{**}}{N})$, $\lambda_2 = 0$, $\lambda_3 = 0$, $\lambda_4 = u_1(Y - \frac{x^{**}}{N}, x^{**}, \frac{x^{**}}{N})[1 - \Phi'(0)]$.The corresponding values of the K-T multipliers are $\lambda_1 = u_1(Y - \frac{x^{**}}{N}, x^{**}, \frac{x^{**}}{N})$, $\lambda_2 = 0$, $\lambda_3 = 0$, $\lambda_4 = u_1(Y - \frac{x^{**}}{N}, x^{**}, \frac{x^{**}}{N})[1 - \Phi'(0)]$, and $\lambda_5 = 0$). The unique Nash Equilibrium is $(c_i, x_i, t_i) = (Y - \frac{x^{**}}{N}, \frac{x^{**}}{N}, 0), \forall i$. ■

B.1.8 Alternative Proof of Proposition 9

If warm-glow preferences exist but are excluded from social welfare and $\Phi'(0) < 1$, then tax avoidance can be eliminated and the Pareto-efficient level of the public good with equal consumption for each taxpayer implemented by setting $\Lambda(x_i) = [\frac{N-1}{N\Phi'(0)} - \frac{u_3(c^*, x^*, \frac{x^*}{N})}{u_1(c^*, x^*, \frac{x^*}{N})\Phi'(0)}]x_i$.

Proof:

Let the government set $\Lambda(x_i) = [\frac{N-1}{N\Phi'(0)} - \frac{v(x_i)}{u_1(c^*, x^*, \frac{x^*}{N})\Phi'(0)}]x_i$, and $T = \Lambda(x_i) = [\frac{N-1}{N\Phi'(0)} - \frac{u_3(c^*, x^*, \frac{x^*}{N})}{u_1((c^*, x^*, \frac{x^*}{N}))\Phi'(0)}]\frac{x^*}{N}$, where x^* is the first-best level of the public good. More specifically, consider, x^* and c^* in which $c^* = Y - \frac{x^*}{N}$. x^* is the unique value s.t.

$u_1(Y - \frac{x^*}{N}, x^*, fracx^*N) = Nu_2(Y - \frac{x^*}{N}, x^*, \frac{x^*}{N}).)$ $x = x^*$ and $c_i = c^*, \forall i \in N$ is the unique allocation which maximizes utilitarian social welfare.

The individual specifically solves the following maximization problem:

$$\begin{aligned} \max_{c_i,x_i,t_i} \quad & u(c_i, x_G + \sum_{j \neq i}^{N} x_j + x_i, x_i) \\ \text{s.t.} \quad & c_i + t_i + x_i + \Phi(T - \Lambda(x_i) - t_i) \leq Y \\ & -c_i \leq 0 \\ & -x_i \leq 0 \\ & -t_i \leq 0. \end{aligned} \tag{B.55}$$

Taking the first-order conditions yield:

$$u_1(c_i, x_G + \sum_{j \neq i}^{N} x_j + x_i, x_i) = \lambda_1 - \lambda_2, \tag{B.56}$$

$$\begin{aligned} & u_2(c_i, x_G + \sum_{j \neq i}^{N} x_j + x_i, x_i) \\ = {} & \lambda_1[1 - \Lambda'(x_i)\Phi'(T - \Lambda(x_i) - t_i)] - \lambda_3, \end{aligned} \tag{B.57}$$

$$\lambda_1[1 - \Phi'(T - \Lambda(x_i) - t_i)] - \lambda_4, \tag{B.58}$$

$$\lambda_1[Y - c_i - t_i - x_i - \Phi(T - \Lambda(x_i) - t_i)] = 0, \tag{B.59}$$

$$\lambda_2[-c_i] = 0, \tag{B.60}$$

$$\lambda_3[-x_i] = 0, \tag{B.61}$$

and

$$\lambda_4[-t_i] = 0. \tag{B.62}$$

If $x_G + \sum_{j\neq i}^{N} x_j = \frac{(N-1)x^*}{N}$, then the solution to the individual problem is $(c_i, x_i, t_i) = (c^*, \frac{c^*}{N}, 0)$. The corresponding values of the K-T multipliers are $\lambda_1 = u_1(Y - \frac{x^*}{N}, x^*, \frac{x^*}{N})$, $\lambda_2 = 0$, $\lambda_3 = 0$, $\lambda_4 = u_1(Y - \frac{x^*}{N}, x^*, \frac{x^*}{N})[1 - \Phi'(0)]$, and $\lambda_5 = 0$). The corresponding values of the K-T multipliers are $\lambda_1 = u_1(Y - \frac{x^*}{N}, x^*, \frac{x^*}{N})$, $\lambda_2 = 0$, $\lambda_3 = 0$, $\lambda_4 = u_1(Y - \frac{x^*}{N}, x^*, \frac{x^*}{N})[1 - \Phi'(0)]$, and $\lambda_5 = 0$). The unique Nash Equilibrium in this game is when each individual chooses $(c_i, x_i, t_i) = (Y - \frac{x^*}{N}, \frac{x^*}{N}, 0)$). ■

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