# ESSAYS ON POLICY DYNAMICS UNDER POLITICAL FRICTIONS

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ESSAYS ON POLICY DYNAMICS UNDER POLITICAL FRICTIONS

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The successful design and implementation of macroeconomic and public policies has an important political dimension. This dissertation, which lies at the intersection of macroeconomics and political economy, focuses on understanding the dynamic fiscal and regulatory policies in the context of political conflicts and special interests.

The first essay, *Curse or Blessing? On the Welfare Consequences of Divided Government and Policy Gridlock*, studies the welfare consequences of divided government by analyzing a dynamic legislative bargaining model with endogenous status quo. By comparing a unified government where the opponent's approval is not needed with a divided government where unanimity rule applies, I show that with divided government tax policy is less responsive due to gridlock and is distorted due to dynamic strategic considerations. However, the welfare consequences of such a policy are mixed because gridlock also reduces policy fluctuations created by political turnover. In the simulated economy, I find that divided government can Pareto dominate unified government. While this phenomenon prevails at various levels of inequality and political polarization, the set of initial status quo tax rates allowing for it shrinks as inequality and political polarization rises. Moreover, as income inequality rises, on average divided government benefits the poor while hurting the rich. This is because households at different income levels trade off potential gains and losses from policy gridlock differently.

The second essay, *Political-Driven Financial Regulatory Cycle*, develops a positive theory of political-driven financial regulatory cycle. The key feature of the model is that the regulatory policy is determined through the interaction of financial sector special interest group, politicians competing for office, and households with time-varying attention on financial regulation. I find that in absence of the special interest group, politicians maximize the utility of households and implement stringent regulation. Once the special interest group is introduced, the politicians are induced to behave as if they were maximizing the weighted sum of utilities of the financial industry and strategic households. In symmetric equilibrium, politicians' policies converge and they choose the regulation such that the electoral loss due to weakened support from households equals the electoral gain created by campaign contribution. Moreover, the equilibrium financial regulation turns out to be pro-cyclical. During financial market expansions, the financial regulation remains largely ignored by the general public. Hence the policymaker cater to the financial interest group and promote loose regulation. Once financial crisis takes place, the public attention on regulation is brought up. For fear of upsetting the voters, the politician is forced to tighten the regulation.

The third essay, *Evaluating Durable Public Good Provision using Housing Prices*, is collaborated work with professor Stephen Coate. Recent empirical work in public finance uses the housing price response to public investments to assess the efficiency of local durable public good provision. This paper investigates the theoretical foundations for this technique. In the context of a novel theoretical model developed to study the issue, it shows that there is limited justification for the technique when a budget-maximizing bureaucrat interacts with rational, forward-looking citizens. A special case in which the bureaucrat faces no voting uncertainty is solved in closed form to show why the technique can falsely predict under-provision. In the generalized model which involves randomness of voting outcomes, we show numerically that the technique may falsely predict both under-provision and over-provision of local durable public good. The technique is valid, however, when citizens have adaptive expectations, believing that whatever provision level that currently prevails will be maintained indefinitely.

## BIOGRAPHICAL SKETCH

Yanlei Ma received her B.A. in Economics in July 2009 from Zhejiang University, China. After graduation, she studied in Msc. Finance and Economics(Research) program at London School of Economics and Political science and graduated with distinction in July 2010. In August 2010, she began her graduate studies in Department of Economics at Cornell University with Sage Fellowship. She received her M.A. in Economics in January 2014 and will receive her Doctor of Philosophy Degree in May 2015.

To my beloved parents Jian Lei and Xinren Ma

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CHAPTER 1

# CURSE OR BLESSING? ON THE WELFARE CONSEQUENCES OF DIVIDED GOVERNMENT AND POLICY GRIDLOCK

## 1.1 Introduction

In the United States, divided government, a situation in which one party controls the White House and the other controls one or both houses of the Congress, has become the political norm rather than the exception. While divided government was rare in the early twentieth century, since the 1970s it has become increasingly common[1]. At the same time, the simultaneous surge of domestic income inequality and political polarization makes it more difficult for lawmakers to find common ground[2]. As a result, coupled with divided government, we see a significant increase in legislative gridlock in recent decades.

Critics have long blamed divided government for slowing down the lawmaking process and exacerbating legislative failures. Yet despite discussions of whether and how divided party control affects legislative performance, there has been no formal and systematic study of its consequences for welfare. Various questions remain unaddressed. Does divided government and its resulting gridlock necessarily make households worse off? How does the impact of divided government vary with recent rising political polarization and income

[1]Since 1900 there have been 48 years in total when the Democrats and Republicans shared control of the White House and Congress. However, 22 of those years of divided government occured within the past 30 years. Unified partisan control of the presidency and both houses of Congress has only taken place less than one third of the time.

[2]It has been widely accepted among political scientists that since the late 1970s there has been a substantial rise in both polarization and inequality persists to this day. For instance, McCarty et al. (2006) documents this trend by measuring political polarization through the ideological distance between average Republican and Democrat in both houses of the Congress using roll call voting records.

inequality? What are the distributional implications of divided government for households at opposing ends of the income distribution?

This paper takes a step towards answering these questions by studying a dynamic legislative bargaining model with an endogenous status quo. By comparing a unified government, where policies simply follow the will of the party with proposal power, with a divided government, where the opposition provides checks and balances, I show that tax policy under divided government is not sufficiently responsive to shocks due to policy gridlock and is distorted due to dynamic strategic considerations. However, the welfare consequences of divided government are mixed. Divided government can Pareto dominate unified government. As inequality and polarization rise, the chance that at least one class of household is better off under unified government increases. Moreover, on average, divided government benefits the poor and hurts the rich.

The model consists of two types of households - rich and poor - differentiated by their income generating abilities. Households consume through labor income and value public goods. There are two political parties, each favoring one type of household. In every time period, one party is randomly selected to propose a linear labor income tax rate to finance public good provisions. For an economy with a unified government, the party with proposal power simply chooses its desired tax rate regardless of the opponent's policy preference. With a divided government, unanimous agreement from the two political parties is required to change the policy. Similar to Romer and Rosenthal (1978a), the party with proposal power confronts the opposing party with a 'take-it-or-leave-it' offer. If the opposing party accepts the proposal, tax policy is immediately implemented as proposed. Otherwise, tax policy remains at the status quo[3]. Differently from Romer and Rosenthal (1978a), however, the policy setter can control the status quo since tax choice in the current period becomes the status quo in the subsequent period. My approach therefore closely follows the dynamic legislative bargaining framework used by Piguillem and Riboni (2012) and Bowen et al. (2012).

My analysis of the model begins with equilibrium tax policy behavior. With unified government, tax policy is independent of the status quo tax rate and responds to shocks regardless of their magnitudes. By contrast, with divided government, policymakers may be gridlocked and fail to reach any consensus on potential policy changes unless the shock magnitude is sufficiently large. Essentially, in this model large economic shocks serve as opportunities for policymakers to convince the opposing party to push policies out of the status quo. This is reminiscent of Drazen and Grilli (1993), who argue that economic crises can accelerate agreement and hence speed up the pace of policy reforms. It also explains the lack of policy response to economic shocks under divided government. Moreover, I find in contrast to unified government, which implements the policy proposer's static ideal tax level, divided government drives the equilibrium tax rate away from such level due to dynamic strategic considerations. By forgoing the current static ideal tax rate, the party with proposal power ensures itself a favorable status quo in the next period and enhanced expected utility in case of future gridlock and political turnover. Essentially, as compared to

[3]The relevance of modeling tax legislation through bargaining with an endogenous status quo can be justified by the fact that the majority of tax legislations, especially those passed after the 1970s, are designed to be permanent. Here I assume away sunset provisions which cease to have effect after a specific date without further legislative action. This is because sunset provisions are seldomly used prior to 2000. Even though they are used more frequently in recent years, they are often pushed back or permanently extended. For instance, while many tax cuts in the Economic Growth and Tax Relief Reconciliation Act of 2001 are designed to be phased out in ten years, part of them are in fact permanently extended in the American Taxpayer Relief Act of 2012.

the static optimum tax rate, the equilibrium tax rate provides the policymaker with higher utility for staying with status quo, and therefore higher bargaining power. The resulting increased disagreement and policy inefficiency is consistent with the findings in Dziuda and Loeper (2010) and Bowen et al. (2012).

A central finding of this paper is that divided government can be preferable to unified government in economies with both aggregate shocks and political power fluctuations. This is because gridlock plays a dual role. On the one hand, it *decreases* welfare by reducing policy responsiveness and creating strategic distortions; on the other hand, it *enhances* welfare by counteracting policy uncertainty generated from exogenous political turnover. In cases where the latter effect dominates the former, divided government Pareto dominates unified government. To further understand these two effects, I disentangle them through an examination of two auxiliary economies. By shutting down exogenous political turnover, I show that in a dynamic setting with only aggregate shocks, while unified government is efficient, divided government turns out to be inefficient due to failure of timely shock response and strategic calculation of future bargaining power. By shutting down aggregate productivity shocks, I highlight that divided government can fully restore efficiency since gridlock removes policy fluctuations. In contrast, unified government is inefficient because of excess policy volatility.

This result sheds new light on the classic work of Wicksell (1896) and Buchanan and Tullock (1962), who argue that unanimity rule guarantees more efficient outcomes than other voting rules in the case of no decision costs. In my model, divided government corresponds to an economy with unanimity rule because consent from both parties is needed to implement proposed policies.

Unified government, where the opponent's approval is not necessary, corresponds to an economy with random dictatorship of the majority. In addition to confirming that unanimity rule can still be more efficient than the dictatorship of the majority in a dynamic setting with both economic and political uncertainty, my paper also shows that under such setting, the desirability of unanimity rule arises from a new channel that is not discussed in these early works; namely, gridlock counteracts policy volatility. Furthermore, such desirability depends on the initial status quo tax policy.

A further result of the paper points to the role of rising inequality and increased political polarization in creating gridlock and affecting welfare. As inequality and polarization increase, average gridlock duration lengthens as a result of increasingly distant ideal policies of the political parties. More importantly, at low inequality and polarization, divided government dominates unified government regardless of the initial status quo tax rate. At high inequality and polarization levels, however, I find a shrinkage in the set of initial status quo tax rates under which both households strictly benefit from divided government. In other words, it becomes increasingly likely that at least one household benefits from unified government. This is because at low inequality and polarization, moderate policy gridlock due to divided government can reduce policy uncertainty while keeping the welfare loss caused by gridlock relatively low. At high inequality and polarization, severe gridlock around the policy proposer's desired tax level causes significant welfare loss to the opponent, which outweighs the welfare gain delivered by reduced policy uncertainty. In such a case, both households are better off with divided government only at status quo tax rates around the average of their desired tax policies.

In terms of distributional consequences, I find households at different income levels tradeoff the welfare-reducing aspect and welfare-improving aspect of policy gridlock differently. While the rich mainly suffer from large income losses, the poor mainly benefit from the policy stability. On average, with divided government and a high inequality level, rich households experience rising welfare losses while poor households experience increasing welfare gains.

***Related Literature*** This paper brings together literature on divided government and legislative performance, dynamic legislative bargaining with an endogenous status quo and dynamic efficiency of political power fluctuations.

My paper contributes to the political science literature on legislative performance under divided government in several aspects. Not only do I provide a formal dynamic framework characterizing how gridlock arises endogenously with divided government, but I also evaluate the desirability of divided government and resulting gridlock through systematic welfare analysis. The conventional wisdom of American politics associates divided government with policy gridlock and low legislative productivity (e.g. Schattschneider (1942), Sundquist (1988), Cox and Kernell (1991)). Since the seminal empirical work of Mayhew (1991), which argues divided government has not resulted in significantly different policy outcomes than unified government, a large literature on the effect of divided government has emerged, including both alternative empirical measures of legislative productivity (e.g. Binder (1999), Kelly (1993), Coleman (1999)) and theoretical models of legislative lawmaking (e.g. Krehbiel (1998)). However, all these works focus only on policy performance and ignore the welfare consequences of divided government. My paper points out that despite the fact that divided government can generate gridlock and reduce legislative productivity, it may still be preferable to unified government from a welfare perspective due to its ability to improve policy stability. This finding also partly justifies the present prevalence of divided government in American politics. Hence even though my paper abstracts away from endogenous constitutional choice decisions, it still complements Fiorina (1992) and Alesina and Rosenthal (1995), which interpret divided government as a natural outcome of voters' rational use of checks and balances to seek moderate policy outcomes.

From a methodology point of view, this paper builds on and extends the rapidly growing literature on legislative bargaining with an endogenous status quo, notably works by Baron (1996), Kalandrakis (2004), Diermeier and Fong (2011), Dziuda and Loeper (2010), Bowen et al. (2012), and Piguillem and Riboni (2012). In contrast to legislative bargaining models where the default option in case of disagreement is not explicitly dependent on previous policies (e.g. Battaglini and Coate (2008) and Barseghyan et al. (2013)), models with endogenous status quo assume that the default option evolves with the policy choice of the previous period. Three recent applications of such a framework that are closely related to this paper include Bowen et al. (2012), Bowen et al. (2014) and Piguillem and Riboni (2012). While Piguillem and Riboni (2012) applies bargaining with endogenous status quo to analyze linear capital taxation, Bowen et al. (2012) and Bowen et al. (2014) focus on mandatory programs and public good provisions.

As far as I know, this paper is the first to use legislative bargaining with an endogenous status quo framework to model the policy making process of a divided government. Such an application is highly relevant because it captures the conflicts and checks and balances between political parties in American politics. Similar to the above papers, in my model, over moderate status quo tax rates, the equilibrium policy stays same as in the previous period, which is the standard result in the take-it-or-leave-it bargaining game. However, since my model introduces aggregate economic shocks in addition to political turnover, I am able to provide a formal explanation for the potential lack of fiscal responses to exogenous shocks under divided government. Moreover, in addition to the insurance motive against power fluctuation as characterized in Bowen et al. (2012), the policymaker in my model has an extra incentive to push the equilibrium policy away from the static ideal level. He or she also takes into account the potential shift of 'gridlock zone' caused by the aggregate economic shocks. In terms of welfare, while I find that divided government can Pareto dominate unified government, Bowen et al. (2012) and Bowen et al. (2014) find that mandatory programs with appropriate flexibility can deliver efficiency. Essentially, both our findings point to the desirability of policies determined by legislative bargaining with an endogenous status quo. However, I further characterize how the relative efficiency of divided government shifts with recent rising inequality and polarization. Moreover, my paper addresses the distributional consequences for households with different income generating ability.

More generally, this paper also relates to the literature on the efficiency of political turnover. The main argument of this literature is that political turnover with no commitment of the policymaker can lead to inefficiency. For instance, Alesina and Tabellini (1990) show that the incumbent overspends on public goods and this leads to over accumulation of public debt. Azzimonti (2011) finds that disagreement and polarization between political parties leads to underinvestment in productive public capital, which slows down the growth of the economy. In line with the above findings, the political turnover in my model leads to excessively volatile policies and creates inefficiencies for both unified and divided government. Under divided government, however, the policy uncertainty created by political turnover is partially counteracted by policy gridlock, which makes households better off than with unified government.

The remainder of the paper is organized as follows. Section 2 outlines the model environment. Section 3 establishes the social planner benchmark. Section 4 compares how policies are determined under unified and divided government. Section 5 parameterizes the model and analyzes the equilibrium policy dynamics. Section 6 compares the welfare consequences of unified and divided government, and highlights the dual role of policy gridlock. Section 7 examines how welfare consequences change with rising inequality and political polarization. Section 8 concludes.

## 1.2 Model Environment

In this section I lay out the economic and political environment of the model. Then I characterize the competitive equilibrium for arbitrary government policies.

### 1.2.1 Households

Time is discrete and infinite horizon, indexed by $t = 0, 1, 2, \cdots$. The economy is populated by two types of infinitely lived households – rich and poor, indexed by $i \in \{r, p\}$. The households differ in their income generating abilities $\bar{\theta}_i$, where

$\bar{\theta}_r > \bar{\theta}_p$. The size of each type of household is normalized to 1. They consume a consumption good $c_t^i$, supply labor $l_t^i$ and benefit from public good $g_t$. Household $i$'s periodic utility is

$$u(c_t^i, l_t^i, g_t) = \frac{1}{1-\gamma}\left(c_t^i - \frac{{l_t^i}^{1+\nu}}{1+\nu}\right)^{1-\gamma} + A\frac{g_t^{1-\xi}}{1-\xi} \tag{1.1}$$

where $\gamma > 0, \nu > 0, A > 0$ and $0 < \xi < 1$. This implies $u(c_t^i, l_t^i, g_t)$ is increasing in $c_t^i$, decreasing in $l_t^i$, increasing in $g_t$, and strictly concave.

There is an economy-wide productivity shock $\epsilon_t$, which follows a log-normal distribution

$$log\epsilon_t = \rho log\epsilon_{t-1} + u_t, \quad u_t \sim N(0, \sigma^2) \tag{1.2}$$

Each period upon realization of the shock $\epsilon_t$, household $i$ supplies $l_t^i$ units of labor and produces $\epsilon_t \bar{\theta}_i l_t^i$ units of private goods. Each unit of private good can be transformed into one unit of public good through a linear technology.

### 1.2.2 Government

The government maintains a balanced budget each period. It finances a public good level $g_t$ through revenues from a linear labor income tax $\tau_t$ imposed on both rich and poor households:

$$g_t = \tau_t(\epsilon_t \bar{\theta}_r l_t^r + \epsilon_t \bar{\theta}_p l_t^p) \tag{1.3}$$

For an arbitrary linear tax $\tau_t$ imposed on labor income $\epsilon_t \bar{\theta}_i l_t^i$, household $i$'s budget constraint is

$$c_t^i = (1 - \tau_t)\epsilon_t \bar{\theta}_i l_t^i \tag{1.4}$$

In this economy, I rule out any borrowing or lending between different types of households as well as between the household and the government in order to isolate the role of the tax rate as the endogenous status quo. Later it will become clear that the tax rate constitutes the only endogenous dynamic link between different time periods.

### 1.2.3 The Political System

The political system of the economy consists of two parties labeled by $j \in \{R, P\}$, each seeking to maximize a weighted sum of households' life-time utility. The periodic utility of each party is given by

$$U^j(\tau_t, g_t, \epsilon_t) = \alpha^j u^r(\tau_t, g_t, \epsilon_t) + (1 - \alpha^j) u^p(\tau_t, g_t, \epsilon_t) \tag{1.5}$$

Here $u^i(\tau_t, g_t, \epsilon_t)$ represents the indirect utility of household $i$ as a function of the government's policy as well as the exogenous shock. The weight party $j$ places on the rich households is $\alpha^j$ and the weight party $j$ places on the poor households is $1 - \alpha^j$. The two political parties are distinguished by their objectives. Party R places more weight on the rich while party P places more weight on the poor, which implies that $\alpha^R$ and $1 - \alpha^P$ exceed 0.5. Moreover, the two parties' objectives are symmetric so that $\alpha^R = 1 - \alpha^P$.

In this model, I abstract from the election process and assume that the political turnover is governed by an exogenous 2-state Markov process. At the beginning of each period, one party is randomly assigned proposal power. With probability $q$, the current policy proposer retains power in the next period; with probability $1-q$, the opponent obtains proposal power.

### 1.2.4 Competitive Equilibrium

Here I define and characterize the competitive equilibrium allocations for arbitrary government policies. In subsequent sections, I will discuss how policies are set under different political institutions based on household's equilibrium behavior as described below.

**Definition 1.** *A competitive equilibrium is an allocation* $\{c_t^i, l_t^i\}_{i\in\{r,p\}}$*, a price system* $\{w_t^i = \epsilon_t \bar{\theta}_i\}_{i\in\{r,p\}}$ *and fiscal policies* $\{\tau_t, g_t\}$ *such that:*

*1. Given the price system and the government's policies, the allocation for each household maximizes its expected discounted life-time utility subject to its budget constraint;*

*2. Given the price system and the allocation, the government maintains a balanced budget each period;*

*3. The goods market clears, i.e.* $c_t^r + c_t^p + g_t = \epsilon_t \bar{\theta}_r l_t^r + \epsilon_t \bar{\theta}_p l_t^p$.

Since households are assumed not to have access to any borrowing or lending facilities, their optimal consumption and labor supply are determined statically within each time period. Maximizing periodic utility of household $i$ subject to its budget constraint (1.4), the equilibrium consumption $C^i(\tau_t, \epsilon_t)$ and labor supply $L^i(\tau_t, \epsilon_t)$ can be characterized as:

$$C^i(\tau_t, \epsilon_t) = \left[(1-\tau_t)\bar{\theta}_i\epsilon_t\right]^{\frac{1+\nu}{\nu}} \tag{1.6}$$

$$L^i(\tau_t, \epsilon_t) = \left[(1-\tau_t)\bar{\theta}_i\epsilon_t\right]^{\frac{1}{\nu}} \tag{1.7}$$

Note that in equilibrium, both households' consumption and labor supply increase with income generating ability $\bar{\theta}_i$ and aggregate productivity shock $\epsilon_t$, and decrease with tax rate $\tau_t$. Household $i$'s periodic utility $v^i(\tau_t, \epsilon_t)$ from consuming private consumption and providing labor supply can thus be expressed as

$$v^i(\tau_t, \epsilon_t) = \frac{1}{1-\gamma}\left[C^i(\tau_t, \epsilon_t) - \frac{L^i(\tau_t, \epsilon_t)^{1+\nu}}{1+\nu}\right]^{1-\gamma} = \frac{\nu^{1-\gamma}}{(1-\gamma)(1+\nu)^{1-\gamma}}\left[(1-\tau_t)\bar{\theta}_i\epsilon_t\right]^{\frac{(1+\nu)(1-\gamma)}{\nu}} \tag{1.8}$$

Using the government budget constraint (1.3) and households' equilibrium labor supply (1.7), the equilibrium level of public good can also be obtained as a function of the tax rate and the aggregate productivity shock

$$G(\tau_t, \epsilon_t) = \tau_t\epsilon_t\bar{\theta}_r L^r(\tau_t, \epsilon_t) + \tau_t\epsilon_t\bar{\theta}_p L^p(\tau_t, \epsilon_t) = \tau_t\bar{\theta}_r\epsilon_t\left[(1-\tau_t)\bar{\theta}_r\epsilon_t\right]^{\frac{1}{\nu}} + \tau_t\bar{\theta}_p\epsilon_t\left[(1-\tau_t)\bar{\theta}_p\epsilon_t\right]^{\frac{1}{\nu}} \tag{1.9}$$

Therefore, we can rewrite the household's indirect utility defined in (1.1) as a function of $\tau_t$ and $\epsilon_t$.

$$
\begin{aligned}
u^i(\tau_t, \epsilon_t) =& v^i(\tau_t, \epsilon_t) + A\frac{G(\tau_t, \epsilon_t)^{1-\xi}}{1-\xi} \\
=& \frac{\nu^{1-\gamma}}{(1-\gamma)(1+\nu)^{1-\gamma}}\left[(1-\tau_t)\bar{\theta}_i\epsilon_t\right]^{\frac{(1+\nu)(1-\gamma)}{\nu}} + \\
& \frac{A}{1-\xi}\left\{\tau_t\bar{\theta}_r\epsilon_t\left[(1-\tau_t)\bar{\theta}_r\epsilon_t\right]^{\frac{1}{\nu}} + \tau_t\bar{\theta}_p\epsilon_t\left[(1-\tau_t)\bar{\theta}_p\epsilon_t\right]^{\frac{1}{\nu}}\right\}^{1-\xi}
\end{aligned} \tag{1.10}
$$

Based on the households' indirect utilities, we can also express the periodic utilities of political parties as a function of $(\tau_t, \epsilon_t)$. These will be denoted as $U^j(\tau_t, \epsilon_t)$, $j \in \{R, P\}$ in the subsequent analysis.

## 1.3 Social Planner Benchmark

Before analyzing policy outcomes and welfare implications under different political systems, I first characterize the tax policy determined by a benevolent social planner as a benchmark. Here the social planner chooses tax policy $\tau_t^O(\epsilon_t)$ in order to maximize the weighted sum of utilities of the households. Let the welfare weights associated with the rich and poor households be $\mu$ and $1-\mu$ respectively, with $\mu \in [0, 1]$. Let $V^O(\epsilon_t)$ denote the life-time utility of the social planner who behaves optimally at shock realization $\epsilon_t$. The social planner's problem can be formulated as follows:

$$
V^O(\epsilon_t) = \max_{\tau_t} \mu u^r(\tau_t, \epsilon_t) + (1-\mu)u^p(\tau_t, \epsilon_t) + \beta\mathbb{E}\left[V^O(\epsilon_{t+1})|\epsilon_t\right] \tag{1.11}
$$

where $u^r(\tau_t, \epsilon_t)$ and $u^p(\tau_t, \epsilon_t)$ are the indirect utilities for rich and poor households. Since tax policy at time $t$ does not affect the expected life-time utility at time

$t+1$, the social planner's problem boils down to a static optimization problem. In other words, the social planner simply chooses the tax rate $\tau_t^O(\epsilon_t)$ to maximize the weighted sum of households' periodic utilities.

The optimal tax rate determined by the social planner is characterized by (1.12), which equates the social marginal benefit of public good on the left-hand side with the cost of financing an additional unit of public good on the right-hand side. More specifically, on the right-hand side of (1.12), the first term denotes the marginal utility loss for additional unit of tax increase in the society. The second term measures the tax raise needed for financing one unit of public good. These two terms together are essentially the direct costs of providing public good. The third term, which is always greater than 1, is the marginal cost of public funds. It captures the distortionary effect created by financing public good through labor income taxation.

$$AG(\tau_t^O,\epsilon_t)^{-\xi} = \left[\mu\frac{dv^r(\tau_t^O,\epsilon_t)}{d\tau_t} + (1-\mu)\frac{dv^p(\tau_t^O,\epsilon_t)}{d\tau_t}\right]\left[\frac{1}{\epsilon_t\bar{\theta}_r L^r(\tau_t^O,\epsilon_t) + \epsilon_t\bar{\theta}_p L^p(\tau_t^O,\epsilon_t)}\right]\left[\frac{1-\tau_t^O}{1-(1+\frac{1}{\nu})\tau_t^O}\right] \tag{1.12}$$

Since there is no inter-temporal link in the optimality condition (1.12), the tax policy of the planner $\tau_t$ will be dependent only on the realization of the aggregate shock $\epsilon_t$ and independent of the previous tax rate $\tau_{t-1}$.

## 1.4 Political Determination of Fiscal Policy

In this section, I discuss and compare how policies are determined under unified and divided government. For each political system, I first describe the policy decision process and then formulate the optimization problem faced by the policymaker.

### 1.4.1 Unified Government

In this political system, the party with proposal power simply implements its desired tax rate, with no regard for the welfare of its opponent. In particular, no approval from the opposition party is needed in the proposer's policy decision-making process. Given the exogenous alternation of proposal power assumed earlier, fiscal policy fluctuates between the desired policies of the two parties.

Let $V_U^j(\epsilon_t)$ denote the life-time utility of party $j$ when it is the policy proposer and $W_U^j(\epsilon_t)$ denote the life-time utility when it is the opponent. Suppose party $j$ is the policy proposer at time $t$ and party $-j$ is the opponent. Then the equilibrium tax policy $\tau_U^j(\epsilon_t)$ implemented by party $j$ is determined through the following optimization problem:

$$V_U^j(\epsilon_t) = \max_{\tau_t} \quad U_j(\tau_t, \epsilon_t) + \beta \left\{ q\mathbb{E}\left[V_U^j(\epsilon_{t+1})|\epsilon_t\right] + (1-q)\mathbb{E}\left[W_U^j(\epsilon_{t+1})|\epsilon_t\right]\right\} \tag{1.13}$$

where

$$W_U^j(\epsilon_t) = U_j(\tau_U^{-j}(\epsilon_t), \epsilon_t) + \beta \left\{ q\mathbb{E}\left[W_U^j(\epsilon_{t+1})|\epsilon_t\right] + (1-q)\mathbb{E}\left[V_U^j(\epsilon_{t+1})|\epsilon_t\right]\right\} \tag{1.14}$$

Similar to the social planner's problem characterized earlier, the equilibrium tax rate can be obtained from an optimality condition equating social marginal benefit of public good to social cost of public funds. However, the optimality condition under unified government differs from (1.12) because the welfare weights vary with the party identity. As a result, for a given shock realization $\epsilon_t$, the equilibrium tax rates proposed by these two political parties differ from each other in general.

$$AG(\tau_U^j, \epsilon_t)^{-\xi} = \left[\alpha^j \frac{dv^r(\tau_U^j, \epsilon_t)}{d\tau_t} + (1-\alpha^j)\frac{dv^p(\tau_U^j, \epsilon_t)}{d\tau_t}\right]\left[\frac{1}{\epsilon_t\bar{\theta}_r L^r(\tau_U^j, \epsilon_t) + \epsilon_t\bar{\theta}_p L^p(\tau_U^j, \epsilon_t)}\right]\left[\frac{1-\tau_U^j}{1-(1+\frac{1}{\nu})\tau_U^j}\right] \tag{1.15}$$

Again, since the current period's tax policy does not affect the expected future life-time utility, the policy proposer's problem is essentially static. The optimality condition implies that the equilibrium tax policy $\tau_U^j(\epsilon_t)$ only depends on the shock realization $\epsilon_t$ and the identity of the policy proposer $j \in \{R, P\}$. It is independent of the status quo tax rate $\tau_{t-1}$.

### 1.4.2 Divided Government

In this political system, the political party with proposal power needs the approval of its opponent in order to change the policy. Therefore, I model the policy determination process through a dynamic take-it-or-leave-it bargaining game between the policy proposer and the opponent.

The status quo tax rate at the beginning of period t is $\tau_{t-1}$, which is the tax rate implemented in previous period. At each time period, upon realization of the policy proposer's identity and the realization of the economy-wide productivity shock $\epsilon_t$, the policy proposer makes a take-it-or-leave-it offer $\tau_t$ to the opposition party given the status quo $\tau_{t-1}$. If the opponent agrees with the proposed policy $\tau_t$, the proposal is implemented immediately as the tax policy for time period $t$; if the opponent fails to agree, the tax policy of time period $t$ stays at the status quo $\tau_{t-1}$. In contrast to the economy with unified government, the previous tax policy $\tau_{t-1}$ affects the proposer's decision by influencing its opponent's acceptance strategy.

In this paper, I focus my analysis on Markov perfect equilibria. I assume that the strategies of each economic agent are dependent on the current state only and each agent is able to correctly forecast the future proposer's tax policy. At time $t$, given the status quo tax rate $\tau_{t-1}$ and the economy-wide shock $\epsilon_t$, the strategy profile of party $j$ who has proposal power is denoted by $T_D^j(\tau_{t-1}, \epsilon_t)$ and the strategy profile of the opposition party $-j$ is denoted by $A_D^{-j}\left[\tau_{t-1}, \epsilon_t; T_D^j(\tau_{t-1}, \epsilon_t)\right] \in \{\text{accept}, \text{reject}\}$.

There are six value functions associated with the economy: $V_D^j(\tau_{t-1}, \epsilon_t)$ denotes the dynamic payoff when party $j$ is the policy proposer, and $W_D^j(\tau_{t-1}, \epsilon_t)$ the payoff when it is the opponent. Finally, $\underline{W}_D^j(\tau_{t-1}, \epsilon_t)$ denotes the dynamic payoff for the opposition party if the economy stays with the status quo tax policy $\tau_{t-1}$ at time t.

Let us start by analyzing the acceptance strategy of the opposition party $-j$. For an arbitrary tax policy proposal $\tau_t$, the opponent will accept the policy if and only if the dynamic payoff obtained from the proposed policy $\tau_t$ is at least as high as staying with the status quo $\tau_{t-1}$, i.e.

$$U^{-j}(\tau_t, \epsilon_t) + \beta\left\{q\mathbb{E}\left[W_D^{-j}(\tau_t, \epsilon_{t+1})|\epsilon_t\right] + (1-q)\mathbb{E}\left[V_D^{-j}(\tau_t, \epsilon_{t+1})|\epsilon_t\right]\right\} \geq \underline{W}_D^{-j}(\tau_{t-1}, \epsilon_t) \quad (1.16)$$

where

$$\underline{W}_D^{-j}(\tau_{t-1}, \epsilon_t) = U^{-j}(\tau_{t-1}, \epsilon_t) + \beta \left\{ q\mathbb{E}\left[ W_D^{-j}(\tau_{t-1}, \epsilon_{t+1}) | \epsilon_t \right] + (1-q)\mathbb{E}\left[ V_D^{-j}(\tau_{t-1}, \epsilon_{t+1}) | \epsilon_t \right] \right\} \tag{1.17}$$

Now consider the decision of party $j$. As the policy proposer, party $j$ chooses the policy $T_D^j(\tau_{t-1}, \epsilon_t)$ that maximizes its dynamic payoff, while taking into consideration the opposition party's strategy, i.e.

$$V_D^j(\tau_{t-1}, \epsilon_t) = \max_{\tau_t} \quad U^j(\tau_t, \epsilon_t) + \beta \left\{ q\mathbb{E}\left[ V_D^j(\tau_t, \epsilon_{t+1}) | \epsilon_t \right] + (1-q)\mathbb{E}\left[ W_D^j(\tau_t, \epsilon_{t+1}) | \epsilon_t \right] \right\} \tag{1.18}$$

$s.t.$

$$U^{-j}(\tau_t, \epsilon_t) + \beta \left\{ q\mathbb{E}\left[ W_D^{-j}(\tau_t, \epsilon_{t+1}) | \epsilon_t \right] + (1-q)\mathbb{E}\left[ V_D^{-j}(\tau_t, \epsilon_{t+1}) | \epsilon_t \right] \right\} \geq \underline{W}_D^{-j}(\tau_{t-1}, \epsilon_t)$$

where

$$\begin{aligned} W_D^j(\tau_{t-1}, \epsilon_t) =& U^j\left(T_D^{-j}(\tau_{t-1}, \epsilon_t), \epsilon_t\right) + \\ & \beta \left\{ q\mathbb{E}\left[ W_D^j\left(T_D^{-j}(\tau_{t-1}, \epsilon_t), \epsilon_{t+1}\right) | \epsilon_t \right] + (1-q)\mathbb{E}\left[ V_D^j\left(T_D^{-j}(\tau_{t-1}, \epsilon_t), \epsilon_{t+1}\right) | \epsilon_t \right] \right\} \end{aligned} \tag{1.19}$$

In this analysis, I focus on equilibria where the equilibrium tax policy proposal is always accepted. Also, the opposition party is assumed to accept any proposal to which it is indifferent between accepting and rejecting. Compared to the optimization problem under unified government, we see the policy proposer not only has to consider how the tax policy affects its current period utility, but also has to take into account the fact that the current tax rate becomes the status quo in the next period, which in turn affects the acceptance strategy of the opponent in the subsequent period.

**Definition 2.** *A* Markov Perfect Equilibrium Under Divided Government *is an allocation* $\left\{c_t^i, l_t^i, g_t\right\}_{i\in\{r,p\}}$*, a price system* $\left\{w_t^i = \epsilon_t\bar{\theta}_i\right\}_{i\in\{r,p\}}$*, a strategy profile* $\left\{T_D^j(\tau_{t-1}, \epsilon_t), A_D^j\left[\tau_{t-1}, \epsilon_t; T_D^{-j}(\tau_{t-1}, \epsilon_t)\right]\right\}_{j\in\{R,P\}}$*, and value functions* $\left\{V_D^j(\tau_{t-1}, \epsilon_t),\right.$ $W_D^j(\tau_{t-1}, \epsilon_t)$*,* $\left.\underline{W}_D^j(\tau_{t-1}, \epsilon_t)\right\}_{j\in\{R,P\}}$ *such that:*

*1. Given the equilibrium tax policy, the allocations are generated through the competitive equilibrium as defined in Definition 1;*

*2. Given the tax policy of the party with proposal power, the opponent accepts the proposal if and only if* (1.16) *is satisfied;*

*3. Given the opponent's acceptance strategy, the party with proposal power chooses the tax policy that solve the problem as described in* (1.18)*;*

*4. Equilibrium value functions are generated by equilibrium strategies, i.e. both equation* (1.18) *and* (1.19) *hold.*

In general, there is no closed-form solution for the equilibrium tax policy under divided government. Hence in subsequent sections discussions of equilibrium tax policies and the welfare will be based on numerical results. In the appendix, I also provide an analytical example with simplified assumptions to illustrate the basic intuition of policy gridlock.

## 1.5 Equilibrium Tax Policies

Before exploring the welfare consequences of the divided government, I illustrate in this section how equilibrium tax policies behave with unified and divided government. To obtain equilibrium tax policies, I parameterize the model and solve it numerically. The procedure involves solving for the fixed points as characterized by the value functions of both political parties through value function iterations. Since the properties of the objective function remains unclear, for given initial guess of value functions, I evaluate the objective at all possible tax grids and choose the one that delivers the maximum value. This generates equilibrium tax policies and new value functions. Such procedure is repeated until convergence is achieved. Table 1.1 summarizes the parameter values used for solving the model. Further details on the model parameterization are provided in the appendix.

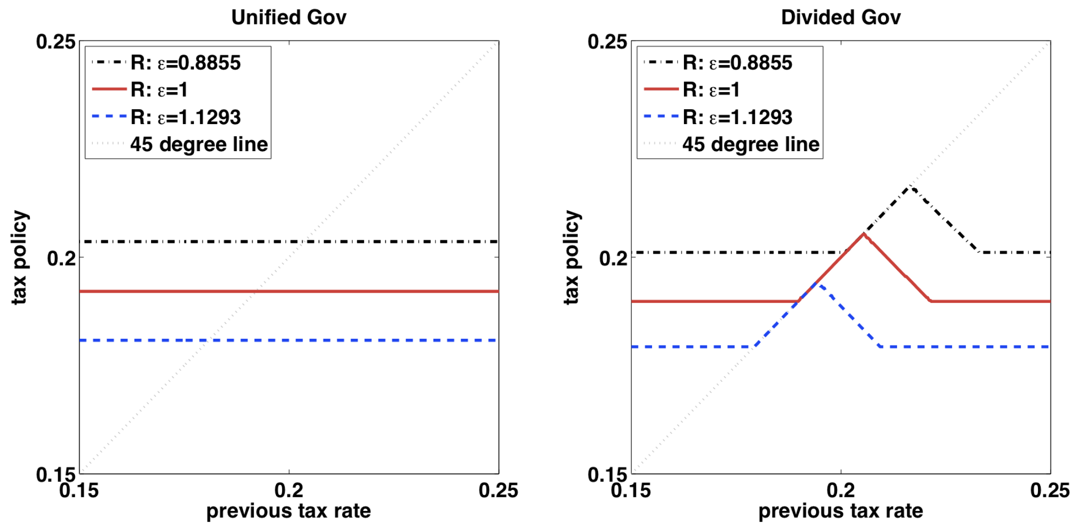
Table 1.1: Parameterization

| Parameter | Interpretation | Value |
|---|---|---|
| $\beta$ | discount factor | 0.9 |
| $\gamma$ | preference parameter | 0.5 |
| $\nu$ | labor elasticity parameter | 1 |
| $\xi$ | spending elasticity parameter | 0.8 |
| $A$ | valuation of public goods | 0.42 |
| $\bar{\theta}_r$ | productivity of rich | 0.97 |
| $\bar{\theta}_p$ | productivity of poor | 0.69 |
| $\rho$ | persistence of productivity shock | 0.95 |
| $\sigma$ | std. deviation of productivity shock | 0.08 |
| $\alpha$ | welfare weight on own constituents | 0.62 |
| $p$ | persistence of power | 0.76 |

### 1.5.1 Policy Gridlock

In Figure 1.1, I compare the equilibrium tax policies of both unified and divided government. For each political system, the equilibrium tax policy of party R is plotted as a function of status quo tax rate $\tau_{t-1}$ at different shock realizations. In the left panel of Figure 1.1, consistent with results in section 1.4, the tax rate chosen by unified government is independent of the status quo tax rate for arbitrary shock realizations. This is because there is no dynamic link between tax policies in different periods. Moreover, unified government responds to the aggregate shock irrespective of the status quo tax rate.

Figure 1.1: Equilibrium Tax Policy Comparison

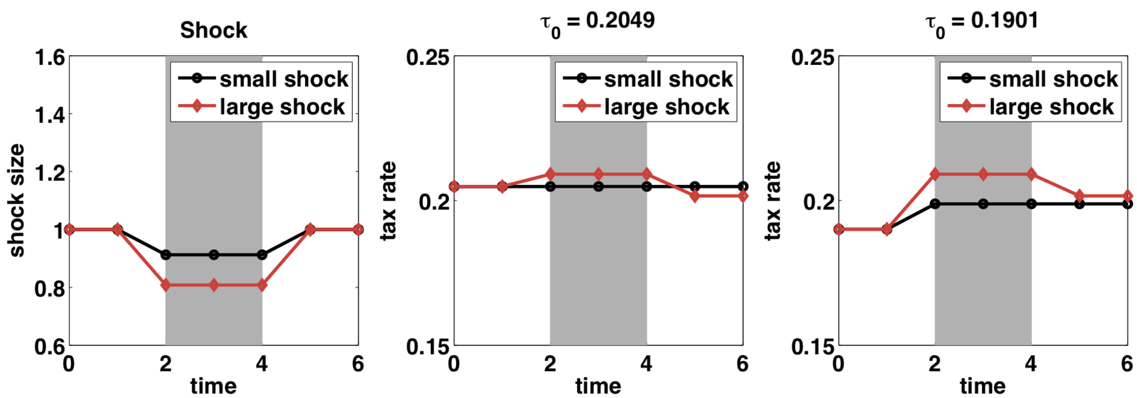



*Notes:* In Figure 1.1, the left panel plots equilibrium tax policies of unified government as a function of status quo tax rates at various shock realizations. The right panel plots equilibrium tax policies of divided government as a function of status quo tax rates at shock realizations same as the left panel. In both figures, the red line denotes tax policy for shock $\epsilon = 1$, the blue line denotes tax policy for shock $\epsilon = 1.1293$ and the black line denotes tax policy for shock $\epsilon = 0.8855$.

The tax policy under divided government behaves differently. In the right panel of Figure 1.1, we see that for an arbitrary shock realization, gridlock and compromise may arise and the equilibrium tax policy depends on the status quo tax rate $\tau_{t-1}$. To be more specific, when the status quo tax rate $\tau_{t-1}$ is below the ideal tax rates of both party R and party P, the two parties agree to raise the tax rate. Since party R is making a take-it-or-leave-it offer and has the full bargaining power, it sets the tax policy around its ideal level. As the status quo $\tau_{t-1}$ moves between the desired tax rate of the party R and party P, party R prefers a lower tax rate while party P prefers a higher tax rate. The two parties end up failing to reach any agreement on policy adjustments so the equilibrium tax policy simply stays at the status quo. This corresponds to the segment of the policy function that overlaps with the 45 degree line. When status quo $\tau_{t-1}$ goes above the desired tax rate of the party P, both parties agree to lower the tax rate. However, as long as the status quo tax rate is close to the ideal tax policy of party P, party P enjoys a relatively high bargaining power and party R is unable to implement its desired tax policy. As the status quo moves further beyond the ideal tax level of party P, party R will regain bargaining power and the equilibrium tax rate will fall. When the status quo is sufficiently high, party R will set the tax policy around its desired tax rate again.

The equilibrium tax policy for the divided government implies that depending on the status quo tax rate, the policymaker may fail to respond to the aggregate shock. When the current shock is of a relatively small magnitude compared to the previous period (e.g. $\epsilon_{t-1} = 1$ and $\epsilon_t = 0.8855$), we see from the right panel of Figure 1.1 the policy functions of party R for $\epsilon_{t-1} = 1$ and $\epsilon_t = 0.8855$ will overlap at status quo tax rates $\tau \in [0.2011, 0.2053]$ and coincide with the 45 degree line. This indicates the policymaker will not respond to the shock at time $t$ if the status quo tax rate is between 0.2011 and 0.2053.

Figure 1.2: Divided Government's Responses to Shocks

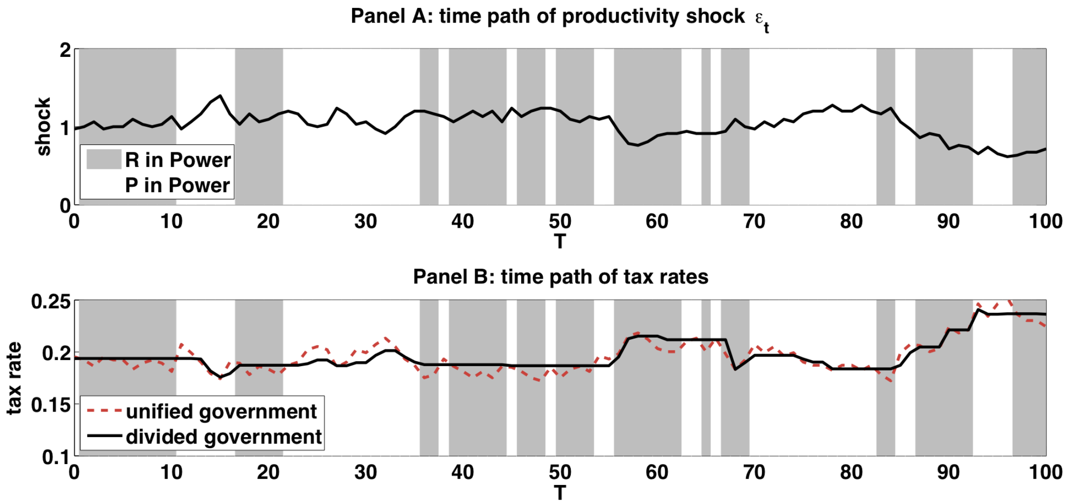



*Notes:* In Figure 1.2, the left panel depicts the time path of shock $\epsilon$. The middle and right panels depict the equilibrium time path of tax policies associated with different initial status quo tax rates. In the middle panel, the initial tax rate is $\tau_0 = 0.2049$; in the right panel, the initial tax rate is $\tau_0 = 0.1901$. For all three panels, gray shade denotes time periods with a negative aggregate productivity shock. The red line is associated with a large negative shock $\epsilon = 0.8083$ and the black line is associated with a small negative shock $\epsilon = 0.9128$.

To further illustrate the idea above, in Figure 1.2 I provide the impulse response of the divided government towards shocks of different magnitudes. Let the economy start with the no shock state at time 0. Starting from time 2, there is a temporary unexpected negative aggregate shock. The shock lasts for three periods and the economy returns to the no shock state afterwards. In addition, I assume party R stays in power from time 0 to time 6. In both the middle and right panels, the red line represents policy response when the size of the negative shock is large (the productivity $\epsilon$ drops from $\epsilon = 1$ to $\epsilon = 0.8083$) and the black line represents the policy response when the size of the negative shock is small (the productivity $\epsilon$ drops from $\epsilon = 1$ to $\epsilon = 0.9128$). Consistent with the analysis above, when the shock is large, despite the status quo tax rate, the two parties manage to achieve an agreement to change the policy. Hence the red lines in both the middle and right panels show response to the shock. When the shock is small, however, depending on the status quo, the two parties may fail to reach any agreement and leave the tax policy unchanged. This is indicated by the flat black line in the middle panel.

In Figure 1.3, I compare the simulated tax series under unified and divided government. Both economies are simulated under the same realization of aggregate shocks and party identities. While the policy by unified government responds to each shock and party alternation, the policy prescribed by divided government experiences periods of inaction despite the shock or political turnover. As summarized by Table 1.2, the simulated tax series from the divided government exhibits higher persistence and lower volatility as compared to the unified government. By forcing policymakers to stay with the status quo, policy gridlock mutes the policy fluctuations generated by political turnover and reduces the uncertainty of the tax policy.

Figure 1.3: Simulated Tax Series




*Notes:* In Figure 1.3, Panel A depicts a typical simulated path of productivity shock $\epsilon$. Panel B compares equilibrium tax rates for unified and divided government corresponding to the shock path in Panel A. The dashed red line represents the simulated tax series of unified government. The solid black line represents the simulated tax series of divided government. In both panels, gray shade marks time periods when party R is in power.

Table 1.2: Summary Statistics of Simulated Tax Series

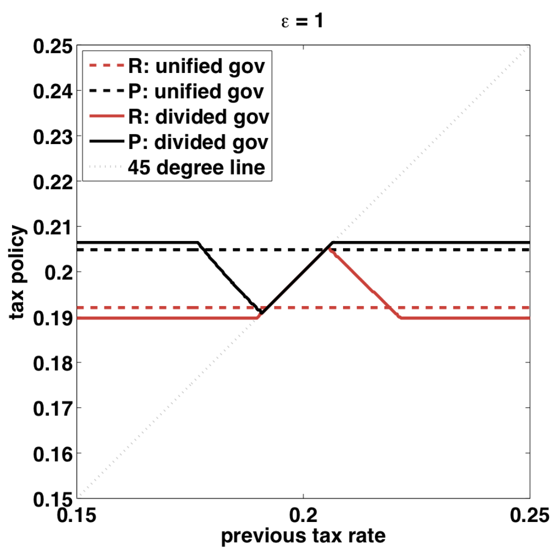
| Economy | $\bar{\tau}$ | $\sigma_\tau$ | $corr(\tau_t, \tau_{t-1})$ |
|---|---|---|---|
| Unified Government | 0.1990 | 0.0246 | 0.9202 |
| Divided Government | 0.1987 | 0.0228 | 0.9639 |

*Notes:* To obtain the summary statistics for the stationary distribution of tax rates in Table 1.2, I simulate each economy for 10000 times and each realization involves 1500 periods. The first 500 periods of each realization are dropped to ensure the economy starts with the stationary distribution.

### 1.5.2 Dynamic Strategic Tradeoffs

In Figure 1.4, I plot the equilibrium tax policies for party R and party P under both political systems. Interestingly, under divided government, for status quo tax rates where no gridlock or compromise occurs, neither party implements the same static ideal tax level as under unified government. At shock realization $\epsilon = 1$, party R chooses a lower tax rate than its static optimum, while party P chooses an even higher tax rate. This is due to the dynamic tradeoffs faced by divided government policymakers. They give up setting the current tax rate at their ideal level in order to increase their leverage and expected utilities in the future. For instance, suppose party R is in power. By choosing a lower than ideal tax rate, it essentially guarantees a low status quo tax level tomorrow. In cases when gridlock occurs in the next period, such strategy on average insures party R a relatively high expected future payoff for staying with the status quo. In contrast, the expected payoff of party P will be even lower because the status quo is pushed further away from the poor's ideal level. This raises the bargaining power of party R and lowers the bargaining power of party P. As a result, party R will be able to choose more favorable tax rates in the future.

Figure 1.4: Dynamic Tradeoff: Unified Vs Divided Government

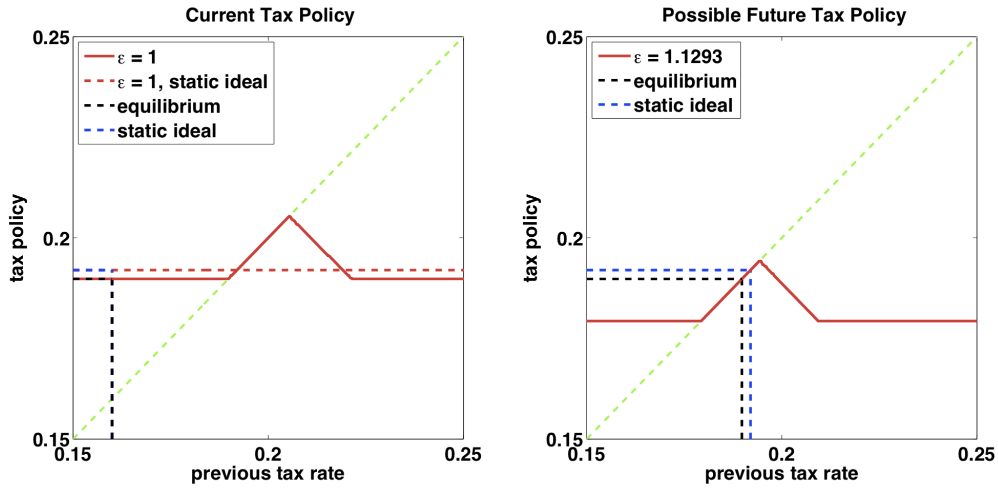



*Notes:* Figure 1.4 compares equilibrium tax policies of unified and divided government at shock realization $\epsilon = 1$. Here solid lines represent equilibrium tax policies of divided government. Dashed lines represent equilibrium tax polices of unified government. Red lines denote policies by party R and black lines denote policies by party P.

To further understand the potential gain for divided government to deviate away from the static ideal tax rate in cases when such tax rate is incentive compatible, I perform a thought experiment using Figure 1.5 and Figure 1.6. I compare the outcome of equilibrium strategy with the outcome of a one-step deviation strategy where the policymaker implements the static optimum tax rate.

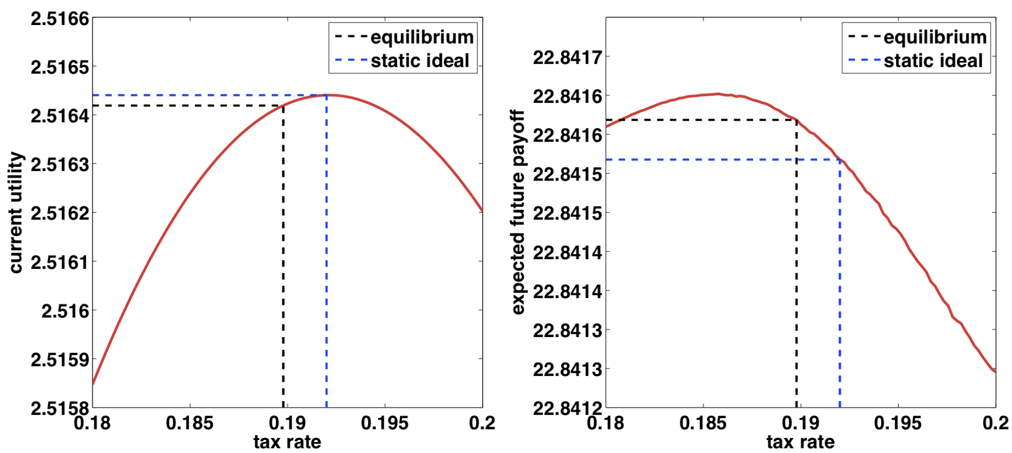
These two figures highlight how the policy chosen at current affects the policy and hence expected life-time utility in the subsequent period. Assume that at time $t$ party R is the policy proposer and the shock realization takes value $\epsilon_t = 1$. Also assume the status quo tax rate is $\tau_{t-1} = 0.16$ such that the static optimum is incentive compatible. In Figure 1.5, I compare the current and subsequent tax policies implied by the equilibrium and the one-step deviation strategy. In the left panel, we see given status quo tax rate $\tau_{t-1} = 0.16$, if the policymaker follows equilibrium strategy as tracked by the dashed black line, the tax rate at time $t$ is $\tau_t = 0.1898$. If the policymaker implements the static ideal level as tracked by the dashed blue line, the tax rate at time $t$ is $\tau_t = 0.1920$, which is higher than the equilibrium tax level. Suppose at time $t+1$ shock realization is $\epsilon_{t+1} = 1.1293$ and party R stays in power. Note that the tax policies determined at time $t$ become the status quo tax rates at time $t+1$. In the right panel, we see if the policymaker previously follows the equilibrium strategy, the tax policy will be gridlocked at $\tau_{t+1} = 0.1898$. If the policymaker previously implements the static ideal tax rate instead, the high status quo will lead to policy gridlocked at a higher level, i.e., $\tau_{t+1} = 0.1920$. This is not desirable because the high tax rate pushes party R further away from its ideal tax level at time $t+1$. In terms of utilities, in Figure 1.6, by comparing the current and expected future utilities under the equilibrium strategy and one-step deviation strategy, we see the expected utility gain in the future for implementing the equilibrium strategy outweighs the current utility loss. Therefore, by giving up the static ideal tax rate, the policymaker guarantees itself higher bargaining power and higher life-time utility in case of future policy gridlock.

Figure 1.5: Trade-offs for Divided Government: Tax Policy




*Notes:* The left panel of Figure 1.5 compares party R's equilibrium tax policy under divided government (represented by the solid red line) with its static ideal level (represented by the dashed red line) at shock realization $\epsilon_t = 1$. The dashed black line and the dashed blue line are used to track above two tax policies at status quo tax rate $\tau_{t-1} = 0.16$. The right panel of Figure 1.5 compares the equilibrium tax policies under divided government in the subsequent period given shock realization $\epsilon_{t+1} = 1.1293$, at the status quo tax rates corresponding to the two tax policy levels in the left panel. This is because the policies in the previous period as tracked by dashed lines in the left panel becomes status quo tax rates in the next period.

Figure 1.6: Trade-offs for Divided Government: Utilities




*Notes:* The left panel of Figure 1.6 compares the current period utility at equilibrium tax level of party R under divided government (denoted by dashed black line) with the current period utility at the static ideal tax level of party R (denoted by dashed blue line) for status quo $\tau_{t-1} = 0.16$ and shock realization $\epsilon = 1$. The right panel of Figure 1.6 compares the discounted expected utility in the future at the equilibrium tax rate and static ideal tax rate same as in the left panel.

## 1.6 Welfare Consequences

This section focuses on understanding the welfare consequences of divided government. I start with analyzing the equilibrium welfare generated by divided government, and then compare it with unified government. To interpret the welfare differences between the two political systems, I break down the welfare-enhancing and welfare-reducing role of policy gridlock through studying two auxiliary economies. Finally, I show both unified and divided governments are inefficient.

### 1.6.1 Welfare Measures

Before focusing on welfare consequences, I first provide the definitions of various welfare measures that will be used in the subsequent analysis. I define the welfare of household $i \in \{r, p\}$ delivered by a benevolent social planner with equal welfare weights on each household as

$$WF_O^i = \mathbb{E} \sum_{t=0}^{\infty} \beta^t u^i \left[ \tau^O(\epsilon_t), \epsilon_t \right] \tag{1.20}$$

where $u^i(\tau_t, \epsilon_t)$ is the indirect utility of household $i$ for arbitrary tax rate $\tau_t$ and shock realization $\epsilon_t$. The tax rate $\tau^O(\epsilon_t)$ is the optimal tax policy of the social planner. To focus on the ex-ante welfare of the households, I take the unconditional expectation over the shock realizations using the time-invariant distribution of the aggregate shock after obtaining the welfare associated with each initial realization. Therefore, the welfare $WF_O^i$ computed above is not conditional on the initial shock realization.

Similarly, the welfare of household $i$ associated with unified government is defined as

$$WF_U^i = \mathbb{E}\sum_{t=0}^{\infty}\beta^t u^i\left[\tau_U^{j_t}(\epsilon_t), \epsilon_t\right] \quad j_t \in \{R, P\} \tag{1.21}$$

Here $\tau_U^{j_t}(\epsilon_t)$ is the equilibrium tax policy chosen by the party with proposal power under unified government. Here the expectation is taken over both the productivity shock realization and the identity of the policy proposer. Again, the welfare measure $WF_U^i$ is unconditional of initial shock and party identity, with the expectation computed with the time-invariant distribution of the aggregate shock and party identity.

Since welfare depends on the initial status quo, I define the welfare of household $i$ under divided government with initial status quo tax rate $\tau_{-1}$ as

$$WF_D^i(\tau_{-1}) = \mathbb{E}\sum_{t=0}^{\infty}\beta^t u^i\left[\tau_D^{j_t}(\tau_{t-1}, \epsilon_t), \epsilon_t\right] \quad j_t \in \{R, P\} \tag{1.22}$$

where $\tau_D^{j_t}(\tau_{t-1}, \epsilon_t)$ is the equilibrium tax policy chosen by the party with proposal power under divided government. Similar to the households' welfare with unified government, the expectation is taken over both the aggregate productivity shock and the party identity. In later analysis, after having fully understood how households' welfare under divided government varies with initial status quo tax rates, I also compute the expected welfare of households using the time-invariant distribution of equilibrium tax rates. This captures the *average* wellbeing of each household when the economy reaches its time-invariant distribution.

To further quantify the efficiency implications of unified and divided government, I measure the welfare *cost of unified government* for household $i$ as the fraction $\chi_u^i$ of the consumption stream under unified government that the household $i$ would need to be as well off as in the benevolent social planner's economy

$$\mathbb{E}\sum_{t=0}^{\infty} u\left\{(1+\chi_u^i)C^i[\tau_U^{j_t}(\epsilon_t),\epsilon_t], L^i[\tau_U^{j_t}(\epsilon_t),\epsilon_t], G[\tau_U^{j_t}(\epsilon_t),\epsilon_t]\right\} = WF_O^i \tag{1.23}$$

where $C^i(\tau_t,\epsilon_t)$, $L^i(\tau_t,\epsilon_t)$, $G^i(\tau_t,\epsilon_t)$ are the competitive equilibrium allocations for arbitrary tax rate $\tau_t$ and shock realization $\epsilon_t$. Similarly, the welfare *cost of divided government* for household $i$ is defined as the fraction $\chi_d^i$ of the consumption stream under divided government that the household $i$ would need to be *on average* as well off as in the benevolent social planner's economy

$$\mathbb{E}\sum_{t=0}^{\infty} u\left\{(1+\chi_d^i)C^i[\tau_D^{j_t}(\tau_{t-1},\epsilon_t),\epsilon_t], L^i[\tau_D^{j_t}(\tau_{t-1},\epsilon_t),\epsilon_t], G[\tau_D^{j_t}(\tau_{t-1},\epsilon_t),\epsilon_t]\right\} = WF_O^i \tag{1.24}$$

Since the expectation is taken over the initial status quo tax rate with the time-invariant distribution, neither $\chi_u^i$ nor $\chi_d^i$ depend on the initial status quo tax rate $\tau_{-1}$[4]. In general, if the above cost measures take *positive* value, it means that the political system incurs a welfare *loss* for the household. On the contrary, if the measures take *negative* values, it indicates the political system is associated with welfare *gain*.

[4]In fact, we can also compute the welfare cost of divided government *conditional* on initial status quo tax rate $\tau_{-1}$. When evaluated at the average tax rate of the time-invariant distribution, the values of welfare cost computed under this alternative measure is almost identical to that computed according to the measure as specified above.

### 1.6.2 Welfare Under Divided Government

Here I analyze how households' welfare varies with the initial status quo tax rate $\tau_{-1}$. In Figure 1.7, I plot the equilibrium welfare associated with the full range of initial status quo tax rates $\tau_{-1}$. The welfare pair for higher initial status quo tax rates is indicated by darker grayness. Since the equilibrium tax policy by divided government at an arbitrary initial status quo tax rate always falls inside the set of tax rates that has positive probability mass in the time-invariant distribution, the equilibrium welfare plotted in Figure 1.7 essentially coincides with the equilibrium welfare associated only with initial status quos that are drawn from the time-invariant distribution of equilibrium tax rates.

To understand the equilibrium welfare behavior, recall the tax policy function in Figure 1.4. First, when the initial status quo tax rate is very low or very high, the first period equilibrium tax policy for both political parties is simply implemented at their desired levels and is independent of the status quo tax rate. This implies the equilibrium welfare for these initial status quo tax rates boils down to one single point, which is indicated by the solid black circle in Figure 1.7. As the initial status quo tax rate rises and the equilibrium welfare moves beyond the solid black circle, at first the rich benefit while the poor suffer. This is because when party P is in power, it compromises with party R and lowers the tax rate, yet when party R is in power, its policy stays at the desired level. Therefore, the equilibrium welfare pair moves northwest. As the status quo tax rate continues to increase while both parties are gridlocked, the welfare of the rich decreases and the welfare of the poor increases. This is because no matter which party is in power, the equilibrium tax policy moves beyond the desired level of the rich and towards the desired level of the poor. Hence the equilibrium welfare shifts from the northwest to the southeast. As the status quo rises further, since the equilibrium tax policy of party R drops while the equilibrium tax policy of party P stays at its desired level, the welfare of the rich increases. This leads the equilibrium welfare pair to move from the southeast towards the black circle.

Figure 1.7: Interpretation

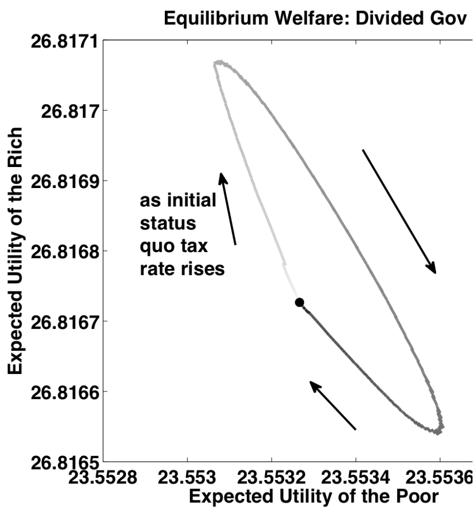
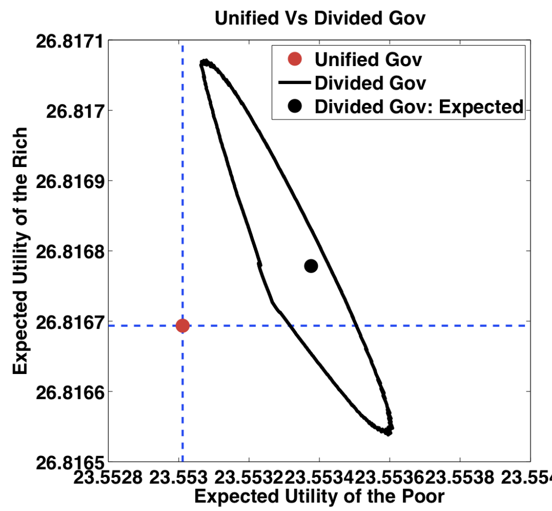



Figure 1.8: Comparison




*Notes:* Figure 1.7 plots equilibrium welfare pairs of divided government at the entire range of initial status quo tax rates. Darker gray colors indicate welfare pairs associated with higher initial status quo tax rates. The black dot marks the equilibrium welfare pair associated with the initial status quo tax rate at which both parties are unconstrained. Figure 1.8 compares the equilibrium welfare between unified government and divided government. The solid red circle represents equilibrium welfare pair of unified government. The solid black line represents equilibrium welfare pair of divided government at various initial status quo tax rates. The solid black circle denotes the expected equilibrium welfare pair of divided government.

### 1.6.3 Comparison Between Unified and Divided Government

In Figure 1.8, I compare the equilibrium welfare delivered by unified and divided government. Under the current model parameterization, there exists a range of initial status quo tax rates under which divided government generates strictly higher welfare than unified government for both rich and poor households. Using the time-invariant distribution of the equilibrium tax rates obtained from the simulated economy, I also compute the expected welfare for the divided government, which is denoted by the solid black circle. Since the average welfare with divided government is to the northeast of the welfare delivered by unified government, it implies on average, both types of households are better off with divided government. Intuitively, policy gridlock in divided government partly counteracts the policy fluctuations generated by the exogenous political turnover. This reduces the excess volatility of households' consumption and therefore makes both households better off. Detailed explanations of this result will be provided in the subsequent section.

One more issue worth mentioning here is the asymmetry of welfare consequences between the rich and the poor. In Figure 1.8, we see the poor are always better off with divided government regardless of the initial status quo, while the rich may experience welfare loss at certain status quo tax rates. Since the rich generate more income than the poor, the welfare loss due to policy gridlock around the opponent's desired tax level will hurt the rich more severely. With the current parameterization, the welfare loss for the rich happens to be so large that it dominates the welfare gain. Yet in general, depending on the inequality and polarization parameterization, other scenarios can occur. In particular, it is possible to have different range of status quo tax rates associated with each household where it is strictly better off with unified government. This will be discussed in section 7.

### 1.6.4 Dual Role of Policy Gridlock: An Explanation

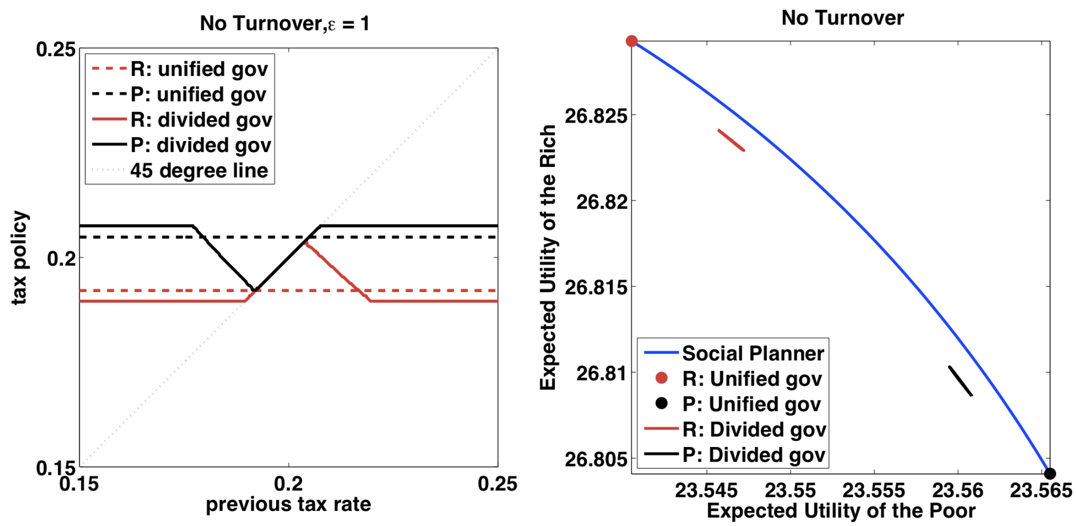
To further understand why divided government can dominate unified government, I analyze the gains and losses associated with policy gridlock – the key phenomenon that distinguishes divided government from unified government. By studying welfare consequences of two auxiliary economies, I find that gridlock in divided government plays both welfare-improving and welfare-reducing roles, which counteract each other.

#### Welfare-Reducing Role

To isolate the welfare reducing aspect of policy gridlock, I compare unified with divided government while assuming away political turnover. In other words, one party is assumed to hold proposal power forever. In the left panel of Figure 1.9, we see the policy gridlock and strategic dynamic considerations arising under divided government turn out to be very similar to Figure 1.4. As long as there exists an aggregate shock, even if there is no uncertainty in the identity of the party with proposal power, the policymaker is concerned with potential gridlock in the future and is willing to forgo its static ideal tax rate. In the right panel of Figure 1.9, we see that while unified government without political turnover achieves efficiency at the expense of equity, divided government incurs inefficiency for arbitrary initial status quo tax rates.

The sources of inefficiency are straightforward. First, the divided government lacks the ability to immediately respond to exogenous shocks. While the unified government always responds to the exogenous shocks regardless of size, the economy with divided government may fail to reach an agreement on a

Figure 1.9: Welfare Reduction Due to Strategic Considerations




*Notes:* In Figure 1.9, the left panel compares equilibrium tax policies of unified and divided government in economy with no political turnover at shock realization $\epsilon = 1$. Here solid lines represent policies by divided government and dashed lines represent policies by unified government. The right panel compares equilibrium welfare of unified and divided government with the Pareto frontier. The blue line represents the Pareto frontier associated with the social planner. The solid red circle and solid black circle denotes equilibrium welfare pairs associated with party R and party P in power under unified government. The red line and the black line denotes equilibrium welfare pairs at various status quo tax rates associated with party R and party P in power under divided government.

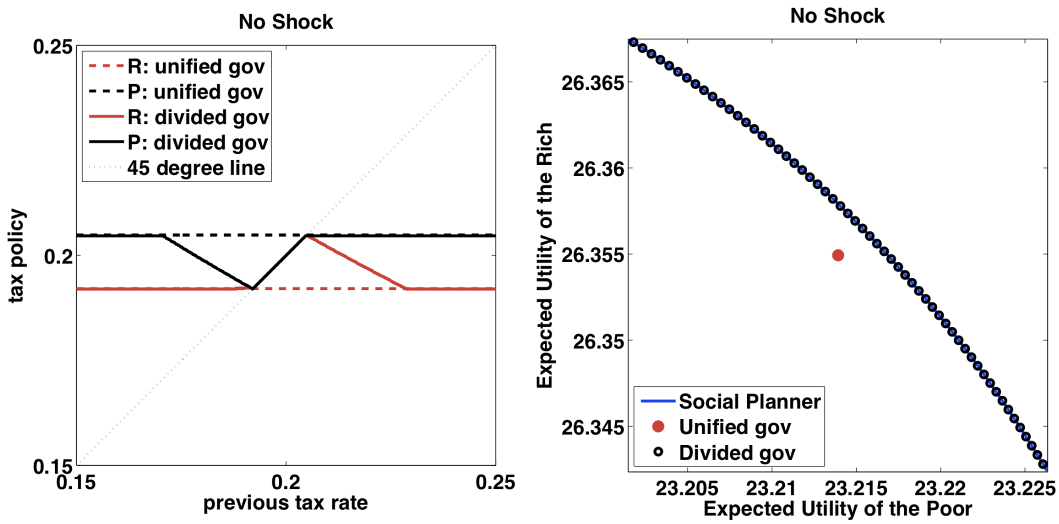
change of policy in the case of small shocks. This forces the tax policy to stay at inefficiently low (or inefficiently high) levels compared to those under unified government. Second, the dynamic strategic considerations distort the policy of the divided government even if the policymaker actively responds to the shock. An important observation from the right panel of Figure 1.9 is that the inefficiency is not limited to the initial status quo tax rates where policy gridlock occurs. Instead, the inefficiency emerges across the entire time-invariant distribution of status quo tax rates, including when the divided government is actively responding to shocks. By forgoing the static ideal policy and implementing a tax rate which could increase its bargaining power tomorrow, the policymaker pushes the equilibrium policy away from that which would be implemented by the unified government. To sum up, both the response failure and the dynamic strategic consideration lead to the inefficiency generated by policy gridlock in the economy with divided government.

**Welfare-Enhancing Role**

To highlight the welfare improving role of the policy gridlock, I compare unified government and divided government assuming away the aggregate productivity shocks, i.e. $\epsilon_t \equiv 1$ for all $t$. Since there is no aggregate shock, the benevolent social planner chooses to keep the tax rate constant. In the economy with unified government, however, the tax policy is forced to switch between the desired levels of the two political parties due to exogenous power fluctuation. In the economy with divided government, on the contrary, the equilibrium tax policy for an arbitrary initial status quo tax rate stays constant over time despite the power fluctuation. This is because the two parties fail to reach any agreement on policy change and end up keeping policy gridlocked at a constant level.

In the left panel of Figure 1.10, we see when there is no gridlock or compromise, the equilibrium tax policy by the divided government coincides with that of the unified government. This is because the constant shock realization removes the incentive of strategic tradeoff for divided government. Moreover, once the tax policy reaches the steady state, it stays constant forever. In the right panel, we see for arbitrary status quo tax rate, the equilibrium delivered by divided government is always efficient and coincides with the Pareto frontier. The equilibrium by the unified government, however, is inefficient due to political turnover. This in essence highlights the benefit of unanimity as a complement of Buchanan and Tullock (1962) and constitutes a stark example of the welfare-improving role of the policy gridlock.

Figure 1.10: Welfare Increase Due to Disagreement




*Notes:* In Figure 1.10, the left panel compares equilibrium tax policies of unified and divided government in economy with political turnover and $\epsilon_t \equiv 1$. Here solid lines represent policies by divided government and dashed lines represent policies by unified government. The right panel compares equilibrium welfare of unified and divided government with the Pareto frontier. The blue line represents the Pareto frontier associated with the social planner. The solid red circle denotes the equilibrium welfare pair under unified government. The black circles denote equilibrium welfare pairs at various status quo tax rates under divided government.

### Discussion

In general, for an economy with aggregate productivity shocks and political turnover, policy gridlock plays both welfare-enhancing and a welfare-reducing roles. In terms of the welfare-reducing role, in the full-fledged economy, the dynamic strategic considerations by the divided government not only involve the concern of potential changes in the aggregate shock, but also include the concern of possible power fluctuation. Namely, the policymaker chooses a tax rate today that will guarantee him a favorable status quo tomorrow in case of losing proposal power. Yet I find that the magnitude of impacts on welfare due to such concern remains relatively small. In terms of the welfare improving role, after introducing the aggregate productivity shock, while the gridlock can still mitigate policy uncertainty, it can no longer fully undo its effect because sufficiently large shocks can foster party agreement and policy changes.

Since the welfare-reducing and welfare-enhancing forces counteract each other, whether or not the divided government delivers higher welfare than the unified government depends on which effect dominates. Under the current parameterization, I find that both the rich and the poor on average benefit from the divided government. Later in the paper I will further discuss how welfare consequences change with key model parameters.

### 1.6.5 Inefficiency of Both Unified and Divided Government

Finally, I compare the efficiency implications of unified and divided government with the social planner benchmark. According to the social planner's optimization problem (1.11), the efficient allocations are characterized by

$$-\frac{u^{r\prime}(\tau_t^O, \epsilon_t)}{u^{p\prime}(\tau_t^O, \epsilon_t)} = \frac{\mu}{1-\mu} \tag{1.25}$$

where $u^{r\prime}(\tau_t^O, \epsilon_t)$ and $u^{p\prime}(\tau_t^O, \epsilon_t)$ are the derivatives of households' indirect utilities with respect to $\tau_t$, evaluated at the tax rate $\tau_t^O$ chosen by the social planner. Since (1.25) holds at any time, it also implies that for two different arbitrary time periods $t$ and $s$, efficient allocations satisfy

$$\frac{u^{r\prime}(\tau_t^O, \epsilon_t)}{u^{p\prime}(\tau_t^O, \epsilon_t)} = \frac{u^{r\prime}(\tau_s^O, \epsilon_s)}{u^{p\prime}(\tau_s^O, \epsilon_s)} \tag{1.26}$$

In contrast, allocations determined by unified and divided governments generally do not satisfy the above condition. For unified government, political power alternations can lead to distinct marginal utility ratios at different time periods. For instance, assume party R is in power at time t and party P is in power at time s. Then

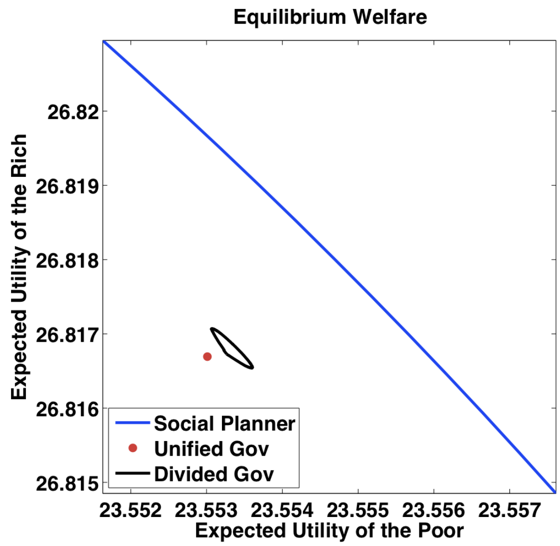
$$-\frac{u^{r\prime}(\tau_t^R, \epsilon_t)}{u^{p\prime}(\tau_t^R, \epsilon_t)} = \frac{\alpha^R}{1-\alpha^R} > \frac{\alpha^P}{1-\alpha^P} = -\frac{u^{r\prime}(\tau_s^P, \epsilon_s)}{u^{p\prime}(\tau_s^P, \epsilon_s)} \tag{1.27}$$

For divided government, in addition to power fluctuations, status quo tax rates also affect equilibrium tax policies, which determine the marginal utility ratios. For two time periods where status quo tax rates differ and the policymaker faces binding incentive compatibility constraints, the ratios of marginal utilities can be different even if the identity of the party with proposal power stays the same.

The findings above imply that in general, neither unified nor divided government is efficient. For the unified government, random power fluctuations create policy uncertainty, which leads to volatile household consumption and reduces households' welfare. For the divided government, while policy gridlock partly counteracts the policy volatility created by political turnover, as long as aggregate uncertainty prevails, the gridlock could not fully undo the inefficiency created by strategic dynamic considerations. In Figure 1.11, I plot the equilibrium welfare for unified and divided government together with the Pareto frontier delivered by a benevolent social planner. As a result, the equilibrium welfare delivered by both unified and divided government turn out to lie within the Pareto frontier.

In Table 1.3, I report the welfare costs associated with the two political systems for each household. Consistent with the findings above, all welfare costs are positive, indicating there are welfare losses associated with each household

Figure 1.11: Inefficiency of Unified and Divided Government




*Notes:* In Figure 1.11, I compare the equilibrium welfare of unified and divided government with the Pareto frontier. The solid blue line represents the Pareto frontier delivered by the social planner. The solid red circle represents the equilibrium welfare pair of unified government. The solid black line represents the equilibrium welfare pair of divided government at various initial status quo tax rates.

in both unified and divided government. Since the divided government generates a slightly lower welfare cost for both households, both the rich and the poor are on average better off with the divided government. In addition, for both political systems, the welfare cost is slightly larger for the rich.

Table 1.3: Welfare Cost of Unified & Divided Government

| Welfare Cost | Rich | Poor |
|---|---|---|
| Cost of Unified Government (%) | 0.0162 | 0.0147 |
| Cost of Divided Government (%) | 0.0154 | 0.0101 |

*Notes:* In Table 1.3, I present welfare cost of unified and divided government for the rich and poor households. The welfare cost of unified government is computed according to (1.23) and the welfare cost of divided government is computed according to (1.24).

## 1.7 On Rising Inequality and Political Polarization

In this section, I address how rising income inequality and political polarization influences the attractiveness of divided government relative to unified government. For each parameter, I first analyze how equilibrium tax policies change, and then discuss how welfare is affected.

### 1.7.1 Income Inequality

To understand the impact of rising income inequality, I first focus on equilibrium tax policies of divided government for economies at different inequality levels as measured by the rich income share[5]. As shown in Figure 1.12, when the rich income share rises, the desired policies of both political parties diverge further. This creates an enlarged gridlock region, which naturally leads to the increase of the average duration of policy gridlock in the simulated economy, as illustrated in Figure 1.13.

In terms of the welfare consequences, I plot the welfare of unified and divided government at different inequality levels in Figure 1.14. By construction, as inequality rises the income grows for the rich and declines for the poor. We see that welfare moves in the same direction. When the income inequality is very low (e.g., rich income share is 0.58), the divided government strictly dominates the unified government for arbitrary status quo tax rate. As inequal-

[5]More specifically, in order to examine the role of income inequality, I vary the income share of the households through introducing a mean preserving spread $\delta \in [0, \bar{\theta}_p)$ to individual productivity levels, i.e. $\tilde{\theta}_r = \bar{\theta}_r + \delta$ and $\tilde{\theta}_p = \bar{\theta}_p - \delta$. This transformation keeps average productivity level constant while increasing the income share of the rich. Also note that under model assumption of households' preferences, it is not difficult to show the rich income share $\frac{\tilde{y}_r}{\tilde{y}_r + \tilde{y}_p}$ as a measure of inequality is independent of $\epsilon$ and it is an increasing function of $\delta$.

Figure 1.12: Tax Policy

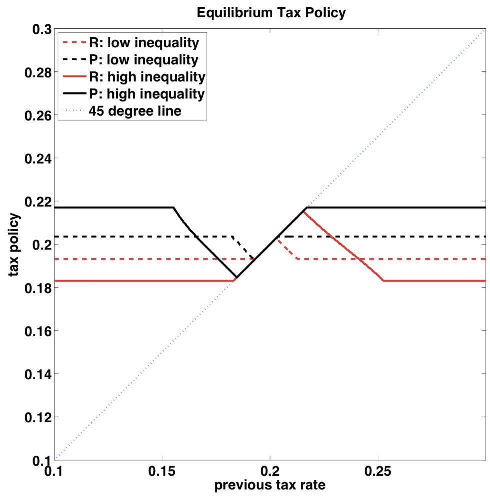
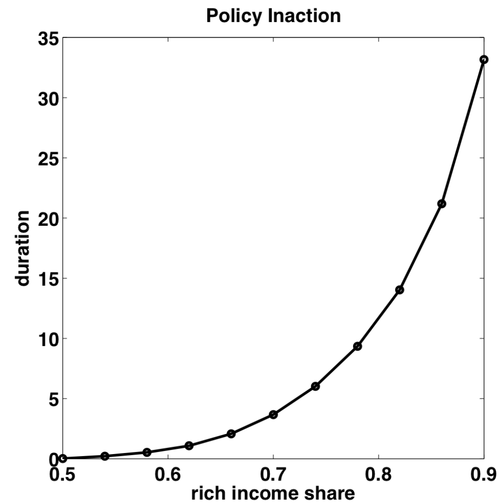


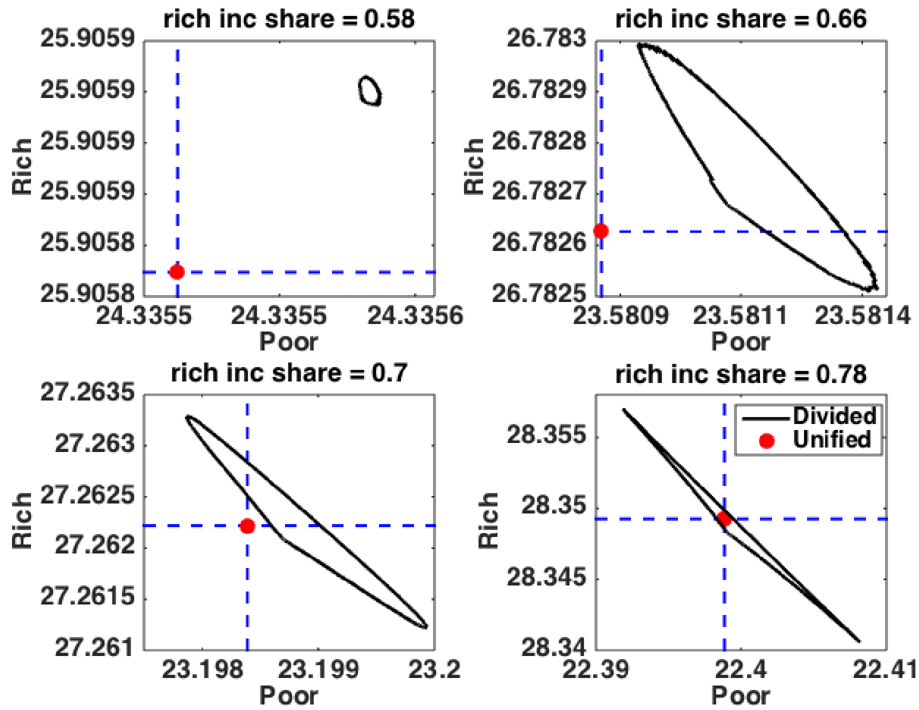

Figure 1.13: Gridlock Duration




*Notes:* Figure 1.12 compares equilibrium tax policies of divided government at high and low income inequality levels for shock realization $\epsilon = 1$. Solid lines represent equilibrium tax policies at the high inequality level. Dashed lines represent equilibrium tax policies at the low inequality level. Figure 1.13 plots simulated policy gridlock duration under divided government as a function of the rich income share.

ity gradually rises, however, the welfare consequences become mixed. For instance, when rich income share is 0.66, the poor are better off with the divided government for an arbitrary status quo tax rate, but the rich could be worse off for certain range of status quo tax rates. When rich income share is 0.7, there is also a certain range of status quo tax rates which make the poor worse off while making the rich strictly better off. As income inequality becomes as high as 0.78, there exist status quo tax rates where the unified government strictly dominates the divided government.

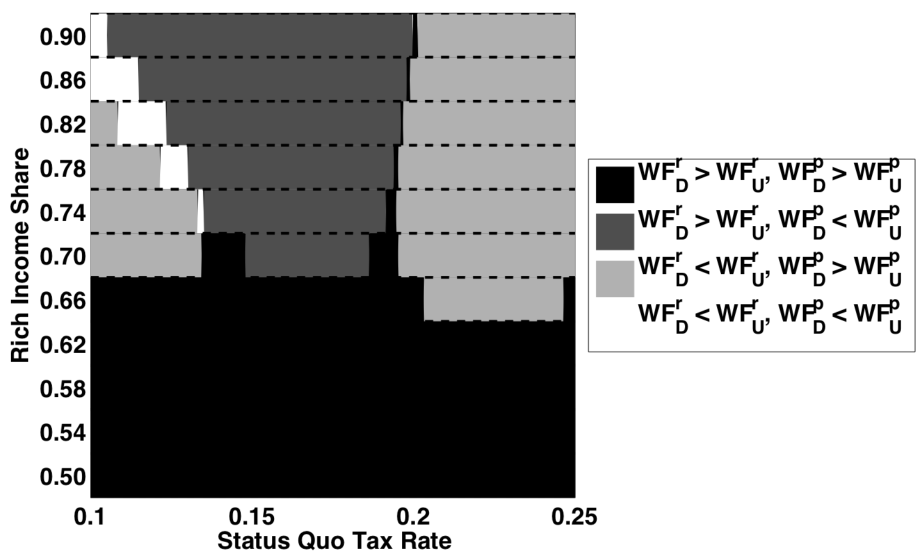
Figure 1.14: Equilibrium Welfare: Role of Inequality




*Notes:* Figure 1.14 plots separately equilibrium welfare pairs of unified and divided government at rich income share 0.58, 0.66, 0.70 and 0.78. Here the solid red circle represents equilibrium welfare of unified government and the solid black line represents equilibrium welfare of divided government at various initial status quo tax rates.

To further characterize the relationship between welfare and initial status quo tax rates, in Figure 1.15 I separate the initial status quo tax rates into four regions at each inequality level: 1) status quo tax rates at which both households are better off under divided government, 2) status quo tax rates at which the rich are better off while the poor are worse off, 3) status quo tax rates at which the poor are better off while the rich are worse off, 4) status quo tax rates at which and both the rich and the poor are worse off. Consistent with Figure 1.14, at low inequality levels, both households are strictly better off with the divided government independent of the status quo tax rates. As inequality rises, the range of status quo tax rates where both households are better off shrinks. However, note that there always exist certain status quo tax rates at which both households are better off with the divided government, even in cases of very high inequality. Moreover, as inequality rises, both the region where both households are better off with the unified government and the region where only the rich benefits from divided government expands.

Figure 1.15: Status Quo Tax Rates for Different Welfare Scenarios

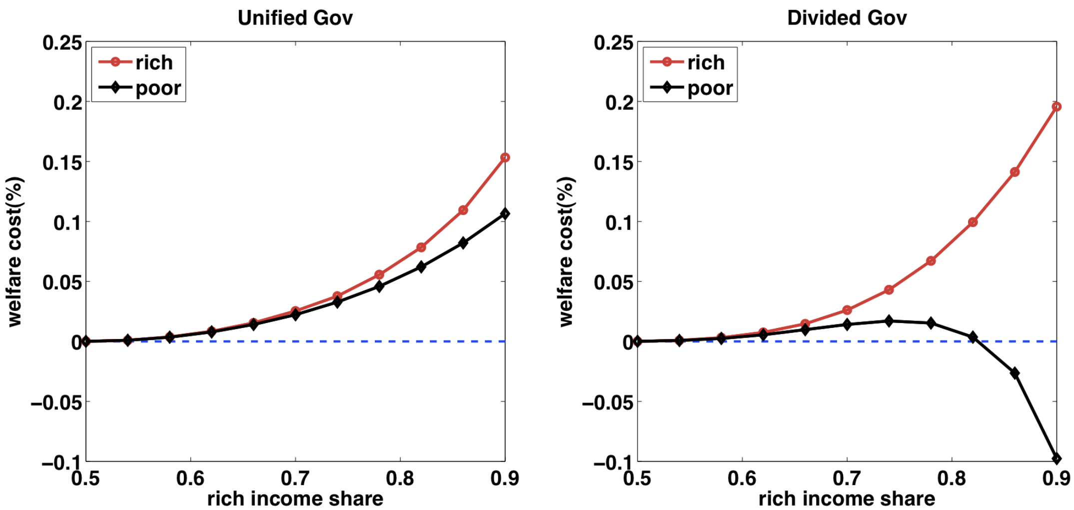



*Notes:* Figure 1.15 marks status quo tax rates associated with different welfare scenarios with different colors at each inequality level. The black color denotes the initial status quo tax rates at which both rich and poor households are better off with divided government. The dark gray color denotes the initial status quo tax rates at which the rich are better off with divided government while the poor are better off with unified government. The light gray color denotes the initial status quo tax rates at which the poor are better off with divided government while the rich are better off with unified government. The white color denotes the initial status quo tax rates where both rich and poor households are better off with the unified government.

Intuitively, as inequality rises, the equilibrium tax policies of both political parties become more polarized. This implies that in the case of tax policies getting gridlocked around opponent's desired level, the welfare loss increases. When inequality is relatively low, such welfare loss remains dominated by the welfare gain coming from the reduction of tax policy uncertainty, hence the divided government dominates the unified government regardless of initial status quo tax rates. As inequality increases, the welfare loss for the opposition party in case of policy gridlock around the desired tax level of the policy proposer becomes so high that it can dominate the welfare gain from reduced policy uncertainty over various initial status quo tax rates. In this scenario, only the status quo tax rates sufficiently close to the average of the two parties' desired policies can make divided government dominate unified government, because these moderate levels of tax rates prevent the welfare loss of one party from rising too high.

In Figure 1.16, I compare the welfare costs of unified and divided government for rich and poor households. Specifically, I examine how much additional consumption should be provided to each household so that *on average* it can be as well off as in a benevolent social planner's economy. For the unified government, as inequality rises, the welfare cost increases for both households. This is because increasingly divided policies lead to significant policy variations, which hurts each household in the economy. For the divided government, as inequality rises, the welfare cost initially increases for both households as well. Yet at high inequality levels, the welfare cost increases for the rich and drops for the poor. In fact, there is even a positive welfare gain associated with the poor at high inequality levels. This asymmetry arises from the fact that households at different income levels tradeoff the gains and losses associated with policy gridlock and policy uncertainty differently. A household with low income is too poor to endure additional potential policy fluctuations. Hence even the stability delivered by policy gridlocked at the 'wrong' level turns out to be preferable than policies that fluctuate around his desired level. As a household's income grows, it incurs increasingly large losses from policies gridlocked at the opponent's desired level, while it is less concerned about policy stability. Therefore, comparing the welfare cost of unified and divided government, I find in general the rich are better off with unified government, while the poor are better off with divided government.

Figure 1.16: Welfare Cost at Different Inequality Levels




*Notes:* In Figure 1.16, the left panel plots the welfare cost of unified government and the right panel plots the welfare cost of divided government. In both panels, the red line represents the welfare cost for the rich and the black line represents the welfare cost for the poor.

### 1.7.2 Political Polarization

In this model, an important parameter that governs the ideological difference between the two political parties is the welfare weight parameter $\alpha$. Large values of $\alpha$ imply intense conflicts between political parties and the implementation of extreme policies. Essentially, the magnitude of $\alpha$ measures the degree of polarization associated with the policymaker.

In Figure 1.17, I compare the tax policies of the divided government at different levels of political polarization. As $\alpha$ rises, the desired policies for both political parties become increasingly polarized, which leads to the expansion of the gridlock and compromise regions. In Figure 1.18, I plot the cumulative density function of policy gridlock at polarization levels that corresponds to Figure 1.17. We see the gridlock duration in the low-polarization economy stochastically dominates that in the high-polarization economy: low polarization implies higher probability of encountering none or very short periods of gridlock, and lower chances of significantly long periods of policy gridlock.

Figure 1.17: Tax Policy

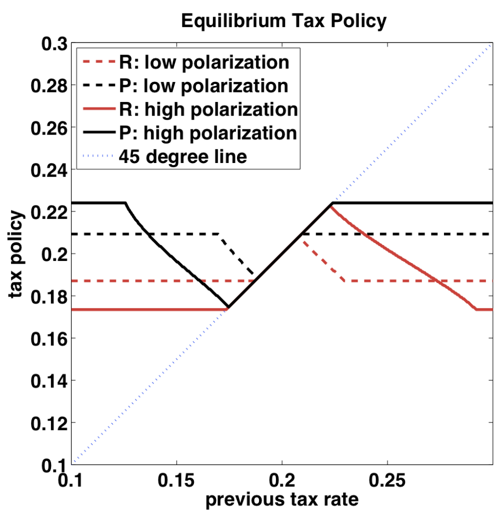
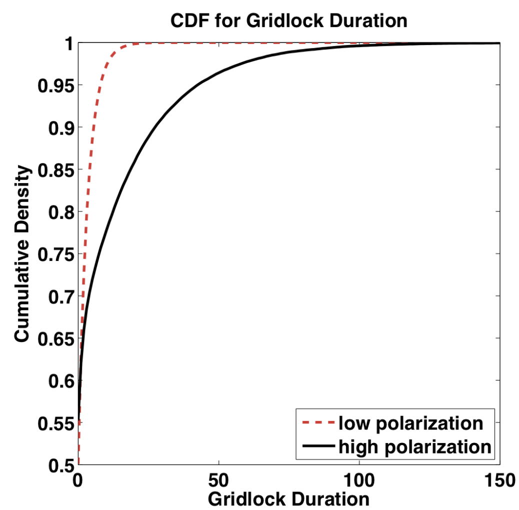



Figure 1.18: Gridlock Duration

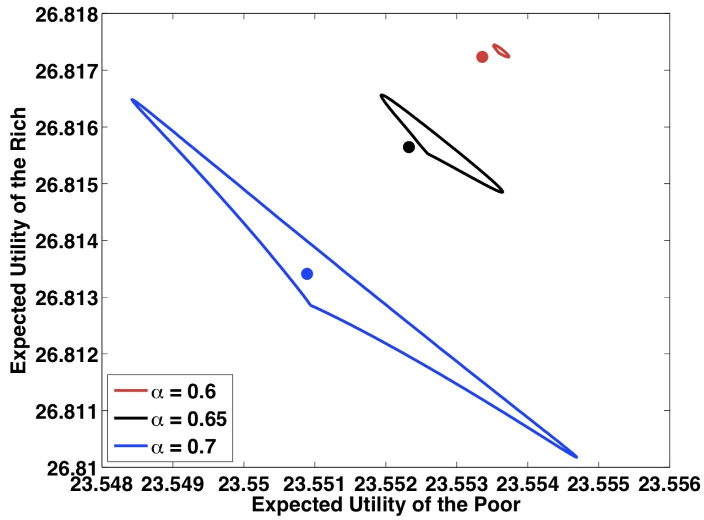



*Notes:* Figure 1.17 compares equilibrium tax policies of divided government at high and low political polarization levels for shock realization $\epsilon = 1$. Solid lines represent equilibrium tax policies at the high polarization level. Dashed lines represent equilibrium tax policies at the low polarization level. Figure 1.13 plots empirical cumulative density functions of policy gridlock duration for high and low polarization economies with divided government.

In Figure 1.19, I examine the welfare implications of political polarization. Here the solid dots indicate the welfare delivered by the unified government, while the solid lines indicate the welfare delivered by the divided government. As polarization decreases, the welfare of both unified and divided governments rises. Moreover, the range of status quo tax rates where divided government dominates the unified government expands. This is because both lower policy polarization and mild policy gridlock reduce the welfare loss created by policy uncertainty, and improve the welfare for each household in this economy.

**Discussion:** Given what happens in the United States today, one natural numer-

Figure 1.19: Equilibrium Welfare at Different Political Polarization Levels




*Notes:* Figure 1.19 plots equilibrium welfare pairs at $\alpha$ = 0.6, 0.65 and 0.7. Solid dots represent welfare pairs associated with unified government. Solid lines represent welfare pairs associated with divided government.

ical exercise would be to examine the welfare consequences at high inequality and polarization. After solving the model by setting rich income share and polarization parameters at various high levels, I found the welfare consequences involve scenarios where one or both types of households are better off with the divided government. Due to the large policy variation, there are no longer scenarios where the unified government strictly dominates the divided government.

In general, the analysis in this section is in line with the contemporary discussion on rising inequality, polarization, and policy gridlock in the United States. In addition to pointing out a potential mechanism linking rising income inequality and deepening policy gridlock, this section has also highlighted how the relative advantage of divided government varies with inequality and polarization. Moreover, the model implies that sending moderate policymakers to Washington can partially undo the welfare losses associated with the high income inequality.

## 1.8 Conclusion

This paper studies the welfare consequences of divided government in a dynamic legislative bargaining framework. By comparing unified with divided government, I show that with divided government the tax policy is both distorted and less responsive. However, since the gridlock arising from divided government also reduces policy uncertainty generated by political turnover, I find that divided government can Pareto dominate unified government. I also find that as political polarization and income inequality rises, despite the continuing existence of some initial status quo tax rates at which both households are better off with divided government, the set of such tax rates shrinks.

There are several avenues for future research based on this framework. One potentially interesting extension would be to consider an economy with divided government where inequality is endogenously determined and analyze whether and how policymakers fight inequality in the context of policy gridlock. Another possible extension would be to introduce government debt to the economy and explore how policy and welfare implications change. Finally, re-election in the current setting is completely exogenous. It would be interesting to endogenize re-election and consider how policymakers in divided government behave after taking into account how policy affects future re-election probabilities.

## 1.9 Appendices

### 1.9.1 Appendix A: Parameterization

This section provides the parameterization of the model. The length of the model period is one calendar year[6].

**Preference** I set discount factor $\beta$ at 0.9, risk aversion parameter $\gamma$ equal to 0.5, labor elasticity parameter $\nu$ equal to 1, and government spending elasticity parameter $\xi$ equal to 0.8. The income generating ability parameters $\bar{\theta}_r$ and $\bar{\theta}_p$ are chosen so that the average income shares of the rich and poor households in the model match the average income shares of the second (as a proxy for the median income of bottom 50%) and fourth quintile(as a proxy for the median income of top 50%) of U.S. before tax income distribution[7]. I choose valuation of public good parameter $A$ so that the average government spending as a share of GDP is around 21% [8]. The weight parameter that political party R places on the rich household $\alpha^R = 1 - \alpha^P$ is determined by matching the average duration of policy gridlock generated by the model with the average length of time when there is no individual income tax legislation change in United States. Using tax legislation changes summarized by Romer and Romer (2010)[9], I obtain that the average spell for U.S. individual income tax legislation inaction is 2.689 years.

**Productivity Shock** The process of log normal economy-wide productivity

[6]For calculation of the empirical counterparts, unless otherwise mentioned, the sample period is 1970 - 2012. This corresponds to the period when the United States experiences simultaneous rise in divided government, inequality, and political polarization.

[7]Source: Congressional Budget Office Website, supplemental data spreadsheet for *The Distribution of Household Income and Federal Taxes, 2008 and 2009*.

[8]Source: Congressional Budget Office website, *the U.S. federal budget: 1971 – 2011*

[9]Modifications have been made to include most recent income tax legislation changes and drop tax changes irrelevant to the individual income taxation.

shock $\epsilon_t$ is approximated with a discrete state Markov chain and the transition probabilities are chosen following Tauchen (1987). The persistence parameter $\rho$ is set at 0.95. The standard deviation parameter $\sigma$ is set at 0.0791 to match the empirical estimates of the variance of wage increase according to Heathcote et al. (2010).

**Power Persistence** To figure out power persistence parameter $q$, I compute the average probability of staying in office for the Republican and Democrat party by looking at historical party control information of White house and both houses of United States Congress from 1969 to 2012. $q = 0.76$ is used as the power persistence parameter in my model.

Table 1.4 summarizes the calibration targets and parameter values.

Table 1.4: Appendix: Parameterization

| Parameter | Interpretation | Target |
|---|---|---|
| $\beta$=0.9 | discount factor | - |
| $\gamma$=0.5 | preference parameter | - |
| $\nu$=1 | labor elasticity parameter | - |
| $\xi$=0.8 | spending elasticity parameter | - |
| $A$ = 0.4158 | valuation of public goods | spending as a share of GDP is on average 21% |
| $\bar{\theta}_r$ = 0.9657 | productivity of rich | average before tax income share of top 50% is around 68.4% |
| $\bar{\theta}_p$ = 0.6886 | productivity of poor | average before tax income share of bottom 50% is around 31.6% |
| $\rho$ = 0.95 | persistence of productivity shock | - |
| $\sigma$ = 0.0791 | std. deviation of productivity shock | estimate from Heathcote et al. (2010) |
| $\alpha$= 0.62 | welfare weight on own constituents | average spell of income tax legislation inaction is 2.689 years |
| $p$=0.76 | persistence of power | average probability of staying in office is 0.76 |

### 1.9.2 Appendix B: A Simple Analytic Example

In this section, I illustrate the occurrence of policy gridlock associated with divided government through a simplified analytical example. Consider an infinite-horizon economy with households' optimization problem and government's budget same as in section 1.2. Assume there is no political turnover and party R has proposal power throughout time. Furthermore, assume each economic agent in this economy is fully informed that the productivity shock realization stays constant forever (i.e. $\epsilon_t \equiv \epsilon, \forall t$). In each period, given status quo tax rate $\tau_{t-1}$, party R makes a take-it-or-leave-it offer $\tau_t$. If party P accepts the proposed policy, $\tau_t$ is implemented immediately. If party P rejects the proposed policy, $\tau_{t-1}$ is implemented instead. Assume in this section $\alpha^R = 1$ and $\alpha^P = 0$ so that each party only cares about welfare of its own constituents. To simplify the analytical derivations, let $\gamma = 0.5$, $\nu = 1$ and $\xi = 0$.

**Proposition 1.** *Assume there is no political turnover and the productivity shock stays constant (i.e. $\epsilon_t \equiv \epsilon, \ \forall t$). Let $\tau_R^*(\epsilon)$ and $\tau_P^*(\epsilon)$ denote the tax policy for party R and party P with unified government and $T^R(\tau_{t-1}, \epsilon)$ denote the equilibrium tax policy* with *divided government. Then*

$$\tau_R^*(\epsilon) = \frac{1}{2}\left[1 - \frac{\sqrt{2}\bar{\theta}_r}{A(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon}\right], \ \tau_P^*(\epsilon) = \frac{1}{2}\left[1 - \frac{\sqrt{2}\bar{\theta}_p}{A(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon}\right]$$

*and*

$$T^R(\tau_{t-1}, \epsilon) = \begin{cases} \tau_R^*(\epsilon) & \text{if } \tau_{t-1} \leq \tau_R^*(\epsilon) \\ \tau_{t-1} & \text{if } \tau_R^*(\epsilon) < \tau_{t-1} \leq \tau_P^*(\epsilon) \\ 2\tau_P^*(\epsilon) - \tau_{t-1} & \text{if } \tau_P^*(\epsilon) < \tau_{t-1} < 2\tau_P^*(\epsilon) - \tau_R^*(\epsilon) \\ \tau_R^*(\epsilon) & \text{if } \tau_{t-1} \geq 2\tau_P^*(\epsilon) - \tau_R^*(\epsilon) \end{cases}$$

*Furthermore, the equilibrium tax policy for this infinite horizon economy coincides with the equilibrium tax policy in a one-period economy with the same economic and political environment.*

*Proof.* In this proof, I first obtain the equilibrium tax policies under unified government and then focus on the equilibrium tax policies under divided government.

**Equilibrium Tax Policy Under Unified Government** As mentioned in Section 4.1, under unified government, the policy proposer simply chooses its desired tax rate and no approval from the opposition party is needed. Since current period's tax policy does not affect the expected future utility, the problem is essentially static.

Assume party R is in power. Under unified government, it simply implements its ideal tax policy by solving the following unconstrained maximization problem:

$$\max_{\tau_t} \quad \sqrt{2}(1-\tau_t)\bar{\theta}_r\epsilon + A\tau_t(1-\tau_t)(\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon^2$$

The optimal tax rate is given by

$$\tau_R^*(\epsilon) = \frac{1}{2}\left[1 - \frac{\sqrt{2}\bar{\theta}_r}{A(\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon}\right]$$

Similarly, if the poor is in power, the ideal tax policy of the poor is given by

$$\tau_P^*(\epsilon) = \frac{1}{2}\left[1 - \frac{\sqrt{2}\bar{\theta}_p}{A(\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon}\right]$$

Note that with unified government, the tax policies $\tau_R^*(\epsilon)$ and $\tau_P^*(\epsilon)$ depend only on shock realization $\epsilon$ and is independent of status quo tax rate $\tau_{t-1}$. As long as $\bar{\theta}_r > \bar{\theta}_p$, we have $\tau_R^*(\epsilon) < \tau_P^*(\epsilon)$.

**Equilibrium Tax Policy Under Divided Government** Now let us focus on the equilibrium tax policy behavior under divided government. For the rest of the proof, I will first derive the solution to a single period problem and use it as a guess for the infinite horizon problem. Then I will verify this guessed policy is the solution to the infinite horizon problem.

Assume party R has the proposal power and party P is the opponent. Let $\tau_0$ denote the initial status quo tax rate. Under divided government, the one-period problem for party R is as follows

$$\max_{\tau} \quad \sqrt{2}(1-\tau)\bar{\theta}_r\epsilon + A\tau(1-\tau)(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2$$

$$s.t.$$

$$\sqrt{2}(1-\tau)\bar{\theta}_p\epsilon + A\tau(1-\tau)(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 \geq \sqrt{2}(1-\tau_0)\bar{\theta}_p\epsilon + A\tau_0(1-\tau_0)(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2$$

1) Full Consensus

Assume the inequality constraint does not bind. Then party R simply implements its ideal tax policy $\tau_R^*(\epsilon)$. In order for $\tau_R^*(\epsilon)$ to be the optimal tax policy under divided government, we also need

$$\sqrt{2}(1-\tau^*)\bar{\theta}_p\epsilon + A\tau^*(1-\tau^*)(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 > \sqrt{2}(1-\tau_0)\bar{\theta}_p\epsilon + A\tau_0(1-\tau_0)(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2$$

which implies

$$\tau_0 < \tau_R^*(\epsilon) \quad \text{or} \quad \tau_0 > 2\tau_P^*(\epsilon) - \tau_R^*(\epsilon)$$

2) Policy Gridlock and Compromise

Assume the inequality constraint binds. Then the optimal tax policy implemented is implied by the solution to the quadratic equation

$$\sqrt{2}(1-\tau^*)\bar{\theta}_p\epsilon + A\tau^*(1-\tau^*)(\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon^2 = \sqrt{2}(1-\tau_0)\bar{\theta}_p\epsilon + A\tau_0(1-\tau_0)(\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon^2$$

the solution of which is given by

$$\tau_1 = \tau_0 \quad \text{and} \quad \tau_2 = 2\tau_P^*(\epsilon) - \tau_0$$

Since the two roots are not equal in general, the optimal tax policy is then determined by the one that generates higher utility. Under the assumption $\bar{\theta}_r > \bar{\theta}_p$, $\sqrt{2}(1-\tau_1)\bar{\theta}_r\epsilon + A\tau_1(1-\tau_1)(\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon^2 \geq \sqrt{2}(1-\tau_2)\bar{\theta}_r\epsilon + A\tau_2(1-\tau_2)(\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon^2$ if and only if $\tau_0 \leq \tau_P^*(\epsilon)$. Hence we have

$$\tau^* = \begin{cases} \tau_0 & \text{if } \tau_0 \leq \tau_P^*(\epsilon) \\ 2\tau_P^*(\epsilon) - \tau_0 & \text{if } \tau_0 > \tau_P^*(\epsilon) \end{cases}$$

First observe that the cutoff point is the ideal tax policy of party P. Also, note that within the interval $[\tau_R^*(\epsilon), \tau_P^*(\epsilon)]$, the optimal tax policy is to implement previous period's tax rate. i.e. the tax policy is completely gridlocked. Within the interval $[\tau_P^*(\epsilon), 2\tau_P^*(\epsilon) - \tau_R^*(\epsilon)]$, the optimal tax policy is implemented at the level that is lower than the status quo.

Summing up above two scenarios, under divided government, the tax policy of party R for the one period problem is characterized by

$$\tau^* = \begin{cases} \tau_R^*(\epsilon) & \text{if } \tau_0 \leq \tau_R^*(\epsilon) \\ \tau_0 & \text{if } \tau_R^*(\epsilon) < \tau_0 \leq \tau_P^*(\epsilon) \\ 2\tau_P^*(\epsilon) - \tau_0 & \text{if } \tau_P^*(\epsilon) < \tau_0 < 2\tau_P^*(\epsilon) - \tau_R^*(\epsilon) \\ \tau_R^*(\epsilon) & \text{if } \tau_0 \geq 2\tau_P^*(\epsilon) - \tau_R^*(\epsilon) \end{cases}$$

Now I will guess and verify that under divided government, the equilibrium solution of party R for the infinite horizon problem $T^R(\tau_{t-1}, \epsilon)$ takes the same form as $\tau^*$, i.e.

$$T^R(\tau_{t-1}, \epsilon) = \begin{cases} \tau_R^*(\epsilon) & \text{if } \tau_{t-1} \leq \tau_R^*(\epsilon) \\ \tau_{t-1} & \text{if } \tau_R^*(\epsilon) < \tau_{t-1} \leq \tau_P^*(\epsilon) \\ 2\tau_P^*(\epsilon) - \tau_{t-1} & \text{if } \tau_P^*(\epsilon) < \tau_{t-1} < 2\tau_P^*(\epsilon) - \tau_R^*(\epsilon) \\ \tau_R^*(\epsilon) & \text{if } \tau_{t-1} \geq 2\tau_P^*(\epsilon) - \tau_R^*(\epsilon) \end{cases}$$

I will proceed the guess-and-verify procedure in three steps. First, I will show the guessed equilibrium policy satisfies $T^R(\tau_{t-1}, \epsilon) \in \left[\tau_R^*(\epsilon), \tau_P^*(\epsilon)\right]$. In particular, for arbitrary $\tau_t \in \left[\tau_R^*(\epsilon), \tau_P^*(\epsilon)\right]$, $T^R(\tau_t, \epsilon) = \tau_t$. Second, I will obtain the value functions associated with the guessed policy functions. Finally, I will show based on these value functions, the equilibrium policy indeed coincides with the guessed value.

**Lemma 1.** *$T^R(\tau_{t-1}, \epsilon) \in [\tau_R^*(\epsilon), \tau_P^*(\epsilon)]$ and $T^R\left(T^R(\tau_{t-1}, \epsilon), \epsilon\right) = T^R(\tau_{t-1}, \epsilon)$ $\forall \tau_{t-1}$.*

*Proof.* When $\tau_{t-1} \leq \tau_R^*(\epsilon)$ or $\tau_{t-1} \geq 2\tau_P^*(\epsilon) - \tau_R^*(\epsilon)$, $T^R(\tau_{t-1}, \epsilon) = \tau_R^*(\epsilon) \in [\tau_R^*(\epsilon), \tau_P^*(\epsilon)]$.
When $\tau_{t-1} \in (\tau_R^*(\epsilon), \tau_P^*(\epsilon)]$, $T^R(\tau_{t-1}, \epsilon) = \tau_{t-1} \in [\tau_R^*(\epsilon), \tau_P^*(\epsilon)]$.
When $\tau_{t-1} \in (\tau_P^*(\epsilon), 2\tau_P^*(\epsilon) - \tau_R^*(\epsilon)]$, $T^R(\tau_{t-1}, \epsilon) = 2\tau_P^*(\epsilon) - \tau_{t-1} \in [\tau_P^*(\epsilon), 2\tau_P^*(\epsilon) - \tau_R^*(\epsilon)]$.

Therefore, for arbitrary $\tau_{t-1}$, we have $T^R(\tau_{t-1}, \epsilon) \in [\tau_R^*(\epsilon), \tau_P^*(\epsilon)]$. Since $T^R(\tau_{t-1}, \epsilon) = \tau_{t-1}$ for $\tau_{t-1} \in [\tau_R^*(\epsilon), \tau_P^*(\epsilon)]$ and $T^R(\tau_{t-1}, \epsilon) \in [\tau_R^*(\epsilon), \tau_P^*(\epsilon)]$, we have $T^R\left(T^R(\tau_{t-1}, \epsilon), \epsilon\right) = T^R(\tau_{t-1}, \epsilon)$. $\square$

Intuitively, what Lemma 1 says is with divided government, for arbitrary status quo tax rate, the guessed equilibrium tax policy will be in-between the desired tax policy of party R and party P. Moreover, once the guessed equilibrium tax policy falls into the gridlock region, it will stay there forever.

**Lemma 2.** *Assume party R is in power. Under divided government, let $V(\tau_t, \epsilon)$ and $W(\tau_t, \epsilon)$ denote the dynamic payoffs for party R and party P at status quo tax rate $\tau_t$ and shock realization $\epsilon$ respectively. Based on the guessed equilibrium tax policy $T^R(\tau_{t-1}, \epsilon)$, we have*

$$V\left(T^R(\tau_t, \epsilon), \epsilon\right) = \frac{1}{1-\beta}\left\{\sqrt{2}(1 - T^R(\tau_t, \epsilon))\bar{\theta}_r\epsilon + AT^R(\tau_t, \epsilon)(1 - T^R(\tau_t, \epsilon))(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2\right\}$$
$$W\left(T^R(\tau_t, \epsilon), \epsilon\right) = \frac{1}{1-\beta}\left\{\sqrt{2}(1 - T^R(\tau_t, \epsilon))\bar{\theta}_p\epsilon + AT^R(\tau_t, \epsilon)(1 - T^R(\tau_t, \epsilon))(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2\right\}$$

*Proof.* By definition, we know that the dynamic payoff for party R at status quo tax rate $T^R(\tau_t, \epsilon_t)$ is

$$\begin{aligned}V\left(T^R(\tau_t, \epsilon), \epsilon\right) = {} & \sqrt{2}\left(1 - T^R(T^R(\tau_t, \epsilon), \epsilon)\right)\bar{\theta}_r\epsilon + AT^R\left(T^R(\tau_t, \epsilon), \epsilon\right)\left(1 - T^R(T^R(\tau_t, \epsilon), \epsilon)\right)(\bar{\theta}_r^2 + \bar{\theta}_p^2) \\ & +\beta V\left(T^R(T^R(\tau_t, \epsilon), \epsilon), \epsilon\right)\end{aligned}$$

Using Lemma 1, we know $T^R\left(T^R(\tau_t, \epsilon), \epsilon\right) = T^R(\tau_t, \epsilon)\ \forall \tau_t$. Hence

$$V\left(T^R(\tau_t, \epsilon), \epsilon\right) = \sqrt{2}\left(1 - T^R(\tau_t, \epsilon)\right)\bar{\theta}_r\epsilon + AT^R(\tau_t, \epsilon)(1 - T^R(\tau_t, \epsilon))(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 + \beta V\left(T^R(\tau_t, \epsilon), \epsilon\right)$$

Rearrange and solve for $V\left(T^R(\tau_t,\epsilon),\epsilon\right)$, we get for arbitrary $\tau_t$,

$$V\left(T^R(\tau_t,\epsilon),\epsilon\right) = \frac{1}{1-\beta}\left\{\sqrt{2}(1-T^R(\tau_t,\epsilon))\bar{\theta}_r\epsilon + AT^R(\tau_t,\epsilon)(1-T^R(\tau_t,\epsilon))(\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon^2\right\}$$

Similarly, we can obtain

$$W\left(T^R(\tau_t,\epsilon),\epsilon\right) = \frac{1}{1-\beta}\left\{\sqrt{2}(1-T^R(\tau_t,\epsilon))\bar{\theta}_p\epsilon + AT^R(\tau_t,\epsilon)(1-T^R(\tau_t,\epsilon))(\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon^2\right\}$$

Essentially, since the equilibrium tax policy of party R stays constant within the gridlock region, the periodic utility of both parties stay the same over time. The dynamic payoff is hence the discounted sum of these periodic utilities. □

Now I will proceed to verify the equilibrium tax policy for party R under divided government coincides with the initial guess. I will show that given $V(T^R(\tau_t,\epsilon),\epsilon)$ and $W(T^R(\tau_t,\epsilon),\epsilon)$, the optimal tax rate for status quo $\tau_{t-1}$ and shock realization $\epsilon$ takes same form as $T(\cdot,\epsilon)$, implying $V(\cdot,\epsilon)$ and $W(\cdot,\epsilon)$ are the fixed points of this problem.

First note that

$$V(\tau_t,\epsilon) = \sqrt{2}(1-T^R(\tau_t,\epsilon))\bar{\theta}_r\epsilon + AT^R(\tau_t,\epsilon)(1-T^R(\tau_t,\epsilon))(\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon^2 + \beta V(T^R(\tau_t,\epsilon),\epsilon) \tag{1.28}$$

Replacing $V(T^R(\tau_t,\epsilon),\epsilon)$ using Lemma2, we get

$$V(\tau_t,\epsilon) = \frac{1}{1-\beta}\left\{\sqrt{2}(1-T^R(\tau_t,\epsilon))\bar{\theta}_r\epsilon + AT^R(\tau_t,\epsilon)(1-T^R(\tau_t,\epsilon))(\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon^2\right\} \tag{1.29}$$

Similarly,

$$W(\tau_t, \epsilon) = \frac{1}{1-\beta}\left\{ \sqrt{2}(1 - T^R(\tau_t, \epsilon))\bar{\theta}_p \epsilon + AT^R(\tau_t, \epsilon)(1 - T^R(\tau_t, \epsilon))(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 \right\} \quad (1.30)$$

Now party R's optimization problem under divided government can be rewritten as

$$\begin{aligned}
V(\tau_{t-1}, \epsilon) = \max_{\tau_t} \quad & \sqrt{2}(1 - \tau_t)\bar{\theta}_r \epsilon + A\tau_t(1 - \tau_t)(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 + \\
& \frac{\beta}{1-\beta}\left\{ \sqrt{2}(1 - T^R(\tau_t, \epsilon))\bar{\theta}_r \epsilon + AT^R(\tau_t, \epsilon)(1 - T^R(\tau_t, \epsilon))(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 \right\} \\
s.t. \quad & \\
& \sqrt{2}(1 - \tau_t)\bar{\theta}_p \epsilon + A\tau_t(1 - \tau_t)(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 + \\
& \frac{\beta}{1-\beta}\left\{ \sqrt{2}(1 - T^R(\tau_t, \epsilon))\bar{\theta}_p \epsilon + AT^R(\tau_t, \epsilon)(1 - T^R(\tau_t, \epsilon))(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 \right\} \geq \\
& \sqrt{2}(1 - \tau_{t-1})\bar{\theta}_p \epsilon + A\tau_{t-1}(1 - \tau_{t-1})(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 + \\
& \frac{\beta}{1-\beta}\left\{ \sqrt{2}(1 - T^R(\tau_{t-1}, \epsilon))\bar{\theta}_p \epsilon + AT^R(\tau_{t-1}, \epsilon)(1 - T^R(\tau_{t-1}, \epsilon))(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 \right\}
\end{aligned}$$

Since $T^R(\tau_{t-1}, \epsilon)$ takes different functional forms depending on $\tau_{t-1}$, I will discuss equilibrium tax policies for different cases. Under each case, because the choice of $\tau_t$ also affects the functional form of $T^R(\tau_t, \epsilon)$, I will solve for the equilibrium tax policy $\tau_t$ under different functional assumptions and then choose the one that delivers the maximum dynamic payoff.

**Case 1:** $\tau_{t-1} \leq \tau_R^*(\epsilon)$

Since $\tau_{t-1} \leq \tau_R^*(\epsilon)$, $T(\tau_{t-1}, \epsilon) = \tau_R^*(\epsilon)$. Hence

$$
\begin{aligned}
W(\tau_{t-1}, \epsilon) &= \sqrt{2}(1-\tau_{t-1})\bar{\theta}_p\epsilon + A\tau_{t-1}(1-\tau_{t-1})(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 + \beta W(T(\tau_{t-1}, \epsilon), \epsilon) \\
&= \sqrt{2}(1-\tau_{t-1})\bar{\theta}_p\epsilon + A\tau_{t-1}(1-\tau_{t-1})(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 + \beta W(\tau_R^*(\epsilon), \epsilon) \\
&= \sqrt{2}(1-\tau_{t-1})\bar{\theta}_p\epsilon + A\tau_{t-1}(1-\tau_{t-1})(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 + \\
&\quad \frac{\beta}{1-\beta}\left\{ \sqrt{2}(1-\tau_R^*(\epsilon))\bar{\theta}_p\epsilon + A\tau_R^*(\epsilon)(1-\tau_R^*(\epsilon))(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 \right\}
\end{aligned}
$$

1) suppose $\tau_t \leq \tau_R^*(\epsilon)$, then $T(\tau_t, \epsilon) = \tau_R^*(\epsilon)$. The optimization problem becomes

$$
\begin{aligned}
V(\tau_{t-1}, \epsilon) = \max_{\tau_t} \;& \sqrt{2}(1-\tau_t)\bar{\theta}_r\epsilon + A\tau_t(1-\tau_t)(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 + \\
& \frac{\beta}{1-\beta}\left\{ \sqrt{2}(1-\tau_R^*(\epsilon))\bar{\theta}_r\epsilon + A\tau_R^*(\epsilon)(1-\tau_R^*(\epsilon)))(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 \right\}
\end{aligned}
$$

$s.t.$

$$
\sqrt{2}(1-\tau_t)\bar{\theta}_p\epsilon + A\tau_t(1-\tau_t)(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 + \frac{\beta}{1-\beta}\left\{ \sqrt{2}(1-\tau_R^*(\epsilon))\bar{\theta}_p\epsilon + A\tau_R^*(\epsilon)(1-\tau_R^*(\epsilon))(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 \right\} \geq
$$

$$
\sqrt{2}(1-\tau_{t-1})\bar{\theta}_p\epsilon + A\tau_{t-1}(1-\tau_{t-1})(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 + \frac{\beta}{1-\beta}\left\{ \sqrt{2}(1-\tau_R^*(\epsilon))\bar{\theta}_p\epsilon + A\tau_R^*(\epsilon)(1-\tau_R^*(\epsilon))(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 \right\}
$$

which could be simplified into following static problem

$$
V(\tau_{t-1}, \epsilon) = \max_{\tau_t} \; \sqrt{2}(1-\tau_t)\bar{\theta}_r\epsilon + A\tau_t(1-\tau_t)(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2
$$

$s.t.$

$$
\sqrt{2}(1-\tau_t)\bar{\theta}_p\epsilon + A\tau_t(1-\tau_t)(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 \geq \sqrt{2}(1-\tau_{t-1})\bar{\theta}_p\epsilon + A\tau_{t-1}(1-\tau_{t-1})(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2
$$

From the result of one period problem, we know that when $\tau_{t-1} \leq \tau_R^*(\epsilon)$, the inequality constraint does not bind and $\tau_t^* = \tau_R^*(\epsilon)$. Since $\tau_t^*$ corresponds to the unconstrained maximum, we know that there does not exist any alternative tax policy that delivers higher dynamic payoff.

**Case 2:** $\tau_{t-1} \geq 2\tau_2^* - \tau_R^*(\epsilon)$

When $\tau_{t-1} \geq 2\tau_2^* - \tau_R^*(\epsilon)$, $T(\tau_{t-1}, \epsilon) = \tau_R^*(\epsilon)$. Again,

$$\begin{aligned}
W(\tau_{t-1}, \epsilon) &= \sqrt{2}(1-\tau_{t-1})\bar{\theta}_p\epsilon + A\tau_{t-1}(1-\tau_{t-1})(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 + \beta W(T(\tau_{t-1}, \epsilon), \epsilon) \\
&= \sqrt{2}(1-\tau_{t-1})\bar{\theta}_p\epsilon + A\tau_{t-1}(1-\tau_{t-1})(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 + \\
&\quad \frac{\beta}{1-\beta}\left\{\sqrt{2}(1-\tau_R^*(\epsilon))\bar{\theta}_p\epsilon^2 + A\tau_R^*(\epsilon)(1-\tau_R^*(\epsilon))(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2\right\}
\end{aligned}$$

Suppose $\tau_t \leq \tau_R^*(\epsilon)$, the optimization problem again boils down to following static problem:

$$V(\tau_{t-1}, \epsilon) = \max_{\tau_t} \quad \sqrt{2}(1-\tau_t)\bar{\theta}_r\epsilon + A\tau_t(1-\tau_t)(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2$$

$$s.t.$$

$$\sqrt{2}(1-\tau_t)\bar{\theta}_p\epsilon + A\tau_t(1-\tau_t)(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 \geq \sqrt{2}(1-\tau_{t-1})\bar{\theta}_p\epsilon + A\tau_{t-1}(1-\tau_{t-1})(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2$$

Using similar argument as in Case 1, we can show when $\tau_{t-1} \geq 2\tau_P^*(\epsilon) - \tau_R^*(\epsilon)$, $\tau_t^* = \tau_R^*(\epsilon)$.

**Case 3:** $\tau_R^*(\epsilon) \leq \tau_{t-1} \leq \tau_P^*(\epsilon)$

When $\tau_R^*(\epsilon) \leq \tau_{t-1} \leq \tau_P^*(\epsilon)$, $T(\tau_{t-1}, \epsilon) = \tau_{t-1}$ and hence

$$\begin{aligned}
W(\tau_{t-1}, \epsilon) &= \sqrt{2}(1-\tau_{t-1})\bar{\theta}_p\epsilon + A\tau_{t-1}(1-\tau_{t-1})(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 + \beta W(T(\tau_{t-1}, \epsilon), \epsilon) \\
&= \sqrt{2}(1-\tau_{t-1})\bar{\theta}_p\epsilon + A\tau_{t-1}(1-\tau_{t-1})(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 + \beta W(\tau_{t-1}, \epsilon) \\
&= \frac{1}{1-\beta}\left\{\sqrt{2}(1-\tau_{t-1})\bar{\theta}_p\epsilon + A\tau_{t-1}(1-\tau_{t-1})(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2\right\}
\end{aligned}$$

1) Suppose $\tau_R^*(\epsilon) < \tau_t \leq \tau_P^*(\epsilon)$. Then $T(\tau_t, \epsilon) = \tau_t$. The maximization problem of party R can be rewritten as

$$V(\tau_{t-1}, \epsilon) = \max_{\tau_t} \frac{1}{1-\beta}\left\{\sqrt{2}(1-\tau_t)\bar{\theta}_r\epsilon + A\tau_t(1-\tau_t)(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2\right\}$$

$s.t.$

$$\frac{1}{1-\beta}\left\{\sqrt{2}(1-\tau_t)\bar{\theta}_p\epsilon + A\tau_t(1-\tau_t)(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2\right\} \geq \frac{1}{1-\beta}\left\{\sqrt{2}(1-\tau_{t-1})\bar{\theta}_p\epsilon + A\tau_{t-1}(1-\tau_{t-1})(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2\right\}$$

Again, the problem simplifies into a static optimization. Using the fact $\tau_R^*(\epsilon) < \tau_{t-1} \leq \tau_P^*(\epsilon)$ and the result for one period problem, we get $\tau_t^* = \tau_{t-1}$.

2) Suppose $\tau_t \leq \tau_R^*(\epsilon)$. Then $T(\tau_t, \epsilon) = \tau_R^*(\epsilon)$. The optimization problem can be simplified into

$$\begin{aligned} V(\tau_{t-1}, \epsilon) = \max_{\tau_t} \ & \sqrt{2}(1-\tau_t)\bar{\theta}_r\epsilon + A\tau_t(1-\tau_t)(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 + \\ & \frac{\beta}{1-\beta}\left\{\sqrt{2}(1-\tau_R^*(\epsilon))\bar{\theta}_r\epsilon + A\tau_R^*(\epsilon)(1-\tau_R^*(\epsilon))(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2\right\} \end{aligned}$$

$s.t.$

$$\sqrt{2}(1-\tau_t)\bar{\theta}_p\epsilon + A\tau_t(1-\tau_t)(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 + \frac{\beta}{1-\beta}\left\{\sqrt{2}(1-\tau_R^*(\epsilon))\bar{\theta}_p\epsilon + A\tau_R^*(\epsilon)(1-\tau_R^*(\epsilon))(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2\right\} \geq$$
$$\frac{1}{1-\beta}\left\{\sqrt{2}(1-\tau_{t-1})\bar{\theta}_p\epsilon + A\tau_{t-1}(1-\tau_{t-1})(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2\right\}$$

If the inequality constraint does not bind, $\tau_t^* = \tau_R^*(\epsilon)$. This, however, could not be true since for $\tau_R^*(\epsilon) < \tau_{t-1} \leq \tau_P^*(\epsilon)$, we have

$$\begin{aligned} &\frac{1}{1-\beta}\left\{\sqrt{2}(1-\tau_{t-1})\bar{\theta}_p\epsilon + A\tau_{t-1}(1-\tau_{t-1})(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2\right\} > \\ &\frac{1}{1-\beta}\left\{\frac{(1-\tau_R^*(\epsilon))\bar{\theta}_p\epsilon}{\sqrt{2}} + A\tau_R^*(\epsilon)(1-\tau_R^*(\epsilon))(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2\right\} \end{aligned}$$

Now consider the case when inequality constraint does bind, i.e.

$$\sqrt{2}(1-\tau_t)\bar{\theta}_p\epsilon + A\tau_t(1-\tau_t)(\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon^2 + \frac{\beta}{1-\beta}\left\{\sqrt{2}(1-\tau_R^*(\epsilon))\bar{\theta}_p\epsilon + A\tau_R^*(\epsilon)(1-\tau_R^*(\epsilon))(\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon^2\right\} =$$
$$\frac{1}{1-\beta}\left\{\sqrt{2}(1-\tau_{t-1})\bar{\theta}_p\epsilon + A\tau_{t-1}(1-\tau_{t-1})(\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon^2\right\}$$

Rearrange, we have

$$\sqrt{2}(1-\tau_t)\bar{\theta}_p\epsilon + A\tau_t(1-\tau_t)(\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon^2 =$$
$$\frac{1}{1-\beta}\left\{\sqrt{2}(1-\tau_{t-1})\bar{\theta}_p\epsilon + A\tau_{t-1}(1-\tau_{t-1})(\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon^2\right\} -$$
$$\frac{\beta}{1-\beta}\left\{\sqrt{2}(1-\tau_R^*(\epsilon))\bar{\theta}_p\epsilon + A\tau_R^*(\epsilon)(1-\tau_R^*(\epsilon))(\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon^2\right\}$$

Using the fact $\tau_R^*(\epsilon) \leq \tau_{t-1}$, we know

$$\frac{1}{1-\beta}\left\{\sqrt{2}(1-\tau_{t-1})\bar{\theta}_p\epsilon + A\tau_{t-1}(1-\tau_{t-1})(\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon^2\right\} -$$
$$\frac{\beta}{1-\beta}\left\{\sqrt{2}(1-\tau_R^*(\epsilon))\bar{\theta}_p\epsilon + A\tau_R^*(\epsilon)(1-\tau_R^*(\epsilon))(\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon^2\right\} \geq$$
$$\left\{\sqrt{2}(1-\tau_{t-1})\bar{\theta}_p\epsilon + A\tau_{t-1}(1-\tau_{t-1})(\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon^2\right\}$$

which implies $\tau_t^* > \tau_{t-1} > \tau_R^*(\epsilon)$. This contradicts the assumption $\tau_t \leq \tau_R^*(\epsilon)$.

3) Suppose $\tau_P^*(\epsilon) < \tau_t \leq 2\tau_P^*(\epsilon) - \tau_R^*(\epsilon)$. Then $T(\tau_t,\epsilon) = 2\tau_P^*(\epsilon) - \tau_t$. The optimization

problem can be rewritten as

$$
\begin{aligned}
&V(\tau_{t-1},\epsilon) = \max_{\tau_t} \quad \sqrt{2}(1-\tau_t)\bar{\theta}_r\epsilon + A\tau_t(1-\tau_t)(\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon^2 \\
&+ \frac{\beta}{1-\beta}\left\{\sqrt{2}[1-(2\tau_P^*(\epsilon)-\tau_t)]\bar{\theta}_r\epsilon + A(2\tau_P^*(\epsilon)-\tau_t)[1-(2\tau_P^*(\epsilon)-\tau_t)](\bar{\theta}_r^2+\bar{\theta}_p^2)\right\} \\
&s.t. \\
&\sqrt{2}(1-\tau_t)\bar{\theta}_p\epsilon + A\tau_t(1-\tau_t)(\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon^2 + \\
&\frac{\beta}{1-\beta}\left\{\sqrt{2}[1-(2\tau_P^*(\epsilon)-\tau_t)]\bar{\theta}_p\epsilon + A(2\tau_P^*(\epsilon)-\tau_t)[1-(2\tau_P^*(\epsilon)-\tau_t)](\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon^2\right\} \geq \\
&\frac{1}{1-\beta}\left\{\sqrt{2}(1-\tau_{t-1})\bar{\theta}_p\epsilon + A\tau_{t-1}(1-\tau_{t-1})(\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon^2\right\}
\end{aligned}
$$

If the inequality constraint does not bind, $\tau_t^* = (1-2\beta)\tau_R^*(\epsilon) + 2\beta\tau_P^*(\epsilon) \in (\tau_P^*(\epsilon), 2\tau_P^*(\epsilon) - \tau_R^*(\epsilon)]$. It is not difficult to show if $\tau_t^* \in (\tau_P^*(\epsilon), 2\tau_P^*(\epsilon) - \tau_{t-1}]$, the value function delivered by $\tau_t^*$ is less than that of $\tau_{t-1}$.
If the constraint binds, it can be simplified to

$$
\sqrt{2}(1-\tau_t)\bar{\theta}_p\epsilon + A\tau_t(1-\tau_t)(\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon^2 = \sqrt{2}(1-\tau_{t-1})\bar{\theta}_p\epsilon + A\tau_{t-1}(1-\tau_{t-1})(\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon^2
$$

which implies $\tau_t^* = 2\tau_P^*(\epsilon) - \tau_{t-1}$ and

$$
\begin{aligned}
V(\tau_{t-1},\epsilon) = &\sqrt{2}[1-(2\tau_P^*(\epsilon)-\tau_{t-1})]\bar{\theta}_r\epsilon + A(2\tau_P^*(\epsilon)-\tau_{t-1})[1-(2\tau_P^*(\epsilon)-\tau_{t-1})](\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon^2 + \\
&\frac{\beta}{1-\beta}\left\{\sqrt{2}(1-\tau_{t-1})\bar{\theta}_r\epsilon + A\tau_{t-1}(1-\tau_{t-1})(\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon^2\right\}
\end{aligned}
$$

We can show that as long as $\tau_R^*(\epsilon) < \tau_{t-1} \leq \tau_P^*(\epsilon)$, above value function is less than the value function derived with $\tau_t^* = \tau_{t-1}$.

4) Suppose $\tau_t \geq 2\tau_P^*(\epsilon) - \tau_R^*(\epsilon)$. Then $T(\tau_t,\epsilon) = \tau_R^*(\epsilon)$. The optimization problem

simplifies into same as 2).

If the inequality constraint does not bind, $\tau_t^* = \tau_R^*(\epsilon)$, which contradicts with the assumption.

If the inequality constraint does bind, we can show by argument similar to 2) that $\tau_t^* < 2\tau_P^*(\epsilon) - \tau_{t-1} < 2\tau_P^*(\epsilon) - \tau_R^*(\epsilon)$, which contradicts our assumption.

**Case 4:** $\tau_P^*(\epsilon) < \tau_{t-1} < 2\tau_P^*(\epsilon) - \tau_R^*(\epsilon)$

Since $\tau_P^*(\epsilon) < \tau_{t-1} < 2\tau_P^*(\epsilon) - \tau_R^*(\epsilon)$, $T(\tau_{t-1}, \epsilon) = 2\tau_P^*(\epsilon) - \tau_{t-1}$. Hence

$$\begin{aligned}
W(\tau_{t-1}, \epsilon) &= \sqrt{2}(1-\tau_{t-1})\bar{\theta}_p\epsilon + A\tau_{t-1}(1-\tau_{t-1})(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 + \beta W(T(\tau_{t-1}, \epsilon), \epsilon) \\
&= \sqrt{2}(1-\tau_{t-1})\bar{\theta}_p\epsilon + A\tau_{t-1}(1-\tau_{t-1})(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 + \beta W(2\tau_P^*(\epsilon) - \tau_{t-1}, \epsilon) \\
&= \sqrt{2}(1-\tau_{t-1})\bar{\theta}_p\epsilon + A\tau_{t-1}(1-\tau_{t-1})(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 + \\
&\quad \frac{\beta}{1-\beta}\left\{\sqrt{2}[1-(2\tau_P^*(\epsilon) - \tau_{t-1})]\bar{\theta}_p\epsilon + A(2\tau_P^*(\epsilon) - \tau_{t-1})[1-(2\tau_P^*(\epsilon) - \tau_{t-1})](\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2\right\} \\
&= \frac{1}{1-\beta}\left\{\sqrt{2}(1-\tau_{t-1})\bar{\theta}_p\epsilon + A\tau_{t-1}(1-\tau_{t-1})(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2\right\}
\end{aligned}$$

1) Suppose $\tau_R^*(\epsilon) < \tau_t \leq \tau_P^*(\epsilon)$. Then $T(\tau_t, \epsilon) = \tau_t$. Party R's optimization problem becomes

$$V(\tau_{t-1}, \epsilon) = \max_{\tau_t} \frac{1}{1-\beta}\left\{\sqrt{2}(1-\tau_t)\bar{\theta}_r\epsilon + A\tau_t(1-\tau_t)(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2\right\}$$

$s.t.$

$$\begin{aligned}
&\frac{1}{1-\beta}\left\{\sqrt{2}(1-\tau_t)\bar{\theta}_p\epsilon + A\tau_t(1-\tau_t)(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2\right\} \geq \\
&\sqrt{2}(1-\tau_{t-1})\bar{\theta}_p\epsilon + A\tau_{t-1}(1-\tau_{t-1})(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 \\
&+ \frac{\beta}{1-\beta}\left\{\sqrt{2}[1-(2\tau_P^*(\epsilon) - \tau_{t-1})]\bar{\theta}_p\epsilon + A(2\tau_P^*(\epsilon) - \tau_{t-1})[1-(2\tau_P^*(\epsilon) - \tau_{t-1})](\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2\right\}
\end{aligned}$$

Note the inequality constraint can be rewritten as

$$\sqrt{2}(1-\tau_t)\bar{\theta}_p\epsilon + A\tau_t(1-\tau_t)(\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon^2 \geq \sqrt{2}(1-\tau_{t-1})\bar{\theta}_p\epsilon + A\tau_{t-1}(1-\tau_{t-1})(\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon^2$$

If inequality constraint does not bind, then $\tau_t^* = \tau_R^*(\epsilon)$, which contradicts the assumption $\tau_R^*(\epsilon) < \tau_t \leq \tau_P^*(\epsilon)$. If inequality constraint does bind, then $\tau_t^* = 2\tau_P^*(\epsilon) - \tau_{t-1} \in (\tau_R^*(\epsilon), \tau_P^*(\epsilon)]$.

2) Suppose $\tau_t \leq \tau_R^*(\epsilon)$. Then $T(\tau_t, \epsilon) = \tau_R^*(\epsilon)$. The optimization problem can be rewritten as

$$\begin{aligned} V(\tau_{t-1},\epsilon) = \max_{\tau_t} \ & \sqrt{2}(1-\tau_t)\bar{\theta}_r\epsilon + A\tau_t(1-\tau_t)(\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon^2 + \\ & \frac{\beta}{1-\beta}\left\{\sqrt{2}(1-\tau_R^*(\epsilon))\bar{\theta}_r\epsilon + A\tau_R^*(\epsilon)(1-\tau_R^*(\epsilon))(\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon^2\right\} \end{aligned}$$

$s.t.$

$$\sqrt{2}(1-\tau_t)\bar{\theta}_p\epsilon + A\tau_t(1-\tau_t)(\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon^2 + \frac{\beta}{1-\beta}\left\{\sqrt{2}(1-\tau_R^*(\epsilon))\bar{\theta}_p\epsilon + A\tau_R^*(\epsilon)(1-\tau_R^*(\epsilon))(\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon^2\right\} \geq$$
$$\frac{1}{1-\beta}\left\{\sqrt{2}(1-\tau_{t-1})\bar{\theta}_p\epsilon + A\tau_{t-1}(1-\tau_{t-1})(\bar{\theta}_r^2+\bar{\theta}_p^2)\epsilon^2\right\}$$

If the inequality constraint does not bind, $\tau_t^* = \tau_R^*(\epsilon)$. This, however, violates the inequality constraint.

If the inequality constraint binds, $\tau_t^* \in (2\tau_P^*(\epsilon) - \tau_{t-1}, \tau_{t-1})$. This contradicts $\tau_t \leq$

$\tau^*_R(\epsilon)$ since $\tau_{t-1} \in (\tau^*_P(\epsilon), 2\tau^*_P(\epsilon) - \tau^*_R(\epsilon))$. To show $\tau^*_t \in (2\tau^*_P(\epsilon) - \tau_{t-1}, \tau_{t-1})$, note that

$$
\begin{aligned}
&\sqrt{2}(1-\tau_t)\bar{\theta}_p\epsilon + A\tau_t(1-\tau_t)(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 \\
&= \frac{1}{1-\beta}\left\{\sqrt{2}(1-\tau_{t-1})\bar{\theta}_p\epsilon + A\tau_{t-1}(1-\tau_{t-1})(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2\right\} - \\
&\quad \frac{\beta}{1-\beta}\left\{\sqrt{2}(1-\tau^*_R(\epsilon))\bar{\theta}_p\epsilon + A\tau^*_R(\epsilon)(1-\tau^*_R(\epsilon))(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2\right\} \\
&> \sqrt{2}(1-\tau_{t-1})\bar{\theta}_p\epsilon + A\tau_{t-1}(1-\tau_{t-1})(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2
\end{aligned}
$$

which implies $\tau^*_t \in (2\tau^*_P(\epsilon) - \tau_{t-1}, \tau_{t-1})$.

3) Suppose $\tau_t \geq 2\tau^*_P(\epsilon) - \tau^*_R(\epsilon)$. Then $T(\tau_t, \epsilon) = \tau^*_R(\epsilon)$. The optimization problem can be rewritten as same in 2).

If the inequality constraint does not bind, $\tau^*_t = \tau^*_R(\epsilon)$. Again, this violates the inequality constraint.

If the inequality constraint binds, similar to 2), $\tau^*_t \in (2\tau^*_P(\epsilon) - \tau_{t-1}, \tau_{t-1})$. Since $\tau_{t-1} < 2\tau^*_P(\epsilon) - \tau^*_R(\epsilon)$, this contradicts with our assumption.

4) Suppose $\tau^*_P(\epsilon) < \tau_t < 2\tau^*_P(\epsilon) - \tau^*_R(\epsilon)$. Then $T(\tau_t, \epsilon) = 2\tau^*_P(\epsilon) - \tau_t$. The optimization problem becomes

$$
\begin{aligned}
V(\tau_{t-1}, \epsilon) &= \max_{\tau_t} \ \sqrt{2}(1-\tau_t)\bar{\theta}_r\epsilon + A\tau_t(1-\tau_t)(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 \\
&+ \frac{\beta}{1-\beta}\left\{\sqrt{2}[1-(2\tau^*_P(\epsilon)-\tau_t)]\bar{\theta}_r\epsilon + A(2\tau^*_P(\epsilon)-\tau_t)[1-(2\tau^*_P(\epsilon)-\tau_t)](\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2\right\} \\
s.t.& \\
&\sqrt{2}(1-\tau_t)\bar{\theta}_p\epsilon + A\tau_t(1-\tau_t)(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2 \geq \sqrt{2}(1-\tau_{t-1})\bar{\theta}_p\epsilon + A\tau_{t-1}(1-\tau_{t-1})(\bar{\theta}_r^2 + \bar{\theta}_p^2)\epsilon^2
\end{aligned}
$$

If the inequality constraint does not bind, then $\tau_t^* = (1-2\beta)\tau_R^*(\epsilon) + 2\beta\tau_P^*(\epsilon)$. We can show that $\tau_t^* > \tau_P^*(\epsilon)$ and $\tau_t^* < 2\tau_P^*(\epsilon) - \tau_R^*(\epsilon)$. If $\tau_t^* > \tau_{t-1}$, inequality constraint is violated. If $\tau_t^* < \tau_{t-1}$, we can show that the life-time utility delivered by this policy is less than that delivered by $\tau_t^* = 2\tau_P^*(\epsilon) - \tau_{t-1}$.[10]
If the inequality constraint binds, $\tau_t^* = \tau_{t-1}$.Again, we can show the life-time utility delivered by this policy is less than that delivered by $\tau_t^* = 2\tau_P^*(\epsilon) - \tau_{t-1}$.[11]

Summarizing above four cases, the equilibrium tax policy of party R under divided government is given by:

$$T^R(\tau_{t-1}, \epsilon) = \begin{cases} \tau_R^*(\epsilon) & \text{if } \tau_{t-1} \leq \tau_R^*(\epsilon) \\ \tau_{t-1} & \text{if } \tau_R^*(\epsilon) < \tau_{t-1} \leq \tau_P^*(\epsilon) \\ 2\tau_P^*(\epsilon) - \tau_{t-1} & \text{if } \tau_P^*(\epsilon) < \tau_{t-1} < 2\tau_P^*(\epsilon) - \tau_R^*(\epsilon) \\ \tau_R^*(\epsilon) & \text{if } \tau_{t-1} \geq 2\tau_P^*(\epsilon) - \tau_R^*(\epsilon) \end{cases}$$

□

[10]To show above argument, note $(1-\beta)V(\tau_{t-1}, \epsilon) = (1-\beta)u[(1-2\beta)\tau_R^*(\epsilon) + 2\beta\tau_P^*(\epsilon)] + \beta u[2\tau_P^*(\epsilon) - ((1-2\beta)\tau_R^*(\epsilon) + 2\beta\tau_P^*(\epsilon))] < u(2\tau_P^*(\epsilon) - \tau_{t-1})$.

[11]To show above argument, note $(1-\beta)u(\tau_{t-1}) + \beta u[2\tau_P^*(\epsilon) - \tau_{t-1})] < u(2\tau_P^*(\epsilon) - \tau_{t-1})$.

CHAPTER 2

# POLITICAL-DRIVEN FINANCIAL REGULATORY CYCLE

## 2.1 Introduction

The 2008-2009 financial crisis has revealed considerable vulnerabilities of our financial system and prompted broad calls for major financial regulation overhaul. Existing pro-cyclical regulatory framework, together with lax oversight, are blamed for facilitating excessive risk taking and credit expansion, which ultimately leads to the worldwide financial meltdown. Nowadays regulatory reform and macro-prudential policies have become the highlight of discussion among both academics and policymakers.

Apart from the well-known regulatory challenges arising from the intrinsic complexities of the financial system, the successful design and implementation of the regulatory reform agenda also has an important political dimension. The regulatory environment, shaped by politician's balance between the pressure from the financial industry and the scrutiny from general public, not only varies with the economic and financial cycles but also determines the policy.

Therefore, it is crucial to understand the incentives that govern politicians' tolerance of risks as well as how these incentives drive the dynamics of the regulatory policy. Despite the consensus that the political calculation contributed to the regulatory failure in the past financial crisis, many important questions remain unaddressed in the literature. For instance, how does the politician tailor the regulatory policies, while confronted with campaign contributions from financial firms and shifting attention of general public? How do the regulations evolve over the financial market's boom and bust cycles? In terms of successful implementing current regulatory reform, what are the potential challenges as electorate's attention on crisis fades and monetary benefits loom?

This paper develops a positive theory of political-driven financial regulatory cycle to address the above questions. By incorporating electoral competition into a dynamic common-agency framework where the financial sector constitutes the only lobby group, I find that the politicians are induced to behave as if they were maximizing the weighted sum of utilities of the financial industry and strategic households. Moreover, I show that the equilibrium financial regulation turn out to be pro-cyclical. Such cycles are generated through two key features: on the one hand, politicians face electoral competition and are influenced by campaign giving; on the other hand, the general public's attention on the financial regulation varies with the financial market performance.

More specifically, the regulatory policy in this model is determined through the interaction of the financial industry pressure group, the general public and the politicians. Self-interested financial firms desire excessive risk and high profits. They organize a special interest group to offer the politicians campaign funds in exchange for lax regulation on risky investment. The general public, who suffers from income loss during the financial crisis, advocates stringent regulation. However, due to limited capacity of information processing, the proportion of general public who base their voting decisions on the financial regulation evolves with the past financial market performance: few households pay attention to financial regulation in good times, yet most people blame regulation in times of crisis. Two politicians are competing for the office. Since their chances of winning the election hinges on both campaign contribution and voter's support, they seek to maximize their probability of getting elected by balancing the policy between the preferences of the household and the financial industry.

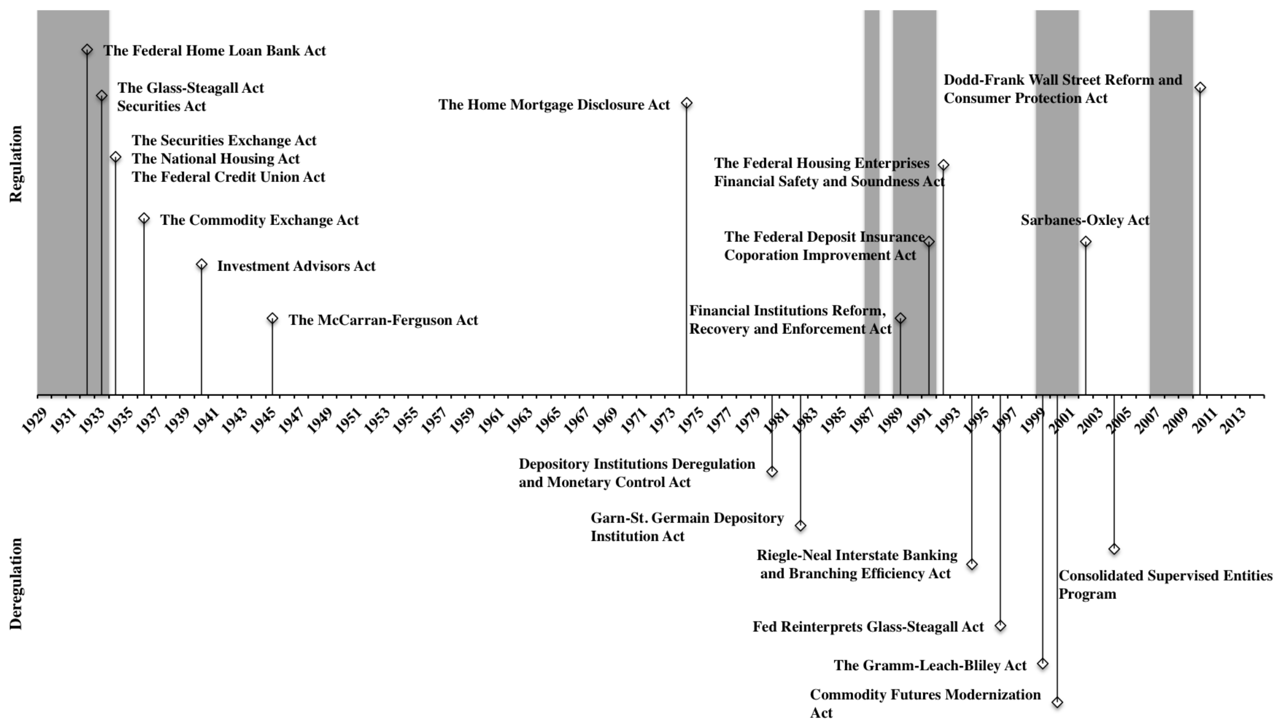
In absence of the pressure group for financial industry, the politicians in equilibrium simply maximizes the utility of general public and implements stringent regulation. After introducing the financial lobby group and campaign contribution, the politicians are induced to choose the financial regulation as if they were maximizing the weighted sum of utilities of the financial industry and the households. In symmetric equilibrium, both politicians' policies converge and the regulation is chosen such that the electoral loss due to weakened support from households equals the electoral gain created by campaign contribution. Since few households incorporate financial regulation concerns into their voting decisions when financial market is performing well, the better the financial market behaves in the past, the more privileged is the financial industry, and hence the more likely the policy tilts towards the interests of the financial sector.

The main contribution of this paper is to highlight the procyclical financial regulatory pattern delivered by economic and political forces of conflicting interests. During economic and financial market expansions, the financial regulatory policy remains largely ignored by the general public. Therefore, the policymaker is willing to cater to the financial interest group and promote loose regulation at the expense of broad social interest. Once a crisis takes place, the public attention on regulation is brought up. Now the politician, for fear of upsetting the general public, is forced to tighten the regulation due to electoral concerns. From a policy perspective, the regulatory cycle generated by varying political incentives in this model emphasizes an additional source of concern in the regulatory reform taking place nowadays. As of right now, since the crisis is recent, the regulation still receives plenty of attention and the prudential policies gain political support. However, as people's attention on regulation gradually fades overtime, it is critical to provide the politician and regulator the right incentives so as to achieve a consistent implementation of the regulatory policy.

Although this paper focuses on the theoretical aspect of the issue, the procyclical regulatory pattern delivered by the model is consistent with the past 85 years of financial regulation and deregulation. In Figure 2.1, major financial regulatory events are listed and categorized according to 'Regulation' or 'Deregulation'. The shaded region indicates the major financial crisis in United States since the start of 20th century – the 1929 Wall Street crash followed by the Great Depression, the 1987 stock market crash, the 1989 -1991 savings and loan crisis, The early 2000s dot-com bubble and the 2007-2008 financial crisis. Clearly, right after each crisis the financial market witnesses periods of regulation tightening. For instance, the Glass-Steagall Act comes after the Great Depression, the Sarbanes-Oxley Act follows the dot-com bust, and the the Dodd-Frank Act follows the 2007-2008 financial crisis. Yet as the crisis fades, de-regulation emerges, which may ultimately induce another financial market crash. The gradual liberal interpretation and final repeal of the Glass-Steagall Act by Gramm-Leach-Bliley Act is a vivid example. In addition to the procyclical regulation, this model also speaks to the empirical fact documented by Moss (2009) that income inequality widens as government regulations ease and bank failures rise.

**Related Literature** This paper is related to several strands of literature. First, it

Figure 2.1: Cycles of Financial Crisis and Regulation in U.S.




is built on a large and growing literature on special interest groups, going back to the seminal work of Bernheim and Whinston (1986) and Grossman and Helpman (1994). Recent works by Bergemann and Välimäki (2003) and Mukoyama and Popov (2014) provide dynamic extension of the framework and apply it to the evolution of industry entry barriers. By bring together campaign contribution and electoral concerns in a dynamic economy where voter's attention evolves with financial market performance, my paper extends the static model of Grossman and Helpman (1996) and Grossman and Helpman (2001). My main contribution to this literature is adopting the special interest group theory in a dynamic context of financial regulation and providing a positive theory of the financial regulatory cycle. By focusing on a setup with one single interest group, the model obtains results that are infinite-horizon analogues of the static model. For instance, in equilibrium, politicians implement policy options offered by the interest group such that the dynamic payoff of the politicians is always equal to that if the politicians were only concerned with the rest of the economy.

This paper also contributes to the growing literature on financial regulation. In the aftermath of 2008 financial crisis, there is a burgeoning literature on macro-prudential regulation, emphasizing that regulation should be designed to internalize pecuniary externalities, such as Bianchi and Mendoza (2013), Farhi and Werning (2013) and Korinek and Kreamer (2014). However, the regulations in these papers are derived through the benevolent social planner. The political failure, which plays a critical role in the financial meltdown, is largely ignored. In contrast to their work, my paper contributes to the debate of financial regulatory reform through the political dimension. Even though my paper does not explicitly characterize the externalities as mentioned above, it does emphasize the conflicting interests of different players and highlights the importance of 'incentive-robust' financial reform. In this sense, this paper constitutes a first step to provide a framework for understanding political frictions in financial regulation.

Finally, one interesting paper that also tackles regulatory capture in financial industry is McCarty (2013). His paper focuses on the trade-off between regulator's expertise and autonomy in a complex policy environment. In light of the regulatory bias towards regulated firms due to policy complexity, the political principal may choose not to delegate power to the agency at all and simply let firms go unregulated. While both his paper and mine involve economic agents' limited ability in processing information, my paper differs from his by emphasizing an explicit capture mechanism through campaign finance. Also, the dynamic environment in my paper allows for interpretation of the historical pattern of financial regulation in the United States.

## 2.2 The Model

In this section, I first describe how the economy behaves under arbitrary financial regulatory policy, and then analyze the political determination of financial regulation under the influence of special interests.

### 2.2.1 Economic Environment

#### The Households

Time is discrete and infinite horizon, indexed by $t = 0, 1, 2, \cdots$. Households derive utility $u(c_t)$ from consumption $c_t$, with $u(\cdot)$ continuous and increasing in consumption. Assume the household has no investment opportunities. At the beginning of each period, the household receives a fixed income $D$. Since it has no access to any savings technology that carries its income to the end of the period when the consumption takes place, the household is willing to lend its entire income $D$ to the financial sector inelastically at exogenous fixed interest rate $r > 0$[1].

[1]This simple debt contract assumption between financial sector and the household can be justified by costly state verification as in Townsend (1979), or alternatively the cost to the household for enforcing contracts where return is contingent on size of the deposit or on asset returns is too high.

### The Financial Sector

The financial sector is risk neutral[2]. It receives fixed non-financial income $y$ at the end of each period[3]. In addition, it can borrow from the household as much as it likes at exogenous interest rate $r$ to make financial investments. These investments are only allowed within each period. To simplify subsequent analysis, neither the financial sector nor households are assumed to have access to intertemporal savings opportunities.

The financial sector has access to two types of assets: the safe asset which pays fixed gross return $r$, and risky asset whose return $R_t$ is i.i.d. and follows distribution $h(\cdot)$ with $\mathbb{E}(R_t) > r$. In each period, the financial sector determines its investment in safe asset $X_t^s \geq 0$ and risky asset $X_t^r \geq 0$ subject to the financial regulation $X_t^r \leq \phi_t$. In case of poor risky asset return realization, the financial sector may not be able to fully repay households at promised return $r$. It defaults and pays back whatever it has made from the financial market. Equating the gain from financial investment $rX_t^s + RX_t^r$ with promised repayment $r(X_t^s + X_t^r)$, we know the threshold risky asset return $R_c$ below which the financial sector defaults can be characterized by

$$rX_t^s + R_c X_t^r = r(X_t^s + X_t^r) \tag{2.1}$$

which implies $R_c = r$. Based on this threshold, we can derive the financial income of the financial sector as well as the consumption of the household. For

[2]Here the risk neutral assumption of the financial sector is made for purpose of analytical characterization of certain model properties. Same results carry through under standard assumption of risk aversion

[3]This assumption is necessary for justifying that the financial sector always have enough funds to make campaign contribution, even in times of financial market failure.

arbitrary $X_t^r$, $R_t$ and $r$, financial sector's net investment gain is

$$R_t X_t^r + r X_t^s - r(X_t^r + X_t^s) 1_{\{R_t \geq r\}} - (R_t X_t^r + r X_t^s) 1_{\{R_t < r\}} = (R_t - r) X_t^r 1_{\{R_t \geq r\}} \tag{2.2}$$

and households' consumption $c_t$ is

$$c_t = (R_t X_t^r + r X_t^s) 1_{\{R_t < r\}} + r(X_t^r + X_t^s) 1_{\{R_t \geq r\}} = rD + (R_t - r) X_t^r 1_{\{R_t < r\}} \tag{2.3}$$

Note that whenever $R_t \geq r$, the financial sector benefits from the excess gains of additional risky investment, whereas the household receives fixed consumption stream $c_t = rD$. Once $R_t < r$, however, the household suffers from the losses incurred by increasing risky investment while the financial sector's net financial gain stays at 0. Given the financial regulation $\phi_t$ and households' income $D$, the desired risky investment level of the financial sector is $X_F^{r*} = min\{\phi_t, D\}$ whereas households' desired risky investment level is $X_H^{r*} = 0$.

Thus, the simple framework above captures the disagreement between the financial sector and the general public in terms of the financial regulation: the financial sector tends to prefer loose financial regulation, while the general public prefers stringent financial regulation. Moreover, since households are forced to lend passively to the financial sector due to its lack of intra-temporal savings technology, the equilibrium risky investment is $X_t^r = min\{\phi_t, D\}$.

### 2.2.2 Political System

#### Politicians & Government

There are two politicians in this economy, denoted by $A$ and $B$. Each politician announces its policy position on financial regulation before the election takes place, and he/she commits to implement the announced policy once elected. Since politicians make announcements before finding out the popularity of their non-policy positions, they choose the regulatory policy to maximize the probability of winning the majority as will be defined in the next section.

#### The voters

The households are the voters. Following Grossman and Helpman (1996) and Grossman and Helpman (2001), I assume there are two types of voters, the strategic voters and the impressionable voters. The welfare of both types of voters are affected by the regulatory policy implemented by the policymaker. However, they vote in different ways.

**Strategic Voter** The strategic voters base their voting decisions on both expected instantaneous utilities derived from the financial regulatory policy $\mathbb{E}u\left(c(\phi_t^I)\right)$ and preferences towards politicians' non-policy positions $v_i^I$, $I = A, B$. The non-policy position captures politician's features unrelated to financial regulation, such as charisma and ideology. The non-policy position is assumed to be fixed, which can not be modified for electoral purpose. Note that here $v_i^I$ is individual-specific and can vary across different strategic voters. Let $v_i = v_i^B - v_i^A$ denote strategic voter $i$'s preference bias towards politician $B$. $v_i$ can take both positive and negative values. A positive(negative) value of $v_i$ implies that voter $i$ is in favor of politician $B(A)$ in terms of non-policy position, while $v_i = 0$ implies that voter $i$ is neutral over non-policy position and only cares about regulatory policy. To facilitate later derivations, assume $v_i$ is uniformly distributed over $\left[-\frac{1}{2f} + \frac{b}{f}, \frac{1}{2f} + \frac{b}{f}\right]$ with mean $\frac{b}{f}$ and density $f$.

The parameter $b$ measures the average relative popularity of politician B in the entire households. Assume politicians have to announce their regulatory policy $\phi_t^I$ before they find out their average popularity in terms of non-policy positions, then ex ante $b$ is random and can take positive or negative values. Again, for purpose of simplicity, assume $b$ is uniformly distributed over $\left[-\frac{1}{2\underline{b}}, \frac{1}{2\underline{b}}\right]$ with mean 0 and density $\underline{b}$.

Based on assumptions above, strategic voter $i$ vote for party A if and only if its utility derived from both policy and non-policy issues by voting for politician A is at least as high as that obtained by voting for politician B[4]

$$\mathbb{E}u\left(c(\phi_t^A)\right) \geq \mathbb{E}u\left(c(\phi_t^B)\right) + v_i \tag{2.4}$$

For a given realization of aggregate shock $b$, the uniform distribution assumption of $v_i$ implies that the fraction of strategic voters voting in favor of politician A is

$$\pi_t^S = \frac{1}{2} - b + f\left[\mathbb{E}u\left(c(\phi_t^A)\right) - \mathbb{E}u\left(c(\phi_t^B)\right)\right] \tag{2.5}$$

Note that the randomness of average relative popularity $b$ ensures that both

[4]Here strategic voters are myopic in the sense that they take into account only the instantaneous utility rather than the life-time utility.

politicians enjoy certain chance of winning the majority, even if their regulatory policy converges.

**Impressionable Voter** The impressionable voters don't understand regulatory policy. They simply vote according to the political campaign. Following Grossman and Helpman (1996), all else equal, the more a politician spends on his/her campaign, the greater fraction of impressionable voters it attracts. Here I assume that the fraction of impressionable voters voting in favor of politician A shares similar structure as the strategic voter

$$\pi_t^U = \frac{1}{2} - b + h(C_t^A - C_t^B) \tag{2.6}$$

which implies that the politicians enjoy the same average popularity in terms of non-policy position among both strategic voters and impressionable voters.

**Time-Varying Attention** At time $t$, there is a fraction $\sigma_t$ of impressionable voters and a fraction $1-\sigma_t$ of strategic voters. In reality, households' attention on financial regulations varies over time. Typically, recent outbreak of a financial crisis leads to increased scrutiny on the financial regulatory policy from the general public. To capture this important feature, in the model I assume that the fraction $\sigma_t$ of impressionable voters in the economy is an increasing function of the previous financial market performance measured by excess return $(R_{t-1} - r)X_{t-1}^r$. In other words, when financial market becomes problematic,i.e., when $(R_{t-1}-r)X_{t-1}^r$ takes low values, households who used to vote according to the campaign now takes financial regulation into their consideration, which leads to a higher fraction of strategic voters. More specifically, to ensure $\sigma_t$ takes values between 0 and 1, I assume $\sigma_t$ takes the following functional form

$$\sigma_t(R_{t-1}, X_{t-1}^r) = \frac{1}{1 + e^{-\lambda(R_{t-1}-r)X_{t-1}^r}} \tag{2.7}$$

Based on (2.5) and (2.6), we can obtain the total fraction $\pi_t$ of voters in favor of politician A by summing up the vote shares from strategic voters and impressionable voters, weighted by $\sigma_t$ and $1 - \sigma_t$ respectively

$$\begin{aligned} \pi_t =& (1 - \sigma_t)\pi_t^S + \sigma_t \pi_t^U \\ =& \frac{1}{2} - b + (1 - \sigma_t) f \left[ \mathbb{E}u\left(c(\phi_t^A)\right) - \mathbb{E}u\left(c(\phi_t^B)\right)\right] + \sigma_t h(C_t^A - C_t^B) \end{aligned} \tag{2.8}$$

Since the vote share $\pi_t$ depends on the realization of $b$, at the time of politicians' policy announcement, $\pi_t$ is also a random variable. Using the fact that $b$ is uniformly distributed over $\left[-\frac{1}{2\underline{b}}, \frac{1}{2\underline{b}}\right]$, politicians can infer their probability of winning the majority. Specifically, the probability of politician A winning the election is

$$p_t^A = F(\pi_t \geq \frac{1}{2}) = \frac{1}{2} + \underline{b}\left\{(1 - \sigma_t) f \left[ \mathbb{E}u\left(c(\phi_t^A)\right) - \mathbb{E}u\left(c(\phi_t^B)\right)\right] + \sigma_t h(C_t^A - C_t^B)\right\} \tag{2.9}$$

and the probability of politician B winning the election is

$$p_t^B = F(\pi_t < \frac{1}{2}) = \frac{1}{2} + \underline{b}\left\{(1 - \sigma_t) f \left[ \mathbb{E}u\left(c(\phi_t^B)\right) - \mathbb{E}u\left(c(\phi_t^A)\right)\right] + \sigma_t h(C_t^B - C_t^A)\right\} \tag{2.10}$$

### The Special Interest Group

The financial sector forms a special interest group to lobby the politicians. Since the financial sector seeks high risk and profits, its pressure group advocates loose regulation on risky investment. One justification for the assumption that financial sector engages in lobby activities while households remain largely unorganized follows from Olson (1965), which says individuals with concentrated interests (e.g. the financial industry) are more likely to form interest groups and take collective actions than those with diffused interests (e.g. the households).

Specifically, the pressure group of the financial sector makes separate contributions $C_t^A$ and $C_t^B$ to politician A and B in exchange for favorable regulatory policies. Let $V^F(\phi_t, R_t)$ denote the dynamic payoff of the financial sector given risky investment regulation $\phi_t$ and risky asset return $R_t$. Then before the election takes place and risky asset return realizes, the expected dynamic payoff of the financial sector can be characterized by

$$\begin{aligned} & p_t^A \int_{R_{min}}^{R_{max}} \left[ y + (R_t - r)X^r(\phi_t^A)1_{\{R_t \geq r\}} - C_t^A - C_t^B + \beta V^F(\phi_t^A, R_t) \right] h(R_t) dR_t \\ + & p_t^B \int_{R_{min}}^{R_{max}} \left[ y + (R_t - r)X^r(\phi_t^B)1_{\{R_t \geq r\}} - C_t^A - C_t^B + \beta V^F(\phi_t^B, R_t) \right] h(R_t) dR_t \end{aligned} \tag{2.11}$$

Here $\beta$ denotes the financial sector's discount factor. The term $y + (R_t - r)X^r(\phi_t^I)1_{\{R_t \geq r\}} - C_t^A - C_t^B$ $(I = A, B)$ represents financial sector's net gain at time $t$ in case of politician $I$ winning the election. Essentially, the expected dynamic payoff of the financial sector involves two components. The first component is the non-financial income subtract lobby expenditures, which are independent of the election outcome. The second component is the current financial gain and the dynamic payoff in the future, both of which are dependent on the election outcome and risky return realization. Hence (2.11) can be rewritten as

$$
\begin{aligned}
& y - C_t^A - C_t^B \\
& + p_t^A \int_{R_{min}}^{R_{max}} \left[ (R_t - r) X^r(\phi_t^A) 1_{\{R_t \geq r\}} + \beta V^F(\phi_t^A, R_t) \right] h(R_t) dR_t \\
& + p_t^B \int_{R_{min}}^{R_{max}} \left[ (R_t - r) X^r(\phi_t^B) 1_{\{R_t \geq r\}} + \beta V^F(\phi_t^B, R_t) \right] h(R_t) dR_t
\end{aligned} \tag{2.12}
$$

### 2.2.3 Timing of Events

Now let's summarize the interactions among politicians, voters and the special interest group organized by the financial sector at each time $t$.

1. At the beginning of period $t$, the economy starts with the risky asset return realization $R_{t-1}$ and risky investment $X^r(\phi_{t-1})$ inherited from previous period, which governs the proportion of strategic and impressionable voters.
2. The special interest group makes separate take-it-or-leave-it offers to politicians to maximize the dynamic payoff of the financial sector. In other words, the financial sector provides campaign contribution $C_t^I$ to politician $I$ if the politician agrees to implement $\phi_t^I$, $I \in \{A, B\}$.
3. The politicians decide whether or not to accept the contribution from the special interest group, and simultaneously announce their policy platforms to maximize chances of getting elected.
4. Elections are held and voters jointly decide the time $t$ incumbent through majority voting. The elected politician implements financial regulation as it announced before the election.

5. The financial sector and the households carry out lending and investment activities according to the newly implemented regulation.
6. At the end of period $t$, the risky asset return $R_t$ realizes.

### 2.2.4 Equivalent Formulation

In optimal fiscal policy literature, primal approach is widely used in the formulation of the problem, where prices and tax policies are first eliminated and government is thought of as directly choosing a feasible allocation consistent with the optimization behavior of households and firms. In a similar spirit, to simplify the problem, I will first eliminate the regulatory policy $\phi_t$ and instead have the politician directly search for the equilibrium risky asset allocation $X_t^r$ subject to lobbying activities and market participants' optimization behavior. Then I will decentralize the allocation. Therefore, in subsequent analysis, variables previously labeled as functions of financial regulatory policy $\phi_t$ such as $c(\phi_t)$ and $V^F(\phi_t, R_t)$ will be replaced by a function of risky investment level $X_t^r$ such as $c(X_t^r)$ and $V^F(X_t^r, R_t)$.

## 2.3 Political Equilibrium Without Lobby Group

First consider a setting without lobby group from the financial sector. According to assumptions in subsection 2.2.2, each politician announces the financial regulation that maximizes its probability of winning the election. In the case of no lobby group, there is no campaign contribution from the financial sector, i.e. $C_t^A = C_t^B = 0$. Thus the fraction of impressionable voters in favor of politician A becomes

$$\pi_t^U = \frac{1}{2} - b \tag{2.13}$$

Since the fraction of strategic voters in favor of politician A stays same as (2.5), the total share of voters supporting politician A becomes

$$\begin{aligned}\pi_t =&(1-\sigma_t)\pi_t^S + \sigma_t \pi_t^U \\ =&\frac{1}{2} - b + (1-\sigma_t) f \left[\mathbb{E}u\left(c(X_t^{r,A})\right) - \mathbb{E}u\left(c(X_t^{r,B})\right)\right]\end{aligned} \tag{2.14}$$

which implies the probability of politician A winning the election is

$$p_t^A = F(\pi_t \geq \frac{1}{2}) = \frac{1}{2} + \underline{b}\left\{(1-\sigma_t) f \left[\mathbb{E}u\left(c(X_t^{r,A})\right) - \mathbb{E}u\left(c(X_t^{r,B})\right)\right]\right\} \tag{2.15}$$

Intuitively, without political campaign, the regulation and the resulting vote share from the strategic voters becomes the key factor driving the election outcome. Recall in (2.3), households' consumption $c_t$ is non-increasing in $X_t^r$. Therefore, given its opponent's strategy $X_t^{r,B}$, the best strategy for politician A to win the election is to ban risky investment at all times, i.e. $X_t^{r,A^*} = 0$. Similarly, given $X_t^{r,A}$, the best strategy for politician B is to set $X_t^{r,B^*} = 0$. Moreover, in such equilibrium, the proportion of strategic voters and impressionable voters turn out to be fixed over time, with $\sigma_t^* = 1 - \sigma_t^* = 0.5$. This is because the financial market won't generate any additional gains or losses without risky investment.

## 2.4 Political Equilibrium With Financial Sector Lobby

This section studies the equilibrium regulatory policy in an economy where the financial sector lobbies politicians through campaign contributions. I will first layout the problem as a common agency problem with financial sector being the only lobby group, and then highlights some properties of this model.

### 2.4.1 Politicians' Participation Constraints

For arbitrary campaign contribution schedule offered by the financial sector, each politician has to decide whether to take the contribution and implement the regulation wanted by the financial sector, or to decline the contribution and implement whatever financial regulation desired by the politician himself/herself.

Take politician A for example. Suppose politician A turns down the contribution from the financial sector, then its chances of winning the election becomes

$$\frac{1}{2} + \underline{b}\left\{(1-\sigma_t)f\left[\mathbb{E}u\left(c(X_t^{r,A})\right) - \mathbb{E}u\left(c(X_t^{r,B})\right)\right] + \sigma_t h(0 - C_t^B)\right\} \tag{2.16}$$

Similar to the benchmark scenario as in section 2.3, given its opponent's strategy $X_t^{r,B}$ and $C_t^B$, politician A maximizes its own chances of getting elected by banning risky investment, i.e. $X_t^{r,A^*} = 0$. Not surprisingly, when the risky investment level will no longer affect the support of the impressionable voters whose votes are determined by the level of contribution $C_t^A = 0$, the policymaker simply maximizes the chances of winning by maximizing support from the strategic voters.

Therefore, in order to convince politician A to implement the risky investment level desired by the financial sector, the lobby group needs to provide sufficient contribution so that the electoral gain from the increased support of impressionable voters compensates for the electoral loss from reduced support from the strategic voters. In other words, politician A accepts the contribution if and only if its probability of winning the election through accepting the contribution is at least as high as the probability obtained from declining the contribution:

$$\begin{aligned}
&\frac{1}{2} + \underline{b}\left\{(1-\sigma_t)f\left[\mathbb{E}u\left(c(X_t^{r,A})\right) - \mathbb{E}u\left(c(X_t^{r,B})\right)\right] + \sigma_t h(C_t^A - C_t^B)\right\} \\
&\geq \\
&\frac{1}{2} + \underline{b}\left\{(1-\sigma_t)f\left[\mathbb{E}u\left(c(X_t^{r,A^*})\right) - \mathbb{E}u\left(c(X_t^{r,B})\right)\right] + \sigma_t h(0 - C_t^B)\right\}
\end{aligned} \tag{2.17}$$

which implies

$$C_t^A \geq \frac{(1-\sigma_t)f\left[\mathbb{E}u\left(c(X_t^{r,A^*})\right) - \mathbb{E}u\left(c(X_t^{r,A})\right)\right]}{\sigma_t h} \tag{2.18}$$

Similarly, party B accepts the contribution if and only if

$$C_t^B \geq \frac{(1-\sigma_t)f\left[\mathbb{E}u\left(c(X_t^{r,B^*})\right) - \mathbb{E}u\left(c(X_t^{r,B})\right)\right]}{\sigma_t h} \tag{2.19}$$

Note that as $X_t^I (I = A, B)$ rises, households' expected utility gap grows larger, which requires higher campaign contribution. Since the proportion of households whose votes are based on financial regulation increases when financial market previously performed poorly, all else equal, higher campaign contribution is needed in case of bad past financial market realizations.

### 2.4.2 Financial Lobby Group's Problem

Given politicians' participation constraints, the financial sector makes separate offers $(C_t^A, X_t^{r,A})$ and $(C_t^B, X_t^{r,B})$ to politicians to maximize the dynamic payoff of the financial sector. Let $V^F(X_{t-1}^r, R_{t-1})$ denote the dynamic payoff of the financial sector who behaves optimally with previous risky asset return $R_{t-1}$ and risky investment level $X_{t-1}^r$. The financial lobby group's optimization problem can be summarized as

$$
\begin{aligned}
&V^F(X_{t-1}^r, R_{t-1}) = \max_{C_t^A, C_t^B, X_t^{r,A}, X_t^{r,B}} \; y - C_t^A - C_t^B \\
&+ p_t^A \int_{R_{min}}^{R_{max}} \left[ (R_t - r) X_t^{r,A} 1_{\{R_t \geq r\}} + \beta V^F(X_t^{r,A}, R_t) \right] h(R_t) dR_t \\
&+ p_t^B \int_{R_{min}}^{R_{max}} \left[ (R_t - r) X_t^{r,B} 1_{\{R_t \geq r\}} + \beta V^F(X_t^{r,B}, R_t) \right] h(R_t) dR_t \\
&s.t. \\
&[1] \; C_t^A \geq \frac{(1-\sigma_t) f \left[ \mathbb{E}u\left(c(X_t^{r,A^*})\right) - \mathbb{E}u\left(c(X_t^{r,A})\right) \right]}{\sigma_t h} \\
&[2] \; C_t^B \geq \frac{(1-\sigma_t) f \left[ \mathbb{E}u\left(c(X_t^{r,B^*})\right) - \mathbb{E}u\left(c(X_t^{r,B})\right) \right]}{\sigma_t h} \\
&[3] \; 0 \leq X_t^{r,A} \leq D, 0 \leq X_t^{r,B} \leq D
\end{aligned}
$$

where

$$p_t^A = \frac{1}{2} + \underline{b}\left\{(1-\sigma_t)f\left[\mathbb{E}u\left(c(X_t^{r,A})\right) - \mathbb{E}u\left(c(X_t^{r,B})\right)\right] + \sigma_t h(C_t^A - C_t^B)\right\}$$
$$p_t^B = \frac{1}{2} + \underline{b}\left\{(1-\sigma_t)f\left[\mathbb{E}u\left(c(X_t^{r,B})\right) - \mathbb{E}u\left(c(X_t^{r,A})\right)\right] + \sigma_t h(C_t^B - C_t^A)\right\}$$

Note that the dynamic link between time $t-1$ and time $t$ occurs through $\sigma_t(X_{t-1}^r, R_{t-1})$, which governs the proportion of households who pay attention to the financial regulation. In addition to the participation constraints, the fraction $\sigma_t$ also enters financial sector's objective through election probability $p_t^A$ and $p_t^B$. Once equilibrium $X_t^{r,A}$ and $X_t^{r,B}$ are obtained, the financial regulation $\phi_t^A$ and $\phi_t^B$ can be backed out by setting $\phi_t^A = X_t^{r,A}$ and $\phi_t^B = X_t^{r,B}$.

### 2.4.3 Equilibrium Definition

In this paper, I focus on stationary Markov Subgame-Perfect Nash Equilibrium. The equilibrium strategy only depends on the payoff-relevant state $X_{t-1}^r$ and $R_{t-1}$, and does not depend on history. As mentioned in subsection 2.2.3, each time period $t$ is associated with a two-stage political game. In the first stage, the financial sector lobby group announce separate contribution schedules to both politicians; in the second stage, two politicians simultaneously choose and announce their policy platforms. After the election outcome is realized, financial market activities take place according to the financial regulation. Note that here the entire contribution schedules for both politicians are assumed to be observed by each politician.

**Definition 3.** *A Political Equilibrium With Financial Sector Lobby consists of a pair of feasible regulatory policies $\{\phi_t^A, \phi_t^B\}_{t=0}^{\infty}$, a pair of financial contribution schedules*

$\{C_t^A(\phi_t^A), C_t^B(\phi_t^B)\}_{t=0}^{\infty}$*, and levels of safe and risky investments* $\{X_t^{s,I}, X_t^{r,I}\}_{t=0}^{\infty}$ *for* $I \in \{A, B\}$ *such that*

1. *given the financial regulatory policy* $\phi_t^I$*, the financial sector invests* $X_t^{r,I}$ *in risky asset and* $X_t^{s,I}$ *in safe asset, for each* $I \in \{A, B\}$*.*
2. *given the strategy of politician I's competitor* $\phi_t^{-I}$ *and the contribution schedules of the financial sector* $\{C_t^I(\phi_t^I), C_t^{-I}(\phi_t^{-I})\}$*, financial regulation* $\phi_t^I$ *maximizes politician I's probability of winning the election, for each* $I \in \{A, B\}$*,* $-I \in \{A, B\}$*, and* $I \neq -I$*.*
3. *the contribution schedules* $\{C_t^A(\phi_t^A), C_t^B(\phi_t^B)\}_{t=0}^{\infty}$ *maximizes the dynamic payoff of the financial sector.*

In above definition, Condition 1 says given the financial regulatory policy, the financial market achieves competitive equilibrium. Condition 2 indicates that the financial regulation announced by the politicians are outcome of Nash equilibrium. Condition 3 guarantees that there is no alternative contribution schedule that delivers higher dynamic payoff to the financial sector than the equilibrium contribution. To further simplify the analysis, I will focus on the symmetric equilibrium in subsequent sections and delegate the discussion of alternative scenarios to the Appendix.

### 2.4.4 Equilibrium Properties

**Proposition 2** (Pure Influence Motive)**.** *In symmetric equilibrium, the contributions from the financial sector satisfy both participation constraints with equality, i.e.*

$$C_t^I = \frac{(1-\sigma_t) f \left[ \mathbb{E}u\left(c(X_t^{r,I*})\right) - \mathbb{E}u\left(c(X_t^{r,I})\right) \right]}{\sigma_t h} \quad I = A, B \tag{2.20}$$

First note that in equilibrium, at least one politician's participation constraint holds with equality. If this was not the case, then we could always reduce both politician's contribution by the same very small amount such that the participation constraint remains non-binding, while keeping the equilibrium election probability and risky investment level same as before. This will always improve the dynamic payoff of the financial sector because it reduces the equilibrium contributions. Due to the symmetry assumption, both participation constraint binds.

In addition, according to (2.3), $u(c_t)$ is a non-increasing function of $X_t^{r,I}$, which implies the contribution schedule $C_t^I$ is non-decreasing in $X_t^I$. Intuitively, the higher risky investment the financial sector wants, the higher 'price' it has to pay to the politicians.

**Proposition 3** (Weighted Sum of Welfare)**.** *In symmetric equilibrium, both politicians behave as if they were maximizing weighted sums of utilities of the financial sector and the strategic voters. The equilibrium financial regulation $\phi_t^I$, $I = A, B$ satisfies*

$$\begin{aligned}\phi_t^I = arg\max_{X_t^{r,I}} \quad & p_t^I \int_{R_{min}}^{R_{max}} \left\{ y + (R_t - r)X_t^{r,I} 1_{\{R_t \geq r\}} + \beta V^F(X_t^{r,I}, R_t) \right\} h(R_t) dR_t \\ & + \frac{(1-\sigma_t)f}{\sigma_t h} \mathbb{E}u\left(c(X_t^{r,I})\right) \end{aligned} \tag{2.21}$$

*where $c(X_t^{r,I}) = rD + (R_t - r)X_t^{r,I} 1_{\{R_t < r\}}$ and $p_t^I = 0.5$ for $I = A, B$. Moreover, in equilibrium, $\phi_t^A = \phi_t^B \triangleq \phi_t$, $X_t^{r,A} = X_t^{r,B} \triangleq X_t^r$ and $C_t^A = C_t^B \triangleq C_t$.*

Proposition 2 follows straightly from Proposition 1. Plugging (2.20) into (2.9) and (2.10), we have

$$F(\pi_t \geq \frac{1}{2}) = F(\pi_t < \frac{1}{2}) = \frac{1}{2} \tag{2.22}$$

This highlights that in equilibrium, since the financial sector lobby group won't contribute more than necessary to neither politician, both politicians enjoy equal probability of winning, which is independent of the contribution schedule and financial regulation. In other words, the financial sector lobby group approaches politicians for pure policy influence purposes.

Using the fact that $p^I = \frac{1}{2}$, plugging (2.20) and (2.22) into the the financial sector's objective as characterized in subsection 2.4.2, it is not difficult to realize that after dropping constant terms, the entire dynamic payoff can be decomposed into two symmetric component, each of which is associated with one politician's equilibrium risky investment allocation $X_t^{r,I}$. More specifically, it is a weighted sum of financial sector's expected dynamic payoff and strategic voter's utility. Since each component only involves one politician's regulatory policy, the full maximization problem can also be decomposed into two symmetric maximization problems of the weighted sum. Moreover, as $p_t^A = p_t^B$ and the structure of the problem is exactly the same for politicians A and B, it is not surprising to find $X_t^{r,A} = X_t^{r,B} \triangleq X_t^r$, which implies that $\phi_t^A = \phi_t^B \triangleq \phi_t$. Plugging $X_t^r$ into (2.20), we also obtain $C_t^A = C_t^B \triangleq C_t$.

**Proposition 4** (Comparative Statics)**.** *All else equal, the higher is the fraction $\sigma_t$ of the impressionable voters, the looser is the financial regulation $\phi_t$, and the higher is the dynamic payoff of the financial sector $V^F(X_{t-1}, R_{t-1})$.*

From (2.9), we see that the higher is the fraction of the impressionable voters

$\sigma_t$ as compared to that of the strategic voters $1 - \sigma_t$, the more productive is the campaign contribution in terms of buying electoral support. Therefore, the politicians place less weight on the strategic voters and cater to the interest of the financial industry by implementing regulation closer to the ideal level of the financial sector. Since the financial sector desires minimal regulation while the impressionable voters desire as much regulation as possible, by placing more weight in front of the objective of the financial sector, the equilibrium regulation is pushed lower and the dynamic payoff for the financial sector increases.

As a result, in case of strong previous financial market performance (i.e. when $(R_{t-1} - r)X^r_{t-1}$ is relatively large), according to the structure of $\sigma(X^r_{t-1}, R_{t-1})$, $\sigma_t$ is relatively large. More households become impressionable – they don't pay much attention to the regulatory process and simply vote according to the campaign. This indirectly encourages the politician to cater to the financial industry and choose loose financial regulation in good economic times.

## 2.5 Numerical Example

To further understand equilibrium financial regulation behavior, in this section I will parameterize the model and solve it numerically. The procedure involves solving for the fixed point as characterized by the value function of the financial sector through value function iterations. Since the properties of the objective function remains unclear, for given initial guess of value functions, I evaluate the objective at all possible grids of risky investment and choose the one that delivers the maximum value. This generates equilibrium risky investment policies and new value functions. Such procedure is repeated until convergence is achieved[5].

### 2.5.1 Parameterization

Since the model is highly simplified, I will present the numerical example for purpose of illustrating equilibrium properties rather than performing calibrated exercises. Specifically, I assume household's utility takes the functional form $u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$, with risk aversion parameter $\gamma = 2$. At the beginning of each period, the household receives a fixed income $D = 1$. The return of safe asset $r = 1$ and the return of risky asset $R_t$ is uniformly distributed between $[0.9, 1.25]$, which satisfies the assumption $\mathbb{E}(R_t) > r$. The financial sector's discount factor $\beta$ is set at 0.9 and its non-financial income $y$ is set at 2, which guarantees the financial sector is always able to provide campaign contribution, even in case of poor financial market performance. As for the non-policy position parameter, I set $f = 2$, $h = 1$ and $\underline{b} = 1$. Furthermore, the parameter $\lambda$ which governs the proportion of strategic voters and impressionable voters is assumed to be 0.3 so that we'll focus on interior solutions. Table 2.1 summarizes the parameter values used for solving the model.

Table 2.1: Parameterization

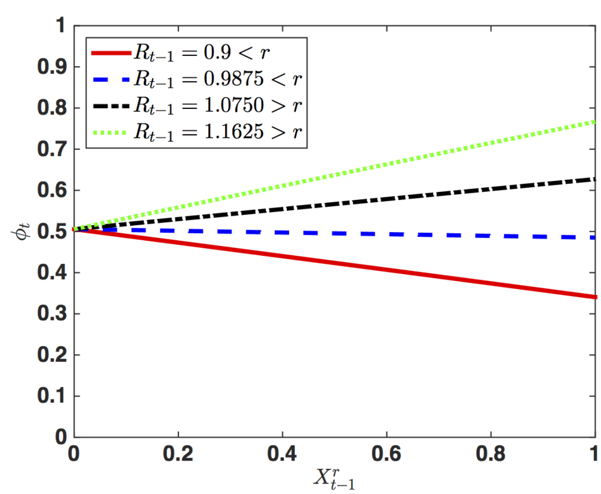
| $\gamma$ | $r$ | $R_{min}$ | $R_{max}$ | $D$ | $\beta$ | $f$ | $h$ | $\underline{b}$ | $\lambda$ |
|---|---|---|---|---|---|---|---|---|---|
| 2 | 1 | 0.9 | 1.25 | 1 | 0.9 | 2 | 1 | 1 | 0.3 |

[5]When solving the problem, I take into consideration possibilities of both symmetric and asymmetric equilibrium outcome. Under the parameterization as specified below, the asymmetric solution is ruled out since it delivers lower dynamic payoff than the symmetric outcome.

### 2.5.2 Results

In Figure 2.2, I plot the equilibrium financial regulatory policy $\phi_t$ as a function of previous risky investment level $X^r_{t-1}$ at different previous financial market realization $R_{t-1}$s. In cases of poor past financial market performance, i.e. $R_{t-1} < r$, the financial sector defaults. Recall from (2.7), given arbitrary $R_{t-1} < r$, the amount the financial sector defaults is increasing in the level of risky investment $X^r_{t-1}$, which shapes the proportion of strategic voters. Since the increased proportion of strategic voters raises the weight in front of strategic voters' utility as shown up in (2.21), all else equal, the higher the previous risky investment $X^r_{t-1}$, the tighter is the current financial regulation $\phi_t$. Intuitively, this is because large financial losses in previous periods bring up voters' attention on financial regulation, which pushes the politician to implement stringent regulation. On the contrary, in cases of financial market expansion, i.e. when $R_{t-1} \geq r$, the higher the previous risky investment $X^r_{t-1}$, the larger is the realized financial gain. This reduces households' attention on the financial regulation and hence allows for even higher level of risky investment.

Figure 2.2: Equilibrium Financial Regulation

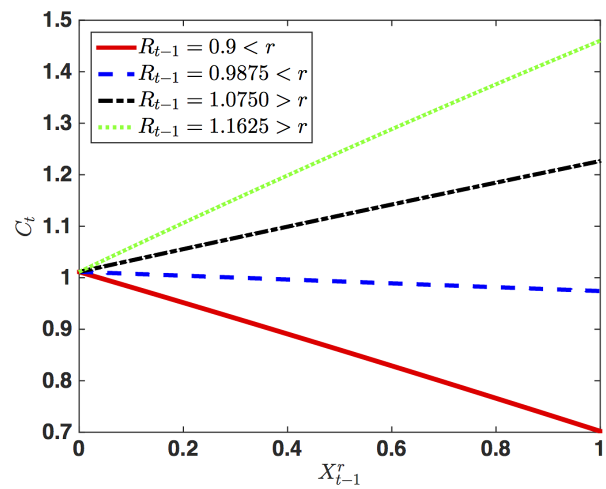


The equilibrium campaign contribution follows from (2.20). In Figure 2.3, I plot the equilibrium campaign contribution $C_t$ as a function of previous risky investment levels $X^r_{t-1}$ at previous financial market realization $R_{t-1}$s same as in Figure 2.2. Note the policy function of the equilibrium contribution resembles that of equilibrium financial regulation $\phi_t$. This is because from (2.20), we can see $C_t$ is a non-decreasing function of $X^r_t$, which takes same value as $\phi_t$ in equilibrium. Yet in addition to $X^r_t$, previous risky investment and risky return realization also affects $C_t$ through the relative proportion of strategic voters and impressionable voters $\frac{1-\sigma_t}{\sigma_t}$. Hence even though $C_t$ and $\phi_t$ shares the same pattern as a function of $X^r_{t-1}$ and $R_{t-1}$, they respond to changes of $X^r_{t-1}$ and $R_{t-1}$ at different magnitudes.

Intuitively, when the financial market is performing well, impressionable voters increase. Knowing that by spending the same amount on the campaign, it will attract a larger share of impressionable voters as compared to the financial crisis period, the financial sector tends to spend more on the political campaign. Moreover, the better the financial market performs, the more effective is the political campaign in terms of attracting impressionable voters. In contrast, when the financial market performs poorly in the past, an increasing number of voters start scrutinize the financial regulation. As a result, the financial sector has less incentive to provide political campaign.

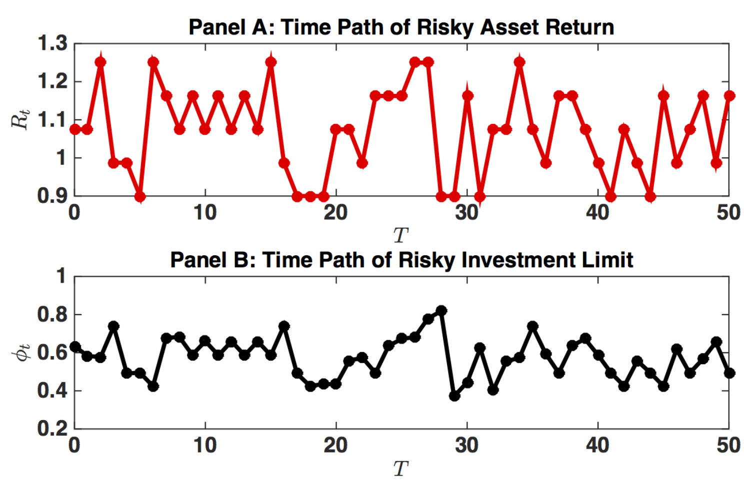
To explore the cyclicality of the financial regulation, in Figure 2.4 I simulate the economy based on the policy functions as defined in (2.21). Comparing the simulated financial regulation series $\phi_t$ with the simulated risky asset realization $R_t$, we can see the financial regulation in the simulated economy is pro-cyclical. In other words, the regulation evolves in a way such that it magnifies the finan-

Figure 2.3: Equilibrium Campaign Contribution




cial market fluctuations. Note that since the regulation depends on both past risky investment $X^r_{t-1}$ and risky asset return realization $R_{t-1}$, depending on past financial regulation $\phi_{t-1}$, for same level of $R_t$ realization, the equilibrium regulation $\phi_t$ may still vary.

Figure 2.4: Simulated Financial Regulation Series



## 2.6 Conclusion

To sum up, this paper studies the cyclicality of financial regulation in a dynamic common agency framework with electoral competition. By comparing financial regulation in an economy without lobbying activities with an economy where the financial sector constitutes the only lobby group in the economy, I show that voters' time-varying attention on financial regulation, together with politicians' electoral concerns, generate pro-cyclical financial regulation. Moreover, I find that in symmetric equilibrium, politicians' electoral loss due to weakened support from the households equals the electoral gain created by campaign contribution from the financial industry. In case of strong past financial market performance, households don't pay much attention to the regulation process and simply vote according to the political campaign. This indirectly encourages the politician to cater to the financial industry and choose loose financial regulation in good economic times.

Since this paper is a first attempt towards understanding the political dimension of financial regulation through a dynamic political economy framework, there remains many potential avenues for future research. One important extension would be to allow for intertemporal asset accumulation and analyze how financial regulation changes in the full dynamic economy. This brings the paper closer to the contemporary discussion of macro-prudential policies by Bianchi and Mendoza (2013), Farhi and Werning (2013) and Korinek and Kreamer (2014). Another natural extension is to relax the myopic assumption of the households and solve the problem with fully forward-looking agents. In this case, household takes into account how current financial regulation affects future evolution of attention as well as their life-time income stream. Finally, following Grossman and Helpman (1996) and Grossman and Helpman (2001), it would be also interesting to explore how electoral bias affects equilibrium contributions and financial regulatory policies.

## 2.7 Appendix: Possibility of Asymmetric Equilibria

This section addresses the possibility of alternative equilibrium outcomes. For purpose of simplicity, I will focus on equilibria with interior solutions. Recall the financial lobby group's problem:

$$
\begin{aligned}
V^F(X_{t-1}^r, R_{t-1}) = &\max_{C_t^A, C_t^B, X_t^{r,A}, X_t^{r,B}} \; y - C_t^A - C_t^B \\
&+ p_t^A \int_{R_{min}}^{R_{max}} \left[ (R_t - r) X_t^{r,A} 1_{\{R_t \geq r\}} + \beta V^F(X_t^{r,A}, R_t) \right] h(R_t) dR_t \\
&+ p_t^B \int_{R_{min}}^{R_{max}} \left[ (R_t - r) X_t^{r,B} 1_{\{R_t \geq r\}} + \beta V^F(X_t^{r,B}, R_t) \right] h(R_t) dR_t \\
&s.t. \\
&[1]\; C_t^A \geq \frac{(1-\sigma_t) f \left[ \mathbb{E}u\left(c(X_t^{r,A^*})\right) - \mathbb{E}u\left(c(X_t^{r,A})\right) \right]}{\sigma_t h} \\
&[2]\; C_t^B \geq \frac{(1-\sigma_t) f \left[ \mathbb{E}u\left(c(X_t^{r,B^*})\right) - \mathbb{E}u\left(c(X_t^{r,B})\right) \right]}{\sigma_t h} \\
&[3]\; 0 \leq X_t^{r,A} \leq D, 0 \leq X_t^{r,B} \leq D
\end{aligned}
$$

where

$$
\begin{aligned}
p_t^A &= \frac{1}{2} + \underline{b} \left\{ (1-\sigma_t) f \left[ \mathbb{E}u\left(c(X_t^{r,A})\right) - \mathbb{E}u\left(c(X_t^{r,B})\right) \right] + \sigma_t h (C_t^A - C_t^B) \right\} \\
p_t^B &= \frac{1}{2} + \underline{b} \left\{ (1-\sigma_t) f \left[ \mathbb{E}u\left(c(X_t^{r,B})\right) - \mathbb{E}u\left(c(X_t^{r,A})\right) \right] + \sigma_t h (C_t^B - C_t^A) \right\}
\end{aligned}
$$

Let $\lambda_t^A$ and $\lambda_t^B$ be the Lagrange multipliers associated with inequality [1] and

[2]. The first order conditions for $C_t^A$ and $C_t^B$ can be derived as follows:

$$\begin{aligned}
&\underline{b}\sigma_t h\Big\{\int_{R_{min}}^{R_{max}}\Big[(R_t-r)X_t^{r,A}1_{\{R_t\geq r\}}+\beta V^F(X_t^{r,A},R_t)\Big]h(R_t)dR_t- \\
&\int_{R_{min}}^{R_{max}}\Big[(R_t-r)X_t^{r,B}1_{\{R_t\geq r\}}+\beta V^F(X_t^{r,B},R_t)\Big]h(R_t)dR_t\Big\} \qquad (2.23)\\
&=1-\lambda_t^A \\
&\underline{b}\sigma_t h\Big\{\int_{R_{min}}^{R_{max}}\Big[(R_t-r)X_t^{r,B}1_{\{R_t\geq r\}}+\beta V^F(X_t^{r,B},R_t)\Big]h(R_t)dR_t- \\
&\int_{R_{min}}^{R_{max}}\Big[(R_t-r)X_t^{r,A}1_{\{R_t\geq r\}}+\beta V^F(X_t^{r,A},R_t)\Big]h(R_t)dR_t\Big\} \\
&=1-\lambda_t^B \qquad (2.24)
\end{aligned}$$

First note that as discussed earlier in the paper, it is impossible to have $\lambda_t^A = \lambda_t^B = 0$. This is because otherwise we would have

$$\begin{aligned}
&\underline{b}\sigma_t h\Big\{\int_{R_{min}}^{R_{max}}\Big[(R_t-r)X_t^{r,A}1_{\{R_t\geq r\}}+\beta V^F(X_t^{r,A},R_t)\Big]h(R_t)dR_t- \\
&\int_{R_{min}}^{R_{max}}\Big[(R_t-r)X_t^{r,B}1_{\{R_t\geq r\}}+\beta V^F(X_t^{r,B},R_t)\Big]h(R_t)dR_t\Big\} \\
=&\underline{b}\sigma_t h\Big\{\int_{R_{min}}^{R_{max}}\Big[(R_t-r)X_t^{r,B}1_{\{R_t\geq r\}}+\beta V^F(X_t^{r,B},R_t)\Big]h(R_t)dR_t- \\
&\int_{R_{min}}^{R_{max}}\Big[(R_t-r)X_t^{r,A}1_{\{R_t\geq r\}}+\beta V^F(X_t^{r,A},R_t)\Big]h(R_t)dR_t\Big\} \\
=&1
\end{aligned}$$

which is a contradiction. In other words, in equilibrium, at least one of the incentive compatibility constraint holds with equality. In fact, there are two possible solutions for the optimization problem: one is the symmetric equilibrium as I characterized earlier, and the other is an alternative solution involving asymmetry. In order to determine the equilibrium regulatory policy, we need to compare the maximized values from these two scenarios.

**Case 1:** $\lambda_t^A > 0, \lambda_t^B > 0$

This corresponds to the symmetric equilibrium as discussed earlier in the paper. Assuming both incentive compatibility constraint binds and using Proposition 1 and Proposition 2, we have $\lambda_t^A = \lambda_t^B = 1$. In this case, the maximized value of the objective function is defined by

$$\bar{V}^F(X_{t-1}^r, R_{t-1}) = y - 2\bar{C}_t + \int_{R_{min}}^{R_{max}} \left[(R_t - r)\bar{X}_t^r 1_{\{R_t \geq r\}} + \beta V^F(\bar{X}_t^r, R_t)\right] h(R_t) dR_t \tag{2.25}$$

where $\bar{X}_t^r$ and $\bar{C}_t$ are as defined by the optimization problem in Proposition 2.

**Case 2:** $\lambda_t^A > 0, \lambda_t^B = 0$ **or** $\lambda_t^A = 0, \lambda_t^B > 0$

Intuitively, an alternative possibility is to have the financial sector contribute to one politician only up to its incentive compatibility constraint, while contribute to the other politician far beyond its incentive compatible level so that the politician implements a highly favorable policy and enjoys high chances of being elected. In this case, we have either $\lambda_t^A > 0, \lambda_t^B = 0$ or $\lambda_t^A = 0, \lambda_t^B > 0$. WLOG, assume that $\lambda_t^A = 0$. Then we have

$$\begin{aligned}
&\underline{b}\sigma_t h \left\{ \int_{R_{min}}^{R_{max}} \left[(R_t - r)X_t^{r,A} 1_{\{R_t \geq r\}} + \beta V^F(X_t^{r,A}, R_t)\right] h(R_t) dR_t - \right. \\
&\left. \int_{R_{min}}^{R_{max}} \left[(R_t - r)X_t^{r,B} 1_{\{R_t \geq r\}} + \beta V^F(X_t^{r,B}, R_t)\right] h(R_t) dR_t \right\} \\
&= 1
\end{aligned}$$

This implies that $\lambda_t^B = 2$. In this case, the maximization problem becomes

$$
\begin{aligned}
& V^F(X_{t-1}^r, R_{t-1}) = \max_{C_t^A, X_t^{r,A}, X_t^{r,B}} \; y - C_t^A - C_t^B \\
& + p_t^A \int_{R_{min}}^{R_{max}} \left[ (R_t - r) X_t^{r,A} 1_{\{R_t \geq r\}} + \beta V^F(X_t^{r,A}, R_t) \right] h(R_t) dR_t \\
& + p_t^B \int_{R_{min}}^{R_{max}} \left[ (R_t - r) X_t^{r,B} 1_{\{R_t \geq r\}} + \beta V^F(X_t^{r,B}, R_t) \right] h(R_t) dR_t \\
& s.t. \\
& [1] \; C_t^B = \frac{(1-\sigma_t) f \left[ \mathbb{E}u\left(c(X_t^{r,B*})\right) - \mathbb{E}u\left(c(X_t^{r,B})\right) \right]}{\sigma_t h} \\
& [2] \; p_t^A = \frac{1}{2} + \underline{b} \left\{ (1-\sigma_t) f \left[ \mathbb{E}u\left(c(X_t^{r,A})\right) - \mathbb{E}u\left(c(X_t^{r,B})\right) \right] + \sigma_t h (C_t^A - C_t^B) \right\} \\
& [3] \; p_t^B = \frac{1}{2} + \underline{b} \left\{ (1-\sigma_t) f \left[ \mathbb{E}u\left(c(X_t^{r,B})\right) - \mathbb{E}u\left(c(X_t^{r,A})\right) \right] + \sigma_t h (C_t^B - C_t^A) \right\}
\end{aligned}
$$

Denote $\tilde{C}_t^A$, $\tilde{X}_t^{r,A}$ and $\tilde{X}_t^{r,B}$ to be the solution to the maximization problem above, and $\tilde{V}^F(X_{t-1}^r, R_{t-1})$ be the value function associated with above maximization problem. Comparing the two value functions, we have: if 1) $\tilde{C}_t^A > \frac{(1-\sigma_t) f \left[ \mathbb{E}u\left(c(X_t^{r,A*})\right) - \mathbb{E}u\left(c(X_t^{r,A})\right) \right]}{\sigma_t h}$ and 2) $\tilde{V}^F(X_{t-1}^r, R_{t-1}) > \bar{V}^F(X_{t-1}^r, R_{t-1})$, then the optimal solution follows the asymmetric equilibrium. Otherwise, the optimal solution follows the symmetric equilibrium.

CHAPTER 3

# EVALUATING DURABLE PUBLIC GOOD PROVISION USING HOUSING PRICES

## 3.1 Introduction

A sizable fraction of government spending is devoted to investment in durable public goods. Such investment is undertaken by all levels of government - federal, state, and local. The goods in question include physical infrastructure (roads, bridges, airports, etc), basic research, defense equipment, environmental clean-ups, parks, and schools. A basic question of interest to economists and policy-makers is how the levels of durable public good provision emerging from the political process compare with socially optimal levels. This question arises in many different policy areas. For example, there seems broad agreement that government substantially underinvests in physical infrastructure and basic research. There is much less agreement concerning defense, environmental, and educational investments, with conservatives and liberals often coming down on opposing sides of the issue. Given the importance of the question, it would be helpful if economic analysis provided convincing ways of answering it.

There is a long tradition in public finance of using house prices to assess the social optimality of local public good provision (see, for example, Brueckner 1979, 1982, Lind, 1973, and Wildasin 1979). The underlying idea is that the demand of potential residents to live in a community will be influenced by the local public goods it provides and the taxes it levies to finance them (Oates 1969, Tiebout 1956). Accordingly, the net surplus generated by local public good provision will be reflected in housing prices.[1] In a well-known and elegant theoretical formulation of the idea, Brueckner develops a model in which if housing prices rise following a small, permanent increase in local public good provision, then it can be inferred that the good is under-provided. Conversely, if housing prices fall, the good is over-provided (Brueckner 1979, 1982). This model has been used as the basis for a number of empirical studies of the optimality of local public good provision (see, for example, Barrow and Rouse 2004, Brueckner 1982, and Lang and Jian 2004).

Can housing prices be used to assess the social optimality of local durable public good provision? In an ambitious and creative paper, Cellini, Ferreira, and Rothstein (2010) employ the approach to detect whether local school districts are over or under-providing public school facilities. Using a static version of Brueckner's model, they argue that if housing prices in a district rise following an investment in public school facilities, then such facilities are under-provided. Conversely, if housing prices fall, facilities are over-provided. To estimate what house prices would be in the counter-factual situation in which an observed investment is not undertaken, Cellini, Ferreira, and Rothstein exploit the fact that investments must be approved by residents in a referendum. Drawing on the regression discontinuity literature, they then compare housing prices in school districts in which referenda have just passed with those in which they have just failed. If prices are higher in the just passing districts, they argue that school facilities are under-provided. This is indeed what they find for California school districts.

The intuitive appeal of this approach not withstanding, there are important

[1] A vast literature investigates the relationship between housing prices and local public good provision empirically, with particular focus on schooling. See Nguyen-Hoang and Yinger (2011) and Ross and Yinger (1999) for useful surveys.

conceptual differences between an investment in a durable public good and a permanent increase in a non-durable public good. First, because of depreciation, the benefits from investment in the durable public good will not be permanent. Rather, they will diminish over time. Second, again because of depreciation, whether or not the investment in question is undertaken, future investments will be made by the community. Moreover, the nature of these investments will depend on the stock of the public good and hence on the fate of the investment in question. This creates a linkage between the current investment and the future investment path in the community. These differences raise the question of whether the logic that underlies the standard test can be applied to justify using the housing price response to investments to evaluate durable public good provision. The purpose of this paper is to investigate this important question.

The paper begins by developing a novel theoretical model in which to study the issue. This model is designed to capture the recurring nature of investment in durable public goods and the linkages between decision-making periods that durability creates. The model is a partial equilibrium model of a single community whose government provides a durable public good. There is a pool of households who, for exogenous reasons, are potential residents of the community. Households move in and out of this pool, creating an active housing market in each period. Public good provision is managed by a bureaucrat who, in any period, can propose investment. Investment is financed by a tax on the residents and, to be implemented, the bureaucrat's proposal must be approved by the majority of residents. The supply of houses in the community is perfectly inelastic, implying that the future surplus a resident is expected to receive from public good provision is fully capitalized into housing prices.

To set the stage for the analysis of durable public goods, the paper first uses the model to review why using the housing price response to a small, permanent increase in provision can be used to evaluate non-durable public good provision. In addition, it extends the approach to show how the test can be used when the assumption of a small increase is not tenable. In particular, it is shown that a non-negative housing price response implies that the public good level without the increase is too low, while a non-positive response implies that the level with the increase is too high.

The paper then investigates whether a similar logic implies that the housing price response to an investment can be used to evaluate durable public good provision. To permit a general analysis, the paper starts out by modelling the behavior of the bureaucrat and residents in a reduced form way with an investment proposal function and a proposal approval probability function. It shows that the key implicit assumption of the test is that the socially optimal level of the public good maximizes the surplus residents are expected to receive from provision *in equilibrium*; that is, when future investment is governed by the investment proposal and proposal approval probability functions. The paper argues that there is no reason to expect this to be the case. The argument is basically an application of the *theory of the second best* (Lipsey and Lancaster 1956). The socially optimal level of the public good is derived under the assumption that all future investments will be socially optimal. This means that all future investments are approved with probability one and any increase in investment today is accommodated by a compensating reduction in the future. It cannot reasonably be presumed that this will be the case in equilibrium. For example, it may be the case that more investment today leads to a more than compensating reduction in the future or reduces the probability that future investments are approved. These second best impacts must be taken into account in the surplus maximization problem and this implies that the public good level that maximizes surplus in equilibrium could be much different from the socially optimal level.

To actually predict the second best impacts of an investment requires a specific model of bureaucrat and resident interaction. To illustrate more concretely how using the housing price response to an investment can provide misleading information, the paper next turns to such a model. Specifically, it assumes that the investment path is generated by the interaction between rational, forward-looking residents and a budget-maximizing bureaucrat who cares about the level of the public good but not its cost. Budget-maximization is a common assumption in the political economy literature and underlies Romer and Rosenthal's *agenda setter model*, the leading alternative to the median voter model of local government spending (Romer and Rosenthal 1978, 1979).[2]

We start our analysis with a simplified model uncertainty in voting outcomes. Using the dynamic agenda setting model laid out in Barseghyan and Coate (2014), we show that the equilibrium price of housing if the bureaucrat's proposed investment is approved (which it will be in equilibrium) exactly equals that if it were not approved. This holds irrespective of the level of the public good prevailing at the time at which the investment is proposed. Intuitively, this reflects the fact that the bureaucrat proposes a level of investment which makes residents indifferent between undertaking it or not. This means that the future value of public good surplus is the same with or without the in-

[2]The budget maximizing assumption was first proposed by Niskanen (1971). For analysis of the relative performance of the median voter and agenda setter models see Romer and Rosenthal (1982), Romer, Rosenthal, and Munley (1992), and, in the specific context of school infrastructure investment, Balsdon, Brunner, and Rueben (2003).

vestment. Since surplus is fully capitalized, this implies that housing prices are the same with or without the investment.

Applying the housing price test, the fact that equilibrium housing prices are the same whether or not the bureaucrat's investment is approved, suggests that the socially optimal level of the public good should lie between the levels that would prevail with and without the investment. However, this is not always the case. Specifically, there exist public good levels at which the bureaucrat proposes an investment, the residents approve it, and the public good level that would prevail without the investment exceeds the socially optimal level. Intuitively, this reflects the fact that the level of public good that maximizes residents' surplus in equilibrium exceeds the socially optimal level. In equilibrium, the public good has a higher marginal value because more units reduce the bureaucrat's future ability to exploit his agenda-setting power. In this situation, therefore, the housing price test falsely predicts under-provision.

To get a deeper understanding of durable public good investment on housing prices, we introduce uncertainty in voting outcomes and solve the model numerically. We find that the housing price test can erroneously predict both under-provision and over-provision of public good. It fails to predict over-provision of public good when there is little aggregate voting uncertainty. Similar to the no uncertainty scenario, it arises from the fact that residents want higher than optimal public good level to control the bureaucrat's agenda setting power. The housing price test can also fail to predict under-provision of public good. This occurs when the bureaucrat is highly risk-averse and there is large aggregate uncertainty. This is because for a highly risk-averse bureaucrat, the additional utility gain from a higher investment level ends up being dominated by the extra loss of residents' support.

Finally, the paper points out that a justification for the housing price test is available if the assumption that residents have rational expectations concerning the future investment path in the community is relaxed. Specifically, the test is shown to work if citizens have adaptive expectations, believing that whatever level of public good they observe in the community at the beginning of a period will be maintained indefinitely. Thus, they observe the current quantity and quality of school facilities, say, and just assume they are at steady state levels. This is a form of myopia that is perhaps not too implausible, particularly for new residents moving into a community. This assumption means that residents perceive a successful investment as permanently increasing provision and this brings us back into the world studied by Brueckner.

Beyond providing a framework to analyze the theoretical question at hand, the model developed here makes a broader contribution. In particular, it provides a simple dynamic model of a housing market in which the market is active in each period and agents are rational and forward-looking. The model highlights the relationship between housing prices and fiscal variables, illustrating the phenomenon of capitalization. In contrast to standard treatments of capitalization in which values of policy variables are frozen through time, the model shows that it is both the current and future values of policy variables that are capitalized into housing prices. Moreover, by endogenizing policy choices via the agenda setter framework, the model derives the dynamic implications of budget maximization for housing prices. More generally, the paper fits in with a growing literature that studies issues in housing markets using dynamic models with rational, forward-looking households (see, for example, Bayer, McMillan, Murphy, and Timmins 2011), particularly those papers that endogenize policy choices with political economy models (Barseghyan and Coate 2013, Epple, Romano, and Sieg 2009, and Ortalo-Magne and Prat 2011).

The paper also contributes to a growing literature on durable public goods. While the vast majority of the public good literature has focused on the provision of non-durable goods, such as firework displays and police protection, in practice many important public goods are durable. Durability not only complicates the conditions for efficient provision but also makes understanding political provision considerably more challenging. This is because today's political choices have implications for future choices, creating a dynamic linkage across policy-making periods. The practical importance of durable public goods and the theoretical challenges they pose is leading to increasing interest in their provision. A number of recent papers have studied the provision of such goods under varying political institutions (see, for example, Battaglini and Coate 2007, Battaglini, Nunnari, and Palfrey, Barseghyan and Coate 2014, and LeBlanc, Snyder, and Tripathi 2000). This paper shows that durability also has important implications for the evaluation of public provision.

The organization of the remainder of the paper is as follows. Section 2 describes the model. Section 3 characterizes the efficient public good provision and reviews the logic underlying using the housing price response to an increase in provision to evaluate non-durable and durable public good provision. Section 4 describes the determination of public good investment through the interaction between a budget-maximizing bureaucrat and rational, forward-looking residents with voting uncertainty. Then it explains why the logic of using the housing price response to an investment to evaluate durable public good provision is generally not justified. Section 5 assumes away voting uncertainty and shows in closed form that using the housing price response to an investment can erroneously predict under-provision. Section 6 parameterize the model and numerically show housing price test can fail to predict both under-provision and over-provision of public goods. Section 7 points out the adaptive expectations justification for the housing price test and Section 8 concludes.

## 3.2 Preliminaries

### 3.2.1 The model

Consider a community such as a municipality or school district. This community can be thought of as one of a number in a particular geographic area. The time horizon is infinite and periods are discrete. There is a pool of potential residents of the community of size 1. These can be thought of as households who for exogenous reasons (employment opportunities, family ties, etc) need to live in the geographic area in which the community is situated. Potential residents are characterized by their desire to live in the community (as opposed to an alternative community in the area) which is measured by the preference parameter $\theta$. This desire, for example, may be determined by a household's idiosyncratic reaction to the community's natural amenities. The preference parameter takes on values between 0 and $\overline{\theta}$, and the fraction of potential residents with preference below $\theta \in [0, \overline{\theta}]$ is $\theta/\overline{\theta}$. Reflecting the fact that households' circumstances change over time, in each period new households join the pool of potential residents and old ones leave. The probability that a household currently a potential resident will be one in the subsequent period is $\mu$. Thus, in each period, a fraction $1-\mu$ of households leave the pool and are replaced by an equal number of new ones.

The only way to live in the community is to own a house. There are a fixed number of houses sufficient to accomodate a population of size $H$ where $H$ is less than the size of the pool of potential residents (i.e., $H < 1$). These houses are infinitely durable.

The community provides a durable public good which depreciates at rate $\delta \in (0,1)$. Provision is managed by a bureaucrat.[3] In any period, the bureaucrat can propose investment. Investment costs $c$ per unit and is financed by a tax on those choosing to reside in the community. To be implemented, the bureaucrat's proposal must be approved by a majority of the residents. Once approved, the investment takes time to build and is not available for use until the next period. The community pays for the investment when it is complete and thus taxes to finance the investment are levied in the next period.[4]

When living in the community, households have preferences defined over the public good and consumption. A household with preference parameter $\theta$ and consumption $x$ obtains a period payoff of $\theta + x + B(g)$ if they live in the community and the public good level is $g$. The benefit function $B(g)$ is increasing, smooth, strictly concave, and satisfies $B(0) = 0$. When not living in the community, a household's per period payoff is $\underline{u}$.[5] Households discount future payoffs

[3]For now, we will not be specific about the bureaucrat's objectives.

[4]As will become clear below, the predictions of the model concerning the impact of an investment on the price of housing would not be changed if the cost of investment was financed by a bond issue rather than a tax.

[5]Note that $\underline{u}$ is both the per period payoff of living in one of the other communities in the geographic area if a household is in the pool and the payoff from living outside the area when a household leaves the pool.

at rate $\beta$ and can borrow and save at rate $1/\beta - 1$. This assumption means that households are indifferent to the intertemporal allocation of their consumption. Each household in the pool receives an exogenous income stream the present value of which is sufficient to pay taxes and purchase housing in the community.[6]

There is a competitive housing market which opens at the beginning of each period. Demand comes from new households moving into the community and supply comes from owners leaving the community. The price of houses is denoted $P$.

The timing of the model is as follows. Each period, the community starts with a public good level $g$ and a tax obligation $T$ (which may be zero). The public good level is the depreciated level from the prior period plus any investment approved in the prior period. The tax obligation is to finance any investment approved in the prior period. At the beginning of the period, those in the pool learn whether they will be remaining and new households join. Households in the pool then decide whether to live in the community. The housing market opens and the equilibrium housing price $P(g, T)$ is determined. The government levies taxes on residents sufficient to meet its tax obligation and residents obtain their payoffs from living in the community. The bureaucrat decides how much investment $I$ to propose and the residents vote. If the proposal is approved, the community's public good level and tax obligation in the next period $(g', T')$ is $((1 - \delta)g + I, cI)$; otherwise, it is $((1 - \delta)g, 0)$.

[6] The assumption that utility is linear in consumption means that there are no income effects, so it is not necessary to be specific about the income distribution.

### 3.2.2 Housing market equilibrium

We now explain how the housing market equilibriates for any given possible path of investment. For the moment being, we summarize the investment path in a reduced form manner with two functions $\pi(g, T, I)$ and $I(g, T)$. The former describes the probability that the investment $I$ will be approved by the residents in a period in which the community starts with a public good level $g$ and a tax obligation $T$. The latter describes the level of investment the bureaucrat proposes when the state is $(g, T)$. We will sometimes use the notation $\pi^*(g, T)$ to denote the probability $\pi(g, T, I(g, T))$. In subsequent sections, we will pin down the investment path by focusing on bureaucrat's decision.

**Decisions of households** At the beginning of any period, households fall into two groups: those who resided in the community in the previous period and those who did not, but could in the current period. Households in the first group own homes. The second group do not. Households in the first group who leave the pool sell their houses and obtain a continuation payoff of

$$P(g, T) + \frac{\underline{u}}{1 - \beta}. \tag{3.1}$$

The remaining households in the first group and all those in the second must decide whether to live in the community. Formally, they make a location decision $l \in \{0, 1\}$, where $l = 1$ means that they live in the community. This decision will depend on their preference parameter $\theta$, current and future housing prices, and public goods and taxes. Since selling a house and moving is costless, there is no loss of generality in assuming that all households sell their property at the beginning of any period.[7] This makes each household's location decision independent of its property ownership state. It also means that the only future consequences of the current location choice is through the selling price of housing in the next period.

To make this more precise, let $V_\theta(g,T)$ denote the expected payoff of a household with preference parameter $\theta$ at the beginning of a period in which it belongs to the pool but does not own a house. Then, we have that

$$V_\theta(g,T) = \max_{l\in\{0,1\}} \left\{ \begin{array}{c} l\left(\theta + B(g) - T/H - P(g,T) + \beta EP(g',T')\right) \\ +(1-l)\underline{u} + \beta[\mu EV_\theta(g',T') + (1-\mu)\frac{\underline{u}}{1-\beta}] \end{array} \right\}, \tag{3.2}$$

where $EP(g',T')$ denotes the expected price of housing next period; i.e.,

$$EP(g',T') = \pi^*(g,T)P((1-\delta)g + I(g,T), cI(g,T)) + (1-\pi^*(g,T))\,P((1-\delta)g, 0), \tag{3.3}$$

and $EV_\theta(g',T')$ denotes the expected payoff of a household in the pool next period; i.e.,

$$EV_\theta(g',T') = \pi^*(g,T)V_\theta((1-\delta)g+I(g,T), cI(g,T))+(1-\pi^*(g,T))\,V_\theta((1-\delta)g, 0). \tag{3.4}$$

Inspecting this problem, it is clear that a household of type $\theta$ will choose to reside in the community if

$$\theta + B(g) - T/H - P(g,T) + \beta EP(g',T') \geq \underline{u}. \tag{3.5}$$

[7]It should be stressed that this is just a convenient way of understanding the household decision problem. The equilibrium we study is perfectly consistent with the assumption that the only households selling their homes are those who plan to leave the community.

The left hand side of this inequality represents the per-period payoff from locating in the community, assuming that the household buys a house at the beginning of the period and sells it the next. This payoff depends on the preference parameter $\theta$, public good surplus, and the current and future price of housing. The right hand side represents the per period payoff from living elsewhere.

**Equilibrium** Given an initial state $(g, T)$, the price of housing $P(g, T)$ adjusts to equate demand and supply. The demand for housing is the fraction of households for whom (3.5) holds. Given the uniform distribution of preferences, this fraction is

$$1 - \frac{\underline{u} - (B(g) - T/H - P + \beta EP(g', T'))}{\overline{\theta}}. \tag{3.6}$$

The supply of housing is, by assumption, perfectly inelastic at $H$. The equilibrium price of housing therefore satisfies

$$1 - \frac{\underline{u} - (B(g) - T/H - P(g, T) + \beta EP(g', T'))}{\overline{\theta}} = H. \tag{3.7}$$

To characterize the housing market equilibrium, define the present value of public good surplus $S(g, T)$ recursively as follows:

$$S(g, T) = B(g) - T/H + \beta\left[\pi^*(g, T)S((1-\delta)g + I(g, T), cI(g, T)) + (1 - \pi^*(g, T))S((1-\delta)g, 0)\right]. \tag{3.8}$$

Intuitively, $S(g, T)$ represents the discounted value of future public good surplus for a household who will be living in the community permanently starting in a period in which the community has public good level $g$ and tax obligation $T$. Then, we have:

**Proposition 1** *In equilibrium, those households for whom* $\theta \in [(1-H)\overline{\theta}, \overline{\theta}]$ *choose to reside in the community and those for whom* $\theta \in [0, (1-H)\overline{\theta}]$ *do not. For households choosing to reside in the community there exists a constant* $\kappa(\theta)$ *such that*

$$V_\theta(g, T) = \kappa(\theta) + S(g, T) - P(g, T), \tag{3.9}$$

*while for households choosing not to reside in the community*

$$V_\theta(g, T) = \frac{\underline{u}}{1-\beta}. \tag{3.10}$$

*Furthermore, there exists a constant* $K$ *such that the equilibrium housing price is given by*

$$P(g, T) = K + S(g, T). \tag{3.11}$$

**Proof:** See Appendix A.

The first part of the proposition tells us that the fraction $H$ of households who choose to reside in the community are those in the pool with the highest preference parameters. This should make good sense intuitively since in all other respects potential residents are identical. The second part gives us expressions for the expected payoffs of the different types of households. These expressions will be useful later in the paper. The final part tells us that the equilibrium price can be expressed as the sum of a constant and the value of public good surplus. Equation (3.11) implies that the value of future public good levels and tax obligations is fully capitalized into the price of housing and follows from the assumption that the supply of houses is fixed. The constant $K$ is tied down by the requirement that the marginal household with preference $(1-H)\overline{\theta}$ is just indifferent between living and not living in the community.[8]

It should be clear from Proposition 1 that households' equilibrium payoffs and the price of housing would be the same if the investment were financed via a bond issue rather than a tax increase. All that matters is the discounted present value of tax obligations and a policy change which held this constant but altered the future timing of taxes would have no impact on the current price of housing.[9] Ricardian Equivalence therefore holds in this model. Similar remarks apply if, once approved, the investment comes on tap over a sequence of future periods rather than all in the next period as assumed here.

## 3.3 Housing Price Test Under Optimal Investment Path

Up to this point, housing market equilibrium have been analyzed with arbitrary investment rule $I(g)$ and probability function $\pi(g, I)$. As $\pi(g, I)$ and $I(g)$ vary, different equilibria are generated, which leads to different housing prices and public good surpluses. In this section, we solve for the socially optimal public good investment policy $I(g)$ which maximizes the residents' payoffs. After that, we discuss the housing price test and show the housing prices generated by socially optimal investment policy correctly predicts the optimal public good provision.

[8]It is straightforward to show that $K$ equals $\kappa((1-H)\overline{\theta}) - \underline{u}/(1-\beta)$. To guarantee that housing always has a positive value, it must be the case that the parameters and investment path are such that $\kappa((1-H)\overline{\theta}) + S(g, T)$ always exceeds $\underline{u}/(1-\beta)$. We will assume this in what follows.

[9]The future housing price path would be impacted by the choice of debt versus taxes. Suppose, for example, that the cost of investment was financed by issuing one period bonds. Then, while the price of housing in the period after the investment was approved would be the same as under tax finance, the price in the subsequent period when the bonds must be repaid would be lower. This is because taxes must be levied, whereas, with tax finance, the investment is already paid for. However, this is irrelevant for the purposes of this paper which is concerned solely with the immediate impact of an approved investment on housing prices.

### 3.3.1 Optimal public good provision

Let's first characterize the path of investment that would be chosen by a bureaucrat that sought to maximize the residents' payoffs. Suppose that the current level of the public good is $g$ and the tax obligation is $T$. From Proposition 1, the residents of the community consist of those households for whom $\theta$ lies between $(1-H)\overline{\theta}$ and $\overline{\theta}$. At the time the bureaucrat is choosing investment, these households all own houses. Thus, the expected continuation payoff of each of these households next period if the bureaucrat chooses $I$ units of investment is given by

$$P((1-\delta)g+I, cI)+\mu V_{\theta}((1-\delta)g+I, cI)+(1-\mu)\frac{\underline{u}}{1-\beta} = \mu\kappa(\theta)+(1-\mu)(\frac{\underline{u}}{1-\beta}+K)+S((1-\delta)g+I, cI). \tag{3.12}$$

To understand the left hand side of (3.12), consider a home-owning household at the beginning of the next period. The public good level and tax obligation will be $((1-\delta)g+I, cI)$. As noted above, we can assume wlog that the household will sell their house and obtain a payoff $P((1-\delta)g+I, cI)$. With probability $\mu$, they will remain in the set of potential residents and obtain a payoff $V_{\theta}((1-\delta)g+I, cI)$ and with probability $1-\mu$ they will exit the pool and obtain payoff $\underline{u}/(1-\beta)$. The right hand side of (3.12) follows immediately from equations (3.9) and (3.11) of Proposition 1 and tells us that the continuation payoff can be written as the sum of a type-specific constant and the value of public good surplus.

It follows from (3.12) that choosing investment to maximize resident payoffs is equivalent to maximizing public good surplus.[10] Letting $S^{o}(g,T)$ denote

[10]This conclusion arises despite the fact that residents may leave the community and thus not get to enjoy the fruits of their investment. The intuition is that, when they leave, residents will sell their homes and the price they get will reflect the future benefits.

maximized surplus, we know that

$$S^o(g,T) = \max_{I \geq 0} B(g) - T/H + \beta S^o((1-\delta)g + I, cI). \tag{3.13}$$

Letting $I^o(g,T)$ denote the surplus maximizing investment rule, we have:

**Proposition 2** *The optimal investment rule is that*

$$I^o(g,T) = \begin{cases} g^o - (1-\delta)g & \text{if } g \leq g^o/(1-\delta) \\ 0 & \text{if } g > g^o/(1-\delta) \end{cases}, \tag{3.14}$$

*where the public good level $g^o$ satisfies the dynamic Samuelson Rule*

$$HB'(g^o) = c\left[1 - \beta(1-\delta)\right]. \tag{3.15}$$

**Proof:** See Appendix A.

Proposition 2 tells us that the optimal investment rule is to get the public good level to $g^o$ as fast as possible and then keep it there. The community's current tax obligation $T$ has no impact on optimal investment because utility is linear in consumption, implying no income effects. The optimal level $g^o$ satisfies the condition that the sum of one period marginal benefits equals the "one period marginal cost". The latter reflects the fact that investing one unit today saves $c(1-\delta)$ in investment costs tomorrow and these future cost savings have a present value of $\beta c(1-\delta)$.

### 3.3.2 The housing price test

#### Non-durable public goods

We begin by reviewing the logic underlying the housing price test in the context of non-durable public goods. The claim to be evaluated is that the housing price response to a small, permanent increase in public good level reveals the efficiency of provision. Suppose therefore that the community currently provides $g$ units of public good per period which generates a tax obligation of $cg$ and consider a small, permanent increase in provision of $\Delta g$ with associated tax obligation $c\Delta g$.

In our model, we capture this scenario by assuming 100% depreciation ($\delta = 1$), which effectively makes the public good non-durable. In addition, we assume that the investment path is such that the proposed investment is always $g + \Delta g$ and the probability that it is approved equals 1. These assumptions imply that the community's public good level and tax obligation each period will be $(g + \Delta g, c\,(g + \Delta g))$. From (3.8), public good surplus is

$$S\,(g + \Delta g, c\,(g + \Delta g)) = \frac{B(g + \Delta g) - c\,(g + \Delta g)\,/H}{1 - \beta}. \tag{3.16}$$

Now let $\Delta P(g, \Delta g)$ denote the difference in housing prices with and without the public good increase.[11] Proposition 1 implies that the price difference is equal to the difference in surplus; that is,

$$\Delta P(g, \Delta g) = S\,(g + \Delta g, c\,(g + \Delta g)) - S\,(g, cg). \tag{3.17}$$

[11] That is, $\Delta P(g, \Delta g) = P(g + \Delta g, c\,(g + \Delta g)) - P(g, cg)$.

The price difference will therefore be positive if the increase has raised surplus, and negative if not. If the increase is small, then the difference in surplus is approximately equal to the total derivative of surplus multiplied by the increase; that is,

$$S(g + \Delta g, c(g + \Delta g)) - S(g, cg) \approx \frac{dS(g, cg)}{d\Delta g}\Delta g. \tag{3.18}$$

From (3.16), the total derivative of surplus is

$$\frac{dS(g, cg)}{d\Delta g} = \frac{B'(g + \Delta g) - c/H}{1 - \beta}. \tag{3.19}$$

The optimal level of the public good $g^o$ satisfies the static Samuelson Rule that the sum of marginal benefits $HB'(g)$ equals the marginal cost $c$. From (3.19) this implies that the derivative of surplus is zero at the optimal level; i.e., $dS(g^o, cg^o)/d\Delta g = 0$. Thus, since surplus is concave in $\Delta g$, we have that

$$\frac{dS(g, cg)}{d\Delta g}\Delta g \gtreqless 0 \Longleftrightarrow g \lesseqgtr g^o. \tag{3.20}$$

From (3.17) and (3.18), therefore, the housing price response to a small, permanent increase in the public good is, to a first approximation, positive if $g$ is less than $g^o$ and negative if $g$ exceeds $g^o$.

The assumption that the increase in public good level is small is important to this logic. Nonetheless, the housing price test is still informative when this assumption is not satisfied. In this case, the public good level *with* the increase (i.e., $g + \Delta g$) must be distinguished from the level *without* (i.e., $g$). Since surplus is strictly concave in $\Delta g$, it follows from (3.17) that

$$\frac{dS\left(g+\Delta g, c\left(g+\Delta g\right)\right)}{d\Delta g}\Delta g < \Delta P(g,\Delta g) < \frac{dS\left(g, cg\right)}{d\Delta g}\Delta g. \tag{3.21}$$

A non-negative price difference therefore signals that $dS\left(g, cg\right)/d\Delta g$ is positive and hence the public good level without the increase is below optimal. By contrast, a non-positive difference signals that $dS\left(g+\Delta g, c\left(g+\Delta g\right)\right)/d\Delta g$ is negative and hence the level with the increase is too high.

**Durable public goods**

We now explore whether a similar logic implies that the housing price response to an investment sheds light on the efficiency of durable public good provision. Suppose the current state is $(g, T)$ and the bureaucrat proposes an investment level $I(g, T)$. Next period's state will be $((1-\delta)g + I(g, T), cI(g, T))$ if the investment is approved and $((1-\delta)g, 0)$ if not. Let the difference in the price of housing that would prevail next period with and without the investment be denoted $\Delta P(g, I(g, T))$.[12] Proposition 1 implies that the price difference equals the difference in surplus; that is,

$$\Delta P(g, I(g,T)) = S\left((1-\delta)g + I(g,T), cI(g,T)\right) - S\left((1-\delta)g, 0\right). \tag{3.22}$$

[12]This price difference corresponds to what Cellini, Ferreira, and Rothstein (2010) refer to in their empirical work as the "intent-to-treat" (ITT) effect of the investment on housing prices. It represents the reaction of housing prices to the investment *assuming that all future investment decisions will be made according to the community equilibrium*. They also discuss a "treatment on the treated" (TOT) effect which is the hypothetical reaction of housing prices to the investment *assuming there were no future investments*. As will be pointed out below in footnote #15, this paper's critique also applies to this latter measure.

A positive price difference implies that the investment has increased surplus, while a negative difference implies that surplus has decreased. Assuming the investment is small, we can approximate the change in surplus as follows:

$$S((1-\delta)g + I(g,T), cI(g,T)) - S((1-\delta)g, 0) \approx \frac{dS((1-\delta)g,0)}{dI} I(g,T). \tag{3.23}$$

If it were the case that the optimal level of the public good $g^o$ satisfied the first order condition that $dS(g^o, 0)/dI$ equals zero and if surplus were concave in $I$, we would have that

$$\frac{dS((1-\delta)g,0)}{dI} I(g,T) \gtreqless 0 \Longleftrightarrow (1-\delta)g \lesseqgtr g^o. \tag{3.24}$$

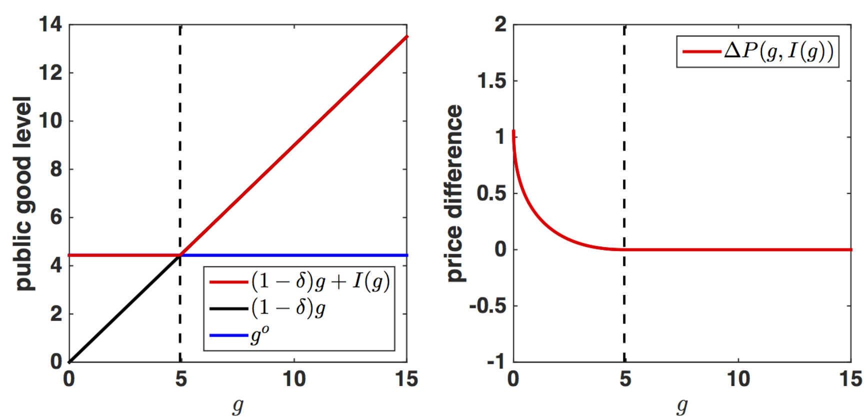
The logic from the non-durable case would then exactly carryover. The price impact of a small investment would be positive if $(1-\delta)g$ is less than $g^o$ and negative if $(1-\delta)g$ exceeds $g^o$.

Again, the assumption that the investment $I(g,T)$ is small is key to this logic. However, this is not a tenable assumption for a durable good subject to depreciation. After all, just to maintain public good levels, it will be necessary to have investment sufficient to offset depreciation. Thus, unless the depreciation rate is infinitessimal, $I(g,T)$ cannot be small in steady state. But, as in the non-durable case, this not a major problem. We just need to distinguish the level of public good with the investment (i.e., $(1-\delta)g + I(g,T)$) and the level without (i.e., $(1-\delta)g$). Since surplus is linear in taxes, if surplus is strictly concave in the public good level, we have that[13]

$$\frac{dS((1-\delta)g + I(g,T), 0)}{dI} I(g,T) < \Delta P(g, I(g,T)) < \frac{dS((1-\delta)g, 0)}{dI} I(g,T). \quad (3.25)$$

Thus, provided the optimal level $g^o$ satisfies the first order condition that $dS(g^o, 0)/dI$ equals zero, a non-negative price difference signals that the public good level without the investment is below optimal, while a non-positive price difference signals that the level with investment is too high. This is the form of the housing price test that is most relevant in the durable good context.

Figure 3.1: Optimal Investment and Housing Price Test




We conclude this section by providing a graphical illustration of Proposition 2 and the validity of housing price test under the optimal investment path. We assume that residents' benefit function takes the functional form $B(g) = \sqrt{g}$. To compute the optimal investment numerically, we assume discount factor $\beta$ equals 0.9, depreciation rate $\delta$ equals 0.1, per unit investment cost $c$ is 1, and housing supply $H$ equals 0.8.

The left panel of Figure 3.1 summarizes Proposition 2. In this figure, the

[13]Note that $dS(g,0)/dI = \partial S(g,0)/\partial g + c\partial S(g,0)/\partial T$. However, $\partial S(g,0)/\partial T$ is equal to $-1/H$. Thus, if $\partial^2 S(g,0)/\partial g^2$ is negative, $dS(g,0)/dI$ is decreasing in $g$.

red line denotes the optimal public good level with investment $(1-\delta)g + I(g)$. The black line denotes the public good level without investment $(1-\delta)g$. The blue line highlights the socially optimal public good level $g^o$. Consistent with Proposition 2, we can see that when $g$ is less than $g^o/(1-\delta)$, the bureaucrat's investment policy $I(g)$ pushes the public good level back to the optimal level $g^o$ in the next period. Once $g$ is above $g^o/(1-\delta)$, the bureaucrat makes no investment and the public good level falls to $(1-\delta)g$ in the next period.

In the right panel of Figure 3.1, we plot the price difference $\Delta P(g, I(g))$ as a function of $g$ in solid red line. The dashed black line denotes the public good level at which $\Delta P(g, I(g)) = 0$. According to the housing price test, as long as $\Delta P(g, I(g))$ is positive, i.e., $g$ is to the left of the black dashed line, the public good is under-provided and the optimal public good level should be above $(1-\delta)g$.Such a prediction is indeed correct. In the left panel, we can see that the black dashed line exactly corresponds to the public good level $g = g^o/(1-\delta)$. Over the range of g where $\Delta P(g, I(g)) > 0$, $(1-\delta)g$ does lie below the socially optimal public good level $g^o$.

## 3.4 Housing Price Test Under Budget-Maximizing Bureaucrat

To illustrate the potential problem with the housing price test, we now analyze what it tells us when the investment path is generated by the interaction between a budget-maximizing bureaucrat who manages the provision of a public good or service for a community and the rational, forward-looking residents of that community. We assume that the level of the public good is chosen by the bureaucrat but is subject to resident approval via a ratification vote. If the bureaucrat's proposed spending level is not approved by a majority of residents, then spending reverts to the depreciated public good level. Similar to the agenda setting model of Romer and Rosenthal (1978, 1979), a tension exists between the bureaucrat and residents, because the former cares just about the size of his budget, while the latter also care about costs. However, this model differs from theirs in two important ways. On the one hand, the durable public good assumption makes the reversion level simply equal to the depreciated current public good level rather than an exogenously set level; on the other hand, the introduction of random voting outcomes allows for the impact of investment on the future probability of proposal passing, which in turn influences expected future public good surplus. Different from section 3.3 where both the probability $\pi(g, I)$ and the investment rule $I(g)$ are set optimal, here they are the endogenous outcome of the equilibrium with probabilistic voting.

Since deriving closed-form solutions for the full-fledged problem turns out to be quite challenging, we proceed in three steps. First, before solving for any specific investment rules or probability functions, we point out how the fundamental problem associated with the housing price test may arise in equilibrium. Such potential problem can be demonstrated without explicit assumptions on the voter and bureaucrat behavior. Second, in subsequent section, an analytical example of the failure of housing price test is provided under the assumption of no voting uncertainty. This special case of the budget-maximizing bureaucrat's problem is essentially the dynamic version of Romer and Rosenthal's agenda control model analyzed by Barseghyan and Coate (2014). We show that the housing price test in this scenario can erroneously predicts under-provision. Finally, we parameterize the full-fledged model and solve it numerically. We show that under the more general setting, housing price test may fail to predict over-provision as well as under-provision.

### 3.4.1 Determination of Equilibrium Investment Path

**Voters' Decision** Consider a typical resident household of type $\theta$. He votes in favor of investment proposal $I$ if the difference in his continuation payoff with and without the proposed investment exceeds the value of a voting preference shock:

$$\left[P((1-\delta)g+I,cI)+\mu V_\theta((1-\delta)g+I,cI)+(1-\mu)\frac{\underline{u}}{1-\beta}\right]\geq$$
$$\left[P((1-\delta)g,0)+\mu V_\theta((1-\delta)g,0)+(1-\mu)\frac{\underline{u}}{1-\beta}\right]+\epsilon_\theta+\eta. \tag{3.26}$$

Here on the left-hand side of the inequality is the expected continuation payoff of the household next period if the bureaucrat chooses $I$ units of the investment. On the right-hand side of the inequality, the term within square bracket is the expected continuation payoff of the household next period if the bureaucrat makes no investment. The additional term $\epsilon_\theta+\eta$ captures the household's bias towards no investment. Such bias consists of an idiosyncratic component $\epsilon_\theta$ and an aggregate component $\eta$. The idiosyncratic component $\epsilon_\theta$ reflects individual resident's personal bias towards no investment. We assume it is the realization of a random variable uniformly distributed on $[-\overline{\epsilon},\overline{\epsilon}]$. The aggregate component $\eta$ measures the average popularity of no investment in the resident population as a whole, which we assume to be the realization of a random variable normally distributed with mean 0 and variance $\sigma^2$.

Using Proposition 1, we can simplify resident's difference of the expected continuation payoff into the difference in terms of public good surplus. Hence the inequality above can be rewritten as

$$S((1-\delta)g + I, cI) - S((1-\delta)g, 0) \geq \epsilon_\theta + \eta. \tag{3.27}$$

Under the assumption that $\epsilon_\theta$ is identically distributed over $[-\overline{\epsilon}, \overline{\epsilon}]$, the fraction of voters voting in favor of the proposal given the aggregate shock $\eta$ is[14]

$$\frac{S((1-\delta)g + I, cI) - S((1-\delta)g, 0) - \eta}{2\overline{\varepsilon}} + \frac{1}{2}. \tag{3.28}$$

Under majority rule, the proposed investment is approved when this fraction exceeds 1/2. The probability that the proposed investment passes $\pi(g, I)$ is therefore the probability that

$$S((1-\delta)g + I, cI) - S((1-\delta)g, 0) \geq \eta. \tag{3.29}$$

Given the assumption that $\eta$ is normally distributed with mean 0 and variance $\sigma^2$, this probability is

$$\pi(g, I) = \Phi\left[S((1-\delta)g + I, cI) - S((1-\delta)g, 0)\right] \tag{3.30}$$

This expression implies a simple relationship between the probability of a proposal passing and the surplus it generates for residents. Note that this probability is independent of the community's current tax obligation $T$, so we have omitted this variable from the probability function.

[14]Here we assume $\overline{\epsilon}$ is sufficiently large so that $\frac{S((1-\delta)g+I,cI)-S((1-\delta)g,0)-\eta}{2\overline{\varepsilon}} + \frac{1}{2} \in [0, 1]$.

**Bureaucrat's Decision** Letting the bureaucrat's value function be denoted $U(g,T)$, the bureaucrat will choose an investment proposal where

$$I(g) = \arg\max \left\{ \begin{array}{c} u(g) + \beta\left[\pi(g,I)U((1-\delta)g + I, cI) + (1-\pi(g,I))\, U((1-\delta)g, 0)\right] \\ s.t.\ I \geq 0 \end{array} \right\}. \tag{3.31}$$

Again, this proposal is independent of the community's current tax obligation, reflecting the absence of income effects. The bureaucrat's value function then satisfies the functional equation

$$U(g,T) = u(g) + \pi(g, I(g))U((1-\delta)g + I(g), cI(g)) + (1-\pi(g, I(g)))\, U((1-\delta)g, 0). \tag{3.32}$$

Using (3.8), the residents' public good surplus function $S(g,T)$ is defined recursively by

$$S(g,T) = B(g) - T/H + \beta\left[\pi(g, I(g))S((1-\delta)g + I(g), cI(g)) + (1-\pi(g, I(g)))S((1-\delta)g, 0)\right]. \tag{3.33}$$

An *equilibrium with probabilistic voting* consists of an investment proposal function $I(g)$, a proposal approval probability function $\pi(g,I)$, and value functions $U(g,T)$ and $S(g,T)$ such that: i) $I(g)$ solves problem (3.31); ii) $\pi(g,I)$ satisfies equation (3.30); iii) $S(g,T)$ satisfies equation (3.33); and iv) $U(g,T)$ satisfies equation (3.56).

### 3.4.2 The problem with housing price test

As mentioned earlier, before solving the bureaucrat's problem, we will first explain the potential problem associated with the housing price test. The fundamental problem with the housing price test analysis lies in the assumption that the socially optimal public good level $g^o$ satisfies the first order condition that $dS(g^o, 0)/dI$ equals zero. This is true for the socially optimal surplus function $S^o(g, T)$ but the *equilibrium* public good surplus function $S(g, T)$ will not in general equal the *optimal* surplus function $S^o(g, T)$. The former assumes that future investment decisions are governed by the equilibrium rules $\pi(g, T, I)$ and $I(g, T)$, while the latter assumes decisions are made optimally. As is well known from the theory of the second best, the fact that some decisions are not optimal typically means that the rules governing the decisions that can be optimized will change.

To see the difficulty formally, note from (3.8) that the derivative of surplus with respect to investment is

$$\frac{dS((1-\delta)g+I,0)}{dI} = \left[B'((1-\delta)g+I) - c/H\right] +$$
$$\beta\left[\begin{array}{c} \frac{d\pi^*(\cdot)}{dI}\left\{S((1-\delta)\left[(1-\delta)g+I\right]+I(\cdot), cI(\cdot)) - S((1-\delta)\left[(1-\delta)g+I\right],0)\right\} \\ +\pi^*(\cdot)\left\{\frac{\partial S((1-\delta)\left[(1-\delta)g+I\right]+I(\cdot),cI(\cdot))}{\partial g}\left[(1-\delta)+\frac{dI(\cdot)}{dI}\right] - c\frac{dI(\cdot)}{dI}/H\right\} \\ +(1-\pi^*(\cdot))\frac{\partial S((1-\delta)\left[(1-\delta)g+I\right]+I(\cdot),cI(\cdot))}{\partial g}(1-\delta) \end{array}\right], \tag{3.34}$$

where to compact notation $\pi^*(\cdot)$ denotes $\pi^*((1-\delta)g + I, 0)$ and $I(\cdot)$ denotes $I((1-\delta)g + I, 0)$.[15] The term in square brackets on the top line of (3.34) measures the immediate consequences of an increase in investment on surplus: public good

[15] Accordingly, the total derivative $d\pi^*(\cdot)/dI$ equals $\partial\pi^*(\cdot)/\partial g + c\partial\pi^*(\cdot)/\partial T$ and the total derivative $dI(\cdot)/dI$ equals $\partial I(\cdot)/\partial g + c\partial I(\cdot)/\partial T$.

benefits go up as do taxes. The second term measures the future consequences and these are evidently quite complicated. In particular, account must be taken of how an increase in current investment will impact the level of next period's investment (i.e., $dI(\cdot)/dI$) and also the probability that next period's investment passes (i.e., $d\pi^*(\cdot)/dI$).

A necessary and sufficient condition for $dS(g^o, 0)/dI$ to equal zero is that the second term in (3.34) equals $\beta(1-\delta)c/H$ when evaluated at $g^o$. For only then will it be the case that

$$\frac{dS(g^o, 0)}{dI} = B'(g^o) - c[1 - \beta(1 - \delta)]/H. \tag{3.35}$$

Given the dynamic Samuelson Rule (3.15), this is required for $dS(g^o, 0)/dI$ to equal zero. Intuitively, when the second term in (3.34) equals $\beta(1 - \delta)c/H$, the future consequence of a marginal increase in current investment is just the discounted value of a compensating decrease in the amount of investment approved next period. With depreciation, a unit of investment this period creates $1 - \delta$ of a unit next period and so a compensating decrease would be $1 - \delta$. This would save each resident $c(1 - \delta)/H$ in taxes and this has a present value of $\beta(1 - \delta)c/H$. Under an optimal investment plan, the second term in (3.34) will indeed equal $\beta(1 - \delta)c/H$. To see this, note from (3.14) that $dI(\cdot)/dI = -(1 - \delta)$ and, since investments pass with probability one, $\pi^*(\cdot) = 1$ and $d\pi^*(\cdot)/dI = 0$. In equilibrium, however, there is no reason to believe that the future consequences of an increase in investment will be so simple.

From equation (3.25), it is clear that if $dS(g^o, 0)/dI$ exceeds zero, a non-negative price difference no longer implies that the public good level without investment is below optimal. Conversely, when $dS(g^o, 0)/dI$ is less than zero, a non-positive price difference no longer implies that the public good level with investment is too high. To understand the potential bias in the housing price test, therefore, it is interesting to understand which of these cases is more likely to arise. The former (latter) case arises when the second term in (3.34) evaluated at $g^o$ exceeds (falls short of) $\beta(1-\delta)c/H$. Differencing these two expressions, we obtain

$$\beta\begin{bmatrix} \frac{d\pi^*(\cdot)}{dI}\left(S\left((1-\delta)g^o + I(\cdot), cI(\cdot)\right) - S\left((1-\delta)g^o, 0\right)\right) \\ +\left(\frac{\partial S((1-\delta)g^o+I(\cdot),cI(\cdot))}{\partial g} - c/H\right)\left(1-\delta+\frac{dI(\cdot)}{dI}\right) \end{bmatrix}. \tag{3.36}$$

When this expression is positive (negative), the future consequence of an increase in investment exceeds (falls short of) $\beta(1-\delta)c/H$. The sign of the first term is unclear because the sign of $d\pi^*(\cdot)/dI$ is uncertain. While *ceteris paribus* having more investment might be expected to reduce the probability of a proposed new investment passing, it will also reduce the size of the proposed investment, so the net effect is uncertain. The sign of the second term is ambiguous because it is not clear how $dI(\cdot)/dI$ will compare with $-(1-\delta)$: that is, will an increase in investment lead to a more or less than compensating adjustment in new investment? Moreover, even if that issue were resolved, the sign of the difference $\partial S/\partial g - c/H$ is not obvious.[16]

All this suggests that the bias in the housing price test depends on the specific interaction of voters and bureaucrat. In next two sections, we will provide examples where $dS(g^o, 0)/dI$ is non-zero and, as a consequence, housing price

[16]Suppose that we instead evaluated public good provision using the hypothetical housing price response to the investment assuming there were no future investments (the TOT effect discussed in footnote #12). With no future investments, public good surplus would be $S(g,T) = \sum_{t=0}^{\infty}\beta^t B(g(1-\delta)^t) - T/H$. It is easily verified that with this surplus function, $dS(g^o, 0)/dI$ exceeds zero. Intuitively, if no investment will take place in the future, the optimal public good level today will be much larger than $g^o$. As a consequence, a positive housing price response would not imply the public good level without investment was below optimal.

test leads to false predictions.

## 3.5 Housing Price Test Failure Without Voting Uncertainty

To analytically demonstrate the possibility of failure in the housing price test, we focus on a special case of section 3.4 where the interaction between residents and bureaucrat follows the dynamic version of Romer and Rosenthal's agenda setter model analyzed by Barseghyan and Coate (2014). Instead of the random voting outcomes, they assume that there is no voting uncertainty and the bureaucrat chooses investment proposals that must be approved by the voters.[17] We derive the equilibrium value of public good surplus in this model and, using Proposition 1, show that whether or not an investment is approved makes no difference in the prices that prevail next period. The housing price test therefore suggests that the socially optimal public good level lies between the levels with and without investment. However, we will see that the socially optimal level can be smaller than the level without investment. This reflects the fact that $dS(g^o, 0)/dI$ is positive.

[17]The model studied here differs from that studied by Barseghyan and Coate (2014) in that it incorporates a housing market. This permits the implications of the equilibrium for housing price dynamics to be derived. Furthermore, the analysis here is limited to deriving the equilibrium public good surplus function, which is all that is necessary to evaluate the performance of the housing price test. By contrast, the point of Barseghyan and Coate (2014) is to provide a comprehensive analysis of equilibrium in the dynamic agenda setter model.

### 3.5.1 Romer-Rosenthal Equilibrium Investment Path

**Voters' Decision** We assume there is no preference shock and $\epsilon_\theta + \eta \equiv 0$ [18]. Following Barseghyan and Coate (2014), residents approve the bureaucrat's investment proposal only if it raises their expected continuation payoffs. According to (3.27), this implies that residents approve the bureaucrat's proposal if it raises public good surplus:

$$\pi(g, I) = \begin{cases} 1 & \text{if } S((1-\delta)g + I, cI) \geq S((1-\delta)g, 0) \\ 0 & \text{if } S((1-\delta)g + I, cI) < S((1-\delta)g, 0) \end{cases}. \tag{3.37}$$

**Bureaucrat's Decision** Letting $U(g, T)$ denote the bureaucrat's value function, he chooses an investment proposal $I(g)$ where

$$I(g) = \arg\max \left\{ \begin{array}{c} g + \beta U((1-\delta)g + I, cI) \\ s.t.\ S((1-\delta)g + I, cI) \geq S((1-\delta)g, 0) \ \&\ I \geq 0 \end{array} \right\}. \tag{3.38}$$

The bureaucrat's value function is defined recursively by the equation

$$U(g, T) = g + \beta U((1-\delta)g + I(g), cI(g)), \tag{3.39}$$

while from (3.8), the residents' public good surplus function is defined by

$$S(g, T) = B(g) - T/H + \beta S((1-\delta)g + I(g), cI(g)). \tag{3.40}$$

An *equilibrium* of the dynamic agenda setter model consists of an investment

[18] Alternatively, we can think of this assumption as a special case of the general model with the variance of the normal distribution $\sigma^2$ converging to 0

proposal function $I(g)$ and value functions $U(g, T)$ and $S(g, T)$ satisfying equations (3.38), (3.39), and (3.40).

Barseghyan and Coate (2014) focuses on a particular type of equilibrium, which they term a Romer-Rosenthal equilibrium. In Romer and Rosenthal's static model, equilibrium involves the bureaucrat proposing the largest level of public spending which leaves the median voter at least as well off as with the reversion level. This is just the reversion level if it exceeds the median voter's optimal level, and otherwise exceeds the reversion level. The Romer-Rosenthal equilibrium is the analogue to this in the dynamic setting. The defining feature is that in each period the bureaucrat proposes the maximum level of investment the residents will approve.[19]

### 3.5.2 Equilibrium Implications

To define the Romer-Rosenthal equilibrium concept formally, let $g^*$ denote the residents' preferred level of the public good given the equilibrium value function $S(g, T)$; i.e., $g^*$ maximizes $S(g, cg)$. When $g$ exceeds $g^*/(1-\delta)$ the residents will prefer the reversion level $g(1-\delta)$ to any higher level. Accordingly, the bureaucrat can propose no investment. When $g$ is less than $g^*/(1-\delta)$, there exist investment levels that will be supported by the residents. In a Romer-Rosenthal equilibrium, the bureaucrat will choose the largest of these. Thus, he will choose an investment level $I(g) > 0$ such that

$$S((1-\delta)g + I, cI) = S((1-\delta)g, 0). \tag{3.41}$$

[19]This is as opposed to holding back in some period to boost the amount of investment that the residents approve in the next period.

Intuitively, at this investment level, the future benefits to the residents are just offset by the tax cost. Accordingly, an equilibrium $(I(g), U(g,T), S(g,T))$ is a *Romer-Rosenthal equilibrium* if $I(g)$ is zero when $g$ exceeds $g^*/(1-\delta)$ and satisfies (3.41) otherwise.

In a Romer-Rosenthal equilibrium, the residents' public good surplus function takes a very simple form.[20] Notice that when $g$ is less than $g^*/(1-\delta)$, equation (3.41) implies that the strategy $I(g)$ is such that surplus with the investment $S((1-\delta)g + I(g), cI(g))$ must equal surplus without $S((1-\delta)g, 0)$. Substituting this equality into (3.40), we see that the surplus function satisfies

$$S(g,T) = B(g) - T/H + \beta S((1-\delta)g, 0). \tag{3.42}$$

Moreover, equation (3.42) also holds when $g$ exceeds $g^*/(1-\delta)$ since $I(g)$ is zero. Applying equation (3.42) repeatedly, we conclude that in a Romer-Rosenthal equilibrium, the public good surplus function is

$$S(g,T) = \sum_{t=0}^{\infty} \beta^t B(g(1-\delta)^t) - T/H. \tag{3.43}$$

Intuitively, the residents get the same level of surplus in equilibrium as they would do if there were never any more investment. This reflects the fact that the bureaucrat extracts all the surplus from any new investment.

[20] The residents' surplus function also takes this form in any equilibrium in which whenever the bureaucrat does invest, he proposes the maximum possible level.

### 3.5.3 Potential Failure of Housing Price Test

With this information, we can now evaluate the performance of the housing price test. Note first that, if investment takes place, the difference in housing prices with and without the investment must be zero. This follows immediately from Proposition 1 and equation (3.41). The housing price test therefore implies that if $g$ is such that investment takes place (i.e., $g$ is less than $g^*/(1-\delta)$) it must be the case that the socially optimal level $g^o$ lies between $(1-\delta)g$ and $(1-\delta)g+I(g)$.[21] However, this is incorrect since it is perfectly possible that $g^o$ is less than $(1-\delta)g$. To see why, note first that given the surplus function (3.43), it is easy to show that

$$\frac{dS(g^o, 0)}{dI} > 0 = \frac{dS(g^*, 0)}{dI}. \tag{3.44}$$

This implies that $g^o$ is less than the residents' preferred level in equilibrium $g^*$. Intuitively, there is an additional benefit of investing in equilibrium: namely, higher public good levels reduce future exploitation by the bureaucrat. It follows that for all $g$ between $g^o/(1-\delta)$ and $g^*/(1-\delta)$, investment takes place but $(1-\delta)g$ exceeds $g^o$. This leads to the following proposition:

**Proposition 3** *Let $(I(g), U(g,T), S(g,T))$ be a Romer-Rosenthal equilibrium. Then, if $g$ is such that $I(g) > 0$, the housing price difference with and without the investment is zero. The housing price test therefore predicts that the socially optimal public good level $g^o$ lies between $(1-\delta)g$ and $(1-\delta)g+I(g)$. However, this is not the case. In particular, while $(1-\delta)g+I(g)$ always exceeds $g^o$, there exists an open interval of public good levels $g$ with the property that $I(g) > 0$ and $(1-\delta)g$ exceeds $g^o$.*

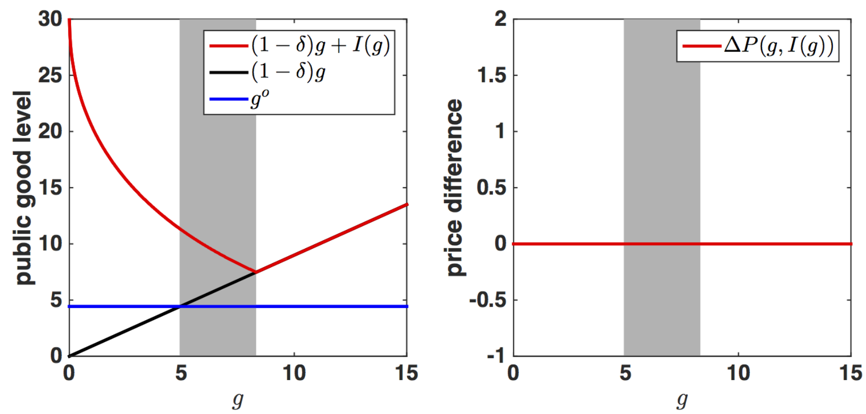
[21] We use the form of the housing price test that applies to large investments since equilibrium investments will only be small when the current level of the public good is close to $g^*/(1-\delta)$.

**Proof:** See Appendix A.

Proposition 3 tells us that, for a range of public good levels, the housing price test will provide misleading information. Specifically, it erroneously predicts that the public good level that would prevail without investment is below optimal. To gain further insight of Proposition 3, let's focus on the example illustrated in Figure 3.2. In this figure, all parameters are assumed to be the same as in Figure 3.1. In the left panel, the red line denotes the public good level with investment $(1-\delta)g + I(g)$ and the black line denotes the public good level without investment $(1-\delta)g$. Consistent with the defining feature of Romer-Rosenthal equilibrium, when public good level is less than $g^*/(1-\delta)$, the bureaucrat makes largest investment that is supported by the residents; when public good level exceeds $g^*/(1-\delta)$, the bureaucrat keeps investment at zero. In the right panel, we can see that the price difference $\Delta P(g, I(g))$ stays at zero for all public good level. According to housing price test, for public good level $g$ within the gray region, the optimal level of public good should be in-between $(1-\delta)g$ and $(1-\delta)g + I(g)$. Yet we can see from the left panel that for public good level $g$ within the gray region, the depreciated public good level $(1-\delta)g$ is in fact above the optimal level of public good. Therefore, housing price test incorrectly predicts under-provision of public good when it is in fact over-provided. In particular, note that the quantitative magnitude of the discrepancy can be substantial. At $g^*/(1-\delta) = 8.30$, the housing price test tells us that $(1-\delta)g = 7.47$ is less than $g^o = 4.43$ despite it being almost 1.7 times as big!

One possible reaction to Proposition 3 is that it will not be very damning if the range of public good levels for which the housing price test provides false information do not arise on the equilibrium path. It would then be the case that

Figure 3.2: Proposition 3




the test would only give misleading results for initial public good levels that were for some reason out of equilibrium. While this is a reasonable point, it does not save the test. Barseghyan and Coate (2014) shows that when benefits are quadratic, the equilibrium public good level converges to a unique steady state $g_s$. What this means is that, in the long run, in every period the community approves an investment of $\delta g_s$. Furthermore, they show that for sufficiently low depreciation rates, $(1-\delta)g_s$ exceeds the optimal level $g^o$. Thus, the difficulty arises for the investments that the community makes repeatedly in long run equilibrium. By simulating the investment path obtained in Figure 3.2, we find depending on the initial level, the equilibrium public good level either converges to $g_s = 7.85$ or oscillates between $g_{s1} = 7.75$ and $g_{s2} = 8.00$, both of which falls into the range of $g$ where housing price test falsely predicts under-provision.

To sum up, the point of this section is to provide a tractable example to illustrate why the socially optimal public good level need not maximize the equilibrium level of surplus and the difficulties this creates for the housing price test. The intuition revealed by the example is very general: namely, with public good levels chosen by a bureaucrat with stronger preferences than the residents, the bureaucrat will exploit a lower reversion level to force through higher investment levels. This effect increases the marginal value of the public good to residents which means that their preferred level in equilibrium exceeds the socially optimal level. To make this point more formally, it is useful to refer back to the expression in equation (3.36) the sign of which determines the direction of bias in the housing price test. In the example, the term on the top line is zero because the bureaucrat's proposals are passed with probability one. The second term is positive. On the one hand, from (3.41) and (3.43), it is clear that $\partial S/\partial g$ is less than $c/H$. On the other, (3.41) and (3.43) also imply that $dI/dI$ is less than $-(1-\delta)$ meaning that a higher public good level creates a more than compensating decrease in investment. It follows that the expression in equation (3.36) is positive, which is why the housing price test can falsely predict under-provision.

## 3.6 Housing Price Test Failure With Voting Uncertainty

To obtain a full picture of the failure of the housing price test, we numerically solve the equilibrium with voting uncertainty as specified in section 3.4. This allows us to see how the bias of housing price test is affected by the impact of investment on future voting outcomes. By varying parameters governing the uncertainty of voting outcomes and the bureaucrat's risk aversion, we find that housing price test may fail to predict both over-provision and under-provision of public good.

The computational procedure for obtaining the equilibrium investment path involves solving for the fixed points as characterized by the value functions

$U(g, T)$ and $S(g, T)$ through value function iterations. Since the behavior of the bureaucrat's objective function remains unclear, for given initial guess of value functions, we construct grids $\mathcal{G}$ for public good $g$ and evaluate the objective function at each grid. Then we choose the grid $g'(g) \geq (1-\delta)g$ that delivers the maximum value. This generates equilibrium investment proposal $I(g) = g'(g) - (1-\delta)g \geq 0$ and new value functions. Such procedure is repeated until convergence is achieved. A detailed description of the solution algorithm is provided in the Appendix. In terms of parameterization, we assume that the bureaucrat's utility function takes the functional form $u(g) = g^{1-\gamma}/(1-\gamma)$. To illustrate two possible scenarios of housing price test failure, we vary the standard deviation of the preference shock $\sigma$ and the bureaucrat's risk aversion parameter $\gamma$ while keeping the remaining parameters same as specified in previous numerical examples.

### 3.6.1 Failure to Predict Over-Provision

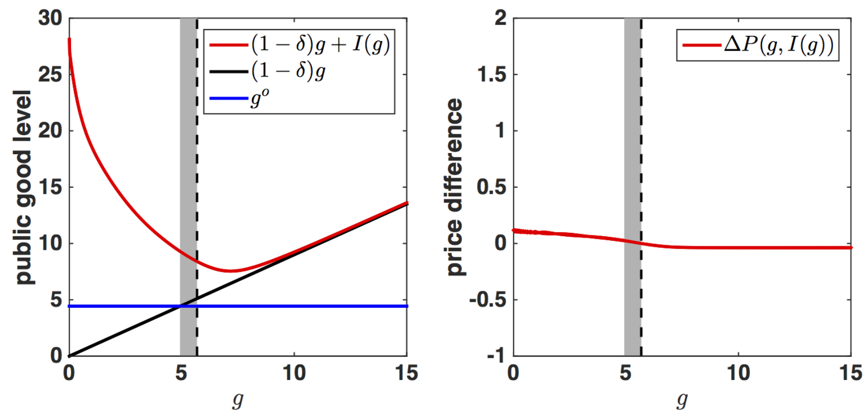
One possibility of the housing price test failure follows naturally from the intuition of the Romer-Rosenthal equilibrium. Note that as $\sigma$ converges to zero, residents' voting decision becomes highly 'accurate'. This implies that the passage of the investment proposal basically hinges on the comparison of public good surplus with and without investment. If the investment raises residents' public good surplus, they are very likely to approve the proposal; otherwise, they would most likely reject it. In such scenario, we would expect that equilibrium investment path mimics the Romer-Rosenthal equilibrium. This can cause housing price test to fail in predicting over provision of public goods.

To capture the intuition above, we set the standard deviation of aggregate preference shock $\sigma$ equal to 0.05 and solve the model numerically. Figure 3.3 provides the equilibrium public good level and the housing price differences. In the left panel of Figure 3.3, the red line denotes the equilibrium public good level with investment if bureaucrat's proposal is approved. The black line denotes public good level after depreciation if bureaucrat's proposal is rejected. Not surprisingly, the behavior of equilibrium investment proposal $I(g)$ is very similar to that obtained in the Romer-Rosenthal equilibrium in Figure 3.2.

One important difference, however, is that the public investment always remains positive, even though it drops as current public good level rises. This is because with the randomness of voting outcomes, residents in the community have limited control over bureaucrat's proposal. The bureaucrat knows that there always exists certain chance that a positive investment proposal be passed 'by mistake' even when $g$ is relatively high. Therefore, instead of setting investment equal to zero, the bureaucrat exploits its agenda-setting power by proposing a positive investment level. In the right panel of Figure 3.3, I plot the equilibrium price difference $\Delta P(g, I(g))$ as a function of current public good level $g$. Due to the random voting outcome, $\Delta P(g, I(g))$ is no longer fixed at zero. As $g$ increases, residents' additional surplus from accepting the investment proposal drops.

Now consider the validity of the housing price test. In both panels, the vertical dashed line denotes the public good level where the price difference $\Delta P(g, I(g))$ hits zero. For any public good level $g$ within the gray region, since the price difference in the right panel stays positive, the housing price test predicts that the optimal public good level should be above $(1-\delta)g$. The left panel reveals that the socially optimal public good level $g^o$, as denoted by the blue line, lies below $(1-\delta)g$ in the gray region. Therefore, the housing price test erroneously predicts under-provision of the durable public good while it is in fact over-provided.

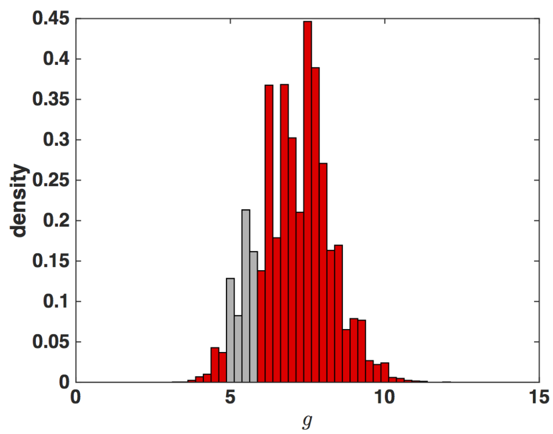
Figure 3.3: Housing Price Test Failure: Scenario One




To further understand why the housing price test falsely predicts under-provision, recall expression (3.36). Since the public good level influences the probability of a proposal passing, the term on the top line no longer stays at zero. We find numerically that under current parameterization, $d\pi^*(g^o)/dg$ is positive, which indicates that the net effect of a higher public good level on the probability of proposal passing is positive. In the meantime, we know from the right panel of Figure 3.3 that the public good surplus difference evaluated at $g^o$ is also positive. Therefore, the term on the top line is positive. As for the term on the bottom line, we find an increase in public good level lead to a more than compensating adjustment in investment, i.e. $1-\delta+dI(g^o)/dg<0$.Since we find $\partial S/\partial g - c/H$ to be positive, the term on the bottom line is negative. Because the magnitude of the term on the top line dominates that of the term on the bottom line, the expression in equation (3.36) takes positive value. This explains why the housing price test can falsely predict under-provision.

To check whether such housing price test failure will occur in the long run, in Figure 3.4 we plot the stationary distribution of public good. In particular, we use gray color to highlight the range of public good level where the housing price test incorrectly predicts under-provision. Note that even in the long run, the housing price test in this economy can fail with perceivable probability.

Figure 3.4: Stationary Distribution of $g$: Scenario One




### 3.6.2 Failure to Predict Under-Provision

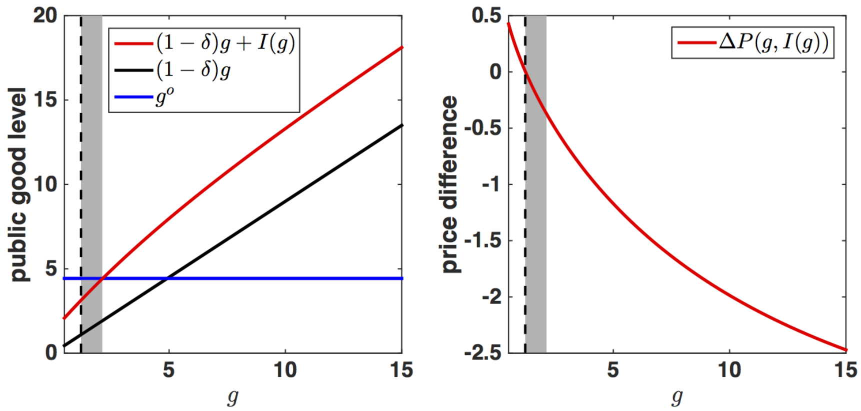
One alternative scenario of the housing price test failure is the erroneous prediction of public good over-provision when it is in fact under-provided. Up to now, in our analysis, the bureaucrat tends to provide more public good than the socially optimal level because the residents are assumed to be more risk averse than the bureaucrat and they face tax burdens. This feature may go in the opposite direction if the bureaucrat is instead assumed to be more risk averse than the residents, because the high risk aversion could make the bureaucrat very cautious in its investment decisions. To examine whether housing price test would fail when public good is under-provided, we set the bureaucrat's risk aversion parameter $\gamma$ equal to 5 while keeping residents' benefit function same as before.

In addition, to magnify the effect of risk aversion, we increase the uncertainty of voting outcome by setting $\sigma$ equal to 5.

In the left panel of Figure 3.5, the public good level with investment if bureaucrat's proposal passes is plotted as a function of current public good level in the red line. Note that the investment policy is driven by two opposing forces. On one hand, the high uncertainty of voting outcome pushes the bureaucrat to make large investment proposals. This is because the chances of such proposal being passed by residents 'mistakenly' is quite high, especially for large current public good levels. On the other hand, the high risk aversion makes the bureaucrat reluctant to risk failure in the election. As compared to a risk neutral bureaucrat facing the same voting uncertainty, for a highly risk averse bureaucrat, the utility gain from high investment levels is dominated by the loss of residents' support. Under current parameterization, the second force dominates the first at low current public good level. This makes the equilibrium level of public good with investment go below the optimal level. In the right panel of Figure 3.5, I plot the equilibrium price difference $\Delta P(g, I(g))$ as a function of current public good level $g$. For any public good level $g$ within the gray region, the housing price test predicts that the public good with investment is provided because $\Delta P(g, I(g))$ is negative. However, in the left panel, the equilibrium public good level with investment corresponding to the range of $g$ within the gray region is actually below the optimal level. Therefore, the housing price test mistakenly predicts over-provision of public good when it is in fact under-provided.

Let's recall the expression in equation (3.36). Again, since the public good level influences the probability of a proposal passing, the term on the top line

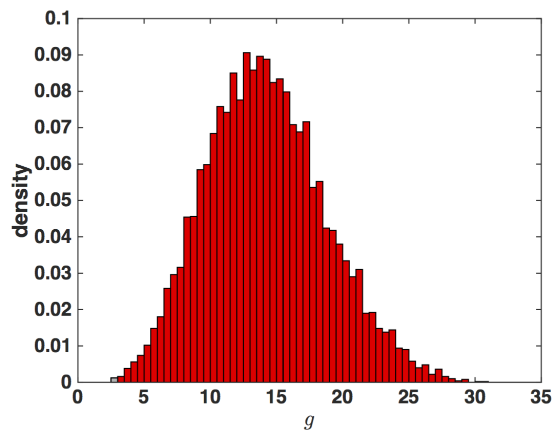
Figure 3.5: Housing Price Test Failure: Scenario Two




no longer stays at zero. We find numerically that in this alternative parameterization, $d\pi^*(g^o)/dg$ is negative. In other words, a higher public good level reduces the probability of a proposed investment passing. From the right panel of Figure 3.5, we know that the public good surplus difference evaluated at $g^o$ is negative as well. Multiplying the two terms, we obtain the term on the top line to be positive. As for the term on the bottom line, we find numerically that $\partial S/\partial g - c/H$ to be negative. However, due to the high risk-aversion of the bureaucrat, we have $dI(g^o)/dg > 0$. This means an increase in public good level lead to a less than compensating adjustment in investment, i.e. $1 - \delta + dI(g^o)/dg > 0$. Hence the term on the bottom line is less than zero. Because the magnitude of the term on the bottom line dominates that of the term on the top line, the expression in equation (3.36) takes negative value. Therefore, the housing price test can falsely predict over-provision.

To check whether above housing price test failure will occur in the long run, we plot the stationary distribution of public good in Figure 3.6. As indicated by the gray color, the housing price test failure occurs at the very left tail of the stationary distribution. Even though the chances of such housing price test failure is relatively low, it does occur with positive probability in the long run. In addition, such failure will also happen in the transition path to the stationary distribution if the economy starts with low level of public good.

Figure 3.6: Stationary Distribution of $g$: Scenario Two



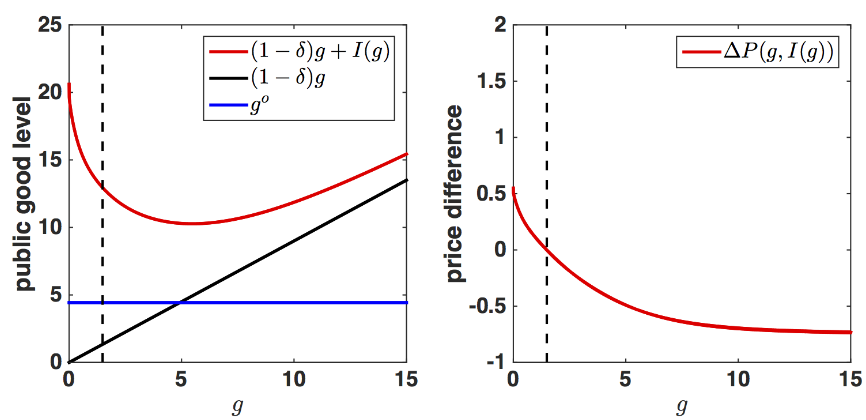

### 3.6.3 Possibility of Housing Price Test Holds

To provide a full picture of the model behavior and the validity of the housing price test, we finally explore an example where the housing price test can be justified all the time. In this example, we assume the bureaucrat is risk neutral ($\gamma = 0$) and there is moderate level of voting uncertainty ($\sigma = 1$).

In the left panel of Figure 3.7, we see that under the assumption of bureaucrat's risk-neutrality and moderate voting uncertainty, equilibrium public good level with investment always stays above the optimal public good level. At low level of current public good level $g$, the bureaucrat proposes high level of investment. As current public good level $g$ increases, the proposed level of investment decreases yet stays positive. In the right panel of Figure 3.7, the housing price difference $\Delta P(g, I(g))$ is plotted as a function of public good level $g$. When

$\Delta P(g, I(g))$ is positive, housing price test correctly predicts that $(1-\delta)g$ is below the optimal level. When $\Delta P(g, I(g))$ is negative, housing price test correctly predicts that $(1-\delta)g + I(g)$ is above the optimal level. In the case of $\Delta P(g, I(g)) = 0$, the housing price test indicates that the optimal level of public good is between $(1-\delta)g$ and $(1-\delta)g + I(g)$, which is also consistent with the left panel. Therefore, in contrast to previous examples, the housing price test in this example indeed delivers correct predictions.

Figure 3.7: No Housing Price Test Failure




The main message from the numerical explorations in this section is that the housing price test could fail to predict both under-provision and over-provision of public good. Even though the test can be valid under certain scenarios, in general we should apply it with great caution, especially when there is significant voting uncertainty or when the bureaucrat is highly risk averse. Since the investment is no longer approved with probability one, this generalized framework allows us to empirically implement the housing price test.

## 3.7 An adaptive expectations justification

The analysis so far has been conducted under the assumption that citizens have rational expectations concerning the future investment path in their community. Thus, they understand the dynamic environment they are in and correctly predict how investment in the public good will evolve over time. This is clear in the game-theoretic budget-maximizing bureaucrat setup as laid out in Section 4. In a more general model, the assumption is reflected in the idea that the functions $\pi(g, T, I)$ and $I(g, T)$ represent accurate predictions about what is going to happen in the future. This section relaxes this assumption and interprets the functions $\pi(g, T, I)$ and $I(g, T)$ as simply representing citizens' beliefs about what will happen. A simple adaptive expectations model is adopted and is shown to provide one way of rationalizing the housing price test.

Specifically, suppose that citizens expect that whatever level of public good they observe in the community at the beginning of a period will be maintained indefinitely. Thus, they observe the current quantity and quality of school facilities, say, and just assume they are at steady state levels. This is a form of myopia that is perhaps not too implausible, particularly for new citizens moving into a community. Formally, this assumption means that when the state is $(g, T)$, citizens expect that investment $I(g, T)$ will equal $\delta g$ and the probability of passing this investment $\pi(g, T, \delta g)$ is equal to 1. These rules imply that citizens believe that the present value of public good surplus is given by

$$S(g, T) = B(g) - T/H + \frac{\beta}{1-\beta}\left[B(g) - c\delta g/H\right]. \tag{3.45}$$

It then follows that the derivative of surplus with respect to investment is

$$\frac{dS((1-\delta)g+I,0)}{dI} = \frac{B'((1-\delta)g+I) - c[1-\beta(1-\delta)]/H}{1-\beta}. \tag{3.46}$$

In particular, therefore, it is the case that $dS(g^o,0)/dI$ is equal to zero. Accordingly, the housing price test will work as advertised. The intuition is straightforward: a successful investment is interpreted as creating a permanent increase in public good levels and hence we are back in a world where the standard logic applies.

In our judgement, this represents the best way to justify the housing price test. But still there is an important caveat. Observe that

$$\frac{dS((1-\delta)g+I,cI)}{dI} = \left(\frac{1}{1-\beta}\right)\left(\frac{dS^o((1-\delta)g+I,cI)}{dI}\right). \tag{3.47}$$

The multiplicative factor $1/(1-\beta)$ means that the change in housing prices overestimates the willingness to pay for investments. Intuitively, this reflects the fact that, under adaptive expectations, citizens interpret a small increase in investment as signaling a permanent increase in the level of the public good. If the good is under-provided, this permanent increase will have a considerably higher value to citizens than a temporary one.

## 3.8 Conclusion

This paper has explored the theoretical foundation for using the housing price response to investments to evaluate local durable public good provision. This is a potentially very useful technique, as the recent analysis of Cellini, Ferreira, and Rothstein demonstrates. Unfortunately, our exploration reveals that there is only limited justification for the idea under the assumption that residents and potential residents have rational expectations about the future investment path in their communities. Low aggregate voting uncertainty and high risk aversion of the bureaucrat can lead to erroneous predictions coming out of the housing price test. The most compelling way of justifying the test appears to be to assume that residents have adaptive expectations. However, even with such a non-standard assumption, caution must be exercised in interpreting the test.

These findings are unquestionably disappointing. Nonetheless, the idea of using the housing price response to investments to infer something about the efficiency of durable public good provision still appears promising. It is just that a more structural approach may be necessary to exploit this connection. Ultimately, inefficient provision must be driven either by bureaucratic objectives or by resident heterogeneity. Different bureaucratic objectives, for example, may have different implications for the behavior of housing prices over the investment cycle. If so, information about objectives and thus efficiency may be recovered from housing price dynamics.

## 3.9 Appendices

### 3.9.1 Appendix A: Proofs

#### Proof of Proposition 1

Consider a period in which the community's level of public good is $g$ and its tax obligation is $T$. Recall that the supply of housing is fixed at $H$. Potential residents just differ in their preference for living in the community $\theta$. Clearly, those with higher $\theta$ will have a greater willingness to pay to live in the community. Thus, in equilibrium, the fraction $H$ of potential residents with the highest preference parameters will live in the community. Since

$$\frac{\overline{\theta} - \overline{\theta}(1-H)}{\overline{\theta}} = H,$$

the marginal resident will have preference parameter $\overline{\theta}(1-H)$. This will be the case in each and every period irrespective of the community's level of public good and tax obligation. It follows that, in equilibrium, potential residents with types $\theta \in [0, \overline{\theta}(1-H))$ never reside in the community. For these types, therefore,

$$V_\theta(g,T) = \frac{\underline{u}}{1-\beta},$$

which yields equation (3.10) of Proposition 1. Types $\theta \in [\overline{\theta}(1-H)), \overline{\theta}]$, on the other hand, will reside in the community as long as they remain in the pool of potential residents. For these types, therefore, irrespective of $g$ and $T$

$$V_\theta(g,T) = \theta + B(g) - T/H - P(g,T) + \beta EP(g',T') + \beta[\mu EV_\theta(g',T') + (1-\mu)\frac{\underline{u}}{1-\beta}].$$

We now show that the value functions of the resident households can be written as equation (3.9) of Proposition 1. Let future periods be indexed by $t = 1, ...., \infty$ and let $(g_t, T_t)$ denote the public good level and tax obligation in period $t = 1, ...., \infty$. If $\theta \in [\overline{\theta}(1-H), \overline{\theta}]$, we know that

$$V_\theta(g,T) = \theta + \beta(1-\mu)\frac{\underline{u}}{1-\beta} + B(g) - T/H - P(g,T) + \beta E\left[P(g_1,T_1) + \mu V_\theta(g_1,T_1)\right], \quad (3.48)$$

where expectations are taken over the possible values of $(g_1, T_1)$; that is, $((1-\delta)g + I(g,T), cI(g,T))$ and $((1-\delta)g, 0)$. But, since the household will reside in the community in period 1 if it remains in the pool, we also know that

$$\begin{aligned}\beta\left[P(g_1,T_1) + \mu V_\theta(g_1,T_1)\right] &= \beta(1-\mu)P(g_1,T_1) \\ &+ \beta\mu\left[\theta + \beta(1-\mu)\frac{\underline{u}}{1-\beta} + B(g_1) - T_1/H + \beta E\left\{P(g_2,T_2) + \mu V_\theta(g_2,T_2)\right\}\right].\end{aligned}$$

Moreover, period 1's housing price $P(g_1,T_1)$ satisfies the equilibrium condition

$$1 - \frac{\underline{u} - (B(g_1) - T_1/H - P(g_1,T_1) + \beta EP(g_2,T_2))}{\overline{\theta}} = H,$$

which implies that

$$P(g_1,T_1) = \overline{\theta}(1-H) - \underline{u} + B(g_1) - T_1/H + \beta EP(g_2,T_2).$$

Substituting this into the above expression, we can write

$$\begin{aligned}
\beta\left[P(g_1,T_1)+\mu V_\theta(g_1,T_1)\right] &= \beta(1-\mu)\left[\overline{\theta}(1-H)-\underline{u}+B(g_1)-T_1/H+\beta EP(g_2,T_2)\right] \\
&\quad +\beta\mu\left[\theta+\beta(1-\mu)\frac{\underline{u}}{1-\beta}+B(g_1)-T_1/H+\beta E\left\{P(g_2,T_2)+\mu V_\theta(g_2,T_2)\right\}\right] \\
&= \kappa_1(\theta)+\beta\left[B(g_1)-T_1/H\right]+\beta^2 E\left[P(g_2,T_2)+\mu^2 V_\theta(g_2,T_2)\right],
\end{aligned}$$

where

$$\kappa_1(\theta)=\beta\left\{(1-\mu)\left[\overline{\theta}(1-H)-\underline{u}\right]+\mu\theta+\beta\mu(1-\mu)\frac{\overline{u}}{1-\beta}\right\}.$$

Again, since the household will reside in the community in period 1 if it remains in the pool, we also know that

$$\begin{aligned}
\beta^2\left[P(g_2,T_2)+\mu^2 V_\theta(g_2,T_2)\right] &= \beta^2(1-\mu^2)P(g_2,T_2) \\
&\quad +\beta^2\mu^2\left[\theta+\beta(1-\mu)\frac{\underline{u}}{1-\beta}+B(g_2)-T_2/H+\beta E\left\{P(g_3,T_3)+\mu V_\theta(g_3,T_3)\right\}\right].
\end{aligned}$$

Equilibrium in the housing market implies that

$$P(g_2,T_2)=\overline{\theta}(1-H)-\underline{u}+B(g_2)-T_2/H+\beta EP(g_3,T_3).$$

Substituting this in, we can write

$$\begin{aligned}
\beta^2\left[P(g_2,T_2)+\mu^2 V_\theta(g_2,T_2)\right] &= \beta^2(1-\mu^2)\left(\overline{\theta}(1-H)-\underline{u}+B(g_2)-T_2/H+\beta EP(g_3,T_3)\right) \\
&\quad +\beta^2\mu^2\left[\theta+\beta(1-\mu)\frac{\underline{u}}{1-\beta}+B(g_2)-T_2/H+\beta E\left\{P(g_3,T_3)+\mu V_\theta(g_3,T_3)\right\}\right] \\
&= \kappa_2(\theta)+\beta^2\left[B(g_2)-T_2/H\right]+\beta^3 E\left[P(g_3,T_3)+\mu^3 V_\theta(g_3,T_3)\right]
\end{aligned}$$

where

$$\kappa_2(\theta) = \beta^2 \left\{ (1-\mu^2)\left[\overline{\theta}(1-H) - \underline{u}\right] + \mu^2 \left[\theta + \beta(1-\mu)\frac{\underline{u}}{1-\beta}\right]\right\}.$$

By similar logic, for all periods $t \geq 3$, we have that

$$\beta^t \left[P(g_t, T_t) + \mu^t V_\theta(g_t, T_t)\right] = \kappa_t(\theta) + \beta^t \left[B(g_t) - T_t/H\right] + \beta^{t+1} E\left[P(g_{t+1}, T_{t+1}) + \mu^{t+1} V_\theta(g_{t+1}, T_{t+1})\right]$$

where

$$\kappa_t(\theta) = \beta^t \left\{ (1-\mu^t)\left[\overline{\theta}(1-H) - \underline{u}\right] + \mu^t \left[\theta + \beta(1-\mu)\frac{\underline{u}}{1-\beta}\right]\right\}.$$

Successively substituting these expressions into (3.48), reveals that

$$\begin{aligned} V_\theta(g,T) &= \theta + \beta(1-\mu)\frac{\underline{u}}{1-\beta} + B(g) - T/H - P(g,T) + \sum_{t=1}^{\infty} E\left\{\kappa_t(\theta) + \beta^t\left(B(g_t) - T_t/H\right)\right\} \\ &= \sum_{t=0}^{\infty} \kappa_t(\theta) + S(g,T) - P(g,T). \end{aligned}$$

Letting $\kappa(\theta) = \sum_{t=0}^{\infty} \kappa_t(\theta)$ yields equation (3.9).

It remains to show that equation (3.11) is satisfied. In equilibrium, it must be the case that the marginal household, which is the household with preference $\overline{\theta}(1-H)$, is just indifferent between residing in the community or not. Thus, it must be the case that

$$\kappa(\overline{\theta}(1-H)) + S(g,T) - P(g,T) = \frac{\overline{u}}{1-\beta}.$$

This implies that

$$P(g,T) = \kappa(\overline{\theta}(1-H)) - \frac{\overline{u}}{1-\beta} + S(g,T).$$

Letting

$$K = \kappa(\bar{\theta}(1 - H)) - \frac{\bar{u}}{1 - \beta},$$

yields equation (3.11). ■

## Proof of Proposition 2

Letting $g'$ denote next period's public good level (i.e., $g' = (1 - \delta)g + I$), we have that

$$S^o(g, T) = \max_{g' \geq (1-\delta)g} B(g) - T/H + \beta S^o(g', c(g' - (1 - \delta)g)). \tag{3.49}$$

Moreover, letting $g'(g, T)$ denote the optimal policy function for (3.49), we have that $I^o(g, T)$ is equal to $g'(g, T) - (1 - \delta)g$. Now note from (3.49) that

$$\frac{\partial S^o(g, T)}{\partial g} = B'(g) - \beta(1 - \delta)c \frac{\partial S^o(g', c(g' - (1 - \delta)g))}{\partial T}, \tag{3.50}$$

and that

$$\frac{\partial S^o(g, T)}{\partial T} = -1/H. \tag{3.51}$$

The first order condition for the optimal policy $g'$ is that

$$\frac{\partial S^o(g', c(g' - (1 - \delta)g))}{\partial g} + c \frac{\partial S^o(g', c(g' - (1 - \delta)g))}{\partial T} \leq 0 \ ( = \ \text{if } g' > (1 - \delta)g).$$

Using (3.50) and (3.51), this can be rewritten as

$$B'(g') - \beta(1 - \delta)c/H \leq c/H \ ( = \ \text{if } g' > (1 - \delta)g).$$

Thus, if $g' > (1-\delta)g$

$$HB'(g') = c\,(1-\beta(1-\delta))\,.$$

Letting $g^o$ satisfy the dynamic Samuelson Rule $HB'(g^o) = c\,[1-\beta(1-\delta)]$, we conclude that the optimal policy function is

$$g'(g,T) = \begin{cases} g^o \ \text{ if } g \leq g^o/(1-\delta) \\ (1-\delta)g \ \text{ if } g > g^o/(1-\delta) \end{cases}.$$

Since $I^o(g,T)$ is equal to $g'(g,T) - (1-\delta)g$, Proposition 2 follows immediately. ■

## Proof of Proposition 3

If $g$ is such that investment takes place, then we know that $g \in (0, g^*/(1-\delta))$. That the housing price difference with and without the investment is zero follows from Proposition 1 and the fact that surplus with the investment must equal that without the investment (by equation (3.41)). To see that $(1-\delta)g + I(g)$ must exceed $g^o$, note from equation (3.43) that

$$\sum_{t=0}^{\infty} \beta^t B([(1-\delta)g + I(g)]\,(1-\delta)^t) - cI(g)/H = \sum_{t=0}^{\infty} \beta^t B([(1-\delta)g]\,(1-\delta)^t).$$

This implies that

$$\sum_{t=0}^{\infty} \beta^t\,[B([(1-\delta)g + I(g)]\,(1-\delta)^t) - B([(1-\delta)g]\,(1-\delta)^t)] = cI(g)/H.$$

Since $B(\cdot)$ is strictly concave, it is clear that

$$\sum_{t=0}^{\infty} \beta^t \left[B'(\left[(1-\delta)g + I(g)\right](1-\delta)^t)\right](1-\delta)^t I(g) < cI(g)/H.$$

But we have that

$$\begin{aligned}\sum_{t=0}^{\infty} \beta^t \left[B'(\left[(1-\delta)g + I(g)\right](1-\delta)^t)\right](1-\delta)^t I(g) &> \sum_{t=0}^{\infty} \left[\beta(1-\delta)\right]^t B'((1-\delta)g + I(g))I(g) \\ &= \frac{B'((1-\delta)g + I(g))I(g)}{1-\beta(1-\delta)}.\end{aligned}$$

Thus, we have that

$$\frac{B'((1-\delta)g + I(g))}{1-\beta(1-\delta)} < c/H,$$

or, equivalently,

$$HB'((1-\delta)g + I(g)) < c\left[1-\beta(1-\delta)\right].$$

Given the definition of $g^o$, this implies that $(1-\delta)g + I(g)$ must exceed $g^o$.

It remains to show that there exists a public good level $\widetilde{g}$ strictly less than $g^*/(1-\delta)$ such that for any $g$ in the interval $(\widetilde{g}, g^*/(1-\delta))$, $(1-\delta)g$ exceeds $g^o$. By definition, $g^*$ maximizes $S(g, cg)$. Given (3.43), this implies that

$$g^* = \arg\max \sum_{t=0}^{\infty} \beta^t B(g(1-\delta)^t) - cg/H.$$

This in turn implies that $g^*$ satisfies the first order condition

$$\sum_{t=0}^{\infty} \left[\beta(1-\delta)\right]^t B'(g^*(1-\delta)^t) = c/H.$$

It follows from this that

$$\sum_{t=0}^{\infty} [\beta(1-\delta)]^t B'(g^*) < c/H,$$

which implies that

$$HB'(g^*) < c[1-\beta(1-\delta)] = HB'(g^o).$$

It follows that $g^* > g^o$, which means that for all $g \in (g^o/(1-\delta), g^*/(1-\delta))$, $(1-\delta)g$ exceeds $g^o$. The proof is now complete. ■

### 3.9.2 Appendix B: Solution Algorithm for Budget-Maximizing Bureaucrat with Probabilistic Voting

In this section, we provide the solution algorithm for the problem faced by budget-maximizing bureaucrat in section 3.4. We first re-formulate the optimization problem so that it involves only one state variable. Then we describe the computational procedure for solving the modified problem.

#### Modified Optimization Problem

Recall the public good surplus function $S(g,T)$ in (3.33) where

$$S(g,T) = B(g) - T/H + \beta[\pi^*(g)S((1-\delta)g + I(g), cI(g)) + (1-\pi^*(g))S((1-\delta)g, 0)]$$

Note that the state variable $T$, which represents the total tax obligation in the past, only affects the instantaneous utility and does not enter the expected surplus in the future. Therefore, we can separate this term from the rest terms in the value function by defining a new function $\tilde{S}(g)$ where

$$\tilde{S}(g) = S(g, T) + T/H \tag{3.52}$$

Substituting (3.52) into (3.33), we can obtain a recursive representation of $\tilde{S}(g)$ which is

$$\tilde{S}(g) = B(g) + \beta\left\{\pi^*(g)\left[\tilde{S}(g'(g)) - \frac{c[g'(g) - (1-\delta)g]}{H}\right] + (1-\pi^*(g))\tilde{S}((1-\delta)g)\right\} \tag{3.53}$$

Here $g'(g) = (1-\delta)g + I(g)$ denotes the equilibrium public good level in the next period. The probability of investment approval can hence be re-written as

$$\pi(g, g') = \Phi\left[\tilde{S}(g') - \frac{c[g' - (1-\delta)g]}{H} - \tilde{S}((1-\delta)g)\right] \tag{3.54}$$

Based on (3.53) and (3.54), the bureaucrat's maximization problem can becomes

$$g'(g) = \arg\max\left\{\begin{array}{c} g + \beta\left[\pi(g, g')U(g') + (1-\pi(g, g'))\,U((1-\delta)g)\right] \\ s.t.\ g' \geq (1-\delta)g \end{array}\right\}. \tag{3.55}$$

where the bureaucrat's value function is given by

$$U(g) = g + \pi^*(g)U(g'(g)) + (1-\pi^*(g))\,U((1-\delta)g). \tag{3.56}$$

and residents' public good surplus function $\tilde{S}(g)$ is as defined in equation (3.53).

## Numerical Procedure

Essentially, the problem above is solved by iteration over value functions $U(g)$ and $\tilde{S}(g)$. Detailed steps are specified as follows:

Step 1. Construct grids $\mathcal{G}$ for public good $g \in [0, g_{max}]$[22].

Step 2. Given the initial guess of public good surplus $\tilde{S}_0(g)$, compute probability of investment approval $\pi(g, g')$ for each $(g, g') \in \mathcal{G} \times \mathcal{G}$. When computing $S_0((1-\delta)g)$, since $(1-\delta)g$ is outside $G$ grid, use spline interpolation.

Step 3. For each grid $g \in \mathcal{G}$, based on $\pi(g, g')$ and initial guess of bureaucrat's life-time utility function $U_0(g)$, compute the bureaucrat's objective function at each $g' \in \mathcal{G}$. Here similar to $S((1-\delta)g)$, we use spline interpolation to obtain $U((1-\delta)g)$. Then choose the $g' \in G$ which maximizes the bureaucrat's objective and satisfies $g' \geq (1-\delta)g$. This generates the equilibrium policy function $g'(g)$ and $I(g)$.

Step 4. Using $g'(g)$, update $\tilde{S}(g)$ and $U(g)$. If the distance between the updated $\tilde{S}(g)$ and $U(g)$ and the initial guess is less than exogenously specified tolerance, then the equilibrium is found; otherwise replace the initial guess with newly obtained $\tilde{S}(g)$ and $U(g)$ and repeat Step 2 to Step 4.

[22]For the case where $u(g) = \frac{g^{1-\gamma}}{1-\gamma}$ with $\gamma \geq 1$, the grids are over $[g_{min}, g_{max}]$ with $g_{max} > g_{min} > 0$

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