ESSAYS ON MULTI-ACTIVITY ANALYSIS

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The dissertation presents marketing issues in which consumers are engaged in more than two activities related to marketing; cross-activity analysis. In particular, the dissertation suggests two marketing models to capture 1) consumers' choices of simultaneous and sequential bundle choices of related categories and 2) consumers' visitation on multiple social network services.

Chapter 2 aims to propose a general framework to describe consumers' sequential bundling of items in related product categories and to compare their choices with those under simultaneous bundling situations. To achieve this, a dynamic model in which strategic consumers can choose a bundle either sequentially or simultaneously is developed.

Chapter 3 presents a bivariate visitation model where users belong to multiple social networking services. In this model, I find various interdependency sources that affect users' visitations across multiple social networks; experience spillover, correlated sensitivities and unobserved components. Among various sources, I also discover a unique dependency source that come from social network structures across different social network services, that is, overlapping friends.

Chapter 4 discusses potential extensions of the approaches that are presented in the dissertation.

BIOGRAPHICAL SKETCH

Hwang Kim was born June 16, 1977 in Seoul, South Korea.. He received the Bachelor of Science in Cloth and Textile from Seoul National University in 2003. After graduation, he worked as a fashion designer in South Korea.

He earned the Master of Science in Business from Yonsei University in 2006 and Master of Professional Study in Statistics from Cornell University in 2009.

After joining the PhD program in Marketing at Cornell University's Johnson Graduate School of Management in 2009, he has served as a Teaching and Research Assistant and lecturer. He will graduate with a PhD degree, with a focus in quantitative marketing.

He looks forward to beginning work as faculty at Chinese University of Hong Kong in August 2015.

To My Parents

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Chapter One

Introduction

This dissertation consists of two independent studies on cross-activity analysis. The first study presents the models when consumers are involved in multiple category bundle choices. After discussing cross-choice analysis briefly, this chapter presents background discussions about two issues from theoretical and managerial perspectives.

The next study proposes an integrated visitation model when users belong to multiple social networking services. I discuss literature on social network analysis; dyadic social relationships and social network structures and address a managerial issue.

1.1. Multi-Category Choice Analysis

First, the heart of the dissertation is consumer's choice behavior across related multiple categories. This consists of two parts; Shopping Basket Analysis and Bundle Choice Analysis (see Seetharaman *et.al.* 2005).

1.1.1. Shopping Basket Analysis

There is an extensive body of literature on the cross-category effects, which is called "shopping basket analysis". During shopping trips, consumers often buy more than one categories (e.g., cereal and milk, bacon and egg). The shopping basket analysis aims to model the composition of products in the basket and then investigate interdependencies that may exist among related product categories. In this sense, the studies assume a shopping situation wherein multiple categories are chosen and purchased simultaneously.

This stream of the research has focused on investigating inter-dependency of purchase decisions that arises when a purchase in one category affects a purchase in another. One way to do so is to develop multivariate choice models to analyze the shopping basket decisions. Manchanda, Ansari and Gupta (1999) studied cross-effects across categories using the multivariate Probit model and found sources of interdependency such as complementarity, co-incidence and heterogeneity. Chib, Seetharaman and Strijnev (2002) found positive complementarity of closely related categories such as cola and non-cola, hot dogs and bacon, and tissue and detergents, using a multivariate Probit model. Hansen, Singh and Chintagunta (2006) developed a multi-category brand-choice model incorporating factor structure of the covariance of the heterogeneity distributions across categories. As an extension, Niraj, Padmanabhan and Seetharaman (2008) investigated not only multi-category choice incidences but also their quantity decisions in an integrated manner. Recently, Mehta and Ma (2012) proposed an integrated model of consumers' purchase incidence, quantity and brand choices by relaxing a restrictive function of utilities to measure accurate cross-category effects. Followed and extended the lines of classical single category choice models proposed by Guadagni and Little (1983), this stream of works has been considered as a popular framework to the very problem.

Shopping basket models of cross categories have been actively applied to aggregate level data. Walters (1991) studied the effect of retail promotions on complementary products using store level data. Song and Chintagunta (2006) developed a choice model to find cross category price effects using store level aggregate data. Interestingly, their modeling framework had considerable similarities with those applied in bundle choice analysis that we will discuss in the next section.

To summarize, mutli-category models such as shopping basket models have provided

important managerial implications to not only retailers who aim to maximize their profits by implementing marketing activities and making price decisions across categories but also manufacturers who seek to design and sell related categories together.

1.1.2. Bundle Choice Analysis

Consumers have a variety of way to choose product across multiple categories. To address it, another stream of studies on multi-category choice analysis is the product bundle choice. Bundling products across different categories has been widely implemented in various areas, for example, flight and hotel, burger and chips, Microsoft Windows and Explorer, cable TV and internet service subscription, etc. As such, there has been an extensive stream of literature on bundling products for decades.

The studies on bundling products have been conducted from various perspectives. One is bundle design. Bundling products is essentially manufacturers' or retailers' strategic decision to compose multiple products as one product. In a broad sense, this is a part of product line design. Thus, the key decision pertains to which products are bundled together.

Farquhar and Rao (1979) pioneered the studies on bundle design. This was applied to assortment choices (Bradlow and Rao 2000) and multi-category bundles of heterogeneous products (Chung and Rao 2003). Their approach is to balancing attributes of products to compose a bundle. The “balance” implies making harmony with different products by adjusting homogeneous attributes that the different products share. This approach brought development in investigating this very problem in product attribute levels and provided important managerial implications of how manufacturers or retailers identify the combinations of products in a bundle which are most appealing to consumers.

Table 1.1 Studies on Multi-Category Analysis and Bundle Choice Analysis

	Studies	Data	Method
Shopping Basket Analysis	Walters (1991)	Individual data of cake mix, frosting, boxed spaghetti, and spaghetti sauce	Linear Regression
	Manchanda, Ansari and Gupta (1999)	Individual data of laundry detergent, fabric softener, cake mix and cake frosting	Multivariate Probit model
	Chib, Seetharaman and Strijnev (2002)	Individual data of bacon, butter, coffee, cola, crackers, detergent, hot dogs, ice cream, non-cola beverages, sugar, toilet tissue and paper towels	Multivariate Probit
	Singh, Hansen and Chintagunta (2006)	Individual data of bathroom tissue, dish washer, foil, ham, mayo, oatmeal, paper towel, peanut butter, tuna and waffle	Multivariate Probit
	Song and Chintagunta (2006)	Aggregate data of liquid laundry detergents , powdered laundry detergents, liquid fabric softeners and sheet fabric softeners	Multinomial Logit to maximize basket utilities
	Niraj, Padmanabhan and Seetharaman (2008)	Individual data of bacon and eggs	Multivariate Probit
	Mehta and Ma (2012)	Individual data of pasta and pasta sauce	Multinomial Logit to maximize basket utilities
Bundle Choice Analysis	Farquhar and Rao (1979)	Survey data of TV shows	Linear Programming to maximize bundle values with balancing effects
	Hanson and Martin (1990)	Survey data of home service bundles (cleaning, laundry, ironing and grocery shopping)	Maximize bundle utilities

Table 1.1 (Continued)

	Studies	Data	Method
Bundle Choice Analysis	Venkatesh and Mahajan (1993)	Survey data of magazine subscriptions, durable goods (videocassette player, video camera)	Maximize bundle utilities
	Jedidi, Jagpal and Manchanda (2003)	Survey data of magazine subscriptions, durable goods (microwave, video camera)	Multinomial Probit to maximize bundle utilities
	Chung and Rao (2003)	Conjoint data of computers, monitors and printers	Multinomial Logit to maximize bundle utilities with balancing attributes
	Derdenger and Kumar (2013)	Aggregate data of game consoles and game titles	Dynamic forward looking logit model (Dynamic BLP)

The second stream in the literature is bundling pricing, which proposes prices at which consumers are most likely choose bundles or individual items. Several approaches have been proposed to tackle this problem. One is probabilistic approach. Ansari, Siddarth and Weinberg (1996) and Venkatesh and Mahajan (1993) proposed the models in which consumers are heterogeneous in uncertainty to the bundle level and they translate this to their reservation prices. The bundle prices can be decided through data from choice experiments. Jedidi, Jagpal and Manchanda (2003) conducted a choice based experiment to derive distributions of reservation prices for bundles and apply them to obtain the optimal prices of the product lines. Chung and Rao (2003) also applied their balancing model of bundle choices to decide the reservation prices of bundles in conjoint setting. As a recent extension, Derdenger and Kumar (2013) developed a dynamic model that assumes that forward looking strategic consumers adopt bundle products

over time using aggregate market level data. We summarize the selected literature on bundle choices and multi-category analysis (i.e., shopping basket analysis) based on data and methodologies in Table 1.1.

It is important to note that similar to the shopping basket analysis, the underlying assumption of the extant literature on bundle is that the composition of bundles are decided simultaneously by manufacturers or retailers and correspondingly consumers evaluate the items in bundles together. This assumption may be limited in various business situations, as discussed in the following section.

1.1.3. Simultaneous vs. Sequential Bundle Choices

Based on the review of the previous literature on shopping basket and bundle analysis, the first chapter of the dissertation intends to answer the following three research questions. First, do consumers behave differently in making choice decisions between simultaneous (and explicit) bundles and sequential (and implicit) bundles, and in what way? Second, in the case of sequential and implicit bundle choices, how is the choice in one category impacted by the prior choice in another category? To be specific, how do the time sequences of choices contribute to consumer asymmetric preferences? Lastly, how should sellers plan and implement pricing strategies for these two different bundling situations? Despite the wide applicability of bundling, this topic has so far not been pursued in the marketing literature.

1.2. Social Network Analysis

A large body of literature in sociology and psychology has emphasized human behaviors in networks. Homophily is considered a central dimension for representing relationships in

networks. Homophily explains that when the characteristics of peers match each other, they may share norms that build their relationships (Heider 1957, Verbrugge 1977, McPherson, Smith-Lovin and Cook 2001). In addition, the frequency of interactions is considered the primary indicator of involvement in these relationships (Adams 1967, Dindia and Canary 1993, Kahanda and Neville 2009). By extension, recent marketing literature has investigated two dimensions of social networks: the dyadic relationship and the network structure.

1.2.1. Dyadic Social Relationship

One strand of studies in marketing defines and models dyadic (peer-to-peer) interactions. Sociometric and demographic characteristics such as gender, region, or age are often used to measure such interactions (Nitzan and Libai 2011, Ansari, Koenigsberg and Stahl 2011).In addition, dyadic relationships can be measured dynamically because strength of relationships in networks may become stronger or weaker over time or even drop out, due to their inactive communication. The quantities of (observable) time-varying interactive activities between network members can be one source; an example would be detailing activities between physicians (Nair, Manchanda and Bhatia 2010).

1.2.2. Social Network Structure

Another stream of studies has characterized the structure of an entire network by using aggregate statistics of the network properties. For example, a simple measure for this is the number of people connected in the network (Stephen and Toubia 2010, Katona, Zubcsek and Sarvary 2011). Also, aggregation of homophily measures of overall connected friends implicitly reflects the network size (Nitzan and Libai 2011). Ma, Krishnan, and Mongomery (2010) used telephone

belltone downloads of connected others to determine network relationships.

Table 1.2 Studies that Measure Social Interaction/Intimacy in Marketing

Studies	Dimensions of social networks	
	Dyadic relationships (Peer-to-peer)	Network size and structures
Trusov, Bodapati and Bucklin (2010)	√	√
Stephen and Toubia (2010)		√
Toubia, Goldenberg and Garcia (2010)		√
Nair, Manchanda and Bhatia (2010)	√	
Katona, Zubscek and Sarvary (2011)		√
Haenlein (2011)		√
Nitzan and Libai (2011)		√
Ansari, Koenigsberg and Stahl (2011)	√	
Yoganarsimhan (2012)		√
Ghose, Han and Iyengar (2012)		√
Lu, Jerath and Singh (2013)	√	

√if present

Table 1.2 shows a summary of selected studies based on which of two dimensions - dyadic relationships or whole network structure—are used to measure social interactions and relationships in networks. The studies investigate either peer-to-peer interactions or whole network structures at an aggregate level. One study listed, Trusov, Bodapati, and Bucklin (2010), in contrast, incorporates correlations of visitations to the social network site as peer-to-peer effects, because their data do not provide information on how and when users interacted and

communicated with one another. This implies that their measure of peer-to-peer effects cannot disentangle the social communication effects between users and the coincidence of their habitual behaviors (Yang and Allenby 2003).

To view the whole picture of the relationships in the network, it is important to consider not only each dyadic relationship, but also the size and structure of the entire network, since these factors represent the distinct nature of social networking behaviors. Yet, it is still unclear how they operate together to characterize social networks. We intend our study to broaden this stream of research by specifying the social networking effects of the interplay between peer-to-peer interactions and whole network structures.

1.2.3. Network Overlaps

People belong to more than one social network in the real world; these include schools, neighborhoods, clubs, Facebook, Twitter and so on. For example, approximately 35% of social network users use more than two social network services (e.g., Facebook and Twitter).[1] From the perspective of firms, 77% of the Fortune Global 100 companies have Twitter accounts, 61% have Facebook pages, and 57% use YouTube to find and select influential and targetable customers.[2] Thus, if dependencies exist between different social networks, limiting the scope of analysis to only one social network sheds only partial light on the effects of social networks on firms.

Thus, an important question arising from the examples above is what is the nature of dependency between different social networks. To be more specific, how this dependency affect consumers' behaviors when navigating multiple social networking services. To find the answers,

[1]Quoted from www.journalism.org, November 13, 2013.
[2]Quoted fromwww.therealtimereport.com, March 18, 2011.

it is crucial to investigate various sources of interdependencies that exist across multiple social networks. The third chapter presents an integrated model to investigate it. Thus, the third chapter of the dissertation is closely related to Chapter 2 in that we also study interdependency across different objects but it differs because the object is not category but social networking platform.

1.3. General Outline

Chapter 2 addresses conceptual backgrounds of two types of bundles; simultaneous and sequential bundles. Based on this, a general framework is proposed to explain consumers' sequential bundling of items in related product categories and to compare their choices with those under simultaneous bundling situations. To this end, a dynamic model in which strategic consumers can choose a bundle either sequentially or simultaneously is developed. In addition, the model enables me to find that optimal bundle prices change considerably depending on combinations of bundle attributes.

Chapter 3 proposes a visitation model where users belong to multiple social networking services. In this model, I find various interdependency sources that affect users' visitations across multiple social networks; experience spillover, correlated sensitivities and unobserved components. Among various sources, I also discover a unique dependency source that come from social network structures across different social network services, that is, overlapping friends.

Finally, Chapter 4 discusses the limitations of the models, data and implications and consequently presents potential extensions of the approaches that are discussed in the dissertation.

Chapter Two

A Conjoint Study of Sequential vs. Simultaneous Bundle Choices

This chapter proposes a general framework to describe consumers' sequential bundling of items in related product categories and to compare their choices with those under simultaneous bundling situations. To achieve this, we develop a dynamic model in which strategic consumers can choose a bundle either sequentially or simultaneously. To empirically test these two models, we conduct conjoint experiments in which respondents select a simultaneous bundle product of a Tablet PC and a Smart TV constructed explicitly by a seller and implicitly select a bundle in a sequential manner.

Our results show that consumers apply different decision processes for these two types of bundle choices. In addition, we propose a new method to find optimal bundle prices under these bundling strategies when consumers are forward looking. We offer valuable implications on design and price strategies of a bundle of related products.

2.1. Background

In order to boost the cross-category purchases by consumers, retailers and manufacturers have practiced various marketing strategies such as bundling products (e.g., Adams and Yellen 1976, Seidmann 1991,Anderson 1993, and Naylor and Frank 2001), offering advertisements or promotions for multiple categories (e.g., Ghose and Yang 2008), and targeting multi-category shoppers (e.g., Heilman and Bowman 2002, Akçura and Srinivasan 2005).

In a broad sense, multi-category bundles are characterized into two strands (see Seetharaman *et. al.* 2005). One is offering multi-category products as a bundle and the other is

offering one product followed by another. The first strand has been popular in many industries such as software, tourism, mobile/internet services, and the fast food industry. Related examples include a bundle of a game console and titles, a bundle of Microsoft Windows and Explorer, a package deal of hotel and flight, a contract for a bundle of internet and cable TV services, and a combo meal of a burger, fries and coke. In this case, the bundle product is explicit in two ways: the combination of products is decided by the seller or manufacturer, and it is priced together as one item. If consumers choose such bundles, they choose products in different categories at one time or simultaneously. We label this strand simultaneous (and explicit) bundle.

On the other hand, consumers often purchase products in different categories one after another. Indeed, consumers frequently make choices from such multiple categories. For instance, they want to buy a blue-ray DVD player after buying a flat screen TV to take full advantage of it. In the fashion industry, consumers who purchase one item (e.g., shoes) want to select a product in another category (e.g., a handbag) to match the first. Also, in a restaurant, people order an appetizer that pairs with a main dish. In this way consumers choose multi- products sequentially by acquiring them one by one. To be specific, they not only implicitly construct the bundle of products but also decide the order (timing) of choice. These characteristics enable us to refer to such an acquisition as a sequential (and implicit) bundle.

With above conceptual categorization illustrated in our introduction, the literature on consumer's multi-category bundles and choice behaviors can be segmented into two streams. The first stream is about the consumer's bundle choice behavior based on the multi-attributes of products. Among a rich body of literature on bundles, our focus in this study is on the consumer decision making process based on the multi-attributes of bundle items. Farquhar and Rao (1976) pioneered the modeling of consumers' bundle choices by developing a balance model. They

categorized balancing attributes into equi-balancing and counter-balancing and also specified a bundle utility function. As an extension, Bradlow and Rao (2000) incorporated the hierarchical Bayesian framework into their balancing model. Similarly, Chung and Rao (2003) proposed a general model of consumer bundle choices among heterogeneous product categories, in which attributes are categorized according to three characteristics: non-comparable, partially-comparable and fully-comparable across product categories. In one sense this study is an extension of this stream of literature.

The second strand of previous research focuses on the consumer multi/cross-category choice behavior, which is analogous to the sequential and implicit bundling. An example of this case is well-known shopping basket analysis. This is the modeling approach to the choice of multiple items in shopping occasions. This stream of the research employs multinomial logit and multivariate Probit models (Manchanda *et al.*1999, Chib *et. al* 2002, Ma and Seetharaman 2004, Song and Chintagunta 2006, Hansen *et al.* 2006). In particular, the shopping basket analysis shares similarities with our context, since we also seek to explore choice behavior toward multiple products. We should point out that our approach differs from the past research in that we investigate not only simultaneous purchase decision making, but also sequential purchase decision making which allows for a time lag between subsequent purchases.

In addition, the simultaneous and sequential multi-product/category shopping in our study is related to two streams of behavioral studies. First is the study which investigates consumers' judgment processes between joint evaluation and separate evaluation (Hsee 1996, Hsee *et.al.* 1999). Second, there is an emerging behavioral marketing literature on balancing and reinforcing response in multiple choice situations (Dhar and Simonson 1999, Huber, Goldsmith and Mogilner 2008). The related behavioral research streams provide insight on application to

bundling choice behaviors. Yet, this approach has been surprisingly under-researched in the quantitative marketing literature on bundling.

One piece of research close to ours is Li, Sun and Wilcox (2005). They investigate the order and pattern of purchases for multiple related products during the lifetime of a customer, and they find that a customer demand for multi-category products evolves over time and is different across customers. Although their research is similar to ours in that it investigates consumers' sequential decisions, our context is more commonly applicable to several situations, such as acquiring durable goods and hedonic product consumption. Our framework does not assume that consumers purchase one product only once during their lifetime or that the order of choices is predetermined. In this respect, our framework of the sequential bundles is related to Bayus and Rao (1989), yet differs from it because we consider that consumers are forward looking in the sequential bundle choices. Therefore, our research offers a generalized framework for multi-product bundle choices.

The review of these two streams of previous literature raises the following three research questions. First, do consumers behave differently in making choice decisions between simultaneous (and explicit) bundles and sequential (and implicit) bundles, and in what way? Second, in the case of sequential and implicit bundle choices, how is the choice in one category impacted by the prior choice in another category? To be specific, how do the time sequences of choices contribute to consumer asymmetric preferences? Lastly, how should sellers plan and implement pricing strategies for these two different bundling situations? Despite the wide applicability of bundling, this topic has so far not been pursued in the marketing literature. To answer these important questions, we formulate two models: (i) a model of simultaneous bundle choices with multiple related categories and (ii) a dynamic model of a sequential bundle choice

that allows for asymmetric preferences between categories. In addition, we allow consumers to choose not only categories, but also the timing of purchases.

To empirically test our proposed models, we design and conduct conjoint experiments under three scenarios; (i) a conjoint experiment of choices of bundles with two related categories, (ii) multi-time period conjoint experiments, in which the subjects first choose a product from one category and then a product from the other category with measure of the timing of adoption. These experiments integrate these two streams of research described earlier.

The implications of this study are significant to marketing practitioners in three ways. First, our study proposes new strategies for bundle design. Specifically by understanding the consumers' different decision-making processes when choosing from simultaneous bundling and sequential bundling items, sellers are able to design and construct the optimal bundle of products in both choice situations.

Second, in this study we show that a prior choice in a related category affects the next choice in the second product category. This leads to a natural question for marketing managers: Which attributes of one category appeal to customers given their past choices in other related categories? Our results on asymmetric relationships among attributes for product categories provide practical methods for effective cross category selling strategies.

Finally, we develop an empirical method to determine bundle-pricing strategies when forward-looking consumers are engaged in choosing either bundles or products singly over multiple time periods. In reality, consumers are often allowed to choose both an explicitly assembled bundle product and products one by one over time. Under this situation, we determine optimal bundle prices of various bundle profiles for different bundling strategies.

This chapter is structured as follows. Next, we present a conceptual foundation in section

2.2 and our modeling specifications for both simultaneous and sequential bundle choices in section 2.3. Then, we present the overviews of our conjoint designs in section 2.4, estimate our proposed model and discuss results on model performance and estimates in section 2.5. We then propose an application to find optimal bundle prices useful in practice in section 2.6. Finally, we summarize our findings and describe limitations as well as our intended future research directions in section 2.7.

2.2. Conceptual Foundation

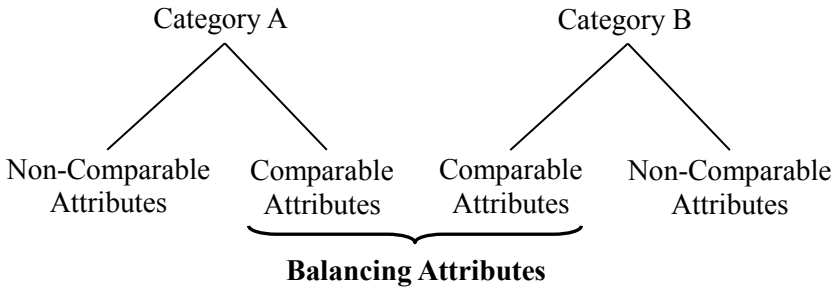
As interdependency of multi-product choices is a heart of our paper, we study various sources of the interdependencies of multi-category choices. First, one way to find it is estimating the correlation of latent utilities of the choices (Manchanda et al. 1999, Russell and Peterson 2000, Chib et al 2002, Singh, Hansen and Chintagunta 2006).Even though interdependency in this form implies the intrinsic relationships among categories, it can often come from contemporaneous multi-product shopping incidences. This explains that this measure of interdependency leads to mostly positive relationships (positive correlation) of different categories. However, unless the purchases incidences occur simultaneously, this synergy defined in the studies above may not be captured. Other source of interdependency of multi-product choices is complementarity. This can be measured by the additional utilities of purchase/consume of the other category products (Ma, Seetharaman, and Narasimhan 2011). Finally, the interdependency can come from the correlated marketing mix sensitivities (Ainslie and Rossi 1998, Seetharaman *et. al.* 1999) or brand preferences (Singh *et. al.* 2005).

While the interdependency describe above has been well applied to various multi-category selling practices, it is worth noting that an important issue for marketing managers for

considering cross-selling is not simply to understand these interdependency measures. That is which specific attributes of multi-categories cause more or less synergy to each other. This is not fully explained by weak or strong correlations of contemporaneous shopping incidences of multiple categories or of marketing mix sensitivities as well as brand preferences. To address this issue more carefully, interdependency of multi-category choices should be defined in another angle; in the product attribute levels.

The balancing model, proposed by Farquhar and Rao (1976), is able to address this very issue. This suggests that consumers adjust products of multiple categories in an attribute level. The rationale behind this is that consumer chooses one product and the other product by trading off the level of the common product attributes with maximizing the within-variance on other attributes, which is progressed to balancing or reinforcing attributes (Huber, Goldsmith and Mogilner 2008). This was applied to assortment choices (Bradlow and Rao 2000) and multi-category bundle choices (Chung and Rao 2003). Figure 2.1 shows its framework.

Figure 2.1 Conceptual Framework for Balancing Attributes



As shown in Figure 2.1, the advantage of balancing attributes over other methods to

specify the relationships of multi-category choices is that it enables us to estimate complementarity (or similarity) directly from attribute levels of multiple categories. This helps marketing managers not only understanding interdependent choice behaviors of multi-products but also finding profitable combinations of product categories to be cross sold. In this respect, we take advantage of this concept of balancing attributes to uncover interdependency in our study.

2.3. Models

We develop our proposed model of simultaneous and sequential bundle choices for the situation wherein a typical consumer i=1,…,I chooses items $j_c = 1_c, \ldots, J_c$in twocategories C={A, B}. These categories are assumed to be closely related to each other in terms of their attributes. To be specific, product j_c in category C contains two types of attributes:$X_{j_c}$is a set of j_c's attributes comparable to categories A and B, which are common to both categories, and that $Z_{j_c}$is a set of j_c's non-comparable attributes, which belong to only one category.

2.3.1. Simultaneous and Explicit Bundle Choices

First, we assume a situation of simultaneous (and explicit) bundle choices of categories A and B. That is, consumer i chooses a multi-category bundle with a predetermined combination. To model this choice task, we specify $u_{it}^1(j_A, j_{\prime B})$, a random utility for a bundle of j_A and j_B from categories A and B for consumer i, as follows.

$$u_{it}^1(j_A, j_{\prime B}) = U_{ij_Aj_B}^* + \frac{\varepsilon_{ij_Aj_B}}{\sigma_{(AB)}}$$
$$U_{ij_Aj_B}^* = \sum_{C=A,B}\left(X_{j_C}\beta_{(AB)i}^C + Z_{j_B}\alpha_{(AB)i}^C\right) + \phi_{(AB)i}Price_{j_A,j_B} + B_{j_A,j_B}, \quad (1)$$

where

$Price_{j_A,j_B}$: bundle price of j_A and j_B;

$\beta^C_{(AB)i}$: a set of consumer i's individual level parameters for comparable attributes of category C={A,B};

$\alpha^C_{(AB)i}$: a set of consumer i's individual level parameters for non-comparable attributes of category C={A,B};

$\phi_{(AB)i}$: consumer i's individual level parameter for the price of the bundle composed from categories A and B;

$\omega_{(AB)i}$: consumer i's individual level parameter for the balancing measure of prices of categories A and B;

$\gamma_{(AB)i}$: a set of consumer i's individual level parameters for balancing comparable attributes of bundle composed from categories A and B;

ε_{ij_A,j'_B}: i.i.d. type I extreme value idiosyncratic error with variance $\pi^2/6$; and

$\sigma_{(AB)}$: scale of the idiosyncratic error term.

We specify the utility for the no-buy option as $U_{i0} = \frac{\varepsilon_{i0}}{\sigma_{(AB)}}$.

The utility specification in Equation (1) includes three components of attributes: a) X_{j_A} and X_{j_B}, comparable attributescommon to both category A and B; b) Z_{j_A} and Z_{j_B}, non-comparable attributeswhich only belong to either category Aor B; and c) balancing attributes B_{j_A,j_B}, specified as $\left|X_{j_A} - X_{j'_B}\right|$, dispersion of comparable attributes between products of categories A and B (Farquhar and Rao 1976, Bradlow and Rao 2000, Chung and Rao 2003).[3]

[3]In the case of more than 2 products (categories) $k = 1, .., K$, the balancing attributes, $B_{1,..,K}$, can be extended to $B_{1,..,K} = \sum_{k=1,..,K} |X_k - \bar{X}|$, where $\bar{X}$ is the mean of attribute levels of products$k = 1, .., K$.

$$B_{j_A,j_B} = \omega_{(AB)i}\left|Price_{j_A} - Price_{j_B}\right| + \left|X_{j_A} - X_{j_B}\right|\gamma_{(AB)i} \quad (2)$$

For the inference of the parameters, the positive (negative) sign of $\beta^C_{(AB)i}$ ($\alpha^C_{(AB)i}$) implies that a higher (lower) value of X_{j_c} (Z_{j_c}) is desirable to consumer i. More importantly, turning to the balancing effects of attributes in Equation (2), the negative sign of $\phi_{(AB)i}$ means that consumer ifavors a low price of the bundle of $j_A$and j_B. In addition, the parameter of balancing attributes $\gamma_{(AB)i}$(or $\omega_{(AB)i}$) represents the consistency of attributes (prices) of the choices in categories A and B.The negative sign of $\gamma_{(AB)i}$ (or $\omega_{(AB)i}$) indicates that consumer i prefers a bundle with similar levels of attributes (or prices) between product categories (equi-balancing), whereas its positive sign implies that she likes to choose a bundle with different levels of attributes (or prices) between them (counter-balancing).

Then, let us denote $C_{(AB)i}$ as consumer i's choice decision on the bundle of categories A and B. Under the assumption of i.i.d. type I extreme value distribution of ε_{ij_A,j_B}, the probability of $P\big(C_{iAB} = (j_A,j_B)\big)$ is given as follows.

$$P\big(C_{iAB} = (j_A,j_B)\big) = \frac{exp\left(\sigma_{(AB)} \cdot U^*_{ij_Aj_B}\right)}{exp\big(\sigma_{(AB)} \cdot U_{i0}\big) + \sum_{(k_A,k_B)\in(K_A,K_B)} exp\big(\sigma_{(AB)} \cdot U^*_{ik_Ak_B}\big)}$$

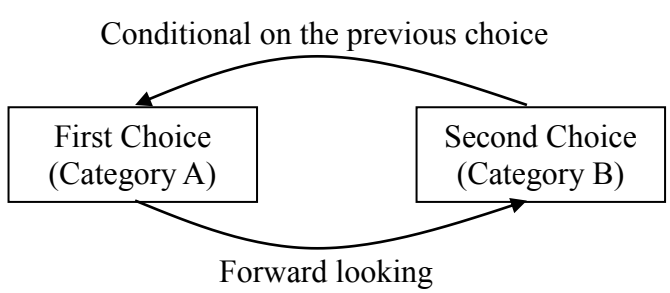
where (K_A,K_B): a set of (explicitly constructed profiles of) bundles of categories A and B, available to consumer i.

2.3.2. Sequential and Implicit Bundle Choices

We now turn to the situation of consumer i choosing products from categories A and B sequentially and implicitly. To be specific, she first chooses a product from category A, and later one from category B. Thus, it is important to note that while she evaluates utilities from category A and B together in the simultaneous and explicit bundle choice situation in the previous section, she evaluates their utilities one by one sequentially in this situation.

We illustrate two sources which drive consumer's sequential choice behavior in Figure 2.2. First, it is reasonable to infer that, when she buys a product, she may consider other related category that she previously purchased. Second, the current choice in one category can be influenced by the expectation of the future choice in the other category. That is, forward-looking behavior can play an important role in the sequential choices. These two behavioral conceptions allow us to construct the following dynamic models.

Figure 2.2 Conceptual Framework for Sequential Bundle Choices



2.3.2.1. Utility of the first choice

We assume the case wherein consumer i chooses a product in category A first. To model this, we begin with a traditional single category choice model, where U_{ij_A} is a latent utility of consumer i

for product j_A in category A as below.

$$u_{it}^{2}(j_A) = U_{ij_A}^{*} + \frac{\varepsilon_{ij_A}}{\sigma_{(A)}}$$
$$U_{ij_A}^{*} = X_{j_A}\beta_{(A)i} + Z_{j_A}\alpha_{(A)i} + \phi_{(A)i}Price_{j_A} \quad (3)$$

where

$Price_{j_A}$: price of product j_A in category A,

$\beta_{(A)i}$: a set of consumer i's individual level parameters for comparable attributes of category A,

$\alpha_{(A)i}$: a set of consumer i's individual level parameters for non-comparable attributes of category A,

$\phi_{(A)i}$: consumer i's individual level parameter for price of product in category A,

ε_{ij_A} : i.i.d. type I extreme value distribution with variance $\pi^2/6$,

$\sigma_{(A)}$: scale of the idiosyncratic error term.

The utility for the no-buy option, $U_{i0} = \frac{\varepsilon_{ij_A}}{\sigma_{(A)}}$.

2.3.2.2. Utility of the second choice given the first choice

Next, consumer i chooses product j_B in category B, given the prior choice j_A in category A. It is important to note that in this situation, consumer's decisions involve not only products in category B but also timing of adoption in category B. It is denoted as t=0 (now), 1 (next period),..,T. We specify that $u_{it}^{3}(j_B|j_A)$ is a latent utility of this choicej_B in category B given the choice j_A. Also, in this stage, the effect of balancing attributes, $B_{j_A,j_B t}$, is realized given the previous choice j_A, in a manner similar to the simultaneous case in Equation (1).

$$u_{it}^{3}(j_B|j_A) = U_{ij_B|j_At}^{*} + \frac{\varepsilon_{ij_Bt}}{\sigma_{(B|A)}}$$
$$U_{ij_B|j_At}^{*} = X_{j_B}\beta_{(B|A)i} + Z_{j_B}\alpha_{(B|A)i} + \phi_{(B|A)i}Price_{j_Bt} + B_{j_A,j_Bt} \quad (4)$$

where

$Price_{j_Bt}$: price of productj_B in category B at time t;

$\beta_{(B|A)i}$: a set of consumer i's individual level parameters for comparable attributes of category B, given the prior choice in category A;

$\alpha_{(B|A)i}$: a set of individual level parameters for non-comparable attributes of category B, given the prior choice in category A;

$\phi_{(B|A)i}$: consumer i's individual level parameter of price for product in category B given the prior choice in category A;

$\omega_{(B|A)i}$: consumer i's individual level parameter for the balancing measure of prices of categories A and B;

$\gamma_{(B|A)i}$: a set of consumer i's individual level parameters for balancing comparable attributes of categories A and B;

$\varepsilon_{ij_{B|A}t}$: i.i.d. type I extreme value distribution with variance $\pi^2/6$; and

$\sigma_{(B|A)}$: scale of the idiosyncratic error term.

As before, the utility for the no-buy option is, $U_{i0} = \frac{\varepsilon_{ij_Bt}}{\sigma_{(B|A)}}$.

Similar to Equation (2), we specify the effect of balancing attributes as;

$$B_{j_A,j_Bt} = \omega_{(B|A)i} \cdot \left|price_{j_A} - price_{j_Bt}\right| + \left|X_{j_A} - X_{j_B}\right|' \cdot \gamma_{(B|A)i}$$

2.3.2.3. Forward looking dynamic model of the sequential choices

As described earlier, we in this section describe the model of consumer's forward looking behaviors when choosing category A. We specify the value function of the first choice in category A as below.

$$V_{it}(j_A) = u_{it}^2(j_A) + E_\varepsilon[V_{it+1}(j_B|j_A)] \tag{5}$$

In Equation (5), the value function consumer i for choice j_A in category A depends on not only its own utility $u_{it}^2(j_A)$ but also the expectation of the future choice of category B given j_A(the choice in category A) as well as the timing of its purchase of $j_B$in category Bat t.Once consumer i made a choice in category A, she chooses a product in category B, given a choice in category A. Thus, the value function of the choice j_B is specified as below.

$$V_{it}(j_B|j_A) = \rho^t \cdot u_{it}^3(j_B|j_A) \tag{6}$$

where ρ is a discount factor, $|\rho| < 1$

In addition to the product choice in category conditional on the choice in category A, consumer i is allowed to decide the timing of the purchase in category B. Since our modeling framework incorporates the durable nature of products, consumers' choice is between purchase at t=0 (now, at the same time as category A) and delay her decision at t=1(in the next period),..,T, based on the assumption that consumers are forward looking (Rust 1987). We further assume $Price_{j_B t>0} < Price_{j_B t=0}$ to indicate that prices decrease over time (as would generally be the case for durable or technology goods). This implies that the primary source of dynamics is

consumer expectation of a future price cut. Depending on this future lower price, consumers may delay the timing of their purchase to the future period (t=1,..,T). In addition, depending on the time interval to the choice in category B, $u_{it}^{3}(j_B|j_{\mathrm{A}})$ is discounted by ρ which is bounded between 0 and 1. Thus, the problem that consumers are faced with is whether to buy the products from two categories at the same time and enjoy their utilities together, or to postpone buying a product in category B until the next periods so as to take advantage of its lower price. This implies that the trade-off between these two options enables her to decide the timing of purchase. Further, as to how consumers perceive future prices, we assume that they perfectly expect the future price, which is price at t=1 in our context. As in Dubé, Hitsch and Jindal (2012), we assume that future prices operate as given information in the timing decision for a purchase. This will become clearer when we describe our conjoint study.

Let us denote C_{iC} as consumer i's choice decision in category C=A, B. As noted above, in her decision making process, consumer i considers not only what to buy among the products in category B, but also when to buy between t=0 (now) and t=1 (the next period). This yields the joint choice probability of C_{iBt} and C_{iA} as follows.

$$P(C_{iA} = j_A, C_{iBt} = j_B) = \frac{\bar{V}_{it}(j_B|j_A)}{\sum_{k_B \in K_B} \bar{V}_{it}(k_B|j_A)} \times \frac{\bar{V}_i(j_A)}{\sum_{k_A \in K_A} \bar{V}_i(k_A)} \tag{7}$$

where,

K_C:a set of product profiles of category C=A,B, available to consumer i.

$$\bar{V}_i(k_A) = V_i(k_A) - \frac{\varepsilon_{ij_A}}{\sigma_{(A)}}$$

$$\bar{V}_{it}(k_B|j_A) = V_{it}(k_B|j_A) - \rho^t \frac{\varepsilon_{ij_Bt}}{\sigma_{(B|A)}}$$

Last, the sequential choices of the reversed order (category B first and category A next) can be modeled in the same manner by exchanging the notations A and B in Equations (3)-(7).

2.3.3. Identification of the Scale Parameters and Discount Rate

It is important to note that both taste parameters $[\beta_{(*)i}, \alpha_{(*)i}, \phi_{(*)i}, \omega_{(*)i}, \gamma_{(*)i}]$ and scale parameters $\sigma_{(*)}$ (where $* = AB, A, B|A, B, A|B$) cannot be identified (Fiebig *et.al.* 2010). It is customary to normalize $\sigma_{(*)}$ as 1 for probit models and $\pi^2/6$ for logit models. This standardization renders the comparison of utilities from different models impossible. Given that one of the primary goals of this study is to examine how consumers make decisions differently between simultaneous and explicit bundle choices and sequential and implicit bundle choices, we should compare not only utilities, but also the attribute preferences of models under different bundle choice situations. Given the situations described earlier, we can estimate individual attribute parameters $[\beta_{(*)i}, \alpha_{(*)i}, \phi_{(*)i}, \omega_{(*)i}, \gamma_{(*)i}]$ and a discount factor ρ, but not the scale parameters $\sigma_{(*)}$. To identify them together, we first fix $\sigma_{(AB)} = 1$ for the simplicity of the estimation. Next, in a similar spirit to Derdenger and Kumar (2013), we assume that the marginal utility of bundle prices of categories A and B specified in Equation (1) is the same as that of prices in single category choices specified in Equations (3) and (4). That is, $\beta_{(AB)i} = \frac{\beta_{(*)i}}{\sigma_{(*)}}$ for $* = A, B|A, B, A|B$. This means that bundle price coefficient $\beta_{(AB)i}$ is related to $\beta_{(*)i}$ by scaling by $\sigma_{(*)}$ for $* = A, B|A, B, A|B$. This special setting allows for the estimation of $\sigma_{(A)}$, $\sigma_{(B)}$, $\sigma_{(A|B)}$and $\sigma_{(B|A)}$. Last, as customary (Gordon 2009), we set the monthly discount rate as 0.99.

2.3.4. Heterogeneity

To incorporate consumer heterogeneity into the model, we employ a latent class model (Kamakura and Russell 1989), which enables us to estimate group level parameters instead rather than individual parameters. We assume that there are g=1,…, G group of consumers that are homogeneous within the group for the category choices. This implies that each group g has its own parameter space $[\beta_{(*)g}, \alpha_{(*)g}, \phi_{(*)g}, \omega_{(*)g}, \gamma_{(*)g}]$. Then, we define $\pi_g$probability that each consumer i belongs to group g respectively, indicating $\pi(i \in g) = P(\beta_{(*)i} = \beta_{(*)g}, \alpha_{(*)i} = \alpha_{(*)g}, \phi_{(*)i} = \phi_{(*)g}, \omega_{(*)i} = \omega_{(*)g}, \gamma_{(*)i} = \gamma_{(*)g})$.

2.4. Empirical Application

In this section, we illustrate the empirical applications in order to estimate our proposed models as specified in the previous sections. To do so, we designed choice-based conjoint experiments in which respondents make the multi-category bundle choices described above. The context of the experiments is two categories in Smart Products. Smart Products from different categories such as Smartphone, tablet PC, Smart Watch and Smart TV are often composed as a bundle by retailers (e.g., Smartphone and Tablet PC bundle in TalkTalk) or manufacturers (e.g., Smart TV and Tablet PC bundle and Smart Watch and Smartphone Bundle by Samsung). As such, consumers purchase and enjoy them together. It is primarily because they share similarities in terms of product attributes and thus create synergy when using together. This implies that there exist considerable dependencies between choices in categories in Smart Products. In this respect, we choose a bundle of Tablet PCs and Smart TVs for the context of our study.

2.4.1. Product Attributes

Table 2.1 Attributes Types and Levels, and Variable Specification

Types		Attributes and Levels	Variables and Coding
Comparable attributes		Brand (Samsung, Sony)	If the brand of Tablet PCj_T (Smart TV j'$_S$) is Samsung, $BR_{j_{S(T)}} = 1$. If the brand is Sony, $BR_{j_{S(T)}} = 0$.
		Screen Size	SC_{j_T} = 7,8,9 inchfor Tablet PC $SC_{j'_S}$= 30,40,50 inch for Smart TV
		Price (current prices)	$Price_{j_T}$= \$200,300,400 for Tablet PC $Price_{j'_S}$= \$300,400,500for Smart TV
Non-comparable attributes	Tablet PC	Weight (0.7lbs or 1.2lbs)	If brand $j_T$weighs0.7lbs, $W_{j_S} = 1$. Otherwise, $W_{j_S} = 1$
	Smart TV	3D Screen	If brand j's supports 3D screen, $D_{j'_T}$= 1. Otherwise, $D_{j'_T}$= 0
Balancing attributes		Brand	$\lvert BR_{j_S} - BR_{j'_T} \rvert$ Whether consumer i chooses the same brand for both Tablet PC and Smart TV.
		Price	$\lvert Price_{j_S} - Price_{j'_T} \rvert$ Difference of the prices of Tablet PC j_S and Smart TV j'$_T$ at time T
		Screen	$\lvert SC_{j_S} - SC_{j'_T} \rvert$ Difference in the screen sizes of Tablet PC j_S and Smart TV j'$_T$ at time T

To decide the focal attributes of a Tablet PC and a Smart TV, we conducted a pilot study in which we interviewed 30 graduate students at business schools in the U.S. The respondents chose brand and screen size as important attributes for both the Tablet PC and Smart TV; weight

for a Tablet PC, and 3D screen for a Smart TV. Therefore, a Tablet PC and a Smart TV each have 4 attributes including current and future prices.[4] To obtain an implementable profile size of conjoint sets, we select two levels for each attribute, for both a Tablet PC and a Smart TV. The details of the attributes and their levels are shown in Table 2.1.

2.4.2. Conjoint Experiments

The experiments of the conjoint studies involve the four choice tasks in which subjects choose three bundles of a Tablet PC and a Smart TV and no-buy option. To elevate the profiles of the bundles, we apply the OPTEX procedure in SAS and develop the four tasks, each containing 12 choice sets of full factorial design with 95% D-efficiency. Table 2.2 shows three conjoint experiments we conducted.

In Task1, the respondents are given both the prices of each component in the bundle, and also the bundle price, which is assumed to be the sum of the prices of the two products. Indeed, consumers often believe that bundled items involve price discounts (Estelami 1999, Janiszewski and Cunha 2004). On the contrary, there are occasions that sellers may not offer any bundle discount. For example, complementary bundles which provide additional value to customers by saving their effort in the decision process can offer no bundle discounts or even charge a premium price (Mazumdar and Jun 1993). Due to the complexity of bundle pricing effects, we assume neither bundle discount nor premium a priori. We will suggest an empirical method to estimate the optimal bundle prices later.

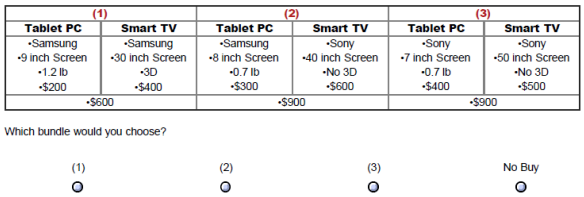
In addition, one of the key drivers for consumers to make sequential choices is the forward

[4]To obtain the realistic future prices for Tablet PCs and Smart TV, we collected price histories of the popular Samsung and Sony products over several months from major retail website and then estimated their monthly discounts, approximately $20 for Tablet PCs and $30 for Smart TVs.

looking behavior. Our assumption that consumers behave in a forward-looking way is realistic and plausible, especially when they choose high-technology products such as Tablet PCs and Smart TVs in our context.

To incorporate this potential behavior, our design in the second choice experiment allows respondents to choose not only one of three profiles of Tablet PCs (or Smart TVs) given their first choice of Smart TVs (or Tablet PCs), but also to make the purchase in the same month as the first choice or the one month after that first choice with a discounted price. Therefore, respondents seem to trade off utilities between two choices of timing: either they want to enjoy the products from both categories at the same time in spite of high prices, or they are willing to enjoy two products together later (so as to save money from the future discounted price). According to the data we collected, about 66.7% of responses report a delay in their purchases so as to take advantage of the future price discounts. This provides good support for the forward-looking assumption of consumer behavior.

Table 2.2 Conjoint Experiments

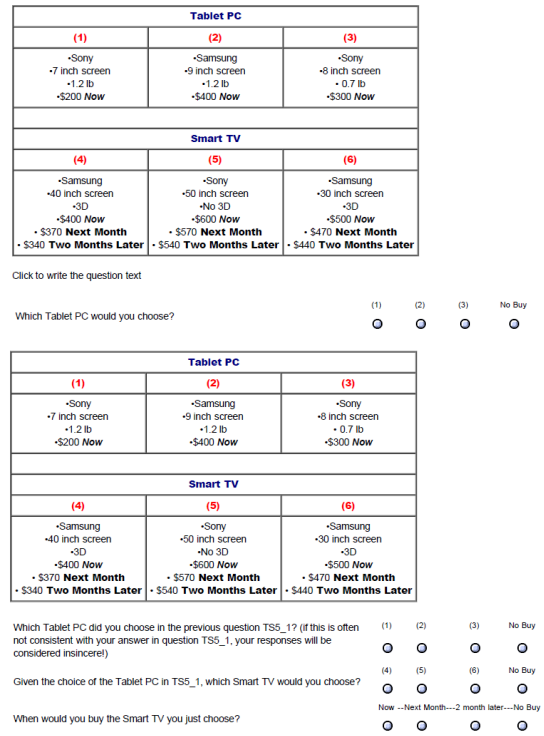
Task	Bundle Type	Choice Sets and Procedure	Example
1	Simultaneous	Respondents are asked to choose one profile among three bundle products of a Tablet PC and a Smart TV, where the combinations are assembled explicitly for them, and a no-buy option.	

(1)		(2)		(3)	
Tablet PC	Smart TV	Tablet PC	Smart TV	Tablet PC	Smart TV
•Samsung •9 inch Screen •1.2 lb •$200	•Samsung •30 inch Screen •3D •$400	•Samsung •8 inch Screen •0.7 lb •$300	•Sony •40 inch Screen •No 3D •$600	•Sony •7 inch Screen •0.7 lb •$400	•Sony •50 inch Screen •No 3D •$500
•$600		•$900		•$900	

Which bundle would you choose?

(1) (2) (3) No Buy

Table 2.2 (Continued)

Task	Bundle Type	Choice Sets and Procedure	Example
2	Sequential	Respondents are asked to choose one option among the 3 profiles of Tablet PCs and a no-buy option, and then to select one from among three Smart TV profiles and a no-buy option, as well as their timing of purchase between current month (t=0) and one/two months later (t=1,2).	
3	Sequential	Similar to Case 2 except for the order of choices. Specifically, respondents are asked to choose one of three Smart TV profiles and then next to choose one from the 3 profiles of Tablet PCs and a no-buy option, and also the timing of purchase.	The choice options are identical to those of Task 2.

2.4.3. Data Collection

We use all permutations of Tasks 1, 2 and 3 depending on their orders in the experiment, leading to 6 combinations of 3 tasks. Each combined set of three Tasks was answered by 25 subjects

older than 18 who reside in the U.S.; this was performed through Amazon Mechanical Turk, which has become increasingly popular in many academic behavioral studies (Mason and Suri 2012). In all, a total of 150 subjects participated in our study.

2.5. Results

The models are estimated in WinBUGS, where the first 80,000 iterations are used for the burning period, and the next 40,000 iterations to draw the posterior distributions of MCMC samples of the individual level parameters. For the hierarchical Bayesian specification, we assume non-informative conjugate prior distributions of parameters and monitor the time series plots across the MCMC iterations to examine the convergence of the posterior distribution.

2.5.1. Number of Latent Segments

Table 2.3 Selection of the Number of Latent Segments

	DIC
Homogeneous	14,240
2 Segments	13,730
3 Segments	13,535

To determine the number of segments, we use DIC (Deviance information criterion) as an sample. Table 2.3 shows that while the model with three latent segments gives the best performance, the gain in increasing segments diminishes after two latent segments. Given this

support, we decide the model with two latent segments as our proposed model. This also gives us managerial interpretation of the segmentation.

2.5.2. Comparison of Models

To validate our proposed model, we estimate the benchmark models shown in Table 2.4; these four models vary on whether or not balancing attributes are included; taste heterogeneity is included and forward looking dynamic structure is included. Model 5 (our proposed model) includes all these modeling components.

Table 2.4 Comparison of Models

	Balancing Attributes	Taste Heterogeneity	Forward Looking	In-Sample (DIC)	Out-of-sample (HIT-RATE)
Model 1				14,187	0.389
Model 2	√			13,691	0.428
Model 3		√		12,163	0.434
Model 4	√	√		11,209	0.560
Model 5 (proposed)	√	√	√	11,001	0.566

√ if present

For in-sample validation, we randomly choose 9 choice sets among 12 for each of the Cases 1, 2 and 3 and compute DIC, which includes both model fits and model complexity (a large number of parameters), by penalizing against higher dimensional models. For out-of-sample validation, we use the remaining 3 choice sets of the 12 and compute the hit rate, given

the estimates from the in-sample validation. This enables us to compare the predictive performance of each model. Table 2.4 shows the DIC and hit-rate for the 4 models. This comparison shows the influence of three modeling features in a step-wise manner.

According to Table 2.4, Model 1 performs the worst in terms of both the criteria: DIC for the in-sample estimation and the hit-rate for the out-of-sample validation. Model 3 shows a lower DIC and a higher hit rate than Model 2 does. This implies that taste heterogeneity contributes more to model-fit and prediction power than balancing attributes do. However, Model 6 shows that both balancing attributes and heterogeneity should important elements that help the model fit. Finally, our proposed model that incorporates not only taste heterogeneity and balancing attributes but also forward looking structures performs best. This allows us to verify that respondents behave forward looking when they choose different categories sequentially.

2.5.3. Estimation Results and Parameter Inferences

In this section, we describe the inferences based on the posterior distributions of our proposed models.

2.5.3.1. Product Attributes

To begin, we found that most coefficients have the same signs in the three Cases 1, 2 and 3 as indicated in Table 2.5. This suggests that consumers' weights for attributes are largely unaffected by either the types of bundle choices (simultaneous and sequential bundles) or the order of choices for sequential bundle cases.

Table 2.5 Parameter Estimates of Attributes (Main Effects)

Bundle Choice	Variable	Segment 1 (87%)			Segment 2 (13%)		
		mean	Low CI	High CI	mean	Low CI	High CI
Case 1: Simultaneous	Tablet brand	**2.99**	**1.77**	**4.15**	1.30	-1.71	4.32
	Tablet screen	1.21	-4.87	7.37	0.06	-5.94	6.45
	Tablet weight	-1.25	-3.60	1.09	-1.23	-5.24	2.99
	TV brand	**2.04**	**0.88**	**3.21**	-0.51	-3.57	2.56
	TV screen	**2.38**	**1.90**	**2.88**	4.63	-1.30	10.09
	TV 3D	**3.44**	**2.34**	**4.58**	**-1.47**	**-4.47**	**1.41**
Case 2: Sequential (TV → Tablet)	Tablet brand	**6.42**	**4.22**	**8.86**	-0.18	-1.01	0.75
	Tablet screen	1.06	-3.95	6.21	-0.93	-7.74	3.43
	Tablet weight	**1.10**	**0.24**	**1.74**	0.58	-0.50	1.81
	TV brand	**0.13**	**0.04**	**0.27**	-0.10	-0.44	0.20
	TV screen	**5.16**	**2.62**	**9.42**	**2.69**	**1.03**	**4.77**
	TV 3D	**0.37**	**0.18**	**0.67**	0.08	-0.23	0.44
Case 3: Sequential (Tablet → TV)	Tablet brand	0.01	-0.01	0.02	-0.01	-0.07	0.05
	Tablet screen	**7.13**	**4.99**	**9.70**	**5.06**	**2.88**	**7.69**
	Tablet weight	**-0.12**	**-0.19**	**-0.07**	0.04	-0.09	0.17
	TV brand	**-1.41**	**-1.72**	**-1.08**	-0.07	-0.45	0.44
	TV screen	**1.92**	**1.41**	**2.27**	**-2.77**	**-3.13**	**-2.51**
	TV 3D	**0.61**	**0.06**	**1.25**	0.01	-0.26	0.31
	Price	-0.02	-0.02	-0.01	-0.02	-0.03	-0.02

Bold indicates that 95% credible interval does not contain zero.

In contrast, it is interesting that TV brand coefficient only in Case 3 is negative while they are positive in Case 1 and 2. That is, when consumers choose a Smart TV first, they prefer Sony to Samsung, while it is quite opposite whey they choose a Tablet PC first or a bundle of two categories.

Last, in terms of segmentation, we found that more coefficients of population means are statistically significant for Segment 1 which accounts for 87% of the subject pool. This indicates that Segment 1 shows more manifest preferences on product attributes.

2.5.3.2. Balancing Attributes

Of more interest is how balancing attributes in the three types of bundles and the sequence of category choices influence consumers' decision processes. In contrast to the results of product attributes above, Table 2.6 reports that balancing attributes play very different roles on consumers' decisions depending not only on the types of bundle choices (simultaneous and sequential bundles), the order of choices for sequential bundle cases, but also on latent segments.

First, Case 1 (simultaneous bundle) and Case 2 (sequential bundle of Smart TV first and Tablet PC next) shows similar signs of balancing effects of price, screen size and brand. To be specific, consumers seem to find prices and brands as a counter-balancing attribute, yet they consider screen size as equi-balancing in Cases 1 and 2. Interestingly, on the other hand, in Case 3 which indicates when consumers choose Tablet PC first and then Smart TV, they equi-balance screen sizes yet counter-balance prices of Smart TV and Tablet PC. Also, Segment 1 of them prefers to buy the different brands, that is, counter-balance brands. The balancing effect of brands for Segment 2 is the only common sign of balancing attributes regardless of which type of bundle and the order of choices for sequential bundle choices.

To sum up, in general different types of bundles and the order of choices seem to evoke different weights for evaluating attributes, and more noticeably so for balancing attributes (or difference of choice processes).

Table 2.6 Parameter Estimates of Attributes (Balancing Effects)

Bundle Choice	Variable	Segment 1 (87%)			Segment 2 (13%)		
		Mean	Low CI	High CI	mean	Low CI	High CI
Case 1: Simultaneous	balance price	-0.53	-1.13	0.06	**-2.57**	**-4.33**	**-0.90**
	balance screen	**2.26**	**1.75**	**2.79**	4.54	-1.35	10.23
	balance brand	**-2.46**	**-3.78**	**-1.08**	**-3.71**	**-6.97**	**-0.46**
Case 2: Sequential (TV → Tablet)	balance price	**-1.32**	**-1.68**	**-1.10**	**-0.80**	**-1.06**	**-0.58**
	balance screen	0.99	-2.54	3.68	**3.60**	**1.37**	**5.80**
	balance brand	**-0.53**	**-1.61**	**0.42**	**-0.78**	**-1.70**	**-0.06**
Case 3: Sequential (Tablet → TV)	balance price	**0.18**	**0.15**	**0.23**	**0.34**	**0.29**	**0.39**
	balance screen	**-2.03**	**-3.16**	**-1.12**	**-0.91**	**-1.34**	**-0.61**
	balance brand	**0.15**	**0.06**	**0.23**	**-0.34**	**-0.55**	**-0.17**

Bold indicates that 95% credible interval does not contain zero.

This result provides an important insight into cross-category dependencies. Formerly, relationships between categories such as complementarity or substitutability were defined based on interdependencies across categories (Manchanda et al. 1999, Russell and Peterson 2000). This was often undertaken at the category level (e.g. cake mix and frosting, detergent and softener). However, our results provide empirical evidence that cross category dependencies are determined not only by brands in categories but also by the attribute levels of the brands; this is

consistent with the findings of the balancing model applications (Farquhar and Rao1976, Chung and Rao 2003). A more noteworthy new finding in our balancing models is that the cross-category interdependencies should be defined by both the types of bundles and the order of choice sequences. That is, how consumers perceive synergy or complementarity between categories is also decided by specific bundle choice situations. Therefore, it is important that interdependencies among categories be examined in greater detail than the prior bundling literature has suggested.

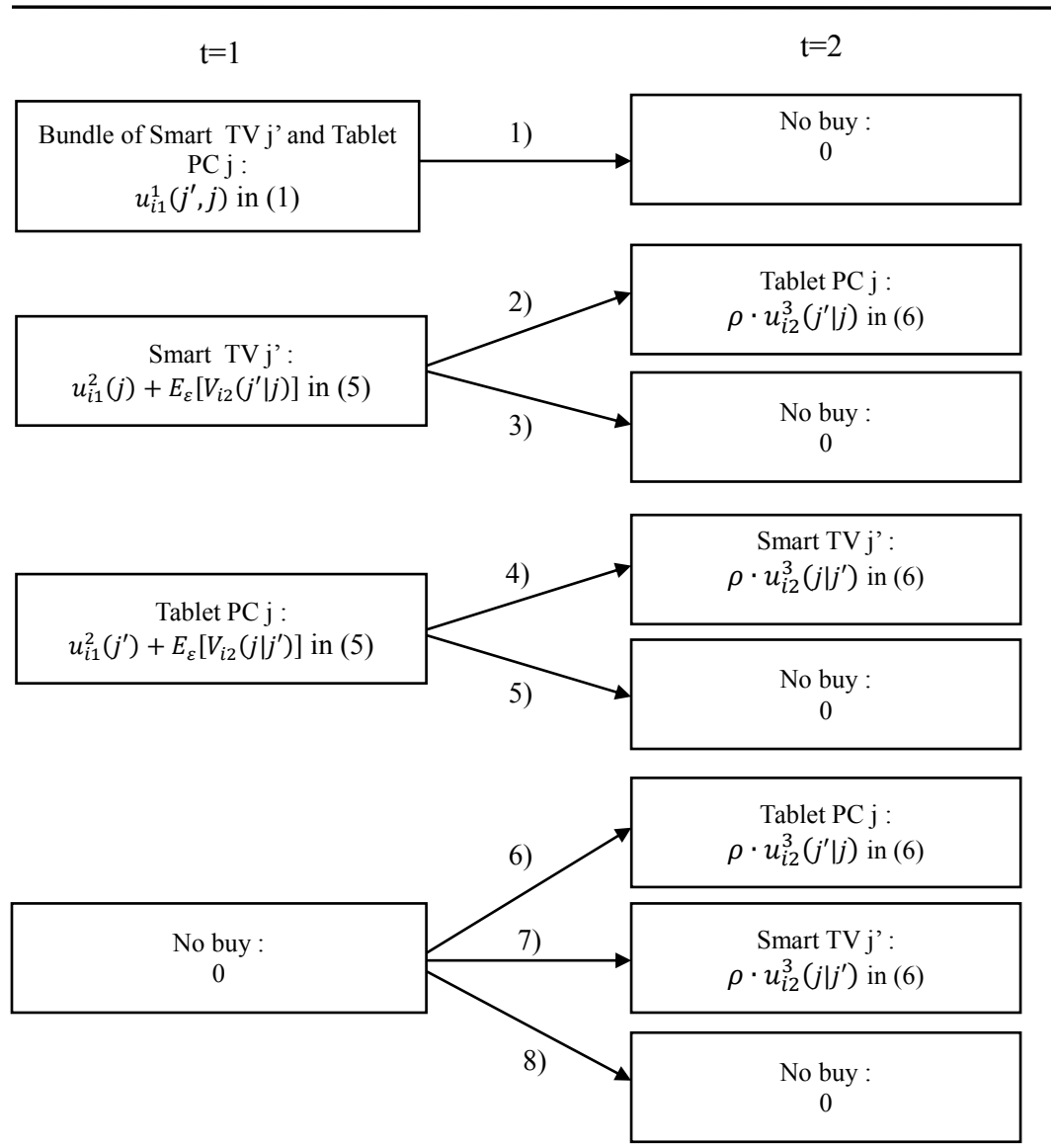
2.6. Optimal Bundle Prices

We now study the optimal bundle pricing strategies of various profiles when both simultaneous and sequential bundles are available to consumers. In practice, finding optimal bundle prices is a notoriously difficult task (see Venkatesh and Mahajan 2009). It is striking that there have been only few studies to investigate the optimal bundle prices in the bundling literature (Hanson and Martin 1990, Venkatesh and Mahajan 1993, Venkatesh and Kamakura 2003). In this study, we specify a realistic choice situation of multi-category products, especially when heterogeneous forward looking consumers are allowed to choose simultaneous or sequential bundle products. To be specific, the hypothetical situation herein is that consumers buy a Tablet PC j and Smart TV j' over two time periods (t=1,2). They can a buy them as a bundle together at t=0 (simultaneous and explicit bundle case), or buy one after the other over two time periods (sequential and implicit bundle case). Thus, consumers have 8 possible choices as shown in Figure 3.

In Figure 2.3, we shows situations; case 1) indicates the simultaneous and explicit bundle choice case, cases 2) and 4) the sequential and implicit bundle cases, cases 3), 5) and 6-7) single

category purchase cases and case 8) is no-purchase option. Therefore, our hypothetical situation faced by a consumer incorporates not only mixed bundling strategies but also consumer's forward-looking behaviors.

Figure 2.3 Possible Choices of Tablet PC and Smart TV over Two Periods and their Utilities



As such, the cross-selling strategy of a seller is to introduce a bundle of Tablet PC j and Smart TV j' at t=1. Additionally, we assume that the prices of the products in these two categories decrease over t=1 and t=2. Similar to our experiments in the previous section, we assume that prices of Tablet PCs and Smart TVs decrease by $20 and $30 respectively at t=2. To maximize the total profit by way of bundling strategies, the seller wishes to determine a bundle price (simultaneous and explicit bundle) and a special price of a product of one category to consumers who have made a purchase of the other category previously (sequential and implicit bundle).

Next, we describe the computation procedure to derive the optimal bundle discounts under the given scenario and propose its implications. Recall the utility functions in Equation (1), (2) and (3). It is worth noting that our modeling framework renders that direct comparison of utilities feasible because we estimated the scale parameters $\sigma_{(A)}$,$\sigma_{(B)}$, $\sigma_{(AB)}$, $\sigma_{(A|B)}$ and $\sigma_{(B|A)}$ in the utility functions. This enables us to specify consumer i's value functions corresponding to situations 1) to 8) in Figure 2.3.

In this setting, we assume that a seller can offer a bundle price of Tablet PC j and Smart TV j', $Price_{jj't=0}^{BUN}{}^{*}$, to consumers who bought one category at t=0. Then, the seller decides an optimal bundle price under the simultaneous and explicit bundle situation, $Price_{jj't=0}^{BUN}{}^{*}$, in Equation (1) in order to maximize the profit. We further assume that the seller is not allowed the bundle price $Price_{jj't=0}^{BUN}{}^{*}$ below the cost and to exceed the sum of prices of the bundle components.

Next, we simulate dynamic choices of 1,000 customers of two segments using the value functions of the estimated segment-level parameters presented in Tables 2.5 and 2.6 and compute their choice probabilities of eight choices 1) to 8) over two periods t=0,1 using forward dynamic

programming. We fix margin[5] as 25% and period cost of inventory holding and display[6] as 5%. Then, we numerically compute the bundle price $Price_{jj't=0}^{BUN}{}^{*}$ which maximizes the profit over two periods.[7]

We derive the bundle prices which maximize seller's profit in various combinations of profiles of Smart TV and Tablet PC. To demonstrate the effects of product attributes on optimal bundle prices, Table 2.7 presents five examples of bundle profiles and their optimal prices.

Table 2.7 Examples of Optimal Bundle Prices

profile	Smart TV				Tablet PC				Original Bundle Price (Profit)	Optimal Bundle Price (Profit)
	brand	price	screen	weight	brand	price	screen	3D		
1	Samsung	$400	30'	1	Samsung	$300	7'	1	$700 ($8.579)	$694 ($8,625)
2	Samsung	$500	50'	1	Samsung	$200	9'	1	$700 ($6,271)	$690 ($6,281)
	When prices are different from Profile 1.									
3	Samsung	$400	50'	1	Samsung	$300	9'	1	$700 ($11,546)	$697 ($11,567)
	When screen sizes are different from Profile 1.									
4	Sony	$400	30'	1	Samsung	$300	7'	1	$700 ($5,550)	$693 ($5,558)
	When Tablet PC brand is different from Profile 1.									
5	Samsung	$400	30'	1	Sony	$300	7'	1	$700 ($5,126)	$693 ($5,167)
	When Smart TV brand is different from Profile 1.									

[5]According to www.digitaltrend.com, the manufacturer margin of Samsung is around 22 to 25%. As we consider retailer's situations, we assume lower margin than that of the manufacturer.
[6]We used a standard "rule of thumb annual inventory cost", which is 25% annually (Richardson 1995). In addition to display cost, we decided 5% of monthly inventory and display costs.
[7]We used the *optimize* function in *R* with convergence tolerance 0.0001.

First, we change the price levels of Smart TV and Tablet PC in Profile 2 but their bundle price is the same as that of Profile 1. We find that the seller should offer a lower bundle price ($690) than that of Profile 1 ($694), which lead to less total profit. Similarly, we also change screen sizes of Smart TV and Table PC in Profile 3. In this case, we discover that less bundle discount ($3) does not need to be offered to consumers than that of Profile 1 ($6). This provides the seller with higher total profit from both categories.

Next, besides changes of attribute levels, marketing practitioners should be interested in brand management when designing a bundle combination. For example, should a bundled product be constructed from the same (manufacturer) brand or the different brands? We refer the former as retailer bundle and the latter as a manufacturer bundle, in that retailers can design a bundle regardless of the brands while manufacturers do not incorporate a brand from other competitive manufacturers to construct a bundle.

Profile 1 indicates situations when the same brand constructs a bundle, whereas different brands construct a bundle in Profiles 4 and 5. Except for the brands, other product attributes are fixed in the profiles. For cases of Profile 4 and 5, the optimal bundle prices are not changed noticeably from that of Profile 1. However, the total profits in both Profiles 4 and 5 decrease dramatically when the seller offers a bundle of the same manufacturer in Profile 1. These examples suggest that constructing a bundle of both Samsung products provides the highest profit to the seller.

In conclusion, our analysis of optimal bundle prices suggests that the discount for the bundle products should be determined depending on the combinations of products in two categories in terms of attributes levels and the amount of discounts for the simultaneous bundle

choices can be markedly different.

2.7. Managerial Implications and Limitations

Our research sheds light on a new domain of cross-selling strategies. We examined an important managerial issue of whether to sell bundles of related products simultaneously or selling related product by allowing a time interval so as to generate more sales and profits. Further, we looked at the decision on the attribute levels for the bundles products.

To enable such resolution for managerial questions, we proposed a dynamic model of the sequential choices and the simultaneous choices for two related categories and tested them using data from conjoint studies. First, in this research we addressed how asymmetric preferences drive consumer's sequential choices on the related categories. We found that consumers show different preferences on brands depending on types of bundles. More interestingly, how they balance attributes of multiple category items is markedly different by not only types of bundles but also the order of choices on categories. In the context of our experiment, while consumers did not balance prices in a simultaneous bundle situation, they counter-balanced prices when choosing a Tablet PC first and a Smart TV later and equi-balanced prices when the order of choices is reversed. Therefore, if marketing managers have information of customer's previous choices for one category, they can use the information to sell other related categories, leading to higher purchase rates.

Next, using the implications of the results, we proposed a framework to decide bundle prices under a realistic situation that forward looking consumers can choose multi-category products by simultaneous bundles as well as sequential bundles. We found that the bundle prices which maximize seller's profit are noticeably different by combinations of multiple category

items. According to our results, higher bundle discounts should be offered when consumers try to buy a low-end item from one category and a high-end item from another category than when they want to buy either low-end or high end items from two categories. In addition, bundling items from the same brands or different brands can lead to different optimal bundle prices. Therefore, our study offers a good understanding of the scope of bundling and multi-category choice behaviors, which leads to better managerial decision making for cross-selling marketing strategies.

Our study is not without limitations. First, the conjoint design does not include consumer's budget constraint. One of the reasons why consumers make sequential choices rather than simultaneous choices may be because of their limited budget. The budget availability along with expectation of lower prices may play a significant role in making decisions sequentially. It would be interesting to develop a special manipulation of the conjoint designs to treat this feature. Second, uncertainty about brand or attributes can play a significant role in their decision making (Meyer 1981, Kahn and Meyer 1991). Especially in our context of durable goods or new product choices, consumers' effort to reduce uncertainty may operate the sequential decision making process. Third, our study restricts consumer's choices to two product categories. However, in real world situations, consumers are engaged in choices of more than two categories; this aspect can be a natural extension of our framework. This is important especially when exploiting consumer's goal completion behaviors in composing a bundle of items (Harlam and Lodish 1995) or making sequential choices (Huber, Goldsmith and Mogilner 2008, Laran2010). We leave these extensions for future research.

Chapter Three

Exploring Dependencies Across Multiple Social Networks: The Role of Overlapping Friends

This chapter proposes an integrated model that accommodates networking activities across social networks and tests the model using data from two social network game services that have considerable overlap among friends. Our model captures various sources of dependencies across social networks, including coincidence, correlated sensitivities, and experience spillover across networks. More important, the model discovers a new source of dependencies that stems from common network members overlapping in different social networks. The model also finds that the spillover effects are asymmetric across networks. Our simulation study provides managerial implications for organizations attempting to target valuable customers and allocate marketing resources across multiple social networks.

3.1. Background

A large body of literature not only in marketing but also in economics, statistics, computer science, psychology, sociology, and so on, has extensively examined social networks. In particular, research in sociology and psychology has emphasized the relationships and bonds in networks, such as Homophily (e.g., Heider1957, Verbrugge 1977, McPherson, Smith-Lovin, and Cook 2001) and social interactions (e.g., Adams 1967, Dindia and Canary 1993, Kahanda and Neville 2009). Economics studies have applied social network analysis to various topics, including network formation theories (e.g., Wasserman and Pattison 1996, Bala and Goyal 2000; Watts 2002) and applications to economic market problems (e.g., Kranton and Minehart 2001;

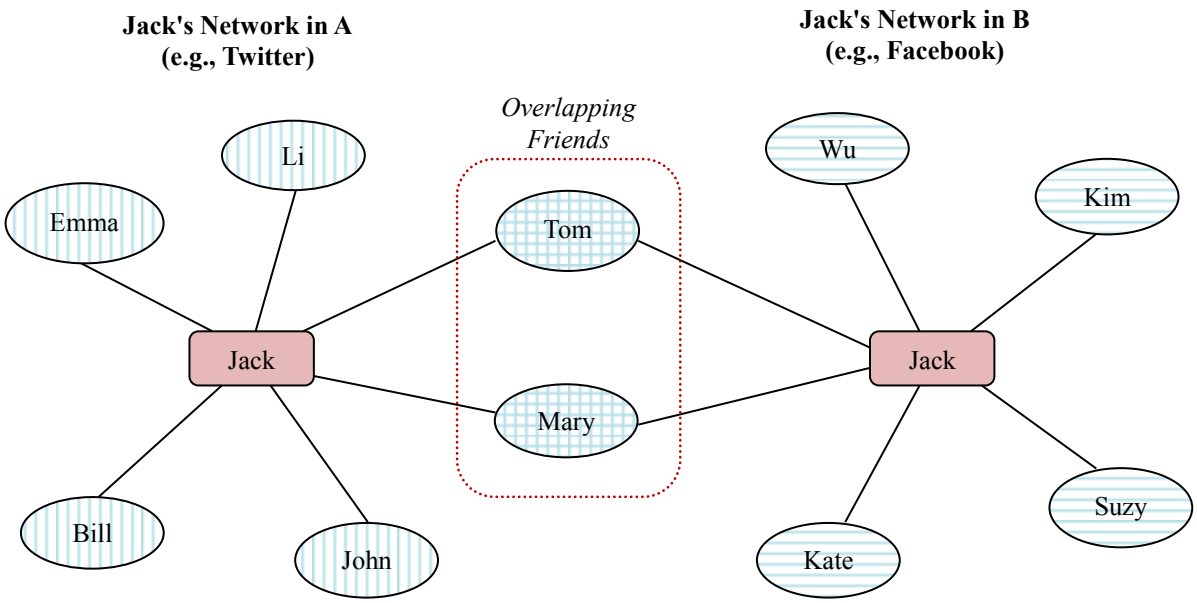
Calvó-Armengol and Jackson 2004).

The booming popularity of social network services such as Facebook, Twitter, and LinkedIn has attracted attention in marketing research. One stream of marketing research applications has focused on measuring the relationships of network members. For example, research has used sociometric and demographic characteristics such as gender, age, and region to measure bonds of connected network members (e.g., Ansari, Koenigsberg, and Stahl 2011, Nitzan and Libai 2011). Another stream of marketing research has characterized the structure of entire networks and their network properties (e.g., degree connectivity, betweenness, closeness) and investigated their effects on market outcomes, such as product adoption and site visitation (e.g., Katona, Zubcsek, and Sarvary 2011, Stephen and Toubia2010, Tucker 2008, Yoganarasimhan 2012). Marketing research on social networks has also extended to examine various topics such as word-of-mouth communication, which spreads through social network platforms (e.g., Godes and Mayzlin 2004; Trusov, Bucklin, and Pauwels 2009) and customer relationship management (e.g., Kumar et al. 2010, Kumar et al. 2013, Nitzan and Libai 2011).

Although this rich and extensive literature has investigated various important aspects of social networks from different approaches and angles for decades, one limitation in common is the underlying assumption of a single social network. That is, prior research has largely explored behaviors occurring in a single social network, such as the workplace (e.g., Tucker 2008), Twitter (e.g., Toubia and Stephen 2013), YouTube (e.g., Yoganarasimhan 2012), or Facebook (e.g., Lewis *et. al.* 2008). However, people in the real world belong to more than one social network, including schools, neighborhoods, clubs, Facebook, Twitter and so on. For example, approximately 35% of social network users use more than two social network services (e.g.,

Facebook and Twitter).[8]

Figure 3.1 An Example of Overlapping Friends



Users' active interplay across different social networks implies that these networks are not completely independent. That is, several sources may induce interrelationships among the networks, including similar network characteristics (e.g., Facebook and Twitter), adjacent locations (e.g., Baseball fan club and local charity society), and related goals (e.g., math club and science club). From a behavioral perspective, the coincidences of activities and correlated individual tendencies (e.g., to be involved in multiple social networks) are examples of dependencies that can exist across social networks. In particular, we are interested in one source

[8]Quoted from www.journalism.org, November 13, 2013.

that emanates from structures across networks—that is, the dependency that arises from common network members who overlap in different networks. Figure 3.1 illustrates an example of the egocentric network structures of Jack across two social networks. Jack has several friends from Twitter, and two friends, Tom and Mary, are also connected on Jack's Facebook page. In this respect, the different networking platforms that Jack belongs to are interrelated and connected through these overlapping friends.

Thus, a natural question arising from this example is how the relationship with these overlapping friends accounts for dependency between different networks. In other words, what is the role of these overlapping friends? Does their influence create synergy or redundancy in the two networks? Despite extensive studies in social networks, much attention has been paid only to a single social network upto this point. Therefore, our research objective is not only to examine various dependency sources across multiple social network platforms but also to explore how overlapping friends play different roles in business measures (e.g., visit) both within and across networks.

This issue is important especially for today's online social networking environments because many firms implement marketing practices across multiple social networks, largely due to the rapid rise of online social networking platforms. For example, 77% of the Fortune Global 100 companies have Twitter accounts, 61% have Facebook pages, and 57% use YouTube to find and select influential and targetable customers.[9]

Thus, if dependencies exist between different social networks, limiting the scope of analysis to only one social network sheds only partial light on the effects of social networks on firms. Therefore, it is crucial to find and understand interdependencies that emanate from

[9]Quoted from www.therealtimereport.com, March 18, 2011.

multiple social networks to obtain a broader picture of social networks for firms. This notion is in line with the important managerial issues of customer relationship management and marketing resource allocation across multiple social network platforms. To tackle these issues, this study provides a generalized framework of empirical analysis on multiple online social networks.

We develop an integrated visit model that decomposes and identifies various sources of dependencies that can arise when consumers engage in or simply belong to multiple social network platforms. We apply the proposed models to unique data of two mobile social network game services, both of which have considerable customer overlap. First, our generalized model enables us to discover various sources of dependencies that induce customers to use multiple social network platforms, such as coincidence, correlated sensitivities, and experience spillover. More important, we find the existence of dependency due to overlapping friends but also that the spillover effects are asymmetric across networks. That is, the direct spillover effect through communications in one network platform positively affects visitation to another social network service, but the interaction between communications with overlapping friends in the two networks is negative.

To verify the managerial importance of our study, we conduct a policy simulation for a hypothetical business problem for a firm that runs business pages in multiple social network platforms. The policy simulation shows that communications with overlapping friends in one social network platform lead to increased user visitation not only to that social network platform but also to the other social network platform. Yet we find that the spillover effects of overlapping friends are *asymmetric* across networks. Thus, we find that the role of overlapping friends can be in general synergistic.

In light of this, the goal of our study is to broaden the literature on social networks by

specifying the effects of network structures on multiple social networks. To the best of our knowledge, this is the first empirical study to explore dependencies of people engaged in multiple online social network platforms.

The rest of this chapter proceeds as follows: We describe conceptual foundation in Section 3.2 and the data in Section 3.3. Then, we propose the models in Section 3.4 and report the estimation results in Section 3.5. Next, we present a simulation study and discuss its implications in Section 3.6. Finally, we conclude by discussing the models' managerial implications and limitations and offering future research directions in Section 3.7.

3.2. Conceptual Foundation and Hypotheses

3.2.1. Conceptual Foundation

In a broad sense, our study lies in the stream of research on cross-choice analysis. At the heart of this stream is the exploration of dependencies that affect consumer behavior across categories, channels, websites, and so on. A large body of literature has investigated sources of dependencies when consumers engage in shopping across categories. For example, Manchanda, Ansariand Gupta (1999) examined cross-category effects using the multivariate Probit model and find sources of interdependency such as complementarity, coincidence, and heterogeneity. Hansen, Singh, and Chintagunta (2006) developed a multi-category brand-choice model, incorporating a factor-analytic structure of the covariance of the coefficient. They specify a parsimonious way to represent dependencies that stem from correlated heterogeneity across categories. In cross-channel analysis, Ansari, Mela and Neslin (2008) measured one source of dependency when consumers choose channels and make transactions, incorporating correlated structures of purchase incidences. Research has also applied correlated incidences and customers' intrinsic

tendencies to explain visitation behaviors across websites (Park and Fader 2004).

However, note that differences stem from distinct features of the domain in the social networks. Unlike consumer behavior across categories, channels, and websites, we examine behavior across social networks that includes not only consumers' own behavior but also that of others around them. This distinction yields a different modeling approach from existing studies as well as a conceptual characterization of interdependency. For example, some dependencies may come only from consumers' activities that occurred in one or another social network, such as coincident activities and correlated behavior tendencies.

Note that some aspects of consumers' behaviors can be explained by their own networks—that is, the people in their networks (Katona, Zubcsek and Sarvary 2011, Trusov, Bodapati, and Bucklin 2010) and those with whom they communicate in the networks (Libai, Mullerand Peres 2013, Trusov, Bucklin, and Pauwels2009). The social network structures are an important source of dependencies across social networks. To address this, we pay special attention to friends who are connected in multiple social networks, especially those who overlap across multiple social networks.

Our concept of overlapping friends emerges from today's environment in which people engage in many online social networks that are increasingly interconnected because of rapid information flow and transaction costs (e.g., Internet, Mobile). Thus, our concept of overlapping friends is not fully explained by the rich body of classical network theories and applications. One closely related concept is the concept of network overlaps suggested in sociology theory. Granovetter (1973, 1983) first conceived of the idea of network embeddedness; overlaps in network structures to explain the strength of the ties of different actors who have mutual friends in networks. Such an overlap indicates two ego-centric networks that share common, connected

agents. The embedded networks play as a bridge through which knowledge and information are transferred to the connected agents (Reagans and McEvily 2003). Coleman (1988) suggests a similar concept of overlapping friends in which two related agents are connected with the same third agent. Provan and Sebastian (1998) further propose overlapping cliques. Their idea is that the entire network is divided into small cliques (i.e., clusters) and that each clique can be involved in different activities (or agents). This notion defines overlapping cliques for network members engaged in multiple, common activities.

The concept of overlapping networks proposed in sociology theories is somewhat similar to our concept but involves just a single network and was developed to identify overlaps of small clusters or networks within it. In contrast, our concept integrates and connects small networks that are formed under different contexts or environments and thus considers overlaps across different multiple networks. In this respect, our conceptualization of overlapping friends across different networks is new and unique and thus is well suited to the contemporary circumstance in which people and firms engage in many connected social network platforms.

3.2.2. Hypotheses

3.2.2.1. Within a Network

As people are interacting and communicating with each other in online communities, firms also strategically take advantage of such interactions as a marketing tool. Previous studies investigated how sharing information in online forum communities influence consumers' purchase decisions. Chevalier and Mayzlin (2006) studied causal relationships of online book reviews on consumer choices and found that an increase in the number of reviews improves sales. Chen and Xie (2008) studied various issues of how sellers benefit from online product

reviews depending on timing, products, information etc. Feng and Zhang (2010) found that impact of consumer reviews varies considerably on the purchase decisions depending on products and consumers' characteristics.

The focus of these studies is on the contents generated by users and consumed by arbitrary users. That is, content generators (i.e., posting reviews) do not target or assume some specific profiles of receivers. However, of our primary interest in this study is the pairwise interaction that a user sends to a particular connected user on online social network services. That is, it is a private way to communicate. In this sense, it is different from social communications such as user generated contents, product reviews etc. Based on sociological theories, direct social interaction leads to similarity and vice versa, interaction creates similarity. This also implies that this direct social communication is an excellent indicator of relationships between pairs of network members (Gilbert and Karahalios 2009).

Although Godes and Mayzlin (2004) found a positive relationship between online Word of mouth and viewership of TV shows and Feng and Zhang (2010) also discovered a positive influence of online reviews on product purchases, the effects of pairwise user interactions though direct communications in online social networking services has not been investigated in academic research. However, in that the direct message is one of the main reasons that users enjoy social network services, most social networking services such as Facebook, Instagram, LinkedIn, etc. offer such communicative services. In this regard, we also expect positive relationship between users' behaviors such as visitation and their communications. We therefore formulate our first hypothesis:

H1a. The effect of communication with (both overlapping and non-overlapping) friends is

positive on service performance within a network

Multiplexity represents that two people are bound to each other in different dimensions of social arenas (Van Den Bulte and Wuyts 2007). An example of a multiplex relationship is that friendship, communications and music download relationships of two people (Ansari, Koenigsberg and Stahl 2011). These relationships are often associated with strong ties. That is, higher levels of multiplexity can also make it more difficult to break ties (Kilduff and Tsai 2003). This allows us to expect that friends connected to different social network platforms are more closely related to users and therefore, their influences should be stronger than friends associated only to one network platform. Thus, we hypothesize the following:

H1b. The effect of communication with overlapping friends is stronger than that of non-overlapping friends on service performance within a network

3.2.2.2. Across Networks

Networks serve as flow of knowledge or information among agents. Fershtman and Gandal (2011) showed existence of knowledge spillover among researches who worked on the same or different projects (Fershtman and Gandal 2011). As they demonstrate this by constructing academic networks, this research shares some similarities to our social network contexts. Likewise, communication between members of firms' networks leads to transmission of knowledge between firms (Cohen and Levinthal1989; Rosenberg 1990).

From this reasoning, if users overlapping in two network platforms have influences on their friends connected to both networks (i.e., H1b: within-network effect), we expect that they

also play a bridging role between two networks based on the information or knowledge spillover theory. That is, their social communication that happens in one network is likely to affect the behavior of users connected to them in the other network. Because we define this as existence of spillover of social communication through overlapping friends, we hypothesize the following:

H2a. The spillover of social communication through overlapping friends exists from one network to another, if its effect within a network is positive.

According to cognitive load theory, people have only limited cognitive processing ability to apply acquired information to new situations (Paas, Tuovinen, Tabbers and Van Gerven, 2003). This cognitive load may result from redundant information that they obtain from different sources of the situations. We apply it to our context of multiple social networking situations.

Information that is received from overlapping friends from both network platforms may lead to redundant effect, which may result from the similarity of the information content. As a result, users are confused by this additional information, which would negatively affect users' ability to process the information. This can lead to cognitive load.

Thus, if overlapping friends play similar roles (i.e., both positive) directly on their connected users both within and across networks, interaction between the communicative activities with the overlapping friends within and across networks is expected to be negative due to redundancy of information that the overlapping friends convey. Based on this discussion, we suggest the following hypothesis:

H2b. The interaction effect of social communication with overlapping friends in two

different networks is negative, if the directed effects of overlapping friends within and across networks are positive.

3.3. Data

Our data come from two mobile social network game services (hereinafter Services A and B) managed by one company that wishes to remain anonymous. Service A debuted in Apple iTunes globally across 100 countries in September 2011, and since then, more than 3.5 million users have downloaded it. With the success of the iOS version, the Android version was launched in February 2013. A second service B was released in February 2012. Both games are on a live service as of 2014.

In terms of content, users run their own (virtual) restaurant for Service A and(virtual) bakery for Service B. For both services, users play the games to complete missions (e.g., upgrading and decorating their virtual restaurant or bakery, hiring staffs). The main driver to induce users to keep playing (i.e., decorating outlets) is their increased satisfaction with creating their own restaurant or bakery. Many social network services, including social network games, contain similar contexts (e.g., uploading friends' profiles, exchanging messages, posting content).

Table 3.1 User Statistics in Services A and B

	Service A	Service B
#of user	29,299	17,433
# of multiusers	4,753(16.2%)	4,753(27.2%)

As Table 3.1 shows, approximately 16% of Service A's users and 27% of Service B's users

enjoy both services. Of particular interest is the social networking structure embedded in each service. Users have a set of friends playing the game who are also their Facebook or Twitter friends.

Figure 3.2 Histograms for the Number of Visits in Networks A and B

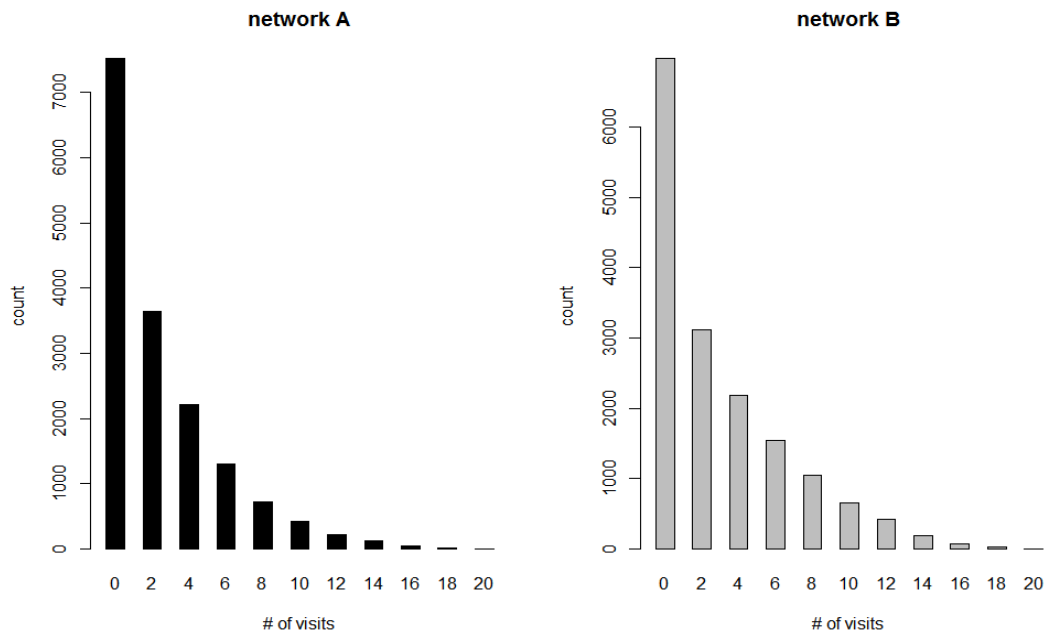


Figure 3.2 visually shows the histograms of daily and individual visits on network services A and B. We find a considerably large number of zero visits in both network services. Specifically, the average of visits on network A is about 2.8 and its standard deviation is 3.2. On the other hand the average on network B is about 3.4 and its standard deviation is 3.7.

Table 3.2 Social Network Structures in Services A and B

	Service A	Service B
Average # of friends	16.27	12.97
St. dev# of friends	19.68	15.19
Max# of friends	152	129
Average # of overlapping friends	6.84	6.84
St. dev# of overlapping friends	8.15	8.15
Max# of overlapping friends	72	72

Table 3.2 shows the summary statistics of the overlapping and nonoverlapping friends of multiusers in their social networks in Services A and B. Note that because both services are managed by the same company, all users are registered in both games with the same IDs (user names and email addresses). This enabled us to identify all the friends in users' networks as well as the multiusers. As such, we assume that users are able to recognize their friends across the different social networks of Services A and B.

According to Table 3.2, the users of Services A and B have large friend networks (average: 16 for A, 12 for B), about half of which are overlapping friends in both services (7 for A and B). The data also include the information of direct communication activities; users can exchange gifts and texts with their friends. Observation of the direct communications enables us to infer users' social relationships. The format of messages in this service is similar to that of other social network platforms (e.g., small pop-up windows). Users can receive gifts in unlimited amounts, and the gifts are not related to users completing missions in the service.

Figure 3.3 Histograms for the Number of Overlapping and Nonoverlapping Friends

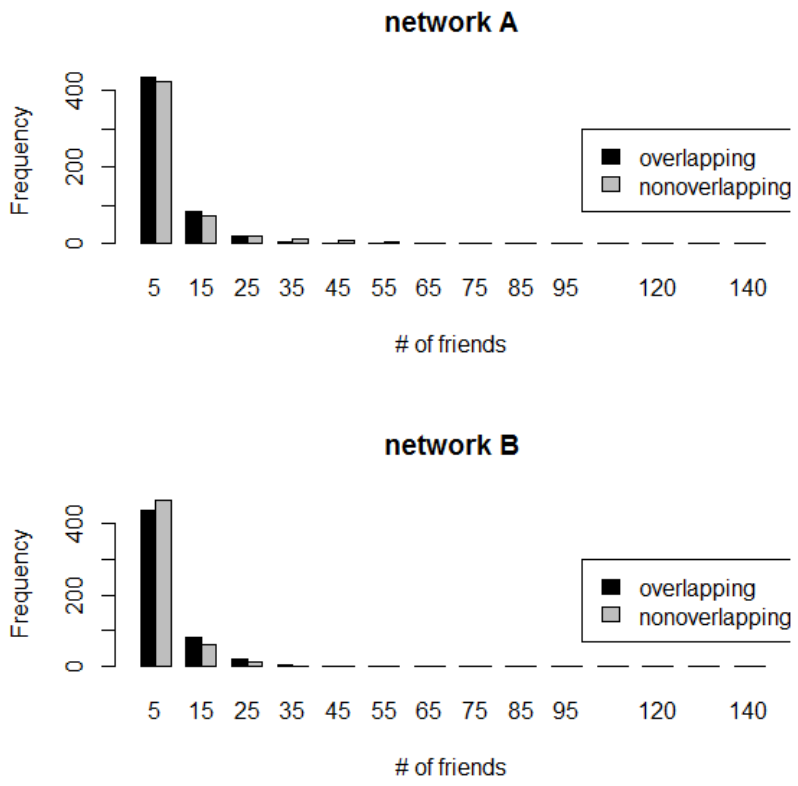


Figure 3.3 visually depicts the distributions of the numbers of overlapping and nonoverlapping friends in Services A and B. It shows considerable dispersion of the distributions. Majority of users have only few overlapping and nonoverlapping friends, thus small social networks. Interestingly, 8.5% of users have no overlapping friends, while 32.5% and 19.6% of users have no nonoverlapping friends in Services A and B respectively.

Table 3.3 reports the summary statistics of social communicative activities with overlapping and nonoverlapping friends of users. According to Table 12, users communicate more with overlapping than nonoverlapping friends in Service B. but they communicate

similarly with both groups in Service A. Though preliminary reasoning, the statistics imply that different roles or relationships exist with overlapping friends in Services A and B. In summary, the large portion of overlapping friends in Services A and B and users' different communication activities between types of friends enable us to pursue our research objectives.

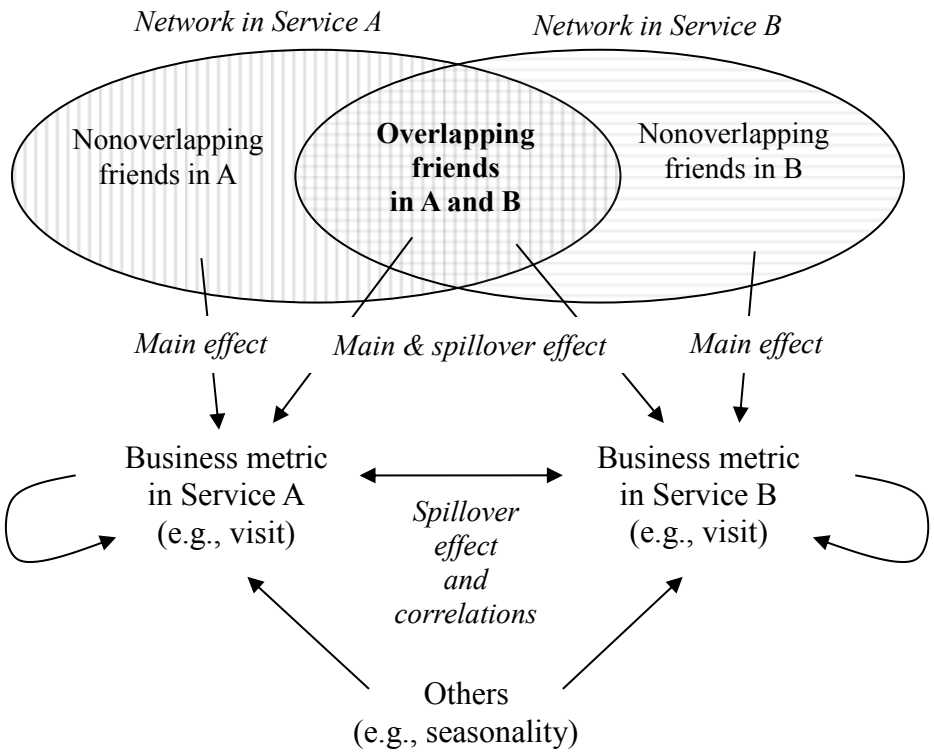
Table 3.3 Differences between Nonoverlapping/Overlapping Friends in Services A and B

	Service A		Service B	
	With Nonoverlapping Friends	With Overlapping Friends	With Nonoverlapping Friends	With Overlapping Friends
Average # of gifts or texts exchanged / day	0.92	0.81	0.45	0.67
St. dev# of gifts or texts exchanged / day	3.90	3.18	2.37	3.04
Max# of gifts or texts exchanged / day	129	78	86	78

3.4. Model

In this section, we develop a generalized model to investigate dependencies between multiple networks when consumers navigate Services A and B, which have different network structures but share overlapping network members. Our model intends to capture how various dependencies operate differently in users' visitation behaviors in Services A and B. Specifically, we assume that users' visitation to the service is driven by seasonality, social communications with friends from two social networks in the focal service, and spillover effects from one network to another. Figure 3.4 depicts our modeling framework.

Figure 3.4 Modeling Framework



3.4.1. Variables

We first define variables used in an integrated visit model in Services A and B that we develop in the next section. To begin, we create variables of social communications of a peer of users i and j in social network service N = A, B. This variable includes a weighted average of the number of communications with friend j that occurs now and in the past with weight $\rho_{1\tau}$.

$$Social_{ijt}^{N} = \sum_{\tau=0}^{T} \rho_{1\tau} \cdot Comm_{ijt-\tau}^{N}, \forall N = A, B,$$

where

$Comm_{ijt-\tau}^{N}$ = the number of messages and gifts that users i and j exchange in social network N = A, B on day t and we specify $\rho_{1\tau}$ as:

$\rho_{1\tau} = \frac{exp(-\tau\omega_1)}{\sum_{\tau=0}^{T} exp(-\tau\omega_1)}$.

In this variable specification, the implication of $\rho_{1\tau}$ is a carryover effect of past communications, similar to loyalty variables (e.g., Guardagni and Little 1984). For example, if ω_1 is large, users weight communications that occurred recently more heavily than those that occurred in the past.

Note that users' past experiences in Services A and B may influence their current behaviors. This is partly explained by state dependence (Guardagni and Little 1984; Seetharaman 2004; Seetharaman, Ainsle, and Chintagunta 1999). More important for our context of multiple social networks, users' experience in Service B can affect their current behaviors in Service A and vice versa. For example, imagine that someone is a member of both a science club and a mathematics club. Her experience in the science club may spark interest in the mathematics club or help her gain familiarity with the mathematics club because of similar contexts between the two clubs. Conversely, imagine that she has both Facebook and Twitter accounts. Her past usage on Twitter may induce her to use Facebook less because she receives similar entertaining content on Twitter. To examine this spillover effect, we develop stock variables of past experience in the services with carryover weight $\rho_{2\tau}$.

$$Past.Experience_{it}^{N} = \sum_{\tau=1}^{T} \rho_{2\tau} \cdot Visit_{it-\tau}^{N}, \forall N = A, B,$$

where

$Visit_{it}^{N}$ = the number of visits of user i to service N = A, B on day t[10] and

$\rho_{2\tau} = \frac{exp(-\tau\omega_2)}{\sum_{\tau=1}^{T} exp(-\tau\omega_2)}$.

Finally, we also take into consideration possible effects not related to the contexts of Services A and B. To do so, we introduce seasonality variables, such as weekends and holidays.

$$Season_t = [Weekend_t, Holiday_t],$$

where

$Weekend_t$= 1 if day t is a weekend and 0 if otherwise and

$Holiday_t$= 1 if day t is a holiday (e.g., Christmas, New Year's Day) and 0 if otherwise.

3.4.2. Visit Model

We propose a visit model to examine how overlapping friends affect visitation behaviors in the services. Let the number of visits of user i to Service A on day t, $Visit_{it}^{A}$, follow a Poisson distribution with mean λ_{it}^{A}:

$$Visit_{it}^{A} \sim Poisson(\lambda_{it}^{A}).$$

We assume that λ_{it}^{A} incorporates seasonality effects, past experience in Services A and B,

[10] The visit measure in our data includes activities that involve some activities related to the game (e.g., leveling, carrying out missions, upgrading). Thus, the visit measure does not include visits in which customers do not perform any actions related to the game.

social communications from friends connected in Service A, and social communications from friends connected in Service B who are also friends in Service A. Following this notion, we model λ_{it}^{A} as follows:

$$
\begin{aligned}
ln(\lambda_{it}^{A}) &= \beta_{i0}^{A} + Season_t \cdot \beta_i^{A} && \text{: effects without networks (A)} \\
&+\gamma_{i1}^{A} \cdot Past.Experience_{it}^{A} && \text{: effects of past experience in Service A (B)} \\
&+\gamma_{i2}^{A} \cdot Past.Experience_{it}^{B} && \text{: spillover effects of past experience in Service B (C)} \\
&+ \sum_{j \in OF_i} \gamma_{ij3}^{A} \cdot Social_{ijt}^{A} + \sum_{k \in NF_i^A} \gamma_{ik4}^{A} \cdot Social_{ikt}^{A} && \text{: effects of communications with overlapping and nonoverlapping friends in the network of Service A (D)} \\
&+ \sum_{j \in OF_i} \gamma_{ij5}^{A} \cdot Social_{ijt}^{B} \\
&+ \sum_{j \in OF_i} \gamma_{i6}^{A} \cdot Social_{ijt}^{A} \cdot Social_{ijt}^{B} && \text{: spillover effects of social communication with overlapping friends from the network of Service B (E)} \\
&+\varepsilon_{it}^{A} && \text{: unobserved component (F)}
\end{aligned}
\tag{8}
$$

where

OF_i = a set of overlapping friends of user i and

NF_i^A = a set of nonoverlapping friends of user i in network A.

In Equation (8), the visiting behavior is driven by six main components: (A) the effect without social effects (i.e., baseline visit tendency and seasonality), (B) past experience in the

focal social network service, (C) spillover effect of past experience in the other social network service, (D) effects from communications with overlapping and nonoverlapping friends in the network of the focal service, (E) spillover effects from communications with overlapping friends that occurred in the network of the other service, and (F) an unobserved component correlated over the visit models in Services A and B. We can specify the visit model for Service B by interchanging superscripts A and B in Equation (8).

3.4.3. Markov Random Field Variable Selection Specification

The Poisson link function in Equation (8) includes three types of spillover effects of communications with user i and his or her connected friends across networks A or B on his or her visitation to a social network service: $\sum_{j \in OF_i} \gamma_{ij3}^A \cdot Social_{ijt}^A + \sum_{k \in NF_i^A} \gamma_{ik4}^A \cdot Social_{ikt}^A$ in (D) and $\sum_{j \in OF_i} \gamma_{ij5}^A \cdot Social_{ijt}^B$ and $\sum_{j \in OF_i} \gamma_{ij6}^A \cdot Social_{ijt}^A \cdot Social_{ijt}^B$ in (E). These terms are difficult to handle because of their possibility of high dimensionality.

More specifically, the dimensions of parameter spaces, $\{\gamma_{ij3}^A \forall\, j \in OF_i\}$, $\{\gamma_{ij4}^A \forall\, j \in NF_i^A\}$, $\{\gamma_{ij5}^A \forall\, j \in OF_i\}$ and $\{\gamma_{ij6}^A \forall\, j \in OF_i\}$ are dependent on the dimensions of OF_i, a set of overlapping friends in networks A and B, and NF_i^A, a set of nonoverlapping friends in network A. Recall that the summary statistics in Table 3.2 report that the average of OF_i is approximately 7, and its maximum is 72. The averages of NF_i^A and NF_i^B are approximately 9 and 6, and their maximums are 80 and 57 in networks A and B, respectively. These high dimensions of OF_i and NF_i^A pose a great challenge of estimating equally high-dimensional parameter spaces $\{\gamma_{ij3}^A \forall\, j \in OF_i\}$, $\{\gamma_{ik4}^A \forall k \in NF_i^A\}$, $\{\gamma_{ij5}^A \forall\, j \in OF_i\}$, and $\{\gamma_{ij6}^A \forall\, j \in OF_i\}$. To overcome this, we apply the Markov random field (MRF) variable selection method (Letovsky and Kasif 2003, Wei

and Li 2007).

We first take into consideration the network with overlapping friends, OF_i, which involves the three sets of terms, that is, $\{\gamma_{ij3}^A \forall\, j \in OF_i\}$ in (D) and$\{\gamma_{ij5}^A \forall\, j \in OF_i\}$ and $\{\gamma_{ij6}^A \forall\, j \in OF_i\}$ in (E) in Equation (8). For ease of presentation, we begin with one parameter γ_{ijh}^A with one friend j, γ_{ijh}^A for $h = 3, 5, 6$, by decomposing it as follows:

$$\gamma_{ijh}^A = \gamma_{ih}^A \cdot I_{ij}^{OF}, \text{ where } h = 3, 5, 6 \text{ and } j \in OF_i. \tag{9}$$

We assume that $I_{ij}^{OF}$is a random variable that takes the value of 0 or 1, to determine whether its corresponding term, $Social_{ijt}^A$, $Social_{ijt}^B$, or $Social_{ijt}^A \cdot Social_{ijt}^B$, is included in the Poisson link function (Equation 8). Most important, we assume that I_{ij}^{OF} is not a priori independent of $I_{ij'}^{OF}$ $\forall j' \neq j \in OF_i$. To do so, we assume that I_{ij}^{OF} isa Markov process given$I_{ij'}^{OF}$ $\forall j' \neq j \in OF_i$. This explicitly incorporates other $I_{ij'}^{OF}$ $\forall j' \neq j \in OF_i$, in the specification of I_{ij}^{OF}.

$$I_{ij}^{OF} | I_{ij'}^{OF} \sim Bernoulli\left(\pi_{ij,(j')}^{OF}\right) \forall j' \neq j \in OF_i$$

where

$$ln\left(\frac{\pi_{ij,(j')}^{OF}}{1 - \pi_{ij,(j')}^{OF}}\right) = \phi_0^{OF} + \phi_1^{OF} \sum_{j' \neq j \in OF_i} I_{ij'}^{OF}. \tag{10}$$

Next, we turn to γ_{ik4}^A, which is associated with the network of nonoverlapping friends in

Service A, NF_i^A. In the same spirit as Equation (10), we decompose γ_{ik4}^A into $\gamma_{i4}^A \cdot I_{ik}^{NF}$ and then assume that $I_{ik}^{NF} | I_{ik'}^{NF} \sim Bernoulli\left(\pi_{ik,(k')}^{NF}\right) \forall k' \neq k \in NF_i^A$. This yields the following logit representation:

$$ln\left(\frac{\pi_{ik,(k')}^{NF}}{1-\pi_{ik,(k')}^{NF}}\right) = \phi_0^{NF} + \phi_1^{NF} \sum_{k' \neq k \in NF_i^A} I_{ik'}^{NF}. \tag{11}$$

The MRF model is useful for both computation and interpretation. First, it allows for resolving the high dimensions of the network data by making $I_{ij}^{OF} \forall j \in OF_i$and $I_{ik}^{NF} \forall k \in NF_i^A$ influential enough to be included in the Poisson link function in Equation (8). Thus, the dimensions of terms (D) and (E) can be reduced so as to render the estimation empirically feasible. Second, from a methodological perspective, the MRF model enables us to capture local dependencies. Again, note that the main objective of our study is to discover dependencies that may exist across multiple social network platforms. This should be preceded by the local dependencies within a network, which in general can be captured by correlations among parameters $\left[\gamma_{i1h}^A, \dots, \gamma_{ijh}^A\right]$ for $h = 3, 5, 6$, where $1, \dots, J \in OF_i$ and $[\gamma_{i14}^A, \dots, \gamma_{iK4}^A]$, where $1, \dots, K \in NF_i^A$. Because the dimensions of sets OF_i and NF_i^A are large, the number of parameters in their correlation terms quadratically increase. In addition to the computational burden, interpretations of the correlation terms become substantially more difficult. The MRF model defines their local dependencies in a simple and interpretable manner. That is, ϕ_1^{OF} and ϕ_1^{NF} explicitly capture the local dependencies. For example, if ϕ_1^{OF} and ϕ_1^{NF} are positive, this implies positive dependencies of influences among both overlapping network members and

nonoverlapping members on users' visitation behavior.

3.4.4. Interdependency Sources Across Network Services

First, we introduce the correlations of visiting behaviors of user i across social network platforms A and B, which cannot be observed by researchers. To do so, we assume that the vector of unobserved components in visit models for Services A and B, $[\varepsilon_{it}^{A}, \varepsilon_{it}^{B}]'$, evolve together[11] and specify that $[\varepsilon_{it}^{A}, \varepsilon_{it}^{B}]'$ follows a multivariate normal distribution as follows:

$$\begin{bmatrix} \varepsilon_{it}^{A} \\ \varepsilon_{it}^{B} \end{bmatrix} \sim MVN\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix} \right) \tag{12}$$

Second, to introduce correlated heterogeneity across customers over networks A and B, we assume that user i's parameter space $[\beta_{i0}^{A}, \beta_{i}^{A}, \gamma_{i1}^{A}, \dots, \gamma_{i6}^{A}, \beta_{i0}^{B}, \beta_{i}^{B}, \gamma_{i1}^{B}, \dots, \gamma_{i6}^{B}]'$ of the visit models of both Services A and B together comes from a multivariate normal distribution with population mean Ψ and covariance Σ as follows:

$$\begin{aligned} &[\beta_{i0}^{A}, \beta_{i}^{A}, \gamma_{i1}^{A}, \gamma_{i2}^{A}, \gamma_{i3}^{A}, \gamma_{i4}^{A}, \gamma_{i5}^{A}, \gamma_{i6}^{A}, \beta_{i0}^{B}, \beta_{i}^{B}, \gamma_{i1}^{B}, \gamma_{i2}^{B}, \gamma_{i3}^{B}, \gamma_{i4}^{B}, \gamma_{i5}^{B}, \gamma_{i6}^{B}]' \\ &\qquad \sim MVN(\Psi', \Sigma). \end{aligned} \tag{13}$$

In this modeling specification, we overlay two types of dependencies of visitation in Services A and B. The first source is the coincidence of visitations to Services A and B. This is

[11] Randomness generated independently in visits to Services A and B can be captured by the Poisson distributions themselves.

captured by covariance term σ_{12} of the multivariate distribution in Equation (12) as applied in cross-category analysis (e.g., Ansari, Mela and Neslin 2008, Chib, Seetharaman and Strijnev 2002, Manchanda, Ansariand Gupta 1999, Ma, Seetharaman and Narasimhan 2012). The second source is the correlation between heterogeneity—for example, if a customer tends to be sensitive to social communications during his or her visitation to both Services A and B or if the correlation between individual baseline visit tendencies drives visitation to Services A and B. These dependencies are captured in Σ in Equation (13). This is conceptually similar to correlated marketing-mix sensitivities (Ainslie and Rossi 1998) or correlated brand preferences (Duvvuri, Ansari, and Gupta 2007, Hansen, Singh, and Chintagunta 2006, Singh, Hansen, and Gupta 2005) in the context of cross-category analysis.

These two sources have been well applied in other cross-choice contexts, such as multi-category choice models and multichannel migration models (e.g., Ansari, Mela, and Neslin 2008). In our specification, we allow for two types of spillover effects from the other social network service. The first is the spillover effect of the past experience in the other network. That is, past experience in one network increases users' familiarity with the other network, making the other network more or less enjoyable. In this sense, the underlying meaning of these effects captured by γ_{i2}^{A} has conceptual similarity to complementarity and substitutability, which are important dependency sources in multi-category analysis (e.g., Manchanda, Ansariand Gupta 1999; Niraj, Padmanabhan and Seetharaman 2008); however, these effects differ in that our concept does not assume that they occur in simultaneous incidences.

As addressed previously, the second important source that is unique only in the context of multiple social network analysis comes from the structures of multiple social networks. To address this, we introduce a spillover effect from other network activities owing to the

overlapping friends in multiple social networks. The rationale behind this is that the effects of communications with overlapping friend j in one network, $Social_{ijt}^{B}$, may spread across networks. We capture this with two terms: γ_{i5}^{A} captures the main effect of the spillover of communications with overlapping friends that occurred in the other network platform, and γ_{i6}^{A} measures the effect of interaction of communications with overlapping friends that user i had in Services A and B. The signs of these two terms help explain whether overlapping friend j is synergistic or redundant in each network platform.

3.4.5. Granger Causality Test

The issue of endogeneity of peer-to-peer influence in social network analysis has been raised in literature (Rogowski and Sinclair 2012, Goldsmith-Pinkhama and Imbensbc 2013). However, this is difficult to empirically verify causal relationships from peer-to-peer interactions (Manski 1993) due to various issues such as presence of homophily (McPherson, Smith-Lovin and Cook 2001) and simultaneity (Godes and Mayzlin 2004). In our context, a major concern is the interpretation of estimates $[\gamma_{ij1}^{A}, \dots, \gamma_{ij6}^{A}]$ and $[\gamma_{ij1}^{B}, \dots, \gamma_{ij6}^{B}]$ in Equation (8). Thus, the issue of possible network endogeneity needs to be considered in our proposed model; it is essentially the causal relationship between users' visitation behaviors and their social communications with friends.

To verify this issue in our models, we test the null hypothesis that visitation to network services does not (Granger-) cause communicative activities among users. To do so, we conduct the Granger Causality test between two variables, $Social_{ijt}^{N}$ and$Visit_{it}^{N}$ $(N = A, B)$ using their 6 lag terms and seasonality variables, $Season_t$. The results of the Chi-square tests for Service A

and B provide an evidence of the rather weak causal relationship from visitation behavior to social communicative activities, That is, we cannot reject the null hypothesis of the causal relationship from visitation behavior to social communicative activities at the 0.1 significant level in both Services A and B.

3.5. Estimation Results and Parameter Inference

For the empirical application, we sample 500 random users who enjoy both Services A and B during the data period from the company's database. We estimate the proposed models across social network services simultaneously using WinBUGS, in which we discard the first 40,000 samples for burning and use the last 30,000 samples for estimation. We assume diffuse density prior distributions of parameters and monitor the time-series plots across the Markov chain Monte Carlo iterations to examine the convergence of the posterior distribution.

3.5.1. Model Validation

We compare the results of our proposed model with those of two benchmark models, omitting variables that operate within and across networks. We first turn off all variables related not only to the focal network but also to the other network for Benchmark 1 and then turn off all variables related to the other network (i.e., spillover variables) for Benchmark 2. Last, our proposed model contains all variables specified in Equation (8).

For in-sample validation, we use data of roughly four weeks, from December 5 to December 28, 2012, to compute the deviance information criterion (DIC) of the various models. In addition, for out-of-sample validation, we holdout about two weeks, from December 29, 2012, to January 11, 2013, and compute the mean absolute error (MAE) of observed visits and

predicted visits of each customer using the estimated individual parameters for in-sample validation as follows:

$$MAE = \sum_{N=A,B} \left(\frac{1}{I} \sum_{i=1}^{I} \left(\sum_{t=1}^{T} \frac{1}{T} \left| Visit_{it}^{N} - \widehat{Visit}_{it}^{N} \right| \right) \right),$$

where$\widehat{Visit}_{it}^{N}$is the prediction of the number of visits of userion day t in network N for customer i=1,...,I; day t=1,...,T; and N=A,B.

Table 3.4 Model Validation

Variable		Model		
		Benchmark1	Benchmark2	Proposed
Interceptor		√	√	√
Weekend		√	√	√
Holiday		√	√	√
Past experience in the focal network			√	√
Communication with overlapping friends			√	√
Communication with nonoverlapping friends			√	√
Spillover from another network	Past experience			√
	Communications with overlapping friends			√
	Interaction of communications with overlapping friends			√
DIC (In-Sample)		75,626	72,993	72,703
MAE (Out-Sample)		2.59	2.10	2.08

√ if present

According to the validation results in Table 3.4, it is no surprise that Benchmark 1 performs the worst, in that it has the highest DIC and the lowest prediction power (DIC=75,626, MAE=2.59). Next, Benchmark 2 shows a lower DIC and a lower error of predicted visits than Benchmark 1 (DIC=72,993, MAE=2.10). Our proposed model, which incorporates all types of spillover effects stemming from overlapping friends across Services A and B, has the lowest DIC and the lowest prediction error of number of visits and thus performs the best (DIC=72,703, MAE=2.08).

Note that given the incremental model fit between Benchmarks 1 and 2 (i.e., difference of DICs and MAEs) and that between Benchmark 2 and the proposed model, the effects of communications in the focal network platform contribute better to model fit than the effects of communications that occur in the other network. In addition, our finding provides evidence that these dependencies exist across network services A and B. Our specification of the dependencies between two networks helps explain the visiting behaviors of users in the social network services.

We also estimate two more benchmark models to verify the contribution of the modeling components. Benchmark 3 is the model which does not incorporate heterogeneity of parameters in Equation (13) and Benchmark 4 is the model which incorporates a stochastic variable selection method (Chipman, Edward, and McCulloch 2001) instead of Markov Random Field Variable Selection Method specified in Equations (10) and (11), which is as follows.

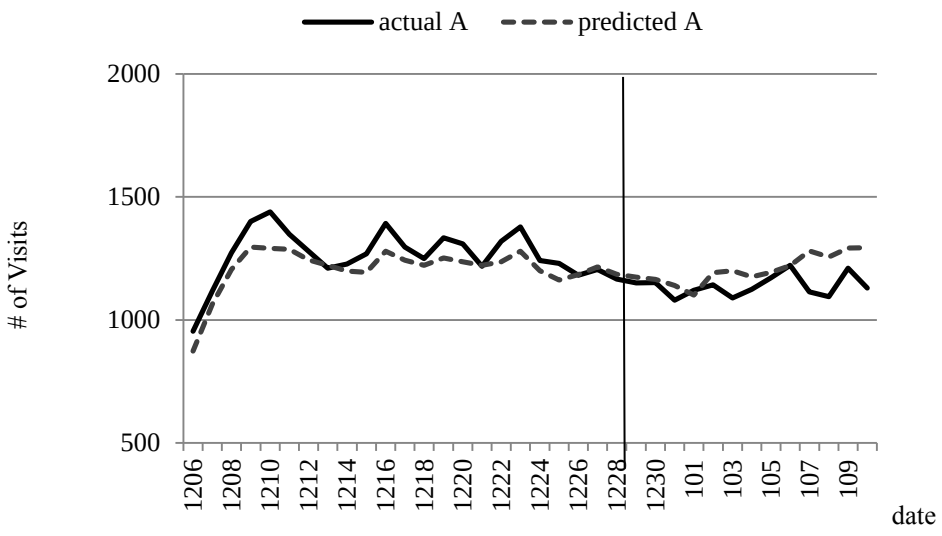
$$I_{ij}^{OF} \sim Bernoulli\left(\pi_{ij}^{OF}\right)$$

, where $\pi_{ij}^{OF} \sim beta(a_{ij}^{OF}, b_{ij}^{OF})$

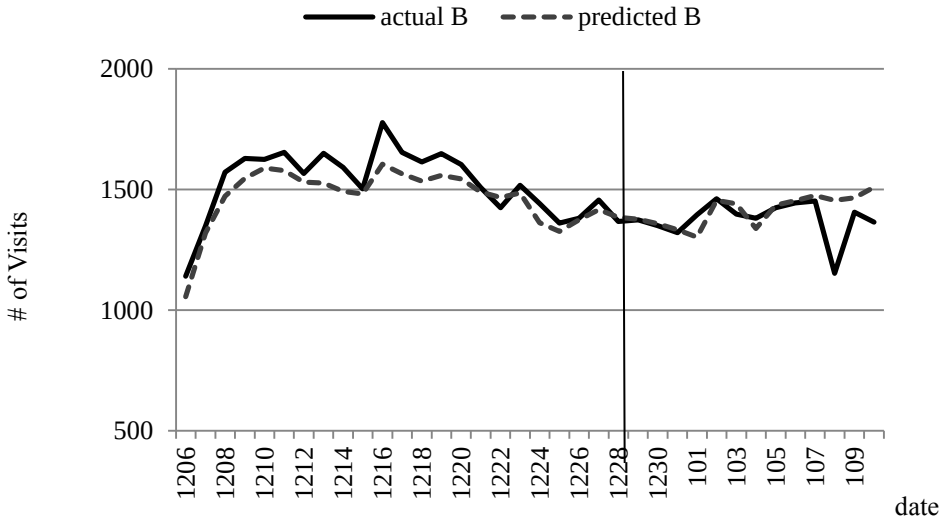
Thus, the main difference between our proposed model and Benchmark 4 is prior assumption of interdependencies among friends j=1, ..., J in user i's network. That is, I_{ij}^{OF} is not conditional on $I_{ij'}^{OF}$, $j' \neq j$ any more. We find that heterogeneity improves model fit as marginal Log-Likelihood of Benchmark 3 is -35,140,while that of our proposed model is -33,590. However, we are not able to find notable difference between our proposed model and Benchmark 4 model as marginal Log-Likelihood of Benchmark 4 is -33,560. Even though our proposed model does not dramatically improve the model fit, this analysis enables us to find out important information of interdependencies among friends in social networks, which lead to managerial implications on social network marketing strategies. We will discuss it more in the section that follows.

Last, in order to assess the performance of the proposed model, we predict the weekly number of visitations on Services A and B over the data period. Figure 3.5 compares the predicted posterior means of the outcome measures to the corresponding observed values, aggregated across the users. As shown, our model tracks the general patterns of visitations. Using the results, we compute the aggregate numbers of visitations over the data period. During the calibration (forecasting) period, the model predicts the aggregate number of visitations at 94.5% and 97.6% accuracy rates, respectively in Services A and B. This model fit results indicate that our proposed model can accurately track the visitation patterns of users.

Figure 3.5 Model Predictions on Daily Visitations on Network Services A and B



Network A



Network B

3.5.2. Estimation Results and Parameter Inference

We begin by describing the inferences of the parameters of our integrated visit model across Services A and B based on their posterior distributions in Table 3.5.

First, we report the estimated parameters of communications that occur within a network. In both networks, the communication effects within a network are positive and significant. More important, the effects of overlapping friends (γ_{i3}^{A}= .91 andγ_{i3}^{B}= 1.37) are higher than those of nonoverlapping friends (γ_{i4}^{A}= .39 and $\gamma_{i4}^{B} = .96$). This implies that overlapping friends are more influential than nonoverlapping friends on users' visitation to a network, thus supporting H1a-b.

Second, we turn attention to spillover effects from the other network. The estimated spillover parameters from past experience in another network ($\gamma_{i2}^{N}, N = A, B$) are positively significant in both network Services A and B, while their magnitudes are smaller than the effects from past experience within the focal network ($\gamma_{i1}^{N}, N = A, B$). Positive and significant γ_{i2}^{N} ($N = A, B$) conceptually explains complementarity between two social network services because users' experience in another network is likely to help them enjoy the focal service more.

As our purpose in this study is to investigate the effects of social communications with overlapping friends that occurred in another social network, we pay attention to the spillover effects of social communications with overlapping friends. These spillover effects are specified in two ways: directly (γ_{i5}^{N},$N = A, B$) and through interactions ($\gamma_{i6}^{N}, N = A, B$) in Equation (8). The direct spillover effects from communications with overlapping friends (γ_{i5}^{N}, $N = A, B$) are positive and significant in both networks. This indicates that social communications with overlapping friends that occur in one network are likely to increase visits to the other network.

Table 3.5 Estimation Result of the Visit Model

Variable		Parameter ($\circ = A, B$)	Service A		Service B	
			Mean	95% CI	Mean	95% CI
Interceptor		β_{i0}°	-1.10*	-1.31, -0.94	-1.75*	-1.98, -1.50
Weekend		β_{i1}°	-0.05	-0.16, 0.04	-0.13	-0.30, 0.01
Holiday		β_{i2}°	-0.18*	-0.30, -0.04	-0.36*	-0.56, -0.15
Past experience in the focal network		γ_{i1}°	3.71*	3.36, 4.08	4.06*	3.72, 4.41
Communication with overlapping friends		γ_{i3}°	0.91*	0.60, 1.22	1.37*	0.74, 1.84
Communication with nonoverlapping friends		γ_{i4}°	0.39*	0.03, 0.67	0.96*	0.57, 1.28
Spillover from another network	Past experience	γ_{i2}°	0.60*	0.35, 0.85	0.36*	0.07, 0.71
	Communications with overlapping friends	γ_{i5}°	0.72*	0.32, 1.13	0.45*	0.15, 0.72
	Interaction of communications with overlapping friends	γ_{i6}°	-0.48*	-0.87, -0.17	-0.43*	-0.84, -0.06
Coincidental visits		$\sigma_{12}/\sigma_1\sigma_2$	0.91			

*95% CI does not contain zero.

However, the interactions between social communications with overlapping friends that occur in networks A and B ($\gamma_{i6}^{N}, N = A, B$) are negative and significant. These estimated coefficients γ_{i5}^{N} and γ_{i6}^{N} ($N = A, B$) allow us to infer that the social communications with overlapping friends in one network that spill over to the other network may increase visitation to the other network service, but it may also reduce the influence of social communications with overlapping friends in the other network. This analysis finds support for H2a-b.

However, these positive main and negative interaction effects of the social communications

with overlapping friends may prevent us from answering our main research question whether the effect of social communications occurring in different social networks is synergistic or redundant. To answer this question, we investigate the overall effects of the overlapping friends using simulation studies, which we describe subsequently.

A more important point is the asymmetric spillover effects of overlapping friends between networks. We found that the social communications with overlapping friends in networks of Service B increase the number of visitations to Service A (γ_{i5}^{A}=.72) more than the corresponding effect of Service A to Service B (i.e., social communications with overlapping friends in networks of Service A increase visitation to Service B (γ_{i5}^{B}= .45)). As addressed in the data description, Service A was introduced earlier and became more popular than Service B. Indeed, the company launched Service B largely because of the overwhelming success of Service A. The intuition underlying the asymmetric effects of the overlapping friends in networks of Services A and B pertains to the difference in popularity or total size of user pools in A and B. While this reasoning is difficult to generalize to other contexts of social network services (e.g., Facebook, Twitter), it still is of interest to find asymmetry of the spillover effects. In summary, overlapping friends are more influential than nonoverlapping friends not only within a network but also across networks. In addition, interdependencies exist between different networks, which stem from their different proportions of overlapping friends in those networks, and these interdependencies can be asymmetric across the networks. Overall, our analysis finds support for H1a-b and H2a-b.

Table 13 also reports the estimated σ_{12} in Equation (12), which captures the effect of coincidental visits in Services A and B. For easier interpretation, we present correlation $\sigma_{12}/\sigma_1\sigma_2$ instead of covariance σ_{12}. The high correlation (0.91) indicates that coincidental visits in

Services A and B are explained by some unobserved reasons. That is, the two social network services can share some similarities that were not incorporated in our model. Given that these social network services were developed in a similar way (e.g., genre and format of games) by the same company, this high correlation of the unobserved components is not surprising.

Next, we attempt to explain the inferences of covariance Σ in Equation (13), which captures dependencies that come from correlated sensitivities within and across two social network services. For easier interpretation, we transform a covariance matrix and a correlation matrix and then report it for variables of interest, $[\gamma_{i1}^{A}, \gamma_{i2}^{A}, \gamma_{i3}^{A}, \gamma_{i4}^{A}, \gamma_{i5}^{A}, \gamma_{i6}^{A}]$ and $[\gamma_{i1}^{B}, \gamma_{i2}^{B}, \gamma_{i3}^{B}, \gamma_{i4}^{B}, \gamma_{i5}^{B}, \gamma_{i6}^{B}]$, to emphasize on sensitivities correlated across networks A and B.

Table 3.6 Correlation Matrix of Coefficients Across Networks A and B

Network A \ Network B			γ_{i4}^{B}	γ_{i3}^{B}	γ_{i1}^{B}	γ_{i2}^{B}	γ_{i5}^{B}	γ_{i6}^{B}
Communication with nonoverlapping friends		γ_{i4}^{A}	0.11	0.34	0.30	0.24	0.37	-0.35
Communication with overlapping friends		γ_{i3}^{A}	-0.06	0.25	0.19	0.65	0.39	-0.41
Past experience in the focal network		γ_{i1}^{A}	0.12	0.41	0.39	0.62	0.40	-0.47
Spillover from another network	Past experience	γ_{i2}^{A}	0.75	0.85	0.72	0.60	-0.61	-0.61
	Communications with overlapping friends	γ_{i5}^{A}	0.67	0.67	-0.26	0.26	-0.30	-0.30
	Interaction of communications with overlapping friends	γ_{i6}^{A}	-0.44	-0.44	-0.29	-0.42	0.48	0.48

According to Table 3.6, correlations among coefficients are mostly positive and rather small. In contrast, correlations with spillover coefficients ($\gamma_{i2}^{A}, \gamma_{i5}^{A}, \gamma_{i6}^{A}$ and $\gamma_{i2}^{B}, \gamma_{i5}^{B}, \gamma_{i6}^{B}$) are either large or negative. In particular, correlations with γ_{i6}^{A} and γ^{B} (and γ_{i6}^{B} and γ^{A}) are mostly

negative, which implies that as the interaction effects of communications with overlapping friends that occur in different network platforms decrease, other effects of social communications not only within but also across networks tend to be increasing.

In addition, most correlations with spillover from past experience in the other network (γ_{i2}^{A} andγ^{B}; γ_{i2}^{B} and γ^{A}) are positive and higher than other correlations. That is, as users put more weight on past experience in another network service, they also put more weight on other effects that exist within and across networks. As a result, these unique findings suggest that these correlated sensitivities across networks need to be included when modeling users' visitations to multiple social network platforms.

Table 3.7 Estimation Results of Carryover and Markov Random Field (MRF)

	Parameter			Mean	95% CI
Carryover Effect	past communication		ω_1	2.03*	1.25,2.71
	past visits		ω_2	-0.39*	-0.50,-0.28
Markov Random Field (MRF)	Among overlapping friends in a network	Interceptor	ϕ_0^{OF}	-1.61*	-1.91,-1.23,
		Local dependency	ϕ_1^{OF}	0.21*	0.15,0.26
	Among nonoverlapping friends in a network	Interceptor	ϕ_0^{NF}	-3.41*	-4.23,-2.61
		Local dependency	ϕ_1^{NF}	3.46*	2.35,4.81

*95% CI does not contain zero.

Finally, Table 3.7 reports the estimation results of carryover parameters of communications and visits and also for the MRF models. First, for empirical application, we apply carryover

effects of four days. The reparameterized carryover of past social communications, ω_1, is 2.03. This indicates that users put most weight only on current communications. Conversely, because the reparameterized carryover of past visits to the service, ω_2, is –.39, the weights for visitations that occurred one to four days ago are .52, .27, .14, and .07. Thus, while the social communications that occurred in the past are hardly influential, the past experience with the services has more gradual effects on recent visitation to the services.

Second, the estimation results of the MRF model support the dependencies that exist locally among sets of overlapping or nonoverlapping friends within a network. This is captured by ϕ_1^{OF} among overlapping friends in Equation (10) and ϕ_1^{NF} among nonoverlapping friends as specified in our MRF specifications in Equation (11). Both estimated coefficients are positive and significant. These results imply that positive local dependencies exist among both nonoverlapping and overlapping friends. In other words, influential nonoverlapping or overlapping friends are more likely to be included in the model. Of particular interest is that these positive interdependencies are much stronger among nonoverlapping friends (ϕ_1^{NF}=3.46) than overlapping friends (ϕ_1^{OF}=0.21). In addition, we found considerable heterogeneity of influence of friends, which is captured by σ_j^2=4.25.

3.6. Managerial Policy Simulation

The many popular social network platforms today (e.g., Facebook, Twitter, YouTube, LinkedIn) have enabled not only large enterprises but also small retailers to actively host their business on multiple social network sites. Especially in the mobile game industry, most firms run several social network games together because of the short time and low cost of developing and

managing a game (e.g., Zynga was running 24 social network games as of 2013). This means that managing customers across multiple social network platforms can be challenging for firms. To show how marketing managers can take advantage of our proposed model regarding their business problems, we conducted a hypothetical simulation based on the estimations in the previous sections. We assumed the following business scenario.

Suppose that a firm is running two social network platforms, A and B. A key aspect in managing customers in social network platforms is encouraging them to engage more in social interactions(see Godes and Mayzlin 2004, Trusov, Bucklin, and Pauwels 2009). For example, a traditional way of doing so is inducing customers to create larger networks by adding more friends (e.g., "People You May Know" in LinkedIn), and many social network sites have recently pushed customers into interacting with existing friends by sending SMS (e.g., Facebook Chat via SMS). Thus, we assume that the firm is using widely practiced promotion tools to encourage customer engagement in either the A or B social network service. In this case, we explore how this tactic helps the firm increase visitation of users, in conjunction with our empirical application in the previous sections. Depending on the nature of a business, other tactics could include enhancing customer value/equity and/or increasing sales revenue.

Suppose that the firm stimulates users to make one more communication with nonoverlapping friends or overlapping friends in social network service A or B every 5, 4, 3 or 2 days,[12] as is similarly done in word-of-mouth communication marketing practices (e.g., SMS, notification) in online social networking platforms. To be specific about the setting, we draw a

[12] We also tried a situation that users communicate one more time every day. However, the simulated number of visits often reaches infinite. Also, given that the average of daily communications per friend is 0.27 in Service A and 0.18 in Service B in our data, encouraging one more communication with a friend every day seems unrealistic. For these reasons, we decided to exclude this case.

random number from a Poisson distribution with mean 0.2, 0.25, 0.33 or 0.5 and add it to daily communications of each of overlapping friends ($\mathrm{Comm}_{ijt}^{N} \forall j \in \mathrm{OF}_i, N = A, B$) or nonoverlapping friends ($\mathrm{Comm}_{ikt}^{N}\ \forall k \in \mathrm{NF}_i^N, N = A, B$) in Section 3.4.2.Given the data we used for estimation and the individual parameters we estimated in the previous section, we compute $\sum_{i=1}^{I} \sum_{t=1}^{T} E(\mathrm{Visit}_{it}^{A})$ and $\sum_{i=1}^{I} \sum_{t=1}^{T} E(\mathrm{Visit}_{it}^{B})$ by simulating visitation of 500 users in A and B. This setting of the hypothetical business situation enables us to investigate the different effects of communications with overlapping and nonoverlapping friends on business outcomes across multiple social network platforms. Table 3.8 reports the simulation results.

According to Table 3.8, if the firm promotes one more communication with nonoverlapping friends in social network service A every two days, visitations would increase by 11.46% in network service A and .12% in B. If the firm promotes one more social communication with nonoverlapping friends in network B every two days, visitation would increase by 7.35% in network service B and .02% in A.

Under the same situation with overlapping friends, if the firm encourages communications with overlapping friends in network A, visitations of users would increase by 14.45% in network service A and 6.77% in B, on account of the spillover effects as specified in (C) and (E) in Equation (8). Similarly, visit rates are higher—14.69% in network service B and 9.24% in A—as a result of increased communications with overlapping friends in B.

The simulation study offers two important managerial implications. First, it enables us to compare influences between overlapping and nonoverlapping friends. We find that communications with overlapping friends in one social network platform would increase users' visitation not only to the social network platform but also to another social network platform, while nonoverlapping friends have only minor spillover effects on both social network platforms.

Table 3.8 Simulation Analysis Results in Services A and B

Firm's Action		Outcome	Changes in Visits in A (%)	Changes in Visits in B (%)
One more communication every 5 days with	overlapping friends	in A	5.39	**2.48**
	nonoverlapping friends		4.25	**0.13**
	overlapping friends	in B	**3.56**	5.18
	nonoverlapping friends		**-0.02**	2.95
One more communication every 4 days with	overlapping friends	in A	6.98	**3.38**
	nonoverlapping friends		5.46	**0.17**
	overlapping friends	in B	**4.18**	6.61
	nonoverlapping friends		**0.13**	3.89
One more communication every 3 days with	overlapping friends	in A	9.16	**4.58**
	nonoverlapping friends		6.79	**-0.04**
	overlapping friends	in B	**5.95**	8.93
	nonoverlapping friends		**0.03**	4.69
One more communication every 2 days with	overlapping friends	in A	14.45	**6.77**
	nonoverlapping friends		11.46	**0.12**
	overlapping friends	in B	**9.24**	14.69
	nonoverlapping friends		**0.02**	7.35

Note: Percentages in bold indicate spillover to another network.

As the estimation results in Table 3.8 show, only overlapping friends can have spillover

effects. The reasoning is that the spillover effect of nonoverlapping friends comes only from this spillover from past experience ((C) in Equation (8)). This is the indirect spillover effect of communications from one network to another yet its effect is negligible. However, overlapping friends not only have this indirect spillover through experience but also have direct spillover effects from their communications in one network to another, as defined ((E) in Equation (8)).

Second, limiting our focus only to overlapping friends, we observe asymmetric spillover effects of overlapping friends in networks A and B. More specifically, the firm will gain more visits when promoting social communications with overlapping friends in network B than in network A. This result holds for cases when firms promote their communications every 2, 3, 4 and 5 days.

This simulation can be applied to other contexts of social networking services. For example, firms, retailers, and even small local businesses want to advertise their products or services on multiple social network services (e.g., Facebook and Twitter). Our firm can expect consequences (e.g., increase visitation of users) in B due to its action in A if there are about the same number of users of A and B who have spillover effects from their interactions from one social network platform to another. This stems from the new proposed source of interdependency between different social networks—that is, the overlapping friends, rather than the traditionally suggested interdependency sources in other contexts (e.g., cross-category/cross-channel analysis), such as coincidence or correlated sensitivities. Given our finding from the simulation study that communications with overlapping friends have larger effects on the firm (e.g., visits) than nonoverlapping friends both within and across networks, we suggest that firms should concentrate more resources on overlapping friends than nonoverlapping friends.

It is particularly crucial for firms to pay attention to the asymmetric spillover effects of

overlapping friends between different network platforms. Given limited marketing resources, the firm may decide to spend resources to stimulate communications strategically only in B, especially if it expects the spillover effects to be larger in B than A, as clearly occurs in our simulation study. In summary, our proposed modeling framework to analyze multiple social network platforms provides important implications for how firms should allocate resources across multiple network platforms to accomplish their business purposes.

3.7. Conclusions and Limitations

This work broadens the stream of research on both social networks and the decomposition of dependencies that may arise in cross-choice behaviors (e.g., cross-category/cross-channel choice behaviors). This chapter provides the first related pieces of evidence of the existence of various sources when consumers engage in multiple social networking activities. The findings shed light on customer relationship management and resource allocations when firms manage or take part in more than one social network service.

To achieve this, we proposed a visitation model that integrates activities across two networks. At the heart of our model is the discovery of various sources of dependencies that may exist in visitation activities across two social networking platforms. We find empirical evidence of dependencies such as coincidence, correlated sensitivities, and complementarity. More important, the dependency source we emphasize is the spillover effects due to the network structures across the two networks. We capture these effects using friends who overlap across networks. We find that communications with overlapping friends in one network have effects on users' visitation not only within but also across social network services.

We also demonstrate how marketing managers can make use of our proposed approach, by

conducting a simulation in which the firm engages in promotional activities on two social network services. Our simulation results reveal that if the firm encourages one more communications with nonoverlapping friends in one social network service every two days, visitations would increase by 9.4% in the network and .07% in the other network, on average. Conversely, if the firm promotes one more social communications with overlapping friends in one network every two days, visitation would increase by 14.4% in the network and 8% in the other network. In addition, the spillover effects of communications with overlapping friends are asymmetric in networks A and B. To be specific, we find that the spillover effect is larger in network service B which contains a large portion of overlapping friends than network service A. These results provide valuable implications for how to select important customers and allocate marketing resources across multiple social networks. This implication is also important to popular social network services, such as Facebook, Twitter, and LinkedIn.

As with any research, our study has some limitations, which also provide opportunities for further research. First, our data only contained static networks during our sample period, which restricted us in the application of existing networks. Thus, in our empirical application, we assumed that the effects of social networks were captured only through direct communications among members. That is, our model does not capture how networks diffuse as more friends sign in or are added (Ansari, Koenigsberg, and Stahl 2011, Trusov, Bucklin, and Pauwels 2009). Thus, it requires different data and modeling approaches to explore how overlapping and nonoverlapping friends contribute to the joint evolution of multiple social networks.

Second, our study demonstrates the effects of overlapping friends under network platforms constructed in two social network games. It is possible that users communicate offline (e.g., face to face), which may affect their behaviors in the games. In addition, considering that mobile

social network game users likely actively adopt and enjoy other social network platforms and communicate there, further research should extend our framework to include more than just network platforms (e.g., social network services of different natures). This would provide new implications for firms that use many social network platforms.

Third, our focus on the structures of two networks is limited to overlappingfriends. This aspect can be extended to a line of studies that explore the effects of network centrality properties, such as connectivity, betweenness, and closeness, on marketing phenomenon (e.g., Katona, Zubcsek, and Sarvary 2011, Stephen and Toubia 2010, Yoganarasimhan 2012). Developing new network centrality properties applicable to more than one network would provide new insights to explore the network structures across multiple social networks.

Last, we incorporate one and half months of data in our empirical application because the data provider collected individual-level data only for that given period. Thus, it remains undiscovered in this study how the social activities and friendships in multiple social networks evolve over time and how these dynamics affect business metrics in the social network platforms. We suggest that future studies designed to investigate such issues use different contexts from online social network platforms, such as workplaces, neighborhoods, schools, and so on. We leave these worthwhile directions for further research.

Chapter Four

Conclusion

This dissertation addresses two issues that arise when customers are engaged in multiple activities related to marketing. This broadens the stream of research on modeling cross-activities (e.g., cross-category/cross-social network behaviors) and discovering dependencies that may arise in them. Specifically, I propose two marketing models;

1. A dynamic model when consumers choose products of multiple related categories over time. To do so, we propose various ways to discover interdependencies across categories including balancing attributes.

2. A bivariate model when consumers visit multiple social network services. This study discovers new sources of interdependencies that exist especially across different social network services by modeling their network structures.

4.1. Findings

4.1.2. Model of Multi-Category Bundle Choices

Consumers are engaged in multi-category choices in various ways. They can buy them sequentially one by one over time (i.e., suit and shoes; smart phone and tablet; game console and game titles), or buy them as a bundle (i.e., sofa and table; jacket and skirt; chips and salsa sauce). Chapter 2 aims to integrate the streams of studies on multi category and bundle analysis. To do so, we develop a dynamic model wherein consumers choose multiple categories one by one

sequentially or choose a bundle of them simultaneously. To introduce complementarity, we employ a balance model which captures how consumers adjust attribute levels that different categories have in common. To apply our proposed model, we design conjoint study where respondents perform multiple tasks of multi-category choices and bundle choices.

We discover asymmetric effects of choices depending on types of bundles (sequential vs. simultaneous) as well as order of sequences of multi-category choices. For example, while consumers did not balance prices in a simultaneous bundle situation, they counter-balanced prices when choosing a Tablet PC first and a Smart TV later and equi-balanced prices when they choose a Smart TV first. This implication helps marketing managers sell multiple categories to consumers who already adopted other related categories.

In addition, we propose a new method to find optimal bundle prices of muti-category products when consumers are engaged in either simultaneous or sequential bundle choices. We find that optimal bundle prices vary significantly depending on combination of attribute levels across multiple related categories. This study sheds light on how retailers construct bundle products and decide their prices over time.

4.1.2. Model of Multi-Social Networks Migration

Due to huge popularity of online social network services, most consumers belong to multiple social networking services (e.g., Facebook and Twitter). Also, most firms use multiple social networking services for various marketing purposes such as acquiring and retaining customers, adverting and promoting products etc. In spite of this, most studies on social networks limit their angles to a single social network and analyze data from it. Thus, Chapter 3 expands the stream of studies on social networks to multiple social networks.

First, we discover various sources of dependencies such as coincidence, correlated sensitivities, and complementarity. Also, we find the empirical evidence on significant spillover through activities with overlapping friends across multiple networks.

In addition, we conduct a simulation study to determine how firms manage multiple social networks and detect influence networking activities. We find positive spillover of effects of networking activities on visitations across networks. More importantly, our analysis shows asymmetric spillover; a small social network services take more advantage of the spillover effects than a large one. This simulation result provides valuable implications for how to select influential groups of users across multiple social networks such as Facebook, Twitter, and LinkedIn.

To summarize, the study in Chapter 3 provides the first related pieces of evidence of the existence of various sources that operate in users' visitation on multiple social network services. The findings shed light on customer relationship management and resource allocations when firms manage multiple social network services.

4.2. Future Research Directions

4.2.1. Multi-Category Analysis and Bundle Choices

Our study in Chapter 2 integrates two streams of literature on multi-category analysis and bundle choices yet it has several limitations.

First, we consider the price decrease as a primary source of dynamic choices. That is, some consumers may be inclined to delay their purchase to take advantage of price discount in future. While it explains considerable dynamic behaviors, other sources of dynamics are likely to be involved in multi-category choices. For example, in case of durable products, budget constraint

can be a reason why some consumers have to postpone their purchases. Another reason is availability of new products (e.g., preannouncement of a new Apple Iphone). Thus, it would be an interesting extension to incorporate various sources of dynamic choice behaviors, especially important to multi-category adoption and bundle choices.

Second, due to the realistic setting of our conjoint study, we assume limited time periods when respondents adopt multi-category products dynamically and develop a dynamic setting of conjoint studies (A. Rao 2015). This new stream of conjoint studies has great potential to investigate dynamic choice behaviors of strategic forward-looking buyers and their inter-temporal preferences.

4.2.2. Social Network Analysis

Our study in Chapter 3 is the first to model customers' visitations on multiple social networks by discovering a new source of interdependency that exists only in this context; Overlapping friends. However, it is not without limitations.

The focus of Chapter 3 is focused on explaining interdependencies which exist in visiting multiple social network services. However, we did not address how consumers formulate (add and drop) friendships across social network services and how the friendship evolve over time. It would be interesting to investigate endogenous network formation behaviors in the context of multiple social networks.

Next, this new angle of multiple social networks brings interesting research opportunities in the line of social network studies on network structure analysis. One of the opportunities is to define new network properties that are important in multiple social networks. Network properties such as connectivity, betweenness, and closeness, on marketing phenomenon are well applied to

single social network analysis. However, it remains unclear which network properties are informative when analyzing data of multiple social networks. These new network centrality properties would provide new managerial insights to manage multiple social network services and find influential users. These limitations also provide opportunities for further research.

REFERENCES

1. Adams, Bert N. (1967), "Interaction Theory and the Social Network,"*Sociometry*, 30 (1), 64-78.

2. Adams, William. J. and Janet L. Yellen (1976), "Commodity Bundling and the Burden of Monopoly,"*Quarterly Journal of Economics*, 90 (3), 475–98.

3. Ainslie, Andrew and Peter E. Rossi (1998), "Similarities in Choice Behavior Across Multiple Categories,"*Marketing Science*, 17 (2), 91-106.

4. Akçura, M. Tolga and Kannan Srinivasan (2005), "Research Note:Customer Intimacy and Cross-Selling Strategy,"*Management Science*, 51 (6), 1007-12

5. Anderson, Simon P. (1993),"Why Firms May Prefer Not To Price Discriminate via Mixed bundling,"*International Journal of Industrial Organization*, 11 (1), 49-61.

6. Ansari, Asim, Carl F. Mela, and Scott Neslin (2008), "Customer Channel Migration,"*Journal of Marketing Research*, 45 (1), 60-76.

7. Ansari, Asim, Oded Koenigsberg, and Florian Stahl (2011),"Modeling Multiple Relationships in Social Networks," *Journal of Marketing Research*, 48 (4), 713-28.

8. Ansari, Asim, S. Siddarth and Charles B,. Weinberg (1996) "Pricing a Bundle Products or Services: The Case of Nonprofits,"*Journal of Marketing Research*, 33 (1), 86-93.

9. Bala, Venkatesh and Sanjeev Goyal (2000), "A Non-cooperative Model of Network Formation," *Econometrica*, 68 (5), 1181-230.

10. Bayus, Barry L. and Vithala R. Rao (1989), "A Hierarchical Utility Model for the Dynamic Acquisition of Heterogeneous Items," *Marketing Letter*, 1 (1), 71-80.

11. Bradlow, Eric T. and Vithala R. Rao (2000), "A Hierarchical Bayes Model for Assortment Choice,"*Journal of Marketing Research*, 37 (2), 259-68.

12. Calvó-Armengol, Antoni and Matthew O. Jackson (2004),"The Effects of Social Networks on Employment and Inequality," *American Economic Review*, 94 (3), 426-54.

13. Chen, Yubo and Jinhong Xie (2008), "Online Consumer Review: Word-of-Mouth as a New Element of Marketing Communication Mix,"*Management Science*, 54 (3), 477 - 91.

14. Chevalier, Judith A. and Dina Mayzlin (2006) "The Effect of Word of Mouth on Sales: Online Book Reviews,"*Journal of Marketing Research*, 43 (3), 345-54.

15. Chib, Siddartha, P.B. Seetharaman, and Andrei Strijnev (2002), "Analysis of Multi-Category Purchase Incidence Decisions Using IRI Market Basket Data," *Advances in Econometrics*, 16 (3–4), 57–92.

16. Chipman, Hugh, Edward I. George, and Robert E. McCulloch (2001), "The Practical Implementation of Bayesian Model Selection (with discussion)," Chapter in Model Selection, P. Lahiri, Ed., Institute of Mathematical Statistics Lecture Notes - Monograph Series.

17. Chung, Jaihak and Vithala R. Rao (2003), "A General Choice Model for Bundles with Multiple-Category Products: Application to Market Segmentation and Optimal Pricing for Bundles,"*Journal of Marketing Research*, 40 (2), 115-30.

18. Cohen, Wesley M. and Daniel A. Levinthal (1989), "Innovation and Learning: The Two Faces of Research and Development," *Economic Journal*, 99 (397), 569-96.

19. Coleman, James S. (1988),"Social Capital in the Creation of Human Capital," *American Journal of Sociology*, 94, 95-120.

20. Derdenger, Timothy and Vineet Kumar (2013), "The Dynamic Effects of Bundling as a Product Strategy," *Marketing Science*, 32 (6), 827-59.

21. Dindia, K. and D. Canary (1993),"Definitions and Theoretical Perspectives on Maintaining Relationships,"*Journal of Social and Personal Relationships*, 10 (2), 163-73.

22. Dubé, Jean-Pierre H., Günter J. Hitsch and Pranav Jindal (2010), "Estimating Durable Goods Adoption Decisions from Stated Choice Data." *Working Paper.*

23. Duvvuri, Sri Devi, Asim Ansari and Sunil Gupta (2007), "Consumers' Price Sensitivities Across Complementary Categories," *Management Science*, 53 (12), 1933-45.

24. Estelami, Hooman (1999), "Consumer Savings in Complementary Product Bundles,"*Journal of Marketing Theory and Practice*, 7 (3), 107-14.

25. Farquhar, Peter H. and Vithala R. Rao (1976), "A Balance Model for Evaluating Subsets of Multi attributed Items,"*Management Science*, 22 (5), 528-38.

26. Feng, Zhu, and Xiaoquan (Michael) Zhang (2010),"Impact of Online Consumer Reviews on Sales: The Moderating Role of Product and Consumer Characteristics,"*Journal of Marketing*, 74 (2), 133-48.

27. Fershtman, Chaim, and Neil Gandal (2011), "Direct and indirect knowledge spillovers: the "social network" of open-source projects," *RAND Journal of Economics*, 42 (1), 70–91.

28. Fiebig, Denzil G., Michael P. Keane, Jordan Louviere and Nada Wasi (2010), "The Generalized Multinomial Logit Model: Accounting for Scale and Coefficient Heterogeneity,"*Marketing Science*, 29 (3), 393-421.

29. Gilbert, Eric and Karrie Karahalios(2009), "Predicting Tie Strength With Social Media", Proceedings of the SIGCHI Conference on Human Factors in Computing System, 211-20.

30. Ghose, Anindya, Sang Pil Han and Raghuram Iyengar (2012), "Network Stability and Social Contagion on the Mobile Internet," *Working Paper*.

31. Ghose, Anindya and Sha Yang (2008),"Modeling Cross-Category Purchases in Sponsored Search Advertising," *Working Paper.*

32. Godes, David and Dina Mayzlin (2004), "Using Online Conversations to Study Word-of-Mouth Communication," *Marketing Science*, 23 (4), 545–60.

33. Goldsmith-Pinkhama, Paul and Guido W. Imbensbc (2013), “Social Networks and the Identification of Peer Effects”, *Journal of Business and Economic Statistics*, 31 (3), 253-64.

34. Gordon, Brett R. (2009), “ A Dynamic Model of Consumer Replacement Cycles in the PC Processor Industry,” *Marketing Science*, 28 (5) 846–67.

35. Granovetter, Mark (1973), “The Strength of Weak Ties,” *American Journal of Sociology*, 78 (6), 1360-80.

36. Granovetter, Mark (1983), “The Strength of Weak Ties: A Network Theory Revisited,” *Sociology Theory*, 1, 201-33.

37. Guadagni, Peter M. and John D.C. Little (1983), “A Logit Model of Brand Choice Calibrated on Scanner Data,” *Marketing Science*, 2 (3), 203-38.

38. Haenlein, Michael (2011), “A Social Network Analysis of Customer-level Revenue Distribution,” *Marketing Letter*, 22, 15-29.

39. Hansen, Karsten, Vishal Singh and Pradeep Chintagunta (2006), “Understanding Store-brand Purchase Behavior Across Categories,”*Marketing Science*, 25 (1), 75-90.

40. Hanson, Ward and R. Kipp Martin (1990), “Optimal Bundle Pricing,”*Management Science*, 36 (2), 155-74.

41. Harlam, Bari A. and Leonard M. Lodish (1995), “Modeling Consumers' Choices of Multiple Items,”*Journal of Marketing Research*, 32 (4), 392-403.

42. Heider, Fritz (1957), The psychology of interpersonal relationship, New York, John Wiley and Sons, Inc.

43. Heilman, Carrie M. and Douglas Bowman (2002), “Segmenting customers using multiple-category purchase data,”*International Journal of Research in Marketing*, 19 (3), 225-52.

44. Huber, Joel, Kelly Goldsmith and Cassie Mogilner (2008), “Reinforcement versus Balance Response in Sequential Choice,”*Marketing Letters*, 19 (3-4), 229-39.

45. Janiszewski, Chris and Marcus Cunha, Jr. (2004), "The Influence of Price Discount Framing on the Evaluation of a Product Bundle,"*Journal of Consumer Research*, 30 (4), 534-46.

46. Jedidi, Kamel, Sharan Jagpal and Puneet Manchanda (2003) "Measuring Heterogeneous Reservation Prices for Product Bundles,"*Management Science*, 22 (1), 107-130.

47. Kahn, Barbara E. and Robert J. Meyer (1991), "Consumer Multiattribute Judgments Under Attribute-Weight Uncertainty,"*Journal of Consumer Research*, 17 (4), 508-22.

48. Kahanda, Indika and Jennifer Neville (2009) "Using Transactional Information to Predict Link Strength in Online Social Networks," Third International AAAI Conference on Weblogs and Social Media.

49. Katona, Zsolt, Peter Pal Zubcsek, and Miklos Sarvary (2011),"Network Effects and Personal Influences: The Diffusion of an Online Social Network," *Journal of Marketing Research*, 48 (3), 425-43.

50. Kilduff , Martin, and Wenpin Tsai. (2003). Social Networks and Organisations. London: Sage.

51. Kranton, Rachel E. and Deborah F. Minehart (2001), "Theory of Buyer- Seller Networks," *American Economic Review*, 91 (3), 485-508.

52. Kumar, V., Lerzan Aksoy, Bas Donkers, Rajkumar Venkatesan, Thorsten Wiesel, and Sebastian Tillmanns (2010), "Undervalued or Overvalued Customers: Capturing Total Customer Engagement Value," *Journal of Service Research*, 13(3), 297-310.

53. Kumar, V., Vikram Bhaskaran, Rohan Mirchandani and Milap Shah (2013), "Creating a Measurable Social Media Marketing Strategy: Increasing the Value and ROI of Intangible and Tangibles for Hokey Pokey,"*Marketing Science*, 32(2), 194-212

54. Laran, Juliano (2010), "Goal Management in Sequential Choices: Consumer Choices for Others Are More Indulgent than Personal Choices,"*Journal of Consumer Research*, 37 (2),

304-14.

55. Li, Shibo, Baohong Sun and Ronald T. Wilcox (2005), "Cross-selling Sequentially Ordered Products: An Application to Consumer Banking Service,"*Journal of Marketing Research*, 42 (2), 233-9.

56. Libai, Barak, Eitan Muller, and Renana Peres (2013),"Decomposing the Value of Word-of-Mouth Seeding Programs: Acceleration Versus Expansion," *Journal of Marketing Research*, 50 (2), 161-76.

57. Letovsky, Stanley and Simon Kasif (2003) "Predicting Protein Function From Protein/Protein Interaction Data: A Probabilistic Approach,"*Bioinformatics*, 19 (1), 197-204.

58. Lewis, Kevin, Jason Kaufmann, Marco Gonzalez, Andreas Wimmer and Nicholas Christakis (2008), "Tastes, Ties, and Time: A New Social Network Dataset Using Facebook.com," *Social Networks*, 20, 330-42.

59. Lu, Yingda, Kinshuk Jerath, Param Vir Singh (2013), "The Emergence of Opinion Leaders in a Networked Online Community: A Dyadic Model with Time Dynamics and a Heuristic for Fast Estimation," *Management Science*, 59 (8), 1783 - 99.

60. Ma, Yu, P.B. Seetharaman, and Chakravarthi Narasimhan (2012), "Modeling Dependencies in Brand Choice Outcomes Across Complementary Categories," *Journal of Retailing*, 88 (1), 47-62

61. Manchanda, Puneet, Asim Ansari and Sunil Gupta (1999), "The Shopping basket: A Model for Multicategory Purchase Incidence Decisions,"*Marketing Science*, 18 (2), 95-114.

62. Manski, Charles F. (1993), "Identification of Endogenous Social Effects: The Reflection Problem"*The Review of Economic Studies*, 60 (3), 531-42.

63. Mason, Winter, and Siddharth Suri (2012), "Conducting Behavioral Research on Amazon's Mechanical Turk,"*Behavior Research Methods*, 44 (1), 1-23.

64. Mazumdar, Tridib and Sung Youl Jun (1993), “Consumer Evaluations of Multiple versus Single Price Change,”*Journal of Consumer Research*, 20 (3), 441-50.

65. McPherson, Miller, Lynn Smith-Lovin and James M. Cook (2001), “Birds of a Feather: Homophily in Social Networks,” *Annual Review of Sociology*, 27, 415-44.

66. Mehta, Nitin and Yu Ma (2012), “A Multicategory Model of Consumers' Purchase Incidence, Quantity, and Brand Choice Decisions: Methodological Issues and Implications on Promotional Decisions,” *Journal of Marketing Research*, 49 (4), 435-51.

67. Meyer , Robert J.(1981), “A Model of Multiattribute Judgments under Attribute Uncertainty and Informational Constraint,”*Journal of Marketing Research*, 18 (4), 428-41.

68. Naylor, Gillian and Kimberly E. Frank (2001), “The Effect of Price Bundling on Consumer Perceptions of Value,”*Journal of Services Marketing*, 15 (4), 270 - 81

69. Nair, Harikesh S., Puneet Manchanda, and Tulikaa Bhatia, (2010) “Asymmetric Social Interactions in Physician Prescription Behavior: The Role of Opinion Leaders,” *Journal of Marketing Research*, 47 (5), 883-95.

70. Niraj, Rakesh, V. Padmanabhan and P. B. Seetharaman (2008) “ A Cross-Category Model of Households’ Incidence and Quantity Decisions,”*Marketing Science*, 27 (2), 225-35.

71. Nitzan, Irit and Barak Libai (2011),“Social Effects on Customer Retention,” *Journal of Marketing*, 75 (6), 24-38.

72. Paas, F., Tuovinen, J., Tabbers, H., and Van Gerven, P. W. M. (2003), “Cognitive Load Measurement As a Means To Advance Cognitive Load Theory,”*Educational Psychologist*, 38 (1), 63-71.

73. Park, Young-Hoon and Peter S. Fader (2004), “Modeling Browsing Behavior at Multiple Websites,”*Marketing Science*, 23 (3), 280-303.

74. Provan, Keith G., and Juliann G. Sebastian (1998), “Networks within Networks: Service

Link Overlap, Organizational Cliques, and Network Effectiveness," *Academy of Management Journal*, 42 (4), 453-63.

75. Rao, Anita (2015) "Online Content Pricing: Purchase and Rental Markets", *Marketing Science*, forthcoming.

76. Reagans, Ray and Bill McEvily (2003), "Network Structure and Knowledge Transfer: The Effects of Cohesion and Range", *Administrative Science Quarterly*, 42 (2), 240-67.

77. Richardson, Helen (1995), "Control Your Costs then Cut Them" Transportation & Distribution, December, 94-6.

78. Rogowski, Jon C. and Betsy Sinclair (2012), "Estimating the Causal Effects of Social Interaction with Endogenous Networks", *Political Analysis*, 20 (3), 316-28.

79. Rosenberg, Nathan, "Why Do Firms Do Basic Research (with Their Own Money)?," Research Policy, 1990, 19, 165–74.

80. Russell, Gary J. and Ann Petersen (2000), "Analysis of Cross-Category Dependence in Market Basket Selection,"*Journal of Retailing*, 76 (3), 367-92.

81. Rust, John (1987), "Optimal Replacement of GMC Bus Engines: An Empirical Model of Harold Zucher,"*Econometrica*, 55 (5), 999-1033.

82. Seetharaman, P. B. (2004), "Modeling Multiple Sources of State Dependence in Random Utility Models: A Distributed Lag Approach,"*Marketing Science*, 23 (2), 263–71.

83. Seetharaman, P. B., Andrew Ainsle, and Pradeep K. Chintagunta (1999),"Investigating Household State Dependence Effects across Categories," *Journal of Marketing Research*, 36 (4), 488-500.

84. Seetharaman,P. B., Siddhartha Chib, Andrew Ainslie, Peter Boatwright, Tat Chan, Sachin Gupta, Nitin Mehta, Vithala Rao and Andrei Strijnev (2005), "Models of Multi-Category Choice Behavior," *Marketing Letters*, 16 (3-4), 239-54.

85. Seidmann, Daniel. J. (1991),"Bundling as a Facilitating Device: A Reinterpretation of Leverage Theory,"*Economica*, 58, 491–99.

86. Singh, Vithal P., Karsten Hansen and Sachin Gupta (2006),"Modeling Preference for Common Attributes in Multi-Category Brand Choice," *Journal of Marketing Research*, 42 (2), 195-209.

87. Song, Inseong and Pradeep K. Chintagunta (2006), "Measuring Cross-Category Price Effects with Aggregate Store Data,"*Management Science*, 52 (10), 1594-609.

88. Stephen, Andrew T., Toubia, Olivier (2010),"Deriving Value from Social Commerce Networks," *Journal of Marketing Research*, 47 (2), 215-28.

89. Toubia, Olivier and Andrew T. Stephen (2013), "Intrinsic vs. Image-Related Utility in Social Media: Why Do People Contribute Content to Twitter?" *Marketing Science*, 32 (3), 368-92.

90. Toubia, Oliver, Jacob Goldenberg and Rosanna Garcia (2010), "Improving Diffusion Forecasts Using Social Interaction Data," *Working Paper*.

91. Trusov, Michael, Anand Bodapati, and Randolph E. Bucklin (2010), "Determining Influential Users in Internet Social Networks," *Journal of Marketing Research*, 47 (4), 643-58.

92. Trusov, Michael, Randolph E. Bucklin, and Koen Pauwels (2009), "Effects of Word-of-Mouth Versus Traditional Marketing: Findings from an Internet Social Networking Site," *Journal of Marketing*, 73 (5), 90–102

93. Tucker, Catherine (2008), "Identifying Formal and Informal Influence in Technology Adoption with Network Externalities," *Management Science*, 54 (12), 2024-38.

94. Van Den Bulte, Christophe and Stefan Wuyts (2007), Social Networks in Marketing, Marketing Science Institute, Cambridge, MA: Marketing Science Institute.

95. Venkatesh, R. and Vijay Mahajan (1993), "A Probabilistic Approach to Pricing a Bundle of

Products or Services,"*Journal of Marketing Research*, 30 (4), 494-508.

96. Venkatesh, R. and Vijay Mahajan (2009), Design and Pricing of Product Bundles: A Review of Normative Guidelines and Practical Approaches, Handbook of Pricing Research in Marketing, Vithala R. Rao (editor). Northampton, MA: Edward Elgar Publishing Company, 232-57.

97. Venkatesh, R. and Wagner Kamakura (2003), "Optimal Bundling and Pricing Under a Monopoly: Contrasting Complements and Substitutes from Independently Valued Products," *Journal of Business*, 76 (2), 211-31.

98. Verbrugge, Lois M. (1977),"The Structure of Adult Friendship Choice,"*Social Forces*, 565, 76-97.

99. Walters, Rockney G. (1991), "Assessing the Impact of Retail Price Promotions on Product Substitution, Complementary Purchase and Inter-store Displacement,"*Journal of Marketing*, 55 (1), 17-28.

100. Wasserman, Stanley and Philippa Pattison (1996),"Logit Models and Logistic Regressions for Social Networks: I. An Introduction to Markov Graphs and P*,"*Psychometrika*, 61 (2), 401-25.

101. Watts, Alison (2002), "Non-myopic Formation of Circle Networks,"*Economics Letters*, 74 (2), 277–82.

102. Wei, Zhi and Hongzhe Li (2007), "A Markov Random Field Model for Network-based Analysis of Genomic Data," *Bioinformatics*, 23 (12), 1537-44.

103. Yoganarasimhan, Hema (2012), "Impact of Social Network Structure on Content Propagation – A Study using YouTube Data," *Quantitative Marketing and Economics*, 10 (1), 111–50.