MODELING A MARKET FOR NATURAL CATASTROPHE INSURANCE

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This dissertation introduces a game theoretic modeling framework and a series of models to examine the interactions between the key stakeholders (property owners, insurers, reinsurers and government) of a natural catastrophe insurance market, which possesses a complicated structure and faces many challenges from the natural catastrophe loss. Specifically, we integrate (1) a utility-based homeowner decision model; (2) a stochastic optimization model to optimize reinsurance decision by the primary insurer(s); (3) a heuristic government intervention model to reduce uninsured losses through price support for insurance purchase and acquisition; and (4) a state-of-the-art regional catastrophe loss estimation model, all within the framework of a static Cournot-Nash noncooperative game assuming perfect information. We allow the number of primary insurers to increase from one (monopoly) to many (oligopoly) within the Cournot-Nash framework, and examines the impacts of competition on market performance from each stakeholder's perspective. An automatic Response-Surface and Trust-Region algorithm is developed to solve the models for real, regional applications. A case study for residential wood frame buildings in Eastern North Carolina is presented. The case study suggests that: (a) private insurance market competition is an efficient mechanism to reduce uninsured loss, which should be facilitated by government; (b) more competition challenges insurers but benefits homeowners, and there exists a balance between insurer profitability and insurance penetration; (c) acquisition, price support and encouraging insurers to keep catastrophe

reserve can all improve market performance and reduce uninsured loss; and (d) catastrophe reserves should be encouraged, which not only help insurers to avoid insolvency, but could also limit competition if imposed as barrier of entry, thus improve their profitability.

BIOGRAPHICAL SKETCH

Yang Gao attended Zhejiang University (Hangzhou, China) and received a Bachelor of Science Degree in Civil Engineering in May, 2008. She then studied in the University of Arizona (Tucson, AZ) and obtained her Master of Science Degree in Civil Engineering with emphasis in transportation engineering in December, 2010. She continued the advanced education at Cornell University afterwards, and completed her PhD in Civil Infrastructure Systems (with minor in Applied Economics and Transportation Engineering) in January, 2014.

I dedicate this dissertation to my beloved parents, for raising me to believe in my ability to concur any obstacle.

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CHAPTER 1

MODELING COMPETITION IN A MARKET FOR NATRUAL CATASTROPHE INSURANCE

1. CATASTROPHE INSURANCE MARKET

Substantial growth in coastal populations has led to a dramatic increase in the consequences of natural disasters (Kunreuther 1998). Natural disaster catastrophe loss insurance is one mechanism to address this growing risk. Unfortunately, this market faces many challenges leaving many properties either uninsured or substantially under insured. Many have proposed possible explanations for challenges to the catastrophe risk insurance market, related to both the nature of catastrophe risk itself and the structure of the market. In particular, catastrophe losses tend to be highly correlated in space and characterized by "fat tail" distributions, making it especially difficult for an insurer to avoid the possibility of insolvency (e.g., Kousky and Cooke 2012). The difficulty in precisely estimating catastrophe risk, limited insurer capacity to cover potentially large losses, biases in building owners manifested in insurance purchase decisions, moral hazard, charity hazard, and tax and agency costs of holding capital all have been noted as possible contributors to the difficulties in establishing a healthy catastrophe insurance market (e.g., Jaffee and Russell 1997; Froot 2001; Kunreuther et al. 2002; Kunreuther 2006; Kousky 2011; Paudel 2012). Also, Grace et al. (1998), Grace et al. (2003), Kleindorfer and Klein (2003), and Kunreuther and Michel-Kerjan (2009) identify a passive supply-demand relationship and likely inefficient and misguided government intervention as conditions that have exacerbated the stress on the market.

With respect to demand, property owners often do not fully insure their property nor do they invest in pre-event mitigation activities that can reduce losses (Kunreuther 1996; Kreisel and Landry 2004; Kunreuther and Pauly 2004; Dixon et al. 2006). Property owners are likely to have insufficient financial resources to recover losses and may demand government relief (Kunreuther and Pauly 2004). Major disasters are often followed by large, unplanned government expenditures that create major difficulties for local and state government budgets. Developed under the time and political pressure of a post-disaster situation, they are often poorly managed, uneven and inefficient (Kunreuther and Pauly 2006). Moreover, property owners' over-reliance on post-disaster relief from the government may crowd out the private insurance market (Kunreuther and Pauly 2004).

On the supply side, insurers receive a stable flow of premiums to address a liability stream that is highly variable. This mismatch between stable receipts and sporadic but very large expenditures must be addressed through intertemporal risk spreading, which is difficult to achieve (Jaffee and Russell 1997). For example, some data from reinsurance companies implies that insurance companies tend to retain, rather than share their large-event risk. In fact, the vast majority of primitive catastrophe risk in the economy is being retained (Froot 2001). Due to the difficulty in managing risk, insurers have limited renewals of existing contracts and issuing of new contracts in at-risk region (US GAO 2007b). In the worst case, disaster events can cause insolvency, as happened, for example, to eight small Florida insurance companies after Hurricane

Andrew (Grier 1996). Despite of the challenges for insurers to survive, state legislators and insurance regulators appear to have suppressed insurance prices, compressed rate differences between high and low risk areas and imposed significant cross-subsidies from low-risk to high-risk areas (Kleindorfer and Klein 2003).

Given the complexity of insurance markets with the potential for catastrophic loss, there is a clear need for comprehensive examination of the interactions between key stakeholders (property owners, insurers, reinsurers, and government) so that effective ways to restructure the catastrophe loss insurance market can be identified. In this study, we use a game theoretic modeling framework to capture the strategic relationship of one or more insurers with the other stakeholders in the market. Within this framework, we examine performance of a voluntary catastrophe insurance market as the level of competition within that market changes. More specifically, we integrate (1) a utility-based homeowner decision model; (2) a stochastic optimization model to optimize reinsurance decisions by the primary insurer(s); and (3) a state-of-the-art regional catastrophe loss estimation model, all within the framework of a static Cournot-Nash noncooperative game assuming perfect information. We allow the number of primary insurers to increase from one (monopoly) to many within the Cournot-Nash framework.[1] In addition, we analyze a cooperative solution for each level of market concentration by symmetric insurers. Although we use a single shot Cournot-Nash game to model the interaction between insurers, by also characterizing the cooperative (joint profit maximizing) solution we can benchmark the range of

[1] A standard exercise of the symmetric Cournot-Nash model with n constant marginal cost firms can demonstrate that when n=1 the monopoly solution obtains, n=2 yields the duopoly solution and as n approaches infinity, Cournot-Nash converges on the perfectly competitive solution.

outcomes viable under a repeated game with trigger strategies for all possible discount rates.

This modeling framework is applied to a full-scale case study for hurricane risk (flood and wind combined) for residential buildings in Eastern North Carolina. We assume heterogeneous homeowners and insurers with identical cost structures, and focus on the equilibrium supply-demand relationship under conditions that range from monopoly and oligopoly to market structures that emulate near perfect competition. Furthermore, we examine impacts of market concentration on the profitability and insolvency of a single insurer, as well as its decisions to hedge risk as the number of insurers increases.

This study is novel in that it represents an integration of a classical form of game theoretic strategic interaction with individual optimization models for key stakeholders, and a data-driven regional catastrophe loss model that provides a disaggregated representation of the risk to be managed. Further, we illustrate how this new model can be applied in a case study.

Following a summary of relevant literature in Section 2, the formulation of the single shot Cournot-Nash model is described in Section 3. Application of the Cournot-Nash model requires defining inverse demand functions and an insurer cost function. In Section 4, we present a modeling framework that includes interacting models of loss, homeowner decisions, and insurer decisions, and show how it can be used to develop those required functions. Section 5 presents inputs required for the case study application for hurricane risk to residential buildings in Eastern North Carolina. The case study results are presented in Section 6, including discussion of how equilibrium price, insurance penetration, insurer's performance, and reinsurance decisions change as the number of insurers in the market increases. Finally, the summary of this research as well opportunities for future work is presented in Section 7.

2. *REVIEW OF INSURANCE MARKET MODELS*

Based on empirical analysis, catastrophe loss insurance is a form of property and liability insurance in which competition is usually characterized by an oligopolistic market structure (Nissan and Caveny 2001; Murat et al. 2002). Furthermore, the catastrophe insurance market is highly regulated in terms of entry and exit, capital availability, and pricing. Given the nature of the product and the current regulatory climate, a voluntary insurance market is generally oligopolistic.

The theoretical insurance literature has focused primarily on a monopoly insurer, monopolistically competitive market structure in which insurers compete with heterogeneous but highly substitutable insurance products, or perfect competition (e.g., Joskow 1973; Rothschild and Stiglitz 1976; Stiglitz 1977; Spence 1978). There are a few models of oligopolistic insurance markets for which the insurers make positive profits. Sonnenholzner and Wambach (2004) gives an excellent overview of these models. Among the papers on oligopolistic insurance markets, Schlesinger and Schulenburg (1991) analyze a model with heterogeneous insurance products and Polborn (1998) applies the Bertrand game to model oligopolistic interactions between risk-averse insurers.

Cournot-Nash and Bertrand models of oligopoly markets can be represented as classical static non-cooperative games (Tirole 1988; Lynne and Dan 2008). The Bertrand framework has been used to represent oligopoly insurance markets—where players/insurers compete based on price (e.g., Polborn 1998; Sonnenholzner and Wambach 2004). For a homogeneous good, the standard Bertrand model of price competition with no capacity constraints implies that the perfectly competitive zero-profit solution with equilibrium price equal to marginal cost will obtain with as few as two players. Generally this result is not consistent with empirical evidence. Therefore, most of the oligopoly insurance models consider product differentiation and/or imperfect information (e.g., Rothschild and Stiglitz 1976; Stiglitz 1977; Spence 1978; Schlesigner and Schulenburg 1991; Polborn 1998; Wambach 2000). In contrast, when there is quantity or Cournot-Nash competition, even for homogeneous products, prices can remain substantially above marginal cost so long as the number of players is not large. Given that there is little evidence of economies of scale in this industry (Hill 1979), we assume Cournot-Nash competition under perfect information with a homogeneous insurance product at the market equilibrium price per unit of expected loss.

The static Cournot-Nash model has players interacting only once. The reality, of course, is that insurers/players are involved in strategic interactions repeatedly. In such a setting, issues such as learning, establishing a reputation, and credibility can become quite important. Dynamic game-theoretic models have been applied to address these issues and model oligopolistic competition (e.g., Abreu 1983; Alos-Ferrer, Ania et al. 1998; Huck, Normann et al. 1999; Kesternich and Schumacher 2009; Abbring and Campbell 2010). In infinitely repeated games, collusive (joint profit maximizing) outcomes can be supported as a stable solution using credible punishment strategies. Playing the static Cournot-Nash strategy forever is a credible punishment strategy in the repeated game and the best outcome for all players in the game is the static collusive solution. Although we have not utilized a dynamic framework, we are able to characterize the range of possible outcomes by identifying the Cournot-Nash solution and the joint profit maximizing solution for each number of players.

Finally, though problems of moral hazard and adverse selection have been addressed in the general insurance industry (e.g., Bernnardo and Chiappori 2003; De Feo and Hindriks 2005; Villeneuve 2005), we do not address them here because they tend not to be as important for catastrophe insurance, since the event creating the risk is beyond the control of those who are insured (Russell and Jaffee 1997).

3. COURNOT-NASH MODEL OF CATASTROPHE INSRUANCE MARKETS

Our base case of oligopolistic behavior in the catastrophe loss market is a single shot noncooperative game, in which each insurer forms a best-response strategy (i.e., volume of business in the region(s)) that maximizes net profit recognizing the best-response strategies (and capabilities) of its competitors. All insurers in the market compete once and make their decisions simultaneously. We assume that there are no disparities in information and all players (private insurers) are homogeneous. In other words, each insurer faces the same cost structure, and is equally capable of handling all levels of demand at market price level. Further, we assume that insurers always offer full coverage so that pricing is expressed as the premium per unit of coverage, which is the same across all insurers; and each property owner, at most, only purchases one insurance contract from a single insurer. Furthermore, by symmetry, total sales at the equilibrium price will be divided evenly geographically and with respect the vulnerability of the building inventory, which will result in the same expected net profit for each insurer.

More specifically, consider n homogeneous private insurers selling catastrophe insurance for residential buildings in a region. We assume the region is divided into smaller risk regions $v \in V$ defined to allow homeowner risk attitudes and insurer premiums to vary geographically. Insurer j's decisions in the Cournot-Nash game—the total coverage offered in each risk region v is denoted as q_{vj}, in terms of annual expected insured loss (from natural disasters). The price (per unit) of insurance policies sold in a risk region is assumed to have a relationship with the total volume offered by all insurers in this region. More specifically, assume that there is a distinct demand function for each risk region: $Q_v = D_v(p_v),\ \forall v \in V$, with its inverse: $p_v = P_v(Q_v) = D_v^{-1}(Q_v),\ \forall v \in V$, where Q_v refers to the total insured loss by all insurers in the entire region and p_v refers to price per dollar coverage. Since the total insured losses actually come from all the insurers in the market, a single insurer j's price-demand relationship also depends on its rivals' actions, and can be presented as: $p_v = P_v(q_{vj}, \sum_{k \neq j} q_{vk}) = P_v(q_{vj}, Q_{-vj})$. The cost function for insurer j to operate the business in all risk regions is denoted as $C_j(\sum_{v \in V} q_{vj})$. Finally, insurer j's annual net profit can be described as the premiums paid to insurer j by those they insure minus the cost of providing the insurance over a year (Eq. 1). Note that the first term price is a function of the coverage provided by all insurers since that determines the price, while the second term cost is a function of only the coverage offered by insurer *j*. Expressing an individual insurer's objective as a function of its decisions/strategies and its competitors' facilitates deduction of the equilibrium conditions.

$$\pi_j(\sum_{v\in V} q_{vj}, \sum_{v\in V} Q_{-vj}) = \sum_{v\in V} q_{vj} P_v(q_{vj}, Q_{-vj}) - C_j(\sum_{v\in V} q_{vj}) \quad q_{vj} \geq 0 \tag{1}$$

We assume each insurer is a net profit maximizer and formulates a best response function that recognizes the strategic interaction with other insurers serving the same locations. If the problem is differentiable, the optimal solution for net profit maximization should satisfy the first order conditions (Eq. 2), based on the envelope theorem (Mas-Colell et al. 1995), which also refers to the relationship that marginal cost equals to marginal revenue:

$$\frac{\partial \pi_j}{\partial q_{vj}} = P_v(q_{vj}, Q_{-vj}) + q_{vj}\frac{\partial P_v(q_{vj}, Q_{-vj})}{\partial q_{vj}} - \frac{\partial C_j(\sum_{v\in V} q_{vj})}{\partial q_{vj}} = 0 \tag{2}$$

Denote insurer *j*'s actions that satisfy the first order conditions as a vector of reaction functions (i.e., coverage provided in each region v, or best responses): $\mathbf{q}_j = (q_{vj}, \forall v \in V)$. Since the insurers are all homogeneous, they will have the same reaction functions. Thus, by symmetry, we would have insurer *j*'s rivals' actions as: $Q_{-vj} = (n-1)q^*_{vj},\ \forall v \in V$. By substituting these relations into the first order conditions (Eq. 2), they can be rewritten as:

$$P_v(nq^*_{vj}) + q^*_{vj}\frac{\partial P_v(nq^*_{vj})}{\partial q_{vj}} - \frac{\partial C_j(\sum_{v\in V} q_{vj})}{\partial q_{vj}} = 0 \quad \forall v \in V \tag{3}$$

Solving Equations 3 explicitly, the reaction function (best response function of insurer *j)*, denoted as $V \times 1$ vector $\mathbf{q}_j = R_j(\mathbf{q}_{-j})$, where $\mathbf{q}_{-j}$ refers to all its opponents' reactions, and each term of the vector has a closed form as:

$$R_j(\mathbf{q}_{-j}): \begin{cases} \cdots \\ q_{vj}^* = \dfrac{[\partial C_j(\sum_{v \in V} q_{vj}) / \partial q_{vj}] - P_v(nq_{vj}^*)}{\partial P_v(nq_{vj}^*) / \partial q_{vj}} \quad \forall v \in V \\ \cdots \end{cases} \tag{4}$$

The reaction functions are an intermediate step to solving for the Nash equilibrium and examining the stability of the equilibrium. Denote the cross partial of reaction function for firm *j* to its rival *i*'s actions as $\dfrac{\partial R_j}{\partial \mathbf{q}_i}, (j \neq i)$, which is a $|V| \times |V|$ matrix. Then, the partial derivative matrix as defined in Zhang and Zhang (1996), denoted as T', has the following form:

$$T' = \begin{bmatrix} 0 & \dfrac{\partial R_1}{\partial \mathbf{q}_2} & \cdots & \dfrac{\partial R_1}{\partial \mathbf{q}_n} \\ \dfrac{\partial R_2}{\partial \mathbf{q}_1} & 0 & & \dfrac{\partial R_2}{\partial \mathbf{q}_n} \\ \vdots & & \ddots & \vdots \\ \dfrac{\partial R_n}{\partial \mathbf{q}_1} & \cdots & \dfrac{\partial R_n}{\partial \mathbf{q}_{n-1}} & 0 \end{bmatrix}_{n|V| \times n|V|} \tag{5}$$

The sufficient condition of a stable Cournot-Nash equilibrium requires the partial derivative matrix of all players to have a norm of less than unity (Zhang and Zhang 1996), i.e., $\|T'\| \leq 1$.

4. HOMEOWNER AND INSURER DECISION MODELS TO COMPUTE INVERSE DEMAND AND COST FUNCTIONS

Application of the Cournot-Nash equilibrium model in Section 3 requires defining inverse demand functions $p_v, \forall v \in V$ and a cost function $C_j(\sum_{v\in V} q_{vj})$, for simplicity, denoted as $C_j(\sum_v q_{vj})$ or C_j. A key contribution of this work is to demonstrate how those functions can be developed using a modeling framework that includes interacting models of loss, homeowner decisions, and insurer decisions. This section describes the framework, and then how it is used to define the functions needed for the Cournot-Nash model. For ease of discussion, the modeling framework is described for hurricane risk and single-family residential buildings specifically, but it could be adapted to earthquakes or other extreme events and other types of buildings.

4.1 Overview of interacting loss and decision models

The framework is illustrated for each primary insurer. It includes three models and represents three main players (Fig. 1.1). The loss model is a simulation that combines hazard, inventory, and damage modules to compute a probability distribution of losses for each group of buildings (defined by location and type) and each possible hurricane in the study area. The inputs required are similar to those that can be obtained from any regional loss estimation model, such as, HAZUS-MH 2.1 (FEMA 2012) or the Florida Public Hurricane Loss Model (2005). The primary insurer and homeowners interact in a way in which the insurer determines what premiums to charge for policies at a specified deductible, and what reinsurance to purchase, and each homeowner (defined by his home's location and type) responds by deciding whether or not to purchase insurance. Specifically, the primary insurer model is a simulation optimization in which the objective is to maximize net profit. Each homeowner's decision-making is modeled as a utility maximization problem by choosing between buying insurance and bearing risk him/herself. Though other options (of dealing with catastrophe risk) such as relocating would be realistic, they are beyond the scope of this study and thus not considered as homeowner's decisions. The homeowner and loss models together are used to develop the inverse demand functions (Section 4.3). The interactions between the insurer model and the homeowners' decisions are then used to develop the cost function (Section 4.4).

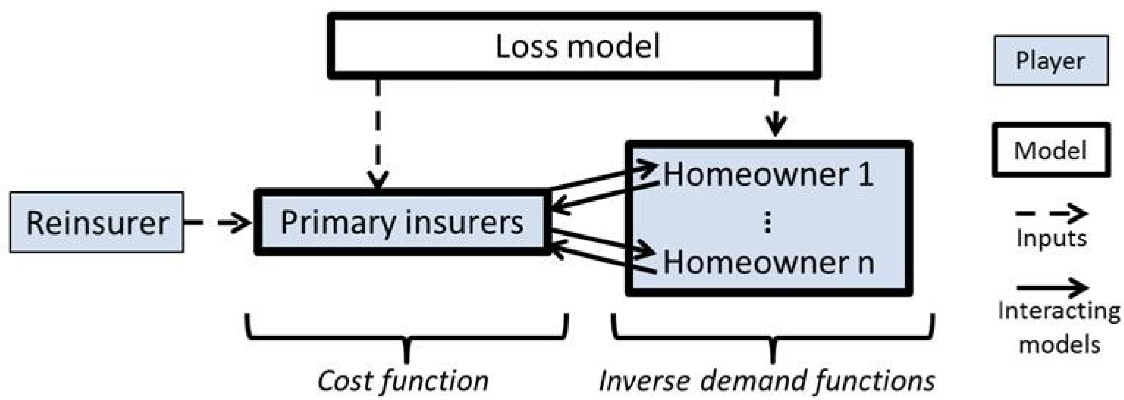

Figure 1.1 Structure of interacting models

4.2 Definitions

Building inventory. The inventory of residential buildings is divided into groups, where each is defined by its geographic area unit or location *i* (e.g., census tract), building category *m*, resistance level *c*, and risk region *v*. Building categories m are defined based on architectural features and are assumed to perform similarly in hurricanes and have similar value (e.g., one-story home with a garage and hip roof). A building's resistance level *c* represents its vulnerability and is a function of structural details that define the probability of damage given wind speed and flood depth. As defined in Section 3, risk regions *v* are larger geographic areas comprised of many area units *i*. They are defined to allow homeowner risk attitudes and insurer premiums to vary by location, but at greater aggregation than area units. The building inventory, which we assume to be constant over time, is defined using X_{imcv}, the number of buildings of type *i, m, c, v*.

Stakeholders. The collection of homeowners in the study area are disaggregated based on their homes' location *i*, building category *m*, building resistance level *c*, and risk region *v*. Since homeowners differ based on their *i, m, c, v* type (and therefore risk), and possibly their risk attitude, the model does not assume they will all make the same decisions but instead captures the heterogeneous behavior of homeowners. We assume all insurers are homogenous net profit maximizers and will purchase one layer of catastrophe risk excess of loss reinsurance to manage/transfer part of their liabilities. Capital markets are not considered for these primary insurers. The reinsurance market is assumed to accommodate primary insurers' insurance demand at a standard price level.

Time. The durations of the time steps *t* vary (a few days to a few weeks). They are defined to be short enough so that we can reasonably assume no two hurricanes occur in the same time period, and so that the probability a hurricane occurs in one time period is equal across time periods. Since hurricane occurrence varies during the year, this means the time periods are shorter, for example, in September when hurricanes are more likely than in June when they are less likely.

Hazard. The model includes coverage for both hurricane-related wind and storm surge flooding. The hurricane hazard is represented by an efficient set of probabilistic hurricane scenarios $h \in (1, \ldots$, defined as tracks with along-track parameters that determine the intensity, including central pressure deficit and radius to maximum winds. Each hurricane scenario has an associated hazard-adjusted annual occurrence probability P^h such that when probabilistically combined, the set of hurricane scenarios represents the regional hazard (Apivatanagul et al. 2011). For each hurricane, wind speeds and surge depths are estimated throughout the study area; in a sense, each hurricane scenario represents all hurricanes that would produce similar wind speeds and surge depths in the study area. The occurrence probability for period *t* is calculated from the annual occurrence probability using the historical relative frequency of events over the course of the hurricane season (Peng 2013).

A series of hurricanes in quick succession can generate very different outcomes for an insurer than the same hurricanes evenly spread over time. We therefore define a long-term (thirty-year) timeline of hurricanes as a scenario $s \in (1, \ldots$. (To avoid confusion, we refer to a single hurricane event scenario as simply a hurricane *h*.) Each scenario *s* is a $1 \times T$ vector, where *T* is total number of time periods, and for each time period *t*, either one of the possible hurricanes *h* occurs, or no hurricane occurs. For ease of notation, we refer to the case of no hurricane as $h = H + 1$. Each scenario has an occurrence probability P^s, such that $\sum_s P^s = 1$. The complete set of scenarios (on the order of hundreds or perhaps thousands) is defined so that it has the same key characteristics as the full set of $(H+1)^T$ scenarios that is theoretically possible. See Peng (2013) for more details on the creation of the suite of scenarios.

4.3 Deriving inverse demand functions

The demand for catastrophe insurance is assumed to be separable by risk region *v*, so there exist *V* independent inverse demand curves, i.e., price-demand relationships $p_v = P_v(Q_v)$. To develop that curve for one region *v*, we solve the homeowner decision model for every homeowner in the region at a specified price level p_v (i.e., price per dollar coverage in risk region *v*) to compute the demand Q_v (i.e., total expected coverage in region *v*) at that price level. Repeating the calculation for many values of p_v, we obtain the paired data, (p_v, Q_v), which we then use to fit an inverse demand curve.

The homeowner decision model is defined on an annual basis and for a single homeowner in location *i*, building category *m*, resistance level *c*, and risk region *v*. It can be solved separately for homeowners in buildings of each type *i, m, c, v*, and thus the computation can be parallelized. For a homeowner of type *i, m, c, v*, the model determines whether or not to buy insurance at the specified price p_v given a specified deductible *d*. Specifically, it yields the binary decision variable w_{imcv}, which is one if insurance is purchased, and zero otherwise.

We denote the loss in the event of hurricane *h*, for a single home of type *i, m, c, v,* as L^h_{imcv}, which is estimated from the loss model. The annual premium, denoted as Z_{imcv}, is defined as the price per dollar coverage times the annual expected loss (Eq. 6), and the amount the homeowner actually pays as a deductible in the event of hurricane *h* is the minimum of the loss experienced and the deductible (Eq. 7).

$$Z_{imcv} = p_v \sum_h P^h L^h_{imcv} \quad \forall i, m, c, v \tag{6}$$

$$B^h_{imcv} = \min\left\{L^h_{imcv},\ d\right\} \quad \forall i, m, c, v \tag{7}$$

We assume each homeowner has a maximum budget for homeowner insurance equal to a specified percentage k_v of his home value V_m (Eq. 8); and insurers will only offer insurance if the premium is greater than some specified value ρ (Eq. 9). Finally, the decision variables w_{imcv} must be zero or one.

$$Z_{imcv} \le k_v V_m \quad \forall i, m, c, v \tag{8}$$

$$Z_{imcv} \ge \rho \quad \forall i, m, c, v \tag{9}$$

$$w_{imcv} = \{0,\ 1\} \quad \forall i, m, c, v \tag{10}$$

We assume the decision is made by maximizing utility, with risk preferences represented by the utility function $U(x) = 1 - e^{-\theta_{imcv} x}$, where θ_{imcv} is the Arrow-Pratt measure of risk aversion for homeowners of type $i,\ m,\ c,\ v$. For $\theta_{imcv} > 0$ homeowners are risk averse, which is necessary for insurance to be a possibility. In the case study, we assume the values of θ_{imcv} within a risk region *v* are lognormally distributed with a specified mean $\log(\theta_v)$ and a standard deviation of one (Eq. 11).

$$\log(\theta_{imcv}) \sim N(\log(\theta_v), 1) \quad \forall i, m, c, v \tag{11}$$

The homeowner's objective function (Eq. 12) is to maximize the probability weighted sum of utilities over all possible hurricanes *h* if he buys insurance (first term) and if he does not (second term). In the first case, the homeowner pays the premium and loss up to the deductible. In the second case, the homeowner pays the loss due to building damage only. Note that when $h = H + 1$, no hurricane occurs, and the loss is zero.

$$Max\ w_{imcv}\left[\sum_h P^h\left(U\left(Z_{imcv} + B^h_{imcv}\right)\right)\right] + (1 - w_{imcv})\left[\sum_h P^h\left(U\left(L^h_{imcv}\right)\right)\right] \tag{12}$$

The homeowner model is defined by optimizing the objective function (12) subject to Constraints (6) to (10). We denote the solution to that model for a homeowner of type *i, m, c, v* as w_{imcv}. If X_{imcv} is the number of buildings of *i, m, c, v* then the total expected insured loss Q_v for insurers at price level p_v and deductible *d* is as given in Eq. 13. Computing Q_v for many different prices p_v, we can then fit an inverse demand function $p_v = P_v(Q_v)$.

$$Q_v = \sum_{imc} w^*_{imcv}\left(\sum_h P^h L^h_{imcv} X_{imcv}\right) \quad \forall v \tag{13}$$

A logical extension of this work would be to replace the homeowner formulation with a discrete choice model or other formulation that accounts for some of the documented homeowner biases and heuristics (Kunreuther 2006).

4.4 Defining cost function

The cost function in the Cournot-Nash model $C_j(\sum_v q_{vj})$ defines the cost for insurer *j* to operate the business in all risk regions *v*, and is a function of all q_{vj}, the policies written by insurer *j* in each risk region *v* (i.e., total expected insured losses in region *v*). We compute this function in three steps. First, given a price per dollar coverage p_v, we use the insurer optimization model to determine how much risk the insurer will transfer (specifically, the attachment point *A* and maximum limit *M* of the excess of loss reinsurance treaty) so as to maximize its profit. As described in Section 4.1, the insurer optimization interacts with the homeowner model, which determines which homeowners will buy insurance w^*_{imcv}, and thus the homeowner demand q_{vj} at the specified price level. Second, given the homeowner demand and reinsurance treaty parameters, we can calculate the cost to the insurer C_j. Third, we repeat that calculation for different price levels p_v and combine it with the homeowner demand to obtain a set of data triplets (p_v, q_{vj}, C_j). Finally, we fit a cost function to that data that relates C_j to q_{vj} for each risk region *v*, which can be used as input in the Cournot-Nash model. In Section 4.4.1, we define insurer cost more precisely, and in Section 4.4.2 we present the insurer decision model formulation.

4.4.1 Definition of insurer cost

We approximate the insurer's total cost in this model as the sum of the operational costs and insurer's liabilities. The operational cost is assumed to be a fixed portion τ (35% in the case study) of annual expected insured losses (i.e., each dollar of coverage costs $0.35 for the insurer to provide). Insurer liabilities include the coverage of insured homeowner losses reduced by the risk transferred to the reinsurer, plus payments to the reinsurer. For the assumption of a single layer of catastrophe risk excess of loss reinsurance (Section 4.2), in the event of a hurricane *h*, the loss to all insured buildings L^h is divided among the homeowners, primary insurer, and reinsurer as in Figure 1.2. The variables *A, M* and β are the attachment point, maximum limit and co-participation percentage of the reinsurance treaty, respectively.

If hurricane h occurs, each homeowner pays the first portion of the loss up to the deductible; the reinsurer pays $\beta\%$ of any loss above the attachment point A and up to a maximum limit $\beta\%$ of $(M - A)$; and the primary insurer pays the remaining loss. The excess of loss reinsurance policy itself requires that the insurer pay the reinsurer a premium of b, and in the event of a hurricane h, an additional reinstatement premium to reinstate the limit M. Overall, the insured losses not covered by the deductibles and the reinsurance policy, plus the payments to the reinsurer are considered the insurer's liability.

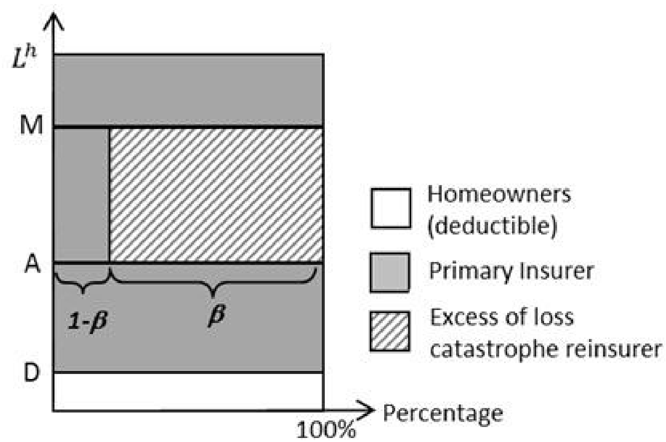

Figure 1.2 Loss structure showing how loss to insured buildings L^h is divided among stakeholders

Purchasing a reinsurance policy with low attachment point and high limit reduces the insured losses/liabilities incurred by the primary insurer thus reducing the risk and stabilizing profit over the long term. However, this must be balanced against the result that a high reinsurance premium reduces the primary insurer's net profit in each time period. Interrelated choices about homeowner insurance pricing that affect homeowner purchase decisions that, in turn, affect the loss distribution ultimately influence the optimal reinsurance strategy for the primary insurer.

4.4.2 Insurer decision optimization model

For a given price level, the insurer determines how much risk to transfer to the reinsurer (i.e., values of *A* and *M*), by maximizing its expected net profit over the set *S* of long-term hurricane scenarios described in Section 4.2. At price level p_v, the homeowners who purchase insurance w^*_{imcv}, and resulting total demand $Q_v = D_v(p_v)$, are determined by the homeowner model (Section 4.3). The total insured loss L^h (Eq. 14) and deductibles B^h incurred by homeowners (Eq. 15) are:

$$L^h = \sum\nolimits_{imcv} w^*_{imcv} L^h_{imcv} X_{imcv} \tag{14}$$

$$B^h = \sum\nolimits_{imcv} w^*_{imcv} B^h_{imcv} X_{imcv} \tag{15}$$

Reinsurance coverage. We define q^h to be the loss above *A* and below *M* for hurricane *h* (Eq. 16). If the loss exceeds the attachment point *A*, then βq^h is recovered from the reinsurer and the primary insurer incurs $(1-\beta)q^h$, where β is a specified input constant (Fig. 2).

$$q^h = \min\left\{\max\left\{L^h - A,\ 0\right\},\ M - A\right\} \quad \forall h \tag{16}$$

For a given scenario *s* and time *t*, which hurricane *h* (or no hurricane) happens is known, so we can define e^{sy}, the loss between attachment *A* and limit *M* for scenario *s* and year *y* in Equation 17, where r^{sth} is a binary indicator variable that is one if hurricane *h* happens in scenario *s* at time *t* and zero otherwise. Since at most one hurricane can happen in a time period *t*, $\sum_h r^{sth} \leq 1 \ \forall s,t$. The set $\omega(y)$ defines the set of time periods t in year y.

$$e^{sy} = \sum_{t \in \omega(y)} \sum_h r^{sth} q^h \quad \forall s, y \tag{17}$$

Reinsurance premium. In each year y, the primary insurer pays the reinsurer a base premium b, and in the event of a hurricane h, it also pays a reinstatement premium to reinstate the limit M. The base premium is computed as the expected loss the reinsurer is responsible for multiplied by one plus a loading factor φ, plus the standard deviation σ of the net reinsurer loss multiplied by β and a user-specified constant g (Eq. 18) (Kunreuther and Michel-Kerjan 2009).

$$b = (1+\varphi)\left[\sum_h P^h \beta q^h\right] + g\beta\sigma \tag{18}$$

The loading factor φ represents the reinsurer's share of the loss adjustment expenses, its own expenses, and its profit, and g represents the reinsurer's risk aversion. The σ is the standard deviation over all scenarios s and years y of the reinsurer's loss, e^{sy}, less the reinstatement premium for scenario s and year y. The reinstatement premium is a pro rata amount of the expected reinsurer loss without adjusting for the length of the treaty's remaining term. That is, it equals the expected loss multiplied by the percentage of the original coverage that was used $(e^{sy}/(M-A))$. The total reinsurance premium for scenario s in year y, therefore, is the sum of the base reinsurance premium and the reinstatement payment:

$$r^{sy} = b + \left(\frac{e^{sy}}{M-A}\right)\left[\sum_h P^h \beta q^h\right] \quad \forall s, y \tag{19}$$

Primary insurer's profit. Equation (20) defines the insurer's net profit, F^{sy}, in scenario s and year y. The terms are, in turn, the total homeowner premiums collected, transaction cost, total loss to insured buildings minus deductibles, actual loss recovered from the reinsurer, and reinsurance premium.

$$F^{sy} = \sum_{v} p_v Q_v - \tau(\sum_{v} Q_v) - \sum_{t\in\omega(y)} \sum_{h} r^{sth} L^h + \sum_{t\in\omega(y)} \sum_{h} r^{sth} B^h + \beta e^{sy} - r^{sy} \quad \forall s, y \tag{20}$$

Primary insurer's accumulated surplus. In reality, the funds available to the insurer at any time would be the policyholder surplus, which is defined as the insurer's admitted assets minus its liabilities, i.e., its net worth. In this model, we treat the profit accumulated in previous periods as the policyholder surplus, and thus we ignore the effect of investments and other lines of business. We assume the company starts its business at time $y = 0$ with a surplus (denoted as μ^{s0}) equal to *k* times the annual premiums Z_{imcv} (Eq. 6) received from homeowners of all building inventories in all risk regions, where *k* is a user-specified constant (Eq. 21).

$$\mu^{s0} = k \sum_{imcv} Z_{imcv} \quad \forall s \tag{21}$$

To determine the surplus μ^{sy} in each year, we make two alternative assumptions, and for each model run, we choose one (i.e., either Equation 22a or 22b is included in the formulation, but not both). In the *capped surplus* version (Eq. 22a), we assume the primary insurer reallocates surplus greater that this amount in each year *y* by reinvesting in other lines of business or distributing it to investors as dividends. The surplus in scenario *s* and year *y* then is the minimum of the sum of the profit in *y* and the surplus in $y-1$, and the maximum allowable surplus $k \sum_{imcv} Z_{imcv}$ (Eq. 22a).

$$\mu^{sy} = \min\left\{\mu^{s,y-1} + F^{sy}, k \sum_{imcv} Z_{imcv}\right\} \quad \forall s, y \tag{22a}$$

Our model allows for an alternative to reinsurance through a *retained taxed surplus* version in which the primary insurer might at least partially self-insure. In that case, we remove the cap on the surplus and instead assume the insurer will reinvest all after-tax surpluses within this business, where the tax rate is χ. As such, these investments are considered to be a reserve available to pay homeowners claims in extreme events, and the surplus in scenario s and year y is defined instead as in Equation 22b. This assumption relates to the argument in (Jaffee and Russell 1997) that the failure of catastrophe insurance market lies in the inability of insurance companies to arrange for the level of capital necessary to settle extraordinarily large losses.

$$\mu^{sy} = \mu^{s,y-1} + (1-\chi)F^{sy} \quad \forall s, y \tag{22b}$$

In both cases, if the accumulated surplus μ^{sy} in year y equals zero or less, we assume that the insurer becomes insolvent, and the profit F^{sy} and surplus μ^{sy} are set to zero for the remaining years $(y+1,\ldots$ of the scenario s.

Insurer Objective function. The objective function is to maximize the average annual profit over the full time horizon, averaged over all scenarios S (Expression 23). The model thus chooses values of the decision variables A and M defining the reinsurance treaty, subject to Constraints (14) to (22). This stochastic optimization model, which is both non-linear and non-convex, is solved using an automatic two-stage Response-Surface and Trust-Region solution procedure in Gao et al. (in progress).

$$\max \frac{1}{SY} \sum_{sy} F^{sy} \tag{23}$$

Note that this objective is equivalent to minimizing the total cost $C_j(\sum_v q_v)$ to the insurer, since total insurer cost equals the total premiums collected from the homeowners as income minus the profit, and the premium income is fixed at the specified price level. In other words, the optimal solution to Expression 23, F^{sy*}, will result in the optimal cost structure $C_j(\sum_v q_v)^* = C_j(\sum_v D_v(p_v))^*$ for the given price levels p_v (Eq. 24).

$$C_j(\sum_v q_v)^* = \sum_v p_v q_v - F^{sy*} \tag{24}$$

5. *CASE STUDY INPUTS*

Eastern North Carolina was used as a case study area to develop and demonstrate the model. The region includes the low-lying coastal part of the state vulnerable to hurricane hazard, extending westward to include half of Raleigh, the state capital. A tropical storm or hurricane is expected to make landfall on the North Carolina coast on average every four years (SCONC 2010). The study focuses on single-family wood-frame homes, the wind and storm surge flooding hazards (not rainfall-induced flooding), and direct losses (structural, non-structural, interior, mechanical, electrical, and plumbing, but no contents or additional living expenses).

The 2010 census tracts are the basic area unit of study, but each of the 143 census tracts that touch the coast was divided into three zones—within one mile of the coastline; one to two miles from the coastline; and the remainder of the census tract to account for zone-specific flood depths within a census tract. The result is 732 locations i. Eight building categories m were defined to represent all combinations of number of stories (one or two), garage (yes or no), and roof shape (hip or gable). Each building is defined as a collection of *components* represented in the damage and loss modeling (e.g., roof covering, openings). Each component in turn is made of many *component units* (e.g., a single window or section of roof covering). For each component a few possible physical *configurations* are defined, each with an associated *component resistance*. The *building resistance c* of each building is then defined by the vector of resistances of its components. The case study includes 192 building resistance levels (Peng 2013).

The component-based loss simulation model is a modified version of the Florida Public Hurricane Loss Model for the wind-related damage (FPHLM 2005); and Taggart and van de Lindt (2009) and van de Lindt and Taggart (2009) for the flood-related damage. Described in detail in Peng et al. (2013), and Peng (2013), the loss model was used to compute the loss L_{imcv}^{h} to a building in location *i* of building type m and resistance level *c* in risk region *v* given hurricane *h*. The building inventory data (X_{imcv}) was estimated using census data, with total building counts allocated among the building resistance levels *c* based on location (coastal or not) and year built relative to major building code and construction practice changes.

We define two risk regions *v*—within two miles of the coast (higher risk *H*) or not (lower risk *L*). The mean risk aversion parameter values θ_v used in the homeowner utility model were estimated using National Flood Insurance Program (NFIP) data (Gao 2014). Specifically, values of θ_v were chosen so that given our assumed utility model, they would result in the penetration rates reported in Dixon et al. (2006). The parameter values are $\theta_H = 3.0(10^{-5})$ and $\theta_L = 1.7246(10^{-5})$ for the higher and lower risk areas, respectively.

We identified the set of 97 probabilistic hurricane scenarios h developed in Apivatanagul et al. (2011) using the Optimization-based Probabilistic Scenario method. For each hurricane scenario, open terrain three-second peak gust wind speeds and surge depths were computed throughout the study region using the storm surge and tidal model ADCIRC (Westerink et al. 2008). This set of hurricanes was shown to result in errors small enough to be inconsequential for regional loss estimation. Using those hurricanes, we developed a set of $S = 2000$ thirty-year scenarios that represent the full set of possible scenarios with minimal error (Peng 2013). There are twenty time steps per year and $T = 600$ time steps per thirty-year scenario s.

Other input parameter values include deductible $d = \$5000$, minimum premium required $\rho = \$100$, and homeowner insurance budgets of $k_H = 5\%$ and $k_L = 2.5\%$ of building value for high and low risk homeowners, respectively. We also assumed primary insurer administrative loading factor $\tau = 0.35$ (personal communication, John Aquino, WillisRe), factor defining capped surplus $k = 3$, tax rate for surplus retained $\chi = 0.4$, co-participation factor $\beta = 95\%$, reinsurer loading factor $\varphi = 0.1$, and two values of reinsurer risk attitude $g = 0.1$ and 0.5, representing soft and hard reinsurance markets, respectively.

6. CASE STUDY RESULTS

6.1 Inverse demand functions and cost function

Using the inputs in Section 5 and the methods described in Sections 4.3 and 4.4, respectively, we developed the inverse demand functions and cost function required for the case study application of the Cournot-Nash model. For each risk region *v*, we compute these functions for the range of price (per dollar of coverage) levels from $1.35 to $5.35 with a step size of $0.001 for the demand function and $0.1 for the cost function. The minimum value of $1.35 assumes the price just covers the transaction costs for the insurer ($0.35 per dollar coverage), and therefore approximates the price at which the insurer makes zero profit. At $5.35 less than 10% of homeowners in both regions are willing to purchase insurance.

The approximated inverse demand curves are shown in Figure 1.3 (based on the calibrated parameter values of the homeowner model in Section 5), where the low risk insurance demand is obviously less sensitive to price than high risk. The main reason lies in our calibrated risk attitude parameters for the homeowner model (based on NFIP data), which suggests that homeowners in low risk region are less risk-averse than those in low risk region $(\theta_L < \theta_H)$, and thus have inelastic insurance demand. Through nonlinear regression, we create polynomial estimates of the inverse demand curves (Eq. 24), each with an adjusted $R^2 > 0.9$.

$$P_H(q_{Hj}) = 2.945(10^{-33})q_{Hj}^4 - 2.59(10^{-24})q_{Hj}^3 + 8.406(10^{-16})q_{Hj}^2 - 1.249(10^{-7})q_{Hj} + 9.287 \tag{24a}$$

$$P_L(q_{Lj}) = 6.309(10^{-15})q_{Lj}^2 - 5.814(10^{-7})q_{Lj} + 14.68 \tag{24b}$$

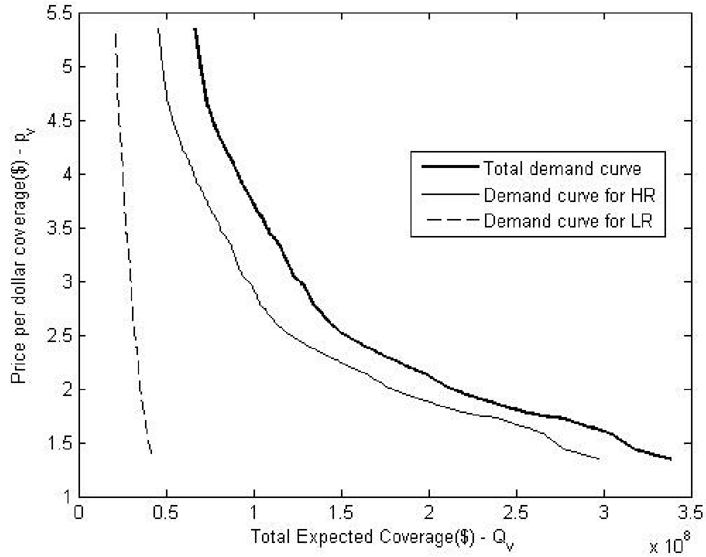

Figure 1.3 Inverse demand curves

Evaluating the insurer cost $C_j(\sum_v q_{vj})$ at each of the 1600 price level combinations (400 for p_H x 400 for p_L), we can plot the cost function as a surface. We create cost functions for three separate cases: (1) capped surplus in a soft reinsurance market (i.e., using Eq. 22a, $g = 0.1$), (2) capped surplus in a hard reinsurance market (i.e., using Eq. 22a, $g = 0.5$), and (3) retain taxed surplus in a hard reinsurance market (i.e., using Eq. 22b, $g = 0.5$). Figure 4 shows the cost function for Case 1, with the capped surplus in a soft reinsurance market. The cost functions for Cases 2 and 3 look similar, with vectors of coefficients $\left[2.272(10^7), 2.24, 2.533, -1.165(10^{-9}), -8.03(10^{-9})\right]$ and $\left[1.966(10^{-7}), 2.197, 2.541, -1.087(10^{-9}), -7.459(10^{-9})\right]$, respectively (Gao 2013). Similarly, we used a polynomial form to approximate the resultant cost surface, with an adjusted $R^2 > 0.9$ (Eq. 25).

$$C(q_{Hj}, q_{Lj}) = 1.087(10^7) + 2.598 q_{Hj} + 3.133 q_{Lj} - 1.779(10^{-9}) q_{Hj}^2 - 1.339(10^{-8}) q_{Hj} q_{Lj} \quad (25)$$

Note that the cross partial derivative of cost to insured loss in high and low risk area equals a negative constant $-1.339(10^{-8})$, which implies that there is some cost complementarity between the two distinct markets. This stems from the fact that a firm can spread its reinsurance cost across both markets which makes it more profitable to serve both markets as opposed to two separate firms (with separate cost functions) serving each market individually.

Figure 1.4 illustrates the costs move more sharply with coverage in the high risk region. That is, for a fixed level of coverage in the high risk region, the insurer's total cost remains at a similar level (shaded color) as the coverage in the low risk area increases. However, the converse is not true. For a fixed level of coverage in the low risk region, cost escalates quickly as coverage in the high risk area rises.

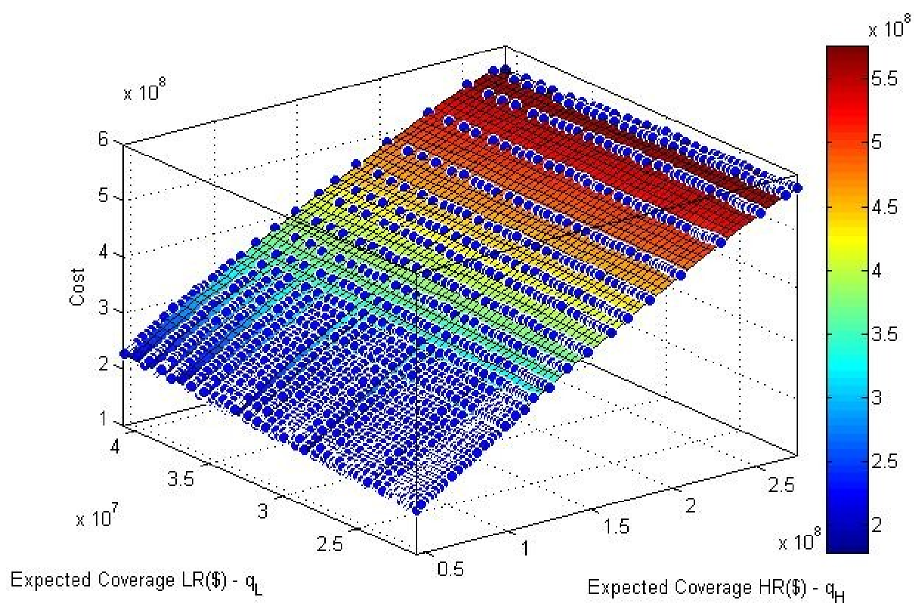

Figure 1.4 Approximated optimal cost surface for insurer with capped surplus and soft reinsurance market

6.2 Cournot-Nash model results

With the fitted inverse demand functions and cost function (Section 6.1), we apply the Cournot-Nash model as described in Section 3 (Eq. 1 to 5), by solving the first order conditions (Eq. 4 and 5). Since the fitted inverse demand and cost functions are in polynomial form, the first order conditions are a system of polynomial equations with two variables. We solve these equations using MATLAB version 7.12.0.635 (2012a) for a specified number of insurers *n*. Since the highest order polynomial is four, there are at most, four solutions that satisfy the first order conditions. From that set we select the solution that results in the highest profit per insurer. Finally, denote the optimal coverage for a single insurer $j \in (1, 2, \ldots$, as (q_{Hj}^*, q_{Lj}^*). We then obtain the optimal price per unit coverage for each region by substituting the coverage into the inverse demand functions: $p_H^* = P_H(nq_{Hj}^*), p_L^* = P_L(nq_{Lj}^*)$.

We check the stability of the Cournot-Nash model by computing the norm of the partial derivative matrix (Eq. 5). More specifically, we calculate the one-norm of the partial derivative matrix for each solution for a given number of insurers. The results suggest that our Cournot-Nash solutions from monopoly to oligopoly (for $n = 1, 2, \ldots$) all satisfy the sufficiency condition for (Cournot-Nash equilibrium) stability, i.e. $\left\| T'(Q_H^*, Q_L^*) \right\|_1 < 1$, where $Q_H^* = (q_{H1}^*, \ldots$ $\ldots$.

The Cournot-Nash model results provide the key trends across these equilibrium solutions as the market structure changes from monopoly ($n = 1$) to less and less concentrated oligopoly ($2 \le n \le 10$), (i.e., as insurer competition increases). In

Sections 6.2.1 to 6.2.3, respectively, we present the following as functions of the number of insurers in the market: (1) equilibrium price (per dollar of coverage) and insurance penetration (i.e., insured loss divided by total loss), (2) insurer's performance in terms of profit, return on equity, and insolvency, and (3) reinsurance decisions. For each section, we present results for the three cases: (1) capped surplus in a soft reinsurance market, (2) capped surplus in a hard reinsurance market, and (3) retain taxed surplus in a hard reinsurance market. In the first case, we examine the market performance under the situation that insurers do not hold extra catastrophe reserve but can more easily transfer risk to the reinsurance market, while in the second case the price for reinsurance is higher making it more difficult to transfer risk to the reinsurance market. In the last case, while the reinsurance market remains expensive, the primary insurers are assumed to keep a portion of the annual surplus as a catastrophe reserve to cover future loss.

6.2.1 Equilibrium price and insurance penetration

Figure 1.5a illustrates the price per dollar of coverage p_H under all three cases in the high risk region. As the number of insurers increases, the price they are able to charge declines quickly. At five primary insurers, the price stabilizes at 116%, 95% and 82% above the fair price of \$1.35, for the capped-soft market (case 1), capped-hard market (case 2), and retain-hard market (case 3) cases, respectively. The stabilization of equilibrium prices above the fair price ensures the profitability for oligopolies, and also implies an imperfect competitive outcome from the game.

Figure 1.5b gives the price per dollar coverage in the low risk region as a function of the number of primary insurers. Due to the cost complementarity between the distinct insurance markets/risk regions (Section 6.1), an insurer could cross subsidize its business in low risk region with its business in high risk region, which enables the insurer(s) to offer coverage with a unit price lower than the fair price. It is interesting to notice that the decline in price as the number of insurance carriers increase is less dramatic for the low risk region than the high risk region. For Case 1, for example, in the low risk area, the price decreases 25% from \$1.20 to \$0.90 as we move from one to ten insurers. In the high risk area, the same change is 42%, from \$5.00 to \$2.90.

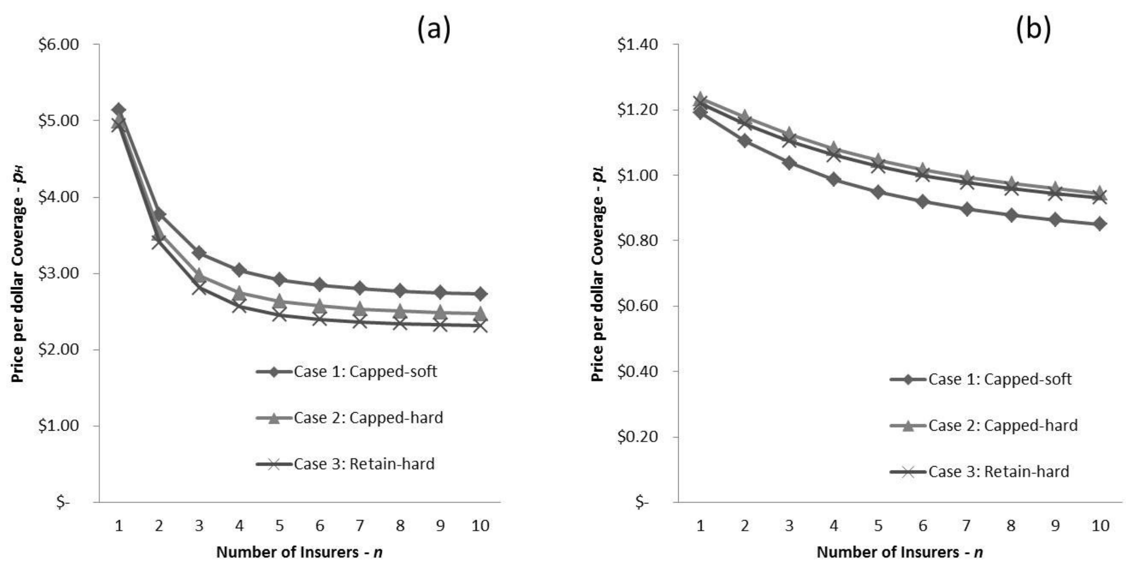

Figure 1.5 Equilibrium price per dollar coverage vs. number of insurers, for (a) high risk region and (b) low risk region, for the three cases

As competition drives down the price in the high risk areas, the total insured loss and insurance penetration increase until the insurance penetration is 35% when there are ten primary insurers in the retain-hard market case (Fig. 1.6a). Comparing the three cases suggests that the insurance penetration is higher in the hard reinsurance market, and in the presence of policies that encourage establishment of larger reserves by the primary insurers (i.e., allowing them to retain profit as in Case 3). This makes sense because the price is lower in those cases (Fig. 1.5). In the low risk area, there are more modest increases in insurance penetration (Fig. 1.6b) as the number of insurers grows because the impact of competition on pricing is more modest than in the high risk region. Note that the penetration is higher for the low risk area than the high risk area because the total loss is smaller in the former, and penetration is defined as insured divided by total loss.

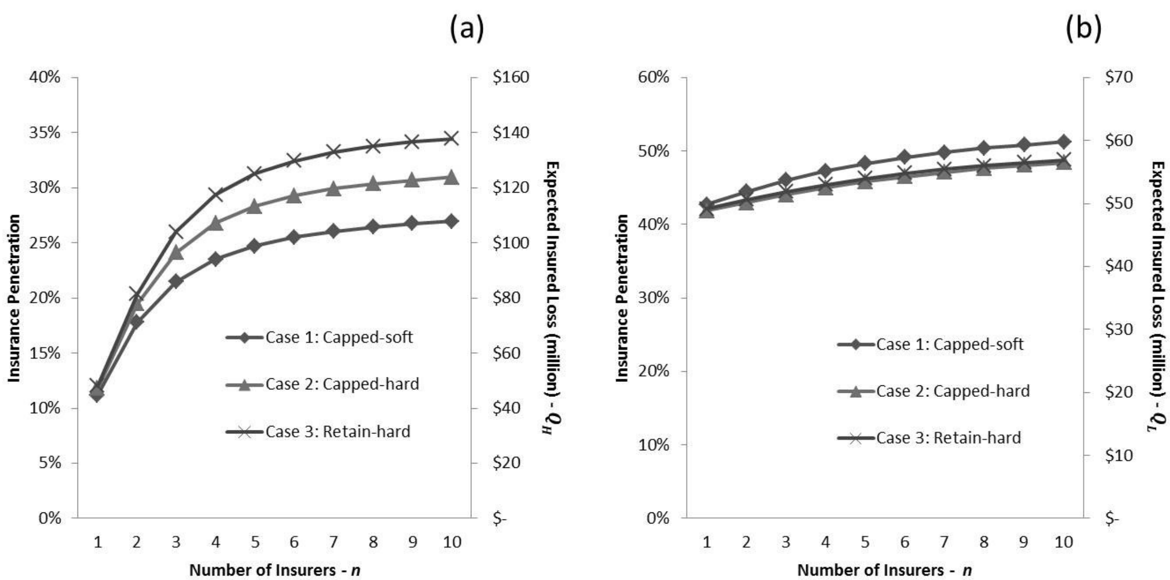

Figure 1.6 Insurance penetration and total expected annual insured loss vs. number of insurers, for (a) high risk region and (b) low risk region, for the three cases

6.2.2 Insurer's performance

We can also examine how each insurer's performance changes with the number of insurers in the market. Here we measure an insurer's performance using three metrics: (1) average annual profit, (2) average annual return on equity, and (3) average annual probability of insolvency. Average annual profit is as given in Equation 20. Return on equity (ROE) measures profitability by showing how efficiently capital is being used. Investors seek a high and stable return on equity, which is defined here as the annual net profit divided by the average surplus between the beginning and end of the year. Here we compute the average annual ROE, denoted as R, for the years η that the insurer is solvent (Eq. 26). To calculate the annual probability of insolvency α, we define ϕ^s to be a binary indicator variable that is one if the insurer becomes insolvent at any time in scenario s and zero otherwise, and take the average over all scenarios s (Eq. 27).

$$R = \frac{1}{S\eta} \sum_s \sum_{y \in \eta} \frac{F^{sy}}{0.5(\mu^{s,y-1} + \mu^{sy})} \tag{26}$$

$$\alpha = \frac{1}{SY} \sum_s \phi^s \tag{27}$$

Figure 7a shows the single insurer's average annual profit as a function of the number of primary insurers. As expected, profit declines quickly as the number of competitors increases. Despite the difference in the penetration of insurance for the three cases in the high risk area, the average annual profits among them have only minor differences (10%).

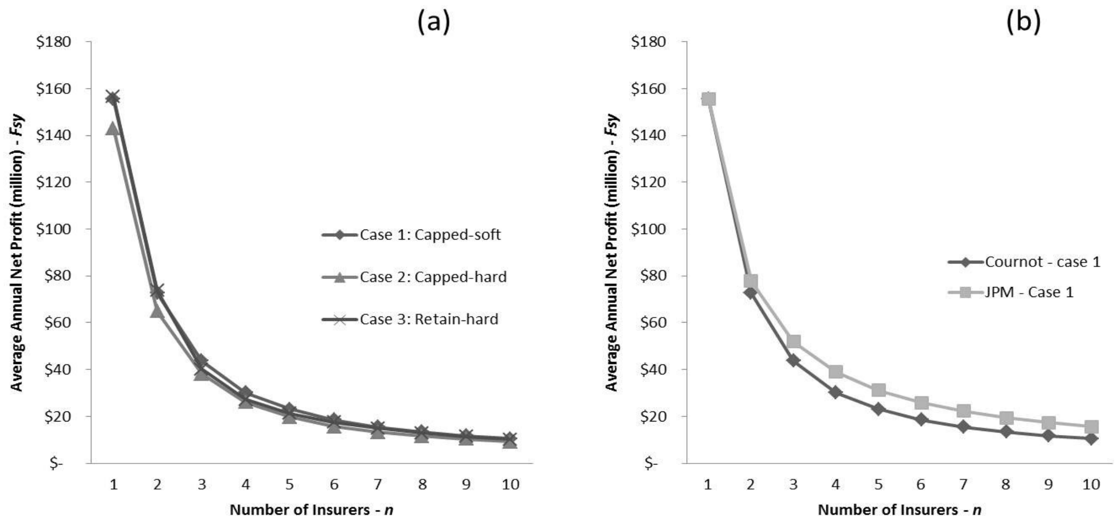

Figure 1.7 Average annual net profit vs. number of insurers (a) for the three cases, and (b) for the Cournot-Nash and Joint Profit Maximization (JPM) solutions for Case 1

Figure 1.7b gives a comparison of the average annual profit for a capped-soft market (Case 1) under Cournot-Nash competition to that under the joint profit maximization (JPM), which assumes all insurers form a cartel and cooperate. The joint profit maximization is obtained by using the monopoly solution (profit level) divided by the number of insurers. The JPM and the single shot Cournot outcome form the support of the set of possible outcomes to the infinitely repeated game. Given the similarity of the outcomes between the JPM and Cournot-Nash game, even under a repeated game with discount rates that support tacit collusion (i.e., JPM), the profitability does not diverge substantively from the single shot Cournot-Nash solution.

Figure 1.8a shows how average annual return on equity (ROE) changes with market competition. Comparing the capped-hard market and retain-hard market cases (Cases 2 and 3) suggests that the ROE is higher for an insurer with a capped surplus than one retaining after-tax profits, because the surplus (and therefore, ROE denominator) is smaller in the former case. This case also reflects the fact that the insurer is retaining more risk. As illustrated in Figure 1.8b, the annual probability of insolvency is substantially higher (more than 50%) with the capped surplus (Case 2) than without (Case 3), all else being equal. Comparing the soft and hard reinsurance markets with capped surplus (Cases 1 and 2) in Figure 8b suggests that the probability of insolvency is higher when it is more difficult to buy reinsurance (Case 2). Even when reinsurance is available in the soft market (Case 1), when more insurers enter the market (especially at more than five insurers), it becomes more expensive relative to their profits, and thus insurers buy less of it and their probability of insolvency increases.

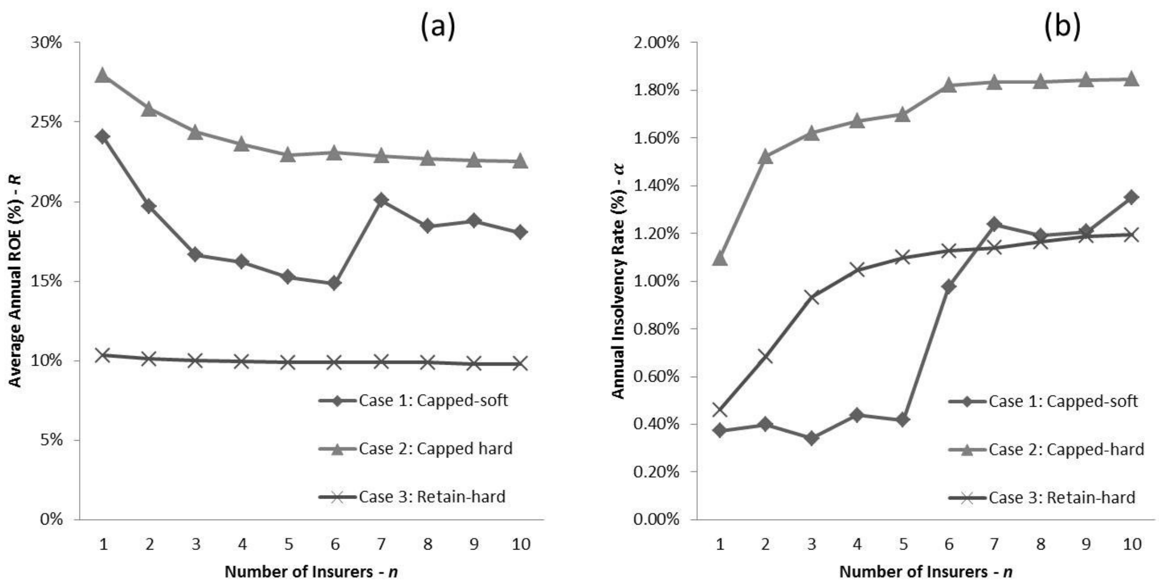

Figure 1.8 (a) Average annual return on equity and (b) annual insolvency rate vs. number of insurers, for the three cases

6.2.3 Reinsurance decisions

As the number of insurers in the market changes, the use of reinsurance changes as well. To first understand how the reinsurance is used, we can compare the three cases. When the reinsurance market is soft, insurers are assumed not to keep an additional surplus of funds; hence they rely on reinsurance to avoid insolvency. By transferring risk, primary insurers obtain a better loss profile. Figure 9 illustrates the cumulative density function (CDF) of the net liability of a single primary insurer for the high risk region when the market includes five primary insurers, where net liability refers to the net loss the primary insurer is liable for after deductible and reinsurance payments are received (Fig. 1.2). Among the three cases, the expenditures are lower in a soft reinsurance market (i.e., Case 1). By contrast, more of the risk is retained by the primary insurer under a hard reinsurance market (with or without capped surplus, i.e., Case 2 and Case 3) (Fig. 1.9). Of course this benefit comes at a cost—the reinsurance expenses are substantially higher (Fig. 1.10). Figure 1.10 shows how the average annual expenditures for reinsurance, including base and reinstatement premiums, changes with competition. An insurer's dependence on reinsurance is dramatically reduced when reinsurance is more expensive (i.e., in a hard market, Cases 2 and 3) and/or under intense competitive pressures (i.e., when there are more insurers in a soft market, Case 1).

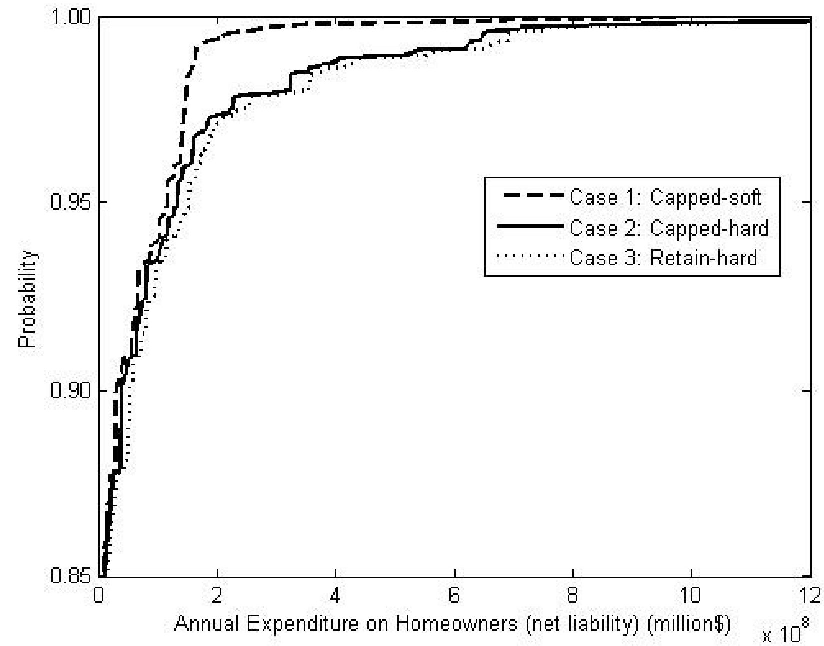

Figure 1.9 Cumulative density function of net liability of a single primary insurer, for high risk region, when n=5 (oligopoly)

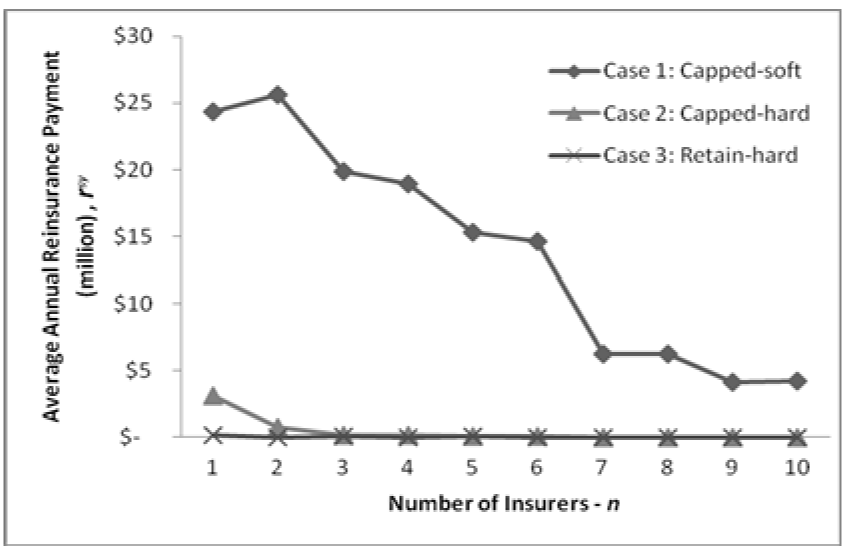

Figure 1.10 Single insurer's annual base reinsurance premium vs. number of insurers, for three cases

Figure 1.11 shows the CDF of the reinsurer's liability under different market configurations (monopoly and oligopoly). As more insurers join the market, the loss/risk from the entire primary insurance market that is transferred to the reinsurer increases as well.

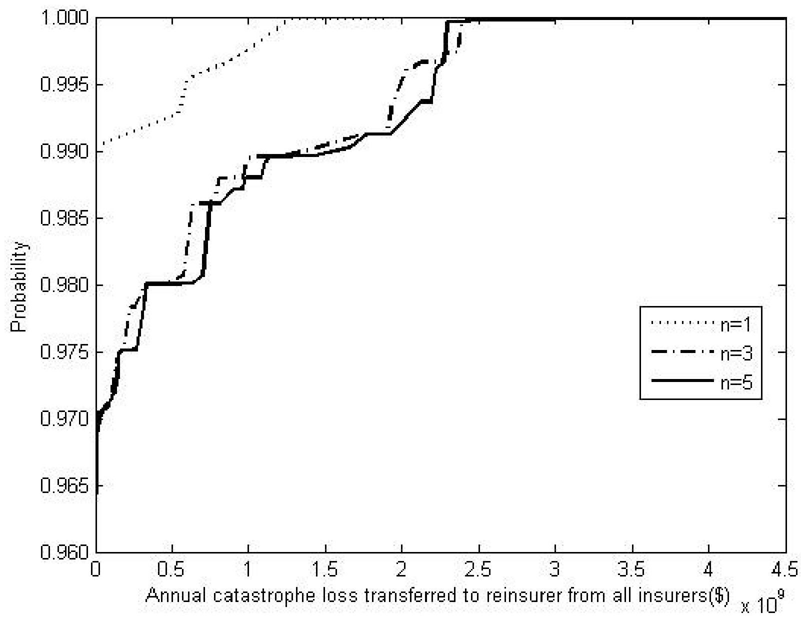

Figure 1.11 Cumulative density function of catastrophe loss transferred to the reinsurance market for three primary insurance market sizes (n=1, 3, 5) (Case 1)

Finally, Figure 1.12 gives the ratio of single insurer's average annual surplus to expected insured loss for three cases. As the number of insurers increases, though the total expected insured losses in both regions are increasing (Fig. 1.6a and 1.6b), the surplus held by the insurer relative to the expected insured loss declines. This simply reflects the decline in the (relative) profitability as a result of competition.

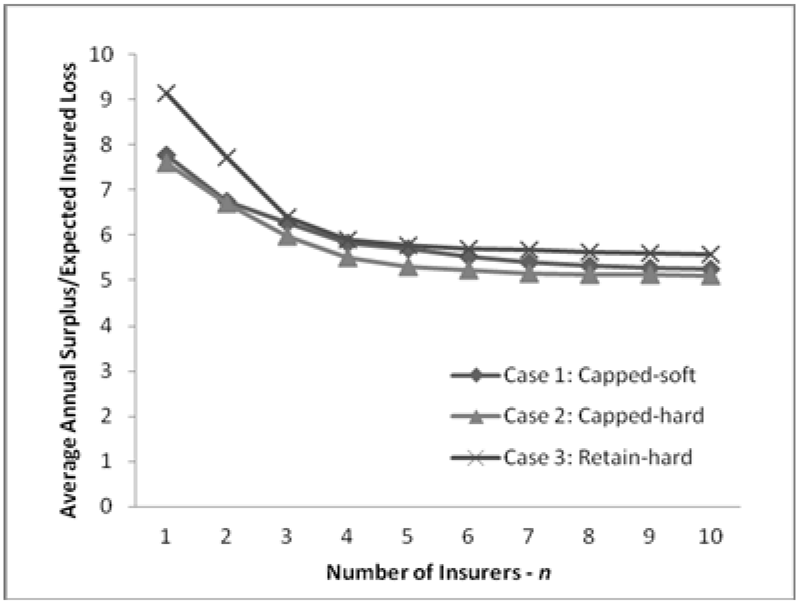

Figure 1.12 Ratio of a single insurer's annual surplus to expected insured loss vs. number of insurers, for the three cases

7. SUMMARY

In this study, we integrated a Cournot-Nash equilibrium framework with individual optimization models for key stakeholders (homeowners and insurers), and a loss estimation model. These tools were applied to a full-scale case study for hurricane risk (flood-wind combined) to residential buildings in Eastern North Carolina. We examined the impact of competition in this voluntary catastrophe insurance market by investigating the market equilibrium price and insurance penetration; insurer's performance in terms of profit, return on equity, and insolvency; as well as its decisions in mitigating financial risk through either reinsurance or self-insurance using cash reserves (retained surplus).

The results from our study indicate that the level of concentration in the primary insurance market can lead to significant differences in the operational decisions for an individual insurance firm. Choices on reinsurance and the magnitude of retained surpluses and therefore resultant insolvency rates can change dramatically with changes in the number of firms competing in the market. There clearly exists a balance between the penetration rate of insurance and the insurers' profitability/solvency. Less market concentration makes it harder for primary insurers to maintain profitability and avoid insolvency, but benefits the homeowners, as the competition drives down the price. Especially for insurers without reserves, competition results in increasing risks to be transferred to the reinsurers, which is difficult if reinsurance prices are high. This is further justification of (existing) reserve requirements. Overall, these findings provide useful information for regulators to make public policy decisions which balance insurer profitability and solvency against insurance penetration. There are opportunities for further work in at least four areas. The first area would include the extension of the modeling structure to include more of the dynamics in the evolution of the building infrastructure over time. For example, building stock is not constant. Coastal populations are growing, and with that growth comes renewal in the building stock and the creation of new building stock. This impacts the spatial pattern and distribution of the losses. The second area would include exploration of different regulatory policies and the introduction of game theoretic treatment of the reinsurance market. The third area is the inclusion of new models of homeowner demand. This paper focused on a utility maximizing model for homeowner decision-making. This modeling could be extended to integrate more ideas from behavioral economics and cognitive psychology. The fourth area would be to relax the assumption that homeowners have complete information about the vulnerability of their losses.

REFERENCES

Abbring, J. H. and Campbell, J. R. (2010). "Last-in First-Out Oligopoly Dynamics." Econometrica **78**(5): 1491-1527.

Abreu, D. (1983). Repeated Games with Discounting: A General Theory and an Application to Oligopoly, Ph. D. Dissertation, Princeton University.

Alos-Ferrer, C., Ania, A. B. and Schenk-Hoppe, K. R. (1998). "An Evolutionary Model of Bertrand Oligopoly." Games and Economic Behavior **33**: 1-19.

Apivatanagul, P., Davidson, R., Blanton, B. and Nozick, L. (2011). "Long -term regional hurricane hazard analysis for wind and storm surge." Coastal Engineering **58**(6): 499-509

Babocock, B. A., Choi, E. K. and Feinerman, E. (1993). "Risk and Probability Premiums for CARA Utility Functions." Journal of Agricultural and Resource Economics **18**(1): 17-24.

Bernnardo, A. and Chiappori, P. A. (2003). "Bertrand and Walras Equilibria under Moral Hazard." Journal of Political Economy **111**(4): 785-817.

De Feo, G. and Hindriks, J. (2005). Efficiency of Competition in Insurance Markets with Adverse Selection. CORE Discussion Paper.

Dixon, L., Clancy, N. Seabury, S. A. and Overton, A. (2006). The National Flood Insurance Program's Market Penetration Rate: Estimates and Policy Implications.

Federal Emergency Management Agency (FEMA). (2012). *HAZUS-MH 2.1 Hurricane Model* Technical Manual, Washington, D.C.

Florida Public Hurricane Loss Model (FPHLM). (2005). Engineering Team Final Report, Vol. I, II, and III. Florida International University, http://www.cis.fiu.edu/hurricaneloss (Nov. 16, 2010).

Froot, K. A. (2001). "The market for catastrophe risk: a clinical examination." Journal of Financial Economics **60**: 529-571.

Gao, Y. (2014) Modeling a Market for Natural Catastrophe Insurance, Ph. D. dissertation, Cornell University

Grace, M. F., Klein, R. W. and Kleindorfer, P. R. (1998). Overview of Catastrophe Insurance Markets in the U.S., The Wharton Catastrophe Risk Management Project.

Grace, M. F., Klein, R. W., Kleindorfer, P. R. and Murray, M. R. (2003). Catastrophe

Insurance: Consumer Demand, Markets and Regulation. Topics in Regulatory Economics and Policy.

Grier J. (1996). Florida residents are still weathering the storm in the aftermath of Hurricane Andrew: The cost of starting over is prohibitive for many. Federal Reserve Bank of Atlanta. Partners in Community and Economic Development, Summer 1996; 6(2). Available at: www.frbatlanta.org/, Accessed on June 26, 2009.

Harrison, C. (2004). Reinsurance Principles and Practices. Malvern, PA, The Institutes, American Institute for Chartered Property Casualty Underwriters.

Hayes, T. L. and Neal, D. A. (2009). FEMA, National Flood Insurance Program Actuarial Rate Review In Support of the Recommended October 1, 2011 Rate and Rule Changes at **3** (2011), available at http://www.fema.gov/library/viewRecord.do?id=4853.

Hill, R. D. (1979). "Profit Regulation in Property-Liability Insurance." The Bell Journal of Economics **10**(1): 172-191.

Huck, S., Normann, H.-T. and Oechssler, J. (1999). "Learning in Cournot Oligopoly - An Experiment." The Economic Journal **109**(454): C80-C95.

Jaffee, D. M. and Russell, T. (1997). "Catastrophe Insurance, Capital Markets, and Uninsurable Risks." Journal of Risk and Insurance **64**(2): 205-230.

Joskow, P. L. (1973). "Cartels, Competition and Regulation in the Property-Liability Insurance Industry." The Bell Journal of Economics and Management Science **4**(2): 375-427.

Kesternich, I. and Schumacher, H. (2009). On the Use of Information in Repeated Insurance Markets. SFB/TR 15 Discussion Paper No. 280.

Kleindorfer, P. R. and Klein, R. W. (2003). Regulation and markets for catastrophe insurance Advances in Economic Design. M. R. Sertel and S. Koray, Springer: 263.

Kesete, Y. Modeling Homeowner Insurance decision making and Insurer-Homeowner Interactions in Managing Natural Disaster Risk, Ph. D. dissertation, Cornell University, in progress.

Kousky, C. (2011) "Managing natural catastrophe risk: State insurance programs in the United States." Review of Environmental Economics and Policy **5**(1): 153–171.

Kreisel, W. and Landary, C. (2004). "Participation in the national flood insurance program: An empirical analysis for coastal properties." Journal of Risk and

Insurance 71(3): 405-420.

Kunreuther, H. (1996). "Migitating Disaster Losses throught Insurance." Journal of Risk and Uncertainty **12**: 171-187.

Kunreuther, H. (1998). "The Role of Insurance in Dealing with Catastrophic Risks from Natural Disasters." Alternative Approaches to Insurance Regulation.

Kunreuther, H., Meyer, R., Zeckhauser, R., Slovic, P., Schwartz, B., Schade, C., Luce, M. F., Lippman, S., Krantz, D., Kahn, B., and Hogarth, R. (2002). “High stakes decision making: Normative, descriptive and prescriptive considerations.” Marketing Letters **13**(3): 259–268.

Kunreuther, H. and Pauly, M. (2004). "Neglecting disaster: Why don't people insure against large losses?" Journal of Risk and Uncertainty **28**(1): 5-21.

Kunreuther, H. (2006). "Disaster mitigation and insurance: Learning from Katrina." The Annuals of the American Academy of Political and Social Science **604**(1):208-227.

Kunreuther H, and Pauly M. (2006) “Rules rather than discretion: Lessons from Hurricane Katrina.” Journal of Risk and Uncertainty **33**(1/2):101-116.

Kunreuther, H. and Michel-Kerjan, E. (2009). At War With the Weather: Managing large-scale risks in a new era of catastrophes. Cambridge, MA, The MIT Press.

Litan, R. (2006). “Sharing and Reducing the Financial Risks of Future ’Mega-Catastrophes’”, Issues in Economic Policy, No. 4, 2006, Washington, DC: The Brookings Institution.

Lynne, P. and Dan, R. (2008). Industrial Organization, South-Western Publishing Co.

Mas-Colell, A., Whinston, M. D. and Green, J. R. (1995) Microeconomic Theory. New York, Oxford, Oxford University Press.

MATLAB and Statistics Toolbox Release (2012a), The MathWorks, Inc., Natick, Massachusetts, United States.

Murat, G., Tonkin, R. and Juttner, D. (2002). "Competition in the General Insurance Industry." Zeitschrift für die gesamte Versicherungswissenschaft **3**: 453-481.

Nissan, E. and Caveny, R. (2001). "A comparison of large firm dominance in property and liability insurance with major industries." Journal of Insurance Issues **24**(1&2): 58-73.

Paudel, Y. (2012) “A comparative study of public-private catastrophe insurance systems: Lessons from current practices.” The Geneva Papers on Risk and

Insurance-Issues and Practice **37**(2):257-285.

Peng, J. (2013) Modeling natural disaster risk management: Integrating the roles of insurance and retrofit and multiple stakeholder perspectives, Ph. D. dissertation, University of Delaware.

Peng, J., Shan, X., Davidson, R., Nozick, L., Kesete, Y., and Gao, Y. "Hurricane loss modeling to support regional retrofit policymaking: A North Carolina case study." 11th International Conference on Structural Safety and Reliability - ICOSSAR'13 New York, NY, June 16-20, 2013.

Polborn, M. K. (1998). "A Model of an Oligopoly in an Insurance Market." The Geneva Papers on Risk and Insurance Theory **23**: 41-48.

Rothschild, M. and Stiglitz, J. (1976). "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information." The Quarterly Journal of Economics **90**(4): 629-649.

Schlesigner, H. and Schulenburg, J.-M. G. v. d. (1991). "Search costs, switching costs and product heterogeneity in an insurance market " The Journal of Risk and Insurance **58**(1):109-119.

Sonnenholzner, M. and Wambach, A. (2004). Oligopoly in insurance markets. Encyclopedia of Actuarial Science. New Jersey, John Wiley & Son.

Spence, A. M. (1978). "Product differentiation and performance in insurance markets." Journal of Public Economics **10**(3): 427-447.

State Climate Office of North Carolina (SCONC). "Hurricanes." North Carolina State University, ⟨http://www.nc-climate.ncsu.edu/climate/hurricane.php⟩ (Nov. 16, 2010)

Stiglitz, J. E. (1977). "Monopoly, Non-Linear Pricing and Imperfect Information: The Insurance Market." The Review of Economic Studies **44**(3): 407-430.

Taggart, M. and van de Lindt, J. (2009) "Performance-based design of residential wood-frame buildings for flood based on manageable loss." Journal of Performance of Constructed Facilities **23**(2): 56-64.

Tirole, J. (1988). The Theory of Industrial Organization, MIT Press.

U.S. Government Accountability Office (U.S. GAO), U.S. Government. Natural Catastrophe Insurance: Analysis of a Proposed Combined Federal Flood and Wind Insurance Program, GAO-08-504. Washington, DC: Government Accountability Office, 2008.

U. S. Government Accountability Office (U.S. GAO), U.S. Government. Natural

Disasters: Public Policy Options for Changing the Federal Role in Natural Catastrophe Insurance. Washington, DC: Government Accountability Office, November, 2007. Available at:http://www.gao.gov/products/GAO-08-7, Accessed on April 28, 2013

Van de Lindt J, and Taggart, M. (2009) "Fragility analysis methodology for performance-based analysis of wood-frame buildings for flood." Natural Hazard Review**10**(3):113-123.

Villeneuve, A. C. B. (2005). "Optimal Risk-Sharing under Adverse Selection and Imperfect Risk Perception." The Canadian Journal of Economics **38**(3): 955-978.

Wambach, A. (2000). "Introducing Heterogeneity in the Rothschild-Stiglitz Model." The Journal of Risk and Insurance **67**(4): 579-592.

Westerink J, Luettich R, Feyen J, Atkinson J, Dawson C, Powell M, Dunion J, Roberts H, Kubatko E, and Pourtaheri H. (2008) "A basin-to-channel-scale unstructured grid hurricane storm surge model as implemented for Southern Louisiana." Monthly Weather Review, **136**: 833-864.

Zhang, A. and Zhang, Y. (1996). "Stability of a Cournot-Nash equilibrium: The multiproduct case." Journal of Mathematical Economics **26**: 441-462.

CHAPTER 2

MODELING GOVERNMENT INTERVENTIONS IN A NATURAL CATASTROPHE INSURANCE MARKET

1. INTRODUCTION AND RELATED LITERATURES

It has been argued that the natural disaster insurance market in the United States has been stumbling for decades (Grace et al. 2003; Kunreuther and Michel-Kerjan 2009). Private insurers face significant insolvency risks with the result that some have failed or simply withdrawn from the market in the past few years (Mills et al. 2001; Kunreuther and Michel-Kerjan 2009; Charpentier and Maux 2010).

The fundamental source of stress in the private insurance market is the lack of capital to finance the highly non-smooth nature of the losses (Jaffee and Russell 1997). Furthermore, tax laws and accounting principles discourage U.S. property and casualty insurers from accumulating funds specifically to cover these large losses should they occur, albeit with low a probability of occurrence (Jaffee and Russell 1997; Cleary and Boutchee 2002). The tax disadvantage of equity financing together with the non-tax costs of capital have limited the scope of the catastrophe insurance and reinsurance market (Harrington and Niehaus 2003), which partially motivates the proposal from the National Association of Insurance Commissioners (NAIC) to facilitate pre-event tax-deferred catastrophe reserves (adopted by several countries, such as Australia, Barbados, Canada, Finland, Germany, Italy, Japan, Switzerland and the United Kingdom) – a proposal that has been advocated by the American insurance industry for at least a decade (Cleary and Boutchee 2002).

These capital challenges do not necessarily imply that these risks cannot be insured (Jaffee and Russell 1997). If the risks can be more easily priced into premiums so as to yield a reasonable profit margin for insurers and those insurers have fewer barriers to raising capital when needed, several authors have speculated that the private catastrophe loss insurance market would become significantly more viable (Cummins 2006; Jaffee and Russell 2006). Rather than focusing on facilitating the private insurance market, the U.S. government has focused on ensuring an "affordable" price for insurance, coupled with federally funded programs to supplement or substitute for private natural catastrophe loss insurance. The implementation of these programs has likely created market distortions that decrease the incentives for private entities to enter the market effectively undermining the private market's ability to effectively manage catastrophe risk (Klein 1998; Grace et al. 2003; Cummins 2006; Kunreuther and Michel-Kerjan 2009; Charpentier and Maux 2011). Though government funded insurance programs (which could be considered as the residual market), seemed successful at first, they ultimately accumulated large deficits (e.g., the National Flood Insurance Program) and require substantial public funding to continue operations (US GAO 2007).

These problems have made it necessary to reexamine government's role in the natural catastrophe insurance market, with the goal of identifying government policies that facilitate, rather than replace, the more efficient private market solutions (Jaffee and Russell 1997; Cummins 2006; Charpentier and Maux 2011). The policy options that affect private insurance markets include regulation/deregulation (i.e., premiums, financial requirements and barriers to entry and exit), mitigation initiatives, insurance price supports, property acquisition as well as the abovementioned tax-favored catastrophe reserve policy.

This paper investigates the impacts of government interventions, which are designed to reduce uninsured losses by facilitating the private insurance market with minimal regulatory intervention in the competitive mechanism. On the demand side, the effects of price support and acquisition programs are explored. On the supply side, the impacts of pre-event tax-favored catastrophe reserves are considered. These interventions are integrated into the modeling of an unaided catastrophe insurance market with explicit representation of the key stakeholders (homeowners, primary insurers and reinsurers) given in Gao et al. (2013). This framework uses a one shot Cournot-Nash noncooperative game structure to model the market.

The next section summarizes the modeling framework. The third section develops a case study based on residential buildings in North Carolina. The fourth section presents conclusions.

2. MODELING FRAMEWORK

2.1 Overview of interacting models

This paper utilizes a Stackelberg framework to represent the government's interaction with the stakeholders in the insurance market, where the government functions as the Stackelberg leader by creating the regulations which govern the interactions between the primary insurers, and homeowners. Figure 2.1 gives an overview of the modeling framework and component models. In this framework, the government decides whether to offer price supports to homeowners and the magnitude of those supports, as well as whether or not to acquire homes that are particularly vulnerable. Both of these decisions affect the demand for insurance.

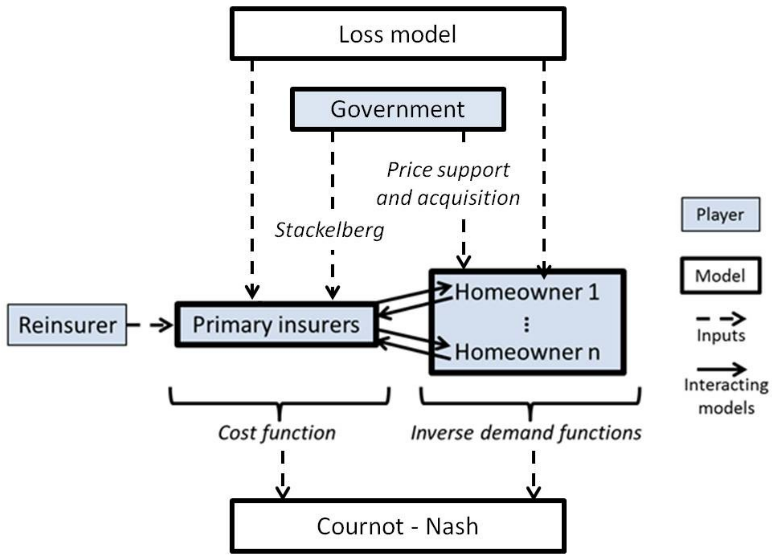

Figure 2.1 Structure of interacting models

Following the framework, individual insurance decisions on the demand side are captured by a utility maximization model with a homeowner budget constraint. This utility model considers homeowner heterogeneity with respect to risk attitude. On the supply side, each primary insurer is assumed to maximize their expected net profit through a stochastic optimization model, to conclude how much risk to transfer to the reinsurer (we assume one layer of catastrophe risk excess of loss reinsurance for simplicity) at any price and demand level (Gao et al. 2013). These elements support the estimation of cost and demand functions which specify the Cournot-Nash model.

A state-of-the-art regional catastrophe loss estimation model is used to estimate homeowner losses to a range of events that represent the regional hurricane hazard (Peng et al. 2013). These events are used to estimate the hazard each homeowner faces. We assume that this information is available to both primary insurers and homeowners. Primary insurers use this information to establish premiums while homeowners use it to make insurance purchase decisions. As stated previously, the government intervenes in the interactions between insurers and homeowners through premium price supports and acquisition decisions, which affects insurance demand and therefore affects the cost structure of the primary insurers.

2.2 Estimating reaction functions

These interacting models are used to estimate reaction functions by varying their inputs and computing their outputs (i.e., optimization result from each model) as the response. More specifically, denote price per unit coverage charged by an insurer in risk region *v* as p_v, the homeowner, government and loss models together are used to derive the insurance demand (in terms of expected coverage) Q_v by region.

We assume that the government supports a portion of the homeowners that cannot afford the premium, subject to a budget constraint on total pre-event government outlays. Therefore, the government aided homeowners' premium will be reduced to their budget level, and they will make the insurance purchase decision based on the subsidized premium they face. However, if purchasing the insurance at the subsidized price does not maximize their expected utility; those homeowners would not purchase insurance. On the other hand, for the homeowners for whom the premium does not exceed their estimated financial resources, they purchase insurance if it maximizes their expected utility.

Denote the total expected insurance coverage Q_v from all homeowners (aided and unaided) in region *v*, at price level Q_v, as $Q_v = D_v(p_v)$, which includes the units of coverage (per dollar of expected loss) fully funded by the homeowner and subsidized by the government. The relationship between Q_v and p_v (demand function $Q_v = D_v(p_v)$ or its inverse $p_v = D_v^{-1}(Q_v) = P_v(Q_v)$, at specific price levels are identified by employing the homeowner model repeatedly at those price levels. Regression is used to establish the continuous form of this relationship.

Similarly, the insurer model interacting with the homeowner, government and reinsurer models are used to develop the cost function. At price level p_v, and corresponding insurance demand level $Q_v = D_v(p_v)$, the insurer's optimal cost of doing business in all risk regions *v*, results from the stochastic optimization model and is denoted as $C(\sum_v Q_v)$. The key decision the insurer makes using this model is whether to purchase reinsurance; and if so, what is the attachment point and maximum payout. These decisions yield the maximum profit level given the price and homeowner reaction to that price (i.e. demand).

This method of transforming these individual optimization models into (reaction) functions has been applied in (Gao et al. 2013), which focuses solely on the private natural catastrophe insurance market (without government interventions) with the same homeowners, insurers and loss model as described in Figure 2.1.

2.3 Cournot-Nash model

Both the inverse demand functions $P_v(Q_v)$ and cost function $C(\sum_v Q_v)$ derived as described above become the input functions in the static Cournot-Nash noncooperation game theoretic model, which captures the strategic interaction of the noncooperative primary insurers We assume perfect information is available to all parties and insurers are homogeneous, i.e., each insurer faces the same basic cost structure, and is equally capable of handling all levels of demand. In the static noncooperative game, each insurer forms a best-response strategy that maximizes net profit recognizing the best-response strategies of its competitors. All insurers in the market make their decisions simultaneously. For simplicity, we assume that insurers always offer full coverage. As in a Cournot game, we assume that the competition is on the quantity of insurance sold at the market equilibrium price. Each property owner, at most, only purchases one insurance contract from a single insurer. Furthermore, by symmetry, total sales at the equilibrium price will be divided evenly geographically and with respect the vulnerability of the building inventory, which will result in the same net profit for each insurer.

More specifically, single insurer *j*'s decision in the Cournot-Nash game is the total coverage offered in risk region *v* is denoted as q_{vj}. This is the annual expected insured loss. Their annual net profit can be described as the premiums received by insurer j by those they insure minus the cost of providing that insurance on an annual basis (Eq. 1). Note that the first term is a function of the coverage provided by all insurers $\sum_j q_{vj} = Q_v$ since that determines the price $P_v(Q_v)$ (i.e., inverse demand function as in Section 2.1), while the second term is a function of only the coverage offered by the insurer *j*, and $C_j(\sum_v q_{vj})$ refers to the cost function (also described in Section 2.1). Expressing the individual insurer's objective as a function of its decisions/strategies and its competitors' facilitates (as given in Eq. (1)) the derivation of the optimization conditions (Eq. 2).

$$\pi_j(\sum_v q_{vj}, \sum_v Q_{-vj}) = \sum_v q_{vj} P_v(q_{vj}, Q_{-vj}) - C_j(\sum_v q_{vj}) \quad q_{vj} \geq 0 \ \forall v \tag{1}$$

We assume each insurer is a net profit maximizer and formulate a best response function that recognizes the strategic interaction with other insurers. If the problem is differentiable, the optimal solution for net profit maximization should satisfy the first order conditions (Eq. 2), based on the envelope theorem (Mas-Colell et al. 1995), which also refers to the relationship that marginal cost equals to marginal revenue:

$$\frac{\partial \pi_j}{\partial q_{vj}} = P_v(q_{vj}, Q_{-vj}) + q_{vj} \frac{\partial P_v(q_{vj}, Q_{-vj})}{\partial q_{vj}} - \frac{\partial C_j(\sum_v q_{vj})}{\partial q_{vj}} = 0, \quad \forall v \tag{2}$$

Denote insurer *j*'s actions that satisfy the first order conditions (for profit maximization) as a vector of reactions (i.e., coverage provided in each region *v*, or best responses) $x_j^* = (q_{vj}^*, \forall v)$. Since the insurers are all homogeneous, they shall have the same optimality conditions and best responses. Thus, by symmetry, insurer *j*'s rivals' actions (under simultaneous optimization) are:

$$x_k^* = x_j^*, \forall k \neq j; \ \ Q_{-vj} = \sum_{k \neq j} x_k^* = (n-1) q_{vj}^*, \forall v.$$

By substitution the first order conditions (Eq. 2), they can be rewritten as:

$$P_v(nq_{vj}^*) + q_{vj}^* \frac{\partial P_v(nq_{vj}^*)}{\partial q_{vj}} - \frac{\partial C_j(\sum_v q_{vj})}{\partial q_{vj}} = 0, \quad \forall v \tag{3}$$

The solution q_{vj}^* that satisfies Equation 3 is firm *j*'s best response to its rivals' best responses, and thus represents the equilibrium condition for this model, i.e., the Nash equilibrium solution for the *n* insurers in the market. The equilibrium price is found via the inverse demand function $p_v^* = P_v(nq_{vj}^*)$.

2.4 Government intervention model

Since the government is assumed to have a fixed 30 year budget for premium support and acquisition we use the following procedure to estimate (1) which homeowners get premium supports and the magnitude of those supports; and (2) which properties are to be acquired and removed from the building inventory.

For a given premium p_H per dollar of insured loss in the high risk region (*H)*, all homes for which the resultant premium would exceed 5% of the value of the home on an annual basis, which is assumed to approximate the homeowner's budget for insurance, are identified. Those homes are then ranked in decreasing order by their benefit to cost ratios. The benefit is assumed to be the expected average annual homeowner losses avoided if the policy is purchased. The cost is assumed to be the annual support required to create a premium payment for the homeowner that is 5% of the value of the home annually. The larger this ratio is, the higher the priority on the part of the government to provide assistance. The homeowners can only receive the support if they are willing to purchase the annual insurance at their maximum budget level (i.e., 5% of the value of the home annually), which is estimated using the homeowner utility model. In other words, if the selected homeowners are not willing to pay up to their budget level for the insurance, they do not receive the subsidy from the government. If the premium support over 30 years exceeds the value of the home, the home is acquired, if that level of benefit to costs falls within the government's budget limit.

3. CASE STUDY

This section describes a full-scale, realistic case study for hurricane risk (flood and wind combined) for single-family residential buildings in Eastern North Carolina. We divide North Carolina into a high risk region *H* (defined as within 3 miles to the coast) and low risk region *L* (beyond 3 miles to the coast). We only consider homes in the high risk region as candidates for the price support and acquisition.

3.1 Relationship between government funds available and insurance demand

Figure 2 gives the increased insurance demand $Q_H = D_H(p_H)$ in the high risk region *H* at price level p_H ranging from \$1.35 to \$5.37 per dollar coverage, due to government premium supports. A risk-based premium approach is adopted in this study, and the fair (minimum) premium is assumed to be \$1.35 per dollar coverage, which is assumed to cover the transaction/administration cost (assumed to be \$0.35 per dollar transaction) and the expected loss (per dollar of expected loss coverage). The original unaided insurance demand $D_H(p_H)$, as described in Section 2.2, stems from homeowners who are willing to purchase insurance and can afford their premium. The affordability requirement imposed on the model is that the annual premium must be less than or equal to 5% of value of the home.

It should be noted that the insurance demand curves with government subsidies plus acquisitions at different total government budget levels overlap when the prices are low (<= $1.7 per dollar coverage), which implies that the demand for insurance are at their highest levels, given homeowner risk attitudes. Figure 2.2 also implies that the saturated demand is not maintained as the price increases because their risk attitudes for many homeowners doesn't support ever increasing expenditures up to their budget levels relative to their exposure. It is useful that with a budget of about $3 Billion, there is a more uniform drop in the expected demand as the price increases than for other budgets. The source of this difference is that the transition from a more price support-centric strategy to a more acquisition centric strategy is somewhat more gradual leading a steadier decline in demand.

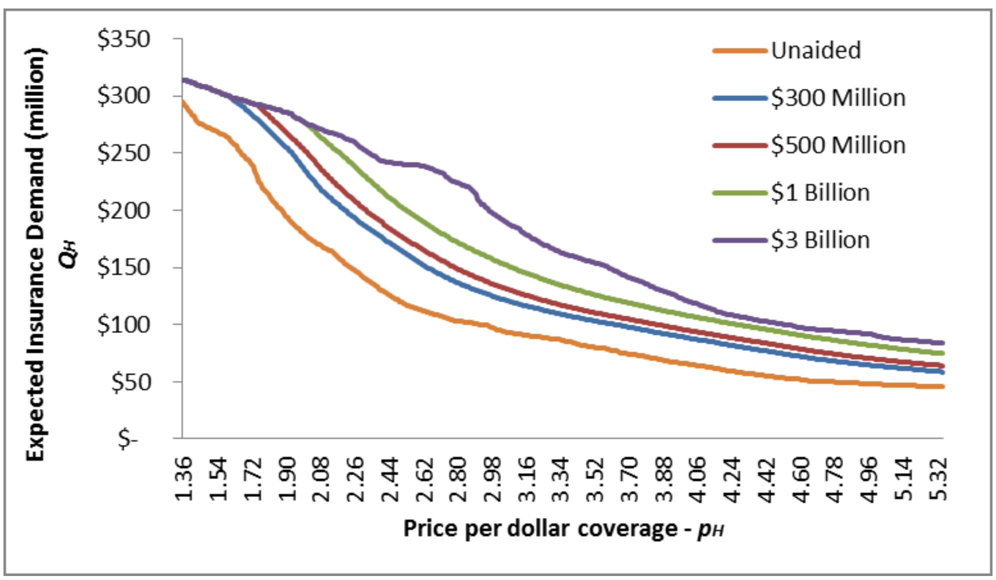

Figure 2.2 Inverse demand functions for varying levels of government funds available

As described in Section 2.4, the government intervention not only includes premium support, but also acquisition when the expenditure on premium support over 30 years would exceed the home value. The impact of the government acquisition program on total expected loss is depicted by Figure 2.3 below. At low price levels (<= $1.8 per dollar coverage), almost no acquisition occurs because price supports are cheaper. As costs rise, acquisition becomes more desirable.

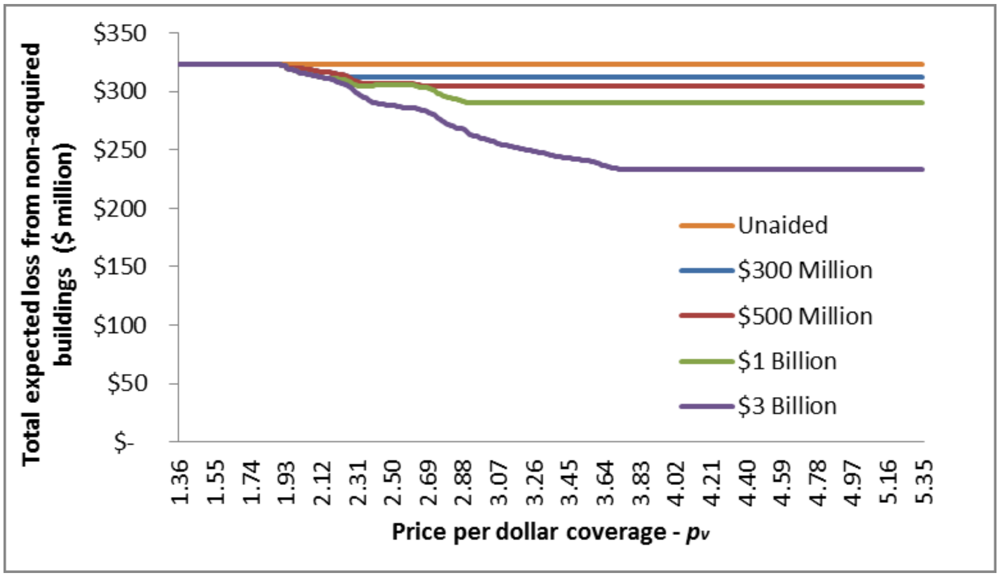

Figure 2.3 Total expected losses for varying levels of government funds available

Finally, the combined impacts of government premium support and acquisition programs on insured loss are illustrated in Figure 2.4, where the insurance demand rate (penetration) is defined as the expected insurance demand divided by total expected loss from non-acquired buildings. It is important to realize that premium supports increases the expected insured loss, while acquisition reduces the total (expected) loss (i.e., the denominator). Both of these contribute to increases in the demand rate. With a $300 million budget over 30 years, the government can raise the demand rate (relative to the unsupported or unaided rate with no price supports or acquisitions) by approximately 10% at all price levels. When the budget increases to $1 billion over 30 years, the increment in the demand rate ranges from 10% to 20% as the price increases from $1.36 to $2 per dollar coverage, and it remains at 20% (or more) for higher price levels. With $3 billion in government intervention funds, the demand rate is above

80% as price increases to \$2.9 per dollar coverage, and the increment (comparing to original unaided) reaches more than 30% for prices higher than \$2.9 per dollar coverage.

Figure 2.4 also illustrates that the insurance demand rate does not reach 100%, regardless of government funds available for these programs. In fact, about 28% of homeowners with their expected loss accounts to 3% of total loss are not willing to pay up to 5% of their home value at the cheapest price level, given the risks their homes face and their risk attitudes assumptions. In other words, even if these homeowners are offered subsidized premiums that are affordable, they will choose not to buy insurance since baring risk results in higher utilities for them than paying for insurance.

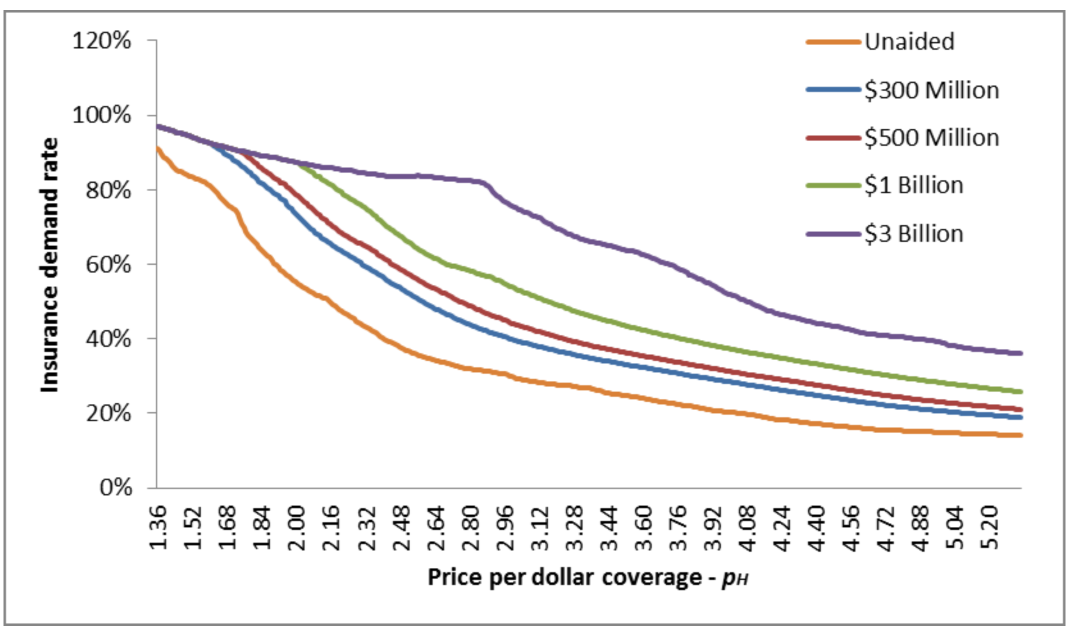

Figure 2.4 Government interventions' combined impacts on insurance demand for varying levels of government funds available

3.2 Inverse demand functions and cost functions for Cournot-Nash model

For simplicity, we focus on the 30-year budgets totaling \$300 million and \$1 billion for government interventions for the remainder of the analysis. The inverse demand functions under the no government intervention and interventions of \$300 million and \$1 billion, which are illustrated in Figure 2.2, were estimated using nonlinear regression and are given in Equations (4), (5) and (6), respectively. All polynomial functions have adjusted R^2 values that exceed 0.9.

$$P_H(q_{Hj}) = 2.291(10^{-33})q_{Hj}^4 - 2.181(10^{-24})q_{Hj}^3 + 7.596(10^{-16})q_{Hj}^2 - 1.187(10^{-7})q_{Hj} + 9.13 \tag{4}$$

$$\begin{aligned} P_H(q_{Hj}) = &-2.044(10^{-41})q_{Hj}^5 - 2.144(10^{-32})q_{Hj}^4 - 8.826(10^{-24})q_{Hj}^3 + 1.802(10^{-15})q_{Hj}^2 \\ &-1.893(10^{-7})q_{Hj} + 14.68 \end{aligned} \tag{5}$$

$$P_H(q_{Hj}) = 2.945(10^{-33})q_{Hj}^4 - 2.59(10^{-24})q_{Hj}^3 + 8.406(10^{-16})q_{Hj}^2 - 1.249(10^{-7})q_{Hj} + 9.287 \tag{6}$$

Since government interventions in the low risk region are not considered, the inverse demand function in this region is unaffected by these demand-side programs. The inverse demand function in the low risk area is also estimated through nonlinear regression and that estimate is given in Equation (7).

$$P_L(q_{Lj}) = 6.309(10^{-15})q_{Lj}^2 - 5.814(10^{-7})q_{Lj} + 14.68 \tag{7}$$

To investigate the impacts of catastrophe reserves, we considered two scenarios when estimating the cost function: (1) insurers retain 75% of their annual surplus as pre-event catastrophe reserve, where the 25% of their annual surplus could be considered as cost/tax (at a favored tax rate by government) for keeping this reserve; and (2) all insurers are assumed to annually reallocate surpluses greater than their initial investment (which is assumed to be 3 times the value of the premiums) by either reinvesting in other lines of business (which are never assumed to be used to address policyholder losses) or distributing it to investors as dividends. As mentioned previously, the insurer model is fully described in Gao et al. (2013).

The cost functions with and without special pre-event reserves under the three sets of inverse demand functions (stemming from no government intervention, \$300 million and \$1 billion dollar government support programs) are estimated using nonlinear regression. All fitted cost functions have adjusted R^2 values that exceed 0.9.

Figure 2.5 below illustrates the insurer's cost function when \$300 million is available in government support and insurers do not maintain reserves in excess of 3 times the value of the policies they have written (which is the initial investment). Equation (8) gives the associated regression equation.

$$C(q_{Hj}, q_{Lj}) = -1.641(10^7) + 2.37q_{Hj} + 2.013q_{Lj} - 1.887(10^{-9})q_{Hj}^2 - 7.162q_{Hj}q_{Lj} \tag{8}$$

These new demand and cost functions are then utilized in the Cournot-Nash (market competition) model (Eq. 1 to Eq. 3), as described in Section 2.3, to solve for the equilibrium $(p_H^*, q_{Hj}^*; p_L^*, q_{Lj}^*)$ under different market structures (from monopoly to oligopoly) through varying the number of insurers.

For ease of discussion, we denote the case of funding government programs with up to \$300 million and insurers with a capped surplus as "\$300M support, capped"; "\$300M support, retain" denotes \$300 million in government funding and insurers that retain a pre-event reserve; “$1B support, capped” denotes up to $1 billion in government funding and insurers have a capped surplus; “$1B support, retain” denotes $1 billion budget and insurers that retain a pre-event reserve; “No support, capped” denotes unaided demand and insurers have a capped surplus; “No support, retain” denotes unaided demand and insurers that retain a pre-event reserve.

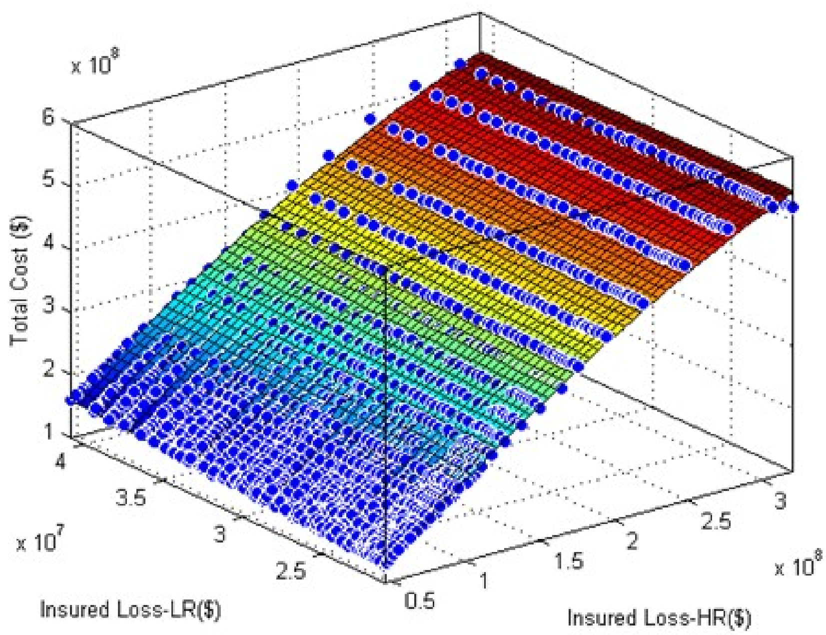

Figure 2.5 Insurer’s cost function for case “$300M support, capped”

3.3 Market performance analysis for Cournot-Nash solutions

To fully understand the impacts of these interventions on market performance, the impacts of on all key stakeholders (i.e., homeowners, primary insurers and government) are considered. More specifically, the impacts of government policies and market competition are examined by considering: 1) insured and uninsured losses; 2) each insurer’s profitability (price, net profit and return on equity), insolvency, and their decisions for managing risk (hedging or retaining); and 3) the government’s expenditures on price support and acquisition programs and how they affect uninsured losses.

3.3.1 Insured loss and uninsured loss

Figures 2.6a and 2.6b display the annual expected insured losses and the penetration rate in each of the six cases. It is clear that competition and both types of government action can reduce uninsured losses. In the best scenario as defined by the highest market penetration for insurance ("\$1B support, retain" case in Fig. 2.6a), that rate reaches 60% of total losses (double that of the unaided market, "No support, capped" case in Fig. 2.6a).

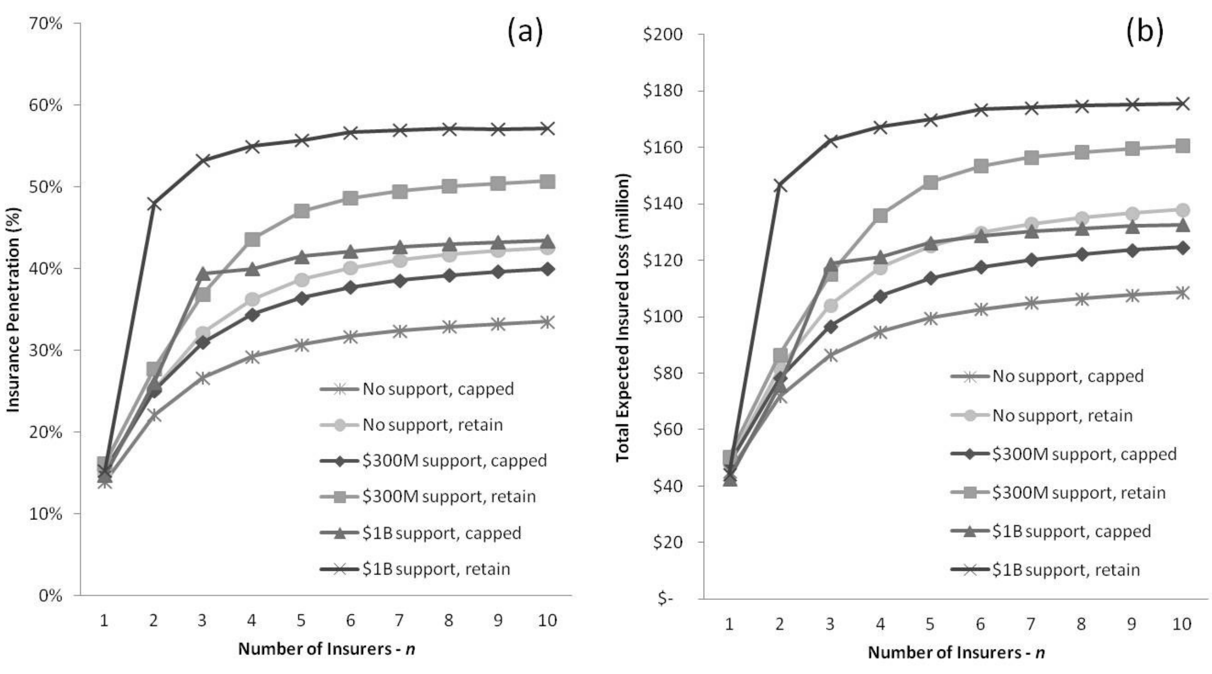

Figure 2.6 (a) Insurance penetration and (b) Expected insured loss vs. number of insurers, for the six cases

In Figure 6a, the penetration rate curve of the "no support, retain" case lies above that of the "\$300M support, capped" case (especially under oligopolies with more than 2 insurers), yet below that of the “$1B support, capped” case, which indicates the potential potency of the catastrophe reserve policy to the effects of demand support with hundreds of million dollars (between $300 million and $1 billion). Furthermore, the curve of “$300M support, retain” partially lies above that of “$1B support, capped”, which again implies that significant benefits are possible with a catastrophe reserve policy. It is important to note however, that government actions will be needed to make these reserve pools viable to the insurers. This is likely to take the form of a tax incentive, which will result a reduction in government revenues.

As for the uninsured losses, we first examined the trends under government interventions (with $300 million and $1 billion budgets) as the market structure changes from monopoly to oligopoly (Fig. 2.7 and Fig. 2.8), since both government interventions and market competition mechanism reduce uninsured losses. In the “$300M support, capped” case, $10 million (3% of total expected loss in high risk region) annual expected loss were avoided or supported by insurance by the government under different market structures (which is equivalent to the money expended via the intervention on an annual basis). But this impact seems less influential than that from the market competition mechanism, which reduces the annual uninsured losses by $99 million (31% of total expected loss) with 3 insurers and $127 million (39% of total expected loss) with 10 insurers (Fig. 2.7a). As a result, the uninsured losses are reduced from 81% to 58% in terms of total expected loss as the market structure changes from monopoly to oligopoly (with 10 insurers). In addition to insurance demand support, the catastrophe reserve policy further facilitates private market competition and reduces more uninsured losses, as indicated in the case of “$300M support, retain”, where the increase in competition reduces annual expected losses by about $117 million (36% of total expected loss) when 3 insurers are added, and $158 million (49% of total expected loss), when 10 insurers are added in the market (Fig. 2.7b). Due to this more efficient mechanism, the uninsured loss are reduced from 81% to 48% (i.e., 10% less than that of “$300M support, capped” case) in terms of total expected loss as the market structure changes from monopoly to oligopoly (with 10 insurers).

The $1 billion budget for government’s interventions over 30 years triples the annual acquired or supported losses from $10 million to $33 million (3% to 10% of total expected loss) as illustrated in Figure 2.8. As competition increases, the uninsured losses are reduced from 76% to 53% (of total expected loss) from monopoly to oligopoly (with 10 insurers) when retained funds are capped in contrast to from 76% to 41% when a reserve is available.

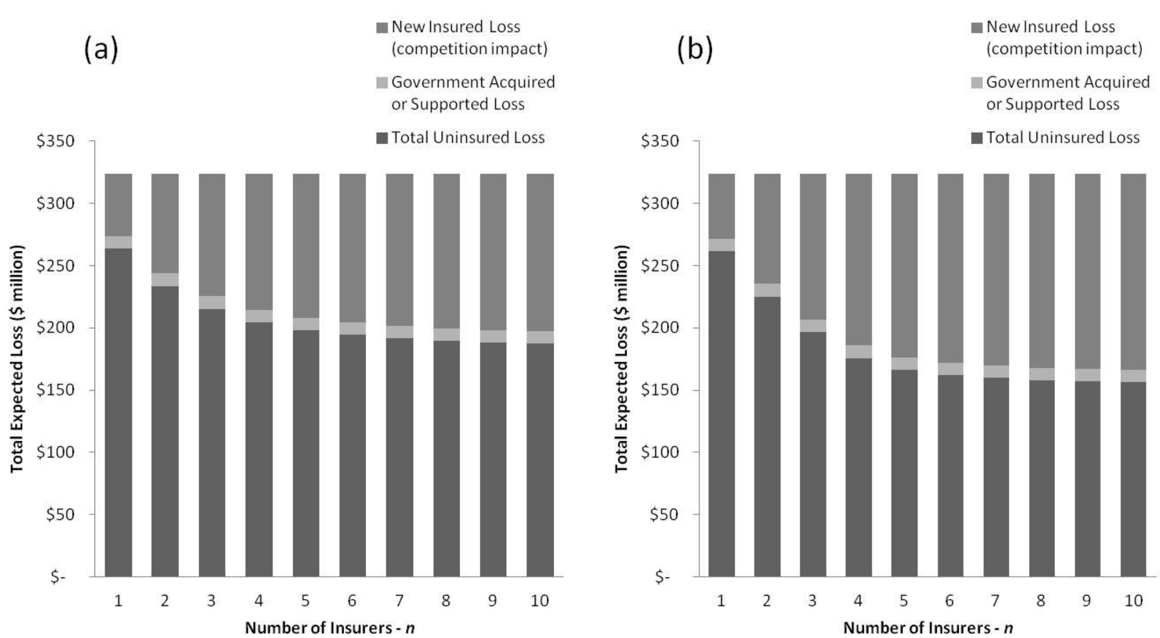

Figure 2.7 Government and competition impacts on annual uninsured loss for (a) \$300M support, capped and (b) \$300M support, retain cases

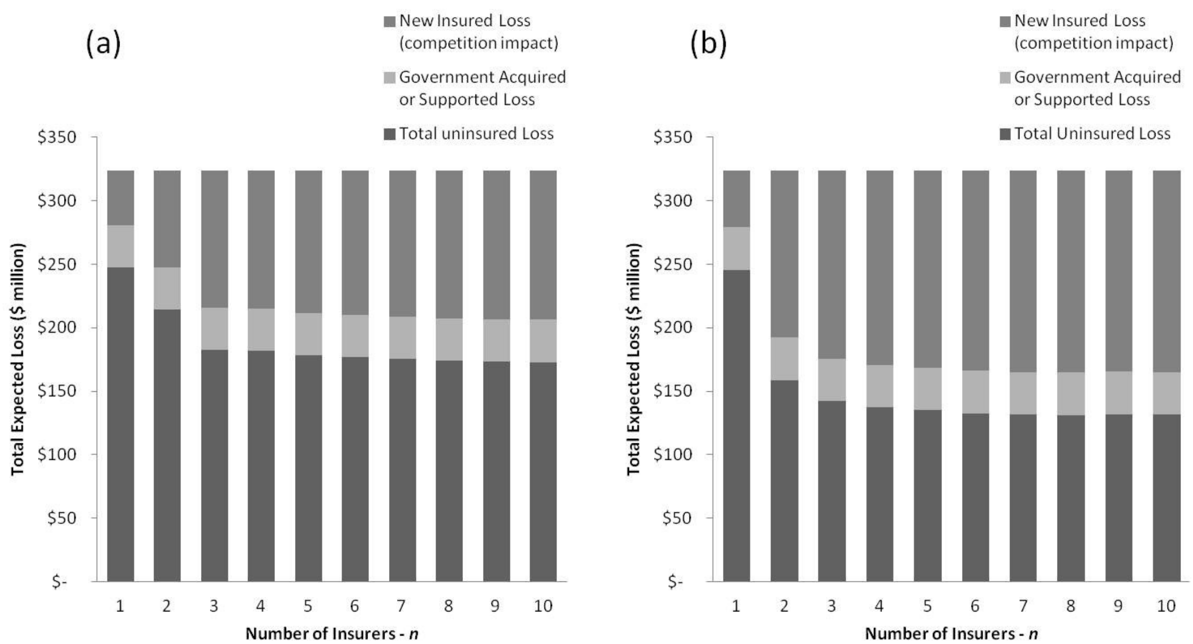

Figure 2.8 Government and competition impacts on uninsured loss for (a) \$1B support, capped and (b) \$1B support, retain cases

Government interventions and market competition not only reduce the expected annual uninsured losses but also improve the risk profile for these expenditures. We examined this impact on the uninsured loss distribution as competition rises. In Figures 2.9 and 2.10, we focus on competition with 1 to 5 insurers in the market (mostly due to the declining impacts of competition beyond 5 insurers). Notice that as competition rises, the tails are reduced on the cumulative density functions (CDF). Under both budget levels (i.e., \$300 million and \$1 billion), the “retain” cases (Fig. 2.9b and Fig. 2.10b) result in smaller tails than their corresponding “capped” case.

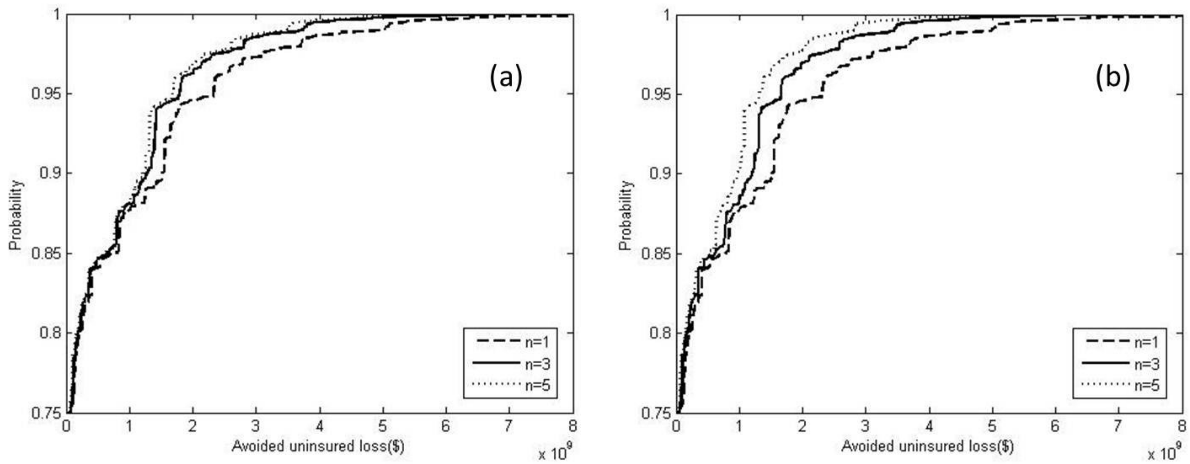

Figure 2.9 CDF of uninsured loss for (a) \$300M support, capped and (b) \$300M support, retain cases

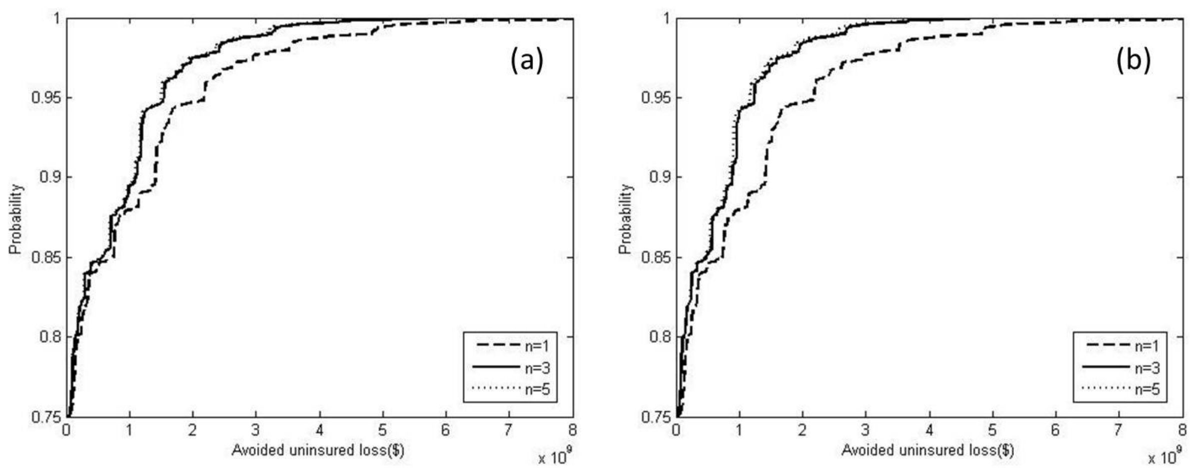

Figure 2.10 CDF of uninsured loss for (a) \$1B support, capped and (b) \$1B support, retain cases

Figure 2.11 gives the CDFs of uninsured losses for all six cases under monopoly and oligopoly market structures. Figure 2.11a implies that under monopoly, the uninsured loss profile under all government demand support cases (with different budget levels and “retain” or “capped” scenarios) are very close and superior to the no support cases (which overlapped with each other). Nevertheless, the benefits from government interventions and competition on the uninsured loss profile become larger as the market structure changes from monopoly to oligopoly. This is illustrated by the shifts in the CDF curves towards the upper-left corner (Fig. 2.11b). It is also interesting to notice that, in Figure 2.11b, the CDF curve of “$300M support, retain” lies to the left of that of “$1B support, capped” case. In other words, the $300 million demand support with catastrophe reserve policy for insurers is roughly slightly better than the $1 billion demand support with capped surplus under and oligopoly market structure (with 5 insurers).

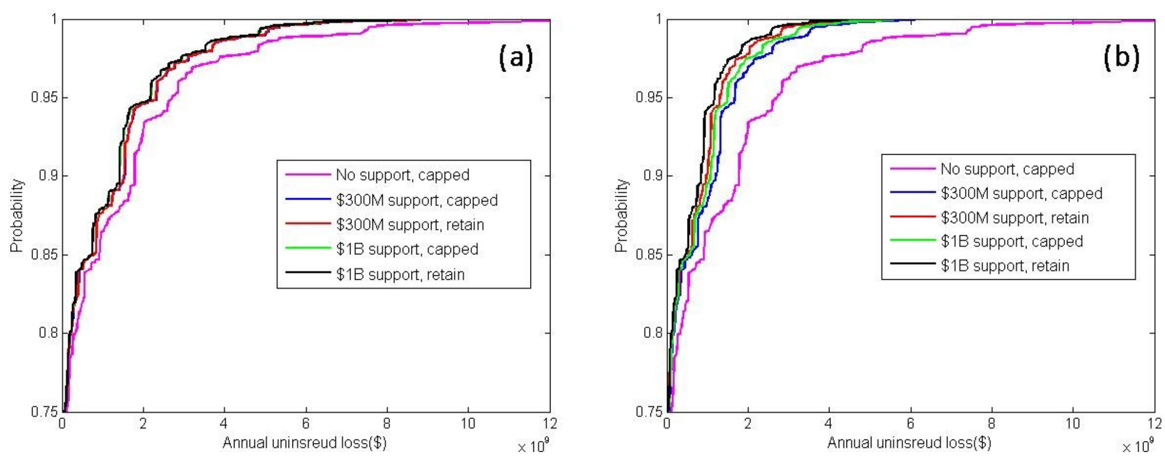

Figure 2.11 CDF of uninsured loss for all six cases under (a) Monopoly and (b) Oligopoly (*n*=5)

3.3.2 Single insurer’s profitability, insolvency and decisions in managing risk

Both the government intervention and market competition effectively reduce equilibrium prices under all scenarios (Fig. 2.12). More specifically, the competition, due to the increment of insurers, causes a similar decreasing pattern in price in all six cases, from approximately $5.3 per dollar coverage under monopoly to approximately $2.5 per dollar coverage under oligopoly ($n >= 5$). Also, the differences between the equilibrium prices across all six cases are modest (< $1 per dollar coverage) as indicated in Figure 2.12. Therefore the insurers have similar levels of net profit under the six cases (Fig. 2.13). Hence, in terms of profitability, the benefits from government interventions do not appear to flow to the supply side.

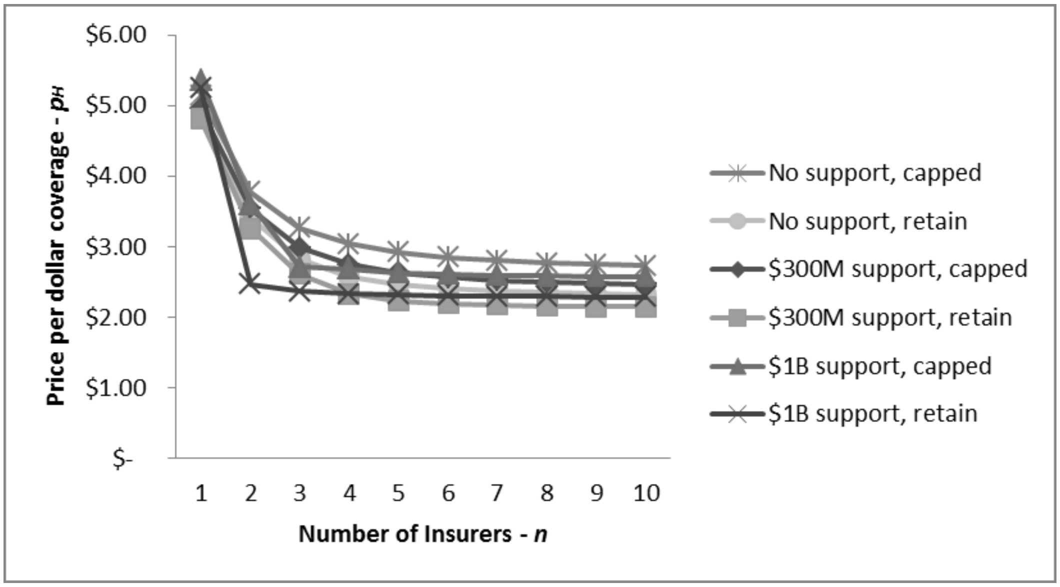

Figure 2.12 Government and competition impacts on equilibrium price for the six cases

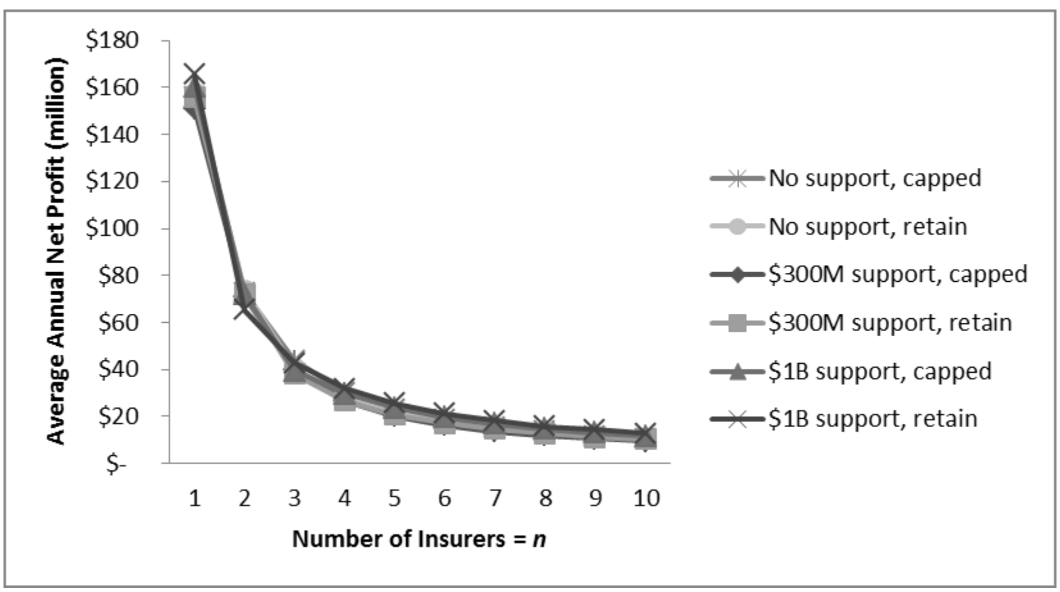

Figure 2.13 Government and competition impacts on net profit for the six cases

Return on equity (ROE) is another important measurement of the desirability of an investment; therefore we focus on a single insurer's ROE under different market structures (Fig. 2.14). Among the six cases, half involve the catastrophe reserve policy, while the others assume that insurers invest their annual surpluses in other lines of business or distribute it as dividends. The catastrophe reserve policy reduces the ROE for insurers, since the reserve (i.e. more investments) is set aside to cover future losses. Since the net profit levels for the cases with or without the catastrophe reserve are all very close (Fig. 2.13), it indicates no obvious benefit from the catastrophe reserve in increasing profitability. As a result, the ROE for the catastrophe reserve cases are lower than when such a large reserve is not present. The stability of the ROE under the "retain" cases also implies that neither government interventions nor competition has significantly affected the efficiency of insurers' investment. On the other hand, for "capped" cases, the market competition mechanism has caused the oscillation of ROE as the market structure changes.

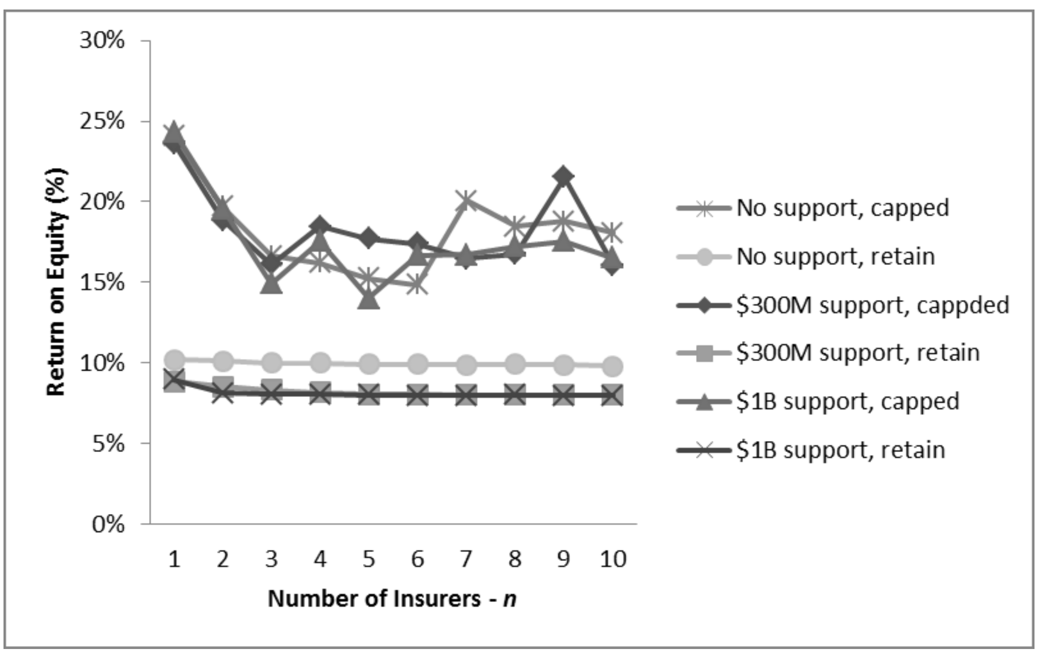

Figure 2.14 Government and competition impact on single insurer's ROE for the six cases

Other than profitability, avoiding insolvency is the biggest challenge for insurers in this market because of the low-frequency but high-consequence nature of the risk. In this study, we focus on excess of loss reinsurance and/or the catastrophe reserves for managing these risks. Figure 2.15 gives the insolvency rates across the 6 cases whereas Figure 2.16 indicates how excess of loss reinsurance is used. For all "capped" cases, insurers rely mostly on reinsurance to transfer risk and avoid insolvency; while for the "retain" case, insurers utilize their catastrophe reserves as well as reinsurance to manage risk and avoid insolvency.

As indicated in Figure 2.15, the annual insolvency rates under all "retain" cases are similar and monotonic as more insurers join in the market, which relates to the impacts of competition. The similarity between the "retain" cases implies no obvious impact of government interventions on insurers' insolvency, since the catastrophe reserve enables insurers to efficiently manage their risk. On the other hand, for all "capped" cases, the market competition also results in an increasing (but not monotonic) insolvency trend; and the insolvency rates do not stabilize across the number of insurers. However, with no more than five insurers in the market, the insolvency rates for "capped" cases are generally smaller those for the "retain" cases. Even though, this advantage is not maintained with more than five insurers. Figure 2.15 also implies that the "capped" cases have more variable insolvency rates than the "retain" cases, which seem smoother as insurer number increases. Since the net profit levels for "capped" and "retained" cases under different number of insures are very similar (Fig. 2.13), the different insolvency patterns reflect that the optimization under "retain" cases (i.e. with catastrophe reserve) is more stable than that under the "capped" cases. In other words, under the same profitable level, it's harder to avoid/control insolvency with capped surplus than retained surplus.

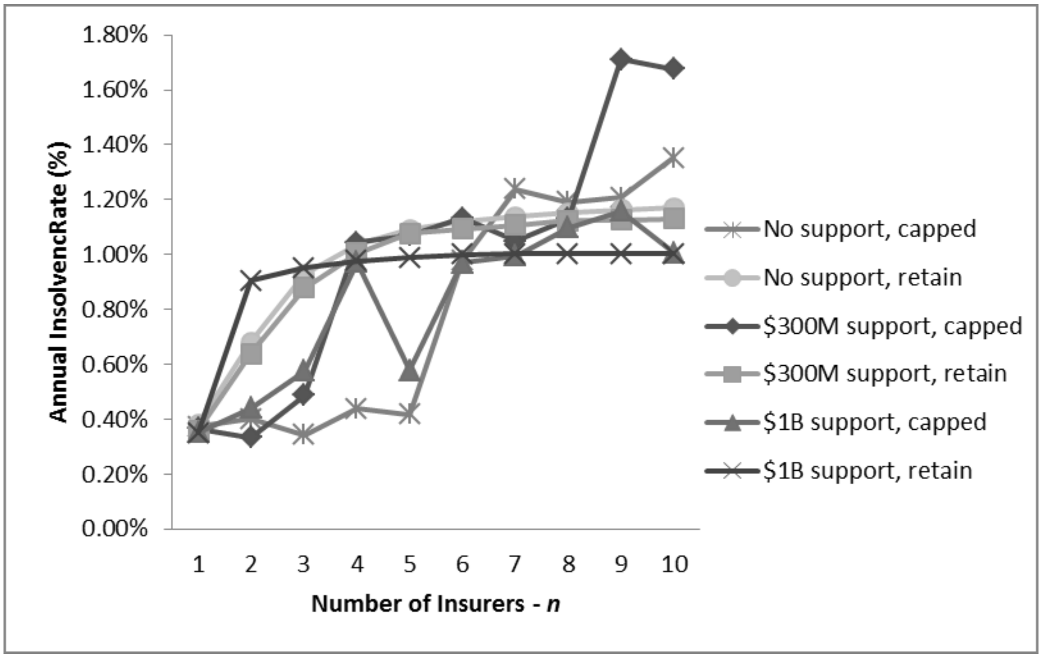

Figure 2.15 Government and competition impacts on single insurer's insolvency rate for the six cases

Figure 2.16 compares the insurers' annual reinsurance payments to total expected insured loss rates for the six cases, which roughly represents the portion of risk transferred to the reinsurer. More specifically, denote the annual expected loss transferred to the reinsurer (from insurer) as L, the corresponding reinsurance premium denoted as b is given in Equation (9) below

$$(1+\varphi)L+g\sigma \tag{9}$$

where φ refers to the loading factor of the insurer which is assumed to be 0.1 in this study (Gao et al. 2013, Kunreuther and Michel-Kerjan 2009). The first term is the burdened expected loss while the second term refers to the standard deviation σ of loss L scaled by a user-specified constant g, which reflects reinsurer's risk aversion attitude. A higher value of g would result in a higher reinsurance premium which make it harder for primary insurers to transfer their risk to reinsurer. A high value of g denotes a "hard" reinsurance market; on the other hand, a low value of g drives down the reinsurance premium and results in a "soft" market (Kunreuther and Michel-Kerjan 2009). In this study we assume a "soft" reinsurance market by setting $g = 0.1$, and the resulted annual reinsurance premium are close to that under fair price level (i.e., $\varphi = 0, g = 0$).

Insurers choose self-insurance over reinsurance under all "retain" cases. As a result, the risk is mostly retained by the primary insurer. By contrast, for all "capped" cases, more than 50% of the risk among all insurers is transferred to the reinsurer under different market structures (from monopoly to oligopoly (*n* <= 10)) and different levels of government intervention budget. However, as more insurers join the market and price competition increases less reinsurance is purchased because of declining revenues against rising costs.

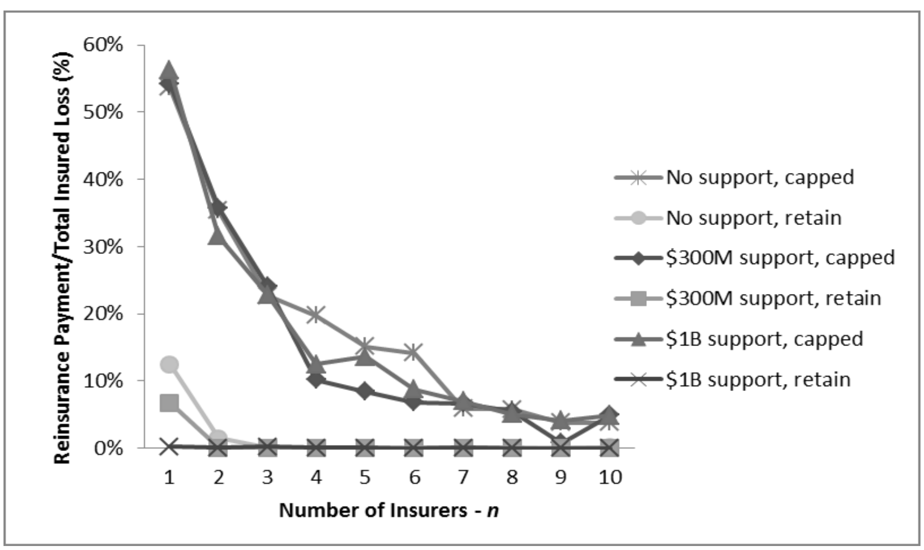

Figure 2.16. Government and competition impacts on single insurer's reinsurance decisions for the six cases

3.3.3 Government's expenditure on price support and acquisition programs and their efficiency

Figures 2.17a to 2.17d give the allocation of the government's total budget (either \$300 million or \$1 billion), under the capped or retain cases, between price support and acquisition programs at equilibrium price levels under different market structures (from monopoly to oligopoly). As competition drives down the equilibrium price, the government's expenditures increase on price supports but decreases on acquisitions (Fig. 2.17b, 2.17c and 2.17d). The one exception is "\$300M support, capped", for which competition does not drive the price of insurance sufficiently low so all \$300 million is spent on acquisition (Fig. 2.17a). Further, Figure 2.17a implies that under monopoly competition all funds are spent on acquisition regardless of government programs and whether or not a reserve fund is created. This is because under monopoly conditions, the equilibrium prices remain relatively high (Fig. 2.12).

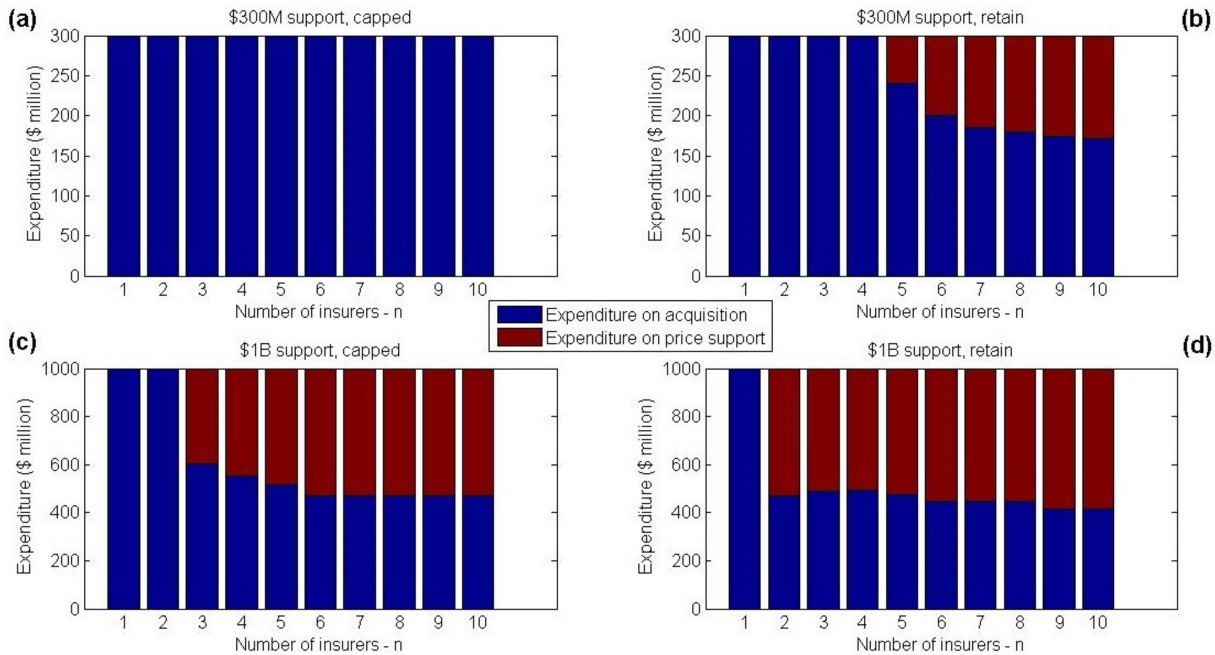

Figure 2.17 Government's expenditure under market equilibrium on price support and acquisition for (a) $300M support, capped, (b) $300M support, retain, (c) $1B support, capped and (d) $1B support, retain cases

Figure 2.18 gives the CDFs of the losses avoided on the properties that are acquired when there is a monopoly. These CDFs indicate that it is possible to avoid uninsured losses of up to $5 billion over 30 years with a $300 million investment over this period, while the $1 billion budget enables the avoidance of uninsured losses on these properties that can rise to about $20 billion dollars.

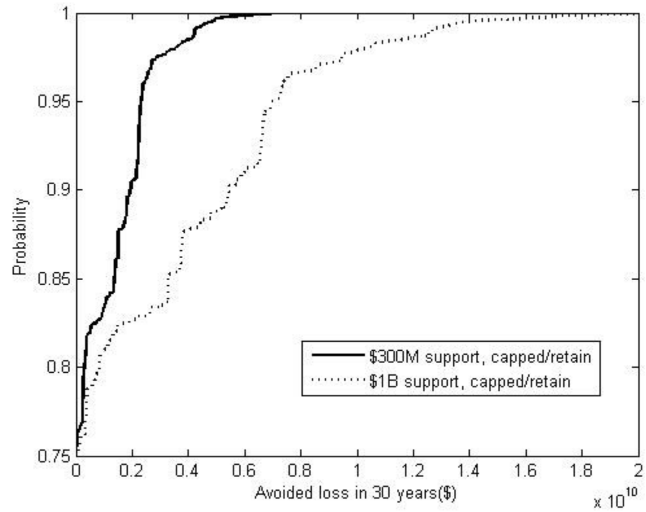

Figure 2.18. CDF of acquired losses by government – Monopoly

Figures 2.19a and 2.19b give the CDFs for these avoided losses when the market is composed of 10 primary insurers. When there are 10 primary insurers there is a mix of price subsidies and acquisition with relatively more money spent on price supports when funds are retained in contrast when these funds are capped at the level of the initial investment. It is important to compare the scales of Figures 2.18 and 2.19. The potential exists to avoid substantially more losses under market competition because the equilibrium prices are lower so the dollars the government makes available in these programs "go farther".

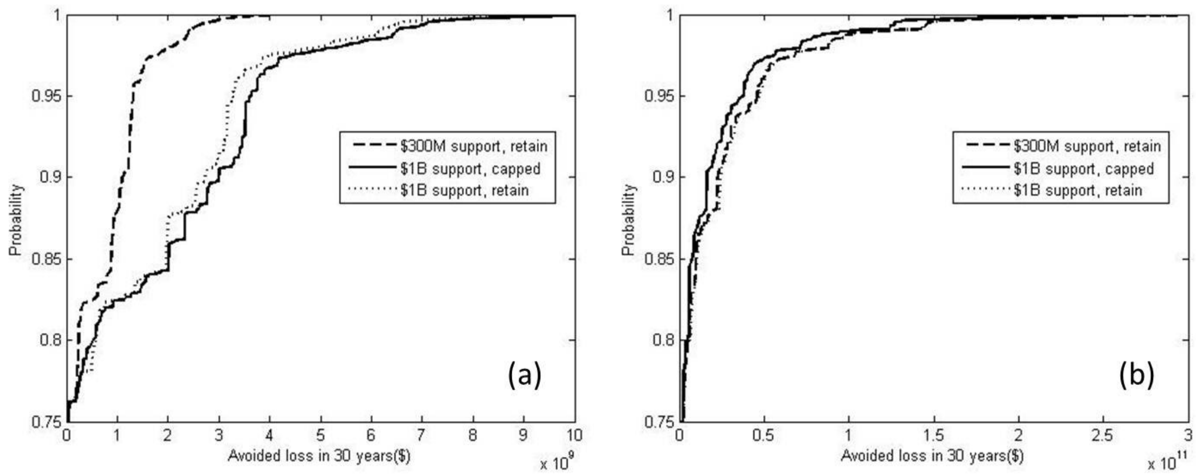

Figure 2.19 CDF of (a) acquired and (b) price supported loss by government under Oligopoly (n=10), for three cases

4. CONCLUDING REMARKS

In conclusion, our results suggests that acquisition, price supports and policies to encourage insurers to create catastrophe reserves do lead to improvements in market performance and a substantial reduction in uninsured losses. For example, under these policies the penetration rate for insurance rises from about 15% to anywhere from 20% to close to 50% under monopoly conditions and from about 30% to about 60% when there are 10 primary insurers in the market. Also, based on our results, the implementation of a catastrophe reserve policy enhances market competition and has a greater impact in reducing uninsured loss than many dollars' worth of subsidization or acquisition by government.

Further, our results also indicate the benefits of these programs are generally do not get passed onto the insurers in the form of higher net profits but rather remain with the homeowners and the government as a substantial reductions in uninsured losses. Finally, this study highlights the importance of competition in this market. With a single primary insurer, prices remain relatively high surprising homeowner demand for insurance and greatly reducing the impacts of government programs. Hence governmental actions are likely to have a “multiplier” effect. That is, actions to address inherent market shortcomings will stimulate competition and that competition will make it easier to address the weaknesses.

There are many avenues for future research. Our analysis is based on a simplified framework of the natural catastrophe insurance market, which does not consider imperfect information, heterogeneity between insurers, competition dynamics as well as complicated insurance regulations. In reality, all of these issues are likely to have effects on the performance of the catastrophe insurance market as well as the government interventions examined in this study.

Also, it is critical to continue to refine the data. Modeling in this domain is very data intensive. Perhaps the place where that need is the highest is that which would provide more insight into consumer preferences.

REFERENCES

Charpentier, A. and Maux, B. L. (2010). "Natural Catastrophe Insurance: When Should the Government Intervene? (November 9, 2010). Available at SSRN: http://ssrn.com/abstract=1706254 or http://dx.doi.org/10.2139/ssrn.1706254."

Charpentier, A. and Maux, B. L. (2011). "Natural Catastrophe Insurance: How Should Government Intervene? (December 13, 2011). Available at SSRN: http://ssrn.com/abstract=1832624 or http://dx.doi.org/10.2139/ssrn.1832624."

Cleary, K. A. and Boutchee, J. P. (2002). "Reserving for catastrophes." The Casualty Actuarial Society Forum **Fall 2002**: 25-76.

Cummins, J. D. (2006). "Should the Government Provide Insurance for Catastrophes?" Federal Reserve Bank of St. Louis REVIEW(July/August): 337-380.

Gao, Y., Nozick, L., Kruse, J. and Davidson, R. (2013) "Modeling Competition in the Natural Catatrophe Loss Insurance Market" The Journal of Insurance Issues. (in progress)

Grace, M. F., Klein, R. W., Kleindorfer, P. R. and Murray, M. (2003). Catastrophe Insurance: Consumer Demand, Markets and Regulation. Topics in Regulatory Economics and Plicy.

Harrington, S. E. and Niehaus, G. (2003). "Capital, corporate income taxes, and catastrophe insurance." Journal of Financial Intermediation **12**: 365-389.

Jaffee, D. M. and Russell, T. (1997). "Catastrophe Insurance, Capital Markets, and Uninsurable Risks." The Journal of Risk and Insurance **64**(2): 205-230.

Jaffee, D. M. and Russell, T. (2006). "Shoud Governments Provide Catastrophe Insurance?" The Economists' Voice **3**(5): 1553-3832.

Klein, R. W. (1998). The Regulation of Catastrophe Insurance: An Initial Overview. The Wharton Catastrophe Risk Project. Center for Risk Management and Insurance Research, Georgia State University.

Kunreuther, H. C. and Michel-Kerjan, E. (2009). At War with the Weather: Managing Large-Scale Risks in a New Era of Catastrophes. Cambridge, MA, The MIT Press.

Mas-Colell, A., Whinston, M. D. and Green, J. R. (1995) Microeconomic Theory. New York, Oxford, Oxford University Press.

Mills, E., Lecomte, E., and Peara, A. (2001). U.S. Insurance Industry Perspectives on Global Climate Change. LBNL-45185.

Peng, J., Shan, X., Davidson, R., Nozick, L., Kesete, Y., and Gao, Y. (2013) "Hurricane loss modeling to support regional retrofit policymaking: A North Carolina case study." 11th International Conference on Structural Safety and Reliability - ICOSSAR'13 New York, NY, June 16-20.

U. S. Government Accountability Office (U.S. GAO), U.S. Government. Natural Disasters: Public Policy Options for Changing the Federal Role in Natural Catastrophe Insurance. Washington, DC: Government Accountability Office, November, 2007. Available at:http://www.gao.gov/products/GAO-08-7, Accessed on April 28, 2013

CHAPTER 3

APPLICATION OF RESPONSE-SURFACE AND TRUST-REGION METHOD IN THE OPTIMIZATION OF A NATURAL CATASTROPHE INSRUANCE MARKET

ABSTRACT

The Response–Surface methodology and Trust–Region methods have been extensively applied in many branches of engineering, but not frequently utilized in economic analysis. This paper presents an application of the combination of these two methods – an automatic Response-Surface Trust-Region method to the constrained simulation optimization of a primary insurer in a natural catastrophe insurance market. The optimization problem involves a utility-based homeowner decision model, a constrained stochastic optimization model to represent the insurer's decisions, as well as a state-of-the-art regional catastrophe loss estimation model. This automatic algorithm was utilized to solve for the equilibrium supply-demand relationship. The models and algorithm were applied to a full-scale case study for hurricane risk to residential buildings in Eastern North Carolina. In comparison with Simulated Annealing, for this problem, the algorithm is shown to be computationally efficient yielding an improved solution more quickly.

1. INTRODUCTION

Response Surface Methodology (RSM) is a collection of statistical and mathematical techniques, (including design of experiments and regression analysis), for the optimization of stochastic functions (Box and Wilson 1951). The fundamental strategy is to sequentially approximate the stochastic 'input-output' relationship of a simulation model in order to optimize the inputs using experimental design and response surface optimization. The concepts and techniques of RSM have been extensively applied in many branches of engineering, especially in the chemical and manufacturing areas (Box et al. 1978). RSM has also become one of the most popular heuristics for simulation optimization over the past three decades (Neddermeijer et al. 2000; Myers and Montgomery 2002; Kleijnen 2008). However, the major disadvantages of RSM are the lack of a convergence guarantee and the non-automatic (with human inputs required at each step) nature of the algorithm.

On the other hand, Trust-Region (TR) methods (Conn et al. 1987) are a class of relatively new algorithms for nonlinear deterministic optimization, with automatic and local convergence properties, and are also local approximation-based. Though RSM and TR both generate steps based on a local approximation model, they do this in different ways. RSM uses a local model (linear or quadratic) to generate a search direction first and then relies on the user to determine a suitable step length along this direction, while TR defines a "trust region" around the current solution and uses an efficient (quadratic) program to solve for the search direction and step length simultaneously. The trust region is adjusted (automatically) from iteration to iteration based on the assessed quality of the local approximation model. Over the past decade, TR has proven to be effective and robust for solving unconstrained/constrained nonlinear optimization problems (e.g., Sadjadi and Ponnambalam 1999; Zhang 2006; Shi and Guo 2008), though its application in stochastic/simulation optimization is somewhat rare.

Recently, the combination of RSM and TR has been proposed for unconstrained simulation optimization with continuous decision variables. This algorithm is referred to as "STRONG" (Chang et al. 2007; Chang et al. 2012). STRONG is a new response surface framework that combines the advantages of traditional RSM with TR methods. More specifically, STRONG follows the iterative "design-experiment" search strategy of traditional RSM, and applies the trust-region updating techniques between iterations to achieve the desired automatic and local convergence properties.

In this paper, we present an application of the combined Response-Surface (with Latin hypercube designs) and Trust-Region method, following a similar structure as STRONG, to the constrained simulation optimization (with continuous variables) problem of a primary insurer in a natural catastrophe insurance market with explicit representation of key stakeholders. The optimization problem has a game theoretical structure and involves an expected utility-based homeowner decision model, a constrained stochastic optimization model for insurer, as well as a state-of-the-art regional catastrophe loss estimation model. The complicated structure of this problem and high dimensional data result in non-smooth functions and expensive function evaluations, which challenge the general non-gradient based heuristic optimization methods (e.g., Genetic Algorithms, Simulated Annealing), as well as the traditional non-automatic RSM. We utilize an automatic Response-Surface and Trust-Region algorithm to optimize the primary insurer's and homeowners' response within the supply-demand relationship embodied in a stochastic economic system, which is a new area of application of RSM and TR. The models and algorithm are applied to a full-scale case study for hurricane risk to residential buildings in Eastern North

Carolina. In comparison with Simulated Annealing, the algorithm is shown to be efficient yielding a better solution more quickly for this problem.

2. PROBLEM DEFINITION

2.1 Modeling framework

In this study, homeowners, primary insurers, reinsurers and the government are assumed to be the key stakeholders in the natural catastrophe insurance market. Individual behaviors and the interactions between the key stakeholders are captured by several interactive models developed in Kesete (in progress). More specifically, homeowners' insurance purchasing behaviors are captured by an expected utility-based decision model, while primary insurer's pricing and risk transfer decisions are optimized through a stochastic optimization model (Figure 3.1). The loss model provides an estimate of the distribution of losses at the level of the individual property to both the insurer and individual homeowners. We assume that the reinsurer simply offers reinsurance based on the liabilities the insurer acquires according to a static formula. Finally, we assume that the government, in order to control insolvency on the part of the primary insurer, places requirements on the cash reserves to be held by the primary insurer proportional to the magnitude of their liabilities. No game theoretic interactions are considered for the government and reinsurer in this modeling framework.

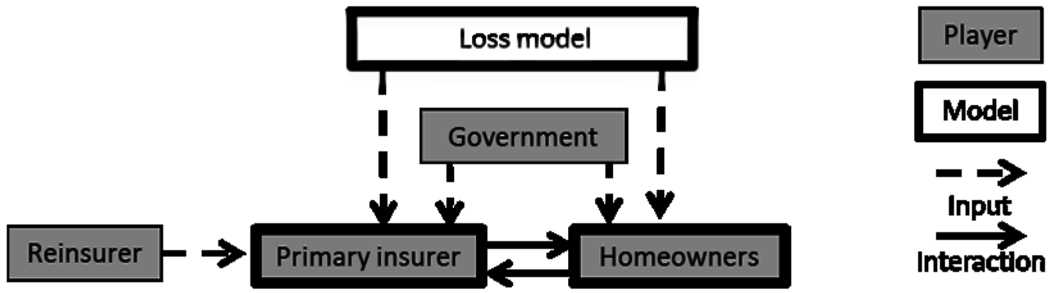

Figure 3.1: Modeling framework

2.2 Loss model

The loss model includes both hazards simulation and loss estimation for each home in the study region. The hurricane hazard is represented by an efficient set of probabilistic hurricane scenarios. Each hurricane scenario h has an associated hazard-adjusted annual occurrence probability P^h such that when probabilistically combined, the set of hurricane scenarios represents the regional hazard (Apivatanagul et al. 2011). We used the set of 97 probabilistic hurricane scenarios developed in Apivatanagul et al. (2011), using the Optimization-based Probabilistic Scenario (OPS) method. Furthermore, how hurricanes occur over time can create very different outcomes for insurers. To address this, we use the collection of 2000 equally possible long-term timelines of hurricane occurrence, called scenarios, developed in Peng (2013). Each scenario spans 30 years with 20 time periods in each year. More specifically, for a given scenario *s*, at time period *t*, which hurricane *h* (or no hurricane) occurs is represented by a binary indicator variable r^{sth} that is one if hurricane *h* occurs under scenario *s* at time *t* and zero otherwise.

The inventory of residential buildings are classified into high (*H*) and low (*L*) risk regions, denoted as $v \in [H, L]$; and further divided into groups, where each is defined by its geographic location *i* (e.g., census tract), building category *m*, and resistance level *c*. Finally, the simulated loss, under hurricane *h*, for a single home of type *i, m, c, v,* as L^h_{imcv}, is estimated from the catastrophe loss model developed in Peng et al. (2013).

2.3 Homeowner model

All homeowners of the same type *i, m, c, v* are assumed to behave the same, but the decisions can vary across different *i, m, c, v* types. We assume each homeowner's purchase decision for insurance maximizes utility represented by a Constant Absolute Risk Aversion (CARA) utility function of the form $U(x) = 1 - e^{-\theta_v x}$, where θ_v is the Arrow-Pratt measure of risk aversion for homeowners in risk area $v \in [H, L]$, and *x* refers to expenditures (negative values) – either from natural disaster damage or insurance payments. More specifically, for a given price p_v per dollar coverage for risk area $v \in [H, L]$, and associated also given deductible level *d*, the homeowner's objective function (Eq. 1) is to maximize the probability weighted sum of utilities over all possible hurricanes *h* with probability P^h, if the homeowners buy insurance (first term) and if they do not (second term), where the binary decision variable representing the insurance purchase decision is denoted as w_{imcv}. In the first case, the homeowner losses are the premium, defined as price per dollar coverage times the expected loss, $\sum_h P^h (L^h_{imcv} - B^h_{imcv})$, and the loss up to the deductible level B^h_{imcv} (Eq. 2). In the second case, the homeowners bear the entire loss L^h_{imcv} resulting from hurricane *h*. If having insurance results in the higher expected utility, w_{imcv} will equal one, indicating insurance is purchased; otherwise, the associated w_{imcv} will equal zero, indicating that not purchasing insurance results in higher expected utility. We also assume each homeowner has a maximum budget for insurance equal to a specified percentage (k_v for people in different regions) of their home value V_m (Eq. 4); and the insurer will not sell insurance to the homeowner if the premium is less than some specified level ρ (Eq. 5).

$$\max_{w_{imcv}}\left\{w_{imcv}\left[\sum_h P^h\left(U\left(p_v\sum_h P^h\left(L^h_{imcv}-B^h_{imcv}\right)+B^h_{imcv}\right)\right)\right]+(1-w_{imcv})\left[\sum_h P^h U\left(L^h_{imcv}\right)\right]\right\} \tag{1}$$

$$B^h_{imcv}=\min(L^h_{imcv},d)\quad \forall i,m,c,v,h \tag{2}$$

$$w_{imcv}\in\{0,1\}\quad \forall i,m,c,v \tag{3}$$

$$w_{imcv}\left(p_v\sum_h P^h L^h_{imcv}\right)\le \kappa_v V_m\quad \forall i,m,c,v \tag{4}$$

$$w_{imcv}\left(p_v\sum_h P^h L^h_{imcv}\right)\ge \rho\quad \forall i,m,c,v \tag{5}$$

2.4 Insurer model

The insurer is assumed to be a net profit maximizer (Eq. 6), who sells insurance with different prices $p_v, v\in[H,L]$ per (expected) unit coverage in each risk region. These prices are assumed to be one plus a specified administrative loading factor τ and a profit loading factor λ_v to be determined by the optimization (Eq. 8). Each policy has a standard deductible d to homeowners which is the same in the two risk regions. Homeowners' decisions of whether to buy the insurance or not are based on the utility-based model described in Section 2.2, and are denoted $O(p_v)$ (Eq. 9). The resulting premium collected from insured homeowners in each risk region is represented as price p_v times the expected insured loss Q_v (Eq. 13). For managing the insured (catastrophe) risk, a single layer of excess of loss reinsurance is assumed as the main option for the primary insurer to hedge risk and avoid insolvency. More specifically, denote A, M, and β as the attachment point, maximum limit, and co-participation percentage of the reinsurance treaty, respectively. In the event of a hurricane h, the loss to insured buildings from both risk regions L^h (Eq. 10) is divided among the homeowners, primary insurer, and reinsurer. The homeowners pay the first portion of the loss up to the deductible d, denoted as B^h (Eq. 11 and Eq. 12); the reinsurer pays β% of any loss above the attachment point A and up to a maximum limit of $(M-A)$, denoted as q^h (Eq. 14); and the primary insurer pays the remaining loss. Based on the loss simulation model (Section 2.1), the annual loss e^{sy} transferred from primary insurer to reinsurer (without timing participation rate) under year y of scenario s could be calculated (Eq. 15). Correspondingly, for the transferred risk, the reinsurance treaty requires the primary insurer to pay an annual premium r^{sy} (Eq. 16), which includes a base premium b (Eq. 17 and Eq. 18) (Kunreuther and Michel-Kerjan 2009), and reinstatement payments defined as pro rata amount of the expected reinsurer loss without adjusting for the length of the treaty's remaining term (Eq. 16). Finally, the insurer's net profit F^{sy} in year y of scenario s refers to its total homeowner premiums collected minus its liabilities, which include the loss adjustment expenses portion of the premiums collected, the insured losses not covered by the deductibles and the reinsurance policy as well as the annual payment to the reinsurer (Eq. 7). We assume the administrative loading factor τ, deductible level d, co-participation percentage β and other reinsurance premium parameters (i.e., φ and g) to be known inputs. The insurer's decisions are then reduced to the profit loading factors $\lambda_v, v \in [H, L]$, the attachment point *A*, and the maximum limit *M* of the reinsurance treaty.

In this study, we further assume the insurer will start its business with an initial investment C^{s0} (under any scenario *s*) that equals *k* (user-specified constant) times the annual premiums received from homeowners of all building inventories in all risk regions (Eq. 19); and in each year *y* under each scenario *s*, it will reallocate its accumulated surplus C^{sy} greater than its initial investment amount by reinvesting in other lines of business or distributing it to investors as dividends (Eq. 20). If insurer's accumulated surplus in year *y* equals zero or less, we assume that the insurer becomes insolvent, and the profit F^{sy} and surplus C^{sy} are set to zero for the remaining years $(y+1,\ldots$ for the scenario *s*.

The problem that the insurer is facing can then be summarized as choosing the appropriate price levels and reinsurance treaty, which together determine its liability and profitability. Furthermore, the credibility for operation by the primary insurer is a small chance of bankruptcy over a 30-years business life as defined in Equation 24, where ϕ^s is a binary indicator variable that is one if the insurer becomes insolvent at any time (during the 30-year period) in scenario *s* and zero otherwise (Eq. 21), and α refers to a user-specified upper limit of insolvency rate over 30 years. Also, the State insurance regulators require the insurer's capacity ratio (also known as the leverage ratio) to be less than three in any year. The capacity ratio is defined as the net written premiums divided by the policyholder surplus (denoted as μ^{sy} in Equation 22). This requirement ensures that the insurer has the ability to honor claims in an extraordinary year. The regulatory capacity ratio requirement is represented as a constraint in the model, as defined in Equation 25, where ψ^{sy} is a binary indicator variable that is one if the insurer's capacity ratio exceeds three in year y of scenarios s and zero otherwise (Eq. 23), and η refers to a user-specified upper bound of this violation probability. Therefore, the insurer's problem is a constrained simulation optimization problem to maximize Equation 6 subject to Equations 7 through 25.

$$\max_{\lambda_H, \lambda_L, A, M} \frac{1}{SY} \sum_{sy} F_{sy} \tag{6}$$

$$F^{sy} = \sum_v p_v Q_v - \tau \sum_v Q_v - \left(\sum_{t \in \omega(y)} \sum_h r^{sth} L^h - \sum_{t \in \omega(y)} \sum_h r^{sth} B^h - \beta e^{sy} \right) - r^{sy} \quad \forall s, y \tag{7}$$

$$p_v = 1 + \tau + \lambda_v \quad \forall v \in [H, L] \tag{8}$$

$$w^*_{imcv} = O(p_v) \quad \forall i, m, c, v \tag{9}$$

$$L^h = \sum_{imcv} w^*_{imcv} L^h_{imcv} X_{imcv} \quad \forall h \tag{10}$$

$$B^h = \sum_{imcv} w^*_{imcv} B^h_{imcv} X_{imcv} \quad \forall h \tag{11}$$

$$B^h_{imcv} = \min\left\{ L^h_{imcv}, d \right\} \quad \forall i, m, c, v \tag{12}$$

$$Q_v = \sum_{imcv} w^*_{imcv} \left(\sum_h P^h L^h_{imcv} X_{imcv} \right) \quad \forall v \in [H, L] \tag{13}$$

$$q^h = \min\left\{ \max\left\{ L^h - A,\ 0 \right\},\ M - A \right\} \quad \forall h \tag{14}$$

$$e^{sy} = \sum_{t \in \omega(y)} \sum_h r^{sth} q^h \quad \forall s, y \tag{15}$$

$$r^{sy} = b + \frac{e^{sy}}{M-A}\left(\sum\nolimits_h P^h \beta q^h\right) \quad \forall s, y \tag{16}$$

$$b = (1+\varphi)\left(\sum\nolimits_h P^h \beta q^h\right) + g\delta \tag{17}$$

$$\delta = std\left\{\beta e^{sy} - \left(\frac{e^{sy}}{M-A}\right)\left(\sum\nolimits_h P^h \beta q^h\right)\right\} \tag{18}$$

$$C^{s0} = k\sum\nolimits_v p_v Q_v \quad \forall s \tag{19}$$

$$C^{sy} = \min\left\{C^{s,y-1} + F^{sy},\ k\sum\nolimits_v p_v Q_v\right\} \quad \forall s, y \tag{20}$$

$$\phi^s = 1 \text{ if } \exists y \in T, \text{ s.t. } C^{sy} \le 0;\ \phi^s = 0 \text{ otherwise } \forall s \tag{21}$$

$$\mu^{sy} = \frac{(1-\tau)\sum_v p_v Q_v - r^{sy}}{C^{sy}} \quad \forall s, y \tag{22}$$

$$\psi^{sy} = 1 \text{ if } \mu^{sy} > 3;\ \psi^{sy} = 0 \text{ otherwise} \tag{23}$$

$$\frac{1}{S}\sum_s \phi^s \le \alpha \tag{24}$$

$$\frac{1}{SY}\sum\nolimits_{sy} \psi^{sy} \le \eta \tag{25}$$

3. SOLUTION PROCEDURE

By substituting the homeowner model with a response function of the form (Eq. 9), the entire optimization problem is represented by the insurer model only (Eq. 6 through Eq. 25), with four continuous decision variables. To apply RSM, these variables are coded as described in Myers and Montgomery (2002).

To solve the constrained simulation optimization problem, we transform the constraints (Eq. 24 through Eq. 25) into penalty functions and add them to the original objective as follows (Eq. 26), where $\lambda_1(\cdot),\lambda_2(\cdot)$ refers to user defined penalty functions for the constrained terms. These penalty functions disappear when the underlying constraint is satisfied but impose a negative penalty on the objective based on the magnitude of the violation when the underlying constraint is not satisfied. The problem is then simplified as an unconstrained simulation optimization with four continuous variables (Eq. 26).

$$\max_{\lambda_H,\lambda_L,A,M}\left(\frac{\sum_{sy}F^{sy}}{SY}+\lambda_1(\alpha-\frac{1}{S}\sum_s\phi^s)+\lambda_2(\eta-\frac{1}{SY}\sum_{sy}\psi^{sy})\right) \tag{26}$$

Consistent with STRONG, we adopt a sequential two-stage solution procedure for this maximization problem, with the first stage constructing and optimizing a first-order polynomial approximation through steepest-ascent search, and the second stage constructing and optimizing a second-order polynomial approximation by efficient quadratic programming methods (e.g., conjugant gradient) (Figure 3.2). However, our solution procedure differs from STRONG in two aspects. Firstly, we utilize Latin hypercube experimental design with random pairing of decision variables in each stage, instead of factorial or fractional factorial design in the first stage and central composite design in the second stage as described in Chang et al. (2007). Since the influence of each variable on the output is not precisely known in advance, factorial design with high levels would result in large number of combinations. For example, a full factorial design of 4 variables with 4 levels of values would result in 4^4=256 design points. On the other hand, in a Latin Hypercube, each factor has as many levels as there are runs in the design; and the levels are spaced evenly from the lower bound to the upper bound of the factor. In other words, a Latin Hypercube design with 256 points would sample each variable from 256 uniformly distributed levels of value, which is very important when the range of a decision variable lies in millions or billions (e.g., A and M of our model). Moreover, Latin hypercube design with random pairing treats all design variables with equal consideration and has been proven to be as efficient as full factorial designs when paired with polynomials in fitting functions (Hussain et al. 2002).

Secondly, our solution procedure simplifies the second stage, due to a heavier emphasis on the redesign of the Latin Hypercube experiments instead of using a sub-program to guarantee improvement/satisfaction in each iteration. In the second stage of STRONG, a sub-algorithm is adopted after trust region update, if the trust region is reduced due to unsatisfactory increase in the ratio. The sub-algorithms keep improving the local solution by adding more design points and re-solving the problem with new trust regions until it reaches a satisfactory increase ratio. It should be noted that the sub-program needs to accumulate design points and track rejected solutions, which requires significant memory. Moreover, the sub-algorithm is useful when the local approximation is poor or there is sampling error, but it is not necessary otherwise. In our study, the Latin Hypercube design with a large number of random pairings typically results in good local model fitness in each iteration, and there is no sampling error in our simulation-optimization problem. As a result, we omit the sub-algorithm in the second stage.

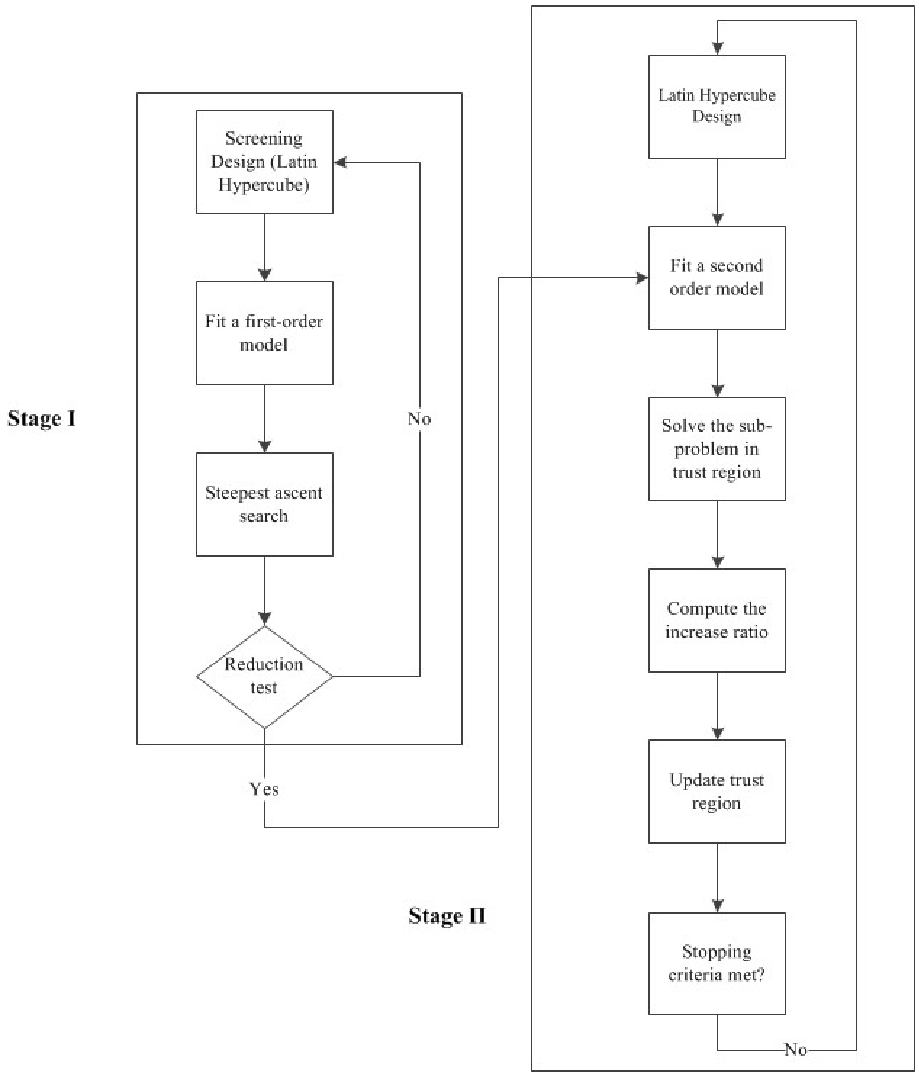

Figure 3.2: Solution procedure (based on STRONG in Chang et al. (2007))

4. CASE STUDY

4.1 Inputs

We conducted a full-scale case study for hurricane risk to residential buildings in Eastern North Carolina. The study area includes 503 census tracts. Those that are on the coast are divided into three zones based on the distance to the coastline. This creates 731 geographic zones *i* that we can reasonably assume are homogeneous in flood depth within each zone. Eight categories *m* of wood-frame buildings with 68 possible resistance levels c are examined for their hurricane risk and considered as the insurance customers. All the residential buildings are then associated with either the high or low risk region v, based on the risk levels as defined by the insurer. The high risk region is assumed to be within 2 miles to the coast, while the low risk region is everywhere else. The resultant building inventory has high dimensional structure and reaches about one million in total.

The risk aversion parameter values θ_v used in the homeowner utility model are estimated using National Flood Insurance Program data. Specifically, values of θ_v are chosen so that given our assumed utility model, they would result in the penetration rates reported in Dixon et al (2006). The final parameter values are $\theta_H = 3.0(10^{-5})$ and $\theta_L = 1.7246(10^{-5})$ for high and low risk areas, respectively (Gao 2013). The homeowner insurance budgets for these two risk areas are assumed to be $k_H = 5\%$ and $k_L = 2.5\%$ of building value. A minimum premium $\rho = \$100$ is required by insurer to sell insurance in this study.

The input parameter values for the insurer model include $d = \$2500$, co-participation factor $\beta = 95\%$, primary insurer administrative loading factor $\tau = 0.35$, reinsurer loading factor $\varphi = 0.1$, reinsurer risk attitude $g = 0.1$, factor defining allowed surplus $k = 3$, maximum allowable thirty-year probability of insolvency $\alpha = 0.1$, maximum allowable capacity ratio violation rate $\eta = 0.1$. Further, based on the results from screening experiments, the weights of penalty terms in the transformed objective function (Eq. 26), are selected to be $\lambda_1 = 7e7, \lambda_2 = 7e7$.

Finally, as described in the loss model definition, the simulation of the hurricane hazards was represented by an efficient set of H=97 probabilistic scenarios, with S=2000 independent thirty-year (600 time steps) timelines of hurricane occurrence (i.e., scenarios s).

4.2 Results

We implemented the combined Response-Surface and Trust-Region method in MATLAB (R2012a) and utilized “rsm” and “trust” statistical packages under R (2.13.0) for regression analysis and the related optimization. The two-stage automatic program was executed in parallel on a Unix machine with 12 cores. In the first stage, the screening experiment was constructed by a Latin hypercube design of 240 pairs of the four decision variables in each iteration; while in the second stage, 120 pairs of Latin hypercube design points were automatically generated in each iteration. During the trust region updating process, if the reduction ratio is below 5%, the radius of the new trust region is reduced by 75% (i.e., 25% of current radius length); if the reduction ratio is above 90%, the radius of the new trust region is increased by 1.5 times (i.e., 2.5 times of current radius length). The algorithm stops either if the size of the trust region radius is below 1e-4, or reaches 30 iterations in the second stage. On average, each trial finished within 80 minutes, and the final result satisfies all the constraints.

To check the result quality and efficiency, we also applied simulated annealing (SA), a non-gradient based heuristic optimization algorithm to solve this problem. Similarly, the optimization was implemented in MATLAB (R2012a) and executed in parallel on the same machine. The initial values of the decision variables and the program parameters were determined based on 100 initial evaluations of average change in cost for an uphill move. Transitions were made randomly within neighborhoods of +/- 0.2 from the current loading factors, λ_L and λ_H and +/- 200 million from the current reinsurance terms, *A* and *M*. Each trial of the algorithm includes 1000 iterations, and requires approximately 3.75 hours. The results converge well with the last 100 iterations of each trial exhibiting only 0.01% improvement in the objective function, and all constraints are satisfied.

Table 3.1 compares these two algorithms in terms of the decision variable values, objective function values, and computation time. The variable values from the two algorithms are each within 10%, and the automatic RSM-TR method produces a better solution than SA (3% better in objective function/net profit value), and uses only a third (35%) of the computational time.

Table 3.1: Comparison between RSM-TR and SA

	RSM-TR	**Simulated Annealing**	**Difference**
λ_H	1.4444	1.4848	-3%
λ_L	1.1562	1.2388	-5%
A	2.3608e8	2.5017e8	-6%
M	2.8911e9	2.7543e9	5%
Objective function value	5.5838e7	5.4272e7	3%
Run time (per trial)	1.33 hours	3.75 hours	-65%

5. *CONCLUSION*

In this study, we applied the combined Response-Surface-Methodology (RSM) and Trust-Region (TR) method (following STRONG structure) to the constrained simulation optimization of natural catastrophe insurance market. The optimization problem has a game-theoretical structure and involves three interacting models. We conducted a full-scale realistic case study for hurricane risk to residential buildings in Eastern North Carolina, with high dimensional input data. The automatic program was utilized to solve the complicated and computationally-demanding optimization problem, and also compared with Simulated Annealing (SA) for result quality and efficiency. The results from both algorithms satisfies the constraints, and the automatic Response-Surface and Trust-Region method outperforms SA in both result quality and computation time.

REFERENCES

Apivatanagul, P., Davidson, B., Blanton, B. and Nozick, L. (2011). "Long-term regional hurricane hazard analysis for wind and storm surge." *Coastal Engineering* 58(6): 499-509.

Box, G. E. P., Hunter, W. G. and Hunter, J. S. (1978). *Statistics for Experimenters: An Introduction to Design, Data Analysis, and Model Building*, John Wiley & Sons.

Box, G. E. P. and Wilson, K. B. (1951). "On the Experimental Attainment of Optimum Conditions." *Journal of the Royal Statistical Society* 13(1): 1-45.

Chang, K.-H., Hong, J. L. and Wan, H. (2007). *Stochastic trust region gradient-free method (STRONG) - A new response-surface-based algorithm in simulation optimization*. Proceedings of the 2007 Winter Simulation Conference.

Chang, K.-H., Hong, L. J. and Wan, H. (2012). "Stochastic Trust-Region Response-Surface Method (STRONG)-A New Response-Surface Framework for Simulation Optimization." *INFORMS Journal on Computing*(Articles in Advance): 1-14.

Conn, A. R., Gould, N. M. and Toint, P. L. (1987). *Trust-Region Methods*, Society for Industrial and Applied Mathematics.

Dixon, L., Clancy, N., Seabury, S. A. and Overton, A. (2006). The National Flood Insurance Program's Market Penetration Rate: Estimates and Policy Implications.

Gao, Y. (2014) Modeling a Market for Natural Catastrophe Insurance, Ph. D. dissertation, Cornell University

Hussain, M. F., Barton, R. R. and Joshi, S. B. (2002). "Metamodeling: Radial basis functions, versus polynomials." *European Journal of Operational Research* 138: 142-154.

Kesete, Y. Modeling Homeowner Insurance decision Making and Insurer-Homeowner Interactions in Managing Natural Disaster Risk, Ph. D. dissertation, Cornell University, in progress.

Kleijnen, J. P. C. (2008). "Response surface methodology for constrained simulation optimization: An overview." *Simulation Modeling Practice and Theory* 16: 50-64.

Myers, R. H. and Montgomery, D. C. (2002). *Response Surface Methodology Process and Product Optimization Using Designed Experiments*, John Wiley & Sons Inc.

Neddermeijer, H. G., Oortmarssen, G. J., Piersma, N. and Dekker, P. (2000). *A framework for Response Surface Methodology for simulation optimization*. Proceedings of the 2000 Winter Simulation Conference, Orlando, FL.

Peng, J. (2013) Modeling natural disaster risk management: Integrating the roles of insurance and retrofit and multiple stakeholder perspectives, Ph. D. dissertation, University of Delaware.

Peng, J., Shan, X., Davidson, R., Nozick, L., Kesete, Y. and Gao, Y. (2013). Hurricane loss modeling to support regional retrofit policymaking: A North Carolina case study. *11th International Conference on Structural Safety and Reliability - ICOSSAR'13*. New York, NY.

Sadjadi, S. J. and Ponnambalam, K. (1999). "Advances in trust region algorithms for constrained optimization." *Applied Numerical Mathematics* 29: 423-443.

Shi, Z.-J. and Guo, J.-H. (2008). "A new trust region method for unconstrained optimization." *Journal of Computational and Applied Mathematics* 213: 509-520.

Zhang, J.-L. (2006). "A robust trust region method for nonlinear optimization with inequality constraint." *Applied Mathematics and Computation* 176: 688-699.

APPENDIX

Homeowner Model Risk Parameters Calibration

1. The NFIP Premium Structure

The National Flood Insurance Act of 1968 separated the flood insurance ratemaking process into two distinct categories, namely, chargeable premium (subsidized) rates and estimated-risk premium (actuarial) rates. Actuarially rated policies are charged premiums that consider the full range of possible losses, including catastrophic levels. The application of these two rates of NFIP depends on whether buildings have been constructed after the issuance of a Flood Insurance Rate Map (FIRM) or before the issuance of the FIRM. New buildings (Post-FIRM) are charged full-risk premiums that contemplate the full range of loss potential including catastrophic levels. Full-risk premiums are also charged to all buildings that are outside the Special Flood Hazard Area. By statute, highly subsidized premiums have been made available for Pre-FIRM buildings in the Special Flood Hazard Areas (SFHA). The distribution of business written in 2010 is anticipated to be 22% at subsidized rates and 78% at full-risk premium rate (refers to Actuarial Rate Review of the NFIP, pages 4-14, 34).

2. Actuarial Rate and Standard Deductible of NFIP in 2010

Prior to the four hurricanes of 2004, program premium were 125% of the historical average loss year. Due to the adverse experience of 2004 and 2005, new rules have been adopted in ratemaking. The written premium based on all rate and rule changes through October 2010 is expected to be 123.5% of the adjusted historical average loss year. This would imply a loading factor of 1.235 for the low risk region.

FEMA estimates that the premiums for subsidized policyholders are between 40% and 45% of fully actuarial premiums. If we take 40% as the subsidization rate, then the loading factor for the SFHA area is going to be 0.341. (Refer to Actuarial Rate Review of the NFIP, pages 4-14, 34)

On October 1, 2009 the NFIP discontinued the $500 deductible for all properties. Pre-FIRM buildings in Special Flood Hazard Areas (SFHAs) will have a $2,000 standard deductible. Post-FIRM buildings and Pre-FIRM buildings rated as Post-FIRM in SFHAs will have a $1,000 standard deductible. Buildings in non-SFHAs will have a $1,000 standard deductible. As a result, $1000 is considered as the standard deductible in our study, and the calibration process is based on this assumption as well. (Refer to FEMA's NFIP rate-revise announcement in 2009)

3. Penetration Rate of NFIP

From the RAND report of penetration rate on NFIP, which used parcel data sampled from communities (in 2004), the national penetration rate for Single-Family Homes (SFHs) based on address matching in SFHA areas is 50% and 1% in non-SFHA areas.

In the South region (where North Carolina is classified into), the penetration rate for SFH(s) with 95% confidential interval is 61% [54~69%] in SFHA and 3% [0.8~5%] in LR regions (refers to RAND report 2006, pages 14-20).

Table A.1 Market Penetration Rate for SFHs Based on Address Matching

TABLE 4.1: Market Penetration Rate for SFHs Based on Address Matching

Location	Sample Size[1]	Number of Single-Family Homes (millions)	95% CI[2]	Number of Single-Family Homes with Policies (millions)	95% CI	Market Penetration Rate (percent)	95% CI
Inside SFHA							
Northeast	631	0.77	[0.45, 1.10]	0.22	[0.10, 0.34]	28	[11, 46]
South	4,069	2.06	[1.56, 2.57]	1.27	[0.89, 1.64]	61	[54, 69]
Midwest	193	0.45	[0.25, 0.65]	0.10	[0.05, 0.15]	22	[15, 30]
West	579	0.29	[0.10, 0.47]	0.17	[0.04, 0.30]	60	[51, 68]
Total U.S.	5,472	3.57	[2.94, 4.20]	1.76	[1.36, 2.16]	49	[42, 56]
Outside SFHA							
Northeast	5,572	20.4	[10.6, 30.2]	0.11	[0. 0.23]	0.6	[0, 1.1]
South	7,487	15.4	[5.6, 25.2]	0.46	[0.27, 0.65]	3	[0.8, 5]
Midwest	4,912	28.0	[15.4, 40.5]	0.11	[0.03, 0.20]	0.4	[0.1, 0.7]
West	4,224	11.9	[2.1, 21.7]	0.12	[0, 0.25]	1	[0.6, 2.5]
Total U.S.	22,195	75.6	[55.9, 95.4]	0.81	[0.55, 1.07]	1	[0.7, 1.4]
Anywhere in NFIP Community							
Northeast	6,203	21.2	[11.0, 31.3]	0.33	[0.10, 0.57]	2	[0.6, 2.5]
South	11,556	17.5	[7.5, 27.4]	1.73	[1.28, 2.17]	10	[4, 16]
Midwest	5,105	28.4	[15.8, 41.0]	0.21	[0.09, 0.34]	1	[0.3, 1.2]
West	4,803	12.2	[2.4, 21.9]	0.29	[0.11, 0.47]	2	[0.8, 4]
Total U.S.	27,667	79.2	[59.3, 99.1]	2.56	[2.03, 3.09]	3	[2, 4]

[1]Denotes the number of observations on which the estimates are based.
[2]95–percent statistical confidence interval for estimate.

FIGURE A1.1: FEMA Regions

The following four geographic regions are used in this study:

Northeast = FEMA regions 1, 2, and 3
South = FEMA regions 4 and 6
Midwest = FEMA regions 5, 7, and 8
West = FEMA regions 9, and 10.

Figure A.1 FEMA Region

FIGURE A2.1: Map of Communities in Sample

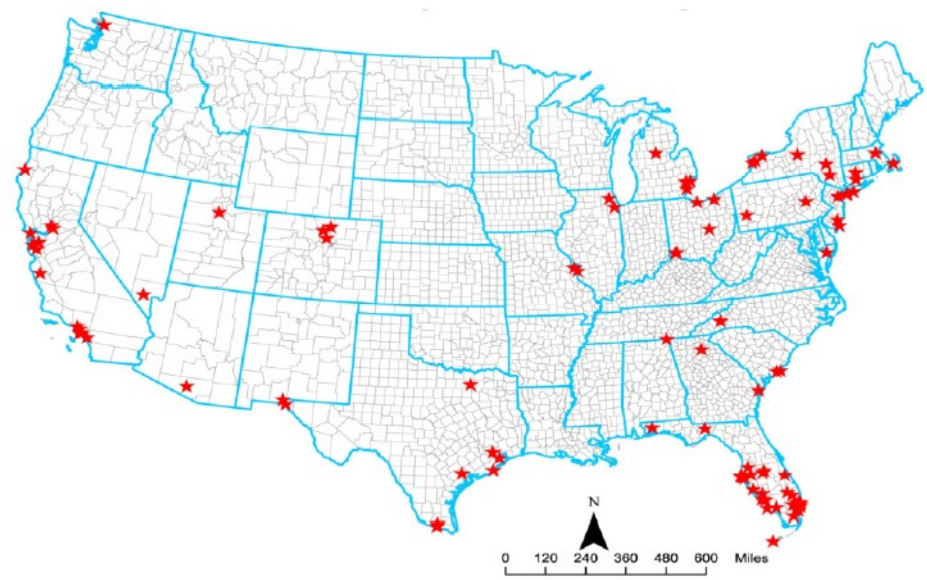

Figure A.2 Map of communities in sample

Until 2010, no reform of NFIP has been adopted by the congress, despite of various proposals it received (FEMA 2010). Thus, we would expect the market penetration rate to stay at the same level. However, as pointed in the Figure A2.1, there are almost no communities sampled in eastern North Carolina. But, North Carolina is included in the "South" region classification of FEMA. As a result, the 60% and 3% penetration rates are not accurate enough to be applied in our study region. Thus, it might be more reasonable to use the national penetration rate (50% for SFHA and 1% in non-SFHA areas) instead.

4. SFHA and non-SFHA Buildings Inventory Classification

In our study area – Eastern North Carolina, consist of 503 census tracts, residential buildings in each census tract are classified into three zones based on their distance to water. However, this definition does not necessarily match the SFHA and non-SFHA areas classification of NFIP. Even though, our 3-zone definition should not differ too much from the NFIP definition. More specifically, all inland buildings (in zone 1) should be classified as non-SFHA areas, while risk-prone areas (zone 2 and 3, 0-2 miles to water) could be considered as SFHA areas.

Besides, since the distribution of business written in 2010 is anticipated to be 22% at subsidized rates and 78% at full-risk premium rate, (i.e. 22% to be SFHA pre-FIRM subsidized buildings, 78% to be low risk buildings). We would need to match this rate by modifying our new definitions.

Under the 3-zones definition, zone 1 (inland) accounts to 69.64% of total building inventory; while zone 2 (1-2 mile to water) accounts to 14.28% of total inventory and zone 3 (0-1 mile to water) accounts to 16.07% of total inventory. (Together, zone 2 and 3 account to 30.36 %.) If we assume subsidize occurs in both zone 2 and 3, and full premium applied in zone 1 only, we would over-subsidize the insurance (since 30.36% > 22%). So, based on the subsidization distribution and building inventory configuration, we made the following adjustment:

a) All buildings in zone 1 (inland) are considered to be in low risk areas (non-SFHA) and are applied with full premium;
b) All buildings in zone 3 (0-1 mile to water) are considered to be in SFHA areas and are apply subsidized premium;
c) Partial of zone 2 (1-2 mile to water) are included in SFHA definition as well for subsidization; the rest of zone 2 is considered as low risk areas without subsidization (43% of zone 2 buildings are subsidized, while the rest 57% are applied with full premium);
d) Under these modifications, 21.6% of total inventory are subsidized while 78.4% are applied full-premium.

5. Risk Parameters Calibration Process

CARA utility function:

$$U(x) = 1 - e^{-\theta x}$$

From the above information, our calibration process is aiming to find the optimal risk parameters that could match the following relationship: (Denote the subsidized premium rate and building inventory as SFHA, the full-premium rate and building inventory as LR)

$$\lambda_L = 1.235,\ p_{imcL} = (1+\lambda_{LR})\sum_h P^h (L^h_{imcL} - B^h_{imcL})\ \forall i,m,c\ (v = L)$$

$$1+\lambda_H = (1+\lambda_L) * (1-40\%) \rightarrow \lambda_H = 0.341$$

$$\theta_H \rightarrow \text{penetration rate at } \lambda_H = 0.341 \rightarrow \frac{\sum_{imc} W_{imcH}}{|i,m,c|} = 50\%\ (v = H)$$

$$\theta_L \rightarrow \text{penetration rate at } \lambda_L = 1.235 \rightarrow \frac{\sum_{imc} W_{imcL}}{|i,m,c|} = 1\%\ (v = L)$$

$$\text{where } \frac{X_{imcH}}{X_{imcH} + X_{imcL}} \approx 22\%;\ \frac{X_{imcL}}{X_{imcH} + X_{imcL}} \approx 78\%$$

After experimenting with 120 pairs of random (θ_H, θ_L) uniformly distributed in [1e-3, 1e-6], and [1e-4, 1e-6] respectively, response surface method (least square regression) is adopted to describe the relationship between θ and penetration rate in order to backward calculate or find the optimal parameter value.

1) Result for low risk (non-SFHA) region

$$\frac{\sum_{imc} W_{imcL}}{|i,m,c|} \sim FO(\theta_L) + SO(\theta_L) + \text{constant}$$

Adjusted R-square = 0.969
Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)	
(Intercept)	-0.024603	0.001982	-12.41	<2e-16	***
x1	0.226548	0.009161	24.73	<2e-16	***
x1^2	-0.095416	0.008752	-10.90	<2e-16	***

Note: x1 refers to the coded value --- $x1 = \frac{\theta_L - 10^{-6}}{10^{-4} - 10^{-6}}$, where $\theta_L \in [10^{-6}, 10^{-4}]$.

Residual standard error: 0.006943 on 117 degrees of freedom
Multiple R-squared: 0.9696, Adjusted R-squared: 0.969
F-statistic: 1864 on 2 and 117 DF, p-value: < 2.2e-16

Finally, by using the fitted relationships and plug the 3% or 1% target value into the equations, the calibrated parameters values are: $\theta_L = 2.7948e-5$ or $\theta_L = 1.7246e-5$ respectively.

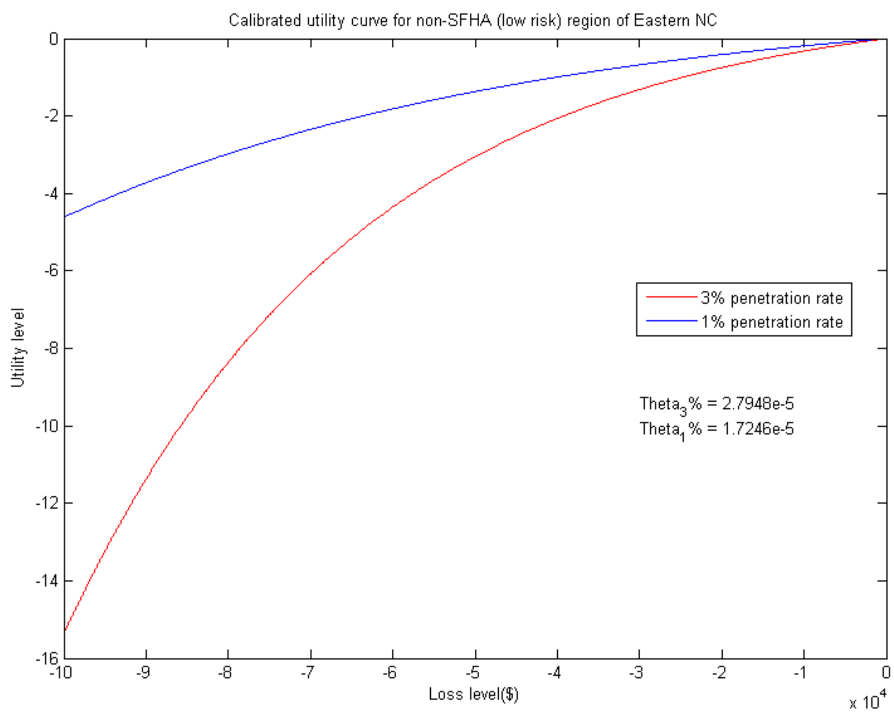

Figure A.3 Calibrated utility curve for non-SFHA (low risk) region of Eastern North Carolina

2) Result for high risk (SFHA) region

$$\frac{\sum_{imc} W_{imcH}}{|i,m,c|} \sim FO(\theta_H) + SO(\theta_H) + \text{constant}$$

Adjusted R-square = 0.7598

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.30833	0.01064	28.98	<2e-16 ***
x1	0.77892	0.04917	15.84	<2e-16 ***
x1^2	-0.59852	0.04699	-12.74	<2e-16 ***

Note: x1 refers to the coded value --- $x1 = \frac{\theta_H - 10^{-6}}{10^{-3} - 10^{-6}}$, where $\theta_H \in [10^{-6}, 10^{-3}]$.

Residual standard error: 0.03727 on 117 degrees of freedom

Multiple R-squared: 0.7639, Adjusted R-squared: 0.7598

F-statistic: 189.2 on 2 and 117 DF, p-value: < 2.2e-16

Though theta ranges from [1e-3~1e-6], the penetration varies between [40~50%] and could not exceed 56% no matter what value of theta. Finally, by plugging the 49% or 56% target value into the equations, the calibrated parameters values are: $\theta_H = 3.052e-4$ or $\theta_H = 5.97e-4$ respectively.

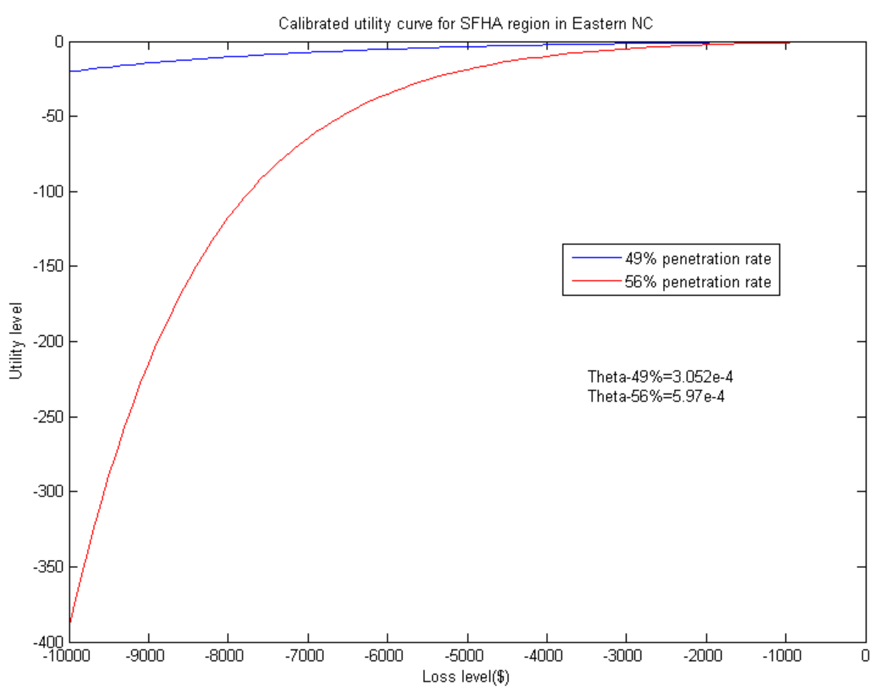

Figure A.4 Calibrated utility curve for SFHA (high risk) region of Eastern North Carolina

6. Risk Parameters Values and Resulted Utility Curves

Based on the results from previous section, we select the risk attitude parameters associated with 1% and 49% penetration rate in low and high risk areas correspondingly for the homeowner model. More specifically, $\theta_{LR} = 1.7246e-5$ and $\theta_{SFHA} = 3.052e-4$. The utility curves based on the selected values are listed below.

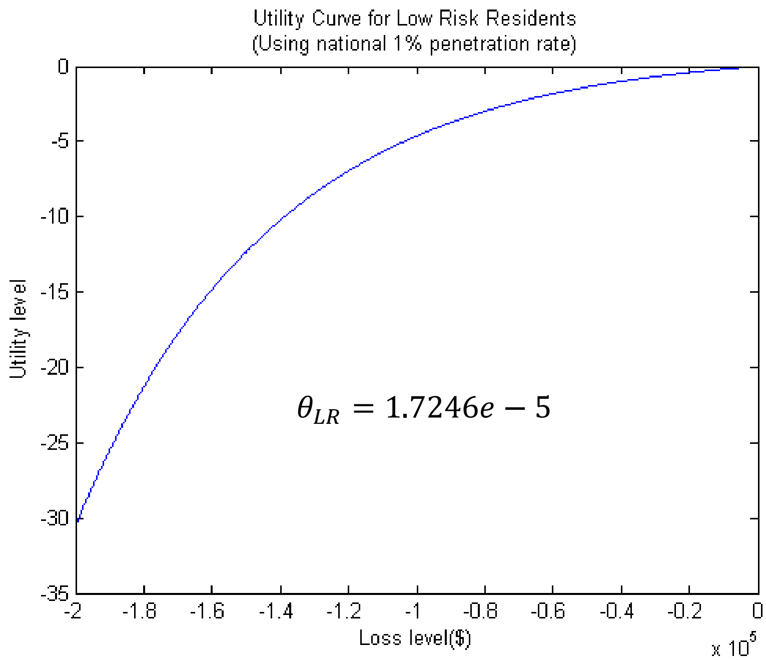

Figure A.5 Selected utility curve for residents in low risk region

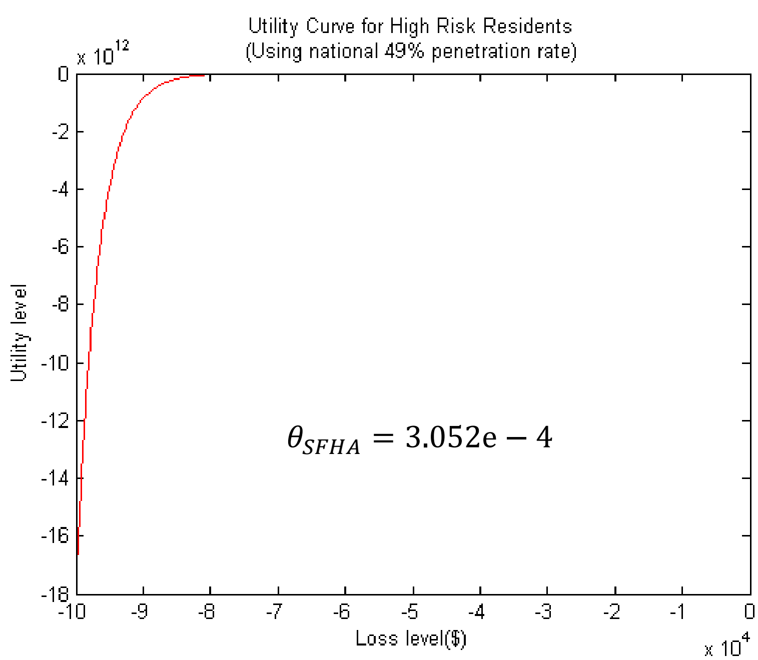

Figure A.6 Selected utility curve for residents in high risk region