

January 9 , 2001

GENDER AND SAY
A Model of Household Behavior with Endogenously-determined Balance of Power

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Abstract

The evidence that the same total income can lead a household to choose different consumption vectors, depending on who brings in how much of the income, has led to an effort to replace the standard unitary model of the household with the ‘collective model’, which recognizes that the husband and the wife may have different preferences and depending on the balance of power between them the household may choose differently. One weakness of this new literature is that it fails to recognize that the household’s choice could in turn influence the balance of power. Once this two-way relation between choice and power is recognized we, are forced to confront some new questions concerning how to model the household. This paper tries to answer these by defining a ‘household equilibrium’, examining its game-theoretic properties and drawing out its testable implications. It is shown, for instance, that a household equilibrium can be inefficient and that (for a certain class of parameters) children will be least likely to work in a household where power is evenly balanced. The paper also draws out the implications for female labor supply.

Keywords : Gender, Power, Household behavior, Female labor supply, Child labor

Acknowledgements: The author is grateful to Chris Barrett, Francois Bourguignon, Nancy Chau, Patrick Francois, Karla Hoff, Ravi Kanbur, Valerie Lechene, Arvind Panagariya, Joo Kyung Ree, and Erik Thorbecke for useful discussions. The paper has benefited from seminar presentations at Cornell University, Florida International University, University of Maryland at College Park, El Colegio de Mexico and the CentER at Tilburg. Thanks also go to the Institute for Social and Economic Change, Bangalore, where the author was Visiting Professor during part of the time that he was working on this paper. The paper is part of the World Bank research project, “Gender and Power Relations in the Household”, being coordinated by Kaushik Basu.

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1. Introduction

The unitary model of the household, which had served mainstream economics well for a long time, has in recent times given way to a more fractious view of the household. This has been an outcome of theoretical advances, empirical investigations and anthropological insights.¹ It is, for instance, now clear that how much say a woman has in the household can vary across households in the same region and with the same total income. A woman's say could depend, for instance, on how much income she contributes to the household's total income.

This recognition has enormous implications for the design of policy. It means that 'how' a certain amount of money is injected by government into households can influence the well-being of individuals significantly. Ten dollars given to the male head of the household and the same money given to his wife can have very different implications for not only the amount of tobacco and alcohol purchased by the household but on child labor, education and health (Kanbur and Haddad, 1995). When a series of policy changes in the United Kingdom (see Walker and Zhu, 1999 for description) from 1976 to 1979 caused household allowance for children to be handed over to the women

¹ Manser and Brown (1980); McElroy and Horney (1981); Folbre (1986); Mencher (1988); Sen (1983, 1990); Thomas (1990); Bourguignon and Chiappori (1994); Browning, Bourguignon, Chiappori and Lechene (1994); Moehling (1995); Udry (1996); Agarwal (1997), Riley (1997); Haddad, Hoddinott and Alderman (1997); Basu (1999).

instead of men, there was a rise in the expenditure on children's clothings (Lundberg, Pollak and Wales, 1997; for related accounts, see Hoddinott and Haddad, 1995, and Quisumbing and Maluccio, 1999).² However to go from this broad recognition to the actual design of policy one needs to understand the relation between the household balance of power and household behavior. There is now a substantial literature on this, some of which was cited in footnote 1 above.

It will be argued in this paper that there is one important lacuna in this new theoretical literature. While this literature models successfully the impact of household power balance on household decision making, it ignores the opposite relation – that is, the effect of household decisions on the balance of power. Modeling both these relations, simultaneously, requires some theoretical inventiveness, as we shall show presently. This demonstration forms the core of this paper. The next section recapitulates the received doctrine and develops the central idea of this paper. The remaining sections are best viewed as corollaries – they develop special cases, draw out the implications of our approach for different areas of economics and suggest new directions for empirical research.

2. Household Decision Making: The Main Model

Consider a household with two adults. There may or may not be children in the household. In the standard "unitary model" of the household, either both adults have the

² For a survey of how micro credit may have contributed to the empowerment of women in Bangladesh, see Rahman (2000).

same preference or one of them takes all the decisions. In any case the upshot is that the household behaves *as if* it were a single or a unitary agent (Becker, 1981).

One way of capturing the fact that household members may cooperate with each other but are nevertheless fractious, is to adopt the "collective approach" to the household. This begins with the recognition that each agent – the woman (1) and the man (2) – has a distinct utility function and the household maximizes a weighted average of these two functions, with the weights capturing the balance of power in the household.

To develop the model formally, let $u_i : \mathfrak{R}_+^n \rightarrow \mathfrak{R}$ be agent i 's utility function, where \mathfrak{R} is the set of real numbers. The argument $x \in \mathfrak{R}_+^n$ of the utility function is a vector of n goods consumed by the household. We can think of goods in very general terms. It includes, for instance, leisure consumed by each person. We also have the option to think of apples for person 1 as a separate good from apples for person 2.

The household's maximand, in this "collective approach" is then given by $\Omega \equiv \theta u_1(x) + (1-\theta)u_2(x)$, where $\theta \in [0,1]$ captures the balance of power in the household. As θ increases, the power of the wife increases.³

It is recognized in this model that θ may in turn, depend on other variables. If, for instance, the wage rate for female workers rises, θ may rise. If the wife brings a lot of inherited wealth into the household, θ could be higher. The value of θ may also depend

³ This is, of course, a simplifying assumption. The power of a woman, like status, is a multi-dimensional concept and in a larger study there would be a compelling case for distinguishing different varieties of it. A woman may have 'access' to resources in the household, without having any 'control'; a woman may have a lot of power in the kitchen, but little outside (see Mason, 1986, for discussion).

on cultural factors.⁴ Let z be the variables which determine θ . Hence, we may write the power function as $\theta(z)$. In the collective model z consists of variables exogenous to the household. This innocuous assumption will be challenged shortly. But let us go along with it for now.

The household's problem can now be written very simply:

$$\text{Max } \theta(z)u_1(x) + (1-\theta(z))u_2(x)$$

$$\text{subject to } x \in \mathfrak{R}_+^n \text{ and } px \leq Y$$

where p is the vector of prices and Y the unearned income of the household. From now on we will refer to the budget set as T . Hence,

$$T = \{x \in \mathfrak{R}_+^n \mid px \leq Y\}$$

With Y and p remaining the same, a household's expenditure pattern can change if z changes, causing a shift in the balance of power.

An important shortcoming of this model is the assumption that z consists of exogenous variables. There is however reason to believe that θ may get affected by changes in the household's choice, x . One variable that is widely acknowledged to be a determinant of θ in the woman's earning power. In the existing literature (see, for instance, Bourguignon and Chiappori, 1994; and Moehling, 1995) this is captured by the

⁴ The second National Family Health Survey, 1998-99, in India has a wealth of information on female autonomy. From the preliminary reports that are beginning to appear (for instance, ISEC and IIPS, 2000) one can see the role of both cultural and economic factors that influence female autonomy. A sharp rise in female autonomy occurs if the female happens to be self-employed. Since the self-employment is the result of deliberate decision, this fact lends support to our claim, made below, regarding the endogeneity of θ .

prevailing market wage for female workers, w_1 . Hence θ is taken to be determined by (among other things) w_1 .

It is however arguable that what determines the woman's bargaining power is not just the female wage rate but what she actually earns. Thus if e_1 is the number of hours the woman works, then, according to this view θ depends on $w_1 e_1$. Since e_1 will typically be a variable the household chooses (that is, it is a part of x), θ gets influenced by the household's decision. This creates some obvious difficulties in modeling household behavior since we need some technique for taking account of this feedback effect.

This theoretical problem cannot be overlooked because it seems eminently reasonable to suppose that whether or not θ depends on the wage rate *per se*, it must also depend on what the woman in the household actually earns. A traditional woman whose social norms prevent her from working surely has less power than a woman who actually works, despite the fact that both may be living in the same region and so confronting the same market wage rate w_1 . There is also some anthropological and sociological evidence (see, for example, Mencher, 1988; Riley, 1997) that a woman's actual contribution to the household budget influences how much say she has in the household decision making. In other words, even if a woman's working in the household or household farm is what enables her husband to go out and earn a wage, she does not have as much power as she would if she did the actual earning herself (see the evidence from Karnataka, India, provided by Desai and Jain, 1994; see also Blumberg and Coleman, 1989).⁵

⁵ As Zelizer (1994, p.140) notes in the context of American labor, "No matter how hard they worked or how much their families depended on their labors, women's housework was defined—and valued—as an emotional task, but hardly of material import."

To keep the model as general as possible at this stage, assume that θ depends not just on z but also on x . Hence, we may write $\theta = \theta(z, x)$.

The household's maximand, as before, is:

$$\Omega(x) = \theta u_1(x) + (1-\theta)u_2(x) \quad (1)$$

The problem now is that θ itself depends on x . So if for a *given* balance of power index, θ , the household maximizes Ω and chooses x , this may in turn cause θ to change. So the household may want to adjust x further.

A natural 'equilibrium' idea (see Basu, 1999), is the stationary point of this process. To define this formally, let us first describe the solution of the household's maximization problem as $x = \eta(p, Y, \theta)$. In other words,

$$\eta(p, Y, \theta) \equiv \arg \max_{x \in T} [\theta u_1(x) + (1 - \theta)u_2(x)]$$

Here is our crucial definition of how a household behaves in equilibrium.

Definition. Given (p, Y, z) a *household equilibrium* is an index of power, θ^* , and a vector of goods, x^* such that $\theta^* = \theta(z, x^*)$, and $x^* = \eta(p, Y, \theta^*)$.

Hence, given the exogenous variables p , Y and z , if we want to predict how a household will behave, we have to identify the household equilibrium (θ^*, x^*) . The household's behavior is given by x^* and its balance of power is given by θ^* . Of course, there may be the problem of multiple equilibria. We discuss that in section 4. An alternative, game-theoretic interpretation of household equilibrium occurs in the next section.

The first question that needs to be answered before we venture to examine the properties of household behavior is whether and under what circumstances a household

equilibrium exists. It will be shown that some fairly standard assumptions turn out to be sufficient to guarantee existence.

Theorem 1. Assume (i) $\theta(z,x)$ is continuous in x , (ii) u_i is strictly concave, continuous and satisfies vector-dominance (i.e. $x > x' \rightarrow u_i(x) > u_i(x')$) and (iii) $Y > 0$ and $p \gg 0$.

Given these assumptions there must exist a household equilibrium.

Proof. Assume (i) – (iii) are valid. It will, first, be shown that f is a function (rather than a correspondence).

Let $x, x' \in T = \{\hat{x} \in \mathfrak{R}_+^n \mid p\hat{x} \leq Y\}$; and let $\lambda \in (0,1)$.

$$\Omega(\lambda x + (1-\lambda)x') = \theta u_1(\lambda x + (1-\lambda)x') + (1-\theta)u_2(\lambda x + (1-\lambda)x')$$

$$> \theta[\lambda u_1(x) + (1-\lambda)u_1(x')] + (1-\theta)[\lambda u_2(x) + (1-\lambda)u_2(x')],$$

since u_i is strictly concave

$$= \lambda\Omega(x) + (1-\lambda)\Omega(x').$$

This shows that Ω is strictly concave. Since u_1 and u_2 are continuous, Ω is continuous. Hence Ω must achieve a maximum at some unique value of x in the domain T . This establishes that $f(p,Y,\theta)$ is a function; and, with p and Y fixed, we can think of it as a function on the domain $[0,1]$.

Fix the values of z , p and Y .

We shall define the mapping

$$\phi : T \times [0,1] \rightarrow T \times [0,1]$$

to be a *response function* if, for all $(x,\theta) \in T \times [0,1]$, $\phi(x,\theta) \equiv (x', \theta')$ is such that

$$x' = f(p,Y,\theta) \text{ and } \theta' = \theta(z,x).$$

Given Assumption (iii), $T \neq \emptyset$. Hence, $T \times [0,1]$ is non-empty and compact. By Assumption (i), θ is continuous. It is obvious that f is continuous in θ . Hence, ϕ is a continuous function. By Brouwer's fixed point theorem, there exists (x^*, θ^*) such that $\phi(x^*, \theta^*) = (x^*, \theta^*)$.

It is easy to verify that a fixed point of the response function constitutes a household equilibrium. ||

With the formal model and results behind us, we are now in a position to explore the implications of our model of household behavior in various special contexts. The purpose of the immediate next section is to consider not just an interesting special case but also to study the game-theoretic foundations of the household equilibrium. It is possible and some readers may prefer to skip directly to section 4.

3. Game-Theoretic Interpretation of Household Equilibria

Two natural modifications worth introducing in the above model are, first, dynamics and, second, some game-theoretic considerations in identifying equilibrium behavior patterns. Both these are attempted in this section and it is shown that equilibrium behavior identified through such an exercise has interesting connections with the 'household equilibrium' discussed in section 2.

It seems reasonable to assume that empowerment is not an instantaneous event. A woman used to domination in the household is unlikely to become powerful immediately if the circumstantial conditions changes in her favor. The process needs time. Let us capture this by assuming that a households' power index in period t , θ^t , depends on the

determinants of power in the previous period, z^{t-1} , x^{t-1} . We will assume that z^t is unchanged over time and so we may suppress it, without loss of generality. Hence, what we have just assumed may be written as:

$$\theta^t = \theta(x^{t-1}) \quad (2)$$

Now let us suppose θ^0 denotes a household's index of power at some period, that we will call the initial period, 0. In period 0, this household chooses a consumption bundle x^0 , by doing the kind of maximization described above, with the power index set equal to θ^0 . This in turn determines the index of power in period 1, $\theta^1 = \theta(x^0)$. In period 1 the household chooses x^1 by maximizing $\Omega(x)$ in (1), with θ being treated as equal to θ^1 . And so on in periods 2, 3 and beyond. How can we predict what a household's profile of power and consumption over time (that is, respectively, $\theta^0, \theta^1, \dots$ and x^0, x^1, \dots) will look like?

The natural way to do this is to think of a household as engaged in playing an extensive-form game. But if we are to take the collective approach to the household, as modeled by Chiappori, Bourguignon (see Chiappori, 1988, 1992; Bourguignon and Chiappori, 1994) and others, seriously *and* want to model it as a game, we face a serious problem: who are the players? Note that in the collective approach the agents are the man and the woman but the decision is taken by a mythical hybrid that is a weighted average of the man and the woman. The line I take is of thinking of this hybrid as the player. Hence, if in period t , the household's index of power is given by θ^t , we will think of the player making a choice in period t as someone endowed with the preference $\theta^t u_1(x)$

+ (1- θ^t) $U_2(x)$. Once this is done there is no loss of generality in referring to θ^t as the player.⁶

Complication arises from the fact that this player will assess all future returns in terms of its own preference. Suppose a household's consumption over time is given by the sequence $\{x^t\}$. Assuming that all agents have a discount factor of $\delta \in [0,1)$, a player θ 's aggregate (present value of) payoff is given by

$$A(\theta, \{x^t\}) \equiv \sum_{t=0}^{\infty} \delta^t [\theta u_1(x^t) + (1-\theta)u_2(x^t)] \quad (3)$$

Since $\theta \in [0,1]$, the set of potential players in this game is infinite and given by $[0,1]$. Each player can at most play in one period⁷ but of course it looks at the entire future stream of returns in choosing his strategy. The intuitive idea behind the equilibrium strategy we are about to formalize is that the household that chooses in period t does so in the awareness that households that will come into existence in the future may not have the same preference as itself. And it evaluates the consumption stream over time that gets generated by the choices in each period *in terms of its (current) preference*. This is in keeping with the literature on rational decision-making by an agent, whose preference changes over time and who is aware of this. As Strotz (1955, p. 173) wrote in his classic paper, “[It is] rational for the man today to try to ensure that he will

⁶ Observe that we are using the symbol θ both as a number in $[0,1]$ to denote the index of power and as a function $\theta:T \rightarrow [0,1]$ which, given $x \in T$, computes the index of power (recall T is the budget set, defined in section 2. This is expositionally convenient and should cause no confusion since it will be obvious from the context whether θ is being used as a number or a function.

⁷ If the same θ occurs in two different periods, we treat the θ in the two periods as distinct players.

do tomorrow that which is best from the standpoint of today's desires." (see also Sally, 2000).

In order to formalize this, let us begin by noting that each player θ 's strategy is to choose a consumption vector x from the budget set T .

Hence, the strategies of all players may be denoted by

$$f : [0,1] \rightarrow T.$$

Hence, each such mapping denotes a **strategy tuple**, which specifies the strategy of every player.

What I now want to do is to identify an equilibrium strategy for every player. In order to do this define $\Delta(\theta^0, x, f)$ to be the aggregate (present-value of) payoff of a player $\theta^0 \in [0,1]$, who chooses a consumption bundle $x \in T$, and when the other players (from then on) are committed to playing strategy f .

If we start from an initial power index, θ^0 , and the household employs a strategy tuple, f , we can generate the household's *consumption path* $\{x^t\}$, in an obvious manner, by repeated application of f and the function θ as described in (2). Thus $x^0 = f(\theta^0)$ and $x^t = f(\theta(x^{t-1}))$, for all $t \geq 1$. We will use P to denote such a function that converts the pair (θ^0, f) to a consumption path. Thus $P(\theta^0, f) = \{x^t\}$, as described above.

Next given a consumption sequence $\{\hat{x}^t\} = (\hat{x}^0, \hat{x}^1, \dots)$, and a consumption vector $x \in T$, define $\langle x, \{\hat{x}^t\} \rangle$ to be the consumption vector $\{x^t\}$ such that $x^0 = x$ and, for all $t \geq 1$, $x^t = \hat{x}^{t-1}$. Now, we can formally define

$$\Delta(\theta^0, x, f) \equiv A(\theta^0, \langle x, P(\theta(x), f) \rangle) \quad (4)$$

To understand this note that if this player θ^0 chooses x now, in the next period the player that comes into existence is $\theta(x)$ (by (2)). Since players are committed to playing f , the consumption stream that occurs from then on is given by $P(\theta(x), f)$.

The subgame perfect equilibrium of this game is now easy to define.

$f^* : [0,1] \rightarrow T$ is a **subgame perfect equilibrium** if, for all $\theta \in [0,1]$,

$$\Delta(\theta, f^*(\theta), f^*) \geq \Delta(\theta, x, f^*), \text{ for all } x \in T.$$

The next theorem states the connection between the idea of a household equilibrium and a subgame perfect equilibrium.

Theorem 2. If (x^*, θ^*) is a household equilibrium, then there exists a subgame perfect equilibrium, f^* , such that $f^*(\theta^*) = x^*$.

Proof. Suppose player θ^* could choose the entire consumption stream of the household, (x^0, x^1, x^2, \dots) . Clearly, the player would choose the stream so as to maximize expression (3) with θ set equal to θ^* . By the definition of household equilibrium, we know it would choose $\{x^t\}$ such that $x^t = x^*$, for all t . Hence, $A(\theta^*, \{x^*\}) \geq A(\theta^*, \{x^t\})$, for all sequence $\{x^t\}$ in T .

Next note that $\Delta(\theta^*, x^*, f^*) = A(\theta^*, \{x^*\})$. Hence $\Delta(\theta^*, x^*, f^*) \geq \Delta(\theta^*, x, f^*)$, for all $x \in T$. Therefore $f^*(\theta^*) = x^*$. ||

Let us now turn to the observable outcomes of this game-theoretic analysis. We will, in particular, be interested in consumption paths generated by subgame perfect equilibrium strategies, that is, in $P(\theta^0, f^*)$, where f^* is subgame perfect, and θ^0 is the index of power that occurs at the start.

From theorem 2 it is clear that if f^* is a subgame perfect equilibrium and (x^*, θ^*) is a household equilibrium, then $P(\theta^*, f^*) = \{x^*\}$, where $\{x^*\}$ is a sequence in which for all t , x^t takes the value of x^* .

Let us call $\{x^t\}$ a *stationary consumption path*, if there exists T such that, for all $t \geq T$, $x^t = x$, for some x . So what I have just shown is that (θ^*, f^*) generates a stationary consumption path where the household settles down on the household-equilibrium consumption level.

Are there other consumption levels (that is, ones which are not a part of a household equilibrium) on which the household can stabilize in a subgame perfect equilibrium? I shall now show that the answer to this is yes, and this is so in an interesting way. In particular, a household can get trapped in a Pareto sub-optimal consumption level (that is, a consumption level where both the husband and the wife are worse off than some $x \in T$). What is interesting is that this occurs in a model where households are modeled along the 'collective approach', which was ostensibly developed to capture the idea that even if members of the household have differing objectives the household will be efficient. It is here shown that, introducing dynamics can result in strategic maneuvering by the husband and the wife, which traps the household in inefficient situations. Hence, the present model could be viewed as a way of reconciling the collective household approach of Chiappori, Bourguignon and others with Udry's

(1996) finding that households typically fail to achieve Pareto optimality (see also Lundberg and Pollak, 1994, and Ligon, 2000).⁸

To demonstrate the Pareto sub-optimality claim, it is useful to reduce the above model to a special case. Consider a case where there are three goods. The number of units of apples consumed by the wife is x_1 ; the number of units of apples consumed by the husband is x_2 ; and the amount of work done by the wife is x_3 . Let us assume that the husband always works and that gives the household an income of $y(> 0)$; $u_1(x) = u(x_1)$; $u_2(x) = u(x_2)$, where $x = [x_1, x_2, x_3]$. In other words, the wife's (husband's) utility depend solely on the wife's (husband's) consumption. Neither of them care about the wife's leisure in itself. Assume $u(0) = 0$, $u'(x_i) > 0$, $u''(x_i) < 0$, $i = 1,2$; $x_3 \in [0,1]$; the price of apples is 1 and the wage rate for 1 unit of work is 1. In addition, assume

$$\theta = \theta(x_3) = \begin{cases} 0 & \text{if } x_3 = 0 \\ 1 & \text{if } x_3 > 0 \end{cases}$$

In any particular period, for a given θ , the household's welfare is given by

$$\theta u(x_1) + (1-\theta)u(x_2)$$

where

$$x_1 + x_2 \leq y + x_3$$

⁸ And for a more extreme statement, delivered with a literary flair social science cannot match, here is August Strindberg in his **The Son of a Servant** (translation by E. Sprinchorn, Anchor Books, 1966 edition, p.20), revealing an unexpected grasp of the idea of returns to scale: “But the family was and still is a very imperfect institution. ... A restaurant could serve hundreds with hardly any more members on its staff.” And later, going a bit over-board (p. 24): “The Family! Home of all social evils, a charitable institution for indolent women, a prison workshop for family breadwinners, and a hell for children!”

Given this budget set, the feasible set of utilities of the husband and the wife is shown in Figure 1.

Suppose, to start with, $\theta = \theta^0 = 0$. Hence, in the beginning the household's preference is the husband's preference. Now consider the following strategy

$$f^*(1) = x = (y+1, 0, 1)$$

$$f^*(0) = x' = (0, y, 0).$$

If households stick to this strategy, the initial household's lifetime utility is

$$\frac{u(y)}{1-\delta}.$$

If the initial household deviates, its lifetime utility is

$$u(y + 1)$$

Hence, f^* is a subgame perfect equilibrium if $\frac{u(y)}{1-\delta} > u(y + 1)$.

Suppose this is true. Then the household that starts at $\theta^0 = 0$, will in each period choose not to send the wife to work and the husband will consume y units of apples. Hence, the household will in each period be at point $u(y)$ in Figure 1, which is clearly inefficient.

Another kind of inefficiency that is now easy to model is the inefficiency of *over* work. It is possible to construct an example along the lines of the above one in which agents of the household work more than they would ideally like to because of their (justified) apprehension that to work less would amount to a diminished say in future household decisions.

To let the wife work, earn more and consume more in this period, would result in relinquishing say in the next period and so being worse off. If δ is sufficiently large so

that this is not worth it, then the household prefers to stagnate in an inefficient outcome. It is worth pointing out that this is in keeping with findings in other areas of economics, especially the study of government and other political institutions (see, for instance, Grossman and Helpman, 1994; Acemoglu and Robinson, 2000).

4. Female Labor Supply

In some economies, at certain times, women participate in the labor market in large numbers. Elsewhere they do not. Given that the participation of women has major implications for an economy's efficiency and progress, it is not surprising that there is a large body of writing that investigates female labor supply. What this literature has not addressed but is germane to our model is the fact that female labor supply is both a matter of household decision and a determinant of the household balance of power, which in turn, influences the supply of female labor.

The model of Section 2 is very well-suited to analyze this problem. It will be shown here that the female labor market can easily have multiple equilibria. Hence, two societies which are innately identical can have very different levels of female labor market participation. It will also be shown that changes in female labor supply participation in response to shifts in exogenous variables can be sudden. Hence, a society in which women do not work can remain that way for a long time, with some exogenous variable shifting all the time. Then as the exogenous variable crosses some threshold level, society can rapidly change with lots of women coming out of their homes to be active participants in the labor market. Of course, in reality the speed of these responses

will be tempered by the force of habit and custom. So it is worth keeping in mind that our model, based, as it is, on pure rationality calculus, may give a somewhat exaggerated picture of the quickness of adjustment. Nevertheless, it points to certain directions of household behavior which have been neglected by the existing literature.

In order to focus on the problem of female labor supply, let us in this section assume that the man always works, the household consumes only one good and the amount of work the woman does, e , is a variable⁹. The amount of leisure, ℓ , consumed by the woman is given by $1-e$. Let us assume, further, purely for reasons of algebraic simplicity, that each person's utility function is separable. In particular $u_i(x, \ell) = x - c_i(e)$, $i = 1, 2$, where $c_i' > 0$, $c_i'' > 0$. We will also here confine attention to the one-period model of section 2.

In other words, both the man and the woman values the good the same way, and both consider the woman's work onerous, though they give different weights to this. Admittedly, we are losing some important and interesting details by virtue of these simplifying assumptions, but for our present purpose the sacrifice seems worth it.

The amount of say that a woman has in household decision making will be assumed to depend on the amount of income she contributes to the household, that is, on

⁹ If there is open unemployment in the economy and a positive probability of the man losing his job, this can effect interesting effects on e (Basu, Genicot, and Stiglitz, 2000). By assuming that the market always clears we stay away from such complications.

ew, where w is the female wage rate prevailing on the market; and as ew increases, the woman's power increases.¹⁰ In brief,

$$\theta = \theta(ew), \quad \theta' \geq 0.$$

Given these assumptions the household's problem reduces to the following.

$$\begin{aligned} \text{Max}_{\{x,e\}} \Omega &= x - [\theta c_1(e) + (1-\theta)c_2(e)] \\ \text{subject to } px &\leq ew + Y \end{aligned}$$

Remember that in this section p and x are scalars. Since the man *always* works, there is no loss of generality in assuming that the income from the man's work is subsumed in Y.

Substituting for x from the constraint (it is easy to see that the constraint will always be binding) we have the following first-order condition:

$$\frac{w}{p} = \theta c_1'(e) + (1-\theta)c_2'(e) \quad (5)$$

Following the definition in Section 2, (θ^*, e^*) is a household equilibrium if it is the solution of (5) and (6):

$$\theta = \theta(ew) \quad (6)$$

Combining these two, we can say that e^* is *part* of a household equilibrium if

$$\frac{w}{p} = \theta(e^* w)c'(e^*) + (1-\theta(e^* w))c_2'(e^*) \quad (7)$$

¹⁰ Citing the work of Blood and Wolfe (1960), Blumberg and Coleman (1989, p.226) observe, "Wives who worked for wages have more [power] than their housewife counterparts had. Further, the more hours a woman worked, the greater her decision-making power." For formal evidence on how household consumption decisions are not separable from the labor supply decision of the man and the woman, see Browning and Meghir (1991).

In analyzing female labor supply response to changes in different exogenous variables it is important to distinguish two different cases

Case I: $c'_1(e) > c'_2(e)$, for all e .

Case II: $c'_1(e) < c'_2(e)$, for all e .

Case I is the 'normal case' where the woman's work is more onerous to the woman herself than to her husband.

Case II is a situation that often prevails in traditional, conservative societies where a man consider's his 'pride' hurt if his wife goes out to work. Since c_i consists of not just the cost of being tired out by work but also social and psychological costs, $c'_2(e)$ can exceed $c'_1(e)$. Another reason why $c'_1(e) < c'_2(e)$ is because a woman who works longer hours outside will have less time for work at home and this could contribute towards a feeling of diminished well-being on the part of the husband. If this feeling is sufficiently strong it could make $c'_2(e)$ very large. We shall refer to Case II as the 'conservative' case.

In Figure 2, first consider equation (6). With w constant, as e increases, θ will increase. We shall call this the 'power-earnings curve', since it relates to the woman's earnings to her power. In the same figure draw the curve representing equation (5). We shall call this the 'effort-supply curve', since this represents the amount of effort that the wife will supply. Let us begin with Case I – the normal case. Since $c'_1(e) > c'_2(e)$, an increase in θ , raises the right-hand side of (5). Hence, for (5) to hold, e must fall (recall $c''_1 > 0$). Hence, in the normal case the effort-supply curve is downward-sloping, as shown in Figure 2. The point of intersection of these two curves represents the household

equilibrium. The woman supplies e^* units of labor and the household balance of power is given by θ^* in equilibrium. The equilibrium is unique. Through an easy exercise of shifting curves the reader can check that in the normal case

(i) a rise in p causes θ and e to decline,

and

(ii) a rise in w causes θ to rise while the impact on e is uncertain

Let us now turn to Case II. It is easy to check that the effort-supply curve is now upward-sloping. As a consequence the equilibrium need no longer be unique.

One particular sub-case is illustrated in Figure 3. The effort-supply curve is given by OABC. Clearly, there are three equilibria at points E_1 , E_2 and E_3 . Of these, let us focus on the two stable equilibria, E_1 and E_3 . At E_1 the wife does not work, at E_3 she works a lot. Interestingly, both outcomes are possible as equilibria. In other words, two households or two societies, one in which women do not work (or work very little) and one in which they do regular, full-time work, can be *ex ante* identical households or societies. Hence, the working and not-working of women *need not be* reflections of fundamental differences.¹¹

An implication of this is that women's work can respond discontinuously to changes in exogenous variables. Consider increases in the female wage rate, w . This will cause the power-earnings curve to move left and the effort-supply curve to move right. Hence, if the household was originally at E_1 , for some time

¹¹ Similar results can be obtained by assuming that women's work is, in part, a matter of social norm that can meet with dissonance and psychological costs (Vendrik, 2000).

nothing will happen. Then suddenly the low-work equilibrium will cease to exist, at which point there is a sudden sharp rise in the woman's labor market participation. There can also be a small rise in women's income and then a sharp fall. Of course the sharpness of the changes will, in the aggregate, be tempered by the heterogeneity of households that one encounters in the real world.

5. Child Labor

There are important links between the status of children and the structure of household decision-making and, not surprisingly, this has been analyzed (see Browning, 1992; Basu and Van, 1998; and Bardhan and Udry, 1999). However, relatively little has been written about the link between the structure of power in the household and the status of children. Analyzing data from early twentieth-century urban America, Moehling (1995) has shown that households, where children contribute a larger share of the aggregate household income, are also the households in which children are likely to get more to consume. Moehling explains this along the lines of the model constructed in this paper. She argues that if one of the agents in the household happens to be a child, the logic of our model remains unchanged and a greater income contributed by the child enhances the child's power (in the same way that a woman's power gets enhanced in our model by a rise in the share of the woman's income). And this, in turn, leads to a greater consumption by the child.

Browning, Bourguignon, Chiappori and Lechene (1994), on the other hand, argue that children are unlikely to have much to say in household decisions (see their footnote

2). One way of reconciling Moehling's empirical finding with this is to argue that (i) a woman tends to internalize her children's preference (that is, her utility function reflects the child's interest); and (ii) as the share of the husband's income in the household decreases the woman's power rises. If (i) and (ii) are true then the fact of a child working could well lead to a higher consumption on the part of the child without the mediating fact of empowerment of the child.

The aim of this section is not, however, to join this debate but to study the relation between a household's power structure and its propensity to send its children to work. It will be shown that the connection between the household power structure and the incidence of child labor is much more intricate than appears at first. Yet it is not intractable. The method I will follow is to start from a very simple structure and then to add complications.

Let us begin by adding some special assumptions to the model of Section 2, so as to narrow our focus down to the essentials. To give θ a direct measurable form, I will equate it with the share of income earned by the woman. Assuming that both the woman's and the man's incomes are given by, respectively, w_1 and w_2 , we have

$$\theta = \frac{w_1}{w_1 + w_2} \quad (8)$$

An important assumption (and not just a simplification for technical reasons), that seems to be realistic and will be maintained here, is that both the man and the woman agree that their child's labor is painful and undesirable, but they have differences concerning what to spend any additional household income on. A simple algebra for capturing this assumption is as follows. Both the man and the woman consider the cost of child labor to be

$$c_i = c(h), \quad c' > 0, \quad c'' > 0 \quad (9)$$

where h is the amount of work done by the child. On the other hand, the woman is only interested in spending money on good 1 and the man's sole interest is good 2. We could, for instance, think of 1 as milk and 2 as alcohol. (This is, admittedly, an insulting and stereotypical depiction of gender difference, though it is not evident who should feel more insulted, the man or the woman, by this characterization.) Hence, using x_i to denote the number of units of good i consumed by the household, we can write agent i 's utility function as:

$$u_i = \phi(x_i) - c(h), \quad (10)$$

where $\phi' > 0$, $\phi'' \leq 0$. It is being assumed that the amount of work done by the adults is fixed. Hence, the household's maximand, following the model of Section 2, is given by:

$$\Omega = \theta\phi(x_1) + (1-\theta)\phi(x_2) - c(h) \quad (11)$$

Taking the price of each good to be 1 and the wage rate of child labor to be w , the budget constraint is given by

$$x_1 + x_2 = hw + w_1 + w_2 \quad (12)$$

The extent of child labor that the household supplies can now be determined by solving the problem of maximizing (11) subject to (12).

An intuitively interesting result emerges in the special case where $\phi(\bullet)$ is linear, for instance if $\phi(x_i) = x_i$. Let me state this result as a 'proposition', using this word to denote a claim that is more informal than a theorem.

Proposition 1: In the model described above, if $\phi(x_i) = x_i$, then as the source of income in a household becomes more diverse, the household is less likely to send its children to

work. Hence, in a household with two adults, as the share of income earned by the mother rises (starting from zero), children will be less likely to work; but as the share continues to rise and heads towards 1, the incidence of child labor is likely to rise.

The intuition is easy to describe. I will do so first and then give a formal proof. Recall the underlying assumption on which the argument is premised. Both the man and the woman prefer (other things remaining the same) not to send the children to work (or, equivalently, they consider their child's labor to be costly); but they have different preferences concerning what to spend any additional household income on.

Consider now the model of Section 2 and, in particular, the special case in which the power of the man and the woman is fairly well-balanced, that is, θ is close to half. Since both the man and the woman are averse to send their child to work, changes in θ will have little effect on this calculation. On the other hand, the benefits of the additional income generated by sending the child to work will not be fully reaped by any agent, since θ being close to half will mean a tussle between milk and alcohol. Hence, neither the man nor the woman will get the full benefit of the additional income generated by a working child. Hence the child is less likely to work. On the other hand if θ goes to 1 or to 0, one agent becomes powerful and so she or he will reap the full benefits of child labor. So child labor will be more likely in such a case.

To prove this formally, note that given the hypothesis of Proposition 1, (11) reduces to

$$\Omega = \theta x_1 + (1 - \theta)x_2 - c(h) \tag{13}$$

Hence, if $\theta > \frac{1}{2}$, the household will spend all its income on good 1. Hence, if $\theta > \frac{1}{2}$, the

household's aim is to choose h so as to maximize

$$\Omega = \theta(hw + w_1 + w_2) - c(h)$$

This is obtained by inserting (12) into (13), after setting $x_2 = 0$. Hence, from the first-order condition we have

$$\theta w = c'(h).$$

It follows that as θ increases h will increase (recall $c''(h) > 0$).

Likewise, if $\theta < \frac{1}{2}$, a rise in θ causes h to fall. This completes the proof of

Proposition 1. Child labor responds in an U-shape to the fraction of household income contributed by women, as shown in Figure 4.

One can inject a little more-realism into this example by drawing on what we know to be empirically true, namely, that women show a stronger preference for the well-being of the children. In developing countries, it has been seen that when a woman has greater say in household matters, the children's nutrition improves. In the above model, we could capture this gender difference by arguing that while the woman feels the cost of child labor to be $c(h)$ the man feels it as $\gamma c(h)$, where $\gamma < 1$. In other words, the woman is more sensitive to the cost of child labor. In such a case, the amount of child labor is less when $\theta = 1$ than when $\theta = 0$. In other words, the incidence of child labor curve, instead of being U-shaped turns out to be tau-shaped, that is an U, with the right-hand up turn being less sharp than the left-hand down turn.

All this happens, when we work with a special case of (11), where ϕ is linear.

What happens if we work with a strictly concave ϕ function? To answer this, we have to

maximize (11) subject to (12). One way of doing this is to solve (12) for x_2 and insert this in (11). Hence, the household's problem reduces to

$$\text{Max}_{\{x_1, h\}} \theta \phi(x_1) + (1 - \theta) \phi(hw + w_1 + w_2 - x_1) - c(h).$$

The first order-conditions are given by:

$$\theta \phi'(x_1) = (1 - \theta) \phi'(hw + w_1 + w_2 - x_1) \quad (14)$$

$$w(1 - \theta) \phi'(hw + w_1 + w_2 - x_1) = c'(h). \quad (15)$$

Using (14) and (15) we get

$$w\theta \phi'(x_1) = c'(h) \quad (16)$$

Hence, we can treat (15) and (16) as the first-order conditions. To see the effect on h of changes in θ , let us take total differentials of (15) and (16):

$$\begin{aligned} w(1 - \theta) \phi''(hw + w_1 + w_2 - x_1)(wdh - dx_1) - d\theta w \phi'(hw + w_1 + w_2 - x_1) &= c''(h)dh \\ w\theta \phi''(x_1)dx_1 + w\phi'(x_1)d\theta &= c''(h)dh \end{aligned}$$

Solving these two equations for $dh/d\theta$ we get:

$$\frac{dh}{d\theta} = \frac{(1 - \theta) \phi''(x_2) \phi'(x_1) w - \theta \phi'(x_2) \phi''(x_1) w}{(1 - \theta) \phi''(x_2) c''(h) + \theta \phi''(x_1) c''(h) - \theta(1 - \theta) w^2 \phi''(x_1) \phi''(x_2)}$$

where $x_2 = hw + w_1 + w_2 - x_1$.

Since $\phi(\bullet)$ is strictly concave and $c(\bullet)$ strictly convex the denominator is always negative. Hence the sign of $dh/d\theta$ is the same as the sign of the term

$$\theta \phi'(x_2) \phi''(x_1) - (1 - \theta) \phi''(x_2) \phi'(x_1).$$

It is now entirely possible that if θ is small, $dh/d\theta$ is positive. Then, as θ rises, $dh/d\theta$ falls and eventually becomes negative. This will be the case if $\phi''(x_1)$ and $c''(h)$ happen to be constants.

Curiously, this flips the child labor incidence curve of Figure 4 into an inverted U.

So what is the overall relation between θ and the incidence of child labor? To answer this observe that when we have a corner solution, that is, either x_1 or x_2 is zero, the relation is U-shaped (as in Figure 4). Otherwise, it is likely to be an inverted U and this is precisely so if ϕ'' and c'' are constants. Since a corner solution is likely when θ is very small or very large, the overall relation is likely to W-shaped, as shown in Figure 5.

Hence, the structure of household power has a determinate effect on child labor. But the precise relation is non-monotonic and very sensitive to the parameters of the model. This is, therefore, one area where empirical work is especially needed.

Note that the empirical estimation of the response of child labor to the balance of power in the household will be quite complex since we will have to, essentially, look for the best fitting polynomial of a fairly high degree; so that we can discover the best W-shaped curve (which includes the case of an U-shaped curve and that of an inverted U-shaped curve as special cases). However, if we are to design policies that control child labor by influencing the balance of power in the household, then it is essential to first conduct empirical research based on the kinds of restrictions derived from the above theoretical investigation.

6. Conclusion

The paper was motivated by the recognition of the fact that while a household's balance of power influences its choices, the choices can in turn affect the household's balance of power. While this feature of households is well-recognized in the descriptive and sociological literature, it has not been formally modeled. Much of the present paper

was devoted to modeling this two-way relation and in deriving its implications for female labor supply, child labor and other aspects of household behavior. This paper may be viewed as spadework for further work in modeling household behavior.

First of all, we could try to build a Nash bargaining model of the household, which allows for asymmetric power and recognizes that not only does the extent of asymmetry and the threat point affect household decisions but they themselves get affected by the decisions.

A more radical direction of research would be as follows. A study of the sociological and anthropological literature draws our attention to another lacuna of the theoretical models of the household, that, at a subliminal level, we all know but our models ignore, namely, that the balance of power within households often manifests itself in the *domains* of control.¹² In other words, a woman's say, captured in our model by 2, is recognized to vary depending on the domain of decision-making. She could have all the power when it comes to choosing the children's clothing and food, but have no say in other matters. A budget may be apportioned to her for expenditures in her domain, with or without additional restrictions being placed on her by her husband (see Guyer, 1988).

In this approach a woman's power would be reflected in part by the size of the domain of her decision. Such a model could raise intricate game-theoretic questions, since how one person chooses over her domain will clearly depend on how she expects the other to choose over this domain and *vice versa*. These are some of the next steps to

¹² As Palriwala (1990, p.41) notes, while summarizing a collection of anthropological papers on gender and work: "One conclusion that can be drawn from these papers is that, even when women are clearly oppressed, there are spheres where they may act and decide."

take in the research venture to map the structure of decision-making and power in the household. And they can influence in an important way how we design policy pertaining to poverty removal, the eradication of child labor, unemployment and social welfare.

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Figure 1

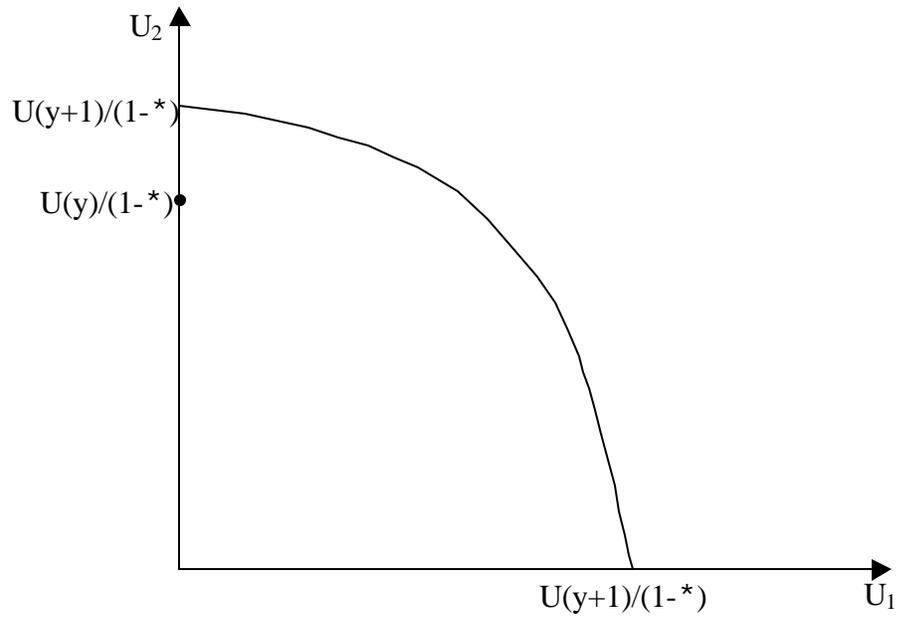


Figure 2

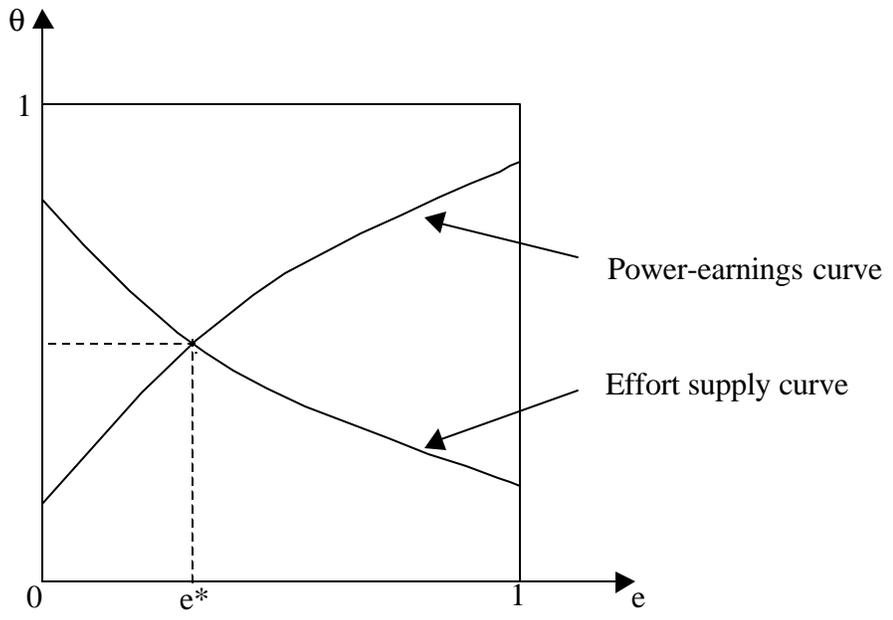
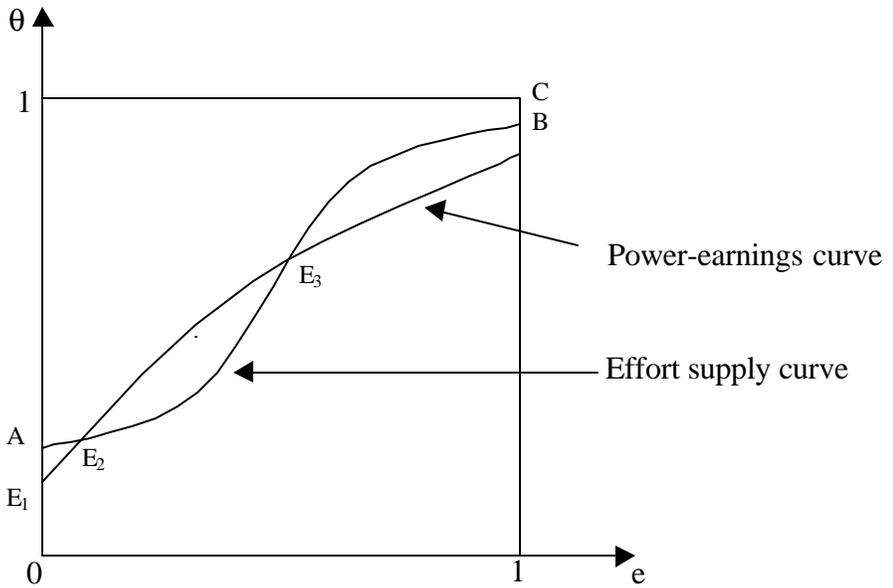


Figure 3



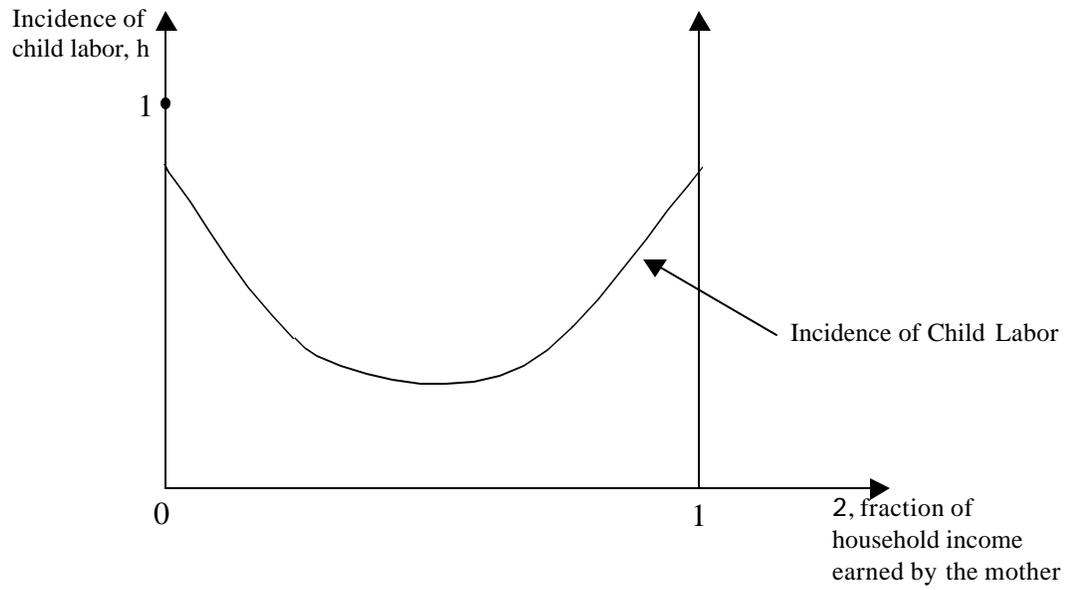


Figure 4

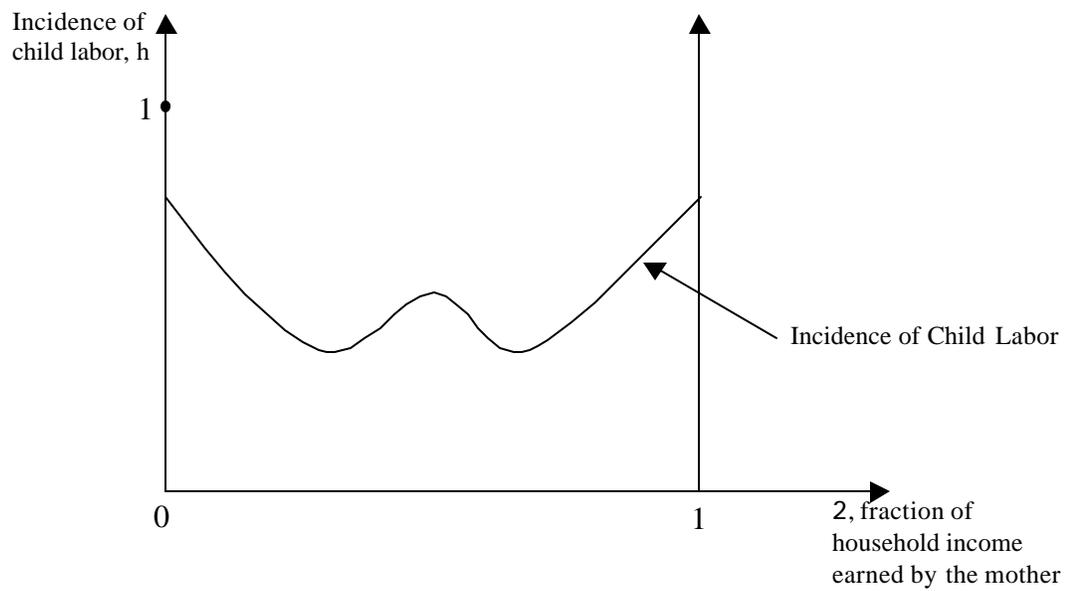


Figure 5