

ESSAYS ON OPTIMAL CONTRACT DESIGN

A Dissertation

Presented to the Faculty of the Graduate School
of Cornell University

in Partial Fulfillment of the Requirements for the Degree of
Doctor of Philosophy

by

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August 2013

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Cornell University 2013

The optimal compensation contract is a very important issue for firms. Some empirical findings of wage structure in internal labor market are puzzling. For example, why are the compensation of workers more compressed than predicted by the classical theories? Why is the wage structure convex in hierarchical firms? This dissertation explores various important factors which may affect the optimal contract in the internal labor market.

The first chapter characterizes the optimal contract when workers in the workplace care not only about their own wage but also their co-workers' wage. Specifically, I assume that workers are inequity averse to their wage differentials or they are status seeking. Building on the inequity aversion model of Fehr and Schmidt (1999), I derive that the optimal wage structure is more compressed with inequity averse workers than with the standard workers. Inequity aversion among workers can also help explain the internal organization of the firms. For example, inequity aversion among workers may lead firms to employ only high productivity workers, even though the marginal product of a low productivity worker is higher than the worker's marginal cost.

Chapter 2 examines two possible realistic explanations for the convex wage structure in the hierarchical firms. Based on the multi-round tournament model of Rosen (1986), we incorporate heterogeneous stage effects. The first extension that can generate the convex wage structure is that the number of workers competing increases with the hierarchical levels. The second explanation is that

the returns to effort increase with the hierarchical levels, which cannot generate the convex wage structure unless further assumptions added on optimal effort levels and cost functions.

The third chapter investigates the underlying assumption in Chapter 1 that people are inequity averse to ex-ante payoff differentials. Specifically, an online survey is conducted to test whether ex ante or ex post fairness views affect people's decision making in a social context. I find that the ex post fairness views do make an important role in people's decision making. The results of the survey data do not support the model of inequity aversion.

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BIOGRAPHICAL SKETCH

Jin Xu was born in Jinxi, Liaoning, China. She earned her bachelor's degree in both economics and mathematics at Wuhan University. She also obtained her master degree in economics at Wuhan University. She began her Ph.D study at Cornell University in 2006. In 2012, She left Ithaca and started to work at Huazhong University of Science and Technology in Wuhan, China as an assistant professor. Her field of study is behavioral economics and applied microeconomic theory.

To my parents.

ACKNOWLEDGEMENTS

I must thank my chair Dan Benjamin. I first met him when I was randomly assigned to be his TA. I was so lucky because I have been his TA twice. He gives me so much help not only in my research but also in my teaching. I think every student around him has benefited from his cheerful encouragement and positive energy. He never loses hope in me even sometimes I doubt myself. Even though he was on leave, he was available to talk on the phone every week. Without his help, I could not have finished this dissertation.

I am extremely grateful to Ted O'Donoghue. He literally sparked my interest in behavioral economics and led me to the behavioral camp. Every meeting with him is incredibly invaluable. He can always provide insightful suggestions and constructive guidance. He has been a true mentor to me.

I am very thankful to Mike Waldman. He has generously shared his ideas with me. He can always show me the intuition behind the models when I was buried in the complex theoretical models. I think I need to hone this skill for the rest of my career. I still remember him saying, 'I never take a break.'

I am extremely fortunate to have Aaron Bodoh-Creed on my committee. He is always available to give any suggestions I need. I was so moved when he helped me revise the paper again and again. One time, our meeting lasted for two hours and we literally went over my paper sentence by sentence. I still keep his manuscript revision of my job market paper as a way to motivate myself.

I am extremely grateful for the generous funding support from LEEDR and I am deeply grateful to Bill Schulze and Kevin Kniffin. Without their help, the experiment could not be run. I must thank Gregory Besharov. Being a teaching assistant for him made my job market year much easier. He shared my grading duties even though he need not have to do so. He has shown great support for

my research and helped me with my funding. Sometimes I think he is on my committee too.

As an international student, writing in English is never easy. I must thank Susan Bland, Annette Bell and especially Keith Hjortshoj, because I have learned so much from their writing courses. Writing and editing are not a tedious and daunting task to me anymore. I must thank Mitra Tapan, John Abowd, Eric Maroney and Jenny Wissink for their guidance during my Ph.D studies. I also must thank my fellow colleagues Jingxian Zheng, Jessica Cao, Shuang Zhang, Ahmed Jaber, Alex Rees-Jones, Kristen Cooper, Xiaomeng Lu, Ying Wu and Pei-Lin Hsieh for their valuable discussions. I also must thank Hengfu Zou and Jacques Cremer. Without their recommendations, I would not have had the chance to begin my Ph.D journey at Cornell. I must thank Joey Anderson. He helped me edit my papers before I finally submitted my dissertation. Last but not least I must thank my parents for their patience and unconditional emotional support.

As always, all the mistakes are my own.

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CHAPTER 1
CONTRACTING WITH INEQUITY-AVERSE WORKERS WITH
HETEROGENEOUS PRODUCTIVITIES

1.1 Introduction

In the workplace, people have heterogeneous productivities. The classical theory assumes that workers are self-interested. Therefore, risk-neutral workers should be paid according to their own marginal productivities. Also, as long as there is no spillover between efforts, it should not matter how firms organize teams of workers with heterogeneous productivities. However, these predictions do not match real world phenomena, i.e., actual wages are more compressed but still increase with workers' productivity (Akerlof and Yellen, 1990). And how teams are organized based on workers' productivities does affect profits (Frank, 1984a and 1984b). Furthermore, the classical theory cannot explain adopted wage secrecy policies (Dickens and Katz, 1986, and Krueger and Summers, 1988) and the consistent wage differentials across industries.

This chapter suggests an explanation for the above phenomena by incorporating other-regarding preferences into the standard principal-agent model. Other-regarding preferences have been widely documented in numerous experimental and empirical studies.¹ The theories of social preferences assume that a person does not only care about his own material payoff but may also be concerned about the material payoff other people receive. I follow Fehr and Schmidt's inequity aversion model because of its simplicity and tractability.

¹See Fehr and Schmidt (2006) for a comprehensive survey. According to their survey, there are three classes of other-regarding preferences: social preferences, interdependent preferences and intention based reciprocity.

Fehr and Schmidt (1999) assume that a person dislikes an inequitable or unfair outcome. Specifically, a person is compassionate or altruistic towards other people (also known as aversion to advantageous inequity), if others' material payoffs are below an equitable benchmark. In addition, a person is envious of other people (also known as aversion to disadvantageous inequity) if others' material payoffs exceed the equitable benchmark level.²

In this model, I analyze an optimal contract design problem under a monopolistic setting where a risk-neutral principal (she) has a project and needs to hire two risk-neutral agents with different productivities. I assume that the principal knows workers' productivities and preferences, i.e., no asymmetric information. I also assume that an agent is inequity averse to wage differences and his reference group is his co-worker. If his wage is higher than his co-worker's, then he suffers disutility from compassion. On the other hand, if his wage is lower than his coworker's wage, he is envious of his coworker. I show that inequity aversion affects the ability of the firm to incentivize the workers through wages.

In the benchmark model, I study the optimal contract design with two purely self-interested agents.³ The first result is that a worker's wage should be based solely on that worker's own productivity. With the optimal contract, the principal's profit is maximized while leaving agents with zero rent. This outcome is socially efficient, where a high productivity worker exerts more effort than a low productivity worker. I refer to this force as the efficiency motive for the firm, which is measured by workers' productivity differences. The efficiency

²When terms such as "equitable", "altruism", "compassion" or "envy" are used here, I disregard any moral philosophical meanings and emotions associated with them as well as any normative implications. I view them as purely positive terms that describe behavior (the same as discussed in Fehr and Schmidt, 2006).

³If the principal can contract with two independent contractors, which means two agents can't compare their payoffs, then inequity aversion doesn't matter.

motive causes wage differentials because a high productivity agent should be paid more in order to induce more effort than a low productivity agent. The stronger the efficiency motive, the greater the wage differential. I also define the optimal effort level as the first best effort level, which can be obtained by linear wage contracts. As shown in section 2, the power of incentives is the same for both agents. Therefore, I can obtain the second standard result that optimal effort levels can also be implemented by selling the project to two workers.

With inequity-averse agents and the optimal wage contract of the benchmark model, inequity aversion affects the ability of the firm to incentivize the workers through wages. Furthermore, the principal is worse off with inequity-averse workers, because to induce the same amount of effort as in the benchmark case, the principal has to pay workers more than the benchmark case because wage differences incur negative utility to workers with different wages. The first best effort levels of the benchmark model are no longer profit maximizing. I refer to this force as the equality motive for the firm, which states that workers should be paid similarly because workers dislike wage differentials. The equality motive is composed of two concerns. Suppose worker 1 is paid more than worker 2. The compassion concern results from the disutility of worker 1 when he is paid more. The envy concern arises from the disutility of worker 2 when he is paid less. Both the compassion concern and the envy concern induce the firm to offer similar wages to each worker *ex post*. As shown in section 2, it is always optimal to pay a high productivity worker more than or the same as a low productivity worker. Therefore, when the principal considers the equality motive, she only needs to consider the compassion from a high productivity worker and the envy from a low productivity worker.

When a principal faces inequity-averse workers, she must balance the efficiency motive and the equality motive. The two motives work in opposite directions. If workers' inequity aversion is large, i.e., the equality motive is strong, it is optimal for the principal to pay equal wages. On the other hand, if workers' productivity differences are large and their inequity aversion is small, i.e., the efficiency motive is strong, it is optimal to offer a high productivity worker more than a low productivity worker. Because of the equality motive, i.e., the compassion concern from a high productivity worker and the envy concern from a low productivity worker, however, the optimal wages are still more compressed than with standard agents. Therefore, with other-regarding preferences, a worker's wage is not only dependent on his own productivity but also depends on his co-worker's productivity. The model offers one explanation for the wage compression phenomenon in internal labor markets.⁴

For the same contract, the power of incentives is different for inequity-averse agents from the power of incentives for standard agents. In other words, inequity aversion affects the power of incentives in a contract. Specifically, a high productivity worker's optimal effort level decreases with the envy concern and the compassion concern, while a low productivity worker's optimal effort level increases with the envy concern and the compassion concern. If I fix the productivity differential and increase inequity aversion i.e., raise the inequity aversion parameters, a high productivity worker's effort level falls, while a low produc-

⁴There are many other explanations for wage compression in the literature. Lazear and Rosen (1981) suggest that it is prohibitively costly to measure the individual productivity in some cases. Stiglitz (1975) argues that agents are more risk averse than the firms. Harris and Holmstrom (1982) offer the insurance explanation. Akerlof and Yellen (1990) develops an fair wage-effort hypothesis. Frank (1984b) argues status seeking workers are self-sorting. Lazear (1989) argues that wage compression results from the importance of cooperation between workers. Alternatively, our paper uses inequity aversion model, initially formulated by Fehr and Schmidt (1999). The notion of our model is closest in spirit to Akerlof and Yellen (1990) and Frank (1984a, 1984b).

tivity worker's effort level rises until they are equal. Therefore, efforts are more compressed. As shown in section 2, when the equality motive is very strong, the optimal effort levels will be the same for the workers. When the efficiency motive is strong, the efforts of workers will be different reflecting their productivities, but will still be more compressed than in the benchmark model. As in the benchmark model, the principal can apply linear wage contracts to implement the optimal effort levels. Unlike the benchmark model, however, the principal applies a high powered incentive to a low productivity agent and a low powered incentive contract to a high productivity agent. In addition, the optimal efforts cannot be implemented by selling the project to the workers.

I also discuss a second case with status-seeking agents. As pointed out by Frank (1984a, 1984b), some people obtain intrinsic value from non-pecuniary elements of compensation such as higher positions in the income hierarchies of the group. In the context of the model, status-seeking workers derive utility when they are paid more than their co-workers. The principal now can implement any effort level with a status-seeking agent using a lower wage than with an inequity-averse agent. Therefore, it is optimal for the principal to pay a status-seeking agent more to induce a much greater effort level than she would pay to an inequity-averse agent. I refer to this force as the status seeking concern, which leads to wage differentials. Hence, the model with status-seeking agents yields a key trade-off between the efficiency motive, the status seeking concern and the envy concern. The first two concerns lead to wage differentials and the last concern leads to wage compression. If the status seeking concern is not strong, the results of wage and effort compression still hold. However, if the status seeking concern is strong, the workers' wage distribution is more dispersed than that of the benchmark case.

Building on the above basic model, I also discuss the issue of optimal internal organization of the firms by endogenizing the number of workers.⁵ When inequity aversion of workers is considerably large, the principal would rather hire one agent instead of two agents. This result has implications on the efficient scale of the firms without assuming spillover between the projects within the firms. I also discuss the sorting problem when the principal faces multiple agents. The principal prefers agents with the same productivity working together if agents are inequity averse. But with strong status-seeking agents, the principal combines a high productivity worker with a low productivity worker. This can imply that even if a task can be accomplished by a high productivity agent, the principal can gain more profit by hiring one more low productivity agent by exploiting the high-productivity status-seeking agent.

In extension of the basic model, I consider synergies between the workers and study the interaction between synergies and inequity aversion. Especially, I discuss the case that synergies take the form of marginal effect or the synergies enter the first order conditions of the workers (as opposed to a level effect of synergies that would not)⁶. Under this circumstance, an increase in one worker's effort can raise the marginal productivity of another worker, then synergies reduce marginal productivity differentials and mitigate inequity aversion among workers. Therefore, synergies provide another explanation for teamwork with inequity-averse workers.

The chapter has several empirical implications and applications in internal organizations. When firms design compensation structures, they need to con-

⁵There is a vast literature explaining firms' optimal internal organizations including models of spillovers between workers, property rights and incomplete information structures.

⁶The synergies with marginal effects here can also be called strategic complements in the literature. See Bulow, Geanakoplos, and Klemperer (1985).

sider inequity aversion among workers. For example, if a firm wants to merge two sectors (or one firm wants to acquire another firm), which are paid very differently, the firm should consider inequity aversion among workers in the two sectors. If inequity aversion is strong, the high-pay sector will be demotivated by the low-pay sector. Therefore the merger may decrease the firm's profit. My finding is consistent with the fact that long-run performances of many acquiring firms after mergers are poor (Agrawal, Jaffe and Mandelker, 1992), because many merger decisions may ignore inequity aversion among workers and only focus on production efficiency. The chapter can generate empirical predictions about the types of workers and optimal payment structure. For example, for some firms with strong status-seeking agents, the wages should be more diverse than wages of inequity-averse agents in some firms. This prediction helps explain the diverse payment structures in labor markets.

This chapter could also shed light on empirical evidence about the issue of wage secrecy.⁷ In the standard model, wage secrecy policies have no impact on the effort levels and firms' profits. This model implies that a wage secrecy policy is more profitable for a firm with inequity-averse agents. And it predicts that with a wage secrecy policy the wages are more diverse than without a wage secrecy policy.⁸ On the contrary, with status-seeking agents the principal may want to adopt a wage transparency policy to gain more profit from a high productivity agent. Usually the pay structure is more transparent in public sectors or in nonprofit organizations; therefore, the wage distribution should be more compressed than in private sectors. This result is consistent with empirical ev-

⁷Bertrand and Mullainathan (2003) propose that with a wage secrecy policy, managers can lead a "quiet" life without any complaints about wage inequality. Danziger and Katz (1997) suggest that wage secrecy enhances the feasibility of risk-shifting contracts by reducing effective labor mobility.

⁸While other papers on behavioral contract theory assume workers compare their rent, there is scarce discussion on how wage secrecy policies would impact the optimal wage contracts.

idence. For example, Borjas (2002) uses data from the U.S. Decennial Census and the Current population to show that since 1970, the wage distribution in the public sector is significantly more compressed than in the private sectors.

The rest of Chapter 1 is organized as follows. The basic model is presented and analyzed in Section 1.2. Section 1.3 investigates the related issues such as sorting. Section 1.4 discusses some extensions of the basic model such as incorporating synergies in the production in agents' preferences. Section 1.5 presents the related literature. Section 1.6 concludes and discusses the robustness of the results. Most proofs are relegated to the appendix.

1.2 Model setup

Suppose a principal (she) has a project that requires two agents to complete. Agents are denoted by $\{1, 2\}$. Assume that agent i 's production y_i is linear in his effort e_i : $y_i = b_i e_i + \epsilon_i, i = 1, 2$, where ϵ_i is an independent and identically distributed noise variable with zero mean and its density function is $f(\epsilon)$. b_i measures agent i 's productivity. Assume that agents are heterogeneous in productivities. Without loss of generality let $b_1 > b_2$. For both agents, the amount of effort e incurs the same convex cost function $c(e) = \frac{e^2}{2}$. Assume that both agents' reservation utility is $\underline{U} = 0$. The principal can observe the output level of each agent but not the effort level; therefore, only outputs are contractible. I also assume the principal and agents are risk neutral to focus on the inequity aversion instead of risk sharing.

The model is a standard principal-agent setup: the principal offers a wage contract and agents choose whether to accept or reject the offer. If an agent

accepts the wage offer, he chooses an effort level and receives a payment according to the contract. I first introduce the concept of inequity aversion. Then I derive the optimal contracts in a special case when the inequity aversion among workers is zero, which I also refer to as the benchmark case with purely self-interested agents. Then I model the case with inequity-averse agents and compare key differences between the optimal contracts under the benchmark model and the model with inequity aversion.

1.2.1 Inequity aversion

There are many ways to model other-regarding preferences. For example, Rabin (1993) develops the intention-based approach to model the interdependent preference (Also see Charness and Rabin (2002), Cox and Friedman (2002) and Levine (1998)). In this chapter, I apply Fehr and Schmidt's (1999) inequity aversion model, because of its tractability and simplicity. Specifically, when there are two people, the utility function for person i is

$$U_i(x) = x_i - \alpha_i \max\{x_j - x_i, 0\} - \beta_i \max\{x_i - x_j, 0\}$$

where $\beta_i \leq \alpha_i$ and $0 \leq \beta_i < 1$, $i \neq j$. In this function, the first term x_i is the person i 's payoff, and the second and third terms capture person i 's inequity aversion (fairness concern). α_i measures the degree of envy (aversion to disadvantageous inequality) and β_i measures the degree of compassion (aversion to advantageous inequality). If person i 's payoff is less than person j , $x_i < x_j$, the utility loss from envy is $\alpha_i(x_j - x_i)$. If person i 's payoff is greater than person j , $x_i > x_j$, the utility loss from compassion is $\beta_i(x_i - x_j)$. $\beta_i \leq \alpha_i$ captures the idea that people suffer more from disadvantageous inequity than from advantageous in-

equity. In this chapter, I also assume $0 < \beta_i < \frac{1}{2}$.⁹ When agents are status seeking, $\beta_i < 0$, which I discuss later. I use the terms "envy" and "compassion" because they may be one of the natural sources driving inequity aversion. But as mentioned in the introduction, "envy" and "compassion" are used in a neutral way to describe behaviors and this model can also be driven by other aspects such as moral concern or altruism. In other words, the emotions associated with the terms and normative implications are completely disregarded in this model.

There are two important questions concerning this other-regarding utility function in the setting of my model. First, with which *reference group* does person i compare himself? For example, for a principal, the reference group can be his subordinate, while for a worker the reference group can be co-workers or his boss.¹⁰ In this model, I assume a worker's reference group is his co-worker.¹¹

Second, what do workers compare, or in other words what does x stand for? The comparison could be between wages w , effort levels e , costs of effort $c(e)$ or rents, $(u(w) - c(e))$. In this model, I assume workers are inequity averse to their ex-ante wage differentials,¹² but the results are robust if workers are inequity averse to the effort cost differentials, which will be checked in the appendix. The assumption of inequity aversion to wage differentials might be reasonable when workers can observe their co-workers' wages but not their effort levels or

⁹If $\beta > \frac{1}{2}$, the person is better off by giving his payment to his reference group, when $U_i = (1 - \beta)x_i + \beta x_{-i}$. In other words, the person cares more than his reference group than himself, which is implausible in workplaces.

¹⁰Some papers take peers or teammates as their reference group. For example, Thakor and Goel (2005) studies the optimal contracts when agents envy each other. Some papers consider the case in which an agent compares his payoff with the principal's payoff (Dur and Glazer (2005) and Englmaier and Wambach (2010))

¹¹See Fehr and Schmidt (2006) for a comprehensive survey of the literature.

¹²There is very scarce experimental and field evidence that differentiates between inequity aversion to ex-ante wage differences and ex-post wage differences. While most literature assumes that workers are inequity averse to ex post wage difference, the ex-ante comparison can greatly simplify our analysis.

even when workers can't observe but can estimate their co-workers' wages.

From the above discussions, the utility function of agent i is

$$EU_i = w_i - c(e_i) - \alpha_i \max(Ew_j - Ew_i, 0) - \beta_i \max(Ew_i - Ew_j, 0)$$

Suppose there are two agents $\{1, 2\}$ and agent 1 earns higher expected wage than agent 2, i.e. $Ew_1 > Ew_2$, agent 1's compassion parameter β_1 comes to play in his utility function, while agent 2's envy parameter α_2 takes effect in his utility function. Agent 1's expected utility is $EU_1 = (1 - \beta_1)Ew_1 + \beta_1Ew_2 - c(e_1)$ and agent 2's expected utility is $EU_2 = -\alpha_2Ew_1 + (1 + \alpha_2)Ew_2 - c(e_2)$. Therefore, given a contract, for agent i only envy concern α_i or only compassion concern β_i matters instead of both. As shown later, the optimal contract only depends on two parameters: envy of one agent (α_i) and compassion of the other agent (β_j), instead of four parameters $(\alpha_i, \beta_i, \alpha_j, \beta_j)$.

1.2.2 Benchmark model (with no inequity aversion)

In this section, I discuss the benchmark model with purely self-interested agents: i.e., $\alpha_i = \beta_i = 0$, for $i = 1, 2$. I assume there is no hidden information and the principal knows the productivity of each agent. The principal implements the optimal efforts from workers to maximize her expected profit by offering optimal wage contract $\{w_1(x_1, x_2), w_2(x_1, x_2)\}$.

The principal's maximization problem becomes

$$\begin{aligned}
& \max_{e_i, w_i(\cdot)} E(x_1 + x_2 - w_1(x_1, x_2) - w_2(x_1, x_2)) \\
& s.t \ E(w_i(x_i, x_j) - c(e_i)) \geq 0, \ i = 1, 2, i \neq j \ (IR_i) \\
& e_i \in \arg \max E(w_i(x_i, x_j) - c(e_i)), \ i = 1, 2, i \neq j \ (IC_i)
\end{aligned}$$

Solving the problem, I derive the following proposition 1-1 (the superscript B denotes the optimal variables in the benchmark case),

Proposition 1-1 *With a standard agent i , $i = 1, 2$., the principal induces the efficient effort level $e_i^B = b_i$ with expected wage $Ew_i^B = \frac{b_i^2}{2}$. The expected profit earned by the principal is $E\Pi^B = \frac{b_1^2}{2} + \frac{b_2^2}{2}$. The principal can implement the optimal effort levels by a linear independent wage contract: $w_i^B = x_i - \frac{b_i^2}{2}$, $i = 1, 2$.*

Participation constraints are binding for both agents, because the principal knows the productivities of workers and leaves zero rent for the agents. The optimal effort level of agent i depends solely on his own productivity: $e_i^B = b_i$. The result shows that a high productivity agent should exert more effort than a low productivity agent. I refer to this force as the efficiency motive, because it maximizes the total social surplus. The efficiency motive is measured by productivity differentials. The efficiency motive is strong when agent 1 and agent 2 differ greatly in productivities. Agent i is expected to receive wage $Ew_i^B = \frac{b_i^2}{2}$, $i = 1, 2$, which is solely dependent on his own productivity. The efficiency motive indicates that a high productivity worker is expected to be paid more than a low productivity worker; therefore, the efficient effort levels are achieved. When the efficiency motive is strong, the expected wage differential ($Ew_1^B - Ew_2^B = \frac{b_1^2}{2} - \frac{b_2^2}{2}$) is greater.

The socially efficient effort levels can be attained by a linear form of wage contract: $w_i(x_i) = a_i x_i + t_i$, where a_i is the marginal bonus payment and measures

the marginal power of incentives. t_i is the transfer or the base payment. From the incentive constraints, I can solve for the optimal linear wage contract:

$$w_i = x_i - \frac{b_i^2}{2}$$

The optimal marginal power of incentives is the same for both agents: $a_i = 1$. Because agents are risk neutral, I can interpret the optimal contract as the principal selling the projects to agent 1 for $\frac{b_1^2}{2}$ and to agent 2 for $\frac{b_2^2}{2}$ so that agents are the residual claimants on the output.

In fact the benchmark model also applies to the following two cases with inequity-averse agents. One is that the principal can contract with two independent contractors. Therefore she can avoid wage comparisons between the agents. The other is that the agent cannot observe his co-worker's wage so there is no inequity aversion in this case. For example, a company's wage privacy policy, even with inequity-averse agents will reduce inequity concern.

1.2.3 Model with inequity aversion

In this part, I assume that agents have heterogenous inequity aversion parameters because I want to analyze which inequity aversion parameters of which workers affect the optimal contract design. The timeline of the setting is the same as in the benchmark case. There is no asymmetric information between workers and the principal; i.e., the principal observes the productivities b_i and inequity aversion parameters α_i and β_i , $i = 1, 2$.

The principal's profit maximization problem becomes

$$\begin{aligned} & \max_{w_1, w_2} E(x_1 + x_2 - w_1 - w_2) \\ & s.t \ Ew_i - c(e_i) - \alpha_i \max(Ew_j - Ew_i, 0) - \beta_i \max(Ew_i - Ew_j, 0) \geq 0 \quad (IR_i) \\ & e_i \in \arg \max Ew_i - c(e_i) - \alpha \max(Ew_j - Ew_i, 0) - \beta \max(Ew_i - Ew_j, 0) \quad (IC_i) \end{aligned}$$

To solve for the optimal contract, I first prove the following Lemma 1-1.

Lemma 1-1 *Any effort levels $(\tilde{e}_1, \tilde{e}_2)$ can be implemented by an independent linear wage contract $w(x_i) = a_i x_i + t_i$.*

Proof: Suppose $(\tilde{e}_1, \tilde{e}_2)$ are a pair of efforts that the principal wants to implement. Consider the linear wage contracts $w_i(x_i) = a_i x_i + t_i, i = 1, 2$. Let $a_i = \tilde{e}_i / b_i, i = 1, 2$, and choose (t_1, t_2) to satisfy $a_1 b_1 \tilde{e}_1 + t_1 = a_2 b_2 \tilde{e}_2 + t_2 \geq \max(c(\tilde{e}_1), c(\tilde{e}_2))$. It is straightforward to prove that the constructed equal wage contracts can implement the effort levels. Analogously, I can also construct an unequal wage contract to implement the effort levels too. (for example let $a_1 = \tilde{e}_1 / (b_1(1 - \beta_1)), a_2 = \tilde{e}_2 / (b_2(1 + \alpha_1))$, and choose (t_1, t_2) to satisfy $a_1 b_1 \tilde{e}_1 + t_1 > a_2 b_2 \tilde{e}_2 + t_2 \geq \max(c(\tilde{e}_1), c(\tilde{e}_2))$).

Lemma 1-2 *Individual rationality constraints $IR_i, i = 1, 2$ are binding for both agents for optimal wage contracts.*

Proof: I only need to consider the linear wage contracts: $w_i(x_i) = a_i x_i + t_i, i = 1, 2$. Suppose neither (IR_i) constraints are binding, I can decrease t_1 and t_2 by a small amount Δ simultaneously so that IR constraints still hold. Because IC constraints remain the same, the new contract can increase profit. Therefore at least one IR constraint is binding. Suppose that IR_1 is binding and IR_2 is not binding. If $Ew_1 > Ew_2$ and $w_1 = a_1 x_1 + t_1, w_2 = a_2 x_2 + t_2$. I can choose a new contract $w_1 = \tilde{a}_1 x_1 + \tilde{t}_1, w_2 = \tilde{a}_2 x_2 + \tilde{t}_2$ ($a_1 > \tilde{a}_1$ and $\tilde{a}_2 > a_2$) so that IR constraints are

still satisfied but more profit can be earned. Analogously, I can prove for other cases such as $Ew_1 = Ew_2$.¹³

Under this monopolistic setting, the principal pays workers just the amount to compensate their effort costs and leave agents with zero rent. Intuitively, if one of the IR constraints is not binding, I can find another more compressed wage contract (or equally compressed if the nonbinding-IR contract is already an equal wage contract) to increase the profit.

From Lemma 1-2, I can derive the following Corollary 1-1.

Corollary 1-1 $Ew_1 \geq Ew_2$, if and only if $e_1 \geq e_2$.

Now I can solve for the optimal contracts. Intuitively, inequity aversion reduces the wage differentials between the workers. I refer to this force as the equality motive. The higher the values of inequity aversion parameters, the stronger the equality motive. To maximize profit, the principal needs to balance the trade-off between the efficiency motive and the equality motive. When the equality motive is strong, it is optimal for the principal to apply an equal wage contract. On the other hand, when the efficiency motive is strong, it is optimal for the principal to apply an unequal wage contract.

I first solve for the case of an equal wage contract and the case of an unequal wage contract. Then I compare the principal's profits in two cases. Thus I can get the optimal contract as a function of exogenous parameters. Add superscript e to equilibrium variables when the optimal contract is equal wage contract and

¹³An alternative proof can be found in the appendix.

add the superscript u to the variables when the optimal contract is an unequal wage contract.

Equal wage contract

When the principal offers an equal wage contract, $Ew_1 = Ew_2$, there is no disutility from wage comparisons between the workers. From Lemma 1-2, the participation constraints are binding for both agents, i.e., $Ew_i = c(e_i)$, $i = 1, 2$. Therefore, the equal wage contract implements an equal effort level $e_1 = e_2$. The principal's profit maximization problem now becomes

$$\max_{e_1, e_2} b_1 e_1 + b_2 e_2 - \frac{(e_1)^2}{2} - \frac{(e_2)^2}{2} \quad s.t \quad e_1 = e_2$$

Solving this problem yields the following lemma, where I add the superscript e to the variables associated with the optimal equal wage contract.

Lemma 1-3 *If an equal wage contract is optimal, the optimal effort levels are the same*

$e_1^e = e_2^e = \frac{b_1+b_2}{2}$; the expected wages are the same $Ew_1 = Ew_2 = \frac{(b_1+b_2)^2}{8}$. The principal earns profit $\Pi^e = \frac{1}{4} (b_1 + b_2)^2$.

As shown in Lemma 1-1, the optimal equal effort levels can be implemented by the independent linear wage contract below

$$w_i(x_i) = a_i x_i + t_i, \text{ where } a_i^e = \frac{b_1 + b_2}{2b_i} \text{ and } t_i^e = -\frac{(b_1 + b_2)^2}{8}, i = 1, 2.$$

Because $b_1 > b_2$, the power of incentives is higher for a low productivity worker than a high productivity worker: $a_1^e = \frac{b_1+b_2}{2b_1} < a_2^e = \frac{b_1+b_2}{2b_2}$. Because $a_1^e < 1 < a_2^e$, unlike in the benchmark case, the principal finds it suboptimal to sell the project to the agents, even when the agents are risk neutral.

Unequal wage contract

The following lemma describes the optimal wage contract if the optimal expected wages are not equal. I add superscript u to the variables associated with the optimal unequal wage contract.

Lemma 1-4 *When $b_1 > \frac{1+2\alpha_2}{1-2\beta_1}b_2$, if an unequal wage contract is optimal, the principal can implement optimal effort levels $e_1^u = \frac{1+\alpha_2-\beta_1}{1+2\alpha_2}b_1$ and $e_2^u = \frac{1+\alpha_2-\beta_1}{1-2\beta_1}b_2$ with profits $\Pi^u = \frac{1+\alpha_2-\beta_1}{2}\left(\frac{(b_1)^2}{1+2\alpha_2} + \frac{(b_2)^2}{1-2\beta_1}\right)$.*

When $Ew_1 > Ew_2$, from Lemma 1-2 and Corollary 1-1, I can derive the optimal effort levels $e_1^u = \frac{1+\alpha_2-\beta_1}{1+2\alpha_2}b_1$ and $e_2^u = \frac{1+\alpha_2-\beta_1}{1-2\beta_1}b_2$. The expected wage contracts are $Ew_1^u = \frac{1}{2}\frac{(1+\alpha_2-\beta_1)(1+\alpha_2)}{(1+2\alpha_2)^2}(b_1)^2 - \frac{1}{2}\frac{(1+\alpha_2-\beta_1)\beta_1}{(1-2\beta_1)^2}(b_2)^2$ and $Ew_2^u = \frac{1}{2}\frac{(1+\alpha_2-\beta_1)\alpha_2}{(1+2\alpha_2)^2}(b_1)^2 + \frac{1}{2}\frac{(1+\alpha_2-\beta_1)(1-\beta_1)}{(1-2\beta_1)^2}(b_2)^2$. The unequal effort levels are attained under the condition $Ew_1^u > Ew_2^u$, i.e., $b_1 < \frac{1+2\alpha_2}{1-2\beta_1}b_2$. Analogously, when $Ew_1 < Ew_2$, I can also derive the optimal effort levels. However, the corresponding expected wage condition $Ew_1^u < Ew_2^u$ violates the assumption $b_1 > b_2$. Hence the principal would never pay a low productivity worker more than a high productivity worker. This result shows that the principal only considers two inequity aversion parameters: the envy concern (α_2) from a low productivity worker and the compassion concern (β_1) from a high productivity worker.

As shown by Lemma 1-1, the independent linear wage contract $w_i(x_i) = a_i x_i + t_i$, $i = 1, 2$ can implement the optimal unequal effort levels, where $a_1^u = \frac{1+\alpha_2-\beta_1}{(1+2\alpha_2)(1-\beta_1)}$ and $a_2^u = \frac{1+\alpha_2-\beta_1}{(1-2\beta_1)(1+\alpha_2)}$. I can derive that the marginal power of incentives of a high productivity worker is lower than the marginal power of incentives of a low productivity worker: $a_1^u < 1 < a_2^u$, which is the same as in the equal wage contract.

Optimal contract

Now I can compare the profits under the equal wage contract and the unequal wage contract. Not surprisingly, $\Pi^u - \Pi^e \geq 0$ if $b_1 > \frac{1+2\alpha_2}{1-2\beta_1}b_2$, and the amount of profit levels are equal when $b_1 = \frac{1+2\alpha_2}{1-2\beta_1}b_2$. Therefore, when $b_1 > \frac{1+2\alpha_2}{1-2\beta_1}b_2$, the principal chooses the unequal wage contract. When $b_2 < b_1 \leq \frac{1+2\alpha_2}{1-2\beta_1}b_2$, the principal chooses the equal wage contract.

Summing up the above results, I have the following proposition:

Proposition 1-2 *A principal contracts with two inequity-averse agents with heterogeneous productivities $b_1 > b_2$. Let $\bar{b}_2 = \frac{1+2\alpha_2}{1-2\beta_1}b_2$.*

- 1. When $b_1 \leq \bar{b}_2$, the optimal effort levels are the same and the principal chooses an equal wage contract.*
- 2. When $b_1 > \bar{b}_2$, the optimal effort levels are different and the principal chooses an unequal wage contract.*

In both of the above cases, the principal earns less profit than in the benchmark case with standard agents. Moreover, effort levels and wage levels are more compressed than those of the benchmark.

The critical value $\bar{b}_2 = \frac{1+2\alpha_2}{1-2\beta_1}b_2$ determines whether the principal should choose an equal wage contract or an unequal wage contract.

Let us analyze the interaction between the efficiency motive and the equality motive. Fix the productivity levels b_1 and b_2 and vary the inequity aversion parameters α_2 and β_1 . For small values of α_2 and β_1 satisfying $b_1 > \bar{b}_2$, or when the inequity aversion parameters are small relative to the productivity differential, the principal will deviate from the benchmark model by compressing effort

levels because of the equality motive. The contract decreases the effort level of a high productivity worker from $e_1^B = b_1$ to $e_1^u = \frac{1+\alpha_2-\beta_1}{1+2\alpha_2} b_1$ and increases the effort level of a low productivity worker from $e_2^B = b_2$ to $e_2^u = \frac{1+\alpha_2-\beta_1}{1-2\beta_1} b_2$. I also find that $Ew_1^u < Ew_1^B$ and $Ew_2^u > Ew_2^B$. To sum up, the efforts and wages are more compressed with inequity-averse agents: $\frac{e_1^u - e_2^u}{e_1^B - e_2^B} < 1$ and $\frac{Ew_1^u - Ew_2^u}{Ew_1^B - Ew_2^B} < 1$.

As the inequity aversion parameters α_2 and β_1 increase and satisfy the equality $b_1 = \bar{b}_2$, I have $e_1^u = \frac{1+\alpha_2-\beta_1}{1+2\alpha_2} b_1 = \frac{1+\alpha_2-\beta_1}{1-2\beta_1} b_2 = e_2^u$. In this case, the optimal unequal wage contract is not feasible for the principal. Therefore, when $b_1 < \bar{b}_2$, i.e., the efficiency motive is strong, the principal chooses an equal wage contract to implement the equal effort levels $e_1^e = e_2^e = \frac{b_1+b_2}{2}$. The wage levels and effort levels are more compressed than in the unequal wage contract or the benchmark model.

From the optimal contract design, not all inequity aversion parameters matter. Only compassion concern β_1 of the high productivity worker and envy concern α_2 of the low productivity worker take effect on the optimal contract design.

Comparisons with the benchmark model and some comparative statics

For this section, I compare differences between key variables in the benchmark model and the model with inequity-averse agents. In addition, I analyze the impact of inequity aversion on the optimal contract.

Figure 1-1 shows the relationship between optimal expected wages in the benchmark model and the optimal expected wages with inequity-averse agents, when I fix b_2, β_1 and α_2 and vary b_1 . The two solid line segments describe the op-

timal expected wage levels with standard agents. The differential between the two solid line segments shows the expected wage differential in the benchmark case. The dashed line denotes the expected wage levels with inequity-averse agents. \bar{b}_2 is the critical value determining when the principal should apply an equal wage contract and an unequal wage contract.

When $b_1 < \bar{b}_2$ or the equality motive is strong, the optimal wage levels are the same for both agents, which is represented by the thin dashed line. When $b_1 > \bar{b}_2$, or the efficiency motive is strong, the optimal wage levels start to diverge as b_1 increases, which is presented by two thick dashed lines Ew_1^u and Ew_2^u . As can be seen, the optimal wage levels are more compressed with the inequity averse agents than with standard agents.

Figure 1-2 describes the optimal effort levels when I fix b_2 , β_1 and α_2 ; and vary b_1 . The upper solid line and the lower flat solid line depict the effort level for the high productivity worker 1 and for the low productivity worker 2 respectively. The dotted line denotes the optimal effort levels with inequity aversion. When b_1 is small, or $b_2 < b_1 < \bar{b}_2$, the equal effort contract is optimal, which is described by the thin dotted segment. When b_1 goes above the threshold \bar{b}_2 , the unequal effort contract is optimal, which is described by the two thick dotted segments. It can be clearly seen that the dashed line lies in between the two solid lines, which implies that the effort levels are more compressed with inequity-averse agents than with standard agents.

The following proposition describes the effects of compassion (β_1) and envy (α_2) on the optimal effort levels (e_1, e_2).

Proposition 1-3 *When the optimal wage contract is an unequal wage contract, then*

1 If β_1 increases, then e_1^u decreases and e_2^u increases.

2 If α_2 increases, then e_1^u decreases and e_2^u increases.

Proof: $e_1^u = \frac{1+\alpha_2-\beta_1}{1+2\alpha_2}b_1$ and $e_2^u = \frac{1+\alpha_2-\beta_1}{1-2\beta_1}b_2$. It is straightforward to show that

$$\frac{\partial e_1^u}{\partial \beta_1} < 0, \frac{\partial e_1^u}{\partial \alpha_2} < 0, \frac{\partial e_2^u}{\partial \beta_1} > 0, \text{ and } \frac{\partial e_2^u}{\partial \alpha_2} > 0.$$

The comparative statics analysis shows that the optimal effort of a high productivity agent decreases in β_1 and α_2 , while the optimal effort of a low productivity agent increases in β_1 and α_2 . For the same contract, the envy α_2 directly increases the marginal incentive of agent 2 compared with the benchmark case, thus increasing the effort level of agent 2. α_2 indirectly decreases agent 1's effort through agent 1's compassion towards agent 2. The compassion β_1 directly reduces the marginal incentive of agent 1 compared with the benchmark case, thus reducing the optimal effort level of agent 1. Lowering worker 1's wage reduces worker 2's disutility from comparison; therefore β_1 indirectly incentivizes agent 2 to work harder than the benchmark case.

This result also relates to the ratchet effect literature.¹⁴ When there is uncertainty concerning how difficult it is for workers to perform the task, workers tend to underproduce in order to avoid demanding schemes in the future. The underproduction of workers is due to workers' prediction of future contracts, while in my model the underproduction of high productivity worker is due to the low powered incentive.¹⁵

Figure 1-3 depicts how α_2 affects the optimal effort levels. I fix b_1 , b_2 , and β_1 and vary α_2 . The upper and lower solid straight lines represent the opti-

¹⁴See Weitzman (1980), Holmstrom (1982) and Freixas, Guesnerie and Tirole (1985).

¹⁵Freixas, Guesnerie and Tirole (1985) also assume that workers have more information about their abilities than the firm.

mal effort levels in the benchmark model. When $\alpha_2 < \bar{\alpha}_2$ ¹⁶ such that $b_1 > \bar{b}_2$ (i.e., the efficiency motive is strong), the optimal efforts are different for two agents. The thick dashed lines represent the optimal unequal effort levels, which can be implemented by an unequal wage contract. When $\alpha_2 > \bar{\alpha}_2$ such that $b_1 < \bar{b}_2$ (i.e., the equality motive is strong), the optimal effort levels are the same for the two agents. To sum up, as α_2 increases, the optimal effort level of a high productivity worker decreases while the optimal effort level of a low productivity worker increases until they are equal.

The principal's profit decreases in agents' inequity aversion parameters: $\frac{\partial \Pi''}{\partial \alpha_2} < 0$ and $\frac{\partial \Pi''}{\partial \beta_1} < 0$. Intuitively, the principal is worse off with equity-averse agents than with standard agents, because the principal needs to compensate agents for their disutility towards the wage difference.

Figure 1-4 shows how α_2 affects profit levels for different levels of β_1 . The three curves represent three cases $\beta_1 = 0$, $\beta_1 = 0.1$ and $\beta_1 = 0.2$ respectively. As can be seen for the three curves, the profit first falls then remains the same. For example, if $\beta_1 = 0$, the profit decreases with α_2 when $\alpha_2 < \bar{\alpha}_2 = 0.1$ ($b_1 > \bar{b}_2$), where the optimal wage contract is an unequal wage contract. The profit then stays constant when $\alpha_2 > \bar{\alpha}_2$ ($b_1 < \bar{b}_2$). In addition, for the same value of α_2 , the profit decreases with β_1 .

1.2.4 Optimal contract with status-seeking agents

In this section, I discuss the case when a principal faces status-seeking agents. In the context of the model, status-seeking agents are the same as inequity-averse

¹⁶ $\bar{\alpha}_2 = \frac{b_1(1-2\beta)}{2b_2} - \frac{1}{2}$

agents when they are paid less than their co-workers; i.e., the envy concern is the same. Differently, when status-seeking agents are paid more than others, instead of the compassion, they derive pleasure from being paid more than others, where $\beta_i < 0$. β_i measures the degree of status seeking. The lower the value of β_i , the stronger the status seeking is. It is straightforward to show that the previous optimal contracts still apply if $\beta_1 \geq -\alpha_2$. The principal is better off with status-seeking agents, i.e., $\frac{\partial \Pi^u}{\partial \beta_1} < 0$, because she can pay a lower wage to a status-seeking agent to induce the same level of effort when the status-seeking agent is paid more than his co-worker.

Some interesting results arise and differ from the model with inequity-averse agents if $\beta_1 < -\alpha_2$. Intuitively, with status-seeking agents, the principal needs to balance the trade-off among three concerns. Two concerns are the efficiency motive and the envy from a low productivity worker, which are the same as discussed in the previous subsection. The third concern is the status seeking from a high productivity worker.

If the status seeking is greater than the envy: $\beta_1 < -\alpha_2$, then $b_1 > b_2 > \bar{b}_2$. According to Proposition 1-2, the optimal effort levels are always different. The principal will use an unequal wage contract to induce a high productivity agent to work more and a low productivity agent to work less than the benchmark case. Both the optimal effort levels and the expected wages are more dispersed.

$$e_1^u = \frac{1 + \alpha_2 - \beta_1}{1 + 2\alpha_2} b_1 > e_1^B; \quad e_2^u = \frac{1 + \alpha_2 - \beta_1}{1 - 2\beta_1} b_2 < e_2^B$$

Comparing with the profit with the benchmark case, I find that the principal earns more profit with status-seeking agents ($\Pi^u > \Pi^B$) when the status seeking is strong ($\beta_1 < -\alpha_2$).

Formally, I have the following proposition,

Proposition 1-4 *With status-seeking agents,*

1. *If $\beta_1 > -\alpha_2$, the optimal effort levels and the optimal contract are the same as described in Proposition 1-2. The optimal effort levels and wage levels are more compressed than in the benchmark case.*
2. *If $\beta_1 < -\alpha_2$, the principal implements the unequal wage contract and earns more profit than in the benchmark case. The optimal effort levels and wage levels are more diverse than in the benchmark case.*

A linear wage contract $w_i(x_i) = a_i x_i + t_i$ can implement the optimal effort levels, where $a_1^u = \frac{1+\alpha_2-\beta_1}{(1+2\alpha_2)(1-\beta_1)}$ and $a_2^u = \frac{1+\alpha_2-\beta_1}{(1-2\beta_1)(1+\alpha_2)}$, where $a_1^u < 1 < a_2^u$.

Discussing the optimal wage contract with status-seeking agents,¹⁷ Frank (1984b) models social preferences by incorporating workers' ranking in firm's income hierarchy. He finds the result that the marginal powers of incentive are flatter for both high productivity and low productivity workers than with standard workers. Different from his model, I assume different incentive contract for both agents. In contrast, my model implies that it is optimal for the principal to adopt a high powered marginal incentive for a low productivity worker and a low powered marginal incentive for a high productivity worker.

1.2.5 Wage secrecy policy VS wage transparency policy

Why do many firms adopt a wage secrecy policy instead of a wage transparency policy?¹⁸

¹⁷Frank (1984b) does not assume that agents are envious of their co-workers if their pay is less than the pay of their co-workers.

¹⁸Card, Mas, Moretti, and Saez (2010) upset many people working in public organizations and universities, when they sent the link containing their compensations as well as their colleagues'.

Several previous studies have offered explanations for wage secrecy. Bertrand and Mullainathan (2003) propose that managers can lead a "quiet" life without any complaints about wage inequality if they keep wages secret. Danziger and Katz (1997) suggest that wage secrecy enhances the feasibility of risk-shifting contracts by reducing effective labor mobility. Beyond these reasons, the above model implies that a wage secrecy policy is also more profitable for the firm with inequity-averse agents. Because if workers cannot compare and estimate their co-workers' wages, they do not incur negative utility from inequity aversion. Proposition 1-3 predicts that with a wage secrecy policy, the principal does not need to consider inequity aversion between agents, therefore, the wages are more diverse and correspond with wages in the benchmark case.

In contrast, with status-seeking agents the principal may want to adopt a wage transparency policy. If the status seeking concern is large enough, the pleasure from a high productivity worker outweighs the disutility from a low productivity worker. It is more profitable for firms to exploit the status seeking concern by making the wage structure transparent.

It may seem contradictory that wage compression exists in many firms with wage secrecy policies. There are huge differences extended to how well a wage secrecy policy is executed in some sense. Even a firm carry a strict wage secrecy policy, workers may discuss their wages with their co-workers.

Other factors may also affect whether a firm adopting a wage secrecy policy or a wage transparency policy. For example, even with a wage secrecy policy, workers may still form some perception of their co-workers' wages. If the perceived wage differential is greater than the real wage differential, a wage trans-

Clark and Oswald (1996) report that people's satisfaction level is inversely related with their comparison wage rates.

parency policy with wage compression may be preferred to a wage secrecy policy. If perceived wage differential is similar to real wage differential, then the model predicts that whether the firm adopts a wage secrecy policy or a wage transparency policy does not matter. Workers' different perception of their co-workers' wages in different firms can also help explain that wage compression exists in some firms while wage secrecy exists in other firms.

1.3 Extension 1: varying the number of workers

For the previous sections, I assumed that the number of workers is exogenous; i.e., the principal has to employ two workers to complete the project. For this discussion I relax this assumption and endogenize the number of workers that the principal can hire. I also consider how the principal organizes teams efficiently when she faces multiple agents with heterogeneous productivities.

1.3.1 Hiring only one high productivity agent

If the project can be completed by one worker, when a principal only hires a high ability worker, the principal's profit is $\Pi^1 = \frac{b_1^2}{2}$. (Superscript 1 denotes the case where only agent 1 is employed.)

I compare the profits with two inequity-averse agents and find that if one worker is considerably more productive than another worker, the firm would rather only employ a high productivity agent. Intuitively, hiring a low productivity worker demotivates a high productivity worker, because of the negative utility from wage differences.

Formally, I have following Proposition 1-5:

Proposition 1-5 *There exists \bar{b}_1 such that for any $b_1 > \bar{b}_1$, the principal would rather hire a high productivity worker instead of both workers, holding all other parameters fixed.*

Proposition 1-5 implies that even when workers have net positive marginal products, with other-regarding preferences among workers, the principal would not hire all agents. This result seems to contradict the standard results without assuming spillovers in production. The classic theory does not generate this result without assuming existence of sabotage between workers. (Lazear, 1989)

1.3.2 Sorting

Next consider four workers: two high productivity workers and two low productivity workers that must be organized into two teams of two workers, because there are two identical projects and each project requires two workers to complete. Suppose the principal knows the workers' inequity aversion parameters and productivities before hiring them. How should the principal pair agents? Should the principal pair a high productivity agent with a high productivity agent or combine a high productivity agent with a low productivity agent? From Proposition 1-4, I derive the following corollary:

Corollary 1-2 1. If agents are inequity averse, the principal will pair a high productivity worker with a high productivity worker and a low productivity worker with a low productivity worker.

2. If agents are highly status seeking, the principal will combine a high productivity worker with a low productivity worker.

The first result is similar to Siemens (2007). The firm can avoid the social comparisons between inequity-averse workers by designing the optimal contract so that workers sort into the workforce with homogeneous productivities. This second result in Corollary 4 corresponds to the result in Frank(1984b). The firm can gain profit by sorting status-seeking workers who are willing to give up monetary compensation for status with workers with lower pay who care less about status. One difference is that Frank (1984b) obtains compressed wage distributions for status-seeking workers because the workers are free choose their coworkers in his model. However, in my model, if the status seeking is very strong, wage dispersion arises. Greater wage dispersion increases the power of incentives for the high productivity worker, hence, the firm can exploit more profit.

1.4 Extension 2: synergies between the agents

One of the main reasons for team production is because of positive spillovers among workers' production. In this section, I extend the basic model and assume that there are synergies between workers. I want to analyze the interaction between synergies and inequity aversion. One interesting question is whether synergies exacerbate or mitigate inequity aversion among workers.

There are two main ways to model synergies. One kind of synergy takes the form of a marginal effect, i.e., one worker's effort can increase other work-

ers' marginal productivity. Second, one worker's effort has a direct impact on the other worker's output level without changing the other worker's marginal product.¹⁹

First, suppose synergies take the first form, i.e., an increase in worker i 's effort can increase marginal productivity of worker j . Assume that the output function for agent i is $x_i = (b_i + se_j)e_i + \epsilon_i, i = 1, 2$, in which $(b_i + se_j)$ represents agent i 's effective marginal productivity.²⁰ With inequity-averse agents, the logic of analysis is similar to Proposition 1-2 and Corollary 1-3 in the appendix derives the optimal contract. Notice that under the optimal contract, the difference between effective marginal productivities of agents is $b_1 + se_2 - (b_2 + se_1) = b_1 - b_2 - s(e_1 - e_2) \leq b_1 - b_2$, because $e_1 \geq e_2$. Intuitively, the marginal effect of synergy reduces the differential between effective marginal productivities; therefore, it mitigates the inequity aversion among workers, which provides another reason for teamwork.

Next, suppose synergies have a level effect without increasing marginal productivity. Assume that the output function for each agent is $x_i = b_i e_i + S e_j + \epsilon_i$, where S measures the synergy effect and it is a level effect on another worker's production. Without inequity aversion, optimal effort levels are $e_1^B = b_1 + S$, $e_2^B = b_2 + S$. Corollary 1-4 in the appendix derives the optimal contract. In this case, synergy does not mitigate inequity aversion among workers.

If the principal can choose to hire one or two inequity-averse workers, there is a trade-off between the synergy effect and inequity aversion. In the above

¹⁹A third way to model synergy is that one worker's effort reduces another worker's marginal productivity but increases the level of production. But we are only interested in the first two ways.

²⁰With standard agents, optimal effort levels are $e_1 = \frac{b_1 + 2sb_2}{1 - 4s^2}$, $e_2 = \frac{b_2 + 2sb_1}{1 - 4s^2}$. To guarantee finite solutions, I assume $s < 0.5$.

model without synergies, the principal could increase her profit by adopting wage secrecy policies or by hiring two independent contractors who cannot meet each other and cannot compare their wages. But this is not necessarily the case if there is a positive spillover effect between agents' efforts (for example, their efforts are complements). The basic idea is that when the synergy effect is strong, the principal will let two inequity-averse agents work together. When inequity aversion is strong, the principal will contract with two independent contractors or adopt a wage secrecy policy.

1.5 Related literature

Chapter 1 is related to two strands of literature.

First, it contributes to behavioral contract theory with other-regarding preference. Englmaier and Wambach (2010) analyze the classical moral hazard problem with a selfish principal and an agent comparing his income with the principal's. They also extend the model to 2 agents' case in which the agent compares his income with the other agent's income as well as principal's. They conclude relative performance evaluation is optimal even if the tasks are technologically independent. Bartling and Siemens (2004) analyze the interaction of behindness aversion and risk aversion in the moral hazard model with two agents. The inequity aversion increases agency costs and inequity aversion renders flat wage contracts optimal. They also shed light on the boundary of the firm. Dur and Glazer (2008) study optimal contracts when a worker envies his boss. Their analysis sheds light on the different incentives between private and public industries. In a private firm, a principal needs to compensate agents more for

inequity aversion while incentive pay is lower in public or nonprofit organization. Thakor and Goel (2005) studies optimal contracts when agents envy each other. The key result is that team incentives are optimal even though other's performance provides no information about one's action. Grund and Sliwka(2005) discuss envy and compassion in tournaments. Bartling and Siemens(2004) studies efficiency in team production with inequity averse agents. Rey-Biel (2008) argues that envy and guilt can be profitably exploited by the employer. Their results imply that sometimes inequity aversion is a reason to form work teams. Demougin, Fluet and Helm (2006) discuss an interesting asymmetric information case where agent 2 in task 2's effort is verifiable while agent 1 in task 1's effort is not. More inequity aversion reduces equilibrium output and effort of agent 1. The effort of agent 2 decreases (increases) if there efforts are complements (substitutes). In terms of key topics discussed, Chapter 1 is most closely to Charness and Kuhn (2004). But I use a different model from theirs and I assume that workers can have different payment structure (different marginal incentive powers and transfers) while they assume the same contract for both workers.

Second, my work relates to the theory of other-regarding preferences, psychological game and lab experiments. The theory of other regarding preferences including Fehr and Schmidt (1999), Bolton and Ockenfels (2000), Rabin (1993), Dufwenberg and Kirchsteiger (2002), Falk and Fischbacher (2000), Charness and Rabin (2002), Cox and Friedman (2002), Levine (1998), and Lazear (1992).²¹

²¹See Fehr and Schmidt (2003) for a comprehensive survey concerning other-regarding preferences.

1.6 Conclusion

I build a parsimonious model incorporating inequity aversion and heterogeneous labor productivities. The main result in Chapter 1 is that inequity aversion causes wage compression. This result depends on the assumption that the agents are inequity averse to ex-ante wage difference. This model can apply to the cases with an information structure in which effort and ex-post wages between the workers are not perfectly observable while ex-ante wages are observable. The results are robust if the workers are inequity averse to effort differences or inequity averse to both effort differences and wage differences. In other words, the model can easily extend to the case where wages are secret but efforts are observable.²² In this case, as shown by Lemma 1-5 in the appendix, inequity aversion to effort differences can also cause wage compression. But if agents compare their ex ante rents, the inequity aversion doesn't matter. Because of symmetric information, the principal can always extract all the rent, so the rent is zero for the agents. This model sheds light on wage secrecy and wage transparency policies. Generally, with inequity-averse agents, the firm earns more profit by adopting a wage secrecy policy.

This model can also predict different incentive schemes within an internal labor market. Due to wage compression, a high-powered incentive contract should apply to a low productivity worker, while a low-powered incentive scheme should apply a high productivity worker. This finding has some empirical implications about power of incentives for workers with different productivities. For example, in a sales department, a senior salesman with more experience is usually a high productivity worker compared with an inexperienced

²²It may apply the case when workers are closely working together and firms adopt wage secrecy policies.

junior salesman. This model can predict that, if workers are inequity averse, the firm should apply a low-powered incentive contract to a senior salesman and apply a high-powered incentive contract to a junior salesman.

As to the future research path, there are several directions I can pursue. The model assumes the complete information structure. One future study is to relax this assumption and to explore the optimal contract under the incomplete information of productivities. Second, I analyze the problem under the monopolistic setting, which can be extended to a competitive setting, especially to study the sorting between the firms and competitive screening of heterogeneous workers. Third, I assume that each worker's payment only depends on his performance. In reality settings, workers' wage may include a team payment component, which is a function of total output. In other words, worker's wage does not only depend on his/her own performance but also depends on the performance of the whole team. Employing such a team payment component may reduce the difference in compensation across workers and might be preferred in the context of this chapter since compensation differences reduce worker's utility. Empirically, I hope to test the results in this chapter. I can collect compensation data in different firms across industries to see if inequity aversion has an impact on the compensation structure and firms' performance. For example, I can compare the performances and the compensation structures of a firm before it adopting a wage secrecy policy and after it adopting wage secrecy (controlling for workers' perception of their co-workers' wages). If the firm's wage compensation becomes compressed after adopting a wage secrecy policy, then the evidence supports the results of the chapter.

CHAPTER 2
ENRICHING MULTI-ROUND TOURNAMENT THEORY

2.1 Introduction

In the organizational literature, the convex wage structure describes the convex relationship between hierarchical levels and promotion prizes. In other words, the promotion prize increases with the hierarchical level. Many empirical studies demonstrate this convex wage structure. For example, Lambert, Larcker, and Weigelt (1993) obtain compensation data for four distinct organizational levels ranging from the plant manager to the corporate CEO in 303 large publicly traded U.S. firms. Their findings support convex wage structures. In particular they find a considerably large increase in the final stage, i.e., the difference in compensation level for the CEO relative to the next lower position in the organizational hierarchy is “extraordinarily” large relative to the compensation increases observed at the lower levels of hierarchy. Baker, Gibbs, and Holmstrom (1994a, 1994b) analyze twenty years of personnel data of a single firm and also find this convex relationship. Eriksson (1999) analyzes actual compensation data of 2600 executives over 260 Danish firms during a four-year period. He finds that as one moves up the corporate hierarchy, the pay difference increases, even when controlling for individual and firm characteristics.¹

Some theoretical literature provides explanations for the convex wage structures documented above, however, in a limited way. One important explanation is proposed by Rosen (1986). He models the promotion in hierarchical firms using a multi-round tournament setting. Specifically, he assumes that a single firm

¹Also see Lazear (1992), O’Reilly, Main, and Crystal (1988), and Main et al. (1993)

hires a number of identical workers who compete in pairs. Each worker's production depends on the effort input and a noise variable. The worker with the higher output is the winner and enters the next round. This process repeats until a final winner is generated. In order to induce optimal effort levels from workers, the firm commits to an ex ante wage structure. A prize is earned when a worker wins a round. The more rounds a worker wins, the more money the worker earns. The money earned in a round can be considered as a wage increase. We also call it the direct monetary prize, the inter-rank spread, or the direct promotion prize with the hierarchical level.

Rosen (1986) shows that the promotion prizes of the optimal wage structure are the same across all previous hierarchical levels with a substantial increase in the final level. Intuitively, except for the last round, the incentive for workers to win a round is not only just the direct monetary prize from winning that round. The incentive also consists of the option value or the expected prize from the possibility of winning subsequent rounds. Because there is no option value for the last round, the direct monetary prize for the last round is greater than the direct monetary prize from winning previous rounds. Rosen (1986) partly explains the convex wage structure in the sense that the promotion prize increases sharply for the last round. Nevertheless this result is not fully consistent with the above empirical findings, because the promotion prize not only increases in the last round, but also increases in the previous rounds too.

The inconsistency is due to the simplified assumption of homogeneous stage effects across rounds, i.e., the assumption that both the number of people competing and the optimal effort level are the same across rounds. Rosen (1986) briefly mentions that some extensions, which allow for varying nature across

stages, may amend this consistency. For example, Rosen (1986) proposes the extensions that, "in a corporate hierarchy the pass-through rate may fall at each successive rank", or "higher-ranking positions are more demanding than lower-ranking ones", may fully explain the convex wage structure. Because of those heterogeneous stage effects, Rosen (1986) explains that "game proceeds, and interrank spreads must be increasing to undo the incentive dilution effects of greater discounting of the future, which otherwise reduces the option value of continuation." His intuition offers possible extensions, which may help explain convex wage structures. However, no one has theoretically derived convex wage structures rigorously under the setting of classical multi-round tournaments.

In this chapter, we theoretically demonstrate that a convex wage structure is optimal by assuming heterogeneous stage effects in the classical multi-round tournament model. Specifically, we make two extensions to Rosen (1986)'s tournament model. The first extension is that the number of workers rises with the hierarchical level or the probability of winning decreases with the hierarchical level. This extension is realistic because it is usually more difficult for workers to be promoted as they move up the career ladder (See Baker, Gibbs, and Holmstrom, 1994a). The second extension is that the return to effort increases in rounds, or the optimal effort increases in rounds. This extension is realistic too because the position in higher levels plays a more significant role than the position in lower levels and thus higher levels require more effort input.

For the first extension, more people competing in further rounds suggests that the promotion rate decreases with hierarchical levels. Baker, Gibbs, and Holmstrom (1994a) used twenty years of personnel data from one firm to show

that promotion rates fall dramatically with the (higher) hierarchical level (from 56% to 11%). Intuitively, if the probability for winning decreases, then the option prize decreases across rounds. To induce the same effort level in all rounds, the optimal direct promotion prize increases monotonically with the hierarchical level. Thus, the optimal wage structure is convex. This extension supports Rosen (1986)'s conjecture. In this extension the key factor is the decreasing promotion rate, but the promotion rate is endogenous in the model. To stay close to Rosen (1986)'s model, we make the primitive assumption about the number of workers competing.

We also discuss the second extension that the returns to effort increase in rounds. The assumption that the returns to effort increase in rounds is valid for many firms because the higher the job level, the higher the returns to effort. Therefore the optimal effort level is increasing across rounds. Intuitively, the promotion prize (the sum of the option value and the inter-rank prize) will increase to compensate the increasing effort cost. However the option value will not necessarily increase or decrease and we cannot reach the conclusion that the inter-rank prize will increase monotonically. In other words, the result of this extension diverges from Rosen's (1986) conjecture that the increasing efficient effort level would yield a more smoothly convex wage structure. We construct an example where the effort level increases moderately and the promotion rates are low, and where the inter-rank prize first falls then rises and the optimal wage structure is not convex.

Therefore, to generate convex wage structures in the second extension, we need further assumptions about the cost functions or the probability of winning functions. One sufficient condition to generate convex wage structures is a de-

creasing option value across rounds. The option prize from winning round n is the difference between the expected earnings from winning subsequent rounds and the cost of effort in round $n + 1$. The expected earnings can be considered as the promotion prize in round $n + 1$ discounted by the promotion rate. If the promotion rate is low or the cost of effort increases sharply across rounds, then the option prize will decrease. We discuss a special case when the cost function is quadratic and the firm's output function is linear in effort. If the production uncertainty is small enough, then the optimal wage structure is convex. Rosen (1982)'s paper is related to this extension, but he assumes workers have heterogeneous talents and discusses the matching of talents with the hierarchical positions of the firm. He finds that it is optimal to assign persons with superior abilities to top positions because of the multiplicative effects that greater talent at top positions will also influence the lower levels, hence increasing productivity by more than the increments of their abilities. He concludes that it is optimal to offer the superior top level manager enormous rewards.

Chapter 2 is organized as follows. Section 2 presents the literature review. Section 3 presents the benchmark model setup, in which we generalize Rosen (1986)'s model by assuming more than two workers competing in each round. Based on the benchmark model, in section 4 we investigate the two realistic extensions discussed above. The first extension is that the number of workers competing increases across rounds. And the second extension is that the returns to effort increase in rounds. Section 5 concludes.

2.2 Literature Review

This chapter contributes to the literature of convex wage structures and enriches the theories of promotion tournaments.

As mentioned in the introduction, many papers have empirically studied the wage structures in hierarchical firms. For example, Lambert, Larcker, and Weigelt (1993), Main, O'Reilly and Wade (1993), Baker, Gibbs, and Holmstrom (1994a, 1994b), and Eriksson (1999), have documented convex wage structures empirically, especially the case where sharp rewards are needed to motivate the highest hierarchical level.

There are two main approaches to model promotions in internal labor markets. One is the classical promotion tournament initially formulated in the seminal analysis by Lazear and Rosen (1981). They assume that firms commit ex ante to future levels of compensation associated with promotions. This ex ante prize structure induces the optimal effort levels of workers. As a consequence, there is a close relationship between efficient effort choices and mechanisms of the task. The second approach is a market-based tournament built on Waldman (1984a)², who also analyzes the case where promotion prizes serve as incentives for workers' efforts. In contrast with the classical tournament model, firms cannot commit to the future wage rates when workers are young. Instead, a promotion serves as a positive signal concerning worker ability. After observing this signal, other potential employers will offer higher wages to the promoted workers, thereby determining the prize of promotion if the current firms want to keep the promoted workers and to reduce the turnovers. Therefore no direct

²See Gibbs (1995), Zabochnik and Bernhardt (2001), Zabochnik (2008), and Ghosh and Waldman (2010).

relationship exists between the promotion prize and the efficient effort levels.

Both classical tournament models and market-based tournament models can help explain many aspects of empirical evidence in internal labor markets, such as promotions associated with large wage increases. But the classical tournament literature can provide more intuitive explanations for the convex wage structure.

Following the assumption that firms commit to an *ex ante* wage structure, many papers provide theoretical explanations for convex wage structures from different views. For example, as we mentioned above, Rosen (1986) extends Lazear and Rosen's (1981) model to a multi-round tournament model to explain that wages increase constantly across hierarchical levels until the final level when wages increase sharply. This chapter generalizes and extends his multi-round tournament model to fully explain the convex wage structure. Rosen (1982) also provides some explanation for why a convex wage structure exists at the top level. Specifically, Rosen (1982) models the assignment of talents to hierarchical positions in a three-level firm and explains the enormous rewards at the top level in some large firms. Calvo and Wellisz (1979) also point out the important role of talent in monitoring and organizing hierarchical production. Different from our model, Rosen (1982) and Calvo and Wellisz (1979) assume that workers are heterogeneous in their talents. They propose that more talented workers will be sorted into higher levels and that these workers also increase the productivity of lower level workers. At top levels the marginal productivities of workers are higher, therefore, the optimal wage increases quickly as workers move up the hierarchy.

2.3 The Benchmark Model

Following Rosen (1986), we analyze the optimal wage structure in a multi-round tournament setting. In the benchmark model, we assume homogeneous stage effect and we generalize Rosen's (1986) model slightly to replicate his key results. Specifically, we assume n workers compete instead of 2 workers as in Rosen's model. And we derive that the optimal wage increases constantly for all levels except a sharp increase for the final level.

Suppose a firm has N hierarchical levels (or N rounds). Suppose n^N homogeneous workers enter the firm and compete to enter higher levels. In each level workers compete in a group of n people. In other words, the number of workers competing is the same across rounds. Round 1 consists of $n^N/n = n^{N-1}$ matches. For each match the worker with the higher output level is the winner, with the winners in other matches entering the competition of the next round. Therefore, n^{N-1} workers enter round 2. After N rounds, only one winner remains.

Assume that a worker's output depends on his effort level (e) and a noise variable (ϵ), which follows a known distribution with expected value 0 and variance σ^2 .³ Let w_k be the reward if a worker only wins k rounds. For example, the overall winner wins w_N .

Let e_k denote the effort exerted in round k and let $c(e_k)$ be the cost associated with the effort level e_k . We assume that the cost function is increasing ($c'(e_k) > 0$) and is convex ($c''(e_k) > 0$). Because we assume that workers have equal talents or abilities, the probability of a worker winning in round k , p_k , depends on his effort e_k given his opponents' effort levels e_{-k} : $p(e_k|e_{-k})$. Because workers are

³As discussed briefly in Lazear and Rosen (1986), when agents are risk neutral, the noise does not have an effect on the optimal solutions.

homogenous, the equilibrium is symmetric and the induced effort is the same for all workers in round k : $e_k = e_{-k}$ ⁴. We assume the optimal effort in round k denoted by e_k^* is exogenous. The optimal wage structure should implement the optimal effort levels, i.e., $e_k = e_{-k} = e_k^*$.

The same effort levels in round k indicate that the probability of winning in round k , denoted by $p(e_k^*)$ ($= p(e_k^*|e_{-k}^*)$), is also the same for all workers. Because the workers compete in a group of n people in each round, the probability of winning for a worker is $\frac{1}{n}$.

We assume the same stage effect in the benchmark model. In other words, we assume that the number of workers competing is the same across rounds and the optimal effort level is also the same across rounds. Let e^* be the optimal effort level in each round and let c^* be the cost associated with e^* . Therefore, the optimal wage structure induces every worker to exert effort level e^* across rounds, i.e., $e_k^* = e^*$. To induce the optimal effort level e^* in each round, the firm chooses the optimal wage structure $\{w_1, w_2, \dots, w_N\}$.⁵

Rosen (1986) derives one key result of this optimal wage structure when the match is a pairwise competition. He also assumes homogeneous stage effects across rounds. Rosen concludes that the incremental prize for winning the last round ($w_N - w_{N-1}$) is much higher than the inter-rank prize or the direct monetary prize in the previous rounds ($w_k - w_{k-1}$, $k = 2, \dots, N - 1$), while the direct monetary prizes ($w_k - w_{k-1}$, $k = 2, \dots, N - 1$) are the same in the previous rounds.

Intuitively, as there are no further rounds beyond the last round, the prize for

⁴Note that e_{-k} is a vector of effort levels of all the other players other than player k . But the symmetric equilibrium indicates the effort levels are the same for all the players. For the sake of simplicity, we denote e_{-k} as a scalar.

⁵It is obvious that the optimal wage structure satisfies that $w_1 < w_2 < \dots < w_{N-1} < w_N$. If not, for example, if $w_k > w_{k+1}$ then workers who won stage k will not exert any effort in round $k + 1$.

winning the last round is only a direct monetary prize, which should be greater than the direct monetary prizes from winning previous rounds, since there is no option value for the last round.

To illuminate the logic of the above results, we present two approaches. The first approach closely follows Waldman (2011)'s concise analysis, which captures the key intuition in a multi-round tournament model. The second approach is the Bellman equation method used in Rosen (1986)'s paper. The first approach can be used to demonstrate that the optimal wage structure in the first extension is convex, but the first approach cannot characterize the optimal wage structure in the second extension. Therefore, we have to use second approach, the Bellman equation.

2.3.1 First approach

Under the setting of our model, proposition 2-1 characterizes the optimal wage structure.

Proposition 2-1 *The inter-rank prize is the same for the previous $N - 1$ rounds*

$$w_{k+1} - w_k = \left(1 - \frac{1}{n}\right)(w_N - w_{N-1}) + c^*, \text{ where } k = 1, 2, \dots, N - 2.$$

The inter-rank prize for the last round is greater than the inter-rank prize for the previous rounds

$$w_2 - w_1 = w_3 - w_2 = w_4 - w_3 = \dots = w_{N-1} - w_{N-2} < w_N - w_{N-1}.$$

The proof is relegated to the appendix.

Intuitively, to induce the same effort level in each round, the promotion prize for winning each round should be the same. Winning round k , except for the last round N , involves two prizes. One is the direct promotion reward (incremental monetary prize or inter-rank prize) from winning round k , *i.e.*, $w_k - w_{k-1}$. The other prize is the option prize, *i.e.*, the expected value from the possibility of winning subsequent rounds. According to the above reasoning, the prize from winning round $N - 1$ is the direct monetary prize, $w_{N-1} - w_{N-2}$, and the expected value of winning round N , which is the difference between the expected earning from round N and the cost of effort, $\frac{1}{n}(w_N - w_{N-1}) - c^*$. The expected earning in round $N-1$, $\frac{1}{n}(w_N - w_{N-1})$, can be considered as the earning in round N , $(w_N - w_{N-1})$, discounted by the promotion rate $\frac{1}{n}$. The sum of these two prizes equals the promotion prize from winning round N ,

$$w_{N-1} - w_{N-2} + \frac{1}{n}(w_N - w_{N-1}) - c^* = w_N - w_{N-1}.$$

Analogously, the prize from winning round $N - 2$ is composed of the inter-rank prize $(w_{N-2} - w_{N-3})$ and the expected value of winning subsequent rounds, which is the difference between expected earning from winning round $N - 1$ and the cost of effort, $(\frac{1}{n}[w_{N-1} - w_{N-2} + \frac{1}{n}(w_N - w_{N-1}) - c^*] - c^*)$. The expected earning from winning round $N - 1$, $\frac{1}{n}[w_{N-1} - w_{N-2} + \frac{1}{n}(w_N - w_{N-1}) - c^*]$ can also be considered as the promotion prize for winning n discounted by the probability of winning $(\frac{1}{n})$, because it is same as the expected earning from winning round N , $\frac{1}{n}(w_N - w_{N-1})$. Therefore, the inter-rank prize is the same for round N and round $N - 1$.

Repeating this procedure yields the following equation

$$w_2 - w_1 = \dots = w_{N-2} - w_{N-3} = (1 - \frac{1}{n})(w_N - w_{N-1}) + c^*. \quad (2.1)$$

The above equation shows that the direct monetary prize is the same for all the rounds except the last round. If we compare the direct monetary prize of the

last round with the direct monetary prize of the previous rounds, we obtain

$$(w_N - w_{N-1}) - (w_k - w_{k-1}) = \frac{1}{n}(w_N - w_{N-1}) - c^*, \text{ where } k = 1, 2, \dots, N - 1. \quad (2.2)$$

The optimal wage structure implies that $\frac{1}{n}(w_N - w_{N-1}) - c^* \geq 0$. Otherwise, no one exerts effort at the last stage because exerting efforts at stage N incurs negative net expected payoff for the workers.

Therefore, it follows from (2.1) and (2.2) that the direct prize from winning increases constantly with a greater increase in the final round. This result is consistent with the result in Rosen (1986)'s paper. The promotion prize $((1 - \frac{1}{n})(w_N - w_{N-1}) + c^*)$ and the option prize $(\frac{1}{n}(w_N - w_{N-1}) - c^*)$ stay the same for all rounds except the last round (no option prize). Therefore, the inter-rank prize, which is the difference between the promotion prize and the option prize, is the same for all rounds except the last round.⁶

2.3.2 Second approach

We can also use Bellman equation approach, as used in Rosen (1986), to analyze the optimal wage structure under the setting of the benchmark model. Define V_k as the value to a worker when $n - k$ possible rounds remain to be played. For an overall winner, $V_N = w_N$. If a worker enters the round k , the Bellman equation is

$$V_{k-1} = \max_{e_k} \{(1 - p(e_k|e_{-k}))V_k + p(e_k|e_{-k})w_k - c(e_k)\}.$$

The first order condition yields $V_k - w_k = \frac{c'(e^*)}{p'(e^*)}$, because the optimal wage structure implements optimal effort level $e_k = e^*$.

⁶The key assumptions driving this result is that the optimal effort level (e^*) and the promotion rate ($\frac{1}{n}$) are the same across rounds.

To derive the optimal wage structure, we first derive the following Lemma 2-1.

Lemma 2-1 Let $\beta = \frac{1}{n} - \frac{p'(e^*)}{c'(e^*)}c^*$. $V_N = w_N$; $V_k = \beta V_{k+1} + (1-\beta)w_k$, where $k = 1, \dots, N-1$.

The formal proof is relegated to the appendix.

It follows from Lemma 2-1 that $V_{N-1} = \beta w_N + (1-\beta)w_{N-1}$, where $\beta = \frac{1}{n} - \frac{p'(e^*)}{c'(e^*)}c^* > 0$. Because if $\beta < 0$, then $V_{N-1} < w_{N-1}$.⁷ In other words, a worker has no incentive to exert any effort in the last round, which contradicts the goal of the firm: to induce positive effort in each round. By Lemma 2-1, we can characterize the optimal wage structure in the following Proposition 2-2.

Proposition 2-2 The inter-rank prize is the same for the previous $N-1$ rounds, i.e.,

$$w_k - w_{k-1} = \left(1 - \frac{1}{n}\right) \frac{c'(e^*)}{p'(e^*)} + c^*, \quad k = 1, 2, \dots, N-1$$

The inter-rank prize for the last round is $w_N - w_{N-1} = \frac{c'(e^*)}{p'(e^*)}$.

$$(w_N - w_{N-1}) - (w_k - w_{k-1}) = \frac{1}{n} \frac{c'(e^*)}{p'(e^*)} - c^* > 0 \text{ since } \beta = \frac{1}{n} - \frac{p'(e^*)}{c'(e^*)}c^* > 0.$$

Therefore the inter-rank prizes are the same for previous $N-1$ rounds with an increase in round N .

$$w_2 - w_1 = w_3 - w_2 = w_4 - w_3 = \dots = w_{N-1} - w_{N-2} < w_N - w_{N-1}.$$

The proof is relegated to the appendix.

The increase of inter-rank prizes for the last round is $(w_N - w_{N-1}) - (w_{N-1} - w_{N-2}) = \frac{1}{n} \frac{c'(e^*)}{p'(e^*)} - c^* > 0$, which is the option value from winning round $N-1$. The option value, $\frac{1}{n} \frac{c'(e^*)}{p'(e^*)} - c^*$, is the same for all rounds except the last round (no option value).

⁷Here $\beta > 0$ accords with the inequality $\frac{1}{n}(w_N - w_{N-1}) - c^* > 0$ in the first approach.

2.4 Enriching the model to explain a convex wage structure

The above two approaches prove Rosen (1986)'s main result in a more general setting. However, as we discussed in the introduction, this optimal wage structure does not capture the real world phenomenon that the inter-rank promotion prize not only increases in the last round, but also increases in previous rounds. Therefore, an interesting question to explore is, under what circumstances is the optimal wage structure convex? In the setting of our model, a convex wage structure is expressed as $w_2 - w_1 < \dots < w_{N-1} - w_{N-2} < w_N - w_{N-1}$. We explore two different realistic extensions which may generate a convex wage structure. The first extension is that the number of people competing increases with the hierarchical level. The second extension is that returns to extra ability or effort increase with the hierarchical level.

2.4.1 Extension 1: the number of people competing increases with the hierarchical level

As discussed in the introduction, if the promotion rate decreases with the hierarchical level, to induce the same effort level across rounds, then the interrank prize will be designed to increase so that the promotion prize will be the same across the rounds. The decreasing promotion rate is the key assumption to generate the convex wage structure. However, this assumption is not about the primitiveness of the model. In this extension, we discuss possible cases may result in decreasing promotion rates.

The promotion rate for a worker is the ratio of the number of positions to

the number of workers competing for the positions, because workers are homogenous in the model. In workplaces it is harder and harder for workers to go up the hierarchy for two main reasons. One is that the number of positions available to workers decreases faster than the number of workers competing for them. For example, in some firms many workers may compete for only one CEO position while there may be many available lower level positions. The second reason is that the number of workers competing increases across the round. For example, in some firms the higher the job levels the larger the pool of candidates.

To stay close to Rosen (1986)'s model, in this extension we make the primitive assumption that the number of workers increases with the hierarchical level;⁸ however, as shown in the proof below, that the key factor generating the convex wage structure is the decreasing promotion rate.

We assume that the total number of workers is $n_1 n_2 \dots n_{N-1}$, where n_k workers compete in each match of round k . We assume that the number of people competing in each match increases with the hierarchical level: $n_1 < n_2 < \dots < n_{N-1} < n_N$. After round k , $n_{k+1} n_{k+2} \dots n_{N-1} n_N$ workers enter round $k+1$. After N rounds, there is one overall winner.

Because workers are homogenous, the probability of winning round k , denoted by p_k , is the same for all participants in that round, i.e., $p_k = \frac{1}{n_k}$. More participants competing in matches in further rounds reduce the probability of winning, i.e., $p_1 > p_2 > p_3 > \dots > p_N$.

As shown above in the benchmark model, because the optimal effort e^* is assumed to be the same across rounds, the optimal promotion prize for each

⁸This assumption is briefly discussed in Rosen (1986) and Waldman (2011).

round is the same across rounds too. We denote c^* to be the cost associated with the optimal effort e^* . As discussed in the first approach, except for the last round, the promotion prize in round k is composed of a direct monetary reward ($w_k - w_{k-1}$) and the option prize or the expected prizes from winning subsequent rounds.

Therefore, the promotion prize in round $N - 1$, $w_{N-1} - w_{N-2} + p_N(w_N - w_{N-1}) - c^*$ equals the promotion prize in round N , $w_N - w_{N-1}$.

Following the same logic as in approach 1, we can derive the relationship between the inter-rank prize in round k and the inter-rank prize in round N , $k = 2, \dots, N - 1$, which is presented in Lemma 2-2.

Lemma 2-2 $w_k - w_{k-1} = (1 - p_{k+1})(w_N - w_{N-1}) + c^*$, $k = 1, 2, 3, \dots, N - 1$,

Proof: To prove the above equation is equivalent to proving

$$w_N - w_{N-1} = w_k - w_{k-1} + p_{k+1}(w_N - w_{N-1}) - c^*, \quad k = 1, 2, 3, \dots, N - 1. \quad (2.3)$$

Because the optimal effort levels are the same across the rounds, promotion prizes are the same across rounds too, i.e., the promotion prize in round N equals the promotion prize in round k , $k = 1, 2, \dots, N - 1$.

Specifically, the promotion prize in round N equals the promotion prize in round $N - 1$, i.e.,

$$w_N - w_{N-1} = w_{N-1} - w_{N-2} + p_N(w_N - w_{N-1}) - c^* \quad (2.4)$$

$$\text{or } w_N - w_{N-1} = \frac{1}{1 - p_N}(w_{N-1} - w_{N-2} - c^*).$$

Hence, equation (2.3) holds for $k = N - 1$.

For the same reason, the promotion prize in round N equals the promotion prize in round $N - 2$,

$$w_N - w_{N-1} = w_{N-2} - w_{N-3} + p_{N-1}[w_{N-1} - w_{N-2} + p_N(w_N - w_{N-1}) - c^*] - c^*.$$

Substituting equation (2.4) into the above equation yields

$$w_N - w_{N-1} = w_{N-2} - w_{N-3} + p_{N-1}(w_N - w_{N-1}) - c^*.$$

Thus equation (2.3) holds for $k = N - 2$ as well. Analogously, we can prove that equation (2.3) holds by induction for $k = 1, 2, \dots, N - 1$.

From Lemma 2, and from the assumption that the probability of winning is decreasing across rounds, i.e., $p_1 > p_2 > p_3 > \dots > p_n$, we can easily see that the inter-rank prize is increasing with the hierarchical level, i.e., $w_{N-1} - w_{N-2} > w_{N-2} - w_{N-3} > \dots > w_2 - w_1$. This convex wage structure is generated directly from the assumption that the probability of winning decreases with the hierarchical level. Intuitively, the option prize for winning round k (except the last round N), $p_{k+1}(w_N - w_{N-1}) - c^*$, decreases as workers enter higher levels because it is more difficult for them to win in higher levels. Therefore, the direct monetary prize must increase with the hierarchical level so that the promotion prize is constant and the same effort level can be induced across rounds.

Equation (2.3) indicates $(w_N - w_{N-1}) - (w_{N-1} - w_{N-2}) = p_N(w_N - w_{N-1}) - c^*$. As discussed above, the optimal wage structure must satisfy $p_N(w_N - w_{N-1}) - c^* > 0$. Otherwise, workers would not exert any effort in the last round. Summarizing the above results, we obtain the following proposition:

Proposition 2-3 *If the number of workers competing increases with the hierarchical level ($n_1 < n_2 < \dots < n_{N-1} < n_N$), the direct monetary prize increases with the*

hierarchical level ($w_2 - w_1 < \dots < w_{N-1} - w_{N-2} < w_N - w_{N-1}$). Therefore, the optimal wage structure is convex.

2.5 Extension 2: returns to effort increase with the hierarchical level

In the previous subsection, we showed that if the number of people competing increases with the hierarchical level, the optimal wage structure is convex. In this subsection, we investigate whether another realistic extension that returns to effort increase with the hierarchical level can also generate a convex wage structure.

Let e_k^* denote the optimal effort level associated with round k . Intuitively, if returns to extra ability increase, then from a basic marginal benefit and cost analysis it is optimal to induce more effort across rounds. Formally, we have the following Lemma 2-3.

Lemma 2-3 *If returns to efforts increase in rounds, then optimal effort level increases in rounds too, i.e., $e_1^* < e_2^* < \dots < e_{N-1}^* < e_N^*$.*

Example 1: Let us check a special case of a firm with only three hierarchies. Assume that $e_1^* < e_2^* = e_3^*$. We can show that the optimal wage structure is convex. The promotion prizes from winning the last two rounds are the same and they are greater than the promotion prizes in the first round, respectively. Intuitively, the option value from winning round 2, $(\frac{1}{n}(w_3 - w_2) - c(e_3^*))$, is the same as the option value from winning round 1, $(\frac{1}{n}((w_2 - w_1) + \frac{1}{n}(w_3 - w_2) - c(e_3^*)) - c(e_2^*)) =$

$\frac{1}{n}(w_3 - w_2) - c(e_2^*)$,⁹ since the effort levels are the same for round 2 and round 3. Because the promotion prize from round 2 is greater than the promotion prize from round 1 due to increasing optimal effort levels, the inter-rank prize increase in round 2 is greater than the inter-rank prize increase in round 1. Therefore, the optimal wage structure is convex.

Example 2: Let's check a second special case of a firm with three effort levels $e_1^* < e_2^* < e_3^*$. We are interested in whether the convex wage structure still holds. It is straightforward to prove that for round 2, $w_3 - w_2 > w_2 - w_1$ because of the option value from winning the last round in the second round. Since $e_1^* < e_2^*$, the promotion prize from winning the second round is greater than the promotion prize from round 1, i.e.,

$$(w_2 - w_1) + \frac{1}{n}(w_3 - w_2) - c(e_3^*) > w_1 + \frac{1}{n}((w_2 - w_1) + \frac{1}{n}(w_3 - w_2) - c(e_3^*)) - c(e_2^*).$$

It follows that $(w_2 - w_1) - w_1 > \frac{1}{n}((w_2 - w_1) + \frac{1}{n}(w_3 - w_2) - c(e_3^*)) - c(e_2^*) - (\frac{1}{n}(w_3 - w_2) - c(e_3^*))$.

However, we cannot conclude that $(w_2 - w_1) - w_1 > 0$. Hence, the optimal wage structure is not necessarily convex. One sufficient condition for the convex wage structure (i.e., the inter-rank increase in level 2 is greater than the inter-rank increase in level 1) is that the option value from winning round 2 is less than or equal to the option value in level 1, i.e., $(\frac{1}{n}((w_2 - w_1) + \frac{1}{n}(w_3 - w_2) - c(e_3^*)) - c(e_2^*)) \geq (\frac{1}{n}(w_3 - w_2) - c(e_3^*))$.

Remark: the option prize is the difference between the expected earning from winning subsequent rounds and the certain effort cost. The expected earnings can be seen as the subsequent inter-rank prize and cost function discounted

⁹The expected earning from winning round 1 and the expected earning from winning round 2 are the same ($\frac{1}{n}(w_3 - w_2)$) since $e_2^* = e_3^*$.

by the promotion rate $\frac{1}{n}$. If the expected earnings are small (or n is large) or the effort cost increases sharply ($c(e_3^*)$ is much larger than $c(e_2^*)$), then the option value will be decreasing. Therefore, to keep the promotion prize increasing the inter-rank prize must increase.

To characterize the optimal wage structure when the optimal effort level increases with the hierarchical level, we apply the Bellman equation approach for its tractability. Let V_k be the value to a worker when $n-k$ possible rounds remain to be played. As discussed in the benchmark model, the optimal wage structure implements the optimal effort e_k^* from all the workers in round k . Let $p(e_k|e_{-k})$ be the probability of winning if a worker exerts effort e_k in round k , while his opponents exert effort e_{-k} . We assume that the probability of winning increases in effort $p'(e_k|e_{-k}) > 0$ and is convex $p''(e_k|e_{-k}) < 0$.¹⁰ The symmetric equilibrium indicates that $p(e_k^*|e_{-k}^*) = \frac{1}{n}$. For simplicity, we denote $p'(e_{k+1}^*) \equiv \frac{\partial p(e_k^*|e_{-k}^*)}{\partial e_k^*}$.

Lemma 2-4 Let $\beta_{k+1} = \frac{1}{n} - \frac{c(e_{k+1}^*)p'(e_{k+1}^*)}{c'(e_{k+1}^*)}$,

$$V_k = \beta_{k+1}V_{k+1} + (1 - \beta_{k+1})w_k, \text{ for } k = 1, 2, \dots, N - 1.$$

It is obvious that $V_{k+1} > V_k > w_k$, otherwise the worker would not exert any effort in the next round $k + 1$. It follows from Lemma 2-4 and $V_{k+1} > V_k > w_k$ that $\beta_{k+1} = \frac{1}{n} - \frac{c(e_{k+1}^*)p'(e_{k+1}^*)}{c'(e_{k+1}^*)} > 0$, i.e., $c'(e_{k+1}^*) - \frac{1}{n} \frac{c'(e_{k+1}^*)}{p'(e_{k+1}^*)} > 0$, which is also the sufficient condition to use Bellman equation. From the Lemma 2-4, we can derive the value of the inter-rank prizes as presented in Corollary 2-1.

¹⁰The assumption about probability of winning is consistent with Rosen (1986). Let $p(e_k) = \frac{h(e_k)}{h(e_k)+h(e_{-k})}$, where $h(e_k)$ is the effective effort and $h(e_{-k})$ is his opponent's effective effort. As in Rosen (1986), it is assumed that $h'(e_k) > 0$ and $h''(e_k) < 0$.

Remark: $p'(e_k) = \frac{h'(e_k)h(e_{-k})}{[h(e_k)+h(e_{-k})]^2} > 0$, $p''(e_k) = \frac{h''(e_k)h(e_{-k})[h(e_k)+h(e_{-k})]-2[h'(e_k)]^2h(e_{-k})}{[h(e_k)+h(e_{-k})]^3} < 0$

Corollary 2-1

$$w_N - w_{N-1} = \frac{c'(e_N^*)}{p'(e_N^*)};$$

$$w_{k-1} - w_{k-2} = \frac{c'(e_{k-1}^*)}{p'(e_{k-1}^*)} - \frac{1}{n} \frac{c'(e_k^*)}{p'(e_k^*)} + c(e_k^*), \quad k = 3, \dots, N.$$

From Corollary 2-1, $(w_N - w_{N-1}) - (w_{N-1} - w_{N-2}) = \frac{c'(e_N^*)}{p'(e_N^*)} - \left(\frac{c'(e_{N-1}^*)}{p'(e_{N-1}^*)} - \frac{1}{n} \frac{c'(e_N^*)}{p'(e_N^*)} + c(e_N^*) \right) > 0$, since $\beta_{k+1} > 0$. The direct monetary reward increases in round N : $w_N - w_{N-1} > w_{N-1} - w_{N-2}$, which is consistent with the result from the first approach. However, $w_{k+1} - w_k > w_k - w_{k-1}$ does not necessarily hold for other rounds. The following proposition 2-4 presents the sufficient and necessary conditions to generate the convex wage structure.

Proposition 2-4 *If 1) returns to effort increase in the hierarchical level, and*

$$2) \frac{c'(e_{k-1}^*)}{p'(e_{k-1}^*)} - \frac{1}{n} \frac{c'(e_k^*)}{p'(e_k^*)} + c(e_k^*) > \frac{c'(e_{k-2}^*)}{p'(e_{k-2}^*)} - \frac{1}{n} \frac{c'(e_{k-1}^*)}{p'(e_{k-1}^*)} + c(e_{k-1}^*), \quad k = 3, \dots, N.$$

then the optimal wage structure is convex.

The option prize for round $N - 1$ is $\frac{1}{n}(w_N - w_{N-1}) - c(e_N^*) = \frac{1}{n} \frac{c'(e_N^*)}{p'(e_N^*)} - c(e_N^*)$.

The option prize for round $N - 2$ is $\frac{1}{n}(w_{N-1} - w_{N-2}) + \frac{1}{n}(w_N - w_{N-1}) - c(e_N^*) - c(e_{N-1}^*) = \frac{1}{n} \left(\frac{c'(e_{N-1}^*)}{p'(e_{N-1}^*)} - \frac{1}{n} \frac{c'(e_N^*)}{p'(e_N^*)} + c(e_N^*) + \frac{1}{n} \frac{c'(e_N^*)}{p'(e_N^*)} - c(e_N^*) \right) - c(e_{N-1}^*) = \frac{1}{n} \frac{c'(e_{N-1}^*)}{p'(e_{N-1}^*)} - c(e_{N-1}^*)$.

Analogously, we can prove that the option prize for round k is $\frac{1}{n} \frac{c'(e_{k+1}^*)}{p'(e_{k+1}^*)} - c(e_{k+1}^*)$.

Because $c'(e_k)$ increases in e_k , and $p'(e_k)$ decreases in e_k , $\frac{c'(e_k)}{p'(e_k)}$ increases in e_k . Since $\frac{c'(e_{k-1}^*)}{p'(e_{k-1}^*)} > \frac{c'(e_{k-2}^*)}{p'(e_{k-2}^*)}$ as the optimal effort level increases with the hierarchical level, Corollary 2-2 presents sufficient conditions for a convex wage structure to hold.

Corollary 2-2 *If 1) the returns to effort increase with the hierarchical level and*

2) the option prize decreases across rounds, i.e., $\frac{1}{n} \frac{c'(e_k^*)}{p'(e_k^*)} - c(e_k^*) \leq \frac{1}{n} \frac{c'(e_{k-1}^*)}{p'(e_{k-1}^*)} - c(e_{k-1}^*)$,
 $k = 2, \dots, N$,

then the optimal wage structure is convex.

The inequality $\frac{1}{n} \frac{c'(e_k^*)}{p'(e_k^*)} - c(e_k^*) \leq \frac{1}{n} \frac{c'(e_{k-1}^*)}{p'(e_{k-1}^*)} - c(e_{k-1}^*)$ holds if and only if $c(e_k^*) - c(e_{k-1}^*) \geq \frac{1}{n} \left(\frac{c'(e_k^*)}{p'(e_k^*)} - \frac{c'(e_{k-1}^*)}{p'(e_{k-1}^*)} \right)$. It holds if the cost function of effort increases faster than the magnitude $\frac{c'(e_k^*)}{p'(e_k^*)}$ increases. In other words, the optimal effort increases sharply across levels ($c(e)$ increases sharply), i.e., the cost function is very steep ($c'(e)$ is large) and $c'(e)$ increases relatively less or $c''(e)$ is relatively small. (Or $p'(e)$ decreases at a slow rate or $p''(e)$ is relatively fat.)

It is straightforward that if the optimal effort level increases sharply with the hierarchical level, then the optimal wage structure will be convex in order to compensate the workers.

Example 2 (Continued): let us check the special example of a three-level hierarchical firm with $e_1^* < e_2^* < e_3^*$. Assume for round k , the output y_k is linear in effort $y_k = e_k + \epsilon_k$, where ϵ_k is a random variable following a uniform distribution in range $[-a, a]$ with mean 0. $f(\epsilon) = \frac{1}{2a}$. As shown in the proof of Corollary 2-3 in appendix, in equilibrium $p'(e_k^*) = \frac{1}{2a}$. Suppose that $c(e_k) = (e_k)^2$. $c'(e_k) = 2e_k$ and $c''(e_k) = 2$. It follows that $\frac{c'(e_k)}{p'(e_k)} = 4ae_k$. Therefore, $w_3 - w_2 = 4ae_3$; $w_2 - w_1 = 4ae_2 - \frac{1}{n}4ae_3 + (e_3)^2$; $w_1 = 4ae_1 - \frac{1}{n}4ae_2 + (e_2)^2$. We have $w_3 - w_2 > w_2 - w_1$ because $e_3 > e_2$ and $\beta_3 = \frac{1}{n} - \frac{e_3}{4a} > 0$. It follows from $\beta_3 = \frac{1}{n} - \frac{e_3}{4a} > 0$ that all effort levels are less than $\frac{1}{n}4a$, i.e., $e_k < \frac{1}{n}4a$, $k = 1, 2$ and 3.

The inter-rank prize difference between the first round and the second round ($w_2 - w_1$) - $w_1 (= 4a(e_2 - e_1) + (e_3 + e_2 - \frac{1}{n}4a)(e_3 - e_2))$ is not necessarily positive. Therefore, we demonstrate that Rosen's (1986) conjecture is incorrect.

We can provide a counterexample that the convex wage structure is not convex. If $e_1 = \frac{a}{16n}$, $e_2 = \frac{2a}{16n}$, $e_3 = \frac{3a}{16n}$ and $n = 3$, then $w_2 - w_1 < w_1$. For this set of parameters, the convex wage structure is not optimal because the effort cost is increasing moderately and the promotion rate ($\frac{1}{3}$) is not small. On the other hand, if $e_1 = \frac{a}{4n}$, $e_2 = \frac{3a}{4n}$, $e_3 = \frac{5a}{4n}$, then the optimal wage structure is convex. As can be seen, the effort cost is increasing more sharply than in the counterexample.

The optimal structure is convex for example 2 if and only if

$$(w_2 - w_1) - w_1 = 4a(e_2 - e_1) + (e_3 + e_2 - \frac{1}{n}4a)(e_3 - e_2) \geq 0$$

One sufficient condition for the above inequality to hold is that $e_3 + e_2 - \frac{1}{n}4a \geq 0$.

If the cost function is quadratic $c(e_k) = b(e_k)^2$, where $b (> 0)$ is a constant, and the noise variable in linear output function follows the uniform distribution, then the second condition in Proposition 2-4 implies that

$$4ab(e_{k-1}^* - e_{k-2}^*) - \frac{4ab}{n}(e_k^* - e_{k-1}^*) + b(e_k^* - e_{k-1}^*)(e_k^* + e_{k-1}^*) > 0.$$

From Corollary 2-2, it is sufficient to have that $e_k^* + e_{k-1}^* > \frac{4a}{n}$. Formally, we have the following Corollary 2-3,

Corollary 2-3 *If the returns to effort increase with the hierarchical level, if the cost function of effort is quadratic: $c(e_k) = b(e_k)^2$, where $b (> 0)$ is a constant, and if output y_k is linear in effort $y_k = e_k + \epsilon_k$, where ϵ_k follows an uniform distribution in range $[-a, a]$ with mean 0, $k = 1, 2, \dots, N$, then the optimal wage structure is convex if the following condition is satisfied: $e_k^* + e_{k-1}^* \geq \frac{4a}{n}$ where $k = 3, \dots, N$.*

Intuitively, when the variance of the production function (or a) is not large compared with the effort levels and when the promotion rate ($\frac{1}{n}$) is low, then the option value decreases across rounds due to increasing effort cost. Therefore, to induce increasing effort levels, the direct monetary prize increases and the optimal wage structure is convex.

2.6 Conclusion and discussion

Empirical evidence shows that the wage structure is usually convex. One sufficient condition for convex wage structure is that the option prize decreases across rounds. We analyze two possible realistic extensions in multi-round promotion tournaments that may generate a convex wage structure. The first extension is that, if the number of workers competing increases with hierarchical levels, then the optimal wage structure is convex, because the option prize decreases across rounds. Second, if the returns to effort increase with the hierarchical level, then the optimal wage structure is not convex. This result proves that Rosen's (1986) conjecture is incorrect. The optimal wage structure is convex if we make additional assumptions about cost functions and promotion functions. For example, if the cost function is quadratic, the production technology is linear in effort, and the noise for production is small, then the direct monetary prize increases with the hierarchical level.

Our research provides empirical predictions for future research. First, for similar firms, if the promotion rate decreases more sharply with hierarchical levels within one firm than within other firms, then according to our theories, the optimal wage structure should be more convex for that firm than for other

firms. Second, if the required effort levels rise more sharply with hierarchical levels in one firm than within other firms, then we predict that the wage structure is more convex. With data available, we can test empirically whether the two extensions proposed above can actually generate convex wage structures.

CHAPTER 3
EX ANTE OR EX POST FAIRNESS CONCERNS: EXPERIMENTAL
EVIDENCE

3.1 Introduction

It is well documented in experimental and empirical studies that people do not only care about their own payoff but also care about other people's payoff. In other words, fairness views play an important part in people's preferences. When people face uncertainties in a social context, how will their fairness concerns and risk attitudes affect their decisions? For example, if a central planner (CP) designs a housing contract for workers, the CP must choose between two options for new housing because of limited resources. On one hand, CP can build an average quality housing, in which case everyone will have the same housing. On the other hand, CP can use the same resources to build new high-quality housing, but not everyone can have the housing. The housing is allocated via lottery and everyone has an equal opportunity to get a high-quality house. How should the CP plan to build the housing? Should the CP design the contract which guarantees everyone the same average quality housing condition ex post (equality of outcome) or should the CP design the high-quality housing and then let workers draw lotteries and decide who will live in a high-quality house (equality of opportunity). Which contract makes workers better off? Whether the CP chooses the first option or the second option depends on workers' fairness views. Consider another situation. If an principal relies on an agent to make a decision choice in a risky context, the agent's choice does not only affect his/her own payoff but also the principal's payoff. Will the agent

choose the allocation that offers both of them equal probability of winnings or will the agent choose the allocation that offers them exactly the same payoff no matter which state occurs? In general, do people focus on ex post allocations or ex ante allocations? In other words, do people value equality of outcome or value equality of opportunity?

The model of inequity aversion by Fehr and Schmidt (1999) is widely applied to the literature of other-regarding preferences. This study can also be conducted under the framework of inequity aversion. If people take the ex post fairness perspective, they choose the allocation which maximizes the expectation of the ex post utility or minimizes the expectation of the ex post payoff differences among people. On the other hand, if they adopt the ex ante fairness perspective, they hope to minimize the expected payoff differences among people. Will an ex ante fairness view or an ex post fairness view affect their decision making? Specifically, if people dislike payoff inequalities or are inequity averse, will they take account of the expectation of payoff differences among people (an ex post fairness view) or the differences of expected payoffs among people (an ex ante view)? Do people prefer equal opportunities among them or prefer equal outcomes?

Many experimental and theoretical studies on social preferences occur in deterministic environments. Whether people in society adopt an ex ante or an ex post fairness view is rarely studied. There are some studies on dictator games under the veil of ignorance and on probabilistic dictator game. (Kariv and Zame, 2009, Karni et al., 2008, Bohnet et al., 2008, Kircher et al. 2009, and Bolton and Ockenfels, 2010). They found that many subjects would assign an equal opportunity of winning both to themselves and to others in a dictator

game. Cappelen et al. (2011) also conducted an experiment to test whether people focus on ex ante opportunities or ex post outcomes when they make decision under uncertainty. In their model, they chose different reference points for an ex ante view and an ex post view. By contrast, this experiment follows standard definitions of ex ante and ex post fairness views and use an online experiment to analyze whether people focus on ex ante or on ex post fairness views. Saito (2012) models preferences for equal opportunities and equal outcomes. Specifically, he introduces one's own expected payoff and the expected payoff of others' into the utility function and provides unified explanations for the dictator game with uncertainties. The issue of social preference under uncertainty has both important theoretical and empirical implications. In the theoretical literature of other regarding preferences, it is often assumed that people maximize the expectation of their ex post utility function. However, there is little evidence for this implicit assumption in the literature. This chapter aims to present experimental evidence in this aspect. Meanwhile, the assumption of an ex ante fairness view could greatly simplify the theoretical analysis if more evidence for an ex ante fairness view was found. Under this assumption we only need to compare expected payoff differences, but for an ex post fairness view we need to consider payoff differences in every case. Empirically, this has important implications for public policies. A better understanding of people's fairness preferences with uncertainties will help policy makers decide whether they should focus on ex ante fairness criterion or ex post fairness criterion.

It is difficult to test ex ante and ex post fairness views with non-experimental data, because other noise variables in the field may also affect people's decisions in a social context such as income, social identity or various demographic factors. Besides, it is hard to tell whether people focus on ex ante allocation

differences or ex post allocation differences given different framing effects. The design of the experiment can avoid these problems, because the experiment can provide a clean environment to focus only on ex ante or ex post fairness views.

Specifically, this online survey includes 12 hypothetical questions and some brief demographic questions. For the hypothetical questions, the subjects were asked to choose allocations not only for himself/herself but also for the other person who was randomly paired with him/her. Specifically, the subjects were asked to compare the option in the question with a list of alternative options. Essentially, the subjects played the dictator game.

To test an ex post fairness view, I designed two sets of questions. The alternative given in each question was uncertain and provided two people with different payoffs in two equally probable states. The subjects compared this uncertain allocation with a list of certain payoffs which could provide the two people equal payoffs. Therefore, I can elicit the subjects' certainty equivalent for the uncertain allocation. By comparing certainty equivalents from the two sets of question, I can evaluate whether the ex ante or ex post fairness views played a role in subjects' decision making. For one set of questions, the alternatives offered the two people equal payoffs in each state. For another set of questions, the alternatives offered the two people unequal payoffs in each state but the same expected payoff. If the participants took an ex post fairness view or derived disutility from ex post unequal payoffs, then the certainty equivalent for the uncertain allocation with equal payoffs for two people in each state (no ex post differences) should be higher than the certainty equivalent for the uncertain allocation with unequal payoffs (with ex post differences) for the two people in each state. The following screenshot is a question in the survey elicit-

ing subjects' certainty equivalent. The alternative given in the question is both ex ante and ex post fair.

Alternative: a coin is tossed.

If it is a Head, you and the other person each will get \$0.

If it is a Tail, you and the other person each will get \$10.

Now you are comparing the above alternative with each of following alternatives. Please select all the alternatives which you prefer (to the above alternative).

- Each of you and the other person gets \$2 for sure.
- Each of you and the other person gets \$4 for sure.
- Each of you and the other person gets \$6 for sure.
- Each of you and the other person gets \$8 for sure.
- Each of you and the other person gets \$10 for sure.

I only focus on the subjects who behaved monotonically with the options in the questions. For example, if the lowest value they chose was \$6 for sure, then the subjects in the sample would also choose \$8 and \$10 (the options with values higher than \$6). In this case the certainty equivalent lies between \$4 and \$6. To simplify the analysis, I took the lowest value of choices as their CE.¹ In the above example, if a subject chose \$6, \$8, \$10, then the certainty equivalent was \$6.

The challenge is to test an ex ante fairness view. Because an ex ante unfair allocation is also ex post unfair, it is difficult to tell whether the subjects adopted an ex ante fairness view or an ex post fairness view when they faced ex ante unfair allocations. I utilize environmental cues to capture the ex ante and ex post fairness views. Specifically, the subjects were randomly divided into three groups by their birthdays. For the control group, there were no remarks about the alternatives the subject faced. For one treatment group, a remark was made

¹I can also choose midpoint of the interval (the lowest value of option chosen and highest value of option nonchosen) as the CE. The statistical results stay the same except estimated CE.

on ex post differences of the alternatives. For the other treatment group, a remark was made about the ex ante differences of the alternatives. These remarks, or environmental cues called "primes", could temporarily render certain factors salient. For example, if people adopted an ex ante fairness view, and if the priming effects did exist in this online survey, then the results from the treatment group priming on ex ante differences would show that the certainty equivalent for the ex post-different-and-ex ante-equal-allocation was closer to the certainty equivalent for the ex post and ex ante equal alternative than the other treatment group and the control group.

The regression results indicate that the ex post fairness concern does affect subjects' decision making. The difference between the certainty equivalent for an ex post fair allocation and the certainty equivalent for its corresponding ex post unfair allocation is statistically significant at the 5% level or the 10% level. For example, an allocation assigns (\$10, \$10) with a 50% probability and (\$0, \$0) with a 50% probability, while the other allocation assigns (\$10, 0) with a 50% probability and (\$0, \$10) with a 50% probability. Then the CE for the second allocation is around \$.51 higher than the first allocation. These results diverge from the inequity aversion model (Fehr and Schmidt 1999). It seemed that the subjects obtained positive utility from payoff differences. Remarks on ex post payoff differences or ex ante differences may slightly change people's behavior. However, the differences are not statistically significant either. I also conduct some demographic analysis on fairness concerns. However, the results are not statistically significant either. The results should be interpreted with caution. The insignificance of the results is because of the relatively small sample size.

The statistical insignificance may be also due to the coarseness of the data. I

also calculate the upper bound of the ex post fairness concerns and the priming effects. For example, if a subject chooses the options with \$6, \$8 and \$10 for sure, when the subject faces the ex post different alternative: (\$10, \$0) with a 50% probability (in the brackets, the first number is the payoff for the chooser and the second number is the payoff for the other person) and (\$0, \$10) with a 50% probability, the lower bound of the certainty equivalent is \$4. If a subject chooses the options that guarantee \$6, \$8 and \$10 for sure when the subject faces the ex post fair alternative: (\$0, \$0) with a probability of 50% and (\$10, \$10) with a probability of 50%. The upper bound of the certainty equivalent is \$6. I compare the lower bound of the certainty equivalent interval of the ex post unfair allocation and the upper bound of the certainty equivalent interval of the ex post fair allocation and I find that the differential between the upper bound and lower bound is statistically significant. This result indicates that the ex post fairness concerns may affect people's decision making.

The chapter is organized as follows. Section 2.2 describes a theoretical framework for testing ex ante and ex post fairness views. Section 2.3 presents the online survey design. Section 2.4 presents the results from the online survey. Section 2.5 concludes Chapter 2. The appendix presents a sample survey.

3.2 A Theoretical Framework

In this section, I outline the inequity aversion model by Fehr and Schmidt (1999) and I analyze ex ante and ex post fairness views under this setting.

Suppose that there are two people denoted by A and B . Suppose (x_A, x_B) is an allocation profile that offers person A x_A and person B x_B . I assume that

people have other-regarding preferences. Specifically, I assume that people are inequity averse to payoff differences and person A 's utility function is

$$U_A(x_A, x_B) = u_A(x_A) + \gamma_A f(x_A, x_B), \quad (3.1)$$

where the function $\gamma_A f(x_A, x_B)$ captures A 's fairness concern for B and the parameter γ_A measures A 's weight on fairness. I assume that $f(x_A, x_B) \begin{cases} = 0 \text{ if } x_A = x_B \\ \neq 0 \text{ if } x_A \neq x_B \end{cases}$. If A and B have equal payoffs, then there is no disutility incurred to A . If A 's payoff is different from B 's payoff, then the different payoff will incur a utility to A . If A is inequity-averse to payoff, then difference of payoff will pose a disutility on A , or $f(x_A, x_B) < 0$. Generally, I assume that the larger the payoff differences the higher the disutility level, i.e., $|f(x_A, x_B)|$ increases as $|x_A - x_B|$ increases. Most other-regarding literature supports this assumption. If A enjoys payoff differences, then the payoff difference will cause positive utility to A , or $f(x_A, x_B) > 0$. Generally, it is assumed that f increases as $x_A - x_B$ increases. Some literature is consistent with this view. For example, status-seeking literature assumes that people enjoy being paid more than their coworkers. Here I assume that $f(x_A, x_B) < 0$ if $x_A \neq x_B$ or workers are inequity averse. I will test this null hypothesis later.

Definition of ex-ante and ex-post fairness views

Ex-post fairness view: If A takes an ex post fairness view, A chooses allocations by maximizing the expectation of the ex post utility, i.e.,

$$\max EU_A^{ex\ post}(x_A, x_B) = \max Eu_A(x_A) + \gamma_A Ef(x_A, x_B).$$

In the literature this ex post form of utility functions is widely applied.

Ex-ante fairness view: If A takes an ex ante fairness view A chooses allocations

by maximizing the utility including the expectation of payoff comparisons, i.e.,

$$\max EU_A^{ex\ ante}(x_A, x_B) = \max Eu_A(x_A) + \gamma_A f(Ex_A, Ex_B).$$

If people take an ex-ante fairness view, then payoffs generated from equal opportunities will not have an impact on their utility.

Let us consider some special allocations (G_1), (G_2) and (G_3) :

(G_1) : An uncertain allocation offers (x_1, x_1) with a 50% probability and offers (x_2, x_2) with a 50% probability. Without loss of generality, let $x_1 > x_2$.

(G_2) : An uncertain allocation offers (x_1, x_2) with a 50% probability and offers (x_2, x_1) with a 50% probability.

(G_3) : A sure allocation (\bar{x}, \bar{x}) .

Both allocations (G_1) and (G_2) offer the same expected payoff ($\frac{1}{2}x_1 + \frac{1}{2}x_2$) to both people. The allocation (G_1) offers equal payoffs to the two people no matter which state occurs, hence allocation (G_1) is both ex ante fair and ex post fair. Put differently, people with an ex ante fairness view and people with an ex post fairness view derive the same utility from (G_1), i.e.,

$$EU_A^{ex\ post}(G_1) = EU_A^{ex\ ante}(G_1) = Eu_A(G_1) = \frac{1}{2}u_A(x_1) + \frac{1}{2}u_A(x_2).$$

An allocation (G_2) offers the same expected payoffs but different ex post payoffs for the two people, so it is ex ante fair but ex post unfair. In other words, the allocation (G_2) provides an equal opportunity for the two people. If A takes an ex post fairness view, then

$$EU_A^{ex\ post}(G_2) = \frac{1}{2}u_A(x_1) + \frac{1}{2}u_A(x_2) + \gamma_A Ef(x_1, x_2).$$

If A takes an ex ante fairness view, then

$$EU_A^{ex\ ante}(G_2) = Eu_A(G_2) = \frac{1}{2}u_A(x_1) + \frac{1}{2}u_A(x_2).$$

The sure allocation (G_2) is both ex ante and ex post fair, so

$$EU_A^{ex\ post}(G_3) = EU_A^{ex\ ante}(G_3) = u_A(\tilde{x}).$$

If A adopts an ex post fairness view and if A is inequity-averse, then A prefers (G_1) to (G_2) since $EU_A^{ex\ post}(G_1) > EU_A^{ex\ post}(G_2)$. On the other hand, if A adopts an ex ante fairness view, then A is indifferent between (G_1) and (G_2) since $EU_A^{ex\ ante}(G_1) = EU_A^{ex\ ante}(G_2)$.

Let $u_A(\tilde{x}_1, \tilde{x}_1) \equiv EU_A^{ex\ post}(G_1)$ and let $u_A(\tilde{x}_2, \tilde{x}_2) \equiv EU_A^{ex\ post}(G_2)$, or I say (G_1)'s certainty equivalent is $(\tilde{x}_1, \tilde{x}_1)$ and (G_2)'s certainty equivalent is $(\tilde{x}_2, \tilde{x}_2)$. It follows from $EU_A^{ex\ post}(G_1) > EU_A^{ex\ post}(G_2)$ that the certainty equivalent for (G_1) is greater than the certainty equivalent for (G_1) from an ex post fairness view, i.e., $\tilde{x}_1 > \tilde{x}_2$. By eliciting and comparing the certainty equivalent from (G_1) and (G_2), I can derive whether people hold ex ante or ex post fairness views. This is the main idea of the survey design.

3.3 Online Survey Design

The survey comprises 12 multiple choice questions, followed by a short survey including some demographic questions. For the online survey, the participants made choices not only for himself/herself but also for another person randomly paired with him/her. Essentially, for this online survey all the subjects played dictator games. The participants were told that the payoffs of all their decisions were final and that there cannot be any transfers. To solely focus on the fairness

concerns and avoid reputation effects, the participants were also informed that their decisions were completely anonymous to the other person.

The first three questions were designed to test whether the subjects have other regarding preferences. The remaining questions were designed to elicit certainty equivalents for different alternatives. Specifically, the subjects were presented with an uncertain alternative in a question and were asked to compare it with a list of certain options, which consists of five or more monotonically increasing options guaranteeing both people equal payoffs. The subjects were asked to choose all the options in the list preferred to the alternative in the question. If the subjects behaved monotonically and if they chose an option in the list, then any option with a sure value greater than the value of that option should be chosen as well. I only focus on the subjects who behaved monotonically so that I can eliminate inattentive responses submitted by the subjects. For the uncertain alternatives, the statement on tossing a coin was used to let participants better understand the probability of each state. The subjects read that "Suppose a coin is tossed. A head and a tail occur with an equal probability. (50%)." To remind the participants that more than one answer may be selected, one example was given on purpose before participants started to answer the questions.

Three pairs of questions are designed to elicit the certainty equivalents for uncertain alternatives of different values. Each pair includes a question about ex ante and ex post fair uncertain alternatives (G_1). Consider the following screenshot of question 4.

Alternative: a coin is tossed.

If it is a Head, you and the other person each will get \$0.

If it is a Tail, you and the other person each will get \$10.

Now you are comparing the above alternative with each of following alternatives. Please select all the alternatives which you prefer (to the above alternative).

- Each of you and the other person gets \$2 for sure.
- Each of you and the other person gets \$4 for sure.
- Each of you and the other person gets \$6 for sure.
- Each of you and the other person gets \$8 for sure.
- Each of you and the other person gets \$10 for sure.

For every question, I offered a five-item list to elicit their cutting point. This multiple-choice list is commonly used in the literature.²

This certainty equivalent would then be compared to the certainty equivalent for the corresponding uncertain alternative (\$0, \$10) with a 50% probability and (\$10, \$0) with a 50% probability. If the subjects disregarded the other person, then the certainty equivalent would be the same for the above two uncertain alternatives, since the payoffs of the choosers were the same for the two alternatives. However, the certainty equivalents might be different if the subjects cared about their partner's payoff. As explained in section 2, if the subjects disliked ex post payoff differences, then the certainty equivalent for the alternative (\$0, \$10) with a 50% probability and (\$10, \$0) with a 50% probability (G_2) should be less than the certainty equivalent for the alternative (\$0, \$0) with a 50% probability and (\$10, \$10) with a 50% probability (G_1).

As discussed in the introduction, to better distinguish whether an ex ante or ex post fairness view affected their decision making, the subjects were divided

²I didn't ask for free response from subjects, because it is difficult to analyze free-response data and to elicit it in a useful way. I was also worried that subjects may offer irrational or unreasoning answers. A list of options may refine their answers to more sensible ones.

to three groups based on their birthdays. The three groups faced the same questions, but their framing is different.

The first group whose birthdays fall between the 1st and 10th day of the month is the control group. The following is a screenshot of a question for the control group. As can be seen, the framing of the question is the same as the question of the first screenshot.

Alternative: a coin is tossed again.

If it is a Head, you get \$10 and the other person gets \$0.

If it is a Tail, you get \$0 and the other person gets \$10.

Now you are comparing the above alternative with each of following alternatives. Please select all the alternatives which you prefer (to the above alternative).

- Each of you and the other person gets \$2 for sure.
- Each of you and the other person gets \$4 for sure.
- Each of you and the other person gets \$6 for sure.
- Each of you and the other person gets \$8 for sure.
- Each of you and the other person gets \$10 for sure.

The second group of subjects whose birthdays fall between the 11th day and 20th day of the month is the first treatment group. They are also called the ex post group because there was a remark describing the ex post differentials of the alternatives in questions. As shown in the screenshot below, the subjects in this treatment were clearly informed how much more and how much less they earn in each state of the alternative. The remark is designed to render ex post payoff differentials more salient. If the effects of priming are salient and if subjects in the treatment group hold an ex post fairness view, then the certainty equivalent for the ex post unfair alternative (G_2) is less than the certainty equivalent for the same alternative (G_2) for the subjects in the control group.

Alternative: A coin is tossed again.

If it is a Head, you get \$10 and the other person gets \$0. (You earn \$10 MORE than the other person.)

If it is a Tail, you get \$0 and the other person gets \$10. (You earn \$10 LESS than the other person.)

Now you are comparing the above alternative with each of following alternatives. Please select all the alternatives which you prefer (to the above alternative).

- Each of you and the other person gets \$2 for sure.
- Each of you and the other person gets \$4 for sure.
- Each of you and the other person gets \$6 for sure.
- Each of you and the other person gets \$8 for sure.
- Each of you and the other person gets \$10 for sure.

Remaining subjects are the second treatment group or the ex ante group. For this group, the subjects were informed explicitly the expected value of the uncertain alternative in the question. As the following screen shot shows, there was a remark about the ex ante payoff differential. This remark is designed to trigger an ex ante fairness concern. If the effects of priming were salient, the differentials of certainty equivalents of (G_2) were smaller for this treatment group than the differentials of certainty equivalents of (G_2) in the control group.

Alternative: a coin is tossed again.

If it is a Head, you get \$0 and the other person gets \$10.

If it is a Tail, you get \$10 and the other person gets \$0.

Note: *the expected payoffs for you and the other person are the same.*

Now you are comparing the above alternative with each of following alternatives. Please select all the alternatives which you prefer (to the above alternative).

- Each of you and the other person gets \$2.
- Each of you and the other person gets \$4.
- Each of you and the other person gets \$6.
- Each of you and the other person gets \$8.
- Each of you and the other person gets \$10.

3.4 Data Descriptions

3.4.1 Subjects

Invitation emails for participating in the online survey were sent to around 4000 potential subjects by Cornell Lab for Experimental Economics and Decision Research. 144 participants submitted the online survey. On average, they spent 10 minutes doing the survey. Most participants (over 90%) are Cornell staff or students. 122 participants behaved monotonically with certainty equivalents and they are the subjects of our study.³ Table 3-1 summarizes some basic demographic data of these 122 subjects.

I also obtained the following information.

Email. The survey was conducted online remotely and thus email address was used to identify participation, so participants were required to leave their email if they hoped to collect their payments. 97 left their email address and more than 92 subjects were Cornell affiliated.

Highest education level. 24 subjects graduated from high schools, 18 subjects are at the Associate level, 46 subjects obtained a Bachelor's degree and 31 subjects are at the graduate level.

Concern for others. I asked them whether they considered others' payoff when they made decisions for the multiple choice questions and rated their concern for others on a scale from 0 to 10, where 0 represents zero weight on the other person's payoff at all, 5 represents an equal weight on him/herself and the other

³I dismissed the survey results of the remaining 22 subjects who did not behave monotonically, because they may have not paid enough attention when they completed the survey.

person, and 10 represents zero weight on themselves and all weight on others. On average, the weight of concern for others is 5.7. Out of 122 subjects, 8 did not consider others' payoff at all and 13 subjects only considered others' payoff when they make choices.

Ex ante fairness concern. The survey also asked our subjects whether they thought it was fair that everyone had an equal probability of winning, but some of them may end up with nothing due to bad luck. They were asked to rate on a scale of 0 to 10, where 0 represents that they thought it was not fair at all and 10 represents that they thought it was very fair. Five subjects chose 0 and considered it to be completely unfair, while 23 subjects chose 10 and considered it to be very fair.

3.4.2 Compensation

Invitation emails informed the potential subjects that every participant who submitted the survey would be eligible to receive \$3 participation fee plus a 12% chance to win \$25. Two 6-hour time windows were scheduled for participants to collect payments. 59 people came to collect their payments and among them 9 people won \$25.

3.5 Results

In this section, I first present some descriptive results, then I check the consistency of subjects' behavior. The main results of the regression are shown at the end of the section.

3.5.1 Basic Descriptive Results

I elicited CE for 6 questions respectively, which can be sorted into three question pairs (QPs):

QP 1: G_1 $(\$0, \$0)50\%+(\$10, \$10)50\%$ and G_2 $(\$10, \$0)50\%+(\$0, \$10)50\%$;

QP 2: G_1 $(\$10, \$10)50\%+(\$20, \$20)50\%$ and G_2 $(\$10, \$20)50\%+(\$20, \$10)50\%$;

QP 3: G_1 $(\$15, \$15)50\%+(\$30, \$30)50\%$ and G_2 $(\$15, \$30)50\%+(\$30, \$15)50\%$.

Because the options given in the questions are not continuous, only the interval of certainty equivalent can be identified. For example, if the subject chose options with values \$6, \$8, and \$10 when his choice set was a list of options with values \$2, \$4, \$6, \$8 and \$10, then his certainty equivalent lies between \$4 and \$6. I take the midpoint of the interval \$5 as the certainty equivalent.

Table 2 presents the basic results of certainty equivalents of six questions and the results are within subjects. The values in the last column are the differentials of the certainty equivalents of the ex post fair alternatives (G_1) and the certainty equivalents of the ex post unfair alternatives (G_2). The standard errors of differential are less than the standard errors of the Mean CE for G_1 and G_2 because subjects' choice of (G_1) and (G_2) are positively correlated. All the differentials are statistically significant at 10%. Interestingly, the certainty equivalent for (G_1) is slightly less than the certainty equivalent for (G_2). The subjects may be in favor of ex post unfairness. This result does not support the model of inequity aversion. (I also test the upper bound of the ex post unfairness concern. Please see the appendix for more detail.)

3.5.2 Consistency Check

In this section, I check whether the subjects behaved consistently through the survey. In general, if subjects behave randomly, for example, people had higher CE for (G_1) with 50% probability and had higher CE for (G_2) with 50% probability, then behaviors of the subjects were just noise and the results from regression would not be conclusive. Therefore, I check whether the subjects behaved consistently by Fisher's exact test. The p-values of Fisher's exact test strongly support that subjects behaved consistently, and that the data from the survey is credible.

First I compare the mean CE for (G_1) and (G_2) in each PQ for every subject. Figure 3-1 summarizes the directions of their preferences with respect to ex post fairness concern. For example, The figure shows that 12 subjects (10% of 122 subjects) had higher CE for (G_1) in QP1 (($\$0$, $\$0$) with 50% and ($\10, $\$10$) with 50%) than CE for (G_2) in QP1 (($\$0$, $\$10$) with 50% and ($\10, $\$0$) with 50%), while 23 subjects behaved otherwise. Note that a majority of people (71%) had the same CE for (G_1) and (G_2).

Because subjects may behave randomly across question pairs, which may undermine the explanation of these results, I check whether the subjects behaved consistently across question pairs. I count the frequency that a subject behaved consistently. For example, if a subject chose higher CE for G_1 in PQ1 and PQ2, but chose higher CE for G_2 in PQ 3, then the subject had higher CE for G_1 with a probability of two thirds.

If a subject behaved independently across QPs, then I can calculate the predicted frequency table (table 3-3) from the histogram figure (3-1). For example,

the number of subjects who never had higher CE for G_1 is $(1 - 10\%)(1 - 16\%)(1 - 8\%) * 122 = 85$ if subjects behaved randomly across PQs.

Table 3-4 summarizes the observed or actual frequency of subjects' preferences across QPs. For example, 91 subjects never chose a higher CE for G_1 in all the three PQs and 25 subjects chose higher CE for G_1 in only one of the three PQs. 4 subjects strictly preferred ex post fair allocations in all three QPs while 8 subjects had higher CE for G_2 in all three PQs.

Now I compare the predicted preferences and the observed preferences. Table 3-5 shows the p-value of Fisher's exact test. According to the p-value of Fisher's exact test, the second and the third comparisons strongly reject the null hypothesis that subjects behaved randomly across the question pairs. Collectively, table 3-5 indicates a strong rejection of the null hypothesis. Hence, the subjects behaved consistently across the question pairs.

3.5.3 Basic Regression Results

These three QPs are pooled together to maximize the statistical power. Three PQs are pooled to run regression.

$$CE = \alpha_i + \mu I(\text{ex post fair})$$

where the dummy variable $I(\text{ex post fair}) = \begin{cases} 1, & \text{if the allocation is an ex post fair allocation} \\ 0, & \text{otherwise.} \end{cases}$

and where α_i is the fixed effect for QP i and μ measures the magnitude of ex post fairness. Table 3-6 shows the results of the regression.

Column (1) to column (4) show the regression results of OLS. Because the

subjects' choice set is a set of intervals, the subjects' certainty equivalent cannot be precisely identified. Therefore, besides OLS, I also use an interval regression (Steward 1983), which offers the Tobit estimator when the dependent variable falls within an interval. The results of the interval regression are listed in column (5) to column (8). The results of OLS and interval regression are very similar both in magnitude and direction, therefore I can only focus on the results of the interval regression, specifically column (8).

1. Ex post fairness concerns

The value of the ex post fairness coefficient is above $-.51$, which is significant at the 5% level. Surprisingly, subjects had lower certainty equivalent for an ex post fair alternative than that for the corresponding ex ante fair alternative. It seemed that for the experiment, people preferred ex post unfair outcome. This finding contradicts some findings of previous experimental literature that people have fairness concerns or are inequity-averse. This result indicates that subjects preferred equal opportunities to equal outcomes, or subjects may dislike payoff equality and may prefer payoff differences.

2. Concern for others

In the survey, subjects were also asked how much they cared for others when they made the decisions on a scale of 1 to 10. The coefficient is $-.20$ and is statistically significantly at the 1% level. The care for others is negatively correlated with the certainty equivalent. In other words, the more people cared for others, the lower the certainty equivalent they asked for.

3. Economic major or minor

Table 6 shows that when subjects with a background of economics major or minor have higher CE by more than \$2, which is statistically significant at the 1% level. This result is consistent with some studies that students who had a background in economics tend to be more self-interested and care for higher payoff.

4. Gender

The male subjects have slightly lower certainty equivalent than the female subjects slightly more than \$.20, but the difference is not statistically significant. Given that the standard errors are above 0.3, the experiment did not have enough statistical power to reject the null hypothesis.

5. Age

Age is positively correlated with the certainty equivalent. The coefficient is .19 and is statistically significant at the 5% level. Senior subjects chose a higher certainty equivalent than young subjects.

6. Ex ante fairness concern

The ex-ante fairness coefficient is -0.1 and is significant at the 10% level. It means the fairer people thought an equal opportunity was, the lower the certainty equivalent.

7. Priming effects

Table 6 also shows that the ex post differential salient treatment reduces the certainty equivalent by \$.34 while the ex ante differential salient treatment has no effect on CE. The direction of priming is consistent with the inequity aver-

sion model, which predicts that with salient ex post differences inequity-averse people would choose lower certainty equivalents. Meanwhile, the ex ante differences are 0 in all three question pairs, and therefore did not affect people's certainty equivalent. But results of both treatments are not statistically significant. Given that the standard errors are above 0.3, the experiment did not have enough statistical power to reject the null hypothesis.

I test whether the priming effects are significant within or between groups. The following table 3-7 presents the certainty equivalents and differentials within subjects. There are 47 observations in the control group, 37 observations in the ex post treatment group and 38 observations in the ex ante treatment group.

The null hypothesis in each PQ is that the mean of certainty equivalent is the same for the alternatives (G_1) and (G_2) within subjects. Under the null hypothesis the differential of the certainty equivalent between (G_1) and (G_2) is zero. From the table, the null hypothesis cannot be rejected at the 0.05 level of significance. Interestingly, for the ex ante treatment group and the control group the mean of certainty equivalent for (G_2) is greater than the certainty equivalent for (G_1). It seems that the subjects enjoyed ex post payoff differentials when the subjects were reminded the expected value for the alternative. In most cases, the mean of the certainty equivalent is greater for the ex post treatment group than that of the control group. In other words, subjects were not averse to the ex post differentials in the survey.

To check whether priming has any effect on the decision of the subjects, I compare the certainty equivalents for the same alternatives with different framing across the three groups. Table 3-8 reports the certainty equivalents of the

three groups when they face different framing of the same questions. The numbers in the parentheses are standard errors.

The null hypothesis is that the means of certainty equivalents are the same for the same alternative with different framing across the three groups. Table 3-9 summarizes the differentials of the certainty equivalents between the three groups. Again the numbers in the parentheses are the P-values (two-tailed test at the 5% significant level). Panel A shows the results when the subjects chose the certainty equivalent for the alternative which offered (\$0, \$10) and (\$10, \$0) with a 50% probability. In the presence of the salient ex post differential, the certainty equivalent is less than the certainty equivalents in the control group. (This property also holds for Panel B and Panel C). It seems that the subjects were more averse to ex post differentials when they were reminded the ex post differentials. The priming of ex post differences did change the subjects' behavior slightly. However, the null hypothesis cannot be rejected at the 5% significant level. As also can be seen in Panel A, when the subjects were told explicitly that the expected payoffs of the alternative were the same for the two people, the certainty equivalent is higher than that of the control group. This result indicates that the priming of the ex ante differential may affect the subjects' decision. However, it is not statistically significant. This is also true for Panel B. But in Table 3-7 the certainty equivalent for the ex ante treatment group is less than the certainty equivalent in the control group. Table 3-9 also shows that the certainty equivalent in the ex ante treatment group is greater than the certainty equivalent in ex post group, which is consistent with the prediction of our model. However, the result is not statistically significant either.

Three more alternatives were designed as both ex ante and ex post unfair.

The differentials of certainty equivalents between subjects are not statistically significant either. (Tables are omitted.) The above analysis from the tables demonstrates that the priming effects are not statistically significant.

3.5.4 Some Regression Results by Demographic Groups

Now I hope to examine results by demographic group to determine whether the differences across groups are significant. The basic results are shown in Table 3-10. According to Panel A, male subjects asked for higher CEs for ex post fair allocations than female subjects. Panel B shows that subjects with a college education or higher asked for higher CE for ex post fair allocations than less educated subjects. However, due to large standard errors, the experiment is under-powered.

3.6 Discussion and Conclusion

The results from this online survey show that ex post and ex ante fairness views are not very important in people's decision making. While priming on the ex post payoff differentials may induce disutility, the result is not statistically significant. Remarks about expected payoff would not change people's behavior in this social context. Interestingly, the results indicate that people may enjoy ex post differentials when subjects are pooled together.

There are some limitations of my design. 1) The subjects were not paid by the actual value in the hypothetical questions. So the fixed payments may not well incentivize the subjects. 2) Some routine experiments should have be

conducted, so that I could compare my results with those of previous studies. Otherwise, I could not show that the subjects in this study are comparable with other subjects in other experiments. 3) For the hypothetical questions, the stakes were small. Ex ante or ex post fairness concern may matter when the subjects face higher stakes. 4) The results of ex ante and ex post fairness concern are based on a within-subject design. The anchoring effect may affect the results. For example, when the subjects chose from the alternatives they preferred to the option $(\$0, \$10)50\%+(\$10, \$0)50\%$, the question $(\$15, \$30)50\%+(\$30, \$15)50\%$ from the previous screen may affect their answer. The anchoring effect might cause the certainty equivalent for $(\$0, \$10)50\%+(\$10, \$0)50\%$ to be greater than the certainty equivalent for $(\$0, \$0)50\%+(\$10, \$10)50\%$. Even though the anchoring effect cannot directly explain why the certainty equivalent for $(\$15, \$30)50\%+(\$30, \$15)50\%$ is greater than the certainty equivalent for $(\$15, \$15)50\%+(\$30, \$30)50\%$, I cannot rule out the anchoring effect on the subjects' decision. If I have a large enough sample, I can use between subject design to avoid the anchoring effect. 5) Because the intervals listed in the multiple choice question are tight and the standard error is very small, the power of the test is almost 1. On the other hand, regression by demographic studies are not significant because they lack statistical power. In the future, I should design questions where the stakes are higher and increase the sample size. 6) Perhaps the finding that people have a larger CE for G_1 may actually reflect some sort of ex post utilitarian concern. For example, under G_1 the ex post sum of surplus is $(\$20, 1/2; \$0, 1/2)$, whereas under G_2 the ex post surplus is $(\$10, 1)$. It's not clear which of these an ex post utilitarian would prefer, but it's a clear difference between G_1 and G_2 . In my study, I ignored this difference. But it may affect my result and I should explore it further in the future.

I discussed the ex post fairness concern by comparing the certainty equivalents of ex post fair and ex post unfair allocations. I could also ask the subjects to compare the ex post fair and ex post unfair allocations directly. Then I can check if the subjects behave consistently and if they have ex post fairness concern.

One interesting direction for future research would be to test the magnitude of ex post and ex ante fairness concerns based on a specific functional form. For example, let $f(x_A, x_B) = \begin{cases} -(x_A - x_B) & \text{if } x_A > x_B \\ -\beta(x_A - x_B) & \text{if } x_B > x_A \end{cases}$, where $\beta > 1$ measures A's loss aversion. I can consider the following utility function

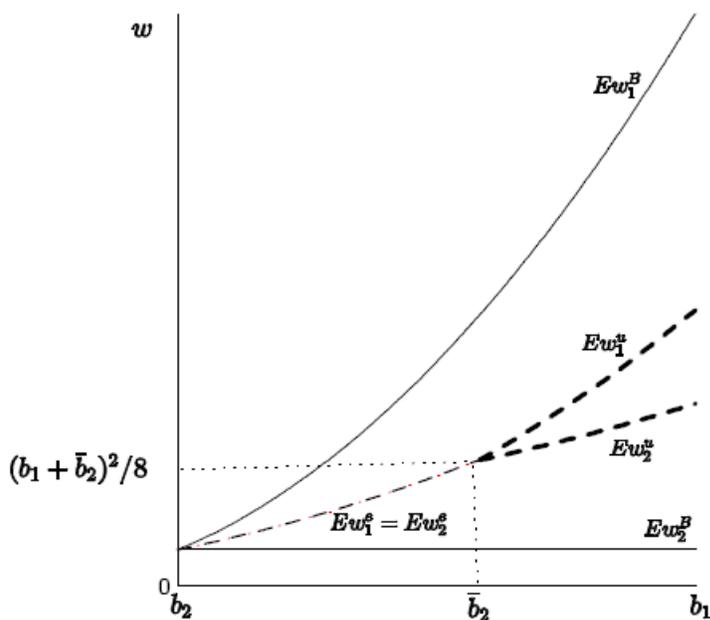
$$Eu_A(x_A) + \gamma_A f(x_A, x_B) + \delta_A f(Ex_A, Ex_B). \quad (3.2)$$

The utility of fairness can be decomposed into two components. The second term in (3.2) measures the ex post fairness concerns and the third term measures the ex ante fairness concerns. If δ_A is significantly different from zero, then an ex ante fairness view plays a role in people's preferences.

APPENDIX A
CHAPTER 1 OF APPENDIX

A.1 Figure 1-1

Figure 1-1 Optimal expected wage when b_1 varies



The two solid line segments describe the optimal expected wage levels with standard agents. The differential between the two solid line segments shows the expected wage differential in the benchmark case. The dashed line denotes the expected wage levels with inequity-averse agents. \bar{b}_2 is the critical value determining when the principal should apply an equal wage contract and an unequal wage contract. When $b_1 < \bar{b}_2$, the optimal wage levels are the same for both agents, which is represented by the thin dashed line. When $b_1 > \bar{b}_2$, the optimal wage levels start to diverge as b_1 increases, which is presented by two thick dashed lines Ew_1^u and Ew_2^u .

A.2 Figure 1-2

Figure 1-2 Optimal effort level when b_1 varies

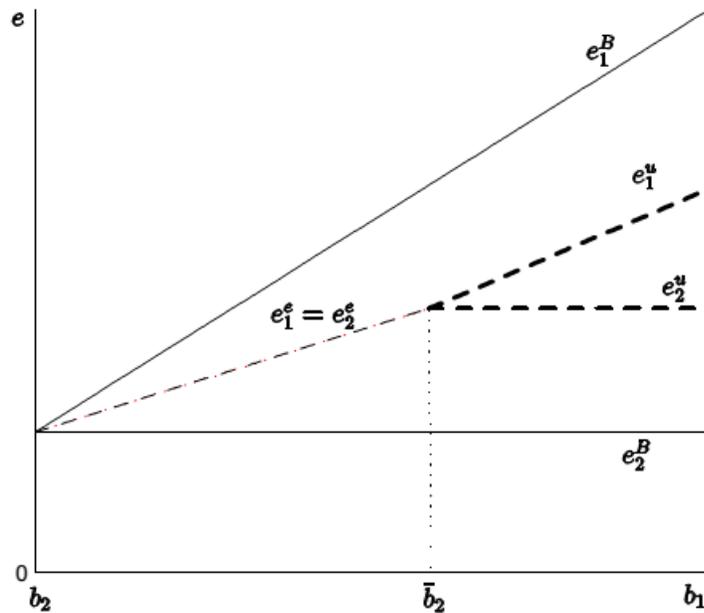


Figure 1-2 describes the optimal effort levels when I fix b_2 , β_1 and α_2 ; and vary b_1 . The upper solid line and the lower flat solid line depict the effort level for the high productivity worker 1 and for the low productivity worker 2 respectively. The dotted line denotes the optimal effort levels with inequity aversion. When b_1 is small, or $b_2 < b_1 < \bar{b}_2$, the equal effort contract is optimal, which is described by the thin dotted segment. When b_1 goes above the threshold \bar{b}_2 , the unequal effort contract is optimal, which is described by the two thick dotted segments. It can be clearly seen that the dashed line lies in between the two solid lines, which implies that the effort levels are more compressed with inequity-averse agents than with standard agents.

A.3 Figure 1-3

Figure 1-3 Optimal effort level when α_2 varies

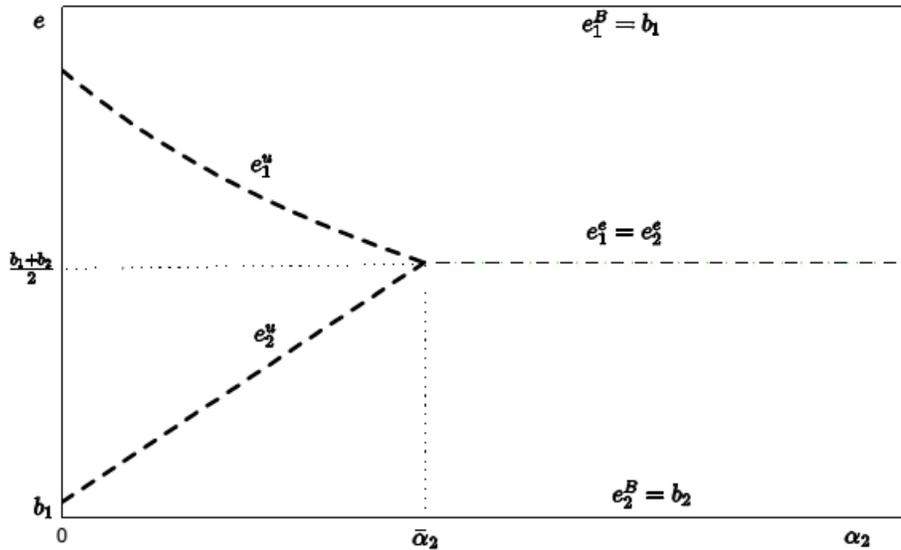
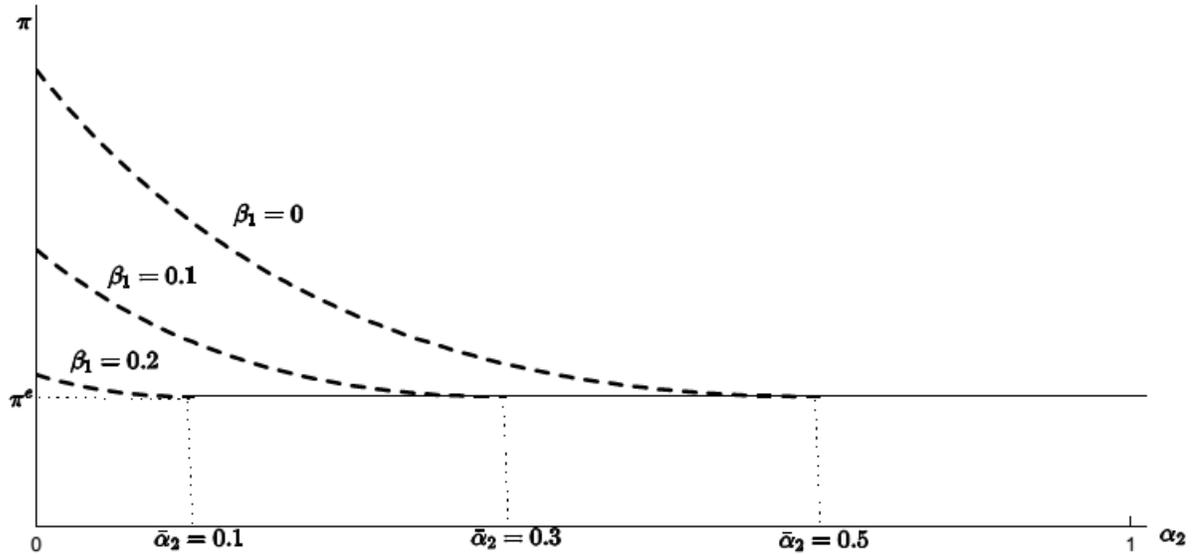


Figure 1-3 depicts how α_2 affects the optimal effort levels. I fix b_1 , b_2 , and β_1 and vary α_2 . The upper and lower solid straight lines represent the optimal effort levels in the benchmark model. When $\alpha_2 < \bar{\alpha}_2$ such that $b_1 > \bar{b}_2$ (i.e., the efficiency motive is strong), the optimal efforts are different for two agents. The thick dashed lines represent the optimal unequal effort levels, which can be implemented by an unequal wage contract. When $\alpha_2 > \bar{\alpha}_2$ such that $b_1 < \bar{b}_2$ (i.e., the equality motive is strong), the optimal effort levels are the same for the two agents.

A.4 Figure 1-4

Figure 1-4 Maximum profit level when β_1 and α_2 varies



The above figure shows how α_2 affects profit levels for different levels of β_1 . The three curves represent three cases $\beta_1 = 0$, $\beta_1 = 0.1$ and $\beta_1 = 0.2$ respectively. As can be seen for the three curves, the profit first falls then remains the same. For example, if $\beta_1 = 0$, the profit decreases with α_2 when $\alpha_2 < \bar{\alpha}_2 = 0.1$ ($b_1 > \bar{b}_2$), where the optimal wage contract is an unequal wage contract. The profit then stays constant when $\alpha_2 > \bar{\alpha}_2$ ($b_1 < \bar{b}_2$). In addition, for the same value of α_2 , the profit decreases with β_1 .

A.5 Proof of Lemma 1-2

Lemma 1-2 *Individual rationality constraints $IR_i, i = 1, 2$ are binding for both agents.*

Proof: I derive optimal contracts under three different cases $Ew_1 > Ew_2$, $Ew_1 = Ew_2$, and $Ew_1 < Ew_2$.

1. If $Ew_1 > Ew_2$, (IR_1), expected utility functions for agents become

$$EU_1 = (1 - \beta_1)Ew_1 + \beta_1 Ew_2 - c(e_1) \geq 0. \quad (\text{A.1})$$

$$EU_2 = -\alpha_2 Ew_1 + (1 + \alpha_2)Ew_2 - c(e_2) \geq 0. \quad (\text{A.2})$$

The above two inequalities and $1 + \alpha_2 - \beta_1 > 0$ imply

$$Ew_2 \geq \frac{\alpha_2 c(e_1) + (1 - \beta_1)c(e_2)}{1 + \alpha_2 - \beta_1}. \quad (\text{A.3})$$

(A.1) and $1 - \beta_1 > 0$ imply $Ew_1 \geq \frac{1}{1 - \beta_1}(c(e_1) - \beta_1 Ew_2)$. Adding Ew_2 on both sides yields

$$Ew_1 + Ew_2 \geq \frac{1 - 2\beta_1}{1 - \beta_1} Ew_2 + \frac{1}{1 - \beta_1} c(e_1).$$

Substituting (A.3) into the above inequality and $1 - 2\beta_1 > 0$ yields

$$Ew_1 + Ew_2 \geq \frac{2\alpha_2 + 1}{\alpha_2 - \beta_1 + 1} c(e_1) + \frac{1 - 2\beta_1}{1 + \alpha_2 - \beta_1} c(e_2). \quad (\text{A.4})$$

Since the principal chooses to maximize the profits, in other words, minimize the costs, here, wages. Hence (A.4) must be binding, i.e., individual rationality constraints are binding for both agents:

$$Ew_1 = \frac{1}{1 + \alpha_2 - \beta_1} ((1 + \alpha_2)c(e_1) - \beta_1 c(e_2)). \quad (\text{A.5})$$

$$Ew_2 \geq \frac{\alpha_2 c(e_1) + (1 - \beta_1)c(e_2)}{1 + \alpha_2 - \beta_1}. \quad (\text{A.6})$$

2. If $Ew_1 = Ew_2$, agents' utility functions are simplified to $EU_1 = Ew_1 - c(e_1) \geq 0$ and $EU_2 = Ew_2 - c(e_2) \geq 0$. Since the cost is minimized in the optimal contract, the above two *IR* constraints are binding. It follows that $Ew_1 = c(e_1)$, $Ew_2 = c(e_2)$ and so $c(e_1) = c(e_2)$.
3. If $Ew_1 < Ew_2$, the analysis is identical to the case (1), i.e., *IR* constraints are binding and

$$\begin{cases} Ew_1 = \frac{1}{1+\alpha_1-\beta_2}(\alpha_1 c(e_2) + (1 - \beta_2)c(e_1)) \\ Ew_2 = \frac{1}{1+\alpha_1-\beta_2}((1 + \alpha_1)c(e_2) - \beta_2 c(e_1)) \end{cases}.$$

A.6 Proof of Corollary 1-1

Corollary 1-1 $Ew_1 \gtrless Ew_2$, if and only if $e_1 \gtrless e_2$.

Proof: From the lemma 1-2,

1. If $Ew_1 > Ew_2$, i.e., $\frac{1}{1+\alpha_2-\beta_1}((1+\alpha_2)c(e_1)-\beta_1 c(e_2)) > \frac{1}{1+\alpha_2-\beta_1}(\alpha_2 c(e_1)+(1-\beta_1)c(e_2))$, which implies $e_1 > e_2$.
2. If $Ew_1 = Ew_2$, i.e., $c(e_1) = c(e_2)$ which implies $e_1 = e_2$.
3. If $Ew_1 < Ew_2$, i.e., $\frac{1}{1+\alpha_1-\beta_2}(\alpha_1 c(e_2)+(1-\beta_2)c(e_1)) < \frac{1}{1+\alpha_1-\beta_2}((1+\alpha_1)c(e_2)-\beta_2 c(e_1))$, which implies $e_1 < e_2$.

A.7 Proof of Lemma 1-3 and Lemma 1-4 and Proposition 1-2

Proposition 1-2 A principal contracts with two inequity-averse agents with heterogeneous productivities $b_1 > b_2$. Let $\bar{b}_2 = \frac{1+2\alpha_2}{1-2\beta_1}b_2$.

1. When $b_1 \leq \bar{b}_2$, the optimal effort levels are the same and the principal chooses an equal wage contract.

2. When $b_1 > \bar{b}_2$, the optimal effort levels are different and the principal chooses an unequal wage contract.

In both of the above cases, the principal earns less profit than in the benchmark case with standard agents. Moreover, effort levels and wage levels are more compressed than those of the benchmark.

Proof:

1. If $Ew_1 > Ew_2$, substituting (A.5) and (A.6) into the principal's maximization problem yields

$$\max_{e_1, e_2} b_1 e_1 + b_2 e_2 - \frac{1+2\alpha_2}{1+\alpha_2-\beta_1} c(e_1) - \frac{1-2\beta_1}{1+\alpha_2-\beta_1} c(e_2).$$

Taking FOCs with respect to e_1 and e_2 , I can derive the optimal efforts:

$e_1^* = \frac{1+\alpha_2-\beta_1}{1+2\alpha_2} b_1$ and $e_2^* = \frac{1+\alpha_2-\beta_1}{1-2\beta_1} b_2$. It follows from Corollary 1-1 $e_1^* > e_2^*$, and so $b_1 > \frac{1+2\alpha_2}{1-2\beta_1} b_2$. This is the parameter constraint that the unequal wage contract is feasible for the principal.

A linear contract $w_i(x_i) = a_i x_i + t_i$ can implement the optimal effort levels.

$$\text{Since } \begin{cases} e_1 = (1 - \beta_1) a_1 b_1 \\ e_2 = (1 + \alpha_2) a_2 b_2 \end{cases}, \text{ it follows } \begin{cases} a_1 = \frac{1+\alpha_2-\beta_1}{(1+2\alpha_2)(1-\beta_1)} \\ a_2 = \frac{1+\alpha_2-\beta_1}{(1-2\beta_1)(1+\alpha_2)} \end{cases}.$$

The firm obtains profit: $\Pi_1 = \frac{1+\alpha_2-\beta_1}{2} \left(\frac{(b_1)^2}{1+2\alpha_2} + \frac{(b_2)^2}{1-2\beta_1} \right)$.

(A.5) and (A.6) imply

$$\begin{cases} Ew_1^u = \frac{1}{2} \frac{(1+\alpha_2-\beta_1)(1+\alpha_2)}{(1+2\alpha_2)^2} (b_1)^2 - \frac{1}{2} \frac{(1+\alpha_2-\beta_1)\beta_1}{(1-2\beta_1)^2} (b_2)^2 \\ Ew_2^u = \frac{1}{2} \frac{(1+\alpha_2-\beta_1)\alpha_2}{(1+2\alpha_2)^2} (b_1)^2 + \frac{1}{2} \frac{(1+\alpha_2-\beta_1)(1-\beta_1)}{(1-2\beta_1)^2} (b_2)^2 \end{cases}.$$

If $Ew_1 = Ew_2$, Corollary 1-1 shows $e_1 = e_2$. The principal's maximization problem becomes $\max_{e_1, e_2} b_1 e_1 + b_2 e_2 - c(e_1) - c(e_2) \quad s.t \quad e_1 = e_2$.

The solution is $e_1^e = e_2^e = \frac{b_1 + b_2}{2}$ and the profit $\Pi^e = \frac{1}{4} (b_1 + b_2)^2$.

The optimal expected wages are $Ew_1^e = Ew_2^e = \frac{1}{2} \left(\frac{b_1 + b_2}{2} \right)^2 = \frac{(b_1 + b_2)^2}{8}$.

3 If $Ew_1 < Ew_2$, the analysis is identical to the first case, and so

$$\begin{cases} e_1^* = \frac{1 + \alpha_1 - \beta_2}{1 - 2\beta_2} b_1 \\ e_2^* = \frac{1 + \alpha_1 - \beta_2}{1 + 2\alpha_1} b_2 \end{cases}.$$

It follows from Corollary 1-1 $e_1^* < e_2^*$ and $b_1 < \frac{1 - 2\beta_2}{1 + 2\alpha_1} b_2 (< b_2)$.

If $\beta_2 > -\alpha_1$, $b_1 < \frac{1 - 2\beta_2}{1 + 2\alpha_1} b_2 < b_2$ contradicts $b_1 > b_2$. Therefore, when agent 1 is more productive than agent 2, the principal would never offer higher wage to agent 2 than agent 1.

When I compare the profits in three cases: 1) benchmark 2) the optimal wages are equal, 3) the optimal wages are unequal. $\Pi^u - \Pi^e = \frac{1}{4} \frac{((1 + 2\alpha_2)b_2 - (1 - 2\beta_1)b_1)^2}{(1 - 2\beta_1)(2\alpha_2 + 1)} \geq 0$ if $b_1 > \frac{1 + 2\alpha_2}{1 - 2\beta_1} b_2$. $\Pi^B - \Pi^u = \frac{1}{2} \frac{\alpha_2 + \beta_1}{(2\alpha_2 + 1)(1 - 2\beta_1)} \left((1 - 2\beta_1)b_1^2 - (1 + 2\alpha_2)b_2^2 \right)$ implies that $\Pi^B - \Pi^u > 0$ if and only if $b_1^2 > \frac{1 - 2\beta_2}{1 + 2\alpha_1} b_2^2$.

A.8 Proof of Proposition 1-5

Proposition 1-5 *There exists \bar{b}_1 such that for any $b_1 > \bar{b}_1$, the principal would rather hire a high productivity worker instead of both workers, holding all other parameters fixed.*

Proof: Let's explore the conditions when hiring a high productivity worker is more profitable than hiring both workers: $\Pi^1 - \Pi^u = \frac{b_1^2}{2} - \frac{1+\alpha_2-\beta_1}{2} \left(\frac{(b_1)^2}{1+2\alpha_2} + \frac{(b_2)^2}{1-2\beta_1} \right) > 0$. Rearrange and simplify the above inequality, I obtain $b_1 > \sqrt{\frac{(1+\alpha_2-\beta_1)(2\alpha_2+1)}{(\alpha_2+\beta_1)(1-2\beta_1)}} b_2$

$$\text{Let } \bar{b}_1 = \sqrt{\frac{(1+\alpha_2-\beta_1)(2\alpha_2+1)}{(\alpha_2+\beta_1)(1-2\beta_1)}} b_2.$$

Note a special case, if $\frac{(1+\alpha_2-\beta_1)(2\alpha_2+1)}{(\alpha_2+\beta_1)(1-2\beta_1)} < \left(\frac{1+2\alpha_2}{1-2\beta_1}\right)^2$ or $\frac{1+\alpha_2-\beta_1}{\alpha_2+\beta_1} > \frac{1+2\alpha_2}{1-2\beta_1}$, then $\bar{b}_1 < \frac{1+2\alpha_2}{1-2\beta_1} b_2$, which means that principal will only hire a high productivity agent for any $b_2 > b_1$.

A.9 Proof of Corollary 1-3

Corollary 1-3 Suppose the synergy effect increases the marginal productivity of each

worker, i.e., $x_i = (b_i + se_j)e_i + \epsilon_i$. Let $m = \frac{2\alpha_2+1}{\alpha_2-\beta_1+1}$ and $n = \frac{1-2\beta_1}{1+\alpha_2-\beta_1}$,

1 If $b_1 > \frac{2s-n}{2s-m} b_2$, the unequal wage contract is optimal and $e_1 = \frac{nb_1+2sb_2}{mn-4s^2}$ and $e_2 = \frac{2sb_1+mb_2}{mn-4s^2}$.

2 If $b_1 < \frac{2s-n}{2s-m} b_2$, the equal wage contract is optimal, and $e_1 = e_2 = \frac{b_1+b_2}{2} + 2s$.

Proof: The logic is identical to that for Proposition 1-2.

$$Ew_1 = \frac{1}{1+\alpha_1-\beta_2} (\alpha_1 c(e_2) + (1-\beta_2)c(e_1)) \text{ and } Ew_2 = \frac{1}{1+\alpha_1-\beta_2} ((1+\alpha_1)c(e_2) - \beta_2 c(e_1))$$

The principal's problem becomes

$$\max_{e_1, e_2} (b_1 + se_2)e_1 + (b_2 + se_1)e_2 - \frac{2\alpha_2+1}{1+\alpha_2-\beta_1} c(e_1) - \frac{1-2\beta_1}{1+\alpha_2-\beta_1} c(e_2).$$

Let $m = \frac{2\alpha_2+1}{\alpha_2-\beta_1+1}$ and $n = \frac{1-2\beta_1}{1+\alpha_2-\beta_1}$. (It is easy to show that $m > 1 > n$.)

The principal's problem simplifies to $\max_{e_1, e_2} b_1 e_1 + b_2 e_2 + 2s e_1 e_2 - \frac{m(e_1)^2}{2} - \frac{n(e_2)^2}{2}$.¹

Solving it yields $e_1 = \frac{nb_1 + 2sb_2}{mn - 4s^2}$ and $e_2 = \frac{2sb_1 + mb_2}{mn - 4s^2}$. Lemma 1-3 shows that $e_1 > e_2$. and so $b_1 > \frac{2s-n}{2s-m} b_2$.

Therefore, if $b_1 > \frac{2s-n}{2s-m} b_2$, the unequal wage contract is optimal.

If $b_1 < \frac{2s-n}{2s-m} b_2$, the equal wage contract is optimal.

A.10 Corollary 1-4

Corollary 1-4 *Suppose the synergy effect increases level of productions among workers, i.e., $x_i = b_i e_i + S e_j + \epsilon_i$ $i = 1, 2$. When $b_1 + S > \frac{1+2\alpha_2}{1-2\beta_1}(b_2 + S)$ and $S > S_1$, the principal will implement an unequal wage contract which implements efforts $\begin{cases} e_1 = \frac{1+\alpha_2-\beta_1}{1+2\alpha_2}(b_1 + S) \\ e_2 = \frac{1+\alpha_2-\beta_1}{1-2\beta_1}(b_2 + S) \end{cases}$ and $S > S_2$, obtains profit $\Pi_1 = \frac{1+\alpha_2-\beta_1}{2} \left(\frac{(b_1+S)^2}{1+2\alpha_2} + \frac{(b_2+S)^2}{1-2\beta_1} \right)$; 2. when $b_2 + S \leq b_1 + S \leq \frac{1+2\alpha_2}{1-2\beta_1}(b_2 + S)$ and the principal will implement an equal wage contract which implements efforts: $e_1 = e_2 = \frac{b_1+b_2}{2} + S$ and obtains profit $\Pi_2 = \frac{1}{4} (b_1 + b_2 + 2S)^2$. 3. When $S < S_2$, the firm will have two independent contractors and implement efforts $e_i = b_i, i = 1, 2$ and obtains $\Pi_0 = \frac{b_1^2}{2} + \frac{b_2^2}{2}$.*

The proof is Omitted since the analysis is identical to the proof of proposition 1-2.

¹To guarantee existence of interior solution, the Hessian matrix must be negative definite. I assume that $mn - 4s^2 > 0$.

A.11 Proof of Lemma 1-5

Lemma 1-5 *If agents are inequity averse to differences in cost of efforts, wage and effort levels are more compressed than in the benchmark case.*

Proof: If agents are inequity averse to differences in cost of efforts, utility function becomes

$$U_i = w_i - c(e_i) - \alpha_i \max(c(e_i) - c(e_j), 0) - \beta_i \max(c(e_j) - c(e_i), 0).$$

Similar analysis to Lemma 1-2 can be applied, so $(IR_i) i = 1, 2$ are binding.

If $e_1 > e_2$, from $EU_1 = Ew_1 - c(e_1) - \alpha_1(c(e_1) - c(e_2)) = 0$ it follows $Ew_1 = (1 + \alpha_1)c(e_1) - \alpha_1c(e_2)$, From $EU_2 = Ew_2 - c(e_2) - \beta_2(c(e_1) - c(e_2)) = 0$ it follows $Ew_2 = (1 - \beta_2)c(e_2) + \beta_2c(e_1)$

The principal's objective function becomes

$$\begin{aligned} & \max_{e_1, e_2} b_1 e_1 + b_2 e_2 - (1 + \alpha_1)c(e_1) + \alpha_1 c(e_2) - (1 - \beta_2)c(e_2) - \beta_2 c(e_1) \\ & \text{or } \max_{e_1, e_2} b_1 e_1 + b_2 e_2 - (1 + \alpha_1 + \beta_2)c(e_1) - (1 - \alpha_1 - \beta_2)c(e_2) \end{aligned}$$

$$\text{Solving this solution yields } \begin{cases} c'(e_1) = \frac{b_1}{(1 + \alpha_1 + \beta_2)} \\ c'(e_2) = \frac{b_2}{(1 - \alpha_1 - \beta_2)} \end{cases}.$$

Since $e_1 > e_2$, it follows $c'(e_1) > c'(e_2)$, i.e., $\frac{b_1}{1 + \alpha_1 + \beta_2} > \frac{b_2}{1 - \alpha_1 - \beta_2}$.

$$\text{So if } b_1 > \frac{1 + \alpha_1 + \beta_2}{1 - \alpha_1 - \beta_2} b_2, \text{ then } \begin{cases} e_1^C = \frac{b_1}{1 + \alpha_1 + \beta_2} \\ e_2^C = \frac{b_2}{1 - \alpha_1 - \beta_2} \end{cases}.$$

It is straightforward to see that $e_2^B < e_2^C < e_1^C < e_1^B$ or $\frac{e_1^C - e_2^C}{e_1^B - e_2^B} < 1$, i.e., the efforts are more compressed than the benchmark case.

If $e_1 < e_2$, from $EU_1 = Ew_1 - c(e_1) - \beta_1(c(e_2) - c(e_1)) = 0$, it follows $Ew_1 = (1 - \beta_1)c(e_1) + \beta_1c(e_2)$. From $EU_2 = Ew_2 - c(e_2) - \alpha_2(c(e_2) - c(e_1)) = 0$, it follows $Ew_2 = (1 + \alpha_2)c(e_2) - \alpha_2c(e_1)$.

The principal's objective function becomes

$$\max_{e_1, e_2} b_1e_1 + b_2e_2 - (1 - \beta_1)c(e_1) - \beta_1c(e_2) - (1 + \alpha_2)c(e_2) + \alpha_2c(e_1)$$

or $\max_{e_1, e_2} b_1e_1 + b_2e_2 - (1 - \alpha_2 - \beta_1)c(e_1) - (1 + \alpha_2 + \beta_1)c(e_2)$

If $e_1 < e_2$, then $c'(e_1) < c'(e_2)$, i.e., $b_1 < \frac{(1 - \alpha_2 - \beta_1)b_2}{1 + \alpha_2 + \beta_1}$ contradicts $b_1 > b_2$.

Therefore, if $b_1 < \frac{1 + \alpha_1 + \beta_2}{1 - \alpha_1 - \beta_2}b_2$, $e_1 = e_2 = \frac{b_1 + b_2}{2}$, and $Ew_1 = Ew_2 = \frac{1}{8}(b_1 + b_2)^2$

Under both cases, the wages and efforts are more compressed.

APPENDIX B
CHAPTER 2 OF APPENDIX

B.1 Proof of Proposition 2-1

Proposition 2-1 *The inter-rank prize is the same for the previous $N - 1$ rounds*

$$w_{k+1} - w_k = \left(1 - \frac{1}{n}\right)(w_N - w_{N-1}) + c^*, \text{ where } k = 1, 2, \dots, N - 2.$$

The inter-rank prize for the last round is greater than the inter-rank prize for the previous rounds

$$w_2 - w_1 = w_3 - w_2 = w_4 - w_3 = \dots = w_{N-1} - w_{N-2} < w_N - w_{N-1}.$$

Proof: To induce the same level effort level in each round, the prize for winning each round should be the same. Winning round k , except for the last round N , involves two prizes. One is the direct monetary reward from winning round k , *i.e.*, $w_k - w_{k-1}$. The other prize is the expected value from the possibility of winning subsequent rounds. According to the above reasoning, the prize from winning round $N - 1$ is the direct monetary prize, $w_{N-1} - w_{N-2}$, and the expected value of winning round N , $\frac{1}{n}(w_N - w_{N-1}) - c^*$. The sum of these two prizes equals the prize from winning round N ,

$$w_{N-1} - w_{N-2} + \frac{1}{n}(w_N - w_{N-1}) - c^* = w_N - w_{N-1}.$$

Expressing the direct prize from winning round $N - 1$ in terms of the prize from winning round N , we have

$$w_{N-1} - w_{N-2} = \left(1 - \frac{1}{n}\right)(w_N - w_{N-1}) + c^*. \tag{B.1}$$

Analogously, the prize from winning round $N-2$ equals the prize from winning round N ,

$$w_{N-2} - w_{N-3} + \frac{1}{n}[w_{N-1} - w_{N-2} + \frac{1}{n}(w_N - w_{N-1}) - c^*] - c^* = w_N - w_{N-1}.$$

Substituting (B.1) into the above equation yields

$$w_{N-2} - w_{N-3} = (1 - \frac{1}{n})(w_N - w_{N-1}) + c^*.$$

Repeating this procedure yields the following equation that

$$w_2 - w_1 = \dots = w_{N-2} - w_{N-3} = (1 - \frac{1}{n})(w_N - w_{N-1}) + c^*. \quad (\text{B.2})$$

Hence, the direct monetary prize is the same for all the rounds except the last round. If we compare the direct monetary prize of the last round with the direct monetary prize of the previous rounds:

$$(w_N - w_{N-1}) - (w_k - w_{k-1}) = \frac{1}{n}(w_N - w_{N-1}) - c^*, \text{ where } k = 1, 2, \dots, N-1. \quad (\text{B.3})$$

It is obvious that $\frac{1}{n}(w_N - w_{N-1}) - c^* > 0$. Otherwise, workers have no incentive to exert effort in the last round.

Therefore, it follows (B.2) and (B.3) that the prize from winning increases constantly with a sharp increase in the final prize, which is constant with the result in Rosen (1986).

B.2 Proof of Lemma 2-1

Lemma 2-1 Let $\beta = \frac{1}{n} - \frac{p'(e^*)}{c'(e^*)}c^*$. $V_N = w_N$; $V_k = \beta V_{k+1} + (1-\beta)w_k$, where $k = 1, \dots, N-1$.

Proof: When only one round remains to be played, the worker chooses his effort e_N , the probability for the worker to enter the final round is $p(e_N|e_{-N})$. If

he wins the final round, his payoff is $w_N - c(e_N)$. If he loses the final round, his payoff is $w_{N-1} - c(e_N)$, therefore, the Bellman equation for worker in round N is

$$V_{N-1} = \max_{e_N} \{(1 - p(e_N|e_{-N}))w_{N-1} + p(e_N|e_{-N})w_N - c(e_N)\}.$$

First order condition yields $p'(e_N|e_{-N})(w_N - w_{N-1}) = c'(e_N)$. Second order condition $p''(e_N|e_{-N})\frac{c'(e_N)}{p'(e_N|e_{-N})} - c''(e_N) < 0$ is always satisfied because $p''() < 0$, $c''() > 0$, $p'() > 0$ and $c'() > 0$. As discussed above, a symmetric equilibrium yields $e_N = e_{-N} = e^*$, $c(e^*) = c^*$ and $p(e_N|e_{-N}) = \frac{1}{n}$. So the inter-rank spread is $w_N - w_{N-1} = \frac{c'(e^*)}{p'(e^*)}$. Plugging it to the V_{N-1} , we obtain

$$V_{N-1} = (1 - \frac{1}{n})w_{N-1} + \frac{1}{n}w_N - c^* = w_{N-1} + \frac{1}{n}\frac{c'(e^*)}{p'(e^*)} - c^*. \quad (\text{B.4})$$

Let $\beta = \frac{1}{n} - \frac{p'(e^*)}{c'(e^*)}c^*$. It follows that $c^* = (\frac{1}{n} - \beta)\frac{c'(e^*)}{p'(e^*)} = (\frac{1}{n} - \beta)(w_N - w_{N-1})$.

Therefore, we can rewrite (B.4) as

$$V_{N-1} = \beta w_N + (1 - \beta)w_{N-1} = \beta V_N + (1 - \beta)w_{N-1}.$$

Analogously,

$$V_k = \max_{e_{k+1}} \{(1 - p(e_{k+1}|e_{-(k+1)}))w_k + p(e_{k+1}|e_{-(k+1)})V_{k+1} - c(e_{k+1})\}.$$

First order condition yields $p'(e_{k+1}|e_{-(k+1)})(V_{k+1} - w_k) = c'(e_{k+1})$. Second order condition $p''(e_{k+1}|e_{-(k+1)})\frac{c'(e_{k+1})}{p'(e_{k+1}|e_{-(k+1)})} - c''(e_{k+1}) < 0$ is always satisfied because $p''() < 0$, $c''() > 0$, $p'() > 0$ and $c'() > 0$. Rearranging first order condition and substituting $e_{k+1} = e^*$ yields $V_{k+1} - w_k = \frac{c'(e^*)}{p'(e^*)}$.

Substituting $c^* (\equiv (\frac{1}{n} - \beta)\frac{c'(e^*)}{p'(e^*)}) = (\frac{1}{n} - \beta)(V_{k+1} - w_k)$ to $V_k = (1 - \frac{1}{n})w_k + \frac{1}{n}V_{k+1} - c^*$ yields

$$V_k = (1 - \frac{1}{n})w_k + \frac{1}{n}V_{k+1} - (\frac{1}{n} - \beta)(V_{k+1} - w_k) = \beta V_{k+1} + (1 - \beta)w_k.$$

B.3 Proof of Proposition 2-2

Proposition 2-2 *The inter-rank prize is the same for the previous $N - 1$ rounds, i.e.,*

$$w_k - w_{k-1} = \left(1 - \frac{1}{n}\right) \frac{c'(e^*)}{p'(e^*)} + c^*, \quad k = 1, 2, \dots, N - 1$$

The inter-rank prize for the last round is $w_N - w_{N-1} = \frac{c'(e^)}{p'(e^*)}$.*

$$(w_N - w_{N-1}) - (w_k - w_{k-1}) = \frac{1}{n} \frac{c'(e^*)}{p'(e^*)} - c^* > 0 \text{ since } \beta = \frac{1}{n} - \frac{p'(e^*)}{c'(e^*)} c^* > 0.$$

Therefore the inter-rank prizes are the same for previous $N - 1$ rounds with an increase in round N .

$$w_2 - w_1 = w_3 - w_2 = w_4 - w_3 = \dots = w_{N-1} - w_{N-2} < w_N - w_{N-1}.$$

Proof: As proved in Lemma 2-2, $w_N - w_{N-1} = \frac{c'(e^*)}{p'(e^*)}$ and $V_{N-1} - w_{N-2} = \frac{c'(e^*)}{p'(e^*)}$.

Substituting (B.4) to the above equation, we obtain $w_{N-1} - w_{N-2} = \left(1 - \frac{1}{n}\right) \frac{c'(e^*)}{p'(e^*)} + c^*$.

From Lemma 2-1, $V_{N-1} = \beta w_N + (1 - \beta)w_{N-1}$ and $V_k = \beta V_{k+1} + (1 - \beta)w_k$, $k = 1, \dots, N - 1$.

By iteration, $V_{N-2} = \beta^2(w_N - w_{N-1}) + \beta(w_{N-1} - w_{N-2}) + w_{N-2}$. Repeating the iteration, we derive

$$V_{k+1} = \beta^{N-k-1}(w_N - w_{N-1}) + \beta^{N-k+1}(w_{N-1} - w_{N-2}) + \dots + \beta(w_{k+2} - w_{k+1}) + w_{k+1}. \quad (\text{B.5})$$

$$V_k = \beta^{N-k}(w_N - w_{N-1}) + \beta^{N-k+1}(w_{N-1} - w_{N-2}) + \dots + \beta(w_{k+1} - w_k) + w_k. \quad (\text{B.6})$$

It follows from (B.6) $- w_{k-1} - \beta * (\text{B.5}) - w_k$, we have $V_k - w_{k-1} - \beta(V_{k+1} - w_k) = w_k - w_{k-1}$. Substituting $V_{k+1} - w_k = V_k - w_{k-1} = \frac{c'(e^*)}{p'(e^*)}$ and $\beta = \frac{1}{n} - \frac{p'(e^*)}{c'(e^*)} c^*$ into the above equation, we obtain $w_k - w_{k-1} = \left(1 - \frac{1}{n}\right) \frac{c'(e^*)}{p'(e^*)} + c^*$. $k = 1, 2, \dots, N - 1$.

B.4 Proof of Lemma 2-4

Lemma 2-4 $V_k = \beta_{k+1}V_{k+1} + (1 - \beta_{k+1})w_k$, where

$$\beta_{k+1} = \frac{1}{n} - \frac{c(e_{k+1}^*)p'(e_{k+1}^*)}{c'(e_{k+1}^*)}, k = 1, 2, \dots, N-1.$$

Proof:

$$V_N = w_N; V_{N-1} = \max_{e_N} \{p(e_N|e_{-N})V_N + (1 - p(e_N|e_{-N}))w_{N-1} - c(e_N)\}$$

$$FOC \Rightarrow p'(e_N)(w_N - w_{N-1}) - c'(e_N) = 0.$$

$$\Rightarrow y_N \equiv w_N - w_{N-1} = \frac{c'(e_N^*)}{p'(e_N^*)} \quad (\text{B.7})$$

$$c_N^* = (w_N - w_{N-1}) \frac{p'(e_N^*)c_N^*}{c'(e_N^*)}. \text{ Let } \beta_N = \frac{1}{n} - \frac{p'(e_N^*)c(e_N^*)}{c'(e_N^*)}.$$

$$c_N^* = \left(\frac{1}{n} - \beta_N\right)(w_N - w_{N-1}) \quad (\text{B.8})$$

$$V_{N-1} = \frac{1}{n}w_N + \left(1 - \frac{1}{n}\right)w_{N-1} - c_N^* \quad (\text{B.9})$$

Substituting (B.8) into (B.9) yields

$$V_{N-1} = \frac{1}{n}(w_N + w_{N-1}) - \left(\frac{1}{n} - \beta_N\right)(w_N - w_{N-1}) = \beta_N w_N + (1 - \beta_N)w_{N-1}.$$

Analogously, $V_k = \max_{e_{k+1}} \{p(e_{k+1}|e_{-(k+1)})V_{k+1} + (1 - p(e_{k+1}|e_{-(k+1)}))w_k - c(e_{k+1})\}$.

FOC yields $p'(e_{k+1}^*)(V_{k+1} - w_k) - c'(e_{k+1}^*) = 0$

$$\Rightarrow V_{k+1} - w_k = \frac{c'(e_{k+1}^*)}{p'(e_{k+1}^*)} \quad (\text{B.10})$$

$$\Rightarrow c(e_{k+1}^*) = \frac{p'(e_{k+1}^*)c(e_{k+1}^*)}{c'(e_{k+1}^*)}(V_{k+1} - w_k)$$

Let $\beta_{k+1} = \frac{1}{n} - \frac{p'(e_{k+1}^*)c(e_{k+1}^*)}{c'(e_{k+1}^*)}$. So $c(e_{k+1}^*) = \left(\frac{1}{n} - \beta_{k+1}\right)(V_{k+1} - w_k)$ and $V_k = \frac{1}{n}V_{k+1} + \left(1 - \frac{1}{n}\right)w_k - c(e_{k+1}^*)$. It follows that $V_k = \beta_{k+1}V_{k+1} + (1 - \beta_{k+1})w_k$.

B.5 Proof of Corollary 2-1

Corollary 2-1

$$w_N - w_{N-1} = \frac{c'(e_N^*)}{p'(e_N^*)};$$

$$w_{k-1} - w_{k-2} = \frac{c'(e_{k-1}^*)}{p'(e_{k-1}^*)} - \frac{1}{n} \frac{c'(e_k^*)}{p'(e_k^*)} + c(e_k^*), \quad k = 3, \dots, N.$$

Proof: Let $y_k \equiv w_k - w_{k-1}$.

From the Lemma 2-4, by iteration, we have

$$V_k = \beta_{k+1}\beta_{k+2}\dots\beta_N y_N + \beta_{k+1}\beta_{k+2}\dots\beta_{N-1} y_{N-1} + \dots + \beta_{k+1} y_{k+1} + w_k \quad (\text{B.11})$$

$$V_{k-1} = \beta_k\beta_{k+1}\dots\beta_N y_N + \dots + \beta_k\beta_{k+2}\dots\beta_{N-1} y_{N-1} + \dots + \beta_k y_k + w_{k-1} \quad (\text{B.12})$$

$$(\text{B.12}) - w_{k-2} - \beta_k((\text{B.11}) - w_{k-1})$$

$$\Rightarrow V_{k-1} - w_{k-2} - \beta_k(V_k - w_{k-1}) = y_{k-1}$$

$$\text{Substituting (B.10) and } \beta_k = \frac{1}{n} - \frac{p'(e_k^*)c(e_k^*)}{c'(e_k^*)} \text{ yield } y_{k-1} = \frac{c'(e_{k-1}^*)}{p'(e_{k-1}^*)} - \frac{1}{n} \frac{c'(e_k^*)}{p'(e_k^*)} + c(e_k^*).$$

B.6 Proof of Corollary 2-3

Corollary 2-3 *If the returns to effort increase with the hierarchical level, if the cost function of effort is quadratic: $c(e_k) = b(e_k)^2$, where $b (> 0)$ is a constant, and if output y_k is linear in effort $y_k = e_k + \epsilon_k$, where ϵ_k follows a uniform distribution in range $[-a, a]$ with mean 0, $k = 1, 2, \dots, N$, then the optimal wage structure is convex if the following condition is satisfied: $e_k^* + e_{k-1}^* \geq \frac{4a}{n}$ where $k = 3, \dots, N$.*

Proof:

$$\begin{aligned}
\Pr(y_i > y_j) &= \Pr(e_i + \epsilon_i > e_j + \epsilon_j) = \Pr(\epsilon_i > \epsilon_j + e_j - e_i) \\
&= \int \Pr(\epsilon_i > \epsilon_j + e_j - e_i | \epsilon_j = x) f(x) dx = \int \Pr(\epsilon_i > x + e_j - e_i) f(x) dx \\
&= \int (1 - F(x + e_j - e_i)) f(x) dx.
\end{aligned}$$

$$\frac{\partial \Pr(y_i > y_j)}{\partial e_i} = \int f(x + e_j - e_i) f(x) dx.$$

Nash equilibrium suggests that $e_j = e_i$, so $p'(e_k^*) = \int f^2(x) dx = \frac{1}{2a}$.

$\frac{\partial^2 \Pr(y_i > y_j)}{\partial e_i^2} = \int (-f'(x + e_j - e_i)) f(x) dx$, so $p''(e_k^*) = -\int f'(x) f(x) dx$. ($f'(x) = 0$, $p''(e_k^*) = 0$.)

Substituting $c(e_k) = b(e_k)^2$ and $p'(e_k^*) = \frac{1}{2a}$ into the second condition in Proposition 2-4

$$\frac{c'(e_{k-1}^*)}{p'(e_{k-1}^*)} - \frac{c'(e_{k-2}^*)}{p'(e_{k-2}^*)} - \frac{1}{n} \left(\frac{c'(e_k^*)}{p'(e_k^*)} - \frac{c'(e_{k-1}^*)}{p'(e_{k-1}^*)} \right) + c(e_k^*) - c(e_{k-1}^*) > 0.$$

We derive that

$$4ab(e_{k-1}^* - e_{k-2}^*) - \frac{4ab}{n}(e_k^* - e_{k-1}^*) + b(e_k^* - e_{k-1}^*)(e_k^* + e_{k-1}^*) > 0.$$

From Corollary 2-2, it is sufficient to have that

$$c(e_k^*) - c(e_{k-1}^*) - \left(\frac{1}{n} \frac{c'(e_k^*)}{p'(e_k^*)} - \frac{1}{n} \frac{c'(e_{k-1}^*)}{p'(e_{k-1}^*)} \right) = b(e_k^* + e_{k-1}^* - \frac{4a}{n})(e_k^* - e_{k-1}^*) \geq 0,$$

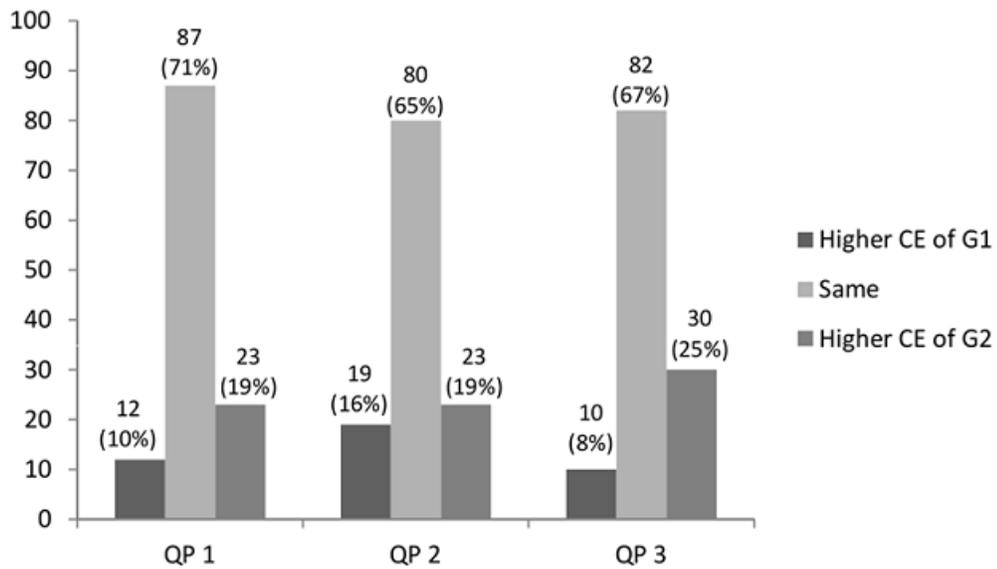
which holds if and only if $e_k^* + e_{k-1}^* > \frac{4a}{n}$.

APPENDIX C

CHAPTER 3 OF APPENDIX

C.1 Figure 3-1

Figure 3-1 Histogram of whether subjects had higher CE for G1 or higher CE for G2



Notes: The numbers in the parentheses are the corresponding percentage of subjects.

Figure 3-1 summarizes the directions of their preferences with respect to ex post fairness concern. For example, The figure shows that 12 subjects (10% of 122 subjects) had higher CE for (G_1) in QP1 ($(\$0, \$0)$ with 50% and $(\$10, \$10)$ with 50%) than CE for (G_2) in QP1 ($(\$0, \$10)$ with 50% and $(\$10, \$0)$ with 50%), while 23 subjects behaved otherwise. Note that a majority of people (71%) had the same CE for (G_1) and (G_2).

C.2 Table 3-1

Table 3-1 Basic demographic information

	Mean	Standard deviation	min	max
Age	35	13	19	66
Fraction of subjects				
Male	27%			
Econ major or minor	7%			
Political views				
Democrats	56%			
Republicans	10%			
Others	34%			

Notes: 122 observations in total.

C.3 Table 3-2

Table 3-2 CE for each question and differential in each question pair.

			Mean CE	Differential
Question Pair 1	G ₁	((0,\$0) with 50% and (\$0,\$0) with 50%)	6.21 (.246)	-
	G ₂	(\$0,\$10) with 50% and (\$10,\$0) with 50%)	6.54 (.228)	-.33* (.187)
Question Pair 2	G ₁	(\$10,\$10) with 50% and (\$20,\$20) with 50%)	14.16 (.326)	
	G ₂	(\$10,\$20) with 50% and (\$20,\$10) with 50%)	14.61 (.317)	-.45* (.257)
Question Pair 3	G ₁	(\$15,\$15) with 50% and (\$30,\$30) with 50%)	20.82 (.455)	
	G ₂	(\$15,\$30) with 50% and (\$30,\$15) with 50%)	21.69 (.439)	-.87*** (.320)

Notes: Standard errors in parentheses. The differential is the difference of the mean CE for G₁ and mean CE for G₂. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

C.4 Table 3-3

Table 3-3 Predicted frequency if subjects behave independently across QPs

Number of subjects	0 out of 3 QPs	1 out of 3 QPs	2 out of 3 QPs	3 out of 3 QPs
Higher CE for G_1	85 (70%)	33 (27%)	4 (3%)	0 (0%)
The same	4 (3%)	26 (21%)	54 (44%)	38 (31%)
Higher CE for G_2	60 (49%)	48 (39%)	13 (11%)	1 (1%)

Notes: The numbers in the parentheses are the corresponding percentages of subjects. The percentages of each row sum to 100%.

C.5 Table 3-4

Table 3-4 Observed frequency across QPs

Number of subjects	0 out of 3 QPs	1 out of 3 QPs	2 out of 3 QPs	3 out of 3 QPs
Higher CE for G_1	91 (75%)	25 (20%)	2 (2%)	4 (3%)
The same	14 (11%)	20 (16%)	35 (29%)	53 (43%)
Higher CE for G_2	78 (64%)	20 (16%)	16 (13%)	8 (7%)

Notes: The numbers in the parentheses are the percentage of subjects. The percentages of each row sum to 100%.

C.6 Table 3-5

Table 3-5 Comparison of predicted preferences and the observed preferences

Number of subjects\QP	0 out of 3 QPs	1 out of 3 QPs	2 out of 3 QPs	3 out of 3 QPs	p-value of Fisher's
Higher CE of G_1 (predicted)	85	33	4	0	exact test
Higher CE of G_1 (observed)	91	25	2	4	0.116
Number of subjects\QP	0 out of 3 QPs	1 out of 3 QPs	2 out of 3 QPs	3 out of 3 QPs	p-value of Fisher's
The same CE (predicted)	4	26	54	38	exact test
The same CE (observed)	14	20	35	53	0.005
Number of subjects\QP	0 out of 3 QPs	1 out of 3 QPs	2 out of 3 QPs	3 out of 3 QPs	p-value of Fisher's
Higher CE of G_2 (predicted)	60	48	13	1	exact test
Higher CE of G_2 (observed)	78	20	16	8	0.000

Table 3-5 shows the p-value of Fisher's exact test. According to the p-value of Fisher's exact test, the second and the third comparisons strongly reject the null hypothesis that subjects behaved randomly across the question pairs. Collectively, table 3-5 indicates a strong rejection of the null hypothesis. Hence, the subjects behaved consistently across the question pairs.

C.7 Table 3-6

Table 3-6: Regression of CE on question pairs and demographic characteristics

	OLS				Interval Regression			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
α_1	6.65*** (.283)	6.69*** (.335)	6.37*** (.729)	7.76*** (.634)	6.46*** (.257)	6.50*** (.303)	7.36*** (.539)	7.45*** (.571)
α_2	14.66*** (.283)	14.70*** (.335)	14.38*** (.335)	15.77*** (.634)	14.73*** (.255)	14.77*** (.301)	15.64*** (.539)	15.72*** (.571)
α_3	21.53*** (.283)	21.57*** (.335)	21.25*** (.729)	22.64*** (.634)	21.54*** (.256)	21.58*** (.302)	22.45*** (.539)	22.54*** (.571)
μ	-.54* (.283)	-.55** (.276)	-.55** (.276)	-.55** (.276)	-.51** (.256)	-.51** (.256)	-.51** (.249)	-.51** (.248)
Age			.20** (.090)	.21** (.091)			.18** (.081)	.19** (.082)
Gender (male)			-.25 (.341)	-.26 (.344)			-.21 (.308)	-.22 (.311)
Econ major or minor			2.38*** (.565)	2.41*** (.567)			2.20*** (.511)	2.22*** (.513)
Political Views								
Democrats			-.45 (.318)	-.49 (.321)			-.36 (.286)	-.40 (.289)
Republicans			-1.28** (.504)	-1.34*** (.507)			-1.13** (.452)	-1.18*** (.455)
Education (college or graduate level)			-.21 (.295)	-.20 (.301)			-.19 (.265)	-.18 (.270)
Concern for others			-.20*** (.052)	-.21*** (.052)			-.19*** (.047)	-.20*** (.047)
Ex ante fairness concern			.01 (.052)	.01 (.052)			.01* (.047)	.01* (.047)
Ex post differential Salient treatment		-.18 (.343)		-.39 (.341)		-.15 (.310)		-.34 (.307)
Ex ante differential Salient treatment		.03 (.341)		-.01 (.346)		.05 (.308)		.00 (.311)
Observations	732	732	732	732	732	732	723	732
R ²	0.9414	0.9414	0.9446	0.9447	-	-	-	-

Notes: Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Age: 1: 18-24; 2: 25-34; 3: 35-44; 4: 45-54; 5: 55-64; 6: 65 or older.

C.8 Table 3-7

Table 3-7 Priming effects within subjects by comparing CE for (G_1) and (G_2)

		(\$0, \$0)50% prob and (\$10, \$10)50% prob	(\$0, \$10)50% prob and (\$10, \$0)50% prob	differential	p-value
PQ1	mean CE (control)	6.04	6.47	-.43	.1054
	mean CE (ex post)	6.22	6.43	-.21	.5787
	mean CE (ex ante)	6.42	6.74	-.32	.3727
		(\$10, \$10)50% prob and (\$20, \$20)50% prob	(\$10, \$20)50% prob and (\$20, \$10)50% prob	differential	p-value
PQ2	mean CE (control)	14.13	14.77	-.64	.1411
	mean CE (ex post)	14.38	14.05	.32	.4677
	mean CE (ex ante)	14.00	14.95	-.95	.0425
		(\$15, \$15)50% prob and (\$30, \$30)50% prob	(\$15, \$30)50% prob and (\$30, \$15)50% prob	differential	p-value
PQ3	mean CE (control)	20.94	21.96	-1.02	.0569
	mean CE (ex post)	20.92	21.24	-.32	.5597
	mean CE (ex ante)	20.58	21.79	-1.21	.0497

Notes: There are 47 observations in the control group, 37 observations in the ex post treatment group and 38 observations in the ex ante treatment group.

C.9 Table 3-8

Table 3-8 CE for (G_2) between subjects

Mean CE	Control group (47 observations)	Ex post treatment 37 observations	Ex ante treatment 38 observations
(\$0, \$10) with 50% prob and (\$10, \$0) with 50% prob	6.47 (.400)	6.43 (.400)	6.73 (.388)
(\$10, \$20) with 50% prob and (\$20, \$10) with 50% prob	14.77 (.539)	14.05 (.556)	14.95 (.553)
(\$15, \$30) with 50% prob and (\$30, \$15) with 50% prob	21.96 (.730)	21.24 (.750)	21.78 (.815)

Notes: The numbers in the parentheses are standard errors.

C.10 Table 3-9

Table 3-9 Priming effects by comparing CE for (G_2) between subjects

Panel A Priming effects by comparing CE for (G_2) in QP 1 between subjects

(\$0, \$10) with 50% and (\$10, \$0) with 50%	Control group	Ex ante group	Ex post group
Control group	-	-	-
Ex ante group	.04 (.9504)	-	-
Ex post group	-.27 (.6357)	-.30 (.5845)	-

Panel B Priming effects by comparing CE for (G_2) in QP 2 between subjects

(\$10, \$20) with 50% and (\$20, \$10) with 50%	Control group	Ex ante group	Ex post group
Control group	-	-	-
Ex ante group	.71 (.3658)	-	-
Ex post group	-.18 (.8164)	-.89 (.2584)	-

Panel C Priming effects by comparing CE for (G_2) in QP 3 between subjects

(\$15, \$30) with 50% and (\$30, \$15) with 50%	Control group	Ex ante group	Ex post group
Control group	-	-	-
Ex ante group	.71 (.5018)	-	-
Ex post group	.17 (.8784)	-.55 (.6236)	-

Notes: The p-value in the parentheses.

C.11 Table 3-10

Table 3-10 Some demographic studies of ex post fairness

Panel A Ex post fairness by gender				Panel B Ex post fairness by education			
		t statistics	p-value			t statistics	p-value
α_1	6.68 (.308)	21.72	0.000	α_1	6.60 (.385)	17.15	0.000
α_2	14.69 (.308)	47.73	0.000	α_2	14.62 (.385)	37.93	0.000
α_3	21.56 (.308)	70.03	0.000	α_3	21.49 (.385)	55.76	0.000
μ	-2.20 (.544)	-2.04	0.042	μ	-.62 (.465)	-1.34	.182
gender(male)	-.13 (.450)	-0.29	0.768	education (college or higher)	.07 (.414)	.16	.874
$\mu * gender$	4.72 (6.37)	0.74	0.459	$\mu * education$.12 (.586)	.20	.838

Notes: The standard errors in the parentheses. 732 Observations.

C.12 The upper bound of the ex post fairness concern

The insignificance may be due to the coarseness of our data and the fact that the values of the options are not continuous. Therefore, I also test the upper bound of the ex post fairness concern. In other words, I choose the lowest value of the certainty equivalent interval as their certainty equivalent for the ex post unfair alternative and the highest value of the certainty equivalent interval as their certainty equivalent for the ex post fair alternative. The results of t-test is presented in Table 3-2. As can be seen from the table, the possible effects of the ex post fairness concerns are statistically significant.

	Mean CE	Differential
(\$0, \$0) with 50% and (\$10, \$10) with 50%	6.21 (.246)	-
(\$0, \$10) with 50% and (\$0, \$10) with 50%	4.54 (.228)	1.67 (.0000)
(\$10, \$10) with 50% and (\$20, \$20) with 50%	13.16 (.326)	-
(\$10, \$20) with 50% and (\$20, \$10) with 50%	11.61 (.317)	1.55 (.0000)
(\$15, \$15) with 50% and (\$30, \$30) with 50%	19.82 (.455)	-
(\$15, \$30) with 50% and (\$30, \$15) with 50%	18.69 (.439)	1.13 (.0006)

Notes: The numbers in the parentheses in the second row are standard errors. The number in the parentheses in the third row are p-value.

C.13 A sample survey

Sample Survey Page 1

Thank you very much for clicking this survey link!

The first 100 participants who submit the survey are eligible to collect payments: a minimum of \$3 plus a 12% chance of earning an additional \$25. As stated in the invitation email, to collect payment, it is necessary to visit the Lab (Warren 37) on Monday June 11th between 12 noon and 6 pm or Monday June 18th between 12 noon and 6 pm.

Once 100 people submit the survey, the survey will be closed for payments. (You will be notified by a different welcome screen. But if you are still interested then, you are welcome to complete this online survey without payments.)

This survey will officially close on June 17, 2012 midnight.

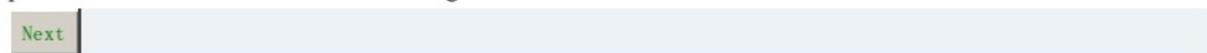
Throughout this survey you will be asked binary choice questions in social context. We anticipate that this survey will take less than 15 minutes. It is very important that you complete this survey on your own and answer truthfully.

Your privacy is very important to us. Your responses will be used solely for research purposes and will be kept strictly confidential. We only use your email to identify you when you visit the Lab to collect payment.

Contact Information: This study is being conducted by Cornell University graduate student: Jin Xu. If you have any questions or concerns about this survey, please contact Jin Xu at jx35@cornell.edu or Assistant Professor Daniel Benjamin at db468@cornell.edu. In addition, you may contact the Cornell University Institutional Review Board for Human Participants with any concerns or complaints about this survey. They may be reached at irbhp@cornell.edu, 607-255-5138, or www.irb.cornell.edu. Concerns or complaints may be filled anonymously through Ethicspoint (www.hotline.cornell.edu) or by calling toll free at 1-866-293-3077.

For your convenience, this contact information will be repeated at the end of survey.

You need to be older than 18 to participate in this survey. If you are older than 18 and agree to fill out this survey, please click the "Next" button below to begin.



Sample Survey Page 2

Imagine that you are making choices in social context.

There are many other participants taking this survey too. Imagine that you are randomly paired with one participant and your choices determine not only your payoff but also the payoff of that person (or your partner). Imagine that your partner's choices will not affect your payoff. Your decisions are completely anonymous and the other person neither knows you nor your decisions.

Imagine that it is impossible to transfer the payoff between you and the other person once you make decisions.

For each question, you are first given an alternative, then you are asked to compare this alternative with a list of alternatives and to select the alternatives from the list you prefer to the this alternative.

Each question is independent. Reminder: once you click "Next", you cannot go back to revise your answers.

Example. you may read a question like this,
(This is an example not a question)

Alternative: each of you and the other person gets \$5.

Now you are comparing the above alternative with each of following alternatives. Please select all the alternatives which you prefer (to the above alternative).

- Each of you and the other person gets \$3.
- Each of you and the other person gets \$4.
- Each of you and the other person gets \$6.
- Each of you and the other person gets \$7.
- Each of you and the other person gets \$8.

(This is an example not a question)

If you prefer every allocation greater than \$5, then you click the last three alternatives.

(This is an example not a question)

Alternative: each of you and the other person gets \$5.

Now you are comparing the above alternative with each of following alternatives. Please select alternatives which you prefer (to the above alternative).

- Each of you and the other person gets \$3.
- Each of you and the other person gets \$4.
- Each of you and the other person gets \$6.
- Each of you and the other person gets \$7.
- Each of you and the other person gets \$8.

(This is an example not a question)

Please start now.

Alternative: each of you and the other person gets \$5.

Now you are comparing the above alternative with each of following alternatives. Please select all the alternatives which you prefer (to the above alternative).

- The other person gets \$0 and you get \$6.
- The other person gets \$2 and you get \$6.
- The other person gets \$5 and you get \$6.
- The other person gets \$6 and you get \$6.
- The other person gets \$7 and you get \$6.
- The other person gets \$10 and you get \$6.
- The other person gets \$20 and you get \$6.

Alternative: each of you and the other person gets \$10.

Now you are comparing the above alternative with each of following alternatives. Please select all the alternatives which you prefer (to the above alternative).

- The other person gets \$12 and you get \$10.
- The other person gets \$0 and you get \$12.
- The other person gets \$4 and you get \$12.
- The other person gets \$8 and you get \$12.
- The other person gets \$12 and you get \$12.
- The other person gets \$16 and you get \$12.
- The other person gets \$30 and you get \$12.

Alternative: each of you and the other person gets \$15.

Now you are comparing the above alternative with each of following alternatives. Please select all the alternatives which you prefer (to the above alternative).

- The other person gets \$0 and you get \$16.
- The other person gets \$3 and you get \$16.
- The other person gets \$7 and you get \$16.
- The other person gets \$11 and you get \$16.
- The other person gets \$15 and you get \$16.
- The other person gets \$18 and you get \$16.
- The other person gets \$20 and you get \$16.
- The other person gets \$40 and you get \$16.

Next

Sample Survey Page 3

Suppose a coin is tossed. A head and a tail occur with equal probability. (50%)

Alternative: a coin is tossed.

If it is a Head, you and the other person each will get \$0.

If it is a Tail, you and the other person each will get \$10.

Now you are comparing the above alternative with each of following alternatives. Please select all the alternatives which you prefer (to the above alternative).

- Each of you and the other person gets \$2 for sure.
- Each of you and the other person gets \$4 for sure.
- Each of you and the other person gets \$6 for sure.
- Each of you and the other person gets \$8 for sure.
- Each of you and the other person gets \$10 for sure.

Alternative: a coin is tossed again.

If it is a Head, each of you and the other person gets \$10.

If it is a Tail, each of you and the other person each gets \$20.

Now you are comparing the above alternative with each of following alternatives. Please select all the alternatives which you prefer (to the above alternative).

- Each of you and the other person gets \$10 for sure.
- Each of you and the other person gets \$12 for sure.
- Each of you and the other person gets \$14 for sure.
- Each of you and the other person gets \$16 for sure.
- Each of you and the other person gets \$18 for sure.
- Each of you and the other person gets \$20 for sure.

Alternative: a coin is tossed again.

If it is a Head, you and the other person each will get \$15.

If it is a Tail, you and the other person each will get \$30.

Now you are comparing the above alternative with each of following alternatives. Please select all the alternatives which you prefer (to the above alternative).

- Each of you and the other person gets \$16 for sure.
- Each of you and the other person gets \$18 for sure.
- Each of you and the other person gets \$20 for sure.
- Each of you and the other person gets \$22 for sure.
- Each of you and the other person gets \$24 for sure.
- Each of you and the other person gets \$26 for sure.
- Each of you and the other person gets \$28 for sure.
- Each of you and the other person gets \$30 for sure.

Next

Sample Survey Page 4

Does does your birthday fall between

- the 1st and 10th day of the month.
- the 11th and 20th day of the month.
- the 21st and the last day of the month.

Next

Sample Survey Page 5

Alternative: a coin is tossed again.

If it is a Head, you get \$10 and the other person gets \$0.

If it is a Tail, you get \$0 and the other person gets \$10.

Now you are comparing the above alternative with each of following alternatives. Please select all the alternatives which you prefer (to the above alternative).

- Each of you and the other person gets \$2 for sure.
- Each of you and the other person gets \$4 for sure.
- Each of you and the other person gets \$6 for sure.
- Each of you and the other person gets \$8 for sure.
- Each of you and the other person gets \$10 for sure.

Alternative a coin is tossed again.

If it is a Head, you get \$10 and the other person gets \$20.

If it is a Tail, you get \$20 and the other person gets \$10.

Now you are comparing the above alternative with each of following alternatives. Please select all the alternatives which you prefer (to the above alternative).

- Each of you and the other person gets \$10 for sure.
- Each of you and the other person gets \$12 for sure.
- Each of you and the other person gets \$14 for sure.
- Each of you and the other person gets \$16 for sure.
- Each of you and the other person gets \$18 for sure.
- Each of you and the other person gets \$20 for sure.

Alternative a coin is tossed again.

If it is a Head, you get \$15 and the other person gets \$30.

If it is a Tail, you get \$30 and the other person gets \$15.

Now you are comparing the above alternative with each of following alternatives. Please select all the alternatives which you prefer (to the above alternative).

- Each of you and the other person gets \$16 for sure.
- Each of you and the other person gets \$18 for sure.
- Each of you and the other person gets \$20 for sure.
- Each of you and the other person gets \$22 for sure.
- Each of you and the other person gets \$24 for sure.
- Each of you and the other person gets \$26 for sure.
- Each of you and the other person gets \$28 for sure.
- Each of you and the other person gets \$30 for sure.

Next

Sample Survey Page 6

Alternative: a coin is tossed again.

If it is a Head, you get \$15 and the other person gets \$3.

If it is a Tail, you get \$2 and the other person gets \$10.

Now you are comparing the above alternative with each of following alternatives. Please select all the alternatives which you prefer (to the above alternative).

- Each of you and the other person gets \$3 for sure.
- Each of you and the other person gets \$5 for sure.
- Each of you and the other person gets \$7 for sure.
- Each of you and the other person gets \$9 for sure.
- Each of you and the other person gets \$11 for sure.
- Each of you and the other person gets \$13 for sure.
- Each of you and the other person gets \$15 for sure.

Alternative: a coin is tossed again.

If it is a Head, you get \$10 and the other person gets \$15.

If it is a Tail, you get \$20 and the other person gets \$5.

Now you are comparing the above alternative with each of following alternatives. Please select all the alternatives which you prefer (to the above alternative).

- Each of you and the other person gets \$8 for sure.
- Each of you and the other person gets \$10 for sure.
- Each of you and the other person gets \$12 for sure.
- Each of you and the other person gets \$14 for sure.
- Each of you and the other person gets \$16 for sure.
- Each of you and the other person gets \$18 for sure.
- Each of you and the other person gets \$20 for sure.

Alternative: a coin is tossed again.

If it is a Head, you get \$13 and the other person gets \$20.

If it is a Tail, you get \$20 and the other person gets \$7.

Now you are comparing the above alternative with each of following alternatives. Please select all the alternatives which you prefer (to the above alternative).

- Each of you and the other person gets \$8 for sure.
- Each of you and the other person gets \$10 for sure.
- Each of you and the other person gets \$12 for sure.
- Each of you and the other person gets \$14 for sure.
- Each of you and the other person gets \$16 for sure.
- Each of you and the other person gets \$18 for sure.
- Each of you and the other person gets \$20 for sure.

Next

Sample Survey Page 7

Thank you for participating in our online survey. Now please answer a few more questions and then click the "Next" button on this page, then you are done.

When you make decisions for above questions, do you consider the other person's payoff too? Please move the slider to rate your concern for others on a scale from 1 to 10.

0 represents that that you don't consider other person's payoff at all, i.e., you put no weight on other person's payoff (all weight on yourself).

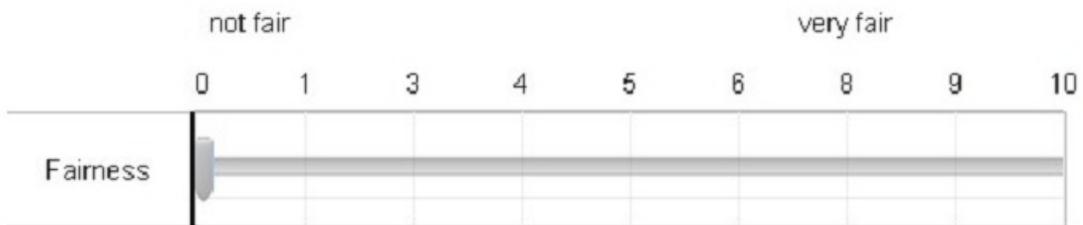
5 represents that you put equal weight on you and the other person.

10 represent that you make your decision only based on the other person's payoff.



Do you think it is fair that everyone has equal probability of winning but some of them may end up with nothing due to bad luck? Please move the slider to rate fairness on a scale of 1 to 10.

0 presents that you think it is not fair at all. 10 presents that you think it is very fair.



Please select your gender.

- Female
- Male

Please enter your age.

Please check the category below that describes your highest level of education completed.

- High School
- Associate's Degree
- Bachelor's Degree
- Graduate Degree
- Others

Are you an econ major (or minor)?

- Yes

- No
- Do not apply

What is your ethnicity?

- Black/African
- Asian
- Latino/Hispanic
- Middle Eastern
- White/Caucasian

Others

Please state your political views.

- Democratic
- Republican
- Others
- Do not apply

Do you have any comments about this survey? If so, feel free to write any comments. (Optional)

Please enter your email address below. (optional)

Thank you very much for your participation. As stated in the invitation email, to collect payment, it is necessary to visit the Lab (Warren 37) on Monday June 11th between 12 noon and 6 pm or Monday June 18th between 12 noon and 6 pm. (If you have any questions, please feel free to email Jin Xu at jx35@cornell.edu). Your email and survey responses will be kept strictly confidential.

Please click "Next" to submit the survey.
Thank you very much for taking our survey.

Contact Information: This study is being conducted by Cornell University graduate student: Jin Xu. If you have any questions or concerns about this survey, please contact Jin Xu at jx35@cornell.edu or Assistant Professor Daniel Benjamin at db468@cornell.edu. In addition, you may contact the Cornell University Institutional Review Board for Human Participants with any concerns or complaints about this survey. They may be reached at irbhp@cornell.edu, 607-255-5138, or www.irb.cornell.edu. Concerns or complaints may be filled anonymously through Ethicspoint (www.hotline.cornell.edu) or by calling toll free at 1-866-293-3077.

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