
TENDINITIS RELIEF:

**THREE DIMENSIONAL MODELING OF COLD THERAPY FOR THE
TREATMENT OF SUPRASPINATUS TENDINITIS**

BEE 4530

Group 1

Kelsey Prucha-Mitchell

Tessa Huffstater

Jeffrey Fitch

Charles McCann

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EXECUTIVE SUMMARY:

Shoulder bursitis and supraspinatus tendinitis are common conditions that result from the inflammation of the supraspinatus tendon. These cause pressure to be placed more heavily on the anterior bursa sac, along with the surrounding bones and nerves. A common treatment is conductive cooling on the affected region, generally in the form of a cold pack. However, if the cold pack is left on for too short of a time period, the cooling may not reach the tendon. The tendinitis would not be adequately treated in this case, as the inflammation could not be reduced if the tendon is not cooled to a temperature close to that of the rest of the body. If it is left on for too long, the patient may be subject to significant pain and the surrounding healthy tissue may be permanently damaged. Therefore, our goal is to identify the ideal treatment time for treating supraspinatus tendon inflammation.

We created a three-dimensional model of the shoulder using COMSOL, with components integrated from Autodesk™ Computer aided design software and Google Sketch-Up. The implementation of our model into COMSOL allowed us to simulate the effects of this type of cold therapy on a human shoulder. A number of major parameter simplifications were required for adequate implementation into our model. Such simplifications included the grouping of the skin and muscle components, approximation of the humerus to a cylinder and a sphere, and grouping of the bursa sac and supraspinatus tendon into one domain because the two structures were too close together to be included in the model individually.

We researched relevant literature to obtain property values for the bone and tissue components of our model. We used our best judgment to approximate the property values for parts of the geometry that were simplified, such as the tendon, muscle, and skin. In order to counteract any inaccuracy that could have resulted from imprecise averaged values, we performed sensitivity analysis to determine how tendon temperature varied with changes in the various material properties. This analysis supported our approximations. To further validate our model, we compared our results with previous research. The results of their study provided us with a guideline as to how much the muscle region should cool in a certain time period. Within that time period, our muscle tissue cooled by the expected amount, which validated our model.

We decided that a 3-dimensional analysis of the supraspinatus tendon was necessary because the shoulder is asymmetric, so a 2-dimensional model would not be capable of capturing the necessary complexity. We determined that the standard suggested practice of treating an injury for a maximum of 20 minutes was not applicable for injuries that are as deep as the supraspinatus tendon. In this amount of time, the tendon's temperature decreased by 3.05 Kelvin, when it needed to decrease by 7 K to be measured as successful cooling. We identified the ideal cold treatment time to be 96 minutes, using COMSOL to determine the point at which the supraspinatus tendon cooled within 1 K of body temperature. Further analysis indicated that the suprascapular nerve would not reach the threshold for cold pain of 288 K. However, other areas closer to the surface would reach temperatures well below this threshold. The cooling of the nerves running through this region could lead to significant pain. Literature shows that cooling for more than an hour could lead to permanent tissue damage, so 96 minutes of continuous cold treatment is not recommended.

While our assumptions limit the clinical significance of this study, our results indicated that the use of cold therapy for only 20 minutes is ineffective because it does not reduce the temperature of the tendon enough to reduce inflammation. Cold treatment would likely be improved by the addition of anti-inflammatory drugs, in an attempt to combat the inflammation in two ways. We would recommend that further studies utilize unique heat transfer properties for the structures present in the shoulder instead of grouping them as one. This would bring the study closer to a clinically significant stage. In addition, further analysis of the likelihood of the pain response could be included by the addition of nerves to the model. Our model could also be used to test other cold or heat therapy technologies and their probable pervasiveness in the human shoulder, specifically the glenoid and sub-acromial regions.

INTRODUCTION:

Tendinitis is a common affliction wherein a tendon becomes inflamed and painful due to overwork and overextension of its associated muscle tissue. The supraspinatus tendon, often referred to as the rotator cuff tendon, lies inferiorly to the acromion process and superiorly to the bursa sac (Figure 1). The tendon passes over the top of the humerus and pulls the bone up when the supraspinatus muscles are engaged to abduct the arms. Supraspinatus tendinitis results from inflammation of the tendon due to friction against the nearby bones, overextension, or general overuse. It is frequently coupled with inflammation of the sub-acromial bursa sac, known as sub-acromial bursitis. This combination can result in severe pain as the bursa sac and

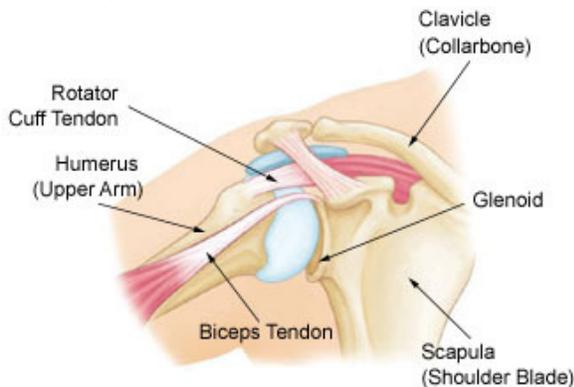


Figure 1 - Shoulder Joint Anatomy. This image illustrates the area of focus for our study, the supraspinatus tendon. The tendon is labeled as "rotator cuff tendon" in this diagram.

tendon push against one another and the bones surrounding them, and the condition can become debilitating. It is extremely common among swimmers, baseball players, and other athletes whose sports involve similar stressful overhead movements (Virtual Medical Centre, 2003).

There are currently three main non-surgical treatment options suggested for tendinitis: 1) rest and protection of the area, 2) application of cold therapy, and 3) introduction of anti-inflammatory medications, including direct cortisone injections (Almekinders, 1998). A combination of these treatments may also be employed, and the use of cold therapy in conjunction with anti-inflammatory drugs is sometimes used to treat early and mild supraspinatus tendinitis. Cold therapy alone is generally used as the first line of defense against

tendinitis and is the most common treatment. As a result, the identification of the optimal parameters for cold pack therapy is amongst the most important goals for medical treatment of shoulder tendinitis. Effective cooling of the tendon can be critical for long-term healing, as it reduces swelling and can help prevent the need for more aggressive treatment options such as steroid injections and even surgery.

There is no general consensus on how cold pack therapy should be approached. The length of cooling, the temperature of the cold pack, the placement of the cold pack, amongst other variables, are all factors that affect whether or not cooling will be a successful treatment option for this type of tendonitis. Another factor to consider is the use of cyclical application of the cold pack, with alternated periods of having the cold source on and off of the skin. Within the literature, there are often conflicting conclusions concerning the optimal length of cooling. One source suggests icing the afflicted area for fifteen to twenty minutes a few times a day (McLaughlin, 2001). Another source writes that as long as the cold pack is at or near the freezing temperature of water, it is unlikely that any long-term tissue damage will occur. As a result, it suggests that the optimal cooling time may be up to an hour (Knight, 2012).

This study will attempt to find the optimal cold therapy treatment time for supraspinatus tendinitis, as current research and analysis varies widely. Using a lifelike model of the rotator cuff implemented through heat transfer models in COMSOL, the benefits of using cold therapy for the various recommended treatment lengths will be evaluated, and the most effective cooling time will be determined.

MODEL DESIGN AND DEVELOPMENT CRITERION

PROBLEM STATEMENT

We proposed the development and optimization of a computational model using COMSOL to accurately predict the necessary therapeutic time for sub-acromial supraspinatus tendon cooling. We based our analysis on a model of cold pack application around the shoulder following diagnosis of supraspinatus tendinitis.

DESIGN OBJECTIVES

- I. Develop a 3 dimensional model to determine the temperature profile for cold treatment of the shoulder to the depth of the supraspinatus ligament
- II. Identify time required for cold therapy to reduce inflammation of the supraspinatus tendon to an appreciable extent
- III. Evaluate feasibility of prolonged treatment and the inherent complications involved
- IV. Suggest future research possibilities and clinical potential of our model

SCHEMATIC

A three-dimensional model was chosen as opposed to a two-dimensional model due to the asymmetric nature of the shoulder joint. Because of this, the cooling will be different depending on where in the shoulder the cooling is being assessed. Similarly, cold pack placement also precludes effective use of two-dimensional modeling. This is because the cold pack is only placed on certain boundaries, with other boundaries having no cooling applied to them at all. Therefore, some points will receive cooling from multiple directions while others will only be significant affected by cooling in one direction. As the shoulder and cooling varies in all three dimensions, a two-dimensional model would not be able to handle the complexities of the problem and therefore could not provide an accurate assessment of cold pack therapy.

Based on currently available literature and textbook schematics, we developed a three-dimensional model of the components of the human shoulder using Google SketchUp, AutoCad, and COMSOL (Figure 2).

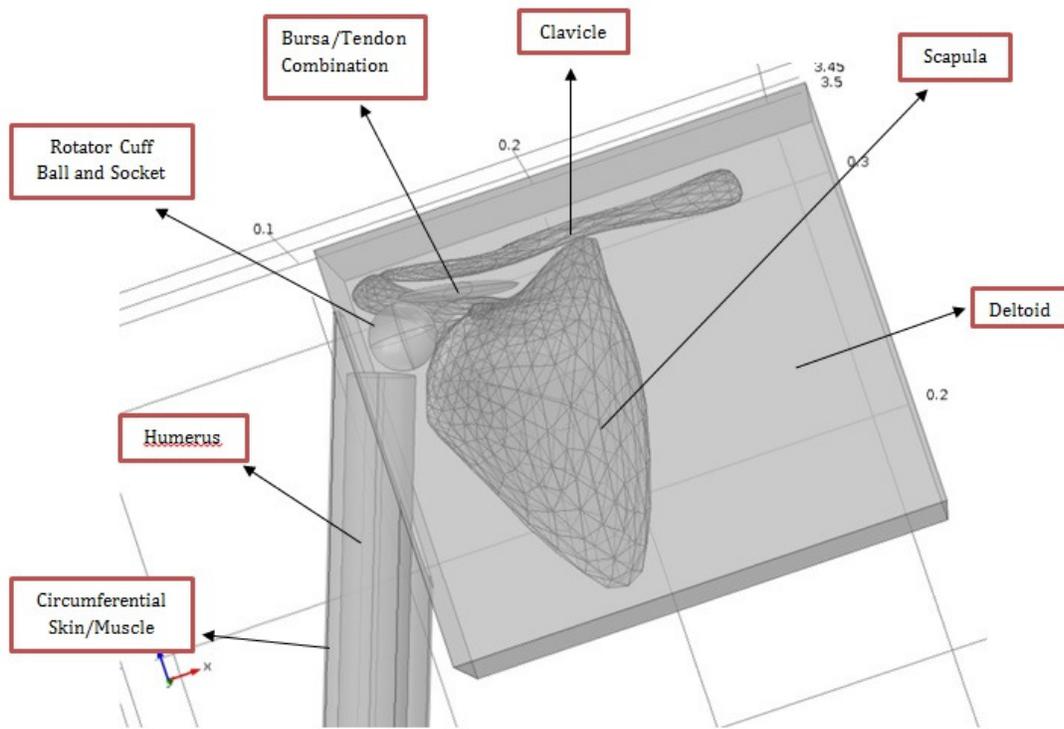


Figure 2 – 3-D Geometry of the Anatomical Components of the Shoulder Joint. This figure is a close-up of the model used in this study, including all relevant musculoskeletal components.

We retrieved the geometries of the scapula and clavicle from the Google SketchUp public database and we then converted them to a COMSOL-compatible file. We scaled the bones to size and positioned them so they fit together as accurately as possible, given the simplifications of the humerus. We combined skin and muscle components and averaged their properties due to their comparable values, allowing for a simpler, COMSOL-compatible domain. We shaped these components as a surrounding box and cylinder in the geometry.

We implemented the boundary and initial conditions to imitate the physical situation of tendon swelling and ice pack application, demonstrated in Figure 3 below. Figure 3 also shows the specific dimensions and conditions inputted into COMSOL.

BOUNDARY CONDITIONS

- Temperature of 273 K at top and back of the shoulder and outer boundaries of the arm
- Flux at all outer boundaries is set at 0 W/m^2

INITIAL CONDITIONS:

- Temperature at 318 K in the inflamed region
- Temperature of 310 K (normal body temperature) in all other regions

GOVERNING EQUATIONS

Based on the literature, we determined that heat transfer in muscle, skin, bursa and the supraspinatus tendon follow the general heat transfer equation with added bio heat to account for temperature change due to blood flow.

EQUATION 1: HEAT TRANSFER EQUATION WITH BIO-HEAT

$$\frac{dT}{dt} = \frac{k}{\rho c_p} \nabla^2 T + \rho_b c_b \dot{V} (T_a - T)$$

EQUATION 2: AVASCULARIZED HEAT TRANSFER EQUATION FOR BONES

$$\frac{dT}{dt} = \frac{k}{\rho c_p} \nabla^2 T$$

We implemented the volumetric blood flow as an equation dependent upon temperature to model how blood vessels contract and expand during temperature fluctuations. We used the following equation as a general heat source term in place of the bioheat generation term, $\rho_b c_b \dot{V} (T_a - T)$, to model blood flow through all regions except the bones.

$$Q = C_{p,muscle} \left(0.45 + 3.55e^{-\frac{(T-45)^2}{12}} \right) * (310 - T)$$

In the above equations, the variables included are as follows in Table 1.

Table 1 – List of heat transfer parameters and properties. Properties listed as “tissue” are the averages of the respective properties for muscle and skin.

Parameter	Definition
k	Thermal Conductivity
c _p	Specific Heat Capacity of Tissue
ρ	Tissue Density
ρ _b	Blood Density
T	Temperature
T _a	Arterial Blood Temperature
c _b	Specific Heat of Blood
\dot{V}	Volumetric Blood Flow

We implemented the above specifications into COMSOL and ran a “heat transfer through solids” model to determine the efficacy of cold therapy for treatment of supraspinatus tendinitis.

RESULTS AND DISCUSSION

20 MINUTE COLD THERAPY

The standard protocol that physical therapists suggest when treating tendinitis with cold therapy is to treat with an applied icepack for 15-20 minutes and they urge the patients not to leave the cold pack on longer. For this reason, our analysis starts with determining how effective cold therapy is after 20 minutes of treatment. Figure 4 shows the temperature profile for the middle of the tendon after running the model for 20 minutes.

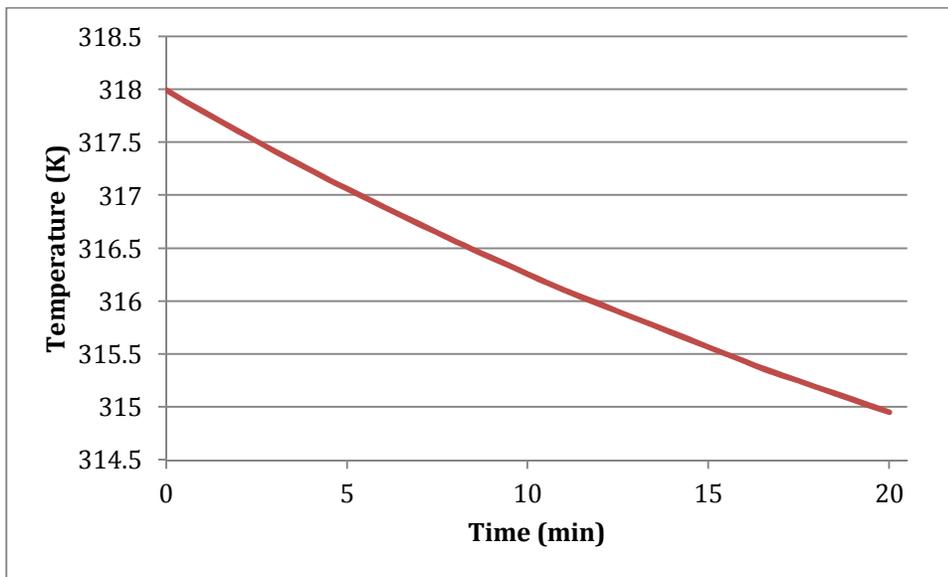


Figure 4 - Temperature versus time plot for 20 minute ice pack treatment. After 20 minutes of cooling at a point within the tendon (0.1609, 3.422, 0.2927), treatment was deemed inefficient at reducing inflammation, only reducing the tendon temperature by 3 K.

Although 20 minutes is the upper limit of what physical therapists and doctors recommend for ice pack for treatment, Figure 4 shows us that after the full time period, the tendon region has only reached 314.95 K, 3.05 K below its initial temperature of 318 K. Consequently, when dealing with deep tendinitis, a 20-minute maximum for icing is ineffective and is not the best treatment option. Though it has some effect, if the patient can handle further icing treatment, prolonged use of cold therapy would better treat the problems associated with inflammation.

After determining that 20 minutes was not enough time for cold therapy to reach the supraspinatus tendon, we calculated approximately how long it would take for the therapy to cool the tendon to the normal body temperature of 310 K (Appendix A). Based on this calculation, we found thermal diffusivity to be $1.047 \times 10^{-7} \text{ m}^2/\text{s}$, which we could use to calculate an approximate necessary cooling time. Assuming a length of 0.0241 m between the supraspinatus tendon and the skin surface, the time for effective cold therapy to lower the tendon temperature within one degree of body temperature was 96 minutes.

96 MINUTE COLD THERAPY

After running our model in COMSOL, we found that the tendon reaches body temperature at approximately 3 hours. However, our goal is to cool the tendon to within one degree of body temperature, which occurs at the treatment time of 96 minutes.

Figure 5 below shows the heating profile for the shoulder after 96 minutes of cold pack treatment. This treatment time was clearly more effective as indicated by the color of the tendon region, which reached the desired temperature of 311 K. Figure 5 shows five slices that run through the geometry, showing how heat transfer changes on the medial lateral axis. One of the slices runs through the tendon region, circled in the figure, which is still the hottest region in the shoulder.

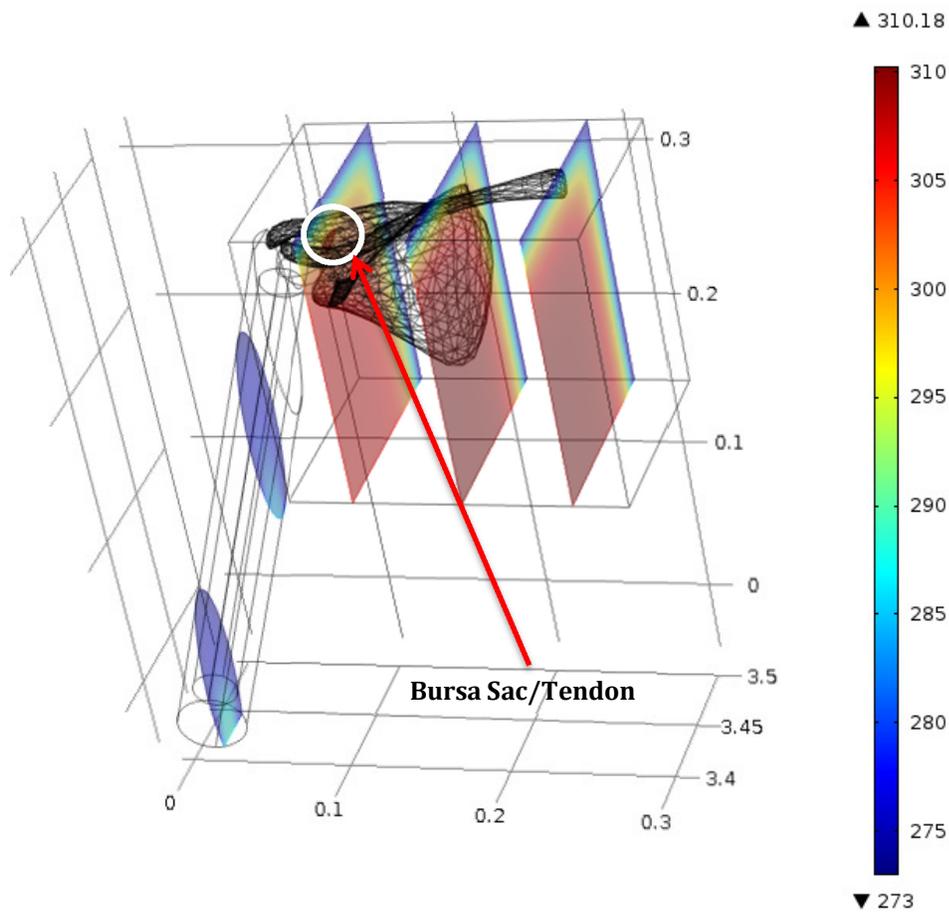


Figure 5 - 3D slice plot at 96 minutes. Generated by COMSOL showing the temperature profile at five different locations on the model at 96 minutes. Note that the temperature of the combined bursa and tendon region is still the hottest region in the system.

As indicated in Figure 5, the temperature of the surrounding musculature decreased to an average temperature of about 280 K. We believe that constriction and cooling of the tissue surrounding the tendon would help decrease tendon inflammation by decreasing local blood flow thus limiting inflammation and pain.

The calculations to determine the length of time for appropriate cooling proved to be accurate, and Figure 6 shows how the cooling of the tendon occurred over the three hours, with the black arrow indicating the 96 minute treatment time to reduce the tendon temperature to 311 K.

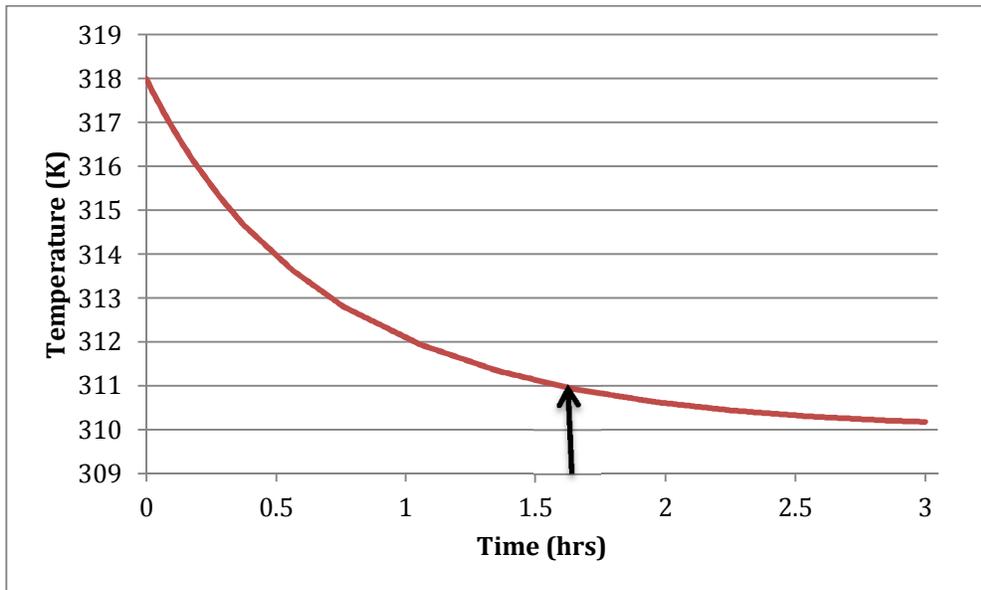


Figure 6 - Temperature versus time plot for 3 hours of ice pack treatment. After 3 hours of cooling at a point within the tendon (0.1609, 3.422, 0.2927), we noticed that the time necessary to cool the tendon within 1 K is 96 minutes, and that the tendon takes almost double that time to cool all the way down to body temperature.

Due to the initial temperature gradient between the skin and the cold pack, the temperature dropped fastest in the first half hour. We determined that to successfully cool the tendon to a point that it was no longer considered inflamed, 96 minutes of treatment was necessary. The final cooling was a temperature drop of 7 K in the tendon, signifying that cooling was successful and inflammation relief had been achieved.

PAIN CONSIDERATIONS FOR PROLONGED TREATMENT

While the patient’s ability to handle prolonged cold therapy is dependent on each individual patient, Figure 7 below identifies a threshold in which ‘cold-pain’ is experienced, on average.

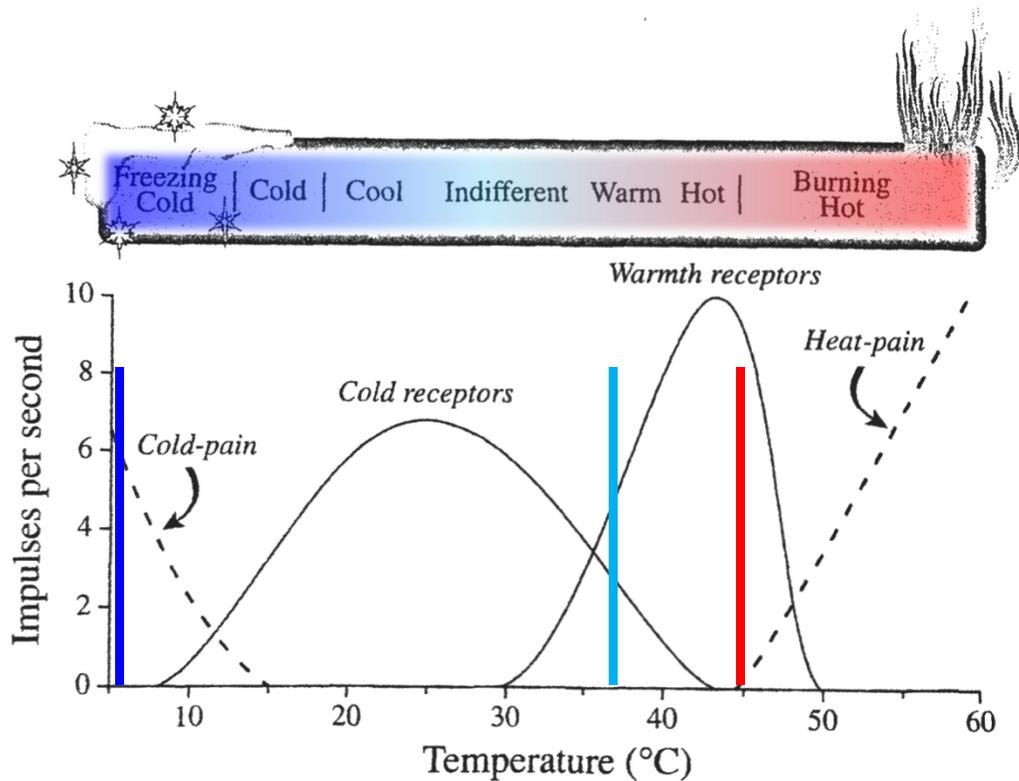


Figure 7 –Strength of pain signals from cold, heat, and pain receptors at different temperatures. The colored lines indicate the various temperature stages of our model throughout treatment (Datta, 2013).

In Figure 7, the red line indicates the inflamed tendon temperature at the start of treatment. It is interesting to note that this temperature is right at the threshold for heat pain. However, most of the pain experienced by the patient is due to impingement of the tendon and bursa sac on the surrounding bones and nerves. The light blue line is indicative of the lowest temperature experienced by the tendon, which is approximately normal body temperature, and this point obviously does not contribute to any thermal pain. The dark blue line illustrates the temperature of the skin and surface tissue after 96 minutes of treatment, and is well below the temperature at which people begin experiencing cold pain. In the shoulder region, we considered the suprascapular nerve, a major nerve located superiorly to the bursa and supraspinatus tendon. Based on our model, the suprascapular nerve remained above 15° C throughout the treatment period, indicating that this large nerve would not experience significant enough cooling for cold pain. Further considerations should be directed to the smaller nerves innervating the surface tissue, as these will most likely be responsible for a significant pain response because they fall below 288 K.

SENSITIVITY ANALYSIS

Sensitivity analysis is an industry standard allowing us to determine how much our model was affected by discrepancies in our model parameters. Our model parameters, previously defined in Table 1, were extracted from scientific literature (Figure 9). We performed sensitivity analysis using a parametric sweep by varying our 7 heat transfer parameters 20% above and below their actual values to determine how changing these parameters affected our final solution. Body temperature was a fairly constant quantity and its variations had limited to no effect on our model. Our model was generally insensitive to variation in bone properties, apparent in Figure 8.

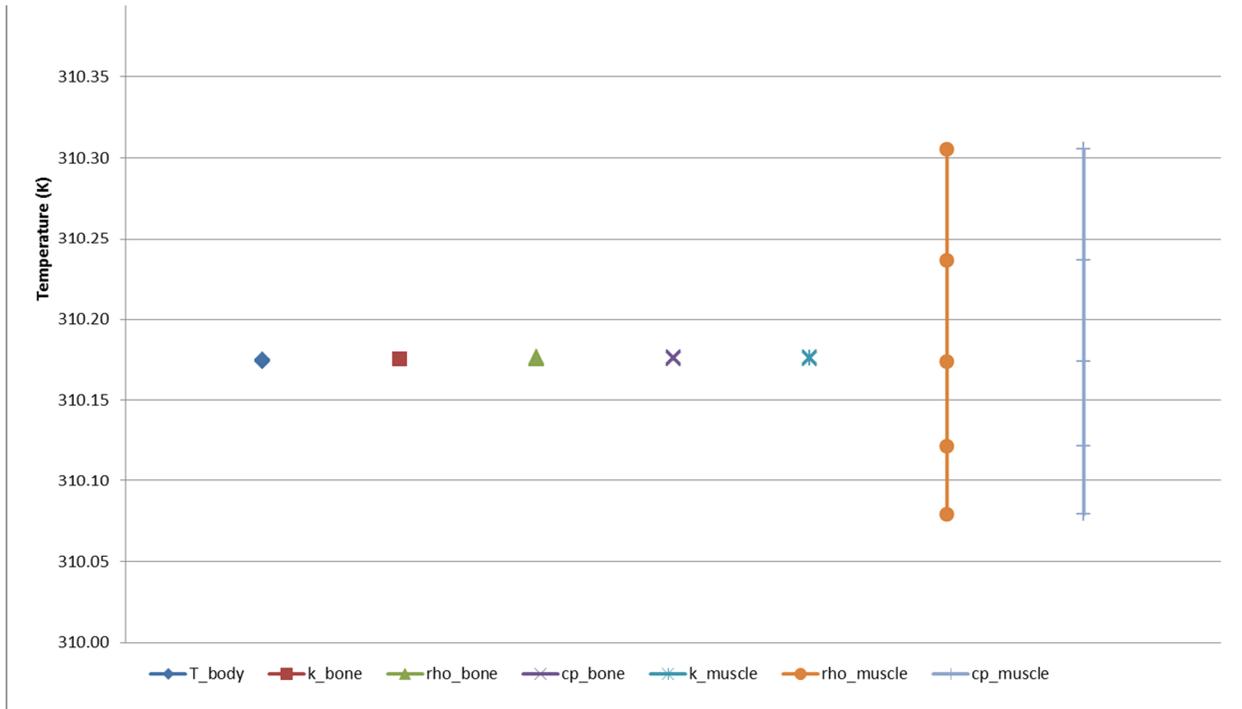


Figure 8 – Sensitivity Analysis from varying 7 parameters. Plot depicting results of sensitivity analysis to determine how temperature changes when altering the parameters at a point within the tendon. A significant spread is only seen with respect to the density and specific heat capacity of muscle tissue.

Our model was most sensitive to the density and specific heat capacity of the combined muscle, skin, and tissue. This finding was what we expected, as it constitutes a majority of the conductive material in our model. In addition, the temperature of this region determined volumetric blood flow rates, further affecting heat transfer rates. It should be noted that despite the apparent wide spread error associated with ρ_{muscle} and $c_{p_{muscle}}$, the temperature spread in reality only ranges from 310.08 K to 310.31 K, a spread of 0.23K. Since our goal was for the tendon/bursa complex to reach a temperature within 1 degree of body temperature, the model is negligibly affected by muscle parameter variation.

MODEL VERIFICATION

To validate our model, we used a study (Knight 2012) that had measured how long it took to decrease tissue by 7 K at a specified point within the muscle. Their results that correspond to our data are shown in Table 2, below.

Table 2 – Time it takes to decrease tissue temperature by 7K at a point 1cm within the muscle and beyond the adipose tissue layer. The first two rows of this table show the time it takes for the tissue to decrease by 7K for two different skinfold thickness ranges. The final row shows our experimental data and the time it takes for our model to decrease by 7K at the specified point.

Skinfold Thickness	Time to Decrease by 7° K
11-20 mm	23.3 +- 6.7 min
21-30 mm	37.8 +- 9.6 min
Avg. Deltoid Thickness Range: 15.0 - 21.6 mm	Our Acceptable Range: 36 +- 12 min

Because the average female ‘Recreationally Active College Student’ has an average deltoid skinfold thickness of 21.6mm and the average male has a mean deltoid thickness of 15.0mm, both of the skinfold thickness ranges presented in Table 2 are relevant to our study. As a result we will average the time values for these ranges. Therefore, the average time to decrease the tissue temperature by 7 K at a point 1cm below the adipose tissue should be 30.6 minutes with a standard deviation of 12 minutes. We will use the mean plus or minus one standard deviation as our acceptable range for validation, giving us a range of 18.6 minutes to 42.6 minutes, which is outlined in Table 2 (Note: although the experimental data is for the thigh and not the deltoid, both are similar in their tissue composition and therefore we assume that Table 2 is applicable). In order to determine the skinfold thickness for the deltoid, we averaged the means from the male and female ‘recreationally active’ groups. This gives us an average skinfold thickness for the deltoid of 18.3mm (Table 5, Appendix).

Table 2 gives the cooling times at a point 0.01m below the adipose tissue. As stated above, we are assuming that the adipose tissue is approximately 0.0183m thick for the deltoid. Therefore, the point we are modeling should be 0.0283m deep. Our point is 0.0283m from both the top and back of our modeled shoulder muscle, and has the coordinates (0.26164, 3.4717, 0.2927). Figure 9 is the temperature profile for this point below the surface.

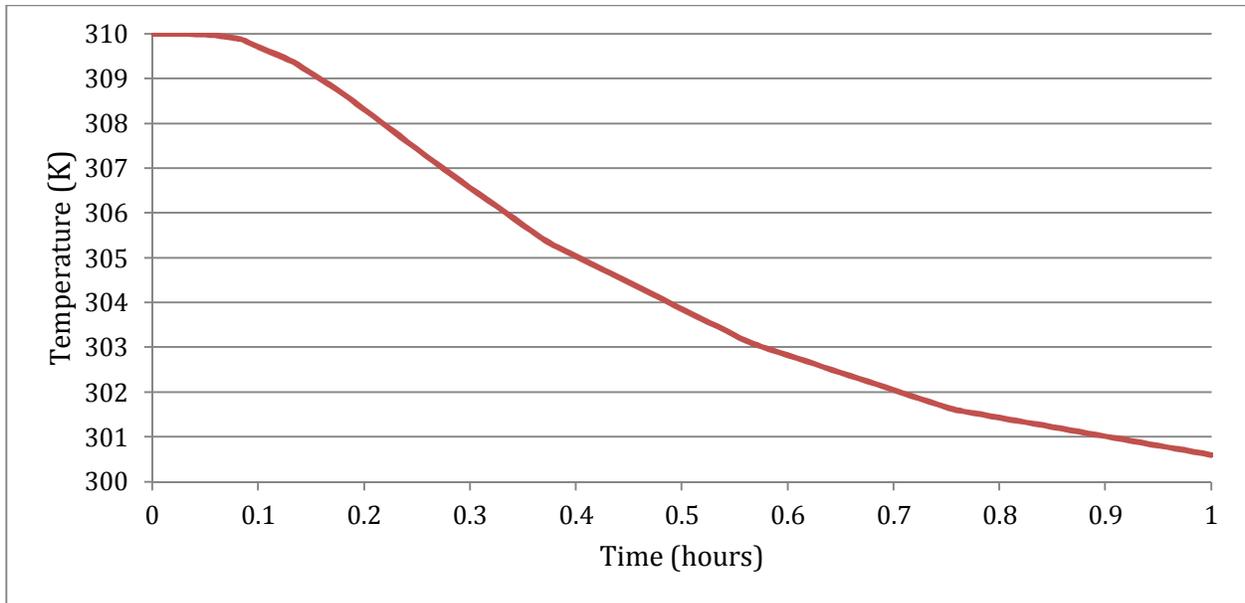


Figure 9 - Temperature versus time plot. Time to cool the tendon by 7 K at a point within the Muscle (0.26164, 3.422, 0.301) occurs at 34.5 minutes.

We are searching for the time at which the temperature at the point has decreased by 7K, which is 303K as this region is initially at 310K. Using the graph we find that this point occurs at 0.575 hours. As 0.575 hours is equivalent to 34.5 minutes, and 34.5 minutes falls within the region of 18.6 to 42.6 minutes as predicted by the above tables, the experiment validates our model.

CONCLUSION & DESIGN RECOMMENDATIONS

RESTATEMENT OF OVERALL OBJECTIVES

Supraspinatus tendonitis is a dynamic injury caused by overuse of the abductor muscles of the shoulder. This injury is common in athletes, especially swimmers, tennis and baseball players who utilize overhead arm movement to an appreciable extent. The purpose of our experiment was to design a three-dimensional model of the shoulder that would allow us to model heat transfer as instigated by cold ice pack therapy. Our ultimate goal was to identify the treatment time necessary to lower the temperature of the inflamed tendon to within 1 degree of body temperature (310K).

INITIAL RESULTS CONCLUSIONS

Based on our model, we found that traditional icing procedures, which consisted of cooling for 20 minutes, were insufficient to successfully bring the supraspinatus tendon and bursa sac down to body temperature. We believe that physical therapists and physicians alike prescribe these short icing procedures as they somewhat numb the area, reducing radiated pain in the shoulder region. It does not, however, affect the inflammation of the tendon itself. This suggestion is usually coupled with rest of the muscle, to allow for inflammation to decrease and the tendon to repair itself. In addition, ice pack therapy could create a cascade effect for reducing inflammation. By reducing inflammation in the outer musculature of the shoulder, external pressure is partially released on the tendon. This therefore suggests that cooling does not necessarily have to bring the tendon down to body temperature. Based on our model, the length of time suggested for cooling therapy is beyond the safe threshold for treatment. Other treatment options may be more realistic, such as anti-

inflammatory drugs coupled with cold therapy. Overall, the previous suggestions by doctors are unrealistic and this protocol for treatment of tendinitis in the shoulder region should be changed.

OPTIMIZED RESULTS CONCLUSIONS

Optimization of the model led to the conclusion that after 96 minutes of continuous cooling, the tendon reached a temperature within 1 K of body temperature. This is an unrealistic treatment time that prompts further revision of our model and viable treatment prospects for practical reasons, as well as pain and tissue damage considerations. However, our model does provide a legitimate basis for expanded medical research in the shoulder region, including other heat transfer problems and even pharmacological studies. While this model is not yet ready for clinical applications, it does provide an accurate depiction of the mechanisms of heat transfer of the shoulder, and the importance of temperature dependent blood flow considerations.

SUGGESTED FUTURE RESEARCH

Future research into our computational model should involve improving accuracy and utilizing enhanced computation power. As indicated by our sensitivity analysis, varying properties of bone had a limited effect on overall heat transfer within the shoulder. The majority of influence is dependent upon muscle properties, explicitly the density and specific heat parameters. By improving our existing model to include more complex geometries involving individual musculature in the region, as well as a separate skin entity from the muscle, our model would prove more accurate for heat transfer analysis as well as other diagnostic applications.

Future studies could also take advantage of known major nerves in the shoulder region, and utilizing temperature changes to predict pain response on an average patient basis.

We would also recommend attempting a model that looks at the treatment option of leaving cold therapy on for twenty minutes, then removing it for an equal amount of time, and repeating until the supraspinatus tendon reaches 311 K. This experiment was attempted during our modeling, but we could not get a realistic solution from our implementation. We had attempted to calculate flux through muscle alone for twenty minutes, and input this flux as a table in COMSOL for the flux through the boundaries that had the cold therapy applied. We then changed the condition to zero flux through these boundaries for the next twenty minutes. What resulted was an inconsistent heating that led to spots of the surface remaining how while the rest of the model reached a lower and constant temperature. There was no temperature gradient, which indicated that the solution was faulty. We did not have enough time to reevaluate our approach in solving, and thus we would suggest that someone look into that in the future because this could limit exposure time of the cold therapy, and possibly reduce pain.

APPENDIX A: MATHEMATICAL STATEMENT OF THE PROBLEM

PARAMETERS

Based on the literature, we determined the following values for our model parameters:

Table 4 - Summary of Parameters. The below variables were implemented in COMSOL model for solution.

Tissue	Parameter	Value	Data Source
Blood			
	$\rho c_p \dot{V}$ (W/m ³ K)	$3469*(0.45 + 3.55*\exp(-((T-45)^2)/12))*(310-T)$	(Datta, 2013)
Bone			
	k (W/mK)	0.58	(Tikuisis, 1991)

	ρ (kg/m ³)	1700	(Kingsmill, 1998)
	c_p (J/kgK)	440	(Davidson, 1999)
Muscle/Skin/Tendon/Bursa			
	k (W/mK)	0.39	(Hubbel, 2004)
	ρ (kg/m ³)	1071.5	(Barber, 1970)
	c_p (J/kgK)	3468.75	(McCall, 2010)

INPUT PARAMETER VALIDATION

All values used in our model were experimentally determined in various other human studies. We decided to combine material properties and parameters for Muscle/Skin/Tendon and Bursa due to their strong similarity in properties in comparison to bone and blood. We averaged their value to determine a more accurate representation of their heat transfer properties. For the bio-heat equation, we utilized a combined blood perfusion coefficient as defined in Problem 6.4.15 in the text.

TIME

The time constraint we chose was based on trial and error and multiple runs of our COMSOL model. We decided to model the treatment under a normal cold-pack usage cycle (15 min) versus the time needed for the supraspinatus tendon to reach body temperature (310K). While wearing a cold pack for that length of time may have a detrimental effect on skin, we were curious as to how long such a treatment would be necessary to bring down the bursa temperature.

CALCULATIONS

THERMAL DIFFUSIVITY CALCULATION:

$$\alpha = \frac{k}{\rho C_p} = \frac{0.39 \left[\frac{W}{mK} \right]}{1074 \left[\frac{kg}{m^3} \right] * 3469 \left[\frac{J}{kgK} \right]} = 1.047E - 7 \text{ m}^2/s$$

TIME FOR COLD THERAPY CALCULATION:

- Assume L = length between tendon and skin surface = 0.0241 m

$$t^* = \frac{L^2}{\alpha} = \frac{0.0241^2}{1.047E - 7} = 5548.53 \text{ seconds} = 92 \text{ minutes}$$

APPENDIX B: SOLUTION STRATEGY

MESH DEVELOPMENT

We are doing a free triangular mesh with a fine element sizes. The meshing will be distributed evenly as there was no good way to have more elements near the domain with the ice pack.

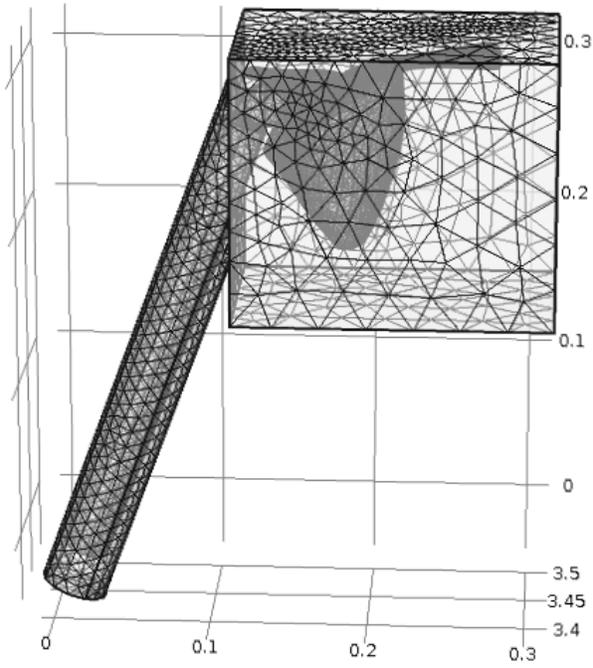


Figure 10 - Mesh development of 3D model. The mesh was created using unstructured free triangular fine meshing parameters.

MESH CONVERGENCE ANALYSIS

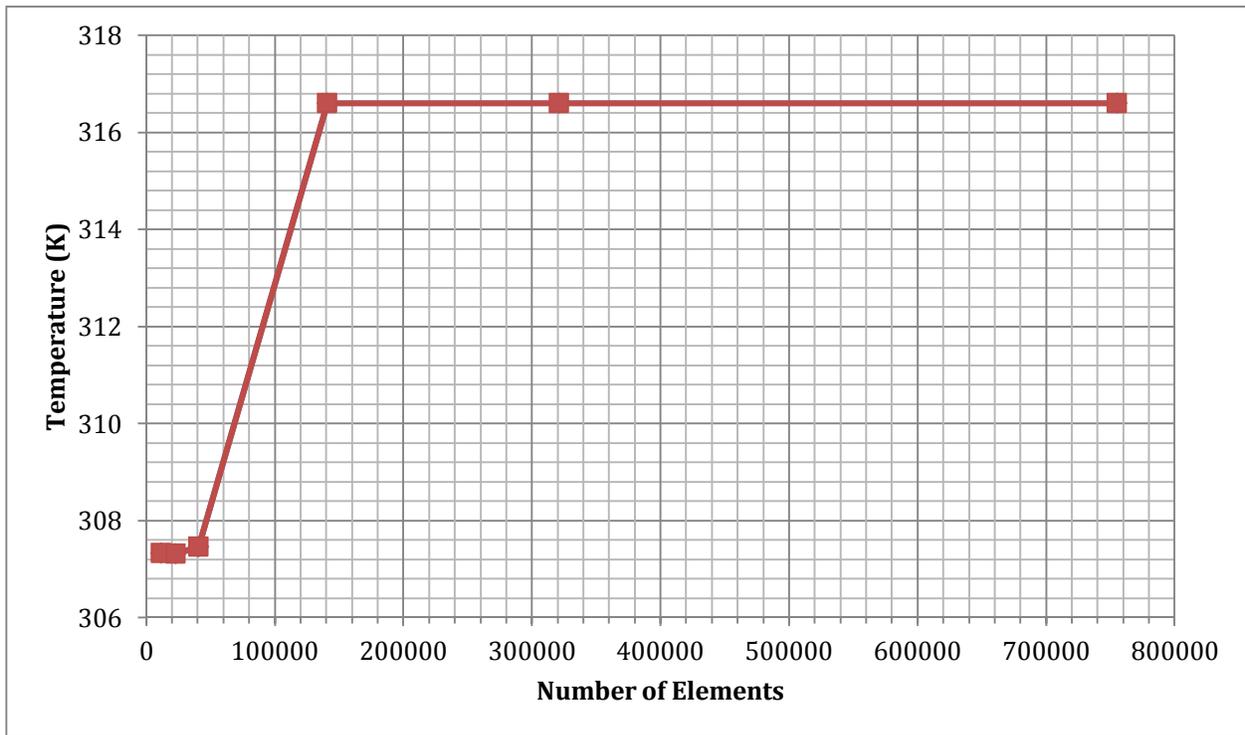


Figure 11 - Plot of mesh convergence at time $t=15$ minutes. The figure indicates broad differences between coarser and fine meshes, but shows convergence once the mesh is fine or smaller.

Our mesh convergence shows that the values for how temperature is varying near the bursa sac and tendon entity change very little when using anything with smaller elements than are present in the fine mesh. When using coarser meshes, the values change drastically because the mesh is more spread out, so the point that is being evaluated ends up being further from the bursa, so the temperature starts at the normal body value of 310 instead of the bursa/tendon value of 318 K. For this reason, we need a finer mesh to make sure there is an evaluation point closer to the point of interest. Because there is no difference between the three levels of fine mesh, we chose the one with fewer elements to speed up computing time and settled on fine for our mesh.

Table 5 - Experimental data outlining the skinfold thicknesses of college-aged students separated by gender and approximate athletic ability. This table was used for its average skinfold thickness of recreationally active college students, and gender was averaged.

TREATMENT SITE	MALE ATHLETES		FEMALE ATHLETES	
	NCAA Div 1 Athletes ($n = 157$)	Recreationally Active College Students ($n = 108$)	NCAA Div 1 Athletes ($n = 39$)	Recreationally Active College Students ($n = 85$)
Thigh	13.0 ± 5.4	15.8 ± 7.2	19.5 ± 4.7	25.7 ± 8.5
Deltoid	16.2 ± 2.4	15.0 ± 5.8	18.8 ± 5.6	21.6 ± 13.1
Scapula	13.4 ± 6.0	14.4 ± 5.4	11.6 ± 4.0	15.6 ± 5.8
MCL*	11.0 ± 4.4	13.1 ± 5.6	17.3 ± 4.6	22.0 ± 7.7

Table 6 – Experimental data relating skinfold thickness in the thigh to time it takes to decrease muscle temperature by 7k at a point 1 cm below the adipose tissue. The table is used to show the time range acceptable for tissue to decrease by 7 K based on different skinfold thicknesses.

SKINFOLD THICKNESS (MM)	TIME TO DECREASE TISSUE TEMP 44.6°F (7°C)
0-10	8.0 ± 3.4 min
11-20	23.3 ± 6.7 min
21-30	37.8 ± 9.6 min
31-40	58.6 ± 11.7 min

APPENDIX C: SUPPORTING VISUALS

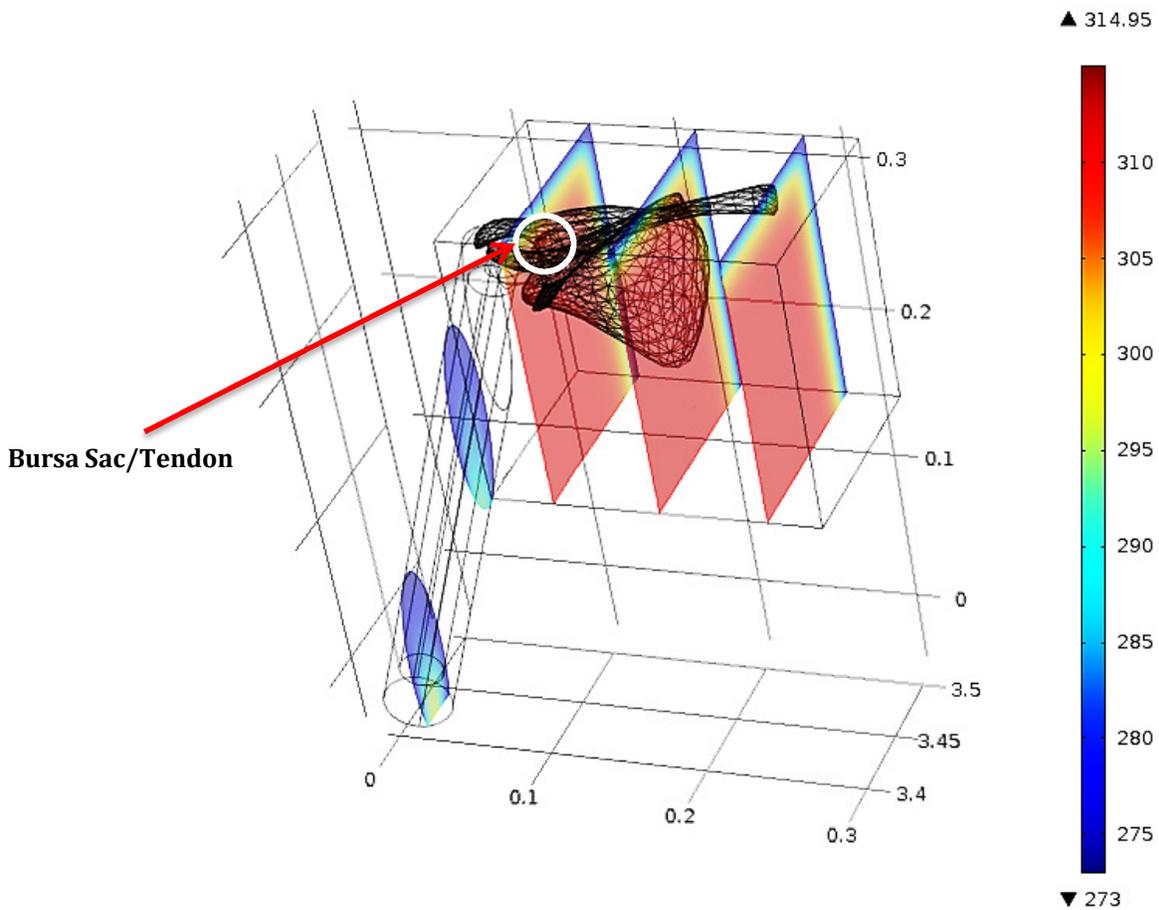


Figure 12 - 3-D Slice Plots at 20 minutes. These 5 slice plots were generated by COMSOL and show the temperature profile at five different locations on the model at t = 20 min

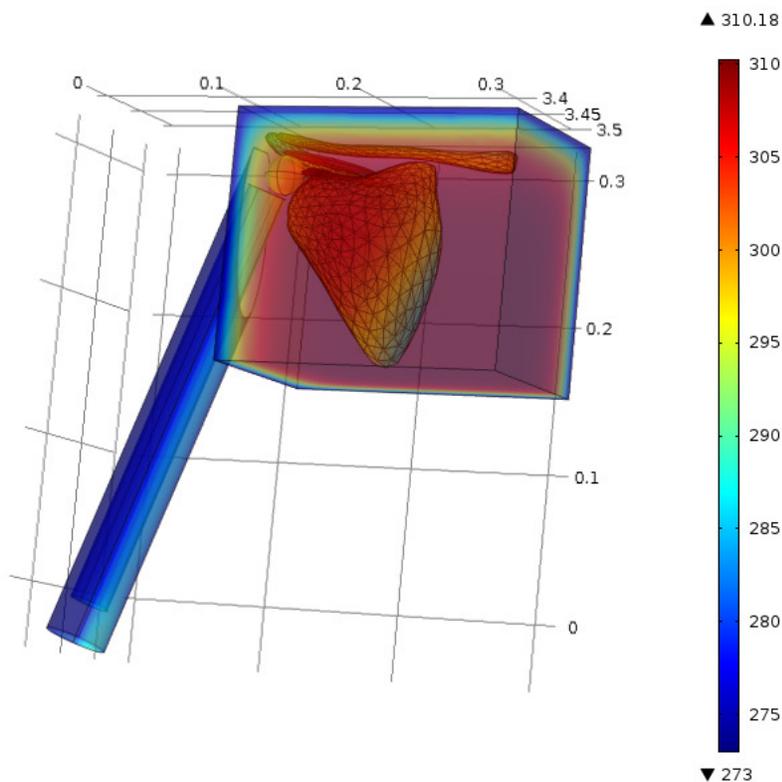


Figure 13- Surface Temperature plot of the entire geometry after three hours of cold therapy. Indicates the pervasiveness of heat transfer throughout the model and ultimate cooling of the tendon to 310 K. Notice skin temperature of the arm and top of shoulder reaches 275 K.

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