LENGTH OF PRODUCTIVE LIFE OF DAIRY COWS:
I. JUSTIFICATION OF A WEIBULL MODEL

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ABSTRACT

The length of productive life of 39,683 grade Holstein cows milked in 150 large herds in New-York State between 1981 and 1986 was analyzed by modeling their hazard, which is a measure of their probability of being culled. Animals still alive when the analysis was performed were assigned a "censored" record equal to the current value of their length of productive life. The concept of hazard allows an adequate statistical treatment of these censored records. The proportional hazards models considered involve a baseline hazard function and log-linear time-dependent explanatory variables affecting culling rate. These include a herd x year effect, a stage of lactation x lactation number effect and a within herd and lactation level of milk production effect (normalized rank based
on 305 ME milk yield).

A semi-parametric analysis - for which the baseline hazard function is completely unspecified (Cox's regression) - showed that the assumption of proportional hazards is appropriate, that all the effects in the model are highly significant and that the baseline hazard function can be closely approximated by a Weibull hazard function of the form $\lambda(t) = \lambda P (\lambda t)^{P-1}$. Such an approximation greatly simplifies computations and facilitates further genetic and nongenetic studies on longevity of dairy cows.

Key words: Holstein-Friesian, stayability, nonlinear model, Cox model, Weibull model, length of productive life

INTRODUCTION

Longevity is a highly desirable quality of a dairy cow: total profit and profit per day of life have been shown to be related to longevity (1, 2, 17, 21): when herdlife increases, fewer heifers need to be raised and replacement costs are decreased. But culling decision usually occurs long before senescence. Consequently, geneticists have developed the concept of stayability (or survivability) to characterize the capability for a cow to remain productive in her herd over time (13, 14, 22).

When reason for leaving the herd is not considered, this ability can be referred to as true stayability. It also measures the dairyman's perception of the value of the cow. However, it may be of interest to distinguish between disposal mostly beyond the control of dairy managers such as the sale of a profitable but sterile cow (involuntary culling) and voluntary disposal of a healthy but not profitable cow. Van Arendonk (26) showed that if involuntary culling is decreased, a higher voluntary culling rate can be applied, resulting in a
larger profit for the farmer. The aptitude to delay involuntary disposal will be
called functional stayability.

Many different measures of stayability have been proposed: age, number of
lactation, length of productive life or lifetime production at time of disposal.
Computation of these measures requires the knowledge of the culling date. But it
is usually impossible or useless to wait until all the animals of interest have
disappeared from the herd before starting any analysis. To overcome this
difficulty, early indicators of true stayability such as the proportion of cows still
alive at a given time $T_0$ (e.g. 48 months) or at the beginning of a given lactation
number have been used. But such measures suffer severe drawbacks: many
different $T_0$ can be chosen and a substantial loss of information exists: cows
culled one day or one year before $T_0$ are treated alike. Also, linear models are
not adequate to analyze such binomial data: at $T_0$, a cow is alive or not (8, 16).

A continuous measure such as the length of productive life (LPL) seems more
desirable. LPL is defined to include animals still alive at the time of the analysis.
The corresponding records, which represent a lower bound of the eventual
LPL's are called censored records and the existence of censored records is
referred to as censoring. Records from cows sold for dairy purposes are also
considered as censored (13).

Specific statistical methods dealing with censoring have been developed (7,
18, 20) but because they are quite complex, they have not been used by animal
breeders until recently (15, 22, 23, 24, 26). The objective of this paper is to show
in which direction the models proposed by these authors may be improved to
more properly describe the culling process as it occurs on the farm. A particular
approach on how functional stayability can be approximately estimated is also
suggested.
MATERIALS and METHODS

General approach

The analysis of censored survival data is based on the use of special modeling distributions such as the hazard function. If T is the nonnegative random variable representing the failure time of a cow, the hazard function \( \lambda(t) \) is defined as:

\[
\lambda(t) = \lim_{\delta \to 0} \frac{\text{Prob}(t \leq T < t + \delta \mid T \geq t)}{\delta}
\]

\[ (18) \]

i.e. \( \lambda(t) \) specifies the instantaneous rate of failure at time \( t \), conditional upon survival up to \( t \). Here, hazard is intuitively synonymous with relative culling rate. In many cases, the exact nature of the density function \( f(t) \) or the survivor function \( S(t) = \text{Prob}(T \geq t) \) is not known in the population under study but some information is available on how the failure rate \( \lambda(t) \) changes over time. Note also that:

\[
S(t) = \exp \left[ -\int_0^t \lambda(u) \, du \right]
\]

\[ (2) \]

The most popular regression model based on the concept of hazard function is the Proportional Hazards (PH) model, for which the hazard \( \lambda(t) = \lambda(t; z_i) \) for animal \( i \) is the product of a time-dependent term \( \lambda_0(t) \) related to the aging process (the baseline hazard function) and a "stress-dependent" term \( e^{z_i' \beta} \) representing how the vector of covariates \( z_i \) influences failure rate, independently of time (5). Hence:

\[
\lambda(t; z_i) = \lambda_0(t) e^{z_i' \beta}
\]

\[ (3) \]

Therefore, the hazards of two animals \( i \) and \( i' \) are assumed to be always proportional with hazards ratio \( e^{(z_i - z_{i'})' \beta} \). The baseline hazard function can
have a known parametric form; e.g.

if \( \lambda_0(t) = \lambda = \text{constant} \), the corresponding

**baseline survivor curve** is exponential: \( S_0(t) = \exp(-\lambda t) \). If \( \lambda_0(t) = \lambda p (\lambda t)^{p-1} \)

for some \( \lambda \) and \( p \), \( T \) follows a **Weibull** distribution: \( S_0(t) = \exp(- (\lambda t)^p) \).

Using the concept of "partial likelihood", Cox (5, 6) proposed a method for the estimation of the effects \( \beta \) in the PH model, which does not require any assumption about the form of \( \lambda_0(t) \). In the Cox's regression, estimates of \( \beta \) are obtained by maximizing the logarithm \( L_1 \) of a "partial" likelihood of the form:

\[
L_1 = \sum_{i \in \{\text{unc.}\}} \left[ z_i \log \left( \frac{1}{\sum_{m \in \text{Risk}(T_{[i]})} e^{z_m \beta}} \right) \right] \tag{4}
\]

where: \( T_{[1]} < \ldots < T_{[n]} \) are the ordered \( n \) observed (uncensored) failure times;

\( \{\text{unc.}\} \) is the set of uncensored cows;

\( \text{Risk} (T_{[i]}) = \{ m : T_m > T_{[i]} \} \) is the set of animals at risk at \( T_{[i]} \), i.e. alive just prior to \( T_{[i]} \).

If, as it often happens in practice, failure times are recorded in a way allowing for ties between some individuals - e.g. same number of days of productive life - an approximation of the partial likelihood is given by:

\[
L_2 = \sum_{i \in \{\text{unc.}\}} \left[ \sum_{k_i \in D(T_{[i]})} z_{k_i} \beta \right] - d_i \log \left( \sum_{m \in \text{Risk}(T_{[i]})} e^{z_m \beta} \right) \tag{5}
\]

(Peto, in (5))

where \( d_i \), the \( k_i \)'s and \( D(T_{[i]}) \) are the number, the indices and the set of cows actually failing at \( T_{[i]} \).
In some cases, the PH assumption is not tenable for all the factors of interest. A possible alternative which retains the simplicity of the PH model and known as stratification is the definition of a different baseline hazard function $\lambda_{0j}(t)$ for each level $j$ of one particular factor. This was the approach chosen by Smith (22, 23) in his analysis of age at disposal of dairy cows: records were "stratified" by year of birth of the cows. Problems related with Smith's model are discussed in (12).

A much more powerful generalization of the PH model is the use of time-dependent covariates. In that case, the exponential part in (3) is allowed to vary with time:

$$\lambda(t; z(t)) = \lambda_0(t) e^{z_i(t) \beta}$$  \hspace{1cm} [7]

Estimation of $\beta$ in a Cox's PH model with time-dependent covariates can lead to extremely tedious computations: at each failure time $T[i]$, the values of $e^{z_m(t) \beta} = e^{z_m(t) \beta}$ in [4] or [5] vary. However, if $z_m(t)$ is a very simple function of time, such as a piecewise constant function, $\sum m \in \text{Risk}(T[i]) e^{z_m(t) \beta}$ in [4] or [5] can be computed in a more efficient way than in the general case (12). In that situation, it is assumed that within each interval for which $z_m(t)$ is constant the PH assumption holds but that the hazards ratio changes from one such interval to the next. But even then, computations are still very tedious and such a model cannot be applied to very large data sets necessary for routine sire evaluation.

On the other hand, when the baseline hazard function $\lambda_0(t)$ in [1] has a
parametric form, estimation of $\beta$ and $\lambda_0(.)$ is generally easier (7, 18). Consequently, the following approach is chosen here: on a data set of moderate size, the Cox's version of the PH model is fit and the baseline survivor function $S_0(t)$ is estimated. Then the PH assumption is checked and the estimate of $S_0(t)$ is compared - using goodness-of-fit and cross-validation tests - to a known parametric form: the Weibull distribution. This choice results from the simplicity of the Weibull survivor function ($S_0(t) = \exp(-\lambda t^\beta)$) allied with its flexibility: a Weibull regression can model constant ($\beta = 1$), increasing ($\beta > 1$) and decreasing ($\beta < 1$) hazard rates. It is the simplest generalization of the exponential survivor distribution. If an approximation of $\lambda_0(t)$ and $S_0(t)$ with a parametric model is possible, further analyses would be greatly facilitated.

Data set

Only grade cows are considered here: culling policies in grade and registered herds are known to be markedly different and should not be treated alike: registered cows are kept longer, are culled less on milk production and more on type or for dairy purposes (10, 11).

In the Northeast Dairy Record Processing Laboratory (DRPL) AI sire file, the exact failure date of cows culled before 1981 was not recorded when failure occurred after more than 305 days of lactation. To avoid the problems associated with such truncated records (12), the period of study was restricted to January 1981 - February 1986, i.e., to the years for which complete information is available.

The data set includes the length of productive life (culling date - first parturition date, in days) of 39,683 grade Holstein cows milked in 150 large herds in New-York state. Admittedly, this data set is not representative of the
whole grade population but this restriction to large herds (190 to 849 cows per herd over the whole period) is a compromise between the need to constrain the estimation problem to a reasonable size by limiting the number of herds and the desire to base conclusions on more precise estimates of the herd x year effect. LPL records of cows sold for dairy purposes or assumed alive on March 1, 1986 were considered as censored: 47% of the total number of records were censored.

Models

Our principal objective was to describe as precisely as possible the main factors affecting the culling process. Two models were envisioned here.

Management practices and culling policies are controlled by the dairy manager and influenced by the herd environment: they are likely to affect the LPL of all the cows in a same herd in a similar fashion. Therefore, a herd effect hj is included in the model and its change over time is simulated by a step function, for which jumps are arbitrarily assumed to occur on January 1, each year.

Stage of lactation is regarded as another essential factor determining the probability for a cow of being culled, i.e., her hazard. For example, during the first months of lactation, milk production is maximum, reproductive status does not affect profitability and salvage value is generally low: culling at that point seems less likely than for cows of the same age but reaching a later stage of lactation. A piecewise constant stage of lactation effect \( p_k(t) \) is defined in order to isolate three "biological periods" ("early", "middle", "end of lactation and dry period").

Finally, two cows may freshen the same day at the same age, one for the xth time and the other for the (x+1)st time. A lactation number effect \( q_l(t) \) is added to treat differently cows managed more or less intensively than others.
The first model *(model A)* is written:

\[ \lambda(t) = \lambda_{jkl}(t) = \lambda_0(t) \exp \{ h_j(t) + p_k(t) + q_l(t) \} \]

where: \( \lambda_0(t) \) is a completely arbitrary baseline hazard function,

\( h_j(t) \) is the \( j \)th herd x year effect,

\( p_k(t) \) is the \( k \)th stage of lactation effect (from day 0 to day 29 after parturition, from day 30 to 249, and from day 250 to the beginning of the next lactation),

\( q_l(t) \) is the \( l \)th lactation number effect (lactation 1, 2, 3 to 5, 6 and more).

Note that \( h_j(t) \) is a function of the calendar time whereas \( p_k(t) \) and \( q_l(t) \) are step functions of biological time, dependent on date of parturition.

Although the estimation of sire genetic merit is our ultimate goal, sire effects are completely ignored in this part: sires are expected to have a rather small effect on the LPL of their daughters. Heritability of stayability is known to be quite low. The other effects described above are intuitively believed to have a more drastic effect on culling rate that the genetic make-up of the cow. Moreover, if sires are to be included in the Cox's PH model, fewer herds have to be selected in order to constrain the estimation problem to a reasonable size. In such a reduced date set, inevitably, each sire would have very few daughters and the precision of their estimate would be very poor. The adequacy of the model would be difficult to assess.

Low milk yield has been described as the major reason for voluntary disposal of a cow. Hence, a correction of LPL for milk production should reveal differences between animals for reasons for disposal other than production: differences in voluntary culling due to type, old age or general health and above
all, differences in involuntary culling caused for instance by infertility, illness, chronic mastitis, etc. Therefore, such a correction would represent a first step toward the study of what was previously defined as functional stayability.

A time-dependent "within herd and lactation level of milk production effect" \( r_m(t) \) is added to the previous model to form model B:

\[
\lambda(t) = \lambda_{jklm}(t) = \lambda_0(t) \exp \{ h_j(t) + p_k(t) + q_l(t) + r_m(t) \} \quad [9]
\]

\( \lambda_0(t), h_j(t), p_k(t) \) and \( q_l(t) \) are defined as in [8]. \( r_m(t) \) is the effect associated with the \( m \)th class of milk production. These classes are defined in a specific way trying to simulate the actual voluntary culling process as it is performed on the farm. In particular, it is believed that relative milk production (compared to the other cows present in the same herd at the same time) plays a larger role in the culling decision than actual yield. In practice, each record (lactation) of a cow is assigned a milk production class in the following way, illustrated in figure 1:

(i) 305 days Mature Equivalent (305ME) records are sorted within herd and year separately for first and later parities.

(ii) ranks within herd-year are standardized by computing their expected normal scores.

(iii) these expected normal scores are divided into 9 classes of equal importance (each of probability 11.1%).

Records for which the 305ME production is not known (mainly lactations not terminated at the end of the study period) are assigned to a tenth group.

**Goodness-of-fit and model validation**

The adequacy of the two models proposed was checked in several ways:

1) A test for the proportional hazards assumption is based on the concept of
generalized residuals developed by Cox and Snell (4). Generalized residuals for observations \( T_i \) are functions \( e_i = g_i( T_i; \beta, z_i ) \) such that the \( e_i \)'s are independent and identically distributed, with known distribution. For example, in the case of failure times, it can be shown that the random variable:

\[
e_i = \int_0^{T_i} \lambda(u; z_i) \, du
\]  

follows an exponential distribution with parameter 1 (4). The generalized residual \( e_i \) represents the sum of the hazards that animal \( i \) encountered during its life.

A test of the proportional hazards assumption is obtained by checking whether the *estimated* generalized residuals \( \hat{e}_i \) constitute a random sample from a unit exponential distribution, where:

\[
\hat{e}_i = \int_0^{y_i} \hat{\lambda}(u; z_i) \, du
\]  

with \( y_i = T_i \) if animal \( i \) is uncensored or \( y_i = C_i \) (censoring time) if the animal \( i \) is censored.

In practice, the ordered (uncensored) \( \hat{e}_i \) are plotted against the expected order statistics of a unit exponential with the same censoring pattern. If the resulting line strongly deviates from a straight line with slope 1 and going through the origin, the proportional hazards assumption is rejected.
2) The need for the inclusion of a particular group of covariates in the model is checked by a forward stepwise procedure based on the large sample likelihood ratio test (19). If \( \hat{\beta}(1) \) represents the maximum likelihood (ML) estimate of \( \beta(1) \) in a reduced model including only covariates \( z(1) \) and if \( \hat{\beta} \) denotes the ML estimate of \( \beta = (\hat{\beta}(1), \hat{\beta}(2)) \) in the extended model including covariates \( z = (z(1), z(2)) \), the procedure to test \( H_0 : \beta(2) = 0 \) is to compare the value of 
\[
2 \left[ L_2(\hat{\beta}) - L_2(\hat{\beta}(1)) \right]
\]
to a \( \chi^2 \) distribution with \( v \) degrees of freedom, where \( v \) is the dimension of \( \beta(2) \).

3) In the case of the Weibull proportional hazards model, we have:
\[
S_0(t) = \exp \left[ - (\lambda t)^{\rho-1} \right]
\]
and therefore:
\[
\log \left[ \log S_0(t) \right] = \rho \log t + \rho \log \lambda \tag{13}
\]
Hence the adequacy of the Weibull model in a study of LPL records can be assessed by looking at the quality of the regression of \( \log \left[ - \log \hat{S}_0(t) \right] \) on \( \log t \), where \( \hat{S}_0(t) \) is the estimated baseline survivor curve, as computed in (6). The slope and the intercept of the regression line also provide crude estimates of the Weibull parameters \( \lambda \) and \( \rho \) (20).

4) To definitely confirm the validity of the Weibull model as an approximation of the Cox's semi-parametric model, a **cross-validation** test was performed: two subsets S1 and S2 of the initial data set were randomly created and Weibull versions of models A and B - i.e., for which the baseline hazard function is a Weibull hazard - were fit on both subsets. The following likelihood function of the observed failures given the model (7,18) was maximized:
L = \left[ \prod_{m \in \{unc.\}} \lambda(y_m ; z_m(y_m)) \right] \left[ \prod_{m' \in \{unc., cens.\}} S(y_{m'} ; z_{m'}(t)) \right] \quad [14]

where \{unc.\} and \{cens.\} are the sets of uncensored and censored cows.

The two sets of estimates for \( \beta \), \( \rho \) and \( \lambda \) are then compared and for each subset, generalized residuals are computed using the ML estimates of \( \beta \), \( \rho \) and \( \lambda \) obtained from the same subset or from the other. The distribution of both sets of generalized residuals is then compared to that of a censored unit exponential. If the model is correct, the same fit should be observed whatever the origin of the estimates.

At the same time, a check for the existence of interactions between Stage of Lactation (SL) and Lactation Number (LN) effects in models A and B was performed through a slight modification of these models: a SL x LN effect \( g_{kl} \) is defined to replace \( p_k \) and \( q_l \) in [8] and [9]. Models A and B are modified as:

\[
\lambda(t) = \lambda_0(t) \exp \{ h_j(t) + g_{kl}(t) \} \quad \text{(Model A*)} \quad [15]
\]

\[
\lambda(t) = \lambda_0(t) \exp \{ h_j(t) + g_{kl}(t) + r_m(t) \} \quad \text{(Model B*)} \quad [16]
\]

In absence of interaction, \( g_{kl}(t) = p_k + q_l \) for all \( k \) and \( l \).

Nine SL x LN classes are defined (3 SL classes defined as previously and only 3 LN classes - first, second, third and more). In contrast with the Cox's model, only records from cows calving \textit{for the first time} after January 1, 1981 can be used in the Weibull model: none of these cows had started a sixth lactation before the end of the study period (February 1986). S1 and S2 include respectively 13,797 and 13,842 LPL records and two-thirds of these records are censored. This proportion of censored records is quite large: it illustrates the need for a different statistical treatment of the two types of records. Indeed, some herd-year "subclasses" include only censored records. It should not be
considered that these subclasses do not contain any information. The absence of uncensored records simply indicates that the average hazard was particularly low in those herd-years.

RESULTS and DISCUSSION

The estimation of 757 effects (750 herd x year + 3 SL + 4 LN effects) and 777 effects (757 + 20 within herd x lactation level of production effects) for the Cox's models A and B was performed by maximum likelihood using a very efficient method for numerical optimization, known as the BFGS algorithm (9), chapter 8). This algorithm mimics the well known Newton's algorithm but replaces the exact evaluation of the matrix of second derivatives of the log-likelihood by an approximation of this matrix in some optimal way.

For the estimation of these same effects when the Weibull models A* and B* are fit, the matrix of second derivatives of the likelihood L in [14] is very sparse and its inversion is simple: therefore, the Newton's algorithm can be used.

The likelihood ratio tests used to check the importance of the factors in models A and B reveal that all the factors included have a very highly significant effect (p<0.001) on a cow's hazard. Estimates of herd x year effects range from -3.96 to 1.32. Note that an estimate of 1.0 means that in the herd considered, the relative culling rate is e^{1.0} = 2.7, i.e. a cow in this herd is 2.7 times more likely to be culled at any time t than a cow in an "average" herd. Figure 2 presents the distribution of herd x year estimates for model A.

(Figure 2 and table 1 here)

The estimates of the stage of lactation and lactation number effects are presented in table 1. As expected, relative culling rates increase considerably with stage of lactation. A cow finishing her lactation has a probability of being
culled \( \exp [0.77 - (-0.63)] = 4.06 \) times larger than a cow of the same productive age starting her lactation. Relative culling rate is also larger in first lactation than later on, especially when differences in milk production between young and old cows are taken into account (model B). A cow finishing her first lactation will be at a much higher risk of being culled than another of the same age in the middle of her second lactation.

(figure 3 here)

Within herd x lactation level of production (WHLP) effects for model B are presented in figure 3. The two curves for first and later lactations are smooth, monotone and almost parallel: there is no interaction between this factor and lactation number. WHLP effects increase continuously at an approximately quadratic rate. But as far as the relative culling rate \( \exp(\hat{r}(t)) \) is concerned, the increase is slow and almost linear from production classes 1 to 7 and then very sharp for the last two classes.

Cows in the last milk production class in first lactation are about 10 times more likely to be culled at any time \( t \) than cows in the first class and almost 4 times more than cows in the seventh class.

This trend was expected but these results suggest that dairymen actually base their voluntary culling decision - maybe only intuitively - on a criterion closely related to the standardized - and therefore artificial - 305ME milk production.

For the tenth class of milk production - which corresponds to cows with unknown 305ME records - the estimates of WHLP effects are extremely low (-1.98 in first lactation, -1.20 in later lactations) because most of the records assigned to this class are from the last lactation of censored cows and therefore, very few failures are actually observed in this category.
A regression of the estimated generalized residuals $\hat{e}_i$ on the expected order statistics $o_i$ of a censored unit exponential distribution leads to the following equations:

$$
\hat{e}_i = -0.0003 + 1.005 o_i \quad R^2 = 0.9997 \quad \text{for model A}
$$

$$
\hat{e}_i = -0.013 + 1.033 o_i \quad R^2 = 0.991 \quad \text{for model B}
$$

The agreement with theoretical prediction when a proportional hazards model is adequate is excellent, especially for model A. The power of such a graphical test based on generalized residuals is unknown. Cox and Oakes (7, p109) warn against an ill-considered positive interpretation of this kind of test for large data sets. However, in a preliminary analysis with some truncated - and therefore incorrect - records, this same test clearly detected a large discordance with the proportional hazards assumption (12). It can be concluded at least that there is no evidence here of a departure from the proportional hazards situation.

The slightly less satisfying behavior of the observed residuals in model B is probably due to an incorrect treatment of the animals with no 305ME record (grouped in the tenth level of production class): the hazard of these animals is compared with the hazard of other cows whose LPL record is adjusted for differences in milk production. However, this discrepancy is rather small: only $0.6\%$ of the residuals deviate significantly from their expected value (see (12) p133).

A weighted regression of $\log [-\log \hat{S}_0(t)]$ on $\log t$ gives the following equations:

$$
\log [-\log \hat{S}_0(t)] = -11.20 + 1.48 \log t \quad R^2 = 0.991 \quad \text{for model A}
$$

$$
\log [-\log \hat{S}_0(t)] = -12.88 + 1.69 \log t \quad R^2 = 0.989 \quad \text{for model B}
$$
Therefore, the baseline hazard function can be well approximated by a Weibull hazard function. The values of the crude estimates of $p$ (1.48 and 1.69) show that the baseline hazard of a cow increases with productive age. Estimates for $\lambda$ are $\lambda = 5.3 \times 10^{-4}$ and $\lambda = 4.9 \times 10^{-4}$ respectively.

Figure 4 presents the values of the estimates of $p^{-1} \beta$ for the SL x LN effects when Weibull model $A^*$ is fit on the two subsets S1 and S2. The estimates obtained for both models are consistent, except for the first period of the first lactation. Indeed, the gap between the two estimates is easily explained by the difference for the number of cows actually failing in S1 and S2 (48 vs 68). This difference is entirely due to sampling. More interestingly, figure 4 shows that an interaction exists between SL and LN: in first lactation, cows are comparatively at a higher risk at the beginning and the middle of their lactation.

Finally, table 2 presents the regression equations of generalized residuals $\hat{e}_i$ for animals in S1 and S2 on the expected order statistics of a censored unit exponential distribution, when models $A^*$ and $B^*$ are fit and when estimates for $\beta$, $p$ and $\lambda$ are obtained either from S1 or S2.

Clearly, the agreement between predicted and observed values is excellent for model $A^*$: very similar results are obtained whatever the origin (S1 or S2) of the estimates used to compute the residuals. For model $B^*$, the agreement is not as good. In particular, the slope corresponding to residuals in S1 computed with estimates from S2 is larger than when these estimates are from S1 itself (1.17 vs 1.03). However, regression equations tends to exaggerate this discordance. This is shown in figure 5: only a small fraction of the residuals strongly deviate from the theoretical straight line with slope 1 and more than 90% of the residuals behave as expected. Again, the observed discrepancies probably originate from
the grouping of LPL records for which the 305ME milk yield is unknown.

CONCLUSION

The results presented here suggest that the Weibull regression is well suited for an efficient analysis of LPL data, especially when its flexibility is enhanced by the use of time-dependent regression variables. The choice of a Weibull model largely alleviates the computation burden which limits the use of the Cox’s model with time-dependent variables. Comparisons between populations are facilitated since the baseline hazard function can be described through only 2 parameters instead of a step function with many jumps. Also, the Weibull model is a particular type of proportional hazards model: an intuitive interpretation of the effects in the models remains very simple, through the concept of relative culling rate.

The inadequate treatment in models B and B* of records for which the 305ME milk yield is not known should be easy to correct: approximate 305ME records can be predicted from early lactation tests. When this is not possible (extremely short lactations), it can be assumed that the corresponding LPL records are censored at the end of the previous lactation. In any case, these models give encouraging results about the possibility of correcting LPL records for voluntary disposal.

Finally, models A* and B* can be extended to include transmitting abilities (i.e. sire effects) in order to detect genetic differences in culling rate of sires’ daughters. Note that although the proportional hazards assumption is found satisfactory here, nothing guarantees that this is still the case for sire effects when they are added to models A* and B*. This will have to be considered as an approximation of the true situation. The validity of this assumption requires further investigation.
REFERENCES


Figure 1: Illustration of the definition of milk production classes.

Figure 2: Distribution of herd x year estimates for model A.

Figure 3: Within herd x lactation level of production estimates for model B.

Figure 4: Estimates of \( p^{-1} \beta_v \) computed from data subsets S1 and S2 for model A*. (\( \beta_v \) is the Stage of lactation x Lactation number (SL x LN) effect and \( p \) is the slope parameter of the Weibull baseline hazard function).

Figure 5: Generalized residuals for model B.
Step 1: ranking

herd k
year j
305 ME

Step 2: rank standardization
expected normal score: 0.656

Step 3: categorization
Figure 2

Histogram showing the number of herd x year effects against estimates.
Figure 3

-2 lact.  
- later lact.

estimate

milk production class

1 2 3 4 5 6 7 8 9
Figure 4

The graph shows the estimate of lactation stage over the stages of lactation, with two lines representing S1 and S2. The x-axis represents the stage of lactation, and the y-axis represents the estimate. There are three stages of lactation indicated: lactation 1, lactation 2, and lactation ≥ 3.
Figure 5

Order statistics from a unit exponential distribution

- MLE from S2
- MLE from S1

2.5% of the residuals

6.5% of the residuals

Generalized residuals
Table 1: Maximum likelihood estimates of Stage of lactation and Lactation number effects for model A

<table>
<thead>
<tr>
<th>Stage of lactation</th>
<th>Lactation number</th>
</tr>
</thead>
<tbody>
<tr>
<td>days after parturition</td>
<td>Model A</td>
</tr>
<tr>
<td>0 - 29</td>
<td>-0.77</td>
</tr>
<tr>
<td>30 - 249</td>
<td>0.18</td>
</tr>
<tr>
<td>250 - 0</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 2: Regression equation of generalized residuals $\hat{e}_i$ for models $A^*$ and $B^*$ on the order statistics $o_i$ of a censored unit exponential distribution

<table>
<thead>
<tr>
<th>Generalized residuals estimate from data subset:</th>
<th>from:</th>
<th>$S1$</th>
<th>$(R^2)$</th>
<th>$S2$</th>
<th>$(R^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model $A^*$:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S1$</td>
<td>$\hat{e}_i = 0.007 + 0.981 , o_i$</td>
<td>(0.999)</td>
<td>$\hat{e}_i = 0.003 + 1.002 , o_i$</td>
<td>(0.999)</td>
<td></td>
</tr>
<tr>
<td>$S2$</td>
<td>$\hat{e}_i = -0.003 + 1.019 , o_i$</td>
<td>(0.999)</td>
<td>$\hat{e}_i = 0.005 + 0.986 , o_i$</td>
<td>(0.998)</td>
<td></td>
</tr>
<tr>
<td>Model $B^*$:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S1$</td>
<td>$\hat{e}_i = -0.013 + 1.031 , o_i$</td>
<td>(0.996)</td>
<td>$\hat{e}_i = -0.049 + 1.167 , o_i$</td>
<td>(0.984)</td>
<td></td>
</tr>
<tr>
<td>$S2$</td>
<td>$\hat{e}_i = -0.026 + 1.078 , o_i$</td>
<td>(0.982)</td>
<td>$\hat{e}_i = -0.021 + 1.051 , o_i$</td>
<td>(0.990)</td>
<td></td>
</tr>
</tbody>
</table>
February 12, 1988

Dr. V. Ducrocq
Animal Science Department
Cornell University
Ithaca, NY 14853

Dear Dr. Ducrocq:

Enclosed is a copy of the manuscript you submitted for publication in the Journal of Dairy Science. The reviewers recommend that the paper be published after you have considered their suggestions. If you can not accept all of the reviewers suggestions, please indicate those that were not accepted and the reasons why in your covering letter. Please consider these, make the necessary changes, and return two copies of the manuscript to me at my Columbus address as soon as possible.

Sincerely,

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Editor
Journal of Dairy Science

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