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of IR Backgrounds**

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At the US Army Ballistic Research Laboratory (BRL), Aberdeen Proving Ground, Md., weapon system analysts use background models in order to: 1) establish "clutter" thresholds for firing algorithms; and, 2) to study the masking and false alarm effect of background in their effort to evaluate the performance of various weapon systems. The BRL has received from US Army Engineer Waterways Experimental Station (WES) several large data bases comprised of blackbody temperatures derived from measurements obtained with an IR sensor. The sensor was mounted on a helicopter and scanned in the cross-track direction perpendicular to the direction of flight (in-track). The data consist of temperatures of scene elements (pixels) for a plowed field, a forested area, and a grassy field. The primary objective of this research is to provide a simple mathematical model which provides simulated data that are consistent with descriptive statistics from the original spatially correlated data base. Such statistics include the mean and standard deviation of temperature, and its "energy spectrum". The Mathematical Sciences Institute (MSI) at Cornell University have suggested time series models and a Spatial Moving Average (SMA) model as two approaches to the problem. One long term objective of this type of investigation is to construct a method for relating parameters in the model to physical constants. If successful, the model may then be extended over the diurnal cycle and seasons.

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ABSTRACT

At the US Army Ballistic Research Laboratory (BRL), Aberdeen Proving Ground, Md., weapon system analysts use background models in order to: 1) establish "clutter" thresholds for firing algorithms; and, 2) to study the masking and false alarm effect of background in their effort to evaluate the performance of various weapon systems. The BRL has received from US Army Engineer Waterways Experimental Station (WES) several large data bases comprised of blackbody temperatures derived from measurements obtained with an IR sensor. The sensor was mounted on a helicopter and scanned in the cross-track direction perpendicular to the direction of flight (in-track). The data consists of temperatures of scene elements (pixels) for a plowed field, a forested area, and a grassy field. The primary objective of this research is to provide a simple mathematical model which provides simulated data that are consistent with descriptive statistics from the original spatially correlated data base. Such statistics include the mean and standard deviation of temperature, and its 'energy spectrum'. The Mathematical Sciences Institute (MSI) at Cornell University have suggested time series models and a Spatial Moving Average (SMA) model as two approaches to the problem. One long term objective of this type of investigation is to construct a method for relating parameters in the model to physical constants. If successful, the model may then be extended over the diurnal cycle and seasons.

I. INTRODUCTION

BRL to date has modeled target signatures in a deterministic manner while background signatures have been treated stochastically. The deterministic model for target signatures is appropriate because under a particular set of conditions, the signature is rather well defined and is amenable to a single characterization. The case is not the same for backgrounds, which are many and varied. Thus, the general approach in modeling backgrounds has been to select a data set of a homogeneous scene, to extract pertinent statistics, such as, the mean temperature, the standard deviation, the 'energy spectrum', the correlation between pixels, etc., and finally, to develop a model, which can simulate a 'typical' background segment with these same statistics.

In most smart weapon simulations, the sensor scans across many square meters of background before any target is encountered. During this time, the sensor's signals are processed by a target discrimination circuit that usually includes some sort of adaptive threshold logic. Usually for this type of discrimination, the signal's Root-Mean-Square (RMS) average is developed as a measure of background 'clutter'. Target

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detections occur when the instantaneous sensor output exceeds a threshold value that is proportional to the average of the output signal. The sensor's output signals produced by scanning the modeled background are thus used to provide a basis for setting the detection threshold; this is perhaps the most important function of the background. The stochastic background modeling approach currently being used at the BRL is based on a normal temperature assumption. It is quite well suited to provide a reasonable estimate of average clutter in many situations, even though the temperature distribution of the pixels is not normal. However, a background model also ought to include some provision for sources of false detection. The simple stochastic background model described here is clearly not capable of fulfilling this objective, for there is only a very remote possibility of a false alarm when the detection threshold is set to some multiple of the RMS signal. What is lacking is a means for incorporating some realistic scene features that would constitute possible sources for false alarms.

Given that a target signature model with a reasonable degree of fidelity is mated with a valid stochastic background signature model, it is possible to predict when and where a target detection is likely to occur. Probabilities of target detection can be inferred and the sensor/processor may be analyzed in terms of performance given a target encounter. This has been the BRL approach for many smart weapon simulations. A different approach must be taken if one wants to make some assessment of the smart weapon's capability for rejecting false targets. Ideally, the background infrared signature model used for this type of performance analysis ought to include a realistic characterization of individual scene elements that might confuse the target discrimination logic. Might it be possible to develop a background signature model that is predictive in nature and includes specific features that are potential false targets? BRL would like such a model if the development effort does not cost us too much, and more importantly if the proposed model does not require so many computer resources as to interfere with those needed for the performance simulation.

An alternative to "modeling" the background signatures either deterministically or stochastically would be to use actual scene measurements as inputs to the smart weapon sensor model. This would require that the measured background signatures be compatible with the sensor model in terms of viewing direction, detector wavelength band, and scene pixel size. Although the existing infrared background signature data base is rather extensive, very few of these sources have the requisite characteristics for smart weapons system evaluations that are currently being conducted. One source of data found to be generally compatible with the type of smart weapons that are being investigated at the BRL is the set of infrared scanner measurements of a rural area near Hunfeld, Germany made by the US Army Engineer Waterways Experiment Station (WES). For these measurements WES employed a helicopter-mounted Daedalus infrared scanner operating in the wavelength band of 8.5 to 12.5 micrometers. The scanner was flown over the test terrain at altitudes of 200 and 600 feet. The sizes of the corresponding ground resolution elements were roughly compatible with the 0.1 meter resolution that is optimum for the BRL's smart munition evaluation efforts, and the site of the measurements and the scene content is quite appropriate. The advantage of modeling this data set is that the model can be checked against the actual data in the simulation of a smart weapons concept.

Up to this point the discussion has been confined to simple scenes, e.g., a grassy field, a plowed field, a forested area, etc. Once a suitable model for a simple scene has

been developed, BRL wants to construct arbitrary scenes from these simple scenes. Thus a forested area of any desired size may be placed next to a plowed field. A road may be added to the scene. This compound scene with these three different kinds of textures could then be used in computer simulations of smart weapon concepts. All kinds of different compound scenes of arbitrary geometry and composition could be constructed from the models of the simple scenes. Thus the ability to construct compound scenes from simple scenes is a desideratum of the modeling effort.

II. DATA BASE

In this paper the time series models were applied to the data of the forested area. The data of the plowed field and grassy area have a similar format. The data base for the forested area is composed of 250 rows of temperatures. Each row contains 500 temperature pixels. Thus, for this data set there are 250 rows times 500 columns or 125,000 pixels of temperature. A row of data (500 pixels) represents one 'cross-track' scan of the sensor, which was mounted on a helicopter that flew in a direction perpendicular to the rows ('in-track'). After processing the data with ground truth information, it was concluded that at the 600 ft altitude the in-track (flight direction) dimension of the pixels was 0.3050m whereas the cross-track dimension was 0.1525m. The data are highly correlated both in-track and cross-track.

III. TIME SERIES MODEL

For each row of 500 observations a (p=1, q=1) autoregressive moving average model, ARMA(1,1) was fitted to the data. If the actual temperature observation was used to forecast the next pixel value for a complete row of simulated data, the forecasted data had the same spatial pattern and statistical characteristics as the actual data. If, however, the forecasted value was used to forecast the next pixel value in the row, the resulting set of forecasted values did not have the same pattern but did have the same characteristics. Thus, to preserve the spatial pattern in the time series approach, the actual data base would have to be used to make the forecasts. It was decided that for most applications it would suffice to have a model with the same statistical characteristics. Therefore, the actual observation of the temperature of the first pixel in each row was used to forecast the 2nd value and thereafter the forecasted value was used to forecast the next pixel value in the row. The ARMA used was

$$\tilde{z}_t = \phi_1 \tilde{z}_{t-1} - \theta_1 a_{t-1} + a_t(\mu_a, \sigma_a), \quad \text{III.1}$$

where

t equals 1,2,3,...,500

z_t temperature of t th pixel in row

\tilde{z}_t temperature of t th pixel in row minus the mean, $(z_t - \mu)$

μ mean temperature of row

ϕ_1 autoregressive parameter of order one

θ_1 moving average parameter of order one

a_t random number for t th pixel from $N(\mu_a, \sigma_a^2)$, called residual or 'shock'

μ_a mean temperature of residuals

σ_a standard deviation of residuals

IV. ENERGY SPECTRUM

Let us represent the the two dimensional array of temperatures as a matrix, whose elements $T(l,m)$ are

$$T(l,m) = z_t^l, \quad \text{IV.1}$$

where

z_t^l is the value of z_t in the l th row

m equals $0, 1, 2, \dots, N_r - 1$

N_r is the number of pixels in a row ($=500$)

l equals $0, 1, 2, \dots, N_c - 1$

N_c is the number of pixels in a column ($=250$).

t equals $m+1$

The discrete Fourier transform (DFT) for a row of temperatures is

$$Z^l(k) = \sum_{m=0}^{N_r-1} T(l,m) \exp[-i(2\pi/N_r)mk], \quad \text{IV.2}$$

where

k equals $0, 1, 2, \dots, N_r-1$,

and for a column of temperatures is

$$Z_m(k) = \sum_{l=0}^{N_c-1} T(l,m) \exp[-i(2\pi/N_c)lk], \quad \text{IV.3}$$

where

k equals $0, 1, 2, \dots, N_c-1$.

The frequency of a row f_r is

$$f_r = m/N_r \Delta_r, \quad \text{IV.4}$$

where

Δ_r is .1525m,

and the frequency of a column f_c is

$$f_c = l/N_c \Delta_c, \quad \text{IV.5}$$

where

Δ_c is .3050m.

The energy of the k th frequency in the l th row $S^l(k)$ is

$$S^l(k) = Z^l(k)Z^{l*}(k), \quad \text{IV.6}$$

and the energy of the k th frequency in the m th row S_m is

$$S_m(k) = Z_m(k)Z_m^*(k), \quad \text{IV.7}$$

where the symbol $*$ denotes the complex conjugate. The cross-track energy spectrum and the in-track energy spectrum are a statistical measure of the correlation of the data, and result when $S^l(k)$ or $S_m(k)$ are plotted against frequency, respectively. (Zero frequency is excluded as the interest is in the the variation from the mean.)

The energy spectrum is symmetrical about the Nyquist frequency, which occurs at $f_r = .5/\Delta_r = 3.279$ cycles per metre and at $f_c = .5/\Delta_c = 1.639$ cycles per metre. Thus, it is common practice to multiply the energy of the k th frequency by a factor of two, and to plot the energy spectrum up to the Nyquist frequency. This convention was used in this paper.

In order to approximate an ensemble average by a spatial average, it is customary¹ to average $S^l(k)$ over the 250 rows and to average $S_m(k)$ over the 500 columns. Thus, the average energy of the k th frequency of the 250 rows $S^r(k)$ is

$$S^r = (1/250) \sum_{l=0}^{249} S^l(k), \quad \text{IV.8}$$

and the average energy of the k th frequency of the 500 columns $S_c(k)$ is

$$S_c = (1/500) \sum_{m=0}^{499} S_m(k). \quad \text{IV.9}$$

V. TWO DIMENSIONAL ARMA MODEL

The criterion for selecting a model was that its mean temperature, its standard deviation, and its energy spectrum, which measures the correlation in the temperature, be in good agreement with the data. The mean temperature and the standard deviation of the data were evaluated. The energy spectrum of the data was evaluated and plotted versus the frequency for the cross-track and in-track directions.

The first two dimensional (2D) model tried was to simulate the 250 rows of temperature by using Equation (III.1) and the appropriate parameter estimates for each row. The mean temperature and its standard deviation were in good agreement. The cross-track energy spectrum for the rows $S^r(k)$ was also in good agreement with the data since the ARMA model was fitted to the rows. However, the in-track energy spectrum for the columns $S_c(k)$ was not in agreement with the data. This was expected because nothing had been done to introduce correlation between adjacent rows. Several approaches based on using the temperatures in the row above to forecast the next forecast in the row below were suggested as a way of introducing correlation. None of these approaches was successful.

After inspection of the spatial temperature variation of several sets of adjacent rows, some trends were noticed. The first was that $T(l,m)$ and $T(l+1,m)$ had similar values and the second was that if $T(l,m+1)$ increased or decreased from $T(l,m)$, then

¹ La Rocca, Anthony J. and Witte, David J., "Handbook of the Statistics of Various Terrain and Water (Ice) Backgrounds from Selected U.S. Locations(U)," DTIC Technical Report Number 139900-1-x, January 1980, pages 2-11 to 2-12.

$T(l+1, m+1)$ would show a similar increase or decrease from $T(l+1, m)$. Perhaps, the shock a_t^l that produced $T(l, m+1)$ was correlated with the shock a_t^{l+1} that produced $T(l+1, m+1)$. Based on this physical evidence, the assumption was made that a_t^l was related to a_t^{l+1} through a bivariate normal distribution $g(a_t^l, a_t^{l+1})$ given by

$$g(a_t^l, a_t^{l+1}) = \left(\frac{1}{2\pi\sigma_a^l\sigma_a^{l+1}\sqrt{1-\rho^2}} \right) \exp \left[-\frac{1}{2(1-\rho^2)} \left(\left(\frac{a_t^l}{\sigma_a^l} \right)^2 - 2\rho \frac{a_t^l}{\sigma_a^l} \frac{a_t^{l+1}}{\sigma_a^{l+1}} + \left(\frac{a_t^{l+1}}{\sigma_a^{l+1}} \right)^2 \right) \right], \quad \text{V.1}$$

where the means of the residuals μ_a^l do not appear since they are approximately equal to zero, and the correlation coefficient ρ has the range

$$-1 < \rho < +1. \quad \text{V.2}$$

The marginal probability density function (pdf) for a_t^l is

$$g_1(a_t^l) = N(0, (\sigma_a^l)^2), \quad \text{V.3}$$

and the marginal pdf for a_t^{l+1} is

$$g_1(a_t^{l+1}) = N(0, (\sigma_a^{l+1})^2). \quad \text{V.4}$$

The conditional distribution for a_t^{l+1} given a_t^l is

$$g_2(a_t^{l+1}/a_t^l) = N \left(\rho \left(\frac{\sigma_a^{l+1}}{\sigma_a^l} \right) a_t^l, (\sigma_a^{l+1})^2(1-\rho^2) \right) \quad \text{V.5}$$

Now, the following procedure was used to find that value of ρ which minimized in the least squares sense the difference between the in-track energy spectrum of the data $S_c(k)$ and the in-track energy spectrum of the simulated data $S_c(k; \rho)$. For a given value of ρ the first row of simulated temperatures was generated from the ARMA model given in Equation (III.1) with the appropriate parameter estimates by using the values of a_t^0 drawn from the marginal distribution given in Equation (V.3). The second row of simulated temperatures was generated from the ARMA model given in Equation (III.1) with the appropriate parameter estimates by using the values of a_t^1 drawn from the conditional distribution given in Equation (V.5). The set of a_t^1 's for the second row were then used to generate the a_t^2 's for the third row through the conditional distribution given in Equation (V.5), etc., until 250 rows of simulated temperatures were generated. Then, the in-track energy spectrum $S_c(k; \rho)$ was evaluated. The process was repeated for several values of ρ and the sum of squares of differences between the in-track energy spectrum for the data and the simulated data was evaluated for each value of ρ . The value of ρ which minimized this sum was chosen as the ρ to be used in this model.

VI. CONCLUSIONS FOR 2D ARMA MODEL

The value of ρ which minimized the difference in the actual and simulated energy spectrum was 0.89. The mean temperature \bar{T} of the data base was 13.1°C and its standard deviation σ was 1.2°C , whereas the simulated data base had a mean temperature of 13.1°C and a standard deviation of 1.1°C . The comparison of the cross-track energy spectrum for the data and for the simulated data can be seen in Figure 1. Similarly, the comparison of the in-track energy spectrum for the data and for the simulated data can be seen in Figure 2. The agreement in both cases is good. Thus, this two dimensional ARMA model can simulate the statistical characteristics of the data, but not the spatial variations. Furthermore, to obtain more than 250 rows use Row 249 parameter estimates for Row 251, Row 248 parameter estimates for Row 252, etc., and

FORESTED AREA (IN-TRACK)-600 FT

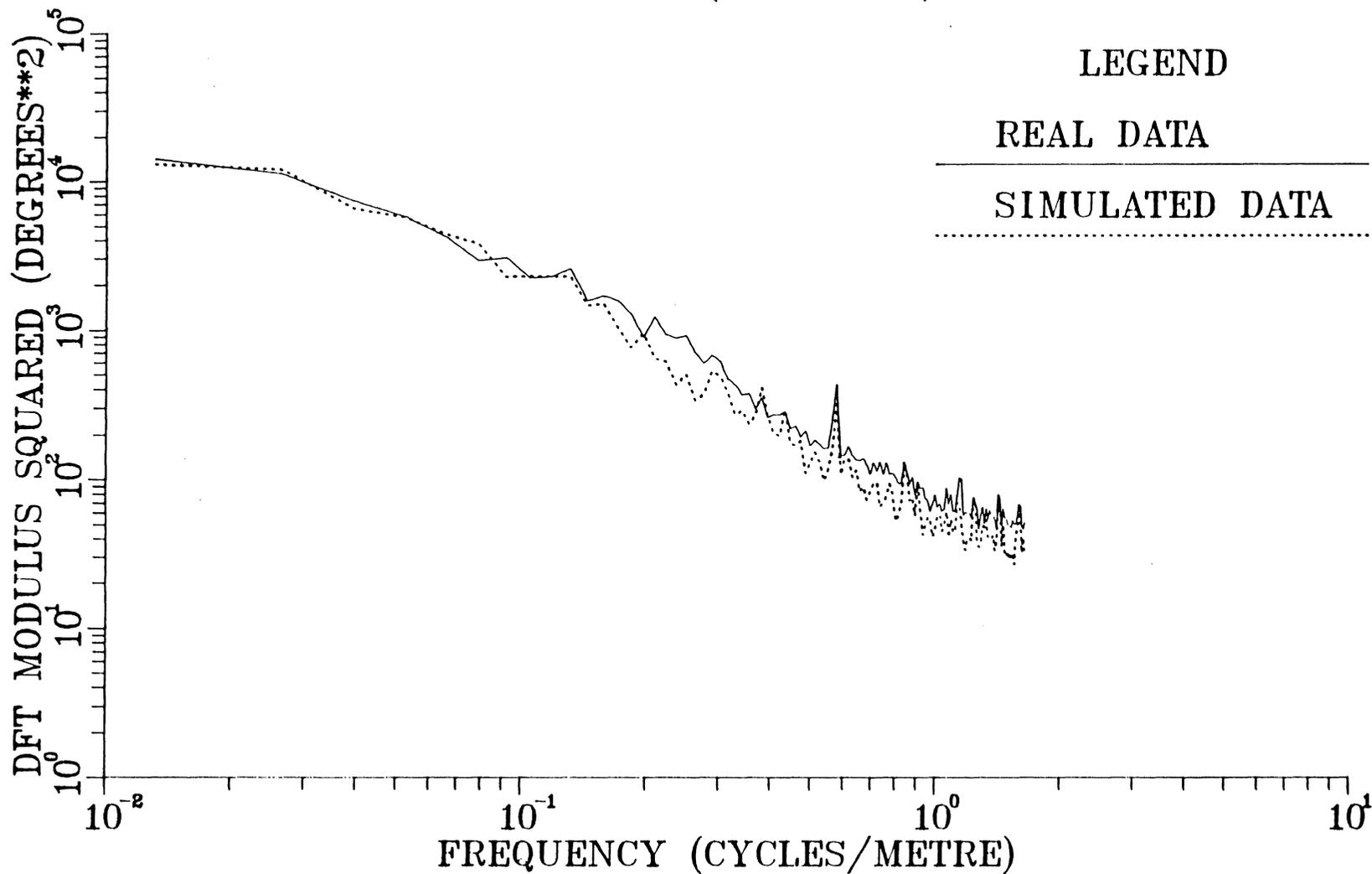


Figure 2: A comparison of the in-track energy spectra of the real and simulated data of a forested area at 600 ft altitude is plotted versus frequency.

3. A_{ij} are chosen by the experimenter such that

$$\sum_i \sum_j A_{ij} = 1 \quad \text{VII.3}$$

Table 1 illustrates the needed coefficients A_{ij} for $p=1, q=1$ that multiply the random variable Z_{nm} in order to obtain a value for $T_{n,m}$ in Equation (VII.1).

TABLE 1. Coefficients of the Spatial Moving Average for Constructing the Datum $T_{n,m}$ Using the NIID Random Variables Z_{ij} .

	m-1	m	m+1
n-1	$A_{-1,-1}$	$A_{-1,0}$	$A_{-1,1}$
n	$A_{0,-1}$	$A_{0,0}$	$A_{0,1}$
n+1	$A_{1,-1}$	$A_{1,0}$	$A_{1,1}$

Some A_{ij} may be chosen to be zero or some other value.

PROBLEM: Optimal determination of A_{ij} in SMA to match marginal spectra from observed process.

VIII. SOME COMMENTS

Our primary objective in this research was to provide a simple mathematical model which provides simulated data that are consistent with descriptive statistics from the original spatially correlated data base. Our 2D ARMA model met our criterion that its mean temperature, its standard deviation, and its energy spectrum, which measures the correlation in the temperature, be in good agreement with the data, even though it did not reproduce the spatial variation in the data. Our assumption that the shocks in adjacent rows be drawn from a bivariate normal distribution was the ingredient that introduced the necessary two dimensional spatial correlation in the simulated data. Some additional approaches for simplifying our 2D ARMA model, which were centered around reducing the number of ARMA parameter estimates needed for simulation, have been suggested in the text. In addition a spatial moving average model has been outlined as an alternative method for this problem.

Our 2D ARMA model is an improvement over the normal models that are currently being used at the BRL, especially since the time series approach naturally forecasts outlier temperatures (false alarms) that are found in the data. In time, after more data are analyzed by ARMA models, methods for relating the parameter estimates to physical constants will be found. If successful, the model may then be extended over the diurnal cycle and seasons. Also, for the theorists, an n-dimensional spatially correlated model is easily constructed.

essentially form a mirror image of the original 250 rows. To make rows longer, just draw more than 500 shocks for each row. An alternative to this procedure would be to use the 250 x 500 array of temperatures as the basic unit and extend it in any direction by mirror reflection.

One untried approach to improve this 2D ARMA model would be to take the average value of the ARMA parameter estimates for the 250 rows or at least several consecutive rows to obtain "representative parameter estimates". Then, randomly perturb these representative parameter estimates within their observed bounds for each row to be simulated, and proceed as before to determine a suitable value of ρ for the simulated temperatures.

Another untried approach to improve this 2D ARMA model might be to fit an ARMA model to every k th row of data. Use the appropriate parameter estimates for Rows 1, $k+1$, $2k+1$, etc.. For the rows in between 1 and k , use a weighted average for the parameter estimates, e.g., Row 2 values are $[(k-1)/k](\text{value of Row 1}) + (1/k)(\text{value of Row } k)$, Row 3 values are $[(k-2)/k](\text{value of Row 1}) + (2/k)(\text{value of Row } k)$, etc. (Note that a small amount of noise could be added to each value.) Proceed as before to determine a suitable value of ρ for the simulated temperatures.

VII. SPATIAL MOVING AVERAGE MODEL

The model described in this section differs from the ARMA models discussed above in that it is a two-dimensional model from the start whereas the others are one-dimensional models adjusted to give a two-dimensional array of spatially correlated observations. It also offers more promise of reproducing the spatial variation of the data, but at present it has not been applied to our problem. The steps for the SMA model are:

1. Generate an array of Z_{ij} , which are independent, identically

distributed normal random variables, NIID($0, \sigma^2$).

2. Use Z_{ij} in a spatial moving average (SMA) to construct

the temperature datum $T_{n,m}$ as

$$T_{n,m} = \bar{T} + \sum_{i=-p}^p \sum_{j=-q}^q A_{ij} Z_{n+i, m+j} \quad \text{VII.1}$$

where $E[T_{n,m}] = \bar{T}$,

and

$$\text{Cov}(T_{n,m}, T_{n+s, m+t}) = 0, \quad \text{if } |s| > p, |t| > q; \quad \text{VII.2.a}$$

$$\text{Cov}(T_{n,m}, T_{n+s, m+t}) = \sigma^2 \sum_{i=-p}^p \sum_{j=-q}^q A_{ij}^2, \quad \text{if } s=0, t=0; \quad \text{VII.2.b}$$

and

$$\text{Cov}(T_{n,m}, T_{n+s, m+t}) = \sigma^2 \sum_{i=-p+s}^p \sum_{j=-q+t}^q A_{ij} A_{i-s, j-t}, \quad \text{otherwise.} \quad \text{VII.2.c}$$

FORESTED AREA (CROSS-TRACK)-600 FT

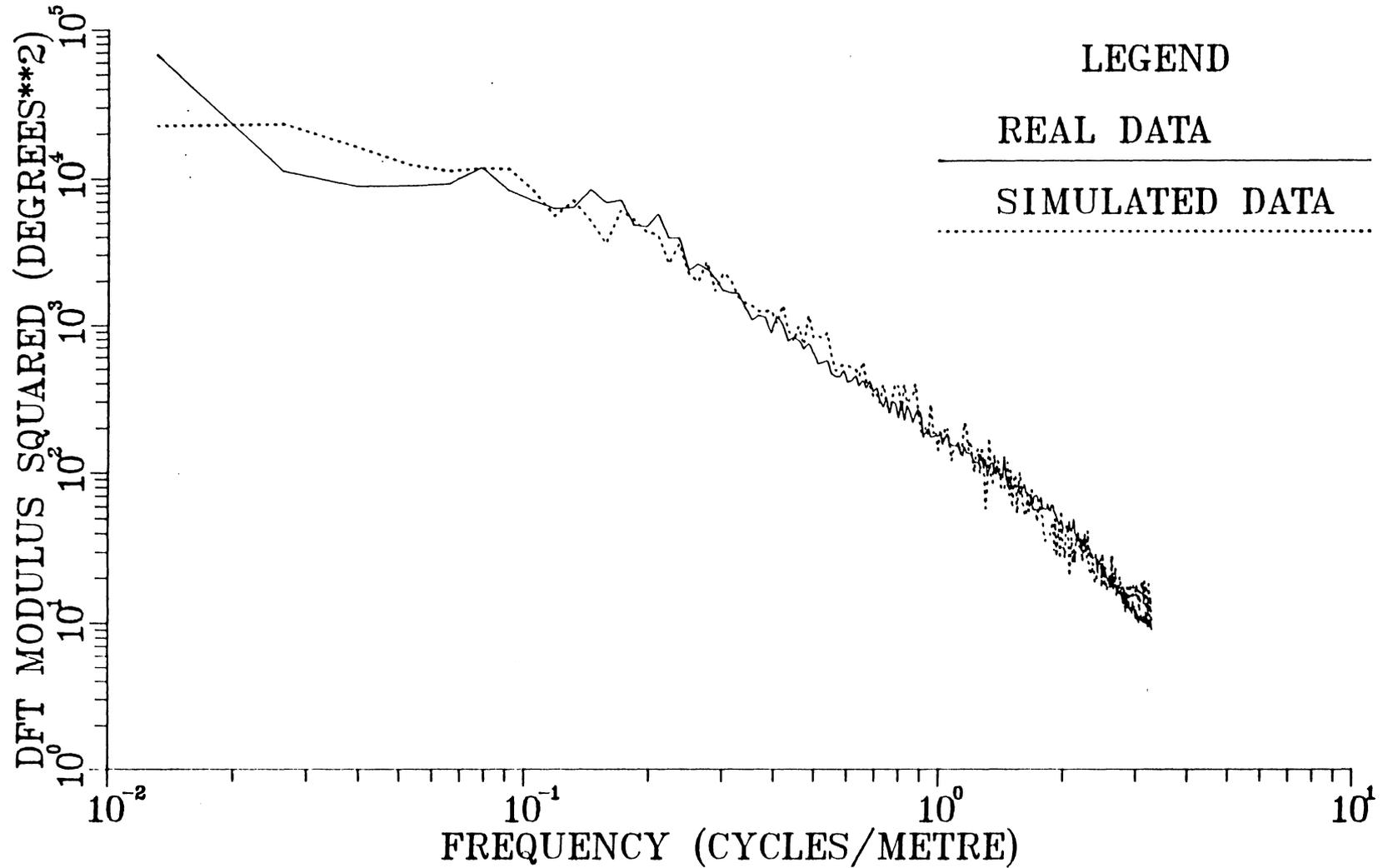


Figure 1. A comparison of the cross-track energy spectra of the real and simulated data of a forested area at 600 ft altitude is plotted versus frequency.