# SOME MODELS FOR SIMULATING SPATIALLY CORRELATED SCENES

by

Michael P. Meredith and Walter T. Federer
Biometrics Unit, Cornell University

and

Denis F. Strenzwilk
BRL, Aberdeen Proving Ground

BU-952-M* December 1987

## ABSTRACT

In order to run simulations on spatially correlated observations, it is necessary to develop models for obtaining the observations. Four such models are proposed. The models will need to be tested to determine how much computer storage and time is required. A model requiring considerable computer storage and/or time would be undesirable. A suggestion is made for extending the models to three dimensions.

## INTRODUCTION

Infrared sensing data are obtained by flying a helicopter over forested areas, roads, plowed fields, fields of crops, etc. These are called *scenes*. The temperature data obtained is highly correlated along the flight path of the helicopter. These data are called *in-track* data. The helicopter then flies a path close to and parallel with the first flight. Adjacent observations are also highly correlated. Several such flights are made. These are called *cross-track*. For example, one set of forest data consists of 500 in-track measurements for each of 250 flights. Thus, for this data set there are 250(500) = 125,000 temperature readings. The data are highly correlated both in-track and cross-track.

---

* In the Technical Report Series of the Biometrics Unit, Cornell University, Ithaca, NY 14853.

Our goal is to find some model to generate data similar to the above. This would allow an investigator to construct a set of data for the desired dimensions, called a scene, and with the desired spatial correlations. Once models have been developed which can construct scenes similar to a forested, plowed, or other area, the models will be investigated to determine their relative computer efficiencies. Any model that utilizes all possible storage of a computer would be undesirable. After a computer-efficient and desirable model has been developed, it would be used to simulate a "smart weapons" system. Many simulations could be run to determine efficiency of a weapons system. For each proposed weapons systems, appropriate modeling would be required for the scanner used.

Testing proposed weapons systems on a computer would be very inexpensive compared to field testing. Savings in time, material, and personnel could be considerable. The above procedure would also be useful in developing new systems, as various items could be computer tested before a system was developed.

We proposed four methods for constructing scenes. All four methods give scenes which have the characteristics of observed data. For each in-track set of 500 observations a (p=1, q=1) autoregressive moving average model, ARMA(1,1) was fitted to the data. If the actual observation was used to forecast the next observation, the forecasted data had the same pattern and characteristics as the actual data. If, however, the forecasted value was used to forecast the next in-tract forecast, the resulting set of forecasted values did *not* have the same pattern but did have the same characteristics. An ARMA(1,1) model was fitted to each of the 250 cross-tracks, giving 250 sets of estimated parameters. The ARMA used was

$$\tilde{Z}_t = \phi_1 \tilde{Z}_{t-1} - \theta_1 a_{t-1} + a_t(\mu_a \sigma_a) \ , \qquad (1)$$

where

$Z_t$ = temperature of the pixel in row,

$\tilde{Z}_t$ = temperature of the pixel in row minus mean,

$\phi_1$ = autoregressive parameter of order one,

$\theta_1$ = moving average parameter of order one,

$a_t$ = random number from $N(\mu_a, \sigma_a^2)$,

$\mu$ = mean temperature of row,

$\mu_a$ = mean temperature of residuals,

and

$\sigma_a^2$ = variance of residuals.

## Simulation Model I

Simulated data may be formed using the following seven steps:

1. From available infrared sensing data, fit an ARMA model to each in-track row (250 rows of data on a forested area).
2. Use forecasted values from ARMA model as values for first row of simulated data.
3. Using parameters of ARMA for Row 2 and forecasted values as data from Row 1, obtain forecasted values for Row 2.
4. Repeat step 3 to obtain forecast values for Row 3 using forecasted values of Row 2 and parameters for ARMA model on Row 3 data.
5. Continue for all 250 rows of data.
6. For more rows simply use Row 249 parameters for Row 251 and essentially form a mirror image of the original 250 rows.
7. To make rows longer, use mirror image of each row.

## Simulation Model II

A second computer simulation model is given below:

1. Instead of fitting an ARMA to each row, fit an ARMA to either Row 1 or to the average row value or to a "representative row."
2. Randomly perturb parameters of ARMA from step 1. Note that bounds on parameters must be observed under the random perturbation.
3. Second, third, etc. rows obtained as for Simulation Model I.

## Simulation Model III

A third computer simulation model is described below:

1. Fit an ARMA model to every kth, say 10th, row of data.
2. Use forecasted values for simulated data for Rows 1, k+1, 2k+1, etc.
3. For the rows between 1 and k, use a weighted average, e.g., Row 2 values are [(k-1)/k](value of Row 1) + (value of Row k)(1/k), Row 3 values are (value of Row 1)[(k-2)/k] + (value of row k)(2/k), etc.*
4. To make a wider scene, continue using ARMA models on Rows 2, k+2, 2k+2, etc. Then do likewise for Rows 3, k+3, 2k+3, etc. This makes a simulated scene.

*Note that a small amount of noise could be added to each value.

## Simulation Model IV

The model described in this section differs from the first three in that it is a two-dimensional ARMA model whereas the first three are one-dimensional models adjusted to give a two-dimensional array of spatially correlated observations. The steps for Model IV are:

1. Generate an array of $Z_{ij}$, which are NIID$(0,\sigma^2)$.
2. Use $Z_{ij}$ in a spatial moving average (SMA) to construct

$$Y_{n,m} = T + \sum_{i=-p}^{p} \sum_{j=-q}^{q} A_{ij} Z_{n+i,m+j}$$

where

$$E[Y_{n,m}] = T$$

$$\text{Cov}(Y_{nm}, Y_{n+s,m+t}) = 0 \quad \text{if} \quad |s| > p, \quad |t| > q$$

$$= \sigma^2 \sum_{i=-p}^{p} \sum_{j=-q}^{q} A_{ij}^2 \quad \text{if} \quad s = 0, \quad t = 0$$

$$= \sigma^2 \sum_{i=-p+s}^{p} \sum_{j=-q+t}^{q} A_{ij} A_{i-s,j-t}, \quad \text{otherwise .}$$

3. $A_{ij}$ chosen by experimenter such that $\sum_i \sum_j A_{ij} = 1$.

To illustrate for s=1, t=1

| n-1 | n | n1 | |
|---|---|---|---|
| $A_{-1,-1}$ | $A_{-1,0}$ | $A_{-1,1}$ | m-1 |
| $A_{0,-1}$ | $A_{0,0}$ | $A_{0,1}$ | m |
| $A_{1,-1}$ | $A_{1,0}$ | $A_{1,1}$ | m+1 |

Some $A_{ij}$ may be chosen to be zero or some other value.

PROBLEM: Optimal determination of $A_{ij}$ in SMA to match marginal spectra from observed process.

## SOME COMMENTS

A slight adjustment of models I to IV would result in models for three-dimensional spatially correlated observations. For example, one would consider a cube instead of a square for Model IV. Also, the parameters in one of the directions could be altered to account for a smaller (larger) correlation between adjacent observations. Three-dimensional models may or may not be of importance for current smart weapons systems but could conceivably be for future systems. For smart weapons systems in water, a three-dimensional model would be needed. Also, for the theorists, an n-dimensional spatially correlated model is easily constructed.

All four models, and perhaps others, need to be critiqued relative to

i) Their ability to create scenes which are realistic.

ii) Their ability to be modified to create various scenes.

iii) Their ability to be modified to insert targets in the various scenes.

iv) Their relative amounts of computer storage and running time needed to simulate scenes.

v) Their ability to be adapted for an inserted scanner in the simulation.

Note that using the last criterion above, it may be possible to simulate rather small scenes just ahead of a scanner. This would virtually eliminate the storage problem.

For further literature on the problems of infrared sensing and spatially correlated data, the reader is referred to the Sharma and Chellappa (1982) paper and references therein. Some additional references for the reader are also given at the end of this paper.

**REFERENCES**

HUDSON, R. E. (1975). The military application of remote sensing by infrared. *Proc. IEEE* **63**, No. 1.

KELTON, G., G. F. LEVY, E. BELL, and L. BIBERMAN (1963). Infrared target and background radiometric measurements concepts, units and techniques. *Infrared Physics* **3**, 139-169.

PAU, L. F. and M. Y. EL NAHAS (1983). *An Introduction to Infrared Image Acquisition and Classification Systems*. Res. Studies Press Ltd., 58B Station Road, Letchworth, Herts., SG6 3BE, England.

SHARMA, G. and R. CHELLAPPA (1982). Two-dimensional spectrum estimation using noncausal autoregressive models. *IEEE Trans. Information Th.* **32**, 268-275.

SOLO, V. (1986). Modeling of two-dimensional random fields by parametric cepstrum. *IEEE Trans. Information Th.* **IT-32**, 743-750.

WINTER, E. M. (1976). Performance evaluation of infrared sensors and associated processing. *SPIE* **95**, 165-175.