

Sequential Procedure for Testing Germination Rates  
of Seeds Stored in Seedbanks

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by

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### Summary

Samples of seeds stored for long-term conservation in seed-banks have to be monitored regularly in order to check the viability status of the seeds. In previous works, each inspection has been regarded as a separate statistical test of the null hypothesis that the sample needs regeneration. Here an overall procedure that treats each inspection as a part of a single process and subjects them to overall error rates will be developed. Properties of the procedure are examined and compared with other procedures.

### Key words

Conservation; Germination test; Overshoot; Power  
I type tests; Seedbanks.

## 1. Introduction

In technologically advanced countries farmers use modern cultivars (high yielding, disease resistant, etc) as opposed to traditional varieties which have commonly been used by farmers in developing countries. But in recent times these latter farmers have been slowly shifting into using introduced cultivars and abandoning the traditional varieties. Continuous use of modern cultivars with desirable characteristics is feasible only if a broad genetic base is retained for each species of crop plants which can be used as a pool for producing new varieties. This shift has exposed the natural gene pool to extinction (Frankel and Bennett, 1970).

In an attempt to control this process of genetic erosion, measures are being undertaken at different levels throughout the world. Seeds of different species of traditional cultivated crops are being systematically collected and stored under conditions believed to prolong the survival of the seeds. Such storage facilities are termed 'genebanks' or 'seedbanks'. This approach is believed to be the cheapest and safest method of conserving crop genetic materials.

Each sample of seeds is given a unique identification number either at point of collection from the fields or time of exchange and is referred to as an accession. All the accessions are kept under similar conditions but each is monitored separately.

Although under proper storage conditions the process of aging is believed to slow down, regular germination tests should be carried out on samples taken from an accession to check if viability has dropped to a level that requires regeneration of the accession. It has been argued that increases in the percentage of cells of surviving seeds which show chromosome aberrations and the incidence of mutant phenotypes in succeeding generations are correlated with loss of viability (Abdella and Roberts, 1968 and 1969). Let  $p$  denote the proportion of viable seeds and  $p_{\min}$  be the minimum  $p$  such that its consequences on surviving seeds which show chromosome aberrations and mutant phenotypes in succeeding generations are within tolerable limits. Hence the accession can be kept in the storage without a need for regeneration as long as  $p$  does not drop below  $p_{\min}$ . But if  $p$  drops to  $p_{\min}$ , then the accession must be regenerated and new seeds stored.

Monitoring viability involves germinating seeds sampled from the accession. Usually the first test is carried out after time  $t_1$  years from initial storage and a formal statistical test is made using the data from the germination test to determine whether or not to regenerate the accession. If the evidence is against regeneration, the seeds are kept in the store until the next regeneration time, regenerated and new seeds stored otherwise.

Thus before regenerating an accession, a number of tests are carried out on groups of seeds sampled from it at different

stages of its life in the store. Since these tests are distinctive, sufficient seeds must be stored initially to insure availability of seeds for exchange, successive tests and regeneration when it is necessary. Hence, it is evident that both frequency of inspection and the number of seeds used for each test are important factors in determining the initial size of an accession. Therefore, adoption of a statistical procedure that requires fewer seeds for tests is highly desirable.

The size of the overall error rates are also essential. The important error rate that has to be controlled is the probability of failing to regenerate the accession. If this rate is high, in the long run the seedbank would be losing some of its most valuable genetic materials. Secondly, it would be desirable if the procedure stops at or close to the true time of regeneration as possible because this could cut on the long-term cost of the seedbank.

A sequential probability ratio test (SPRT) for testing percentage germination of seeds has been suggested for use in seedbanks (Ellis, Roberts and Whitehead, 1980 and Whitehead, 1981). SPRT and also fixed sample approach consider inspections at different times as unrelated statistical problems rather than part of an overall process and result in separate significant statements (inspection wise error rates). Although in both cases, inspection wise error rates are known the overall error rates are unknown. nevertheless, it is possible to estimate the unknown overall error

rates for each of these approaches from computer simulation for comparison purpose.

At inspection time  $t_i$ , the new procedure makes use of information from all inspections up to time  $t_{i-1}$  and updates it with current information from germination test. Based on this cumulated information about viability condition of the seeds, a decision is made whether or not to regenerate the accession. Hence the whole monitoring process is treated as a single act.

The method is based on the assumption that, for any fixed time period  $t$ , the number of germinating seeds out of  $n$  tested is binomially distributed with probability of germination  $p(t)$ . In addition, it is assumed that the logit of  $p(t)$  is a linear function of  $t$ . The test procedure is developed with some modification analogous to the power 1 type tests of Darling and Robbins (1967, 1968) for iid normal random variables.

## 2. Formulation of the Problem

Let  $p(t_i)$  denote the germination rate of the accession at time  $t_i$  and  $T$  be the true time of regeneration ( $T$  is unknown).

Next let

$$p_o = p(t_o)$$

and

$$p_{\min} = p(T)$$

$p_o$  is the initial germination rate and  $p_{\min}$  is the terminal germination rate. Hence  $T$  denotes the true time it takes for  $p(t_i)$  to drop from  $p_o$  to  $p_{\min}$ .

An each-inspection time germination test is made and the following hypothesis assessed:

$$H_0: p \leq p_{\min}$$

$$H_A: p > p_{\min}$$

The accession is kept in the store as long as evidence supports  $H_A$  and there are sufficient seeds for future testing.

Now consider a case where tests carried out on a single seed basis and let  $t_1, t_2, \dots, t_i, \dots$  denote predetermined inspection times (note that the  $t_i$ 's need not be all different since in practice test are carried out on a number of seeds at any given inspection time). Define

$$x_i = \begin{cases} 1, & \text{if a seed planted at } t_i \text{ germinated} \\ 0, & \text{otherwise} \end{cases}$$

If

$$P(x_i=1) = p(t_i)$$

then

$x_i$  is a Bernoulli random variable with parameter  $p(t_i)$ .

The loglikelihood of  $p(t_i)$  is given by:

$$\ell(p(t_i)) = \sum x_i \log \{p(t_i) / (1 - p(t_i))\} + \sum \log \{1 - p(t_i)\} \quad 2.1$$

Let

$$\log(p(t_i)) = \log\{p(t_i)/(1 - p(t_i))\} \quad 2.2$$

be denoted by  $R(t_i)$ .

Assume that  $R(t_i)$  has the following form:

$$R(t_i) = R_0 - \beta t_i \quad 2.3$$

where  $R_0$  is the logit of  $p_0$ .

$\beta$  is the rate of deterioration of seeds per unit time on a logistic scale. It is a general parameter that includes the true rate of deterioration.

Hence the loglikelihood of  $p(t_i)$  reparameterized in terms of  $\beta$  is:

$$\ell(\beta) = \sum x_i (R_0 - \beta t_i) - \sum \log\{1 + \exp(R_0 - \beta t_i)\}. \quad 2.4$$

Under this parameterization, it is desirable to regenerate the accession when  $R(t_i)$  drops to  $R_1 (=R(T))$ , and maintain accession in the store otherwise.  $R_1$  is the logit of  $p_{\min}$ .

### 3. Test Procedure

The test statistics are defined and the stopping rule is given below. An approximate overshoot correction is incorporated into the procedure.

#### 3.1 Derivation of Test Statistics

If  $S$  denotes the current time, it is desirable to regenerate the accession when  $S$  coincides with  $T$  where  $S < T$ .

Suppose that each time an inspection is made it is pretended that 'it is now time to regenerate the accession'. Let  $\beta_s$  denote the rate of deterioration of seeds under this pretense. Hence, at time  $s$  we have the following logistic regression line:

$$R_s(t_i) = R_0 - \beta_s t_i \text{ for } t_i = t_1, t_2, \dots, s \quad 3.1.1$$

Where

$$\beta_s = (R_0 - R_1)/S \quad 3.1.2$$

The true logistic regression line is

$$R_T(t_i) = R_0 - \beta_T t_i \text{ for } t_i = t_1, t_2, \dots, T \quad 3.1.3$$

Where

$$\beta_T = (R_0 - R_1)/T \quad 3.1.4$$

$\beta$  includes all  $\beta_s$ 's and  $\beta_T$ .

The hypothesis now can be expressed as in terms of  $\beta$  as follows:

$$H_0: \beta_s \leq \beta_T$$

$$H_A: \beta_s > \beta_T$$

Figure 3.1.1 shows the relationship between  $R_s(t_i)$  and  $R_T(t_i)$ .

(Figure 3.1.1 goes here)

Now from (3.1.2) and (3.1.4)

$$\beta_S > \beta_T \text{ as long as } S < T.$$

Hence it is desirable to regenerate the accession when  $\beta_S = \beta_T$ .

Otherwise, define

$$Z = \sum t_i (Y_i - E_S(Y_i)) \tag{3.1.5}$$

and

$$V = \sum t_i n_i E_S(Y_i/n_i) [1 - E_S(Y_i/n_i)]. \tag{3.1.6}$$

Summation is over all inspection times up to the current time S.

$Y_i$  is the number of germinating seeds among the  $n_i$  seeds tested at time  $t_i$  and  $E_S$  is the expectation under the pretended assertion 'it is now time to regenerate the accession' (refer to appendix B). Hence

$$E_S(Y_i) = n_i p_S(t_i). \tag{3.1.7}$$

$p_S(t_i)$ 's are computed from the logits derived from  $R_S(t_i)$ .

$$E(Z) \begin{cases} > 0 \text{ for all } S < T \\ = 0 \text{ at } S = T \\ < 0 \text{ for } S > T. \end{cases}$$

So  $E(Z)$  is a decreasing function of  $t$  and has different distributions at each time  $t_i$ .

Now, by analogy to Darling and Robbins (1967, 1968) procedure (Appendix A) and modifying (Appendix B) to serve the requirements of seedbanks, the following stopping rule can be used.

regenerate the accession if

$$Z \leq a(v) \tag{3.1.8}$$

continue otherwise

where

$$a(v) = \{(v+1)[\log(v+1) - 2\log 2\alpha]\}^{\frac{1}{2}} \tag{3.1.9}$$

$\alpha$  is type I error of Darling and Robbins procedure and it can be chosen as small as desired. Then the following hold

$$p(\text{stopping too late}) < \alpha \tag{3.1.10}$$

$$p(\text{stopping too early}) \rightarrow 0 \text{ as } n \rightarrow \infty \tag{3.1.11}$$

The test terminates with probability 1 as  $n \rightarrow \infty$  (refer to Appendix c for proofs).

So at each inspection time,  $Z$  and  $a(v)$  are computed and based on the evidence either the accession is regenerated or sampling continued.

The procedure controls the probability of stopping too late as desired. And secondly the test terminates with probability 1 as  $n$  increases at  $t_i = T$ .

### 3.2 Correction for Overshoot

Examination of the properties of the procedure indicates that it is certainly conservative. The probability of failing to stop is lower than the desired level  $\alpha$  and secondly for small sample size the procedure could lead to early stoppings. Therefore, an approximate correction is incorporated into the procedure by analogy to Siegmund (1979) and Whitehead (1981).

At current inspection time  $s$ , information increases at rate  $I_s$ , where

$$I_s = R_s S^2 p_s(s) (1 - p_s(s)). \tag{3.2.1}$$

$$p_s(s) = p_{\min}$$

Then an approximate correction is

$$O_s = 0.583\sqrt{I_s}. \quad 3.2.2$$

The procedure (3.1.8) becomes regenerate the accession if

$$Z_c \leq a(v) \quad 3.2.3$$

continue otherwise. Where

$$Z_c = Z + O_s \quad 3.2.4$$

The correction increases at smaller rate than  $V$ , and therefore, the properties (3.1.10) and (3.1.11) still hold. The effect of the correction factor can be specially effective when small sample sizes are used.

#### 4. Discussion

Computer simulation was used to examine the properties of the procedure and to make comparison between different tests. Table 4.1 gives estimated error probability ( $\hat{\alpha}$ ) for 1000 replicates each for two different sample sizes. T was set at 100 years and  $p_{\min}$  at 0.85. The value used for  $\alpha$  was 0.05.

Table 4.1

Estimated error probabilities ( $\alpha$ ) for two initial germination rates  $p_o = 0.99$  and  $0.95$ .

n	$p_o$	
	0.95	0.95
100	0.001	0.002
1000	0.001	0.000

For each of these simulations, inspection intervals of equal sizes of five years were used starting the first inspection at year five. The overall error rate was considerably smaller than  $\alpha$  as expected. Also it is important to note that the sample size has no appreciable effect on the error rate.

Table 4.2 gives estimates of the probability of stopping too late for SPRT, the new approach and the fixed sample case. For each case an estimate of  $\hat{\alpha}$  based on 1000 runs is given. Two initial germination rates were used. A group of 40 seeds were used for SPRT which lead to the use of an average of 116 and 194 seeds for  $p_o = 0.99$  and  $0.95$  respectively at any given inspection time. For fixed sample case 467 seeds were used

per test. Inspection interval of 20 years was used starting with year 20 until the test terminated. T was fixed at 100 years.

Table 4.2

Estimates of probability of stopping too late for the three procedures

Tests	$p_o$	
	0.95	0.99
SPRT	0.02	0.049
New Procedure*	0.009	0.01
Fixed Sample	0.25	0.058
New Procedure (n=467)	0.004	0.004

\* The new approach's estimates are based on 194 and 116 sample sizes for  $p_o = 0.95$  and  $0.99$  respectively which is the same as the average for the SPRT.

The fixed sample requires 467 seeds to achieve the same result as SPRT. In fact an elaborate comparison of SPRT and fixed sample approach is given by Ellis and others (1980). The fixed sample approach is extremely wasteful as compared to the other two.

The SPRT approach stops too late on average about 3.5 times more often than the new approach for the same average sample size. Hence the new approach shows a higher performance in this respect than SPRT.

The use of the error rate to compare different procedures without considering the effect of inspection times could be unsatisfactory.

It would be interesting to see the magnitude of such an error when the inspection grid misses the desired time of regeneration. In fact this is one of the serious problems of predetermined inspection times. If the last inspection is carried out at  $t_m$ , when  $t_m > T$ , the error rate should be higher for any procedure. The size of course depends on the difference  $t_n - T$ .

Simulation was carried out to study the effect of inspection times on error rates for SPRT and the new approach (Table 4.3). Inspections were made at equal intervals of 20 years starting at 20 years for both cases.  $T$  was fixed at 90 years and initial germination rate of 0.99 was used. 1000 replicated runs were made for both approaches. Group of 40 seeds were used for SPRT which led to the use of an average of 145 seeds per inspection time. So 145 seeds per test were used in the simulation for the new procedure.

Table 4.3

Frequency of stoppages at different times of inspection out of 1000 replicates each for SPRT and new procedure.

Inspection times (yrs)	Frequency	
	SPRT	New Procedure
20	0	0
40	0	0
60	0	39
80	260	745
100	738	216
120	2	0

When the last inspection is carried out after the true time of regeneration, which could happen in practice if pre-

determined times of inspections are used, the SPRT will stop more frequently at the first time after T the last time before T. For the same average sample size, the new procedure however, will stop more frequently at the last time before T than the first time after T.

Although adoption of statistical procedures with desirable properties such as seed saving and ideally smaller error probabilities, their vulnerability to changes in inspection times must as well be accounted for. In practice this is a more serious problem because for thousands of accessions of different species of crop plants, the desirable times of regeneration were not known. An objective method of estimating these inspection times should be sought for.

Certainly the new procedure indicates better performance in terms of smaller error rate than the SPRT which uses the same average sample size per inspection. Even if the last inspection is carried out after the true time of regeneration, fewer accessions will be regenerated after T years if the new procedure is used. But another important property of the new procedure is that it enables stochastic estimation of inspection times using germination information. Therefore, the procedure is a powerful statistical tool.

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References

- Abdella, F. H. and Roberts, E. H. (1968). Effects of temperature, moisture and oxygen on the indirection of chromosome damage in seeds of barley, broad beans and peas during storage. Annals of Botany 32, 119-136.
- Abdella, F. H. and Roberts, E. H. (1969). The effects of temperature and moisture on the induction of genetic changes in seeds of barley, broad beans and peas during storage. Annals of Botany 33, 153-167.
- Bekele, I. (1981). Monitoring Accessions in Seedbanks. M.S. Thesis (unpublished). University of Reading: Reading.
- Darling, D. A. and Robbins, H. (1967). Inequalities for the sequences of sample means. Proceedings of National Academy of Science 57, 1577-1580.
- Darling, D. A. and Robbins, H. (1967). Confidence sequences of sample mean, variance and median. Proceedings of National Academy of Science 58, 66-68.
- Darling, D. A. and Robbins, H. (1968). Some further remarks on inequalities for sample sums. Proceedings of National Academy of Science 60, 1175-1182.
- Ellis, R., Roberts, E. H. and Whitehead, J. (1980). A new more economic and accurate approach to monitoring the viability of accessions during storage in seedbanks. Plant Genetic Resources - Newsletter 41, 3-17.
- Frankel, O. H. and Bennet, E. (1970). Genetic Resources in Plants - their Exploration and Conservation. Oxford: Blackwell Scientific Publications.
- International Board for Plant Genetic Resources (1976). Report of the IBPGR Working group on Engineering, Design and Cost aspect of long-term seed storage Facilities. Rome: IBPGR.
- Siegmund, D. (1979). Corrected diffusion approximation in certain random walk problems. Advances in Applied Probability 11, 701-719.
- Whitehead, J. (1981). The use of the sequential probability ratio test for monitoring the percentage germination of accessions in seedbank. Biometrics 37, 129-136.

Appendix A

Power One type tests for Normal Random Variables

Let

$x_1, x_2, \dots$  be iid normal random variables with  $E(x_1) = \theta$  and  $v(x_1) = 1$ . Suppose interest lies in testing

$$H_0: \theta \leq 0$$

versus

$$H_A: \theta > 0$$

Assume it is desirable to continue with sampling as long as  $H_0$  is true and quit sampling otherwise and take some appropriate action.

Darling and Robbins have suggested the following type procedure: continue with sampling as long as

$$S_m < a(m)$$

where

$$S_m = x_1 + \dots + x_m, \tag{A.1}$$

and

$$a(m) = \{(m+1)[\log(m+1) + 2\log 2\alpha]\}^2 \tag{A.2}$$

Under  $H_0$ :

$$S_m \sim N(0, m).$$

Each time a sample is drawn, both  $S_m$  and  $a(m)$  are computed and compared. The procedure calls for termination of inspection when

$$S_m \geq a(m).$$

Darling and Robbins show that:

$$P_{H_0}(S_m \geq a(m) \text{ for some } m \geq 1) \leq \alpha \quad \text{A.3}$$

$$P_{H_1}(S_m \geq a(m) \text{ for some } m \geq 1) \rightarrow 1 \text{ as } m \rightarrow \infty. \quad \text{A.4}$$

In the next section a modified version of this procedure to suit the special case of monitoring percentage viability is given.

Appendix B

Derivation of test statistics

First transitional test statistics  $Z^1$  and  $v^1$  are derived and test procedure outlined with analogy to Darling and Robbins (1967, 1968) procedure. Then the statistics  $Z$  and  $v$  of section 3 are formally derived.

I. For  $\beta$  close to  $\beta_0$ , the loglikelihood of  $\beta$  can be expanded approximately as:

$$l(\beta) \doteq l(\beta_0) + (\beta - \beta_0)l'(\beta_0) + \frac{1}{2}(\beta - \beta_0)^2 l''(\beta_0) \quad \text{B.1}$$

where

$$l'(\beta_0) = d\ell/d\beta|_{\beta_0}$$

and

$$l''(\beta_0) = d^2\ell/d\beta^2|_{\beta_0}$$

Next let

$$\theta = \beta_0 - \beta$$

then

$$l(\beta) \doteq l(\beta_0) - \theta l'(\beta_0) + \frac{1}{2}\theta^2 l''(\beta_0) \quad \text{B.2}$$

$T_0$  test

$$H_0: \theta \leq 0$$

versus

$$H_A: \theta > 0$$

The statistics  $Z^1$  and  $v^1$  can be used, where

$$Z^1 = -l'(\beta_0)$$

$$Z^1 = -\ell'(\beta_0)$$

and

$$v^1 = -\ell''(\beta_0).$$

$Z^1$  is a linear function of the efficient score and  $v^1$  is Fisher's information.

If  $n_i$  seeds are used for germination test at time  $t_i$  and  $y_i$  denotes the number of seeds that germinated, then

$$Z^1 = \sum t_i (Y_i - n_i p_T(t_i)) \quad \text{B.3}$$

and

$$v^1 = \sum t_i^2 n_i p_T(t_i) (1 - p_T(t_i)). \quad \text{B.4}$$

The sequential test is based on  $S_m$  and  $m$  of Darling and Robbins test replaced by  $Z^1$  and  $v^1$ , respectively. This analogy is reasonable since under  $H_0$

$$Z^1 \sim AN(0, v^1).$$

Then by analogy to (A.3)

$$P_{H_0}(Z^1 \geq a(v^1) \text{ for some } v^1 > 0) < \alpha \quad \text{B.5}$$

where

$$a(v^1) = \{(v^1 + 1) [\log(v^1 + 1) - 2\log(2\alpha)]\}^{\frac{1}{2}}. \quad \text{B.6}$$

Where  $\alpha$  can be chosen as small as desired. Use of the statistics  $Z^1$  and  $v^1$  requires knowledge of  $\beta_0$ . To overcome restrictions arising from this, the approach can be modified as follows:

- II. Suppose at time  $t_1$ ,  $Z^1$  and  $v^1$  were evaluated and decision arrived to continue with sampling.

Now let  $\beta_1$  satisfy:

$$-\ell'(\beta_1) = a\{-\ell''(\beta_1)\} \quad \text{B.7}$$

where

$$\ell'(\beta_1) = d\ell/d\beta|_{\beta_1}$$

and

$$\ell''(\beta_1) = d^2\ell/d\beta^2|_{\beta_1}$$

$\ell(\beta)$  is the loglikelihood of  $\beta$ .

If  $S$  denotes the current time, then

$$Z^1 < a(v^1) \Leftrightarrow S < (R_0 - R_1)/\beta_1 = T_1 \tag{B.8}$$

where  $R_0$  and  $R_1$  are the logits of  $p$  at  $t_0$  and  $T$ .

Given information on the status of the seeds in storage up to time  $t_1 (=s)$ , then  $T_1$  is the future time for which it would be necessary to undertake regeneration of the accessions if  $T_1 = T$ . Then from (B.8)

Continue with inspection as long as

$$\beta_1 < (R_0 - R_1)/s \tag{B.9}$$

as long as

$$-\ell'((R_0 - R_1)/s) > a\{-\ell''((R_0 - R_1)/s)\} \tag{B.10}$$

Then

$$-\ell'((R_0 - R_1)/s) = \sum t_i \{Y_i - E_s(Y_i)\} \tag{B.11}$$

$$-\ell''((R_0 - R_1)/s) = \sum t_i^2 n_i E_s(Y_i/n_i) [1 - E_s(Y_i/n_i)] \tag{B.12}$$

Where  $E_s(Y_i)$  is the expectation of  $Y_i$  under the pretense that it is now time to regenerate the accession. (B.10) holds for all time  $t_i < T$ . B.11 and B.12 are  $Z$  and  $V$  of 3.1.5 and 3.1.6.

Appendix C

Properties of the test Procedure

Now

$$Z = \sum t_i (Y_i - n_i p_S(t_i)) = Z^1 + \Delta \quad \text{C.1}$$

where

$$\Delta = \sum t_i n_i (p_T(t_i) - p_S(t_i))$$

and

$$V = \sum t_i^2 \bar{n}_i p_S(t_i) (1 - p_S(t_i)) = v^1 + s \quad \text{C2}$$

where

$$s = \sum t_i^2 n_i \{ (p_S(t_i) - p_S^2(t_i)) - (p_T(t_i) - p_T^2(t_i)) \}.$$

Under  $H_0$

$$E(Z) \sim AN(0, v).$$

Since at  $t_i = T$

$$\Delta = 0$$

$$\delta = 0$$

$$a(v) = a(v^1 + \delta) = a(v^1) + d(\delta).$$

Hence,

$$a(v) \begin{cases} > a(v^1) & \text{for } t_i < T \\ = a(v^1) & \text{for } t_i = T \\ < a(v^1) & \text{for } t_i > T. \end{cases}$$

Properties

$$p(\text{stopping too late}) < \alpha \quad \text{C.3}$$

$$p(\text{stopping too early}) \rightarrow 0 \text{ as } n \rightarrow \infty \quad \text{C.4}$$

$$p(\text{stopping at desired time } T) \rightarrow 1 \text{ as } n \rightarrow \infty. \quad \text{C.5}$$

Proofs

(C.3):

$$\begin{aligned} p(\text{stopping too late}) &= P_{H_0}(Z > a(v) \text{ for all } t_i \leq T) \\ &< P_{H_0}(Z > a(v) \text{ for } t_i = T). \end{aligned}$$

But at  $t_i = T$

$$P_{H_0}(Z > a(v)) = P_{H_0}(Z^1 > a(v^1)) \leq \alpha.$$

Hence the result

(C.4):

$$\begin{aligned} p(\text{stopping too early}) &= P_{H_A}(Z \leq a(v) \text{ for any } t_i < T) \\ &= P_{H_A}(Z^1 \leq a(v) - \Delta \text{ for any } t_i < T). \end{aligned}$$

At any given time  $t_i$ ,  $a(v)$  and  $\Delta$  are increasing functions of  $n$ .

$a(v)$  increases by an order of  $n^{\frac{1}{2}}$  and  $\Delta$  by  $n$ .  $\Delta > 0$  for all  $t_i < T$ .

When

$$a(v) - \Delta \rightarrow -\infty \text{ as } n \rightarrow \infty$$

$$P_{H_A}(Z^1 \leq (a(v) - \Delta) \text{ for any } t_i < T) \rightarrow 0 \text{ as } n \rightarrow \infty$$

as required.

(C.5):

$$\begin{aligned} P(\text{stopping at time } T) &= P_{H_A}(Z \leq a(v) \text{ for } t_i = T) \\ &= P(Z^1 \leq (a(v) - \Delta) \text{ for } t_i = T) \\ &\rightarrow 1 \text{ as } n \rightarrow \infty \end{aligned}$$

Noting that  $\Delta = 0$  at  $t_i = T$  and  $a(v) \rightarrow \infty$  as  $n \rightarrow \infty$ .

It follows that the test terminates with probability 1 as  $n \rightarrow \infty$ .

- Title: Stochastic Estimation of Inspection times for Monitoring Viability of seeds in "Gene-banks".
- Summary: A procedure for estimating inspection times based on techniques to monitor viability of seeds suggested by Bekele (1984) is explained.
- Introduction: Background of the problem is summarized. The importance of objective estimations of the inspection times is explained. The distributional assumptions about survival of seeds are given and the model applied developed.
- Test Procedure: The test statistics are briefly defined and the decision process explained. The properties of test outlined.
- Estimation of In-  
spection times: Technique for estimating confidence sequences is given. The use of the confidence intervals for estimating inspection times is explained. Properties of the confidence interval are given.
- Discussion: Predetermined and estimated inspection times are compared simulation results. Modification of estimated times is suggested.

