FRACTIONAL REPLICATION AND FEEDING FISH

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Abstract

Six different fish foods were to be combined into mixtures of equal proportions of the foods to ascertain their effect on growth of fish in tanks. Four fish tanks were available and each tank could be divided into eight units each such that a given mixture could be fed to the fish in one unit. There are 2^6-1 combinations of one food, two foods, ···, six foods in a mixture. The 2^6 factorial combinations with the 000000 combination omitted produces the mixtures if zero means the food is not present and one means the food is present. Since only 32 units were available, it was necessary to use a one-half fraction of the complete 2^6 factorial. The fraction used aliased the six-factor interaction with the mean. Then, one two-factor and two three-factor interactions were used to obtain the treatments allocated to the four tanks.

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1. The Problem

An experimenter wishes to compare six different fish foods in mixtures to assess whether mixtures of one, two, ···, six foods would produce faster growth of fish. The experimenter has four tanks available for the experiment, and each tank is divided into eight distinct units such that one mixture could be given to the fish in one unit of the tank. A constant amount (weight) of food will be given to each unit in each of the four tanks, i.e., in each of the 32 units. Each food in a mixture will appear in equal proportions even though the total amount of food given might increase as the fish become larger. Thus, if two foods appear in a mixture, each one will constitute one-half of the total weight; if three foods appear in a mixture, each food will make up one-third of the total weight; and so forth for other numbers of foods in a mixture. Now, how should one select combinations of foods in order to assess the effects and interactions of the various foods?

2. A Solution

Since there are six foods which are either present or absent in a mixture, the total number of combinations correspond to the combinations in a 2^6 factorial. Since only eight combinations can appear in any one tank and since there are only 32 total units, it will be necessary to obtain one-eighth fractions for each tank and a one-half replicate fraction for the entire experiment. Also, we might wish to select fractions which have either two, four, or six feeds in a mixture or fractions which have one, three, or five feeds in a mixture. For the former we would use a one-half replicate wherein the mean was aliased with $(ABCDEF)_0$, the six factor interaction effect at the zero level; the other one-half fraction would be obtained from $(ABCDEF)_1$, where either one, three or five ones appear in a combination. One should note that the combination 000000 is inadmissible because this would mean a treatment with no food, which would be disasterous for the fish. We would replace the 000000 combination in $(ABCDEF)_0$ with one of the other combinations, e.g., 111111. The effects that can be obtained from a one-half replicate of a 2^6 are (= means completely confounded with):

(mean)	I = ABCDEF	CD = ABEF
	A = BCDEF	CE = ABDF
	B = ACDEF	CF = ABDE
	C = ABDEF	DE = ABCF
	D = ABCEF	DF = ABCE
	E = ABCDF	EF = ABCD
	F = ABCDE	ABC = DEF
	AB = CDEF	ABD = CEF
	AC = BDEF	ABE = CDF
	AD = BCEF	ABF = CDE
	AE = BCDF	ACD = BEF
	AF = BCDE	ACE = BDF
	BC = ADEF	ACF = BDE
	BD = ACEF	ADE = BCF
	BE = ACDF	ADF = BCE
	BF = ACDE	AEF = BCD

If we use the following scheme of confounding in the four tanks, we can estimate the remaining effects

Tank 1: $(AB)_0$, $(ACE)_0$, $(BCE)_0$

Tank 2: $(AB)_0$, $(ACE)_1$, $(BCE)_1$

Tank 3: $(AB)_1$, $(ACE)_0$, $(BCE)_1$

Tank 4: $(AB)_1$, $(ACE)_1$, $(BCE)_0$

In determining which effects can be confounded with tanks, one may chose any two, but the third must be the interaction of the chosen two. Also, one could partially confound effects, using a different set for each tank.

Since the combination 000000 cannot be used, and one of the other combinations substituted for it, the resulting fraction is not a one-half replicate. Also, if the experimenter wishes to use the foods alone and in mixtures, he would use the $(ABCDEF)_1$ fraction instead of the $(ABCDEF)_0$ fraction. This would result in mixtures of one food, three foods and five foods, and no substitution would be required. If the experimenter did use the fraction $(ABCDEF)_0$ with 000000 replaced by 111111, then 30 of the 31 effects could be estimated and the effect omitted would be aliased with the remaining 30. (See, e.g., Banerjee and Federer (1963), Annals Math. Stat. 34:1068-1078.)

If the experimenter knows that three-factor interactions are unimportant and that these contrasts could be used as experimental error, then there would be eight degrees of freedom available for the error variance if one used the fraction $(ABCDEF)_1$. There are ten degrees of freedom for these factor interactions in the one-half replicate, and two of these were confounded with differences between tanks.

3. Additional Comments

The combinations of a 2^6 factorial divided into mixture designs follow:

$(ABCDEF)_0$		$(ABCDEF)_1$	
000000		100000	011111
110000	001111	010000	101111
101000	010111	001000	110111
100100	011011	000100	111011
100010	011101	000010	111101
100001	011110	000001	111110
011000	100111	mixtures of 1	mixtures of 5
010100	101011	111000	000111
010010	101101	110100	001011
010001	101110	110010	001101
001100	110011	110001	001110
001010	110101	101100	010011
001001	110110	101010	010101
000110	111001	101001	010110
000101	111010	100110	011001
000011	111100	100101	011010
mixtures of 2	mixtures of 4	100011	011100
111111 = mixture of 6		mixtures of 3	

If one had only two tanks for a total of 16 observations, the combinations in the first column above omitting 000000 could be used as 15 mixtures of two foods and one mixture of six. If one wanted mixtures of four foods, then the 15 combinations in the second column plus the combination 111111 could be utilized. Likewise, there are six combinations with one food each, six combinations with five foods in a mixture, and there are 20 combinations of three foods in a mixture.

The statistical analysis for mixture experiments of this type could be considerably different from that for a factorial treatment design. Interest could center on the specific combination as an entity in itself rather than on main effects and interactions. Alternatively, one could perform statistical analyses in the same manner as for diallel and triallel crosses. (Federer (1979), Agronomy J. 71(5):701-706, and Federer and Wijesinha (1979), Contributed Papers, International Statistical Institute, 42nd Session, pages 167-170.)