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A Note on the Distribution of a Single Species Along a Single Gradient

D. L. Solomon

It is common practice in phytosociology to pretend that a species responds to an environmental gradient by a Gaussian distribution of abundance (importance value, biomass, ...) across the gradient. From a curve-fitting perspective, the model is convenient, being determined by only two parameters (in addition to the total abundance) and computer programs for their estimation are readily available. It does not appear however that the literature contains an argument, from biological first-principles, which leads to the Gaussian distribution. In particular, none of the usual arguments for Normality, i.e. those based on the Central Limit Theorem seem to be applicable. The intent of this note is to offer some "biological assumptions" which indeed ~~predict~~ ^{predict} a Gaussian abundance distribution with a hope that ecologists will assess the plausibility of these

assumptions. At the least, the characterization provides an implication of adopting the Gaussian model which may shed light on situations in which the model fails.

Thus we denote by t the value of some environmental gradient and by $y(t)$ the abundance (or some other appropriate importance value) of some species at gradient value t . We wish to describe the function $y(t)$. We shall make ~~two~~ two suppositions and then appeal to parsimony.

Supposition 1: It is meaningful to transform abundance to a logarithmic scale. Perhaps an argument can be made based on the multiplicative nature of growth. (We will return to this supposition again.) Thus, we will be characterizing $g(t) \equiv \ln y(t)$.

Supposition 2: The species has an optimal location on the gradient. Denote by μ this optimal value of t . Equivalently, we have supposed that as we move away from μ in either direction, the "desirability" of the gradient value decreases.

~~The rate of change of $g(t)$ increases as t moves away from μ .~~

Thus for $t < \mu$, $g(t)$ should increase with increasing t (positive slope), and for $t > \mu$ $g(t)$ should decrease with increasing t (negative slope).

Parsimony: A simple model would have the rate of ~~change~~^{decline} of $g(t)$ be linear in $t - \mu$.

Symbolically:

$$(1) \quad \frac{d}{dt} g(t) = -c(t - \mu) \quad \text{for some constant } c.$$

Thus, as in supposition 2, the right hand side is positive for $t < \mu$ and negative for $t > \mu$. Solving the differential equation (1) gives

$$(2) \quad g(t) = -\frac{c}{2}(t - \mu)^2 + D$$

where D is a constant of integration to be determined by the total abundance of the population. Recalling the definition of $g(t)$, (2) becomes

$$\ln y(t) = -\frac{c}{2}(t - \mu)^2 + D$$

or \Leftrightarrow

$$y(t) = K e^{-\frac{c}{2}(t - \mu)^2} \quad \text{where } K \equiv e^D.$$

This is of course proportional to a Gaussian

probability density function with mean μ and variance $\frac{1}{c}$. Also, the total abundance of the population is

$$\int_{-\infty}^{\infty} y(t) dt = \int_{-\infty}^{\infty} K e^{-\frac{c}{2}(t-\mu)^2} dt = K \sqrt{\frac{2\pi}{c}}$$

Notice that one can work backwards from (3) to (1), so that if a Gaussian curve is adequate in a given situation, it follows that log-abundance falls-off linearly with departure from the optimum, and that if the Gaussian model does not fit, then this is not the case.

Concerning the first supposition, note that the left hand side of (1) can be written

$$\frac{d}{dt} g(t) \equiv \frac{d}{dt} \ln y(t) = \frac{\frac{d}{dt} y(t)}{y(t)}$$

so that when measuring abundance on the original scale, (1) suggests that the rate of decline of abundance per unit abundance (e.g. per individual) is proportional to the gradient distance from the optimum.

Finally, it might be of interest to note that this characterization of the Normal distribution was given by early astronomers in their study of "error distribution."