STATISTICAL ANALYSES OF YIELDS FROM UNIBLENDS AND BIBLENDS OF EIGHT DRY BEAN CULTIVARS

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ABSTRACT

Response model equations and corresponding statistical analyses are presented for experiments involving mixtures of pairs of cultivars (biblends), both when the individual yields in a biblend and when only the total yields are available. These were applied to yield data for eight dry bean (Phaseolus vulgaris L.) cultivars. Data were obtained from four experiments conducted in two years (1966 and 1967) at two locations (Ithaca and Aurora, New York). Concepts of general mixing effects and specific mixing effects are discussed and are related to the concepts of general combining ability and specific combining ability in diallel crossing experiments. In diallel crossing experiments, only the total of the two components is available, whereas in the bean experiments, individual yields of the two cultivars in a biblend were available. It is shown how this additional information may be used for further evaluation of the mixtures.

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5/81

INTRODUCTION

Cropping systems such as intercropping, relay-cropping, successive cropping, sequential cropping, etc. are a centuries-old tradition in agriculture. These systems involve the growing of mixtures of cultivars or lines of a cultivar, simultaneously or sequentially on the same tract of land. This area of investigation had received relatively little attention from researchers in the past. Recently, however, interest in this area has been stimulated from a variety of causes, for example, their importance in tropical agriculture, the high cost of energy and fertilizer, their current use by farmers, increased income, etc. Studies on designing, analyzing, and evaluating results from experiments involving various cropping systems are being reported in the literature. The advantages and disadvantages of cropping systems used for centuries is only now being scientifically assessed.

Several approaches to the quantification of results from investigations have been made. For example, Mead et al. (1980,1980) discuss the use of treatment design and spatial arrangements; McGilchrist and Trenbath (1971) deal with response models for competition experiments, etc. Our interest in this paper centers on developing and investigating response models for cultivars grown alone (uniblend, sole crop) and in mixtures of two cultivars (biblends) in equal proportions. Response model equations and associated statistical analyses are presented for two different experimental situations. The first model is applicable to a situation wherein the biblend responses cannot be separated into the individual cultivar responses; only the uniblend responses and the combined responses of the biblends are available for statistical analyses. Inferences must pertain to the mixture as a unit rather than to the individual responses for the two cultivars in a biblend. Ideas and statistical analyses developed for diallel crossing experiments (see, for example, Eberhart and Gardner, 1966, Jensen and Federer, 1965, and the bibliography in Randall, 1975) are used for this model. A second or alternate response model is considered for the case wherein the individual responses for each cultivar in the biblend are available. It is shown how to utilize this additional information.

Yield data were available from four experiments involving eight uniblends and 28 biblends of eight dry bean (Phaseolus vulgaris L.) cultivars grown at two different locations for two years. Since the bean seeds differed in color, seed shape, and/or distinguishable plant types, it was possible to obtain dry bean yields for each cultivar in the biblend. The data were analyzed using both response models. However, since the individual yields were available, the second or alternate model was considered to be more appropriate for these experiments; and the analyses of the data using this model are presented here for each experiment and for the four experiments combined. The implications of the results obtained from applying both models to the data are discussed, and recommendations are given for other applications of the analyses using these models.

MATERIALS

The eight dry bean cultivars included in the experiment were: 'Black Turtle Soup', 'Michelite 62 Pea Bean', 'Perry Marrow', 'Red Kidney', 'Seaway Pea Bean', 'Steuben Yellow Eye', 'Sutter Pink', and 'White Kidney'. The seed colors and/or seed shape and size for these cultivars were all different, except for Michelite 62 Pea Bean and Seaway Pea Bean which had distinguishable plant types; this allowed the seeds of the biblend to be separated into two groups at harvest time. There were $v(v-1)/2 = 28$ biblends and the $v = 8$ uniblends, making up the 36 entries of the treatment design in the experiment. The experiment design was a randomized complete block with $r = 4$ blocks in each of two locations (Ithaca and Aurora, New York) in each of two years (1966 and 1967). The experimental unit (the plot) was a single row 3.0 meters in length with a 0.8 meter spacing between rows. The seeds of a biblend were in a 1:1 (50 seeds of each genotype) ratio and were thoroughly mixed prior to planting in the experimental unit. The row spacing was sufficiently wide to eliminate or greatly reduce interrow competition, thus most of the competition was intrarow. Yield of mature seed was obtained for each plot. Bean yields were obtained for each cultivar in the 27 biblends by handsorting into two component cultivars. For the 28^{th} mixture, Michelite 62 Pea Bean and Seaway Pea Bean, the components were identified before harvest by their respective vine and bush plant types, and were then harvested separately. However, subsequent progeny tests showed that separations of the combination often were not accurate. Consequently, a type of "missing plot" calculation was applied as follows for each replication for this combination. Let 3 = Seaway Pea Bean and 6 = Michelite 62 Pea Bean. The calculation for the "missing plot" in each replicate was computed as:

$$\hat{Y}_{3(6)} = \frac{Y_3}{Y_3+Y_6} \times Y_{36} \quad \text{and} \quad \hat{Y}_{6(3)} = \frac{Y_6}{Y_3+Y_6} \times Y_{36},$$

where $\hat{Y}_{3(6)}$ = computed yield of 3 in 3-6 mixture,

$\hat{Y}_{6(3)}$ = computed yield of 6 in 3-6 mixture,

Y_3 = uniblend yield of 3,

Y_6 = uniblend yield of 6,

and Y_{36} = total biblend yield in 3-6 mixture.

We could have followed several procedures in handling this one non-separable mixture. However, since our primary purpose was to demonstrate certain other concepts of analysis, we used the "missing plot" values as described above and used 28 mixtures and 8 uniblends of the 8 cultivars. Thus, there were $4(8+2(28)) = 256$ yields obtained in each experiment, 8 of which were calculated values as described above. Since four "mixed-up" plot values were used to compute eight values for each experiment, it could be suggested that the "error" degrees of freedom be reduced by four in each experiment. This was not done here, as the method of calculating the yields was not the usual least squares for mixed-up plot values. The values computed in this manner could be expected to contribute as much or more to the error sum of squares than actual values. A second reason was that the results obtained would be little changed from those obtained by decreasing the error degrees of freedom.

RESPONSE MODEL EQUATIONS AND STATISTICAL ANALYSES

The yields of uniblends are assumed to have the following response model equation for a randomized complete block design:

$$Y_{hiiu} = \mu + \rho_h + \tau_i + \epsilon_{hii} \;; \qquad [1]$$

the total yield of a biblend when individual yields are not available is assumed to have a response model equation of the following form:

$$Y_{hijb} = \mu + \rho_h + (\tau_i+\tau_j+\delta_i+\delta_j)/2 + \gamma_{ij} + \epsilon_{hij} \;. \qquad [2]$$

where u refers to uniblend and b to biblend yields, μ is considered to be a general mean effect common to every observation, ρ_h is the h^{th} block effect, τ_i is an effect due to the i^{th} cultivar $(i=1,2,\cdots,v)$ when grown as a uniblend, δ_i represents a general mixing effect (gme) of the i^{th} cultivar grown in a biblend, γ_{ij} represents a specific mixing effect (sme) of cultivars i and j when grown together in a mixture for $i < j = 2,\cdots,v$ with $\gamma_{ij} = \gamma_{ji}$, and $\gamma_{ii} = 0$, and ϵ_{hii} and ϵ_{hij} are random error components distributed with mean zero and variance σ_ϵ^2. Under the model restriction $\Sigma_{h=1}^{r}\rho_h = 0$, $\Sigma_{i=1}^{v}\tau_i = 0$, and $\Sigma_{j=1,j\neq i}^{v}\gamma_{ij} = 0$, solutions for the resulting normal equations are obtained as:

$$\hat{\mu} = Y_{...u}/rv = \bar{y}_{...u} \qquad [3]$$

$$\hat{\tau}_i = (Y_{.iiu}-r\hat{\mu})/r \qquad [4]$$

$$\hat{\delta}_i = [2Y_{.i.b} - (v-2)Y_{.iiu} - 2Y_{...b}/(v-1)]/r(v-2) \qquad [5]$$

$$vr(v-1)\bar{\delta} = r(v-1)\Sigma_1^v\hat{\delta}_i = 2Y_{...b} - (v-1)Y_{...u} \qquad [6]$$

$$\hat{\gamma}_{ij} = [Y_{.ijb} - r\hat{\mu} - r(\hat{\tau}_i+\hat{\tau}_j+\hat{\delta}_i+\hat{\delta}_j)/2]/r \qquad [7]$$

$$\hat{\rho}_h = [2Y_{h...} - v(v+1)\hat{\mu} - v(v-1)\bar{\delta}]/v(v+1) \;. \qquad [8]$$

Note that the block effects are orthogonal to the other effects in a randomized complete block design. Also, note that $\Sigma_{i=1}^{v}\hat{\delta}_i$ is not required to equal zero, as each δ_i could be positive (negative); this means that all cultivars could yield higher (lower) in biblends than they yield as uniblends. If cultivars in mixtures yield higher than when grown alone, then there would be an advantage in growing mixtures. Such was the case in oats (Avena sativa) (see Jensen and Federer, 1964) and in wheat (Triticum aestivum L.) (see Jensen, 1978).

In relating sums of squares among biblends to corresponding ones for general combining ability (gca) and specific combining ability (sca) in a diallel crossing experiment, one should note that comparisons for gca are the same as among $\hat{\tau}_i + \hat{\delta}_i - \bar{\delta}$ = "cultivar effects" and that the sca sum of squares corresponds to interactions among the "cultivar effects". Also, if only biblend values are available, one can obtain solutions for the sums $\mu + \bar{\delta}$ and $\tau_i + \delta_i$ only, but not for the individual components of these sums. This is the usual situation considered in diallel cross experiments when only the $v(v-1)/2$ possible crosses are present in the experiment.

An analysis of variance for total yields is given in Table 1. It relates the totals for biblends and for uniblends. An orthogonal partitioning of the $v(v+1)/2-1$ degrees of freedom for uniblends and biblends is $(v-1)$ among uniblend totals, one for the contrast of uniblend with biblend yields, and $v(v-1)/2-1$ among biblend totals. The degrees of freedom for biblends may be partitioned in the same manner as is done for diallel crosses to obtain $(v-1)$ degrees of freedom for comparisons of $\tau_i + \delta_i - \bar{\delta}$, the "cultivar effects", for which the sum of squares is computed in the same manner as for general combining ability, and to obtain $v(v-3)/2$ degrees of freedom for interactions of "cultivar effects", for which the sum of squares is computed in the same manner as for specific combining ability. In addition, when uniblend yields are available, one can obtain solutions for $\hat{\delta}_i$, the gme for cultivar i;

under the model restrictions $\Sigma_{j=1, j\neq i}^{v} \gamma_{ij} = 0$, tests for the hypotheses $\delta_i = 0$ can be obtained, but one should note that these are not independent.

AN ALTERNATE MODEL

The results for the preceding model on the eight dry bean cultivars showed wide variability among the four sets of data. The parameter estimates, under the model restrictions $\Sigma_{h=1}^{r}\rho_h = \Sigma_{i=1}^{v}\tau_i = \Sigma_{j=1,j\neq i}^{v}\gamma_{ij} = 0$, $\gamma_{ii} = 0$, and $\gamma_{ij} = \gamma_{ji}$ varied widely from year to year and location to location. It appeared that some form of model inadequacy was present. Although the individual yields in a biblend were available, the analysis only incorporated them in terms of total yields in a biblend; thus not all the available information in the data sets was used in estimating parameters. Therefore, it was decided to perform a further analysis, using a different model, which appeared more realistic (see Jensen and Federer, 1964, 1965).

The yields of uniblends from a randomized complete block design are assumed to have response model equation [1]. However, the individual yields in a biblend are assumed to have response model equations of the form:

$$Y_{hi(j)b} = (\mu+\rho_h+\tau_i+\delta_i)/2 + \gamma_{i(j)} + \epsilon_{hi(j)} \qquad [9]$$

and

$$Y_{h(i)jb} = (\mu+\rho_h+\tau_j+\delta_j)/2 + \gamma_{(i)j} + \epsilon_{h(i)j} \;, \qquad [10]$$

where $Y_{hi(j)b}$ denotes yield for cultivar i when grown in biblend with cultivar j, μ is a general mean effect common to every observation, τ_i is the effect due to the i^{th} cultivar $(i=1,2,\cdots,v)$ when grown in a uniblend, δ_i represents a general mixing effect (gme) of the i^{th} cultivar when grown in a biblend, $\gamma_{i(j)}$ represents a specific mixing effect (sme) on the yield of cultivar i, when grown with cultivar j $(i=1,2,\cdots,v, j \neq i)$ and $\gamma_{ii} = 0$. (Here we do <u>not</u> assume $\gamma_{i(j)} = \gamma_{j(i)}$; since the effect of one cultivar on another does not require an identical reciprocal effect, we may have $\gamma_{i(j)} > 0$ for all j for a particular i while $\gamma_{(i)j}$ may take any value.) The h^{th} block effect is represented by ρ_h $(h=1,2,\cdots,r)$, and $\epsilon_{hi(j)}$ and $\epsilon_{h(i)j}$ are random error components with distributions given by $\epsilon_{hi(j)}, \epsilon_{h(i)j} \sim (0,\sigma_\epsilon^2/2)$. (This takes account of the fact that individual cultivar yields from biblends are obtained from one-half the plot size for that of uniblends.) Under the model restrictions $\Sigma_{h=1}^{r}\rho_h = 0$, $\Sigma_{i=1}^{v}\tau_i = 0$, $\Sigma_{j=1,j\neq i}^{v}\gamma_{i(j)} = 0$, we obtain best linear unbiased estimates for the parameters as:

$$\hat{\mu} = Y_{\cdots u}/rv \,, \qquad [11]$$

$$\hat{\rho}_h = \frac{2}{v(v+1)}\left[Y_{h\cdot\cdot u} + Y_{h\cdot\cdot b} - \frac{Y_{\cdots u}+Y_{\cdots b}}{r}\right] \qquad [12]$$

$$\hat{\tau}_i = \frac{Y_{\cdot iiu}}{r} - \hat{\mu} \,, \qquad [13]$$

$$\widehat{\tau_i+\delta_i-\bar{\delta}} = \frac{2}{r(v-1)}\left[Y_{\cdot i(\cdot)b} - \frac{Y_{\cdot\cdot(\cdot)b}}{v}\right] , \qquad [14]$$

$$\hat{\gamma}_{i(j)} = \frac{Y_{\cdot i(j)b}}{r} - \frac{Y_{\cdot i(\cdot)b}}{r(v-1)} \,, \qquad [15]$$

and

$$\widehat{\mu+\bar{\delta}} = \frac{2Y_{\cdot\cdot(\cdot)b}}{rv(v-1)} \,. \qquad [16]$$

Thus,

$$\hat{\bar{\delta}} = \widehat{\mu+\bar{\delta}} - \hat{\mu} \qquad [17]$$

and

$$\widehat{\delta_i-\bar{\delta}} = \widehat{\tau_i+\delta_i-\bar{\delta}} - \hat{\tau}_i \,. \qquad [18]$$

An analysis of variance for testing some of the effects of interest is given in Table 2. An orthogonal partitioning of the sums of squares provides tests for contrasts among the uniblends τ_i's, the "cultivar effects" $\tau_i + \delta_i - \bar{\delta}$'s, the sme's, the block effects, and the $\bar{\delta}$ effect, which is the uniblends vs biblends effect. Further, since uniblends are available, tests for contrasts among the $\delta_i - \bar{\delta}$'s could also be constructed, although these would not be independent tests. Since individual cultivar yields of biblends were available for the given sets of data, it seemed more realistic in this case to apply the alternate model which incorporated the additional information on the components of the biblends, rather than the model of the previous section, which only made use of the total yields. The analysis, using the model of this section, is presented next.

ANALYSIS OF EXPERIMENTAL DATA USING ALTERNATE MODEL

Using response model equations [1], [9], and [10], an analysis of variance, Table 3, was prepared for the combined data from all four experiments. Using the block × treatment-within-locations-and-years mean square as the error mean square, F-statistics were computed and compared with tabulated F-values at prescribed levels. All F-statistics except those involving $\bar{\delta}$, the comparison of uniblend and biblend mean yields, were larger than the tabulated F-value at the one percent level. In an attempt to pinpoint reasons for large interactions of the uniblend, cultivar, and specific mixing effects with years and locations, a combined analysis for each of the two years and each of the two locations was obtained (Table 4). Using the block × treatment within location (year), as the error mean square, F-statistics were computed and compared with tabulated values. In Table 4 it may be noted that all F-values not involving $\bar{\delta}$, except for the uniblend × location interaction in 1967, were larger than the tabulated F-values at the 0.05 probability level. In the individual analyses (Table 5), the size of the F-statistics varied widely over the four experiments. This contributed to the interactions with locations and years. For example, the uniblend F-ratios were 2.21, 3.46, 5.37, and 25.50, indicating considerable differences for this effect. The uniblend × year F-ratio of 9.86 at Aurora and the uniblend × location F-ratio of 8.90 for 1966 both involved the large F-ratio 25.50 and one of the smaller ones. Likewise, the small F-ratio 0.76 for uniblend × location in 1967 involved the two rather similar F-ratios 3.46 and 5.37.

From the above, it is concluded that the sizes of the uniblend effects, the cultivar effects, and the specific mixing effects depend upon the climatic conditions, and conditions peculiar to the location such as soil type, drainage, planting date, etc. (see Table 6). A general recommendation for the area over years should not be made from these data. The ranks of the $\hat{\tau}_i$ and of the $\widehat{\delta_i - \bar{\delta}}$ varied considerably over the four experiments. More needs to be known about the factors that make one bean cultivar or one mixture yield better than another, as the parameter estimates showed no stability over years and locations. Hence, it should be emphasized that an adequate model fit in one experiment should not be extrapolated to other climatic and soil situations.

CONCLUSION

Two response model equations and corresponding statistical analyses are presented here for experiments involving mixtures of pairs of cultivars (biblends). The first model is suggested for experiments in which the total yield in a biblend cannot be separated into individual cultivar yields. The alternate model is suggested for experiments in which biblends are separable, thus producing additional information about the cropping systems. The alternate model requires the estimation of more parameters than the first model, as it incorporates the additional information. Here, specific mixing effects are obtained for each cultivar in a pair, whereas in the first model, only a single specific mixing effect for the combination is available. The interpretation of the general mixing effects in the two cases is necessarily different, due to the difference in the structure of the data in each case.

As the experimental data was obtained in a form that could utilize the more informative alternate model, the results of this analysis are presented here. Although the first model was also applied to the same data, the results are not reported here, but are available on request. It is to be recommended that the alternate model be used wherever possible. However, in cases where <u>only</u> yield totals are available from a mixture, the first model provides an analysis which utilizes the only information available from the data.

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Table 1. **Analysis of variance for totals of biblend and uniblend yields** for a randomized complete block design.

Source of variation	Degrees of freedom	Sum of squares
<u>On totals</u>		
Total	$\frac{rv(v+1)}{2}$	$\sum_{h=1}^{r}\sum_{i=1}^{v} Y^2_{hiiu} + \sum_{h=1}^{r}\sum\sum_{i<j} Y^2_{hijb}$
Correction for mean	1	$2(Y_{\cdots u}+Y_{\cdots b})^2/rv(v+1) = \mathrm{CFM}$
Blocks	r-1	$2\Sigma_1^r(Y_{h\cdot\cdot u}+Y_{h\cdot\cdot b})^2/v(v+1) - \mathrm{CFM}$
Entries	$\frac{v(v+1)}{2} - 1$	$(\Sigma_1^v Y^2_{\cdot iiu}+\sum_{i<j}\Sigma Y^2_{\cdot ijb})/r - \mathrm{CFM}$
Uniblends	v-1	$\Sigma_1^v Y^2_{\cdot iiu}/r - Y^2_{\cdots u}/rv$
Uniblends vs biblends	1	$Y^2_{\cdots u}/rv + 2Y^2_{\cdots b}/rv(v-1) - \mathrm{CFM}$ $= r(v-1)(\Sigma\hat{\delta}_i)^2/v(v+1)$
Biblends	$\frac{v(v-1)}{2} - 1$	$\sum_{i<j}\Sigma Y^2_{\cdot ijb}/r - 2Y^2_{\cdots b}/rv(v-1)$
Cultivar effects	v-1	Same formula as for general combining ability $= r(v-2)\Sigma_1^v(\hat{\tau}_i+\hat{\delta}_i-\bar{\delta})^2/4$ $= 4\Sigma_1^v(\frac{v}{2}Y_{\cdot i\cdot b}-Y_{\cdots b})^2/rv^2(v-2)$
Interaction	$\frac{v(v-3)}{2}$	Same formula as for specific combining ability or by subtraction
Blocks × entries	$\frac{(r-1)(v-1)(v+2)}{2}$	by subtraction

Table 2. Analysis of variance for individual biblend and uniblend yields for a randomized complete block design, alternate model.

Source of variation	Degrees of freedom	Sum of squares
Total	rv^2	$\sum_{h=1}^{r}\sum_{i=1}^{v} Y^2_{hiiu} + 2\sum_{h=1}^{r}\sum_{i=1}^{v}\sum_{\substack{j=1\\ j\neq i}}^{v} Y^2_{hi(j)b}$
Correction for mean	1	$\frac{2}{rv(v+1)}[Y_{\cdots u}+Y_{\cdots b}]^2 = CFM$
Blocks	r-1	$\frac{2}{v(v+1)}\sum_{h=1}^{r}[Y_{h\cdot\cdot u}+Y_{h\cdot\cdot b})^2 - CFM$
Uniblends	v-1	$\sum_{i} \frac{Y^2_{\cdot iiu}}{r} - \frac{Y^2_{\cdots u}}{rv}$
Uniblends vs biblends	1	$\frac{Y^2_{\cdots u}}{rv} + \frac{2Y^2_{\cdots b}}{rv(v-1)} - CFM$
Cultivar effects	v-1	$2\sum_{i} \frac{Y^2_{\cdot i(\cdot)b}}{r(v-1)} - \frac{2Y^2_{\cdots b}}{rv(v-1)}$
Specific mixing effects	v(v-2)	$2\sum_{i}\sum_{j} \frac{Y^2_{\cdot i(j)b}}{r} - 2\sum_{i} \frac{Y^2_{\cdot i(\cdot)b}}{r(v-1)}$
Residual	$(v^2-1)(r-1)$	(by subtraction)

Table 3. F-statistics and error mean square for the combined analysis of four experiments.

Source of Variation	Degrees of Freedom	F-statistic
Uniblend vs biblends ($\bar{\delta}$)	1	0.1
Uniblend (U)	7	11.4
Cultivar effect (C)	7	75.5
Specific mixing effect (SME)	48	3.5
$\bar{\delta}$ x location	1	0.0
U x location	7	3.0
C x location	7	7.6
SME x location	48	1.7
$\bar{\delta}$ x year	1	1.4
U x year	7	8.8
C x year	7	55.0
SME x year	48	2.5
$\bar{\delta}$ x location x year	1	0.3
U x location x year	7	3.2
C x location x year	7	16.3
SME x location x year	48	1.8
Error mean square	756	8,106

$F_{.05}(1,756) = 3.85$ $F_{.05}(7,756) = 2.02$ $F_{.05}(48,756) = 1.37$

$F_{.01}(1,756) = 6.67$ $F_{.01}(7,756) = 2.66$ $F_{.01}(48,756) = 1.56$

Table 4. F-statistics and error mean squares for combined analyses of years within locations and locations within years.

Source of variation	Degrees of freedom	F-statistics Ithaca	Aurora	1966	1967
Uniblends = U	7	2.90	13.40	16.10	7.67
U X year or location	7	3.36	9.86	8.90	0.76
Uniblends vs biblends = D	1	0.05	0.01	0.40	0.07
D X years or locations	1	0.20	1.89	0.40	0.07
Cultivar effects = C	7	33.90	52.60	51.60	70.80
C X years or locations	7	37.30	33.30	15.50	10.50
SME = S	48	2.77	2.35	3.70	2.75
S X years or location	48	2.01	2.37	1.49	1.83
Error mean square	378	9590.9	6622.6	4670.8	11542

$F_{.05}(1,378) = 3.9$ $F_{.05}(7,378) = 2.0$ $F_{.05}(48,378) = 1.4$

Table 5. F-statistics and error mean squares for analyses of individual experiments.

Source of variation	Degrees of freedom	F-statistics 1966 Ithaca	F-statistics 1966 Aurora	F-statistics 1967 Ithaca	F-statistics 1967 Aurora
Uniblends	7	2.21	25.50	3.46	5.37
Uniblends vs biblends	1	0.02	1.40	0.13	0.76
Cultivar effects	7	22.70	47.20	40.40	41.05
Specific mixing effect	48	1.41	4.06	2.75	1.59
Error mean square	189	5208.4	4133.2	13972	9112.0

$F_{.05}(1,189) = 3.9$ $F_{.05}(7,189) = 2.0$ $F_{.05}(48,189) = 1.4$

Table 6. Parameters estimates for alternate model under model restrictions $\Sigma_i \tau_i = \Sigma_h \rho_h = \Sigma_j \gamma_{ij} = 0$.

Parameter	Ithaca 1966	Ithaca 1967	Aurora 1966	Aurora 1967	Parameter	Ithaca 1966	Ithaca 1967	Aurora 1966	Aurora 1967	Parameter	Ithaca 1966	Ithaca 1967	Aurora 1966	Aurora 1967	Parameter	Ithaca 1966	Ithaca 1967	Aurora 1966	Aurora 1967
μ	447	775	359	653	γ_{12}	-15	65	-7	-32	γ_{37}	8	-51	13	13	γ_{64}	-19	-34	-4	28
$\bar{\delta}$	-2	9	-15	17	γ_{13}	8	4	77	43	γ_{38}	51	78	13	-5	γ_{65}	6	45	-42	17
τ_1	19	87	22	118	γ_{14}	-7	-75	-75	32	γ_{41}	-30	-43	-70	-59	γ_{67}	26	-68	-2	-2
τ_2	84	141	181	190	γ_{15}	-13	-11	-59	-6	γ_{42}	20	75	-31	20	γ_{68}	11	73	11	-36
τ_3	-59	27	66	-17	γ_{16}	43	78	63	-36	γ_{43}	-5	-63	31	-5	γ_{71}	-18	-80	-20	-8
τ_4	-52	62	-193	58	γ_{17}	-24	-82	-47	-22	γ_{45}	-50	112	-64	30	γ_{72}	34	-73	-5	-41
τ_5	27	-137	-224	-68	γ_{18}	7	22	48	20	γ_{46}	14	-39	88	14	γ_{73}	14	-34	17	34
τ_6	2	-132	171	-117	γ_{21}	-24	-70	-34	-90	γ_{47}	8	-17	38	7	γ_{74}	-44	47	12	-12
τ_7	-64	63	-134	-77	γ_{23}	-25	48	88	31	γ_{48}	44	-24	8	-7	γ_{75}	9	88	-41	29
τ_8	43	-111	109	-87	γ_{24}	27	-18	-36	71	γ_{51}	-26	-77	-24	-51	γ_{76}	14	-4	47	3
$\delta_1-\bar{\delta}$	53	251	47	104	γ_{25}	-20	60	-38	5	γ_{52}	48	45	11	17	γ_{78}	-9	55	-10	-5
$\delta_2-\bar{\delta}$	10	-65	-86	60	γ_{26}	35	31	40	-76	γ_{53}	-4	-34	45	83	γ_{81}	-29	-122	-41	-5
$\delta_3-\bar{\delta}$	-36	-41	-45	-143	γ_{27}	27	-25	6	8	γ_{54}	-42	24	-20	31	γ_{82}	-30	0	27	-40
$\delta_4-\bar{\delta}$	108	-80	104	-44	γ_{28}	-20	-27	-25	51	γ_{56}	29	72	22	-31	γ_{83}	81	-2	62	33
$\delta_5-\bar{\delta}$	-22	-166	79	-95	γ_{31}	-31	-66	-27	-44	γ_{57}	11	-68	-26	-54	γ_{84}	-25	6	-47	73
$\delta_6-\bar{\delta}$	-40	56	-133	102	γ_{32}	-21	-19	40	-32	γ_{58}	-16	39	-8	4	γ_{85}	-36	11	-94	31
$\delta_7-\bar{\delta}$	-100	115	-24	79	γ_{34}	-12	73	-37	84	γ_{61}	-25	-129	-31	-45	γ_{86}	54	151	38	-60
$\delta_8-\bar{\delta}$	26	-70	58	-63	γ_{35}	-12	77	-21	9	γ_{62}	-18	-22	1	-30	γ_{87}	-13	-45	55	-33
					γ_{36}	16	-92	18	-25	γ_{63}	19	134	67	69					

Table 1. Analysis of variance for totals of biblend and uniblend yields for a randomized complete block design.

Table 2. Analysis of variance for individual biblend and uniblend yields for a randomized complete block design, alternate model.

Table 3. F-statistics and error mean square for the combined analysis of four experiments

Table 4. F-statistics and error mean squares for combined analyses of years within locations and locations within years.

Table 5. F-statistics and error mean squares for analyses of individual experiments.

Table 6. Parameters estimates for alternate model under model restrictions $\Sigma_{i=1}^{v}\tau_i = \Sigma_{h=1}^{r}\rho_h = \Sigma_{j=1,j\neq i}^{v}\gamma_{ij} = 0$.