A METHOD FOR ESTIMATING COMBINING ABILITY COMPONENTS OF VARIANCE IN INCOMPLETE BLOCK DESIGNS*

by

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ABSTRACT

A method is described for estimating components of variance for specific and general combining ability from breeding experiments arranged in incomplete block designs. Utilizing published results estimates of these components of variances were computed for a potato breeding experiment arranged in a rectangular lattice design with the parameters v=200 entries, k=20 entries per incomplete block, r=6 complete blocks, and b/r=20 incomplete blocks within each complete block. The experiment was grown at three different locations thus yielding three estimates of the components of variance for combining ability. These results were used in the computations of expected genetic progress in the preceding paper of this Journal. The algebraic results were arranged in table form to facilitate printing.

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A METHOD FOR ESTIMATING COMBINING ABILITY COMPONENTS OF VARIANCE IN INCOMPLETE BLOCK DESIGNS [1]

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A potato breeding experiment containing 190 testcrosses and ten check varieties (including six tester parents) was designed as a rectangular lattice with parameters v=200, k=10, r=6, and b=120. The details of the design and the analysis were reported previously by the authors [2]. Some of the results are presented in the preceding paper in this Journal [5]. The experiment was conducted at three locations -- Ithaca and Riverhead, New York and Clear Lake, Iowa. The 190 testcrosses resulted from the available crosses between 45 lines and the six tester parents. Only eight of the lines were crossed with all six testers; each line was crossed with at least one tester parent. There was no restriction in the experiment on the allocation of the testcrosses with a given tester parent; the 190 testcrosses were randomly alloted to the groups of entries in a given incomplete block arrangement.

The specific purpose of this paper is to present a method for estimating variance components for specific combining ability and for general combining ability from $m_{..}$ testcrosses arranged in an incomplete block design. For purposes of generality, it is assumed that $v \geq m_{..}$ = number of testcrosses grown in the experiment with r replicates on each testcross (one could assume r_{ij} replicates on the ijth cross, but this was not done for the particular experiment involved), that there are $\underline{c}$ lines crossed to at least one of the $\underline{a}$ tester parents, that estimates of the within incomplete blocks and among incomplete blocks components of variance (i.e., σ_e^2 and σ_b^2) are available from the analysis of variance for the incomplete block design,[2] that the number of testcrosses in an incomplete

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[2] If the design were a lattice square, the estimates of σ_c^2 = the column component of variance, σ_r^2 = the row component of variance, and σ_e^2 = the intra-row and column component of variance would be obtained from the analysis of variance for the lattice square design and the model in Table 1 would have to be changed to include row and column effects within replicates in place of incomplete block effects within replicates.

block is less than or equal to k = size of the incomplete block, that each testcorss appears once in one of the incomplete blocks of each replicate (this condition could also be modified if desired), and that the other conditions described in Table 1 hold. In Table 2, testcross totals $Y_{..ij}$, the tester parent totals $Y_{..i.}$, the line totals $Y_{...j}$, and the total of the $m_{..}$ testcrosses are described in terms of the effects given in Table 1. An analysis of variance for the totals in Table 2 is presented in Table 3.

If expectations of the sums of squares in Table 3 are obtained under the "random effects" model (Table 4), the resulting expectations may be equated to the sums of squares of the data and estimates for the various components of variance may be obtained, provided estimates of σ_e^2 and σ_b^2 are available. Estimates of the latter two components of variance may be from the analysis of variance for an incomplete block design; this analysis is of the form:

Source of variation	df	Expected value
Among complete blocks	r-1	--
Among entries (ignoring incomplete blocks)	v-1	--
Entries x complete blocks (ignoring incomplete blocks)	(r-1)(v-1)=f	--
Among incomplete blocks within complete blocks (eliminating entry effects)	b-r	$\sigma_e^2+K\sigma_b^2$
Intrablock error	f-b+r	σ_e^2

The coefficient of σ_b^2, i.e., K, has been obtained in general terms [1] and is simple in form for a number of incomplete block designs. Thus, the method proposed here is a combination of Henderson's methods 1 and 3 [4]. If the tester parent effects are considered to be fixed effects, then it would be necessary to estimate the tester parent effects and correct the $Y_{..ij}$ totals for these effects and then to proceed as described above (Henderson's [4] methods 2 and 3). The various models described by Griffing [3] may be utilized in conjunction with Henderson's methods to estimate the various components of variance.

Returning now to the breeding experiment on potatoes, the totals, $Y_{..ij}$, for the $m_{..}$=190 testcrosses are presented in Tables 1, 2, and 3 of the preceding paper [5]. In order to illustrate the method described in this paper, Tables 5 and 6 have been prepared. (It should be noted that there were three basic

Table 1. Model and conditions for testcross experiment

Yield for $ghij^{th}$ observation is equal to

$Y_{ghij}=m_{ij}n_{ghij}(\mu+\rho_g+\beta_{gh}+\alpha_i+\gamma_j+\alpha\gamma_{ij}+\epsilon_{ghij})$ where

n_{ghij} = 1, if ijth testcross appears in hth incomplete block of replicate g
= 0, otherwise;

m_{ij} = 1, if jth line is crossed with ith tester parent
= 0, otherwise;

g = 1,2,•••,r= number of complete blocks; h=1,2,•••,b/r=2k=number of incomplete blocks within complete blocks; i=1,2,•••,a=number of tester parents;

j = 1,2,•••,c=number of lines; μ=general mean effect;

ρ_g = effect of gth replicate or complete block with the ρ_g independently and identically (II) distributed with mean zero and variance σ^2_ρ;

β_{gh} = effect of hth incomplete block within gth complete block with the β_{gh} II distributed with mean zero and variance $\sigma^2_\beta=\sigma^2_b$;

α_i = effect of ith tester parent with the α_i II distributed with mean zero and variance σ^2_α;

γ_j = effect of jth line with the γ_j II distributed with mean zero and variance $\sigma^2_\gamma=\sigma^2_g$ = variance component for general combining ability;

$\alpha\gamma_{ij}$ = interaction effect of jth line crossed with ith tester with the $\alpha\gamma_{ij}$ II distributed with mean zero and variance $\sigma^2_{\alpha\gamma}=\sigma^2_s$ = variance component for specific combining ability;

ϵ_{ghij} = random effect II distributed with mean zero and variance $\sigma^2_\epsilon=\sigma^2_e$.

$\sum_{i=1}^{a}\sum_{j=1}^{c_i} m_{ij}=m_{..}$=number of testcrosses=190 testcrosses in the potato experiment

$\sum_{i=1}^{a} m_{ij}=m_{.j}$=number of tester parents crossed with jth line

$\sum_{j=1}^{c} m_{ij}=m_{i.}$=number of lines crossed with ith tester parents

$\sum_{gh}\sum n_{ghij}=n_{..ij}=r=6$ for the potato breeding experiment

$\sum_{h=1}^{2k}\sum_{ij=1}^{v} n_{ghij}=n_{g...}$=number of entries in gth replicate=v

$\sum_h n_{ghij}=n_{g.ij}=1$

$\sum_{gh}\sum n_{ghij}m_{ij}=rm_{ij}=6m_{ij}$

$\sum_{gh}\sum\sum_{ij} n_{ghij}=m_{ij}=rm_{..}=6(190)=1140$

Table 2. <u>Designation of totals used in the testcross analysis</u>*

Tester parent	Line 1	2	...	j	...	c	Total
1	$Y_{\cdot\cdot 11}$	$Y_{\cdot\cdot 12}$		$Y_{\cdot\cdot 1j}$		$Y_{\cdot\cdot 1c}$	$Y_{\cdot\cdot 1\cdot}$
2	$Y_{\cdot\cdot 21}$	$Y_{\cdot\cdot 22}$		$Y_{\cdot\cdot 2j}$		$Y_{\cdot\cdot 2c}$	$Y_{\cdot\cdot 2\cdot}$
⋮							
i	$Y_{\cdot\cdot i1}$	$Y_{\cdot\cdot i2}$		$Y_{\cdot\cdot ij}$		$Y_{\cdot\cdot ic}$	$Y_{\cdot\cdot i\cdot}$
⋮							
a	$Y_{\cdot\cdot a1}$	$Y_{\cdot\cdot a2}$		$Y_{\cdot\cdot aj}$		$Y_{\cdot\cdot ac}$	$Y_{\cdot\cdot a\cdot}$
Total	$Y_{\cdot\cdot\cdot 1}$	$Y_{\cdot\cdot\cdot 2}$		$Y_{\cdot\cdot\cdot j}$		$Y_{\cdot\cdot\cdot c}$	$Y_{\cdot\cdot\cdot\cdot}$

*$Y_{\cdot\cdot ij} = 0$ for any $m_{ij}=0$ (i.e., no yield was observed if the cross was not included in the experiment).

In terms of effects, the various totals are equal to:

$$Y_{\cdot\cdot ij} = rm_{ij}(\mu+\alpha_i+\gamma_j+\alpha\gamma_{ij})+m_{ij}\sum_{g=1}^{r}\rho_g+m_{ij}\sum_{g=1}^{r}\sum_{h=1}^{b/r}n_{ghij}(\beta_{gh}+\epsilon_{ghij})$$ = total for ijth testcross.

$$Y_{\cdot\cdot i\cdot} = rm_{i\cdot}(\mu+\alpha_i)+m_{i\cdot}\sum_{g=1}^{r}\rho_g+r\sum_{j=1}^{c}m_{ij}(\gamma_j+\alpha\gamma_{ij})+\sum_{g=1}^{r}\sum_{h=1}^{b/r}\sum_{j=1}^{c}m_{ij}n_{ghij}(\beta_{gh}+\epsilon_{ghij})$$

= total for ith tester parent over all lines.

$$Y_{\cdot\cdot\cdot j} = rm_{\cdot j}(\mu+\gamma_j)+m_{\cdot j}\sum_{g=1}^{r}\rho_g+r\sum_{i=1}^{a}m_{ij}(\alpha_i+\alpha\gamma_{ij})+\sum_{g=1}^{r}\sum_{h=1}^{b/r}\sum_{i=1}^{a}m_{ij}n_{ghij}(\beta_{gh}+\epsilon_{ghij})$$

= total for jth line over all tester parents.

$$Y_{\cdot\cdot\cdot\cdot} = m_{\cdot\cdot}r\mu+m_{\cdot\cdot}\sum_{g=1}^{r}\rho_g+r\sum_{i=1}^{a}m_{i\cdot}\alpha_i+r\sum_{j=1}^{c}m_{\cdot j}\gamma_j+r\sum_{i=1}^{a}\sum_{j=1}^{c}m_{ij}\alpha\gamma_{ij}$$

$$+\sum_{g=1}^{r}\sum_{h=1}^{b/r}\sum_{i=1}^{a}\sum_{j=1}^{c}m_{ij}n_{ghij}(\beta_{gh}+\epsilon_{ghij})$$

= total of $m_{\cdot\cdot}$ testcrosses over r replicates.

Table 3. Analysis of variance on testcross totals

Source of variation	d.f.	Sum of squares
Total (uncorrected)	$m_{..}$	$\sum_{i=1}^{a} \sum_{j=1}^{c} m_{ij} Y^2_{ij..}/r = T$
Correction for mean	1	$Y^2_{....}/rm_{..} = CT$
Among tester parents	a-1	$\sum_{i=1}^{a} \frac{Y^2_{..i.}}{rm_{i.}} - CT = A$
Among lines	c-1	$\sum_{i=1}^{c} \frac{Y^2_{...j}}{rm_{.j}} - CT = C$
Tester parent x line	(a-1)(c-1)	T-A-C-CT=I

Table 4. Expected values of sums of squares in Table 3

Sum of Squares	Component						
	μ^2	σ^2_ρ	σ^2_α	$\sigma^2_\gamma=\sigma^2_g$	$\sigma^2_{\alpha\gamma}=\sigma^2_s$	σ^2_β	σ^2_ϵ
$\Sigma \frac{m_{ij}Y^2_{\cdot\cdot ij}}{r}$	$rm_{\cdot\cdot}$	$m_{\cdot\cdot}$	$rm_{\cdot\cdot}$	$rm_{\cdot\cdot}$	$rm_{\cdot\cdot}$	$m_{\cdot\cdot}$	$m_{\cdot\cdot}$
$\sum_{i=1}^{a} \frac{Y^2_{\cdot\cdot i\cdot}}{rm_{i\cdot}}$	$rm_{\cdot\cdot}$	$m_{\cdot\cdot}$	$rm_{\cdot\cdot}$	ar	ar	$\frac{1}{r}\sum\sum\sum_{igh} \frac{(\sum_j n_{ghij}m_{ij})^2}{m_{i\cdot}}$	a
$\sum_{j=1}^{c} \frac{Y^2_{\cdot\cdot\cdot j}}{rm_{\cdot j}}$	$rm_{\cdot\cdot}$	$m_{\cdot\cdot}$	cr	$rm_{\cdot\cdot}$	cr	$\frac{1}{r}\sum\sum\sum_{jgh} \frac{(\sum_i n_{ghij}m_{ij})^2}{m_{\cdot j}}$	c
$\frac{Y^2_{\cdot\cdot\cdot\cdot}}{rm_{\cdot\cdot}}$	$rm_{\cdot\cdot}$	$m_{\cdot\cdot}$	$\frac{r}{m_{\cdot\cdot}} \sum_{i=1}^{a} m^2_{i\cdot}$	$\frac{r}{m_{\cdot\cdot}} \sum_{j=1}^{c} m^2_{\cdot j}$	r	$\frac{1}{rm_{\cdot\cdot}} \sum\sum_{gh} (\sum\sum_{ij} n_{ghij}m_{ij})^2$	1

Expected value of A is

$$E(A)=(a-1)\sigma^2_\epsilon+\frac{1}{r}\sigma^2_\beta\Big[\sum_{i=1}^{a}\frac{1}{m_{i\cdot}}\sum_{g=1}^{r}\sum_{h=1}^{b/r}(\sum_{j=1}^{c}n_{ghij}m_{ij})^2-\frac{1}{m_{\cdot\cdot}}\sum_{g=1}^{r}\sum_{h=1}^{b/r}(\sum_{i=1}^{a}\sum_{j=1}^{c}n_{ghij}m_{ij})^2\Big]$$

$$+r\sigma^2_{\alpha\gamma}(a-1)+r\sigma^2_\gamma(a-\frac{1}{m_{\cdot\cdot}}\sum_{j=1}^{c}m^2_{\cdot j})+r\sigma^2_\alpha(m_{\cdot\cdot}-\frac{1}{m_{\cdot\cdot}}\sum_{i=1}^{a}m^2_{i\cdot})$$

$$E(C)=(c-1)\sigma^2_\epsilon+\frac{1}{r}\sigma^2_\beta\Big[\sum_{j=1}^{c}\frac{1}{m_{\cdot j}}\sum_{g=1}^{r}\sum_{h=1}^{b/r}(\sum_{i=1}^{a}n_{ghij}m_{ij})^2-\frac{1}{m_{\cdot\cdot}}\sum_{g=1}^{r}\sum_{h=1}^{b/r}(\sum_{i=1}^{a}\sum_{j=1}^{c}n_{ghij}m_{ij})^2\Big]$$

$$+r\sigma^2_{\alpha\gamma}(c-1)+r\sigma^2_\alpha(c-\frac{1}{m_{\cdot\cdot}}\sum_{i=1}^{a}m^2_{i\cdot})+r\sigma^2_\gamma(m_{\cdot\cdot}-\frac{1}{m_{\cdot\cdot}}\sum_{j=1}^{c}m^2_{\cdot j})$$

$$E(I)=(m_{\cdot\cdot}-a-c+1)(\sigma^2_\epsilon+r\sigma^2_{\alpha\gamma})+\sigma^2_\beta\Big[m_{\cdot\cdot}-\frac{1}{r}\sum_{i=1}^{a}\frac{1}{m_{i\cdot}}\sum_{g=1}^{r}\sum_{h=1}^{b/r}(\sum_{j=1}^{c}n_{ghij}m_{ij})^2$$

$$-\frac{1}{r}\sum_{j=1}^{c}\frac{1}{m_{\cdot j}}\sum_{g=1}^{r}\sum_{h=1}^{b/r}(\sum_{i=1}^{a}n_{ghij}m_{ij})^2+\frac{1}{rm_{\cdot\cdot}}\sum_{g=1}^{r}\sum_{h=1}^{b/r}(\sum_{i=1}^{a}\sum_{j=1}^{c}n_{ghij}m_{ij})^2\Big]$$

$$+r\sigma^2_\alpha(\frac{1}{m_{\cdot\cdot}}\sum_{i=1}^{a}m^2_{i\cdot}-c)+r\sigma^2_\gamma(\frac{1}{m_{\cdot\cdot}}\sum_{j=1}^{c}m^2_{\cdot j}-a)$$

Table 5. Values of $\sum_{j} n_{ghij} m_{ij}$ with corresponding sums and sums of squares

Incomplete Block	2 Reps with Z grouping (g=2 and 5): Tester Parent i=1	2	3	4	5	6	$\sum\sum_{ij} n_{ghij} m_{ij}$	2 Reps with Y grouping (g=3 and 4): Tester Parent 1	2	3	4	5	6	$\sum\sum_{ij} n_{ghij} m_{ij}$	2 Reps with X grouping (g=1 and 6): Tester Parent 1	2	3	4	5	6	$\sum\sum_{ij} n_{ghij} m_{ij}$
h = 1	1	2	2	2	0	2	9	3	3	1	2	1	0	10	0	2	1	4	0	1	8
2	2	3	0	1	2	1	9	3	2	2	1	1	0	9	2	2	1	4	0	1	10
3	4	3	0	1	1	1	10	1	2	3	0	1	3	10	3	3	0	0	3	1	10
4	4	2	1	1	1	1	10	1	3	0	0	5	1	10	1	2	2	2	3	0	10
5	2	2	0	2	2	2	10	1	2	3	2	2	0	10	1	2	2	2	1	2	10
6	2	3	1	1	2	1	10	2	3	1	2	0	1	9	4	1	1	1	1	1	9
7	3	1	2	3	1	0	10	3	2	3	2	0	0	10	1	1	2	1	3	2	10
8	1	1	3	2	1	2	10	0	0	2	6	1	1	10	3	0	3	1	2	1	10
9	0	3	5	0	1	0	9	3	0	1	0	2	3	9	1	4	2	1	0	2	10
10	0	0	3	3	3	1	10	2	3	1	1	1	2	10	3	3	3	0	1	0	10
11	1	0	4	1	1	0	7	0	0	1	4	3	2	10	0	4	1	2	0	2	9
12	2	4	0	2	0	1	9	1	2	3	2	0	1	9	2	3	0	1	1	2	9
13	2	3	1	1	2	0	9	1	3	3	1	1	0	9	3	0	1	3	3	0	10
14	2	1	3	1	2	1	10	2	2	2	2	0	2	10	3	0	3	1	1	1	9
15	1	1	2	2	3	0	9	2	1	0	2	2	2	9	0	1	6	1	1	0	9
16	1	1	1	1	2	4	10	1	2	1	2	1	1	8	2	1	1	1	1	3	9
17	1	1	2	2	1	2	9	2	2	1	0	2	2	9	0	4	0	1	1	3	9
18	1	4	1	2	0	2	10	2	3	3	0	1	0	9	0	1	3	2	2	2	10
19	0	2	3	3	1	1	10	1	1	4	3	0	1	10	1	2	3	2	1	0	9
20	2	2	3	1	0	2	10	1	3	2	0	2	2	10	2	3	2	2	1	0	10
$m_{i.}$	32	39	37	32	26	24	190	32	39	37	32	26	24	190	32	39	37	32	26	24	190

$m_{..}=190$

$$\sum_i \frac{1}{m_{i.}} \sum\sum_{gh} \left(\sum_j n_{ghij} m_{ij}\right)^2 = 2\left\{\frac{1^2+2^2+\cdots+2^2}{32} + \frac{2^2+3^2+\cdots+3^2}{39} + \cdots + \frac{2^2+1^2+\cdots+0^2}{24}\right\} = 85.96$$

$$\frac{1}{m_{..}} \sum\sum_{gh} \left(\sum\sum_{ij} n_{ghij} m_{ij}\right)^2 = \frac{1}{190}(9^2+9^2+\cdots+10^2) = 57.26$$

arrangements (X, Y, and Z) for the rectangular lattice used here; each arrangement was utilized twice to obtain the six complete blocks.) Utilizing the expectations in Table 4 divided by degrees of freedom and the values in Tables 5 and 6, remembering that $m_{..}=190$, $a=6$, $c=45$, and $r=6$, the coefficients of the various components of variance are computed (Table 7). The intrablock and interblock components of variance and the mean squares for the three sources of variation (line, tester parent, and interaction) at each of the three locations are also given in Table 7. The estimated specific (σ_s^2) and general (σ_g^2) combining ability components of variance and the components of variance for tester parents are presented in Table 8 for each of the three locations.

The use and interpretation of the components of variance for combining ability depend upon the particular type of experiment involved; Sprague and Tatum [6], Griffing [3], the preceding paper in this Journal [5], etc. illustrate the use of these components of variance and the calculation and use of the effects in plant breeding experiments.

Summary

A method is described for estimating components of variance for specific and general combining ability from breeding experiments arranged in incomplete block designs. Utilizing published results estimates of these components of variances were computed for a potato breeding experiment arranged in a rectangular lattice design with the parameters v=200 entries, k=20 entries per incomplete block, r=6 complete blocks, and b/r=20 incomplete blocks within each complete block. The experiment was grown at three different locations thus yielding three estimates of the components of variance for combining ability. These results were used in the computations of expected genetic progress in the preceding paper of this Journal.

Table 6. Values of $\sum_i n_{ghij} m_{ij}$ for 2 reps. with X grouping

Incomplete Block	Line j = 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
h = 1	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	1	0	1	0	0	0	0	1	1	0	0	1	1	0	0	0	0	0	0
3	0	0	0	0	2	0	0	0	0	0	0	0	0	2	0	0	0	1	0	0	0	1	0
4	1	0	0	0	0	2	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	0	0
5	0	0	0	0	0	0	0	1	1	0	0	1	0	0	0	1	0	0	2	0	1	0	0
6	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	1
7	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	2	0	0	0	0
8	0	0	0	0	1	0	0	0	0	0	2	0	0	1	0	0	0	0	0	0	0	0	0
9	1	0	1	0	0	1	0	1	0	0	0	1	1	0	1	0	0	0	0	0	0	0	0
10	0	1	1	0	0	0	0	0	0	1	1	0	1	0	1	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	2
12	0	0	1	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	2	0	0	1	0	0	1	0	1	0	0	1	0	0	0	0	0	0	0	0
14	0	0	1	0	0	0	0	0	0	0	0	0	2	0	0	1	0	1	1	0	0	0	0
15	0	0	0	0	0	1	1	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	1	0	0	1	0	0
17	0	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	1	0	1	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	1
19	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	1	0
20	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
$m_{\cdot j}$	4	5	4	6	6	6	6	2	1	4	6	5	5	4	3	4	2	5	6	5	3	3	4

Incomplete Block	Line j = 24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
h = 1	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	1	0	1	0	0
2	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1
3	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0
4	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
5	0	0	0	0	0	2	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	1	0	1	0	0	1	0	0	1	0	0	0	0	0	0
7	0	0	0	0	0	0	2	0	1	1	0	0	0	0	0	1	0	0	0	0	0	0
8	1	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	2	0	1	0
9	0	0	0	0	0	2	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
10	1	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1
11	1	1	0	1	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
12	0	0	0	0	1	0	0	0	1	0	0	1	0	0	1	0	1	1	0	0	0	0
13	0	0	1	0	0	0	0	1	0	0	0	0	0	1	0	0	1	0	0	0	0	0
14	0	0	0	0	0	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0	1	0	0	1	0
16	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0	0	0	1	0	0	1	0
17	2	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	1
18	0	0	0	0	0	0	0	0	0	1	1	1	0	1	0	0	1	0	0	0	0	0
19	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0
20	0	0	0	0	1	0	0	0	0	0	0	1	1	0	0	0	0	1	0	2	0	0
$m_{\cdot j}$	5	4	4	3	4	6	6	5	4	5	4	5	4	2	1	5	3	6	3	5	4	3

Table 6 (cont'd). Values of $\sum_i n_{ghij} m_{ij}$ for 2 reps. with Y grouping

Incomplete Block	Line j =																						
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
h = 1	1	1	0	0	2	1	1	0	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0
2	0	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0	1	0	0
3	0	0	1	1	0	0	0	0	0	0	1	1	1	0	0	0	1	1	0	0	0	0	0
4	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
5	0	1	0	0	0	0	0	1	0	0	0	0	0	2	1	0	0	1	1	0	0	0	0
6	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	0	0	0	0	0
7	0	1	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	1	0	0	0	0	0
9	1	0	0	0	0	0	0	0	0	0	2	1	0	0	0	0	0	0	1	1	0	1	0
10	0	0	0	1	1	1	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1
11	0	0	0	1	0	1	0	0	0	0	1	0	1	0	1	0	0	0	0	1	0	0	0
12	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	1	0	1	0	1
13	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0
14	1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
15	0	0	0	0	0	1	0	0	0	1	0	2	0	0	0	0	0	1	0	0	0	0	1
16	0	0	0	0	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	1	0	0	0
17	0	0	0	1	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
19	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0	1
20	0	0	1	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
$m_{\cdot j}$	4	5	4	6	6	6	6	2	1	4	6	5	5	4	3	4	2	5	6	5	3	3	4

Incomplete Block	Line j =																					
	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
h = 1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	1	0	1	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0
4	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1	2
5	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0
6	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	2	0	0	1	1	0	0
7	0	1	0	0	0	1	0	1	1	0	0	0	1	0	0	0	0	0	0	0	1	0
8	1	1	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0
9	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0
10	1	0	0	1	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0
12	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0
13	1	0	0	0	0	0	0	1	0	1	0	2	0	0	0	0	1	0	0	0	1	0
14	0	0	1	1	0	0	1	0	0	0	1	1	0	0	0	0	0	1	0	0	0	0
15	0	1	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	1	1	0	0
17	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	1	1	0	0	1	0	0
18	1	0	1	0	0	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
19	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	1	0	0	0	0	0	1
20	1	0	1	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0
$m_{\cdot j}$	5	4	4	3	4	6	6	5	4	5	4	5	4	2	1	5	3	6	3	5	4	3

Table 6 (cont'd). Values of $\sum_i n_{ghij} m_{ij}$ for 2 reps with Z grouping

Line j = 1–23

Incomplete Block	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
h = 1	1	1	0	0	2	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
2	0	0	0	0	1	0	0	1	1	0	1	0	0	0	1	0	1	0	0	0	0	0	0
3	0	0	0	0	0	1	0	0	0	0	2	0	0	1	0	1	0	0	0	0	0	0	0
4	0	0	1	1	1	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1	0	0	0	1
6	0	2	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	1	1	0	0	0
7	0	0	0	1	0	1	0	0	0	0	0	0	1	0	1	1	0	0	1	0	0	1	0
8	2	0	0	1	1	0	0	0	0	1	0	1	0	1	0	0	0	0	0	0	1	0	0
9	0	0	1	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	1	1	0	0	0
10	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	2
13	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	1	0	0
14	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
15	1	0	0	1	0	2	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0	1	1
16	0	1	0	0	0	0	2	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0
17	0	0	2	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
18	0	1	0	0	0	1	2	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	1	0	0	0	0
20	0	0	0	0	1	0	0	0	0	2	0	0	1	0	0	0	0	0	1	0	0	1	0
$m_{\cdot j}$	4	5	4	6	6	6	6	2	1	4	6	5	5	4	3	4	2	5	6	5	3	3	4

Line j = 24–45

Incomplete Block	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
h = 1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
2	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0
3	1	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	1
4	0	1	0	0	0	0	0	1	0	0	1	0	0	0	0	2	0	0	0	0	0	0
5	0	0	0	0	1	0	1	1	0	2	0	0	0	0	0	0	0	0	0	0	0	1
6	0	1	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0
7	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	1	0
8	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
9	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
10	0	0	0	1	1	2	0	0	2	1	0	0	0	0	0	1	0	0	0	0	0	0
11	0	1	0	0	0	0	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0
12	0	0	2	0	0	0	0	0	0	0	2	0	0	0	0	0	1	1	0	0	0	0
13	0	0	0	0	1	0	0	0	0	0	0	1	1	0	0	1	0	1	0	0	0	1
14	1	0	0	1	0	1	1	1	1	0	0	1	0	1	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	1	0	1	0	0
17	1	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	1	1	0
18	1	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	1	1	0	0	0
19	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	2	1	0
20	0	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0
$m_{\cdot j}$	5	4	4	3	4	6	6	5	4	5	4	5	4	2	1	5	3	6	3	5	4	3

Table 7. Expected values of mean squares for the potato breeding experiment

Source of variation	d.f.	Expected value of mean square
Among tester parents	5	$\sigma_e^2 + .96\sigma_\beta^2 + 6\sigma_{\alpha\gamma}^2 + 188.91\sigma_\alpha^2 + 1.63\sigma_\gamma^2$
Among lines	44	$\sigma_e^2 + .95\sigma_\beta^2 + 6\sigma_{\alpha\gamma}^2 + 1.69\sigma_\alpha^2 + 25.28\sigma_\gamma^2$
Tester parents x lines	140	$\sigma_e^2 + .96\sigma_\beta^2 + 6\sigma_{\alpha\gamma}^2 - .53\sigma_\alpha^2 - .06\sigma_\gamma^2$

Source of variation	Location		
	Riverhead	Ithaca	Clear Lake
σ_e^2 = intrablock variance component	9.36	16.70	10.10
σ_b^2 = interblock variance component	5.75	8.83	2.31
Tester parent mean square	199.50	234.70	68.75
Line mean square	64.15	59.42	57.99
Tester parents x lines mean square	48.95	32.48	27.54

Table 8. Estimates of the variance components

Component estimated	Location			Average
	Ithaca	Riverhead	Clear Lake	
σ_α^2	.79	1.06	.21	.69
$\sigma_\gamma^2 = \sigma_g^2$	.53	.97	1.18	.89
$\sigma_{\alpha\gamma}^2 = \sigma_s^2$	4.04	3.04	2.57	3.22

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