ON THE EFFICIENCY OF THE NEW GENETIC SELECTION INDEX

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Abstract

A new genetic selection index was proposed by the author in [1] for the case of unknown $E(Y)$. Here, the efficiency of this index is compared with the index for known $E(Y)$. 
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1. Introduction

Consider the genetic model

\[ \mathbf{y} = \mathbf{X}\beta + \mathbf{Zu} + \mathbf{e}; \quad (\mathbf{e} | \mathbf{u}) \sim N[0, \begin{pmatrix} \mathbf{E} & \mathbf{0} \\ \mathbf{0} & \mathbf{G} \end{pmatrix}] \]. \quad \text{--- (1)}

Suppose we want to predict a linear combination of the 'genetic' components of the model, say \( \mathbf{g}'u \). When \( \beta \) is known the best predictor for \( \mathbf{g}'u \) is given by

\[ \hat{\mathbf{u}}_1 = \mathbf{GZ'Z}^{-1}( \mathbf{y} - \mathbf{X}\beta) \] \quad \text{--- (2)}

and

\[ \mathbf{A} = \mathbf{ZGZ'} + \mathbf{E} \).

A new selection index \( \hat{\mathbf{u}}_2 \) was proposed in [1], viz.

\[ \hat{\mathbf{u}}_2 = \mathbf{GZ'(PAP')^{-1}P'Y} \] \quad \text{--- (3)}

where

\[ \mathbf{P} = (\mathbf{I} - \mathbf{X}(\mathbf{X}'\mathbf{A}^{-1}\mathbf{X})^{-1}\mathbf{X}'\mathbf{A}^{-1}) \]

and hence a predictor of \( \mathbf{g}'u \) is \( \mathbf{g}'\hat{\mathbf{u}}_2 \).

We shall now show that even when \( \beta \) is known, (3) can be used and still achieve at least the same efficiency as (2) in terms of variances of the predicted values.
2. The efficiency

For any \( \xi \), we have

\[
V(\xi'\hat{u}(1)) = \xi'GZ'A^{-1}ZG\xi
\]

--- (4)

\[
V(\xi'\hat{u}(2)) = \xi'GZ'P'(PAP')^{-1}PZG\xi
\]

--- (5)

and

\[
\text{Cov}(\xi'\hat{u}(1), \xi'\hat{u}(2)) = \xi'GZ'A^{-1}AP'(PAP')^{-1}ZG\xi
\]

\[
= \xi'GZ'P'(PAP')^{-1}ZG\xi
\]

\[
= V(\xi'\hat{u}(2))
\]

--- (6)

\[
\therefore [V(\xi'\hat{u}(2))]^2 = \left[ \text{Cov}(\xi'\hat{u}(1), \xi'\hat{u}(2)) \right]^2
\]

\[
\leq V(\xi'\hat{u}(1)) \cdot V(\xi'\hat{u}(2))
\]

--- (7)

Since \( V(\xi'\hat{u}(2)) \geq 0 \), (7) gives

\[
V(\xi'\hat{u}(2)) \leq V(\xi'\hat{u}(1)) \quad \text{for all } \xi
\]

Thus \( \xi'\hat{u}(2) \) is a more efficient predictor of \( \xi'\hat{u} \) than \( \xi'\hat{u}(1) \) in general. It is easy to see that, when \( \beta \) is known, \( V(\xi'\hat{u}(1)) = V(\xi'\hat{u}(2)) \) since \( \xi'\hat{u}(1) \) is the minimum variance predictor in that case. The generalization to the case, where

\[
V(\xi'\hat{u}) = \begin{pmatrix} E & C \\ C & G \end{pmatrix}; \ C \neq 0
\]

is straightforward.

Reference