A NOTE ON POWERS OF THE SAMPLE MEAN

by

E. J. Carney*

Cornell University

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Abstract

Dayhoff gives a formula for the expansion of powers of sample means as linear functions of polykays, but gives no proof for the correctness of the formula in general. A simple proof utilizing the relation between power sums and polykays represented as functions of ordered partitions is given here.

* On leave from the University of Rhode Island

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1. Introduction

Dayhoff [3] exhibits the formula

$$\bar{x}^t = \sum \frac{1}{n^{t-\rho}} \frac{t!}{(p_1!)^{\Pi_1} \cdots (p_m!)^{\Pi_m} \Pi_1! \cdots \Pi_m!} k_P \tag{1}$$

for the expansion of $\bar{x}^t$ as a linear function of sample polykays, k_P. Here P indicates the partition $p_1^{\Pi_1} p_2^{\Pi_2} \cdots p_m^{\Pi_m}$ of t and $\rho = \rho(P) = \Pi_1 + \Pi_2 + \cdots + \Pi_m$ is the number of parts of P. The summation extends over all partitions P of t. (1) is more conveniently written as

$$\bar{x}^t = \sum \frac{1}{n^{t-\rho}} C(P)\, k_P \tag{2}$$

using the combinatorial coefficient [5] of the partition P

$$C(P) = \frac{t!}{(p_1!)^{\Pi_1} \cdots (p_m!)^{\Pi_m} \Pi_1! \cdots \Pi_m!} . \tag{3}$$

(2) leads immediately to the result

$$E(\bar{x}^t) = \sum \frac{1}{n^{t-\rho}} C(P)\, K_P$$

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since $E(k_P) = K_P$, the population polykay for the partition P . Dayhoff was encouraged to pursue use of polykays in his work on variance components [4] by the simplicity of the formula (1) but found it difficult to prove in general and does not give a proof.

2. Power Sum Expansions

When the symmetric functions are represented (redundantly) in terms of ordered partitions the power sums of degree t may be expressed in terms of polykays by

$$[\underset{\sim}{\ }] = A(\underset{\sim}{\ }) \quad . \tag{5}$$

$[\underset{\sim}{\ }]$, $(\underset{\sim}{\ })$ denote vectors of power sums and polykays, respectively, written in terms of ordered partitions $\alpha_1, \alpha_2, \cdots, \alpha_s$, say, with the convention that α_i is not a subpartition of α_j if $i < j$. The matrix A has elements $a_{ij} = n^{\varphi_{ij}}$ [2], where φ_{ij} is the number of parts of the lub partition of α_i and α_j [1].

Let α_s denote the t part ordered partition. Then $\phi_{si} = \rho_i$, since α_s is a subpartition of each ordered partition of weight t . The last row of A is then

$$n^{\rho_1} \ n^{\rho_2} \ \ldots \ n^{\rho_s}$$

which gives the identity

$$(\Sigma x)^t = [\alpha_s] = \Sigma \ n^{\rho_i}(\alpha_i) \ ,$$

where (α_i) denotes the polykay for the ordered partition α_i . Since C(P) is the number of order partitions of the partition P, we have

$$(\Sigma x)^t = \Sigma \ n^{\rho_i} \ C(P) \ k_P \ ,$$

which, when divided by n^t gives (2).

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