A NOTE ON A METHOD OF CONSTRUCTION OF DESIGNS FOR

TWO-WAY ELIMINATION OF HETEROGENEITY

D. Raghavarao and G. Nageswararao
Punjab Agricultural University

ABSTRACT

In this note we present a mathematical proof for a method of constructing a balanced generalized two-way elimination of heterogeneity design. The method was given in a paper by H. Agrawal in 1966.

*Visiting professor, Cornell University

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INTRODUCTION

Of the methods of construction of two-way elimination of heterogeneity designs, Agrawal (1966) had no mathematical proof for the following construction procedure:

Method. Let $D$ be a symmetrical balanced incomplete block (BIB) design (cf. Raghavarao [1971], pp. 63) with parameters $v = b, r = k, \lambda$. Let $D_1$ be the derived design (cf. Raghavarao [1971], pp. 65) of $D$ with parameters $v_1 = v-k, b_1 = v-1, r_1 = k, k_1 = k-\lambda, \lambda_1 = \lambda$

and let $D_2$ be the residual design (cf. Raghavarao [1971], pp. 65) of $D$ with incidence matrix $N_2$ having the parameters $v_2 = k, b_2 = v-1, r_2 = k-1, k_2 = \lambda, \lambda_2 = \lambda-1$.

In the incidence matrix of the complementary design (cf. Raghavarao [1971], pp. 65) of $D_2$, by replacing 1's in the $i^{th}$ column by the symbols of the $i^{th}$ set of $D_1$ and rearranging the symbols in such a way that each of the $v-k$ symbols occurs exactly once in each row, a balanced generalized two-way elimination of heterogeneity design will be obtained.

The proof that the symbols could be arranged such that every row is a complete replication of $v-k$ symbols will be supplied by us in this communication with the help of systems of distinct representatives (SDR). For the definition and results of SDRs, we refer to chapter 6 of Raghavarao (1971).
PROOF OF THE METHOD

A moment's consideration reveals that we will be through with the proof if we show that SDRs exist for the sets of $D_2$ in which each of the symbols of the first set of $D$ does not occur in $D_1$. Let $\theta_1, \theta_2, \ldots, \theta_k$ be the $k$ symbols in the first set of $D$ and let $\phi_1, \phi_2, \ldots, \phi_{v-k}$ be the other $v-k$ symbols. Let $B_1^{(1)}, B_2^{(1)}, \ldots, B_v^{(1)}$ be the sets of $D_2$ not containing $\theta_1$ in the corresponding sets of $D_1$. Since $(\theta_1, \phi_j)$ pair occurs exactly $\lambda$ times in $D$, each $\phi_j$ can occur at most $k-\lambda$ times in $B_1^{(1)}, B_2^{(1)}, \ldots, B_v^{(1)}$ for $j = 1, 2, \ldots, v-k$. One can easily show that the necessary and sufficient condition for the existence of SDR will be satisfied by sets $B_1^{(1)}, B_2^{(1)}, \ldots, B_v^{(1)}$ and this SDR will be a permutation of $\phi_1, \phi_2, \ldots, \phi_{v-k}$. Bringing these symbols in the first column and leaving blanks in the first column of other sets, we omit the first column and analogously show that for the deleted sets $B_1^{(2)}, B_2^{(2)}, \ldots, B_v^{(2)}$ where $\theta_2$ does not occur in the corresponding sets of $D_1$, an SDR exists. This could then be brought to the second position leaving blanks in the other sets. Following this method we can show that the rearrangement as given by the method could be achieved.

It may be noted that this series of designs could be constructed by omitting a column and the symbols in it from the corresponding Youden Square designs. However, in this method also we use SDRs to show the existence of Youden Squares from symmetrical BIB designs.

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BIBLIOGRAPHY
