

EXAMPLES OF OUTPUT FROM A COMPUTER PROGRAM FOR CALCULATING ESTIMATES OF
VARIANCE COMPONENTS AND THEIR SAMPLING VARIANCES

BU-462-M

by

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The computer printout attached hereto shows at least one example of each kind of output available from a program package developed for calculating estimates of variance components and their sampling variances. References in the output such as "LM page 234" are page references to Searle [1971].

Documentation of the program package is available in Searle and Corbeil [1973].

Development of this program was supported by Grant number GJ-31766 from the National Science Foundation.

Searle, S. R. [1971] Linear Models, Wiley, N. Y.

Searle, S. R. and Corbeil, R. R. [1973] Estimation of variance components and their sampling variances: documentation of computer routines, Version 7. Paper BU-442-M of the Biometrics Unit, Cornell University, Ithaca, N. Y.

Example 1:
(3 pages)

1-way classification

Data from Searle, "Linear Models" p. 229

Output includes

optional output for fixed effects model

sampling variances with assigned components

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*** ESTIMATION OF VARIANCE COMPONENTS AND THEIR SAMPLING VARIANCES ***
CONTROL CARD AS READ: CLASS 1, DATA 4, A 3, B 0, C 0, LB 0, NUB 0, VARS 3, OUTPUT 2

1 WAY LM P.229

OPTIONAL OUTPUT FOR FIXED EFFECTS MODEL

FOR ESTIMATES OF ESTIMABLE FUNCTIONS

CELL MEANS CORRESPONDING TO N-PATTERN, SHOWN BELOW AS N(I,J,K)

MEANS

0.730000000D 02 0.780000000D 02 0.890000000D 02

GRD. : 0.790000000D 02 (7)

ANALYSIS OF VARIANCE (LM PAGE 234 OR 253)

	DF	SS
SSM=R(MU)	1	0.4368700D 05
SSRM=R(AIMU)	2	0.3100000D 03
SSE	4	0.8200000D 02
TOTAL	7	0.4407900D 05

ESTIMATED VARIANCES AND/OR SAMPLING VARIANCES OF ESTIMATORS

1 WAY LM P 229

1 WAY NESTED CLASSIFICATION

NAMES OF FACTORS :

A= EDUCATION LEVEL

NO. OF LEVELS OF A :

3

N(I,J,K)

3 2 2

UNCORRECTED SUMS OF SQUARES (T'S) EG. EQ 19 LM PAGE 426

FACTOR A	0.4399700D 05
DUE TO MEAN	0.4368700D 05
TOTAL	0.4407900D 05

ESTIMATED COMPONENTS AND SAMPLING VARIANCES USING THOSE ESTIMATES
ANALYSIS OF VARIANCE METHOD

ESTIMATES		VAR(ANOVA ESTIMATORS)	
A	58.843750	4692.857315	-91.929688
E	20.500000	-91.929688	210.125000

		VAR(LARGE SAMPLE ML ESTIMATORS)		EFFICIENCY
A	3114.051211	-92.546349		0.66
E	-92.546349	210.033519		1.00

'MIVQUE' 3 ITERATION(S)

ESTIMATES		VAR(ESTIMATES)		BASIC MINQUE		VAR(ESTIMATES)	
A	58.155970	4560.875068	-92.802408	58.520295	4615.338359	-94.425868	
E	20.480334	-92.802408	209.691995	20.398524	-94.425868	208.549236	

SAMPLING VARIANCES USING PRE-ASSIGNED COMPONENTS GIVEN AS INPUT

ASSIGNED COMPONENTS		VAR(ANOVA ESTIMATORS)	
A	10.000000	110.599609	-0.218750
E	1.000000	-0.218750	0.500000

		VAR(LARGE SAMPLE ML ESTIMATORS)		EFFICIENCY
A	72.810079	-0.221617		0.66
E	-0.221617	0.499979		1.00

MIVQUE

ASSIGNED COMPONENTS		VAR(ESTIMATES)		BASIC MINQUE		VAR(ESTIMATES)	
A	10.000000	109.175662	-0.221924	10.000000	109.739712	-0.324131	
E	1.000000	-0.221924	0.499993	1.000000	-0.324131	0.518513	

ASSIGNED COMPONENTS		VAR(ANOVA ESTIMATORS)	
A	1.000000	2.177734	-0.218750
E	1.000000	-0.218750	0.500000

		VAR(LARGE SAMPLE ML ESTIMATORS)		EFFICIENCY
A	1.472554	-0.217184		0.68
E	-0.217184	0.498807		1.00

MIVQUE			BASIC MINQUE		
ASSIGNED COMPONENTS	VAR(ESTIMATES)		ASSIGNED COMPONENTS	VAR(ESTIMATES)	
A	1.000000	2.173985	1.000000	2.173985	-0.219926
E	1.000000	-0.219926	1.000000	-0.219926	0.499631

ASSIGNED COMPONENTS	VAR(ANOVA ESTIMATORS)	
A	0.500000	0.978516
E	1.000000	-0.218750

	VAR(LARGE SAMPLE ML ESTIMATORS)	EFFICIENCY
A	0.673410	0.69
E	-0.213873	0.99

MIVQUE			BASIC MINQUE		
ASSIGNED COMPONENTS	VAR(ESTIMATES)		ASSIGNED COMPONENTS	VAR(ESTIMATES)	
A	0.500000	0.978485	0.500000	0.980564	-0.219353
E	1.000000	-0.218589	1.000000	-0.219353	0.499420

Example 2:
(3 pages)

3-way nested classification

Data from Bliss, "Statistics in Biology", Vol. 1, p. 354

Output includes

optional output for fixed effects model

*** ESTIMATION OF VARIANCE COMPONENTS AND THEIR SAMPLING VARIANCES ***
 CONTROL CARD AS READ: CLASS 5, DATA 1, A 11, B 3, C 2, LB 1, NUB 2, VARS 0, OUTPUT 2

BLISS P. 354 3 WAY NESTED UNBALANCED

OPTIONAL OUTPUT FOR FIXED EFFECTS MODEL

FOR ESTIMATING ESTIMABLE FUNCTIONS

CELL MEANS CORRESPONDING TO N-PATTERN, SHOWN BELOW AS N(I,J,K)

		MEANS	
0.4750000000 00	0.4600000000 00	B:A :	0.4675000000 00 (4)
0.2600000000 00	0.5400000000 00	B:A :	0.4000000000 00 (2)
0.5200000000 00		B:A :	0.5200000000 00 (1)
		A :	0.4557142860 00 (7)
0.5450000000 00	0.4850000000 00	B:A :	0.5150000000 00 (4)
0.4000000000-01	0.4300000000 00	B:A :	0.2350000000 00 (2)
0.1080000000 01		B:A :	0.1080000000 01 (1)
		A :	0.5157142860 00 (7)
0.3000000000 00	0.3850000000 00	B:A :	0.3425000000 00 (4)
0.2500000000 00	0.3800000000 00	B:A :	0.3150000000 00 (2)
0.2900000000 00		B:A :	0.2900000000 00 (1)
		A :	0.3271428570 00 (7)
0.2100000000 00	0.3000000000 00	B:A :	0.2550000000 00 (4)
0.1300000000 00	0.2500000000 00	B:A :	0.1900000000 00 (2)
0.1000000000 00		B:A :	0.1000000000 00 (1)
		A :	0.2142857140 00 (7)
0.8550000000 00	0.5550000000 00	B:A :	0.7050000000 00 (4)
0.9500000000 00	0.8400000000 00	B:A :	0.8950000000 00 (2)
0.9200000000 00		B:A :	0.9200000000 00 (1)
		A :	0.7900000000 00 (7)
0.5900000000 00	0.4750000000 00	B:A :	0.5325000000 00 (4)
0.3300000000 00	0.2600000000 00	B:A :	0.2950000000 00 (2)
0.4100000000 00		B:A :	0.4100000000 00 (1)
		A :	0.4471428570 00 (7)
0.6650000000 00	0.5550000000 00	B:A :	0.6100000000 00 (4)
0.4400000000 00	0.5000000000 00	B:A :	0.4700000000 00 (2)
0.4400000000 00		B:A :	0.4400000000 00 (1)
		A :	0.5457142860 00 (7)
0.9050000000 00	0.6650000000 00	B:A :	0.7850000000 00 (4)
0.6400000000 00	0.5400000000 00	B:A :	0.5900000000 00 (2)
0.3600000000 00		B:A :	0.3600000000 00 (1)
		A :	0.6685714290 00 (7)
0.5850000000 00	0.5650000000 00	B:A :	0.5750000000 00 (4)
0.6000000000 00	0.7100000000 00	B:A :	0.6550000000 00 (2)
0.9200000000 00		B:A :	0.9200000000 00 (1)
		A :	0.6471428570 00 (7)
0.5500000000 00	0.5000000000 00	B:A :	0.5250000000 00 (4)
0.4600000000 00	0.5200000000 00	B:A :	0.4900000000 00 (2)
0.5200000000 00		B:A :	0.5200000000 00 (1)
		A :	0.5142857140 00 (7)
0.3000000000 00	0.3900000000 00	B:A :	0.3450000000 00 (4)
0.5300000000 00	0.5000000000 00	B:A :	0.5150000000 00 (2)
0.3900000000 00		B:A :	0.3900000000 00 (1)
		A :	0.4000000000 00 (7)
		GRD. :	0.5023376620 00 (77)

ANALYSIS OF VARIANCE. (LM PAGE 234 OR 253)

	DF	SS
R(MU)	1	0.19430420 02
R(A MU)	10	0.28321540 01
R(B:A U,A)	22	0.99174640 00
R(C:BI U,A,B)	22	0.35757500 00

FACTOR A 0.2127083D 02
 FACTOR B 0.2226257D 02
 FACTOR C 0.2262015D 02
 DUE TO MEAN 0.1943042D 02
 TOTAL 0.2284100D 02

ESTIMATED COMPONENTS AND SAMPLING VARIANCES USING THOSE ESTIMATES
 ANALYSIS OF VARIANCE METHOD

ESTIMATES

VAR(ANOVA ESTIMATORS)

A	0.017497	0.000145	-0.000021	0.000000	0.000000
B	0.015005	-0.000021	0.000054	-0.000006	-0.000001
C	0.004143	0.000000	-0.000006	0.000015	-0.000006
E	0.010039	0.000000	-0.000001	-0.000006	0.000009

VAR(LARGE SAMPLE ML ESTIMATORS)

EFFICIENCY

A	0.000121	-0.000017	0.000000	0.000000	0.83
B	-0.000017	0.000052	-0.000006	-0.000001	0.98
C	0.000000	-0.000006	0.000014	-0.000006	0.97
E	0.000000	-0.000001	-0.000006	0.000009	0.95

Example 3:
(3 pages)

2-way crossed classification, with interaction, random model

Data from Federer, Memoir 350 (y-values of Table 7), N. Y. Agric. Exp. Stat., 1959

Output includes

optional output for fixed effects model

sampling variances of ANOVA estimators with assigned components

estimates from fitting constants method

*** ESTIMATION OF VARIANCE COMPONENTS AND THEIR SAMPLING VARIANCES ***

CONTROL CARD AS READ: CLASS 4, DATA 4, A 6, B 3, C 0, LB 0, NUB 0, VARS 3, OUTPUT 2

DATA SET 12 FEDERER COVARIANCE MEMOIR

OPTIONAL OUTPUT FOR FIXED EFFECTS MODEL

FOR ESTIMATES OF ESTIMABLE FUNCTIONS

CELL MEANS CORRESPONDING TO N-PATTERN			(CELL MEAN =OBSERVATION WHEN N(I,J)=1)	ROW MEANS
0.644000000000	01	0.506000000000	0.510000000000	0.535200000000
0.520000000000	01	0.369000000000	0.508000000000	0.441500000000
0.855000000000	01	0.858000000000	0.541000000000	0.778000000000
0.790000000000	01	0.681000000000	0.648000000000	0.727250000000
0.917000000000	01	0.544000000000	0.686500000000	0.708500000000
0.471500000000	01	0.678000000000	0.600500000000	0.583333333333
COLUMN MEANS				GRAND MEAN
0.682375000000	01	0.604700000000	0.587888888889	0.622111111111

ANOVA FOR FITTING CONSTANTS (LM PAGE 275 OR 298-9)

	A BEFORE B		B BEFORE A	
	DF	SS	DF	SS
R(MU)	1	0.10449600 04	1	0.10449600 04
R(A MU)	5	0.34854580 02	2	0.42625800 01
R(B MU,A)	2	0.29846380 01	5	0.33576640 02
R(AB MU,A,B)	10	0.21228300 02	10	0.21228300 02
SSE	9	0.27654950 02	9	0.27654950 02
TOTAL	27	0.11316820 04	27	0.11316820 04

ESTIMATED VARIANCES AND/OR SAMPLING VARIANCES OF ESTIMATORS

DATA SET 12 FEDERER COVARIANCE MEMOIR

2-WAY CLASSIFICATION WITH INTERACTION, RANDOM MODEL

NAMES OF FACTORS :

A= ROWS

B= COLUMNS

NO. OF LEVELS OF A :

6

NO. OF LEVELS OF B :

3 3 3 3 3 3

N(I,J,K)

1 2 2
 1 2 1
 1 2 1
 2 1 1
 1 1 2
 2 2 2

UNCORRECTED SUMS OF SQUARES (T'S) EG. EQ 19 LM PAGE 426

FACTOR A 0.10798150 04
 FACTOR B 0.10492230 04
 FACTOR AB 0.11040280 04
 DUE TO MEAN 0.10449600 04
 TOTAL 0.11316820 04

ESTIMATED COMPONENTS AND SAMPLING VARIANCES USING THOSE ESTIMATES
 ANALYSIS OF VARIANCE METHOD

ESTIMATES	VAR(ANOVA ESTIMATORS)			
A 1.142534	1.030275	0.023062	-0.200520	0.077030
B 0.011906	0.023062	0.073888	-0.114929	0.041596
C -0.747072	-0.200520	-0.114929	1.514200	-1.505461
E 3.072772	0.077030	0.041596	-1.505461	2.098206

SAMPLING VARIANCES USING PRE-ASSIGNED COMPONENTS GIVEN AS INPUT

ASSIGNED COMPONENTS	VAR(ANOVA ESTIMATORS)			
A 10.000000	52.099576	3.115219	-3.626760	0.008158
B 5.000000	3.115219	33.232484	-3.455323	0.004405
C 2.000000	-3.626760	-3.455323	4.894259	-0.159444
E 1.000000	0.008158	0.004405	-0.159444	0.222222

ASSIGNED COMPONENTS	VAR(ANOVA ESTIMATORS)			
A 5.000000	13.415969	0.656303	-0.861491	0.008158
B 2.000000	0.656303	5.946965	-0.779572	0.004405
C 1.000000	-0.861491	-0.779572	1.410313	-0.159444
E 1.000000	0.008158	0.004405	-0.159444	0.222222

ASSIGNED COMPONENTS	VAR(ANOVA ESTIMATORS)			
A 1.000000	1.128857	0.124535	-0.317183	0.008158
B 1.000000	0.124535	1.780018	-0.203215	0.004405
C 1.000000	-0.317183	-0.203215	0.821417	-0.159444
E 1.000000	0.008158	0.004405	-0.159444	0.222222

FITTING CONSTANTS METHOD

ESTIMATES

3 SETS OF ESTIMATES, DEPENDING ON WHICH EQUATIONS ARE USED.
 SEE LM PAGES 483(WITH INTERACTION) AND 488(WITHOUT INTERACTION)

FOR EQUATIONS:

	USING IA	USING IB	USING IA AND IB
A	1.117645	1.067369	1.067369
B	-0.064159	-0.001418	-0.064159
C	-0.671907	-0.671907	-0.671907
E	3.072772	3.072772	3.072772

Example 4: 2-way crossed classification, no interaction, mixed model
(3 pages) Data from Searle, "Linear Models", Table 7.6

Output includes

ANOVA estimators, assuming random model

variance components and fixed effects estimators using
fitting constants method

Thompson's iterative method

Henderson's method 2

sampling variances of fitting constants estimators, using assigned components

*** ESTIMATION OF VARIANCE COMPONENTS AND THEIR SAMPLING VARIANCES ***
 CONTROL CARD AS READ: CLASS 6, DATA 4, A 4, B 3, C 0, LB 0, NUB 0, VARS 3, OUTPUT 2

2 WAY X'D WITHOUT INTERACTION MIXED MODEL CASE 2 ASSIGNFD COMPONENTS

OPTIONAL OUTPUT FOR FIXED EFFECTS MODEL

FOR ESTIMATES OF ESTIMABLE FUNCTIONS

CELL MEANS CORRESPONDING TO N-PATTERN (CELL MEAN =OBSERVATION WHEN N(I,J)=1)			ROW MEANS
0.100000000000D 02	0.900000000000D 01	0.0	0.960000000000D 01
0.0	0.130000000000D 02	0.800000000000D 01	0.105000000000D 02
0.120000000000D 02	0.0	0.150000000000D 02	0.140000000000D 02
0.900000000000D 01	0.0	0.120000000000D 02	0.110000000000D 02
COLUMN MEANS			GRAND MEAN
0.100000000000D 02	0.110000000000D 02	0.117500000000D 02	0.110000000000D 02

A SOLUTION VECTOR, FOR MU, ALPHAS(4 OF THEM), AND BFTAS(3 OF THEM)
 0.0 0.9785714D 01 0.9928571D 01 0.1435714D 02 0.1135714D 02-0.1071429D 01 0.1142857D 01 0.0

SUBMATRICES OF G - EQUATION 21, LM PAGE 268

C(-1)
 0.3928571D 00 0.2142857D 00
 0.2142857D 00 0.5714286D 00

-MC(-1)
 -0.3214286D 00-0.3571429D 00
 -0.1071429D 00-0.2657143D 00
 -0.1309524D 00-0.7142857D-01
 -0.1309524D 00-0.7142857D-01

D(-1)+MC(-1)M'
 0.5357143D 00 0.1785714D 00 0.1071429D 00 0.1071429D 00
 0.1785714D 00 0.3928571D 00 0.3571429D-01 0.3571429D-01
 0.1071429D 00 0.3571429D-01 0.3769841D 00 0.4365079D-01
 0.1071429D 00 0.3571429D-01 0.4365079D-01 0.2103175D 00

ANOVA FOR FITTING CONSTANTS (LM PAGE 275 OR 298-9)

	A BEFORE B		B BEFORE A		
	DF	SS	DF	SS	
R(MU)	1	0.2178000D 04	R(MU)	1	0.2178000D 04
R(A MU)	3	0.3760000D 02	R(B MU)	2	0.1050000D 02
R(B MU,A)	2	0.9485714D 01	P(A MU,B)	3	0.3678571D 02
SSE	12	0.9071429D 02	SSE	12	0.9071429D 02
TOTAL	18	0.2316000D 04	TOTAL	18	0.2316000D 04

ESTIMATED VARIANCES AND/OR SAMPLING VARIANCES OF ESTIMATORS

2 WAY X'D WITHOUT INTERACTION MIXED MODEL CASE 2 ASSIGNED COMPONENTS

2-WAY CLASSIFICATION WITHOUT INTERACTION, MIXED MODEL

NAMES OF FACTORS :

A= VARIETY

B= TREATMENTS

NO. OF LEVELS OF A :

4

NO. OF LEVELS OF B :

3 3 3 3

N(I,J,K)

3 2 0
0 2 2
1 0 2
2 0 4

UNCORRECTED SUMS OF SQUARES (T'S) EG. EQ 19 LM PAGE 426

FACTOR A 0.22158000 04
FACTOR B 0.21885000 04
DUE TO MEAN 0.21780000 04
TOTAL 0.23160000 04

ESTIMATED COMPONENTS AND SAMPLING VARIANCES USING THOSE ESTIMATES

ANALYSIS OF VARIANCE METHOD

ESTIMATES

VAR(ANOVA ESTIMATORS)

ANALYSIS ASSUMING RANDOM MODEL

A	1.305731	5.944628	-0.560335	-1.209267
B	-0.690562	-0.560335	1.146499	-1.476716
E	7.571480	-1.209267	-1.476716	8.832458

ANALYSIS ASSUMING MIXED MODEL, WITH B EFFECTS FIXED

FITTING CONSTANTS METHOD

ESTIMATES

VAR(ESTIMATORS)

A	1.322545	8.734049	-2.678738
E	7.559524	-2.678738	9.524400

FIXED EFFECTS ESTIMATORS

0.10101440 02 0.11460490 02 0.11644780 02

ITERATIVE METHOD (LM P 490 AND PAPER BU-450-M)

COMPUTED SIGMA VALUES

	A	E
1	1.323	7.560
2	1.118	7.705
3	0.983	7.801
4	0.886	7.870
5	0.815	7.921
6	0.760	7.959
7	0.717	7.990
8	0.682	8.015
9	0.654	8.035
10	0.631	8.051

FIXED EFFECTS ESTIMATORS

0.1005688D 02 0.1124724D 02 0.1169633D 02

HENDERSON'S METHOD 2 (SEE PAPER BU-450-M)

A E
1.196 0.083

FIXED EFFECTS ESTIMATORS

0.1009087D 02 0.1140842D 02 0.1165772D 02

ASSIGNED COMPONENTS		VAR (ESTIMATORS)	
A	10.000000	73.444824	-0.046875
E	1.000000	-0.046875	0.166667

ASSIGNED COMPONENTS		VAR (ESTIMATORS)	
A	1.000000	1.137207	-0.046875
E	1.000000	-0.046875	0.166667

ASSIGNED COMPONENTS		VAR (ESTIMATORS)	
A	0.500000	0.427490	-0.046875
E	1.000000	-0.046875	0.166667