A Monte Carlo Study of the Small Sample Validity of Confidence Interval Estimators Which Are Based on Large Sample Approximations: An M.S. Thesis Problem

BU-459-M April, 1973

D. S. Robson

Abstract

The statistical practitioner resorts to large sample approximations in a wide variety of interval estimation problems, usually without benefit of any firm evidence regarding small sample validity. Reliability of this practice could be improved through the accumulation of computer simulated evidence revealing rates of approach to asymptotic validity in a variety of cases. As a Master's thesis project such an investigation has the advantages of being open-ended (because of the variety of cases which could be studied), utilitarian, and highly statistical if Monte Carlo methods are employed to reduce costs of simulation. In simulating sampling distributions (of interval estimators) the investigator is confronted with a challenging array of idealized statistical problems to tax (his) her ingenuity in applying statistical techniques to his (her) own experiments.

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Perhaps the most commonly used large sample approximations for interval estimators are of the form $\hat{\theta}_i \pm 1.96\ \hat{\sigma}_{\hat{\theta}_i}$ where $\hat{\underset{\sim}{\theta}} = \hat{\underset{\sim}{\theta}}(\underset{\sim}{X})$, and the covariance matrix of $\hat{\underset{\sim}{\theta}}$ is estimated by

$$\hat{V}_{\hat{\underset{\sim}{\theta}}} = \hat{\Delta}' V_x(\hat{\underset{\sim}{\theta}})\hat{\Delta}$$

where $V_x(\underset{\sim}{\theta})$ is the (possibly approximate) covariance matrix of $\underset{\sim}{X}$ and where $\hat{\Delta} = \partial\hat{\underset{\sim}{\theta}}/\partial\underset{\sim}{X}$. Virtually any applied statistician could identify particular problems within this class for which the small sample properties would be of special interest, so that even within this class the scope of the study would be limited only by the investigator's resources.