TAG-RECAPTURE ESTIMATION OF SURVIVAL RATES IN A CONTINUOUS FISHERY

(Consultee: George Spangler)

BU-380-M D. S. Robson July, 1971

Abstract

Some commercial fisheries operate with fixed gear, permanently located, which fishes continuously with periodic removal of the accumulated catch. A tag-recapture experiment on a population fished in this manner yields information on mortality rates between lifts of the catch, including the separately identifiable components of mortality due to natural causes and to the fishery itself. The number of tagged fish alive and free immediately prior to a lift can be estimated by the so-called Jolly method; a sequence of such estimates at successive lifts, together with the known numbers of tagged fish returned after each lift, provide estimates of survival rates between lifts. The numbers of (tagged) fish captured in each lift provide estimates of exploitation rates which may then be transformed into estimates of (instantaneous) fishing mortality rates.

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Estimation of Survival Rates in a (Time) Discrete Tag-recapture Process

The sampling model we consider here is not strictly applicable to continuous fishery since we shall assume that samples are drawn instantaneously at discrete points in time. The present development, however, does lay the groundwork for approaching the problem of a continuous fishery, which will be the subject of a subsequent note. Since catch is removed at discrete times in a continuous fishery there is a close relationship between the two models, but the time-discrete model (like the Jolly model) presumes, in effect, that the gear is not fishing during the intervals between lifts, and rather that the catch is made at the instant the gear is lifted. The following is, in fact, an application of the Jolly model to the tagged population, ignoring the capture of untagged fish which are presumed to be too numerous to count.

We thus assume that instantaneous catches are made at the successive time points $t_1, t_2, t_3, \cdots, t_k$, and that for a fish alive at time t_{i-1} the probability of survival to time t_i is S_i, and that for a fish alive at time t_i the probability of being captured at that instant is P_i. Since the gear is not fishing during the interval (t_{i-1}, t_i) then S_i measures only natural mortality, and the probability of surviving (t_{i-1}, t_i) and being captured at t_i is then P_iS_i; if the gear were fishing continuously during (t_{i-1}, t_i) then the product P_iS_i is only an approximation — though this approximation improves rapidly as the interval (t_{i-1}, t_i) is

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shortened. Unfortunately, estimation of survival rates over very short intervals is not feasible unless we specify a survival model relating the survival rates in successive intervals (e.g., $S_i = S^{(t_i - t_{i-1})}$) and the Jolly model does not do this.

The Jolly model provides us with an estimate of the number of tagged fish alive at the instant the i^{th} sample is to be drawn:

$$\hat{N}_i = \#\text{ recaps at } t_i + \frac{\left(\begin{array}{c}\#\text{ tagged fish}\\ \text{released at } t_i\end{array}\right)\left(\begin{array}{c}\#\text{ that were tagged before } t_i\text{, not recaptured}\\ \text{at } t_i\text{, but recaptured after } t_i\end{array}\right)}{\#\text{ fish that were released at } t_i \text{ and later recaptured}} .$$

With this estimate of the number of tagged fish available at the time the i^{th} sample is to be drawn we can now estimate P_i, the probability of capture, by:

$$\hat{P}_i = \frac{\#\text{ recaps at } t_i}{\hat{N}_i} .$$

In order to estimate S_i we note that the <u>expected</u> number of tagged fish available immediately prior to t_i is expressible in terms of the numbers m_1, m_2, $\cdots$, m_{i-1} of tagged fish released at t_1, t_2, $\cdots$, t_{i-1}:

$$\begin{aligned} &E(N_i | m_1, m_2, \cdots, m_{i-1}) \\ &\quad = S_i m_{i-1} + m_{i-2}(1 - P_{i-1})S_{i-1}S_i + m_{i-3}(1 - P_{i-2})S_{i-2}(1 - P_{i-1})S_{i-1}S_i \\ &\qquad + \cdots + m_1(1 - P_2)S_2(1 - P_3)S_3 \cdots (1 - P_{i-1})S_{i-1}S_i \\ &\quad = S_i \sum_{j=1}^{i-1} m_j \prod_{\nu=j+1}^{i-1} (1 - P_\nu)S_\nu . \end{aligned}$$

We thus obtain the (maximum likelihood) estimate

$$\hat{S}_i = \frac{\hat{N}_i}{\sum\limits_{j=1}^{i-1} m_j \prod\limits_{\nu=j+1}^{i-1} (1 - \hat{P}_\nu)\hat{S}_\nu}$$

so that in order to estimate S_i we must first calculate estimates of $P_2, \cdots, P_{i-1}$ and $S_2, \cdots, S_{i-1}$. The formula for $\hat{S}_2$ reduces to $\hat{S}_2 = \hat{N}_2/m_1$.

Estimation of Instantaneous Rates of Fishing and Natural Mortality from Tag-recaptures in a Continuous Fishery

We now consider a fishery in which the gear is assumed to be fishing continuously with periodic, instantaneous removal of the accumulated catch. Again starting at time t_1 when a batch of tagged fish is released, we assume that catch is lifted at successive times $t_2, t_3, \cdots$ whereupon tagged fish are identified and recorded, some or all of the tag-recaptures are released, and possibly an additional batch of untagged fish are tagged and released.

Fish survival is assumed to follow a continuous (in time) Markov process which is homogeneous within an interval (t_{i-1}, t_i) with instantaneous fishing rate F_i and natural mortality rate M_i. The fishing gear may consist of a number of units, say n_i, having instantaneous fishing rates $F_{i1}, F_{i2}, \cdots, F_{in_i}$, in which case

$$F_i = F_{i1} + F_{i2} + \cdots + F_{in_i} ,$$

and some of these units may kill fish on capture to disallow the possibility of releasing tag-recaptures.

For a fish alive at time t_{i-1} the probability of escaping all gear and surviving to time t_i is then

$$s_i = e^{-(M_i+F_i)(t_i-t_{i-1})} \tag{1}$$

while the probability of being captured in gear unit j is

$$p_{ij} = \frac{F_{ij}}{M_i + F_i} [1 - s_i] \tag{2}$$

and the probability of dying from natural causes before t_i is

$$\frac{M_i}{M_i + F_i}(1 - s_i) .$$

If we now define $U(t_i^-)$ as the number of tagged fish at large at time t_i^- (i.e., immediately prior to lifting the gear at time t_i), excluding those which are currently in the gear that is about to be lifted, then

$$E\left\{U(t_i^-) \mid U(t_{i-1}^-), m_{i-1}\right\} = \left[m_{i-1} + U(t_{i-1}^-)\right]s_i .$$

Thus, at the start of the interval (t_{i-1}, t_i) there were $m_{i-1} + U(t_{i-1}^-)$ tagged fish at large, and a fraction s_i of these are expected to be still alive and free at time t_i^- . Since the Jolly model provides us with estimates of $U(t_i^-)$:

$$\hat{U}(t_i^-) = \frac{\begin{pmatrix}\text{\# tagged fish}\\ \text{released at } t_i\end{pmatrix}\begin{pmatrix}\text{\# recaptured after } t_i \text{ that were tagged before } t_i\\ \text{and not caught at } t_i\end{pmatrix}}{\text{\# fish that were released at } t_i \text{ and later recaptured}}$$

then we can estimate s_i by

$$\hat{s}_i = \frac{\hat{U}(t_i^-)}{m_{i-1} + \hat{U}(t_{i-1}^-)}$$

and so, from (1),

$$\widehat{M_i + F_i} = \frac{\log_e(\frac{1}{\hat{s}_i})}{t_i - t_{i-1}} .$$

In order to estimate the separate components of $M_i + F_i$ we note that if r_{ij} is the number of tag-recaptures in gear unit j at time t_i then

$$E\left\{r_{ij} \mid U(t_{i-1}^-), m_{i-1}\right\} = \left[m_{i-1} + U(t_{i-1}^-)\right]p_{ij}$$

so that

$$\hat{p}_{ij} = \frac{r_{ij}}{m_{i-1} + \hat{U}(t_{i-1}^-)} \quad .$$

Further, from (2),

$$\hat{F}_{ij} = \frac{\widehat{(M_i + F_i)}}{1 - \hat{s}_i} \hat{p}_{ij}$$

and

$$\hat{M}_i = \widehat{M_i + F_i} - \sum_j \hat{F}_{ij} \quad .$$

As shown in the attached appendix, all of these estimators are maximum likelihood (as well as moment estimators).

All estimates depend basically upon the sequence $\hat{U}(t_i^-)$, and the goodness of these estimators depends strongly on the numbers of tagged fish released at t_i. In applying this method one would therefore be well advised to combine successive catch records in such a manner that the boundary points t_i of the (combined) intervals are times at which large tag releases were made, but taking care not to destroy the assumption that instantaneous rates are constant within a (combined) interval. The size of a tag release at t_i can be further enlarged by including releases made within a day or two of t_i. Within a (combined) interval no fish can be counted more than once.