A Statistical Test for Recruitment in the k-Sample Tag-Recapture Experiment: A Preliminary Report

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BU-376-M June, 1971

Abstract

If S_2 is the number of tag-recaptures in the second sample of a 3-sample tag-recapture experiment then an optimal test of whether or not new individuals are recruited into the population between dates of sampling is the one-tailed test

$$\varphi(S_2|R_1,R_2) = \begin{cases} 1 & \text{if } S_2 \geq s_2(\alpha|R_1,R_2) \\ 0 & \text{otherwise} \end{cases}$$

Letting η_i denote the collection of individuals captured in the i'th sample, and $n_i = \#\{\eta_i\}$, then the statistic (R_1,R_2) given by

$$R_1 = \#\{\eta_1 \cap (\eta_2 \cup \eta_3)\}$$

$$R_2 = \#\{\eta_2 \cap \eta_3\}$$

is minimal sufficient with respect to the hypothesis of no recruitment (H_o) and

$$P_{H_o}(S_2|R_1,R_2) = \binom{n_2}{S_2}\binom{n_3-R_3}{R_1-S_2} \Big/ \binom{n_2+n_3-R_3}{R_1}$$

The upper tail of this hypergeometric distribution is an optimal critical region.

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INTRODUCTION

The procedure employed in estimating survival rates from a k-sample tag-recapture experiment is dependent upon the assumed structure of the population dynamics during the course of the experiment. If the population is known to be closed to recruitment and other forms of immigration then the population size at any sampling point t_i is estimated by the familiar formula $\hat{N}_i = n_iC_i/R_i$ where n_i is the number of elements captured and released at time t_i, C_i is the number of distinct elements captured after t_i and R_i is the number of elements in n_i that are later recaptured. Survival rate between t_i and t_{i+1} is then estimated by $\hat{N}_{i+1}/\hat{N}_i$. If recruitment is admitted into the model then an element captured after t_i is no longer known to have been present in the population at time t_i unless it bears a tag indicating that it was captured and released on or before t_i. The effective sample size for estimating N_i is thus substantially reduced, and the estimators (developed by J. M. Jolly) in this case are substantially less efficient. In situations where the question is in doubt, a statistical test for recruitment thus becomes a critical step in determining the estimation procedure.

A test which is statistically independent of the estimators $\hat{N}_i$ under the null hypothesis of "no recruitment" can be constructed by defining a critical region in the sample space of a statistic which is minimally sufficient with respect to the alternative hypothesis and such that this critical region has size α with respect to the conditional probability distribution determined by fixed values of $R_1,\ldots,R_{k-1}$. The statistic $(R_1,\ldots,R_{k-1})$ is minimal sufficient with respect to H_o, the vector $(C_0,\ldots,C_{k-1})$ being uniquely determined by $C_{i-1} = (n_i-R_i) +\ldots+ (n_{k-1}-R_{k-1})$. In this preliminary report we outline the development of such a test for the case k=3.

A CONDITIONAL TEST FOR RECRUITMENT

A 3-sample tag recapture experiment, in which each sample is returned to the population after tagging all untagged elements appearing in the sample, generates $2 + 2^2 = 6$ random variables subject to the (linear) constraints

$$n_2 = X_{01} + X_{11}$$

$$n_3 = X_{001} + x_{011} + x_{101} + X_{111}$$

where X_{101}, for example, is the number of elements which were tagged and released at t_1, were not recaptured at t_2, and were recaptured at t_3. A minimal sufficient statistic with respect to H_o is then

$$R_1 = X_{11} + X_{101} \qquad\qquad R_2 = X_{011} + X_{111}$$

When recruitment is admitted into the model the minimal sufficient statistic increases in dimension to include R_1, R_2 and X_{11}, or in general $R_1,R_2,\ldots,R_{k-1}$ and $S_2,\ldots,S_{k-1}$ where S_i is the number of tagged elements captured in the i'th sample $(R_1 +\ldots+ R_{k-1} = S_2 +\ldots+ S_k)$.

For fixed R_1,R_2 the conditional probability distribution of X_{11} $(S_2 = X_{11})$ under the null hypothesis is readily found to be the hypergeometric

$$P_{H_o}\left(x_{11} \mid R_1, R_2\right) = \binom{n_2}{x_{11}} \binom{n_3 - R_2}{R_1 - x_{11}} \Big/ \binom{n_2 + n_3 - R_2}{R_1} .$$

In order to determine an optimal critical region in the space of X_{11} we note that if recruitment does occur then the population size N_{i+1} at time t_{i+1} is given by

$$N_{i+1} = B_i + N_i(1-a_i)$$

where a_i is the total mortality rate between t_i and t_{i+1}, and B_i is the number of recruits entering the population in this interim and surviving to t_{i+1}. If we loosely define capture probabilities by $p_i \doteq n_i/N_i$ then

$$\frac{n_2}{n_2+n_3-R_2} \doteq \frac{p_2}{p_2+p_3(1-p_2)(1-a_2)+p_3(B_2/N_2)} < \frac{p_2}{p_2+p_3(1-p_2)(1-a_2)} \doteq \frac{x_{11}}{R_1}$$

and the optimum critical region of size α is therefore determined by the upper 100α percent tail of $P_{H_o}(x_{11} \mid R_1, R_2)$.