Applicability of Dunnett's procedure for the multiple comparison of several treatments with a control in a balanced incomplete block design using only intrablock information.

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Dunnett (1955) published tables for a one- or two-sided multiple comparison of p treatments with a control. His tables are constructed for the case where the p treatment means $\bar{X}_1, \cdots, \bar{X}_p$ and the control mean $\bar{X}_0$ are independently and normally distributed with common variance σ^2/K estimated by s^2/K, where K is a known constant and s^2/σ^2 is chi-square distributed with v degrees of freedom, independent of the p + 1 means. Under these assumptions the chance variables

$$Z_i = \frac{\bar{X}_i - \bar{X}_0 - (\mu_i - \mu_0)}{\sqrt{2/k}} \quad ,$$

for $i=1,\cdots,p$, have the multivariate normal distribution with means 0 and variances σ^2 and with a correlation of 1/2 between Z_i and Z_j. Utilizing tables prepared by the National Bureau of Standards, Dunnett then tabulated the values of t for which

$$(1) \qquad \begin{aligned} P_1 &= \text{Prob } (Z_1 < ts,\ Z_2 < ts,\ \cdots,\ Z_p < ts) \\ P_2 &= \text{Prob } (|Z_1| < ts,\ |Z_2| < ts,\ \cdots,\ |Z_p| < ts) \end{aligned}$$

for P_1, P_2 = .95 and .99, p = 1,2,$\cdots$,9, and v = 5(1)20, 24, 30, 40, 60, 120, ∞.

The application of Dunnett's test to the adjusted treatment means from a balanced incomplete block design (utilizing only the intrablock information) might at first appear to be questionable because in this case the p + 1 adjusted means $\bar{X}_0, \bar{X}_1, \cdots, \bar{X}_p$ are not independent. Because of the balance condition, however, the covariance $\sigma(\bar{X}_i, \bar{X}_j)$ is the same for all pairs i,j where $i \neq j$, $0 \le i,j \le p$; in fact, by appropiately choosing the constant C (see, for example, Calvin (1954)) we may write

$$\sigma(\bar{X}_i, \bar{X}_j) = \sigma^2/C \quad .$$

Likewise, the variance $\sigma^2(\bar{X}_i)$ is independent of i; i.e.,

$$\sigma^2(\bar{X}_i) = \sigma^2/K$$

for $0 \leq i \leq p$. Consequently, the chance variables

$$Z_i = \frac{\bar{X}_i - \bar{X}_0 - (\mu_i - \mu_0)}{A} \quad ,$$

where A is another constant, all have the same variance

$$\begin{aligned}
\sigma^2(Z_i) &= \frac{1}{A^2}\left[\sigma^2(\bar{X}_i) + \sigma^2(\bar{X}_0) - 2\sigma(\bar{X}_0,\bar{X}_1)\right] \\
&= \frac{1}{A^2}\left(\frac{\sigma^2}{K} + \frac{\sigma^2}{K} - \frac{2\sigma^2}{C}\right) \\
&= \frac{2}{A^2}\left(\frac{\sigma^2}{K} - \frac{\sigma^2}{C}\right) \quad .
\end{aligned}$$

Likewise, the covariance of Z_i and Z_j is the same for all i,j, $i \neq j$,

$$\begin{aligned}
\sigma(Z_i,Z_j) &= \frac{1}{A^2}\,\sigma(\bar{X}_i - \bar{X}_0, \bar{X}_j - \bar{X}_0) \\
&= \frac{1}{A^2}\left[\sigma(\bar{X}_i,\bar{X}_j) - \sigma(\bar{X}_i,\bar{X}_0) - \sigma(\bar{X}_0,\bar{X}_j) + \sigma^2(\bar{X}_0)\right] \\
&= \frac{1}{A^2}\left(\frac{\sigma^2}{C} - \frac{\sigma^2}{C} - \frac{\sigma^2}{C} + \frac{\sigma^2}{K}\right) \\
&= \frac{1}{A^2}\left(\frac{\sigma^2}{K} - \frac{\sigma^2}{C}\right)
\end{aligned}$$

The correlation ρ_{ij} between Z_i and Z_j is therefore

$$\rho_{ij} = \frac{\sigma(Z_i,Z_j)}{\sigma(Z_i)\sigma(Z_j)} = \frac{\frac{1}{A^2}\left(\frac{\sigma^2}{K} - \frac{\sigma^2}{C}\right)}{\frac{2}{A^2}\left(\frac{\sigma^2}{K} - \frac{\sigma^2}{C}\right)} = 1/2$$

for all i,j such that $i \neq j$, $1 \leq i,j \leq p$. Consequently, if we assume the intrablock errors to be normally (as well as independently and identically) distributed and choose the constant A so that

$$\sigma^2(Z_i) = \frac{2}{A^2}\left(\frac{\sigma^2}{K} - \frac{\sigma^2}{C}\right) = \sigma^2 \quad ,$$

where σ^2 is the expectation of the intrablock error mean square, then the chance variables Z_i, $i=1,\cdots,p$ satisfy all the assumptions outlined by Dunnett and are independent of the intrablock error mean square s^2. Hence, his tables are directly applicable to the balanced incomplete block experiment with p = number of treatments, excluding the control, and v = intrablock error degrees of freedom.

By exactly the same argument it follows that Dunnett's procedure is applicable to the doubly balanced incomplete block design and, in particular, to the design and model described by Calvin (1954) for taste testing when treatment effects are correlated. Dunnett's test would seem particularly appropriate to taste testing experiments where it is desired to learn whether the proposed new varieties of a food item are better than the variety currently marketed.

The same argument also implies to the applicability of Tukey's hsd test.

References

Dunnett, Charles W. (1955), "A multiple comparison procedure for comparing several treatments with a control," Journal of the American Statistical Association, Vol. 50, pp. 1096-1121.

Calvin, Lyde D. (1954), "Doubly balanced incomplete block designs for experiments in which the treatment effects are correlated," Biometrics, Vol. 10, No. 1, pp. 61-88.