PERSPECTIVES ON COMBINATORIAL ASPECTS OF SATURATED MAIN EFFECT PLANS OF THE 2^n FACTORIAL[1/]

by

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0. **Full model:** $\underset{2^n \times 1}{E[Y]} = \underset{2^n \times 2^n}{X} \quad \underset{2^n \times 1}{\beta}$

1. **Sme model:** $\underset{(n+1)\times 1}{E[Y_1]} = \underset{(n+1)\times(n+1)}{X_{11}} \quad \underset{(n+1)\times 1}{\beta_1} + \underset{(n+1)\times(2^n-n-1)}{X_{12}} \cdot \underset{(2^n-n-1)\times 1}{\beta_2}$

2. **Normal equations:** $X_{11}'X_{11}\hat{\beta}_1 = X_{11}'Y_1$

3. **Alias matrix:** $[X_{11}'X_{11}]^{-1}X_{11}'X_{12} = X_{11}^{-1}X_{12}$

4. **Optimal sme plan:** if $\det X_{11}'X_{11}$ is max., i.e., $|\det X_{11}|$ is max.

5. **Implications:** $X_{11}(-1,1)$ $\Updownarrow$ $X_{11}^*(0,1) = [\underset{(n+1)\times 1}{1} \; \vdots \; \underset{(n+1)\times n}{D}]$ $\longrightarrow$ $\det X_{11} = 2^n \cdot \det X_{11}^*$

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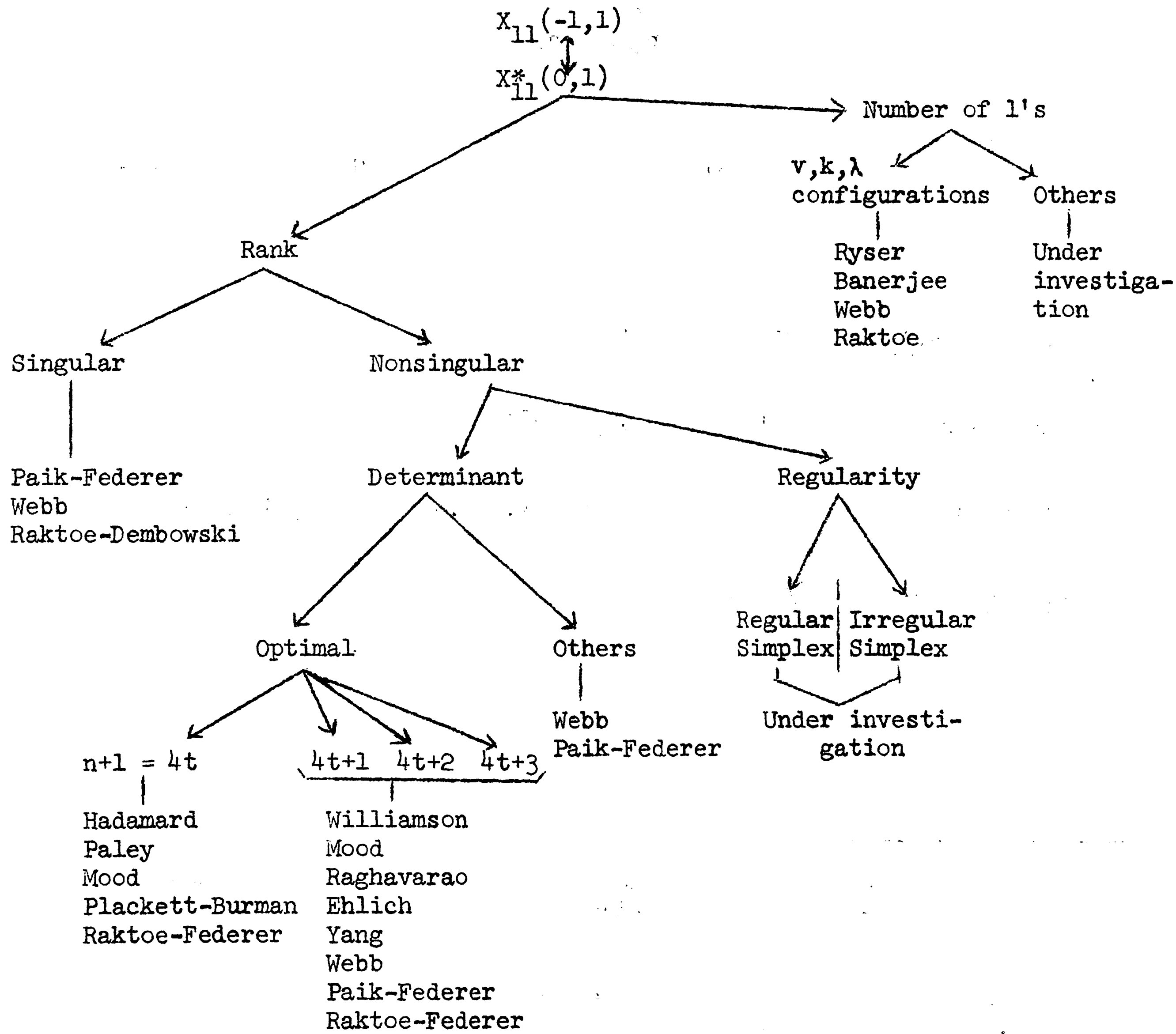
6. <u>Possible plans</u>: $\{D_1, D_2, \ldots, D_\alpha\}$, $\alpha = \binom{2n}{n+1}$

7. <u>Enumeration problems</u>:

   (i) What are the possible values of $|\det X^*_{11}|$?

   (ii) How many X^*_{11}'s (or equivalently how many D's) belong to each possible value of $|\det X^*_{11}|$?

8. <u>Possible avenues of attack</u>:



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