Research Designs: Experimental and Statistical Controls*

by Walter T. Federer

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ABSTRACT

Examples of alternative analyses for standard designs and of some new designs are discussed. Analyses of ranks and ranges, non-additivity, analyses for differential gradients in the rows and columns, and analyses of row × treatment interaction in row by column designs are discussed. Designs with the incomplete block size k greater than v treatments and k row by b column latin rectangle designs are presented. Particular attention is paid to RB/RB, RB/BIB, BIB/BIB, and direct product latin rectangle designs.

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INTRODUCTION

Most of you are familiar with the randomized complete blocks and the latin square designs and the standard analyses for these designs, but many of you are not familiar with the several alternative analyses associated with such designs and with the several classes of new experimental designs that have been constructed and are available for use by the experimenter. A few examples of alternative analyses and of new designs will be presented in this lecture. In particular, let us consider the following topics:

<u>Analyses for standard designs</u>

- i) Analysis of ranks and ranges
- ii) Non-additivity
- iii) Differential gradients in the rows of a latin square design
- iv) An example of a period by treatment interaction in a Youden square design

<u>Some new designs</u>

- i) BIB designs with the block size, k, larger than the number of treatments, v
- ii) RB/RB designs (latin square, etc.)
- iii) RB/BIB design (Youden square, etc.)
- iv) BIB/BIB
- v) Other k row by b column designs

The designs in the last four groups were taken from Zelen and Federer [1964]

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and Federer and Zelen [1964]. Kurtzer [1965] has constructed designs with two-way elimination of heterogeneity for $2 \leq v$ treatments ≤ 10 and $2 \leq r$ replicates ≤ 10 which are as nearly balanced in rows and columns as is possible.

ANALYSES

1. Analyses of ranks. Suppose that the experimenter wishes to run an analysis of the ranks of treatments from an experiment designed as a completely randomized design (i.e. no blocking). This may be accomplished rather easily as illustrated below for the data of Example IV-1 of my text (Federer [1955]).

Feeding treatments (actual weights and rank)

	A		B		C		D	
	actual	rank	actual	rank	actual	rank	actual	rank
	55	7	61	8	42	3.5	169	20
	49	5	112	16	97	15	137	17
	42	3.5	30	2	81	10	169	19
	21	1	89	12	95	14	85	11
	52	6	63	9	92	13	154	18
Sum = R_i	219	22.5	355	47	407	55.5	714	85
Range	34	-	82	-	55	-	84	-

An analysis of variance and an F test on the actual weights results in:

Source of variation	d.f.	Mean square	F	F_{01}
Among treatments	$3 = v - 1$	8745.07	12.1	5.99
Within treatments	$16 = f_e$	722.42		

An alternative test of the hypothesis of equality of means may be obtained using the ranks (e.g. see Bradley [1955])

$$X^2(v\text{-}1\ df) = \frac{12}{N(N+1)} \sum_{i=1}^{v} \frac{R_i^2}{r_i} - 3(N+1)$$

$$= \frac{12}{20(20+1)} \left(\frac{22.5^2+47^2+55.5^2+85^2}{5} \right) - 3(20+1)$$

$$= 74.4 - 63 = 11.4 \quad ,$$

where $r_i=5$ and ties are ignored. The tabulated value of X^2 at 1 percent level for $v - 1 = 3$ degrees of freedom is 11.34.

Suppose that instead of tests of significance one wishes to compute confidence intervals. The standard experimentwise (e.g. see Federer [1961]) 95 percent confidence intervals are computed as:

$$q_{\alpha,v,f_e} s_{\bar{x}} = q_{.05,4,16} s_{\bar{x}} = 4.05 \sqrt{\frac{722.42}{5}} = 48.6 \quad .$$

Alternatively, we may utilize the sum of the ranges ($34 + 82 + 55 + 84 = 255$) instead of sums of squares to compute a 95 percent experimentwise confidence interval as follows:

$$\frac{255}{r=5} \left(\begin{array}{c} .96 = \text{value from Table II-4} \\ \text{in Federer [1955] for } v=4,\ r=5 \end{array} \right) = 49.0$$

which is nearly identical to the value obtained above. These procedures are available for some other designs as well.

2. Non-additivity. Professor John W. Tukey [1949] proposed a unique procedure for detecting the presence of a certain type of non-additivity in such designs

as the randomized complete blocks and latin square designs. If this type of non-additivity is present a transformation of the form X^n, including log X for n=0, will suffice to make the data additive. For example, consider the following sets of data:

	Column (set 1)			Column (set 2)		
Row	1	2	3	1	2	3
1	36	81	144	2	4	12
2	0	9	36	4	8	24

The analyses of variance for the data are:

		Sum of Squares	
Source of variation	d.f.	Set 1	Set 2
Row	1	7776	54
Column	2	5292	252
Row × Column	2	1296	28
TNA	1	1269.6	28
Residual	1	26.4	0

Now if we had used the square root transformation on Set 1 and the logarithmic transformation on Set 2, the Row × Column interaction sums of squares would be zero. Thus, we note that Tukey's one degree of freedom for non-additivity can be quite useful for detecting certain types of non-additivity.

(The formula for computing the TNA sum of squares may be found in the above reference, in Federer [1955], etc.)

3. <u>Differential gradients</u>. The standard analysis of variance for a k × k latin square design as given in statistics books has the following form:

Source of variation	Degrees of freedom
Row	k-1
Column	k-1
Treatment	k-1
Error	(k-1)(k-2)

Such an analysis of variance assumes additivity of effects and no row × column, treatment × row, or treatment × column interactions. If the gradients from column to column depend upon the row then there is a row × column interaction. Also, if the gradients are perpendicular and random then it would be highly unlikely that the experimenter would be so fortunate as to orient the latin square such that the rows were perpendicular to one gradient and the columns perpendicular to the other gradient. If one or both gradients could be considered to be no higher degree than quadratic the following analysis of variance might be the appropriate one to use:

Source of Variation	Degrees of Freedom
Total	k^2
Correction for the mean	1
Column effects (linear + quadratic ignoring treatment effect)	2
Row (linear + quadratic ignoring treatment effect)	2
Rows effects × Column effects (ignoring treatment)	4
Treatments (eliminating other effects)	k-1
Remainder = error	k^2-k-8

In a different situation let us suppose that fatigue curves for subjects are quadratic in nature but that some individuals tire much more rapidly than others. This means automatically that the time gradients vary from subject to subject. For example, suppose that the rows represents time and that the columns represent subjects. Then there will be a differential gradient within each subject. If, for example, the part of the fatigue curve we are experimenting over is linear the following form of the analysis of variance would be appropriate (see Cox [1958]):

Source of Variation	Degrees of Freedom
Total	k^2
Correction for mean	1
Column	k-1
Gradient within columns (ignoring treatment)	k
Treatment (eliminating gradient)	k-1
Remainder = error	(k-1)(k-2)-1

4. An example of a period by treatment interaction in a Youden square design. In a marketing experiment on the effect of different sized containers on the sale of peaches during the "peach season", it was believed that the "peach season" would last three weeks. Seven treatments involving 2(A), 3(B), 4(C), 6(D), 8(E), 10(F), and 12(G) pound containers and seven stores were utilized in the experiment. The standard analysis of variance is:

Source of Variation	Degrees of Freedom
Row = period	2
Store = column (ignoring treatment)	6
Treatment (eliminating store)	6
Remainder = error	6

The design was:

Period	Stores = columns 1	2	3	4	5	6	7
Week 1	G	B	A	F	D	E	C
2	A	C	B	G	E	F	D
3	C	E	D	B	G	A	F

The standard analysis was rendered inappropriate because of a period by treatment interaction occassioned by the fact that peaches were priced 4 pounds for 29¢ in week 1, 3 pounds for 29¢ in week 2, and 2 pounds for 29¢ in week 3. If this price change had been suspected, an experiment would have been completed within each of the three weeks. Since the price change was not anticipated something needs to be done to salvage the information in the experiment. From the nature of the treatments we could postulate that a second degree regression of sales on container size would account for most of the variation in sales for the treatments. Also, from previous work we know that a covariance on total store sales often removes a good share of the store to store variation. Consequently let us consider the following covariance analysis:

Source of variation	d.f.	Sum of products yy	xy	xx	x^2y	x^2x	x^2x^2
Total	21						
Correction for mean	1						
Period = P	2						
Treatment (linear + quadratic) = T	2						
P × T	4						
Remainder = R	12						
Remainder (adjusted for covariates)	10						
R + T (adjusted for covariates)	12						
T (adjusted for covariates)	2						

where Y = sales of peaches and X = total store (or produce) sales. There are several alternative analyses that might be appropriate depending upon what facts and assumptions are available for an experiment of this type.

NEW DESIGNS

1. BIB designs for $k > v$. In statistical literature there is a convention that the size of the block should be less than or equal to the number of treatments. Because of this convention it has been customary to define an incomplete block size = k to be less than v = number of treatments. There is no need to restrict ourselves to this convention. To illustrate, let us suppose that we have v = 5 treatments and that we have 5 litters of rats with 6 rats in each litter. Most experimenters and statisticians would delete one animal from each litter and run a randomized complete blocks design with 5 blocks (litters). Now there is no need for eliminating the additional rat per litter. Instead let us use the following schematically arranged design for treatments A, B, C, D, and E:

Litter (blocks)

1	2	3	4	5
A	A	A	A	A
B	B	B	B	B
C	C	C	C	C
D	D	D	D	D
E	E	E	E	E
A	B	C	D	E

Thus, each treatment occurs twice in one of the litters and once in all the other litters to yield r=6 replicates on each treatment. The analysis of this design follows that for a balanced incomplete block (BIB) design.

2. RB/RB designs. The standard latin square design is known to many scientists. This design for 4 treatments (A,B,C,D) is:

	Column			
Row	1	2	3	4
1	A	B	C	D
2	B	C	D	A
3	C	D	A	B
4	D	A	B	C

Here we note that each treatment occurs once in a row and once in a column. The sums of squares for treatments is computed in the same way as for randomized complete blocks (RB) design for the rows as blocks and also when the columns are considered to be blocks. Thus we shall call the latin square design an RB/RB design to indicate that the relation of treatments and rows is RB and that the relation of treatments and columns is RB.

Now suppose that we have the following design (before randomization):

	Columns							
Row	1	2	3	4	5	6	7	8
1	A	B	C	D	A	B	C	D
2	B	C	D	A	B	C	D	A
3	C	D	A	B	C	D	A	B
4	D	A	B	C	D	A	B	C

Each treatment occurs twice in a row and once in a column. Since the design is not "square" we shall denote the design as a latin rectangle. This design consists, in general, of k rows and b columns with v treatments each (usually) replicated r times such that vr = bk.

RB/RB designs are either latin square designs or latin square designs tied together to form a tied latin square or a simple change-over design. The row and column effects are orthogonal to each other and to the treatments in these designs in the same manner as they are for a randomized complete block design with the treatments occurring p times in each block.

A listing of the k row by b column designs for $2 \le v, r \le 10$ is given below (Federer and Zelen [1964]):

v	k	b	r	design
2	2	2	2	2 × 2 latin square
	2	4	4	simple change-over
	2	6	6	simple change-over
	2	8	8	simple change-over
2	2	10	10	simple change-over
	4	4	8	4 × 4 latin square
3	3	3	3	latin square
	3	6	6	simple change-over
	3	9	9	simple change-over
4	4	4	4	latin square
	4	8	8	simple change-over
5	5	5	5	latin square
	5	10	10	simple change-over
6	6	6	6	latin square
7	7	7	7	latin square
8	8	8	8	latin square
9	9	9	9	latin square
10	10	10	10	latin square

3. RB/BIB designs. Certain k row by b column designs may be arranged in such a manner as to have the randomized complete block property in rows and the balanced incomplete block property in columns. Such designs are denoted as RB/BIB designs. The Youden square design has such properties. Also, the extended Youden squares and tied Youden squares are of this type. These designs for $2 \le v,\ r \le 10$ are listed below (Federer and Zelen [1964]):

v	k	b	r	design
3	2	3	2	Youden square
	2	6	4	tied Youden square
	2	9	6	tied Youden square
	2	12	8	tied Youden square
	2	15	10	tied Youden square
	4	3	4	extended Youden square
	4	6	8	tied extended Youden square
	5	3	5	extended Youden square
	5	6	10	tied extended Youden square
	7	3	7	extended Youden square
	8	3	8	extended Youden square
	10	3	10	extended Youden square
4	2	12	6	-
	3	4	3	Youden square
	3	8	6	tied Youden square
	3	12	9	tied Youden square
	5	4	5	extended Youden square
	5	8	10	tied extended Youden square
	7	4	7	extended Youden square
	9	4	9	extended Youden square
5	2	10	4	-
	2	20	8	-
	3	10	6	-

	4	5	4	Youden square
	4	10	8	tied Youden square
	6	5	6	extended Youden square
	9	5	9	extended Youden square
6	2	30	10	-
	5	6	5	Youden square
	5	12	10	tied Youden square
	7	6	7	extended Youden square
7	2	21	6	-
	3	7	3	Youden square
	3	14	6	tied Youden square
	3	21	9	tied Youden square
	4	7	4	Youden square
	4	14	8	tied Youden square
	6	7	6	Youden square
	8	7	8	extended Youden square
8	7	8	7	Youden square
	9	8	9	extended Youden square
9	2	36	8	-
	4	18	8	-
	5	18	10	-
	8	9	8	Youden square
10	3	30	9	-
	9	10	9	Youden square

4. <u>BIB/BIB</u> <u>design</u>. If the relation of treatments with both rows and columns is the same as that for balanced incomplete blocks (BIB) then the design is denoted as a BIB/BIB design. No systematic listing is given here for $2 \le v$, $r \le 10$, but these may be obtained from the work of Kurtzer [1965]. An example of a design in this class is given below for v=4 treatments with r=9 replicates in a 6×6 latin square:

A	B	C	D	A	C
D	A	B	C	B	D
C	D	A	B	C	B
B	C	D	A	D	A
A	B	C	D	A	B
B	C	D	A	C	D

5. <u>Other k row $\times$ b column designs.</u> Federer and Zelen [1964] have obtained many classes of latin rectangle designs. A particularly interesting group of designs (to the authors, at least) is the one obtained by taking direct products of designs in the preceding sections. For example, suppose that we take the direct product of a Youden square design (RB/BIB) and the same Youden square turned on end, i.e. a BIB/RB design. Then we obtain a GD/GD design which has known analyses. For example the direct product of an RB/BIB design and a BIB/RB design for v=3 treatments is given as:

1	2	3
3	1	2

×

1	3
2	1
3	2

=

11	13	21	23	31	33
12	11	22	21	32	31
13	12	23	22	33	32
31	33	11	13	21	23
32	31	12	11	22	21
33	32	13	12	23	22

The resulting design for v=9 treatments repeated r=4 times is a 6 row by 6 column latin square.

The procedure of taking direct products of known designs to form new designs allows anyone to construct designs. Also, the development and exploitation of the "calculus of factorials" by Dr. Marvin Zelen and co-workers enables one to obtain the analyses for these new designs with relative ease. Despite the availability of these methods, it would be wise to contact a competent statistician on the design and analysis prior to experimentation. He may be able to help.

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