A TWO-PERIOD DESIGN FOR TREATMENT × PERIODS INTERACTION

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SUMMARY

The construction and analysis of an experimental design which is a modification of the standard change-over design is presented. This design, in which there are t^2 experimental units for t treatments, is an efficient design where the treatments are applied to an experimental unit in a sequence over two periods and where an estimate of the treatments × periods interaction is required. Least-squares estimators for the various effects are obtained and the analysis of variance of a numerical example is presented to illustrate the application of the results to experimental data.

I. INTRODUCTION

The simple change-over and double change-over designs are described in a number of standard texts on experimental design, e.g. on page 441 and 445 of Federer (1955). For three treatments (A,B,C) the simple change-over design has the form shown in Table 1. With four treatments, the double change-over design for measuring residual effects is presented in Table 2. This is obtained from three 4x4 orthogonal latin squares.

(Insert Tables 1 and 2)

If in each of these designs, the experiments were discontinued at the end of the second period, it is found that each treatment in the first period is followed by each of the other treatments. A modification of the above designs has been used at the University of Sydney for animal nutrition trials on young lambs where there is reason to suspect that treatment effects in one period of time may be different from the treatment effects at a latter period of time. For t treatments the design requires t^2 experimental units (in this case animals) and thus has practical value in lamb nutritional experiments for either 4 or 5 treatments. However, in other fields this practical limitation may not exist. The design and analysis is illustrated for 4 treatments and 16 animals. The experimental design is presented in Table 3. For the first 12 animals, the experiment is that which would result from Table 2 if this were discontinued at the end of the second period. For animals 13 to 16 it can be thought that the treatments applied here are the first two of the sequences AAAA,BBBB,CCCC, DDDD. Cochran and Cox (1957), in which is contained many references to designs in this area, state on page 141 of their text that experiments containing sequences of this type "have been used to some extent on perennial crops and on crop rotations in agriculture". Additional and more recent references to designs in this field are found in Federer and Atkinson (1964). A review of the literature shows no experimental design of the type used here which has some of the features of a latin square design (Table 4). Extension of this table to a $t \times t$ square determines the t^2 sequences to apply in a 2 period $\times$ t^2 experiment.

Many papers, among these those of Sheehe and Bross (1961) and of Williams (1949,1950) consider the estimation of residual effects. Residual effects are not considered here as a rest period is inserted between the two periods of experimentation.

(Insert Tables 3 and 4)

II. RANDOMIZATION

Having chosen the design the sequences of treatments for the different experimental units is fixed. Thus there can be no randomization among the rows; the randomization is restricted to randomly allocating the sequences to the experimental units (lambs) and the letters to the treatments. As there is no stratification among the experimental units receiving the different sequences, the following randomization procedure is adequate:

i) Randomly assign the t treatments to the t letters A,B,C,D,···

ii) Randomly assign the t^2 experimental units to the t^2 different sequences.

This randomization procedure is identical with that presented by Federer and Atkinson (1964).

III. LEAST SQUARES ESTIMATORS OF EFFECTS

Suppose that the observational yield Y_{ijk} on the i^{th} animal in the j^{th} period for the k^{th} treatment is such that it is expressible in the following linear additive form:

$$Y_{ijk} = n_{ijk}(\mu + \alpha_i + \pi_j + \tau_k + \delta_{jk} + \epsilon_{ijk}) \quad \begin{bmatrix} i=1,2,\cdots,16 \\ j=1,2;\ k=1,2,3,4 \end{bmatrix}$$

where $n_{ijk} = 1$ if the i^{th} animal is, in the j^{th} period, given the k^{th} treatment and equals zero otherwise; thus $\sum_i\sum_j\sum_k n_{ijk} = 32$; where μ is an effect common to all observations; α_i is the effect of the i^{th} animal; π_j is the effect of the j^{th} period; τ_k is the effect of the k^{th} treatment; δ_{jk} is the interaction effect of the j^{th} period and k^{th} treatment; and ϵ_{ijk} are identically independently distributed random variates with mean zero and common variances σ^2.

The residual sum of squares is

$$R = \sum_i\sum_j\sum_k [n_{ijk}(Y_{ijk} - \mu - \alpha_i - \pi_j - \tau_k - \delta_{jk})]^2$$

Partial differentiation of the residual sum of squares with respect to the various parameters and equation of the results to zero lead to the normal equations presented below if the following restraints are used

$$\sum_i \hat{\alpha}_i = \sum_j \hat{\pi}_j = \sum_k \hat{\tau}_k = \sum_k \hat{\delta}_{jk} = \sum_k \hat{\delta}_{jk} = 0$$

<u>For $\hat{\mu}$:</u>

$$32\hat{\mu} = Y_{\cdot\cdot\cdot} = \text{grand total} \qquad (1)$$

For the $\hat{\alpha}_i$ (animal effects):

$$2(\hat{\mu}+\hat{\alpha}_i) + \sum_j \sum_k n_{ijk}(\hat{\tau}_k+\hat{\delta}_{jk}) = \sum_j \sum_k Y_{ijk}$$

$$= Y_{i..} = i^{th} \text{ animal total} \qquad (2)$$

which for i=1,4,5,7,9,10,13 become

$$2(\hat{\mu}+\hat{\alpha}_1) + \hat{\tau}_1 + \hat{\tau}_2 + \hat{\delta}_{11} + \hat{\delta}_{22} = Y_{1..} \qquad (3)$$

$$2(\hat{\mu}+\hat{\alpha}_4) + \hat{\tau}_4 + \hat{\tau}_1 + \hat{\delta}_{14} + \hat{\delta}_{21} = Y_{4..} \qquad (4)$$

$$2(\hat{\mu}+\hat{\alpha}_5) + \hat{\tau}_1 + \hat{\tau}_3 + \hat{\delta}_{11} + \hat{\delta}_{23} = Y_{5..} \qquad (5)$$

$$2(\hat{\mu}+\hat{\alpha}_7) + \hat{\tau}_3 + \hat{\tau}_1 + \hat{\delta}_{13} + \hat{\delta}_{21} = Y_{7..} \qquad (6)$$

$$2(\hat{\mu}+\hat{\alpha}_9) + \hat{\tau}_1 + \hat{\tau}_4 + \hat{\delta}_{11} + \hat{\delta}_{24} = Y_{9..} \qquad (7)$$

$$2(\hat{\mu}+\hat{\alpha}_{10}) + \hat{\tau}_2 + \hat{\tau}_1 + \hat{\delta}_{12} + \hat{\delta}_{21} = Y_{10,..} \qquad (8)$$

$$2(\hat{\mu}+\hat{\alpha}_{13}) + \hat{\tau}_1 + \hat{\tau}_1 + \hat{\delta}_{11} + \hat{\delta}_{21} = Y_{13,..} \qquad (9)$$

Nine equations similar to the above are obtained for i=2,3,6,8,11,12,14,15,16

For the $\hat{\pi}_j$ (period effects):

$$16\hat{\mu} + 16\hat{\pi}_j = \sum_i \sum_k Y_{ijk} = Y_{.j.} = j^{th} \text{ period total} \qquad (10)$$

For the $\hat{\tau}_k$ (treatment effects):

$$8\hat{\mu} + \sum_i \sum_j n_{ijk}(\hat{\alpha}_i+\hat{\tau}_j) = \sum_i \sum_j Y_{ijk} = Y_{..k} = k^{th} \text{ treatment total} \qquad (11)$$

which for k=1 becomes

$$8\hat{\mu} + \hat{\alpha}_1 + \hat{\alpha}_5 + \hat{\alpha}_9 + \hat{\alpha}_{13} + \hat{\alpha}_4 + \hat{\alpha}_7 + \hat{\alpha}_{10} + \hat{\alpha}_{13} + 8\hat{\tau}_1 = Y_{\cdot\cdot 1} \qquad (12)$$

Three equations similar to (12) are obtained for k=2,3,4.

For the $\hat{\delta}_{jk}$ (interaction effects):

$$4\hat{\mu} + \sum_i n_{ijk}(\hat{\alpha}_i + \hat{\pi}_j + \hat{\tau}_k) + 4\hat{\delta}_{jk} = \sum_i Y_{ijk} = Y_{\cdot jk}$$

= total for k^{th} treatment in the j^{th} period (13)

which for j=1, k=1 becomes

$$4\hat{\mu} + \hat{\alpha}_1 + \hat{\alpha}_5 + \hat{\alpha}_9 + \hat{\alpha}_{13} + 4\hat{\pi}_1 + 4\hat{\tau}_1 + 4\hat{\delta}_{11} = Y_{\cdot 11} \qquad (14)$$

Seven equations similar to equation (14) are obtained for j=1, k=2,3,4 and for j=2, k=1,2,3,4.

Unique estimators of the treatment, interaction and animal parameters can be obtained in the following manner. For the treatment effects, addition of equations (3) to (8) together with twice (9) gives

$$16\hat{\mu} + 2(\hat{\alpha}_1 + \hat{\alpha}_5 + \hat{\alpha}_9 + \hat{\alpha}_{13} + \hat{\alpha}_4 + \hat{\alpha}_7 + \hat{\alpha}_{10} + \hat{\alpha}_{13}) + 8\hat{\tau}_1$$

$$= Y_{1\cdot\cdot} + Y_{5\cdot\cdot} + Y_{9\cdot\cdot} + Y_{13,\cdot\cdot} + Y_{4\cdot\cdot} + Y_{7\cdot\cdot} + Y_{9\cdot\cdot} + Y_{13,\cdot\cdot} \qquad (15)$$

Multiplication of equation (12) by 2 gives

$$16\hat{\mu} + 2(\hat{\alpha}_1 + \hat{\alpha}_5 + \hat{\alpha}_9 + \hat{\alpha}_{13} + \hat{\alpha}_4 + \hat{\alpha}_7 + \hat{\alpha}_{10} + \hat{\alpha}_{13}) + 16\hat{\tau}_1$$

$$= 2(Y_{111} + Y_{511} + Y_{911} + Y_{13,11} + Y_{421} + Y_{721} + Y_{10,21} + Y_{13,21}) \qquad (16)$$

Subtraction of equation (15) from (16) gives

$$8\hat{\tau}_1 = Y_{..1} - (Y_{414} + Y_{713} + Y_{10,12} + Y_{13,11}) - (Y_{122} + Y_{523}$$

$$Y_{924} + Y_{13,21})$$

$$= Y_{..1} - \{Y^c_{\cdot(21)}\} - \{Y^c_{\cdot(11)}\}$$

$$= Y_{..1} - Y^c_{\cdot(\cdot 1)}$$

$$= Y^c_{..1} \qquad (17)$$

where $Y^c_{\cdot(21)}$ is the total of the observations made in the first period on animals which in period 2 recieved treatment 1,

$Y^c_{\cdot(11)}$ is the total of the observations made in the second period on animals which in period 1 received treatment 1,

and $Y^c_{\cdot(\cdot 1)} = Y^c_{\cdot(21)} + Y^c_{\cdot(11)}$

Generally,

$$8\hat{\tau}_k = Y_{..k} - Y^c_{\cdot(\cdot k)} = Y^c_{..k} \qquad (18)$$

Estimates of the 8 interaction effects are found in a manner similar to that which is now used for finding $\hat{\delta}_{11}$.

Multiplying equation (14) by 2 gives

$$8\hat{\mu} + 2(\hat{\alpha}_1 + \hat{\alpha}_5 + \hat{\alpha}_9 + \hat{\alpha}_{13}) + 8\hat{\pi}_1 + 8\hat{\tau}_1 + 8\hat{\delta}_{11} = 2Y_{\cdot 11} \qquad (19)$$

Subtracting equations (3), (5), (7), (9) from (19) gives

$$8\hat{\pi}_1 + 4\hat{\tau}_1 + 4\hat{\delta}_{11} = 2Y_{\cdot 11} - (Y_{1..} + Y_{5..} + Y_{9..} + Y_{13,..})$$

whence

$$8\hat{\delta}_{11} = 4Y_{\cdot 11} - 2(Y_{1\cdot\cdot} + Y_{5\cdot\cdot} + Y_{9\cdot\cdot} + Y_{13,\cdot\cdot}) - (Y_{\cdot 1\cdot} - Y_{\cdot\cdot\cdot}/2) - Y^{c}_{\cdot\cdot 1}$$

$$= 2(Y_{\cdot 11} - Y^{c}_{\cdot(11)}) - (Y_{\cdot 1\cdot} - Y_{\cdot\cdot\cdot}/2) - Y^{c}_{\cdot\cdot 1}$$

$$= 2Y^{c}_{\cdot 11} - (Y_{\cdot 1\cdot} - Y_{\cdot\cdot\cdot}/2) - Y^{c}_{\cdot\cdot 1} \qquad (20)$$

and, in general,

$$8\hat{\delta}_{jk} = 2Y^{c}_{\cdot jk} - (Y_{\cdot j\cdot} - Y_{\cdot\cdot\cdot}/2) - Y^{c}_{\cdot\cdot k} \qquad (21)$$

Animal effects can now be estimated from the 16 equations of the type of equation (2), e.g.

$$2\hat{\alpha}_1 = Y_{1\cdot\cdot} - \hat{\tau}_1 - \hat{\tau}_2 - \hat{\delta}_{11} - \hat{\delta}_{22} - \hat{\mu}$$

For the general case with t^2 experimental units and t treatments, equations (1), (10), (18) and (21) respectively become

$$2t^2\hat{\mu} = Y_{\cdot\cdot\cdot}$$

$$t^2(\hat{\mu}+\hat{\pi}_j) = Y_{\cdot j\cdot}$$

$$2t\hat{\tau}_k = Y^{c}_{\cdot\cdot k}$$

$$2t\hat{\delta}_{jk} = 2Y^{c}_{\cdot jk} - (Y_{\cdot j\cdot} - Y_{\cdot\cdot\cdot}/2) - Y^{c}_{\cdot\cdot k}$$

IV. SUMS OF SQUARES AND THE ANALYSIS OF VARIANCE

From the model assumed for this experiment, if there are t treatments and t^2 experimental units

$$\sum_i\sum_j\sum_k Y^2_{ijk} = 2t^2\hat{\mu}^2 + 2\sum_{i=1}^{t^2}(\hat{\alpha}_i)^2 + t^2\sum_{j=1}^{2}(\hat{\pi}_j)^2 + 2t\sum_{k=1}^{t}(\hat{\tau}_k)^2 + t\sum_{j=1}^{2}\sum_{k=1}^{t}(\hat{\delta}_{jk})^2$$

+ Residual Sum of Squares.

The analysis of variance takes the form given in Table 5. The residual or error sum of squares for the various effects from the sum of squares for the total (uncorrected) set of data. The residual sum of squares divided by $(t-1)^2$ is an estimate of the error variance, σ^2.

(Insert Table 5)

The following example (Table 6) with artifical data [$\mu = 10$, $\pi = -3$, $\pi_2 = +3$, $\tau_1 = -5$, $\tau_2 = -1$, $\tau_3 = +2$, $\tau_4 = +4$, $\delta_{11} = 3$, $\delta_{12} = 1$, $\delta_{13} = 0$, $\delta_{14} = -4$, $\delta_{21} = -3$, $\delta_{22} = -1$, $\delta_{23} = 0$, $\delta_{24} = 4$ and $e_{313} = 4$, $e_{511} = 1$, $e_{713} = -3$, $e_{911} = -4$, $e_{13,11} = 3$, $e_{15,13} = -1$ and Error S/S = 52] is used to illustrate the numerical computations for the sums of squares in the analysis of variance and for the preparation of a table of adjusted or corrected means (Table 7).

(Insert Tables 6 and 7)

V. VARIANCES OF MEANS AND EFFICIENCY OF DESIGN

The variances of the treatment and period-treatment means, and differences between two means are found in the following manner.

(i) variance of $\bar{y}_{...} = \text{var}[Y_{...}/32] = \sigma^2/32$

(ii) variance of $\bar{y}_{.j.} = \text{var}[Y_{.j.}/32] = \sigma^2/16 = 2\sigma^2/32$

(iii) variance of $\bar{y}^c_{..k} = \text{var}\left[\frac{Y^c_{..k}}{8} + \bar{y}_{...}\right] = 7\sigma^2/32$

e.g. variance of $\bar{y}^c_{\cdot\cdot 1}$ $= \text{var}\Big[1/8\{Y_{111} + Y_{511} + Y_{911} + Y_{13,11} + Y_{421} + Y_{721}$

$+ Y_{10,21} + Y_{13,21}\} - 1/8\{Y_{122} + Y_{523} + Y_{924}$

$+ Y_{13,21} + Y_{414} + Y_{713} + Y_{10,12} + Y_{13,11}\}$

$+ 1/32\{\sum_i\sum_j\sum_k Y_{ijk}\}\Big]$

$= \text{var}\Big[5/32\{Y_{111} + Y_{511} + Y_{911} + Y_{421} + Y_{721} + Y_{10,21}\}$

$- 3/32\{Y_{122} + Y_{523} + Y_{924} + Y_{414} + Y_{713} + Y_{10,12}\}$

$+ 1/32$ {sum of 20 observations}$\Big]$

$$= \left[\frac{25\times 6}{32^2} + \frac{9\times 6}{32^2} + \frac{20}{32^2}\right]\sigma^2 = \frac{7}{32}\sigma^2$$

(iv) variance of $\bar{y}^c_{\cdot jk}$ $= \text{var}\left[\frac{Y^c_{\cdot jk}}{4} - \bar{y}_{\cdot j\cdot} + 2\bar{y}_{\cdots}\right]$

$$= \frac{7}{16}\sigma^2 = 14\sigma^2/32$$

e.g. variance of $\bar{y}^c_{\cdot 11}$ $= \text{var}[1/4\{Y_{111} + Y_{511} + Y_{911} + Y_{13,11}\}$

$- 1/4\{Y_{122} + Y_{523} + Y_{924} + Y_{13,21}\}$

$- 1/16\{\sum_i\sum_k Y_{i1k}\} + 1/16\{\sum_i\sum_j\sum_k Y_{ijk}\}]$

$= \text{var}[1/4\{Y_{111} + Y_{511} + Y_{911} + Y_{13,11}\}$

$- 1/4\{Y_{122} + Y_{523} + Y_{924} + Y_{13,21}\} + 1/16\{\sum_i\sum_k Y_{i2k}\}]$

$$= \left\{\frac{1}{16}\times 4 + \frac{1}{16^2}\times 4 + \frac{1}{16^2}\times 12\right\}\sigma^2$$

$$= \frac{7}{16}\sigma^2$$

(v) variance of $(\bar{y}^c_{\cdot jk} - \bar{y}^c_{\cdot jk'})$ $= var[1/4\{Y^c_{\cdot jk} - Y^c_{\cdot jk'}\}]$

$= \sigma^2$

e.g. variance of $(\bar{y}^c_{\cdot 11} - \bar{y}^c_{\cdot 12})$ $= var[1/4\{(Y_{111} + Y_{511} + Y_{911} + Y_{13,11})$

$- (Y_{122} + Y_{523} + Y_{924} + Y_{13,21})$

$- (Y_{212} + Y_{612} + Y_{10,12} + Y_{14,12})$

$+ (Y_{223} + Y_{624} + Y_{10,21} + Y_{14,22})\}]$

$= \sigma^2$

(vi) variance of $(\bar{y}^c_{\cdot jk} - \bar{y}^c_{\cdot j'k})$ $= var[\{Y^c_{\cdot jk}/4 - \bar{y}_{\cdot j\cdot} + 2\bar{y}_{\cdots}\}$

$- \{Y^c_{\cdot j'k}/4 - \bar{y}_{\cdot j'} + 2\bar{y}_{\cdots}\}]$

$= \frac{7}{8}\sigma^2$

e.g. variance of $(\bar{y}^c_{\cdot 11} - \bar{y}^c_{\cdot 21})$ $= var[(Y_{111} + Y_{511} + Y_{911} + Y_{13,11})/4$

$- (Y_{122} + Y_{523} + Y_{924} + Y_{13,21})/4$

$- (\sum_i\sum_k Y_{i1k})/16 - (Y_{421} + Y_{721} + Y_{10,21}$

$+ Y_{13,21})/4 + (Y_{414} + Y_{713} + Y_{10,21}$

$+ Y_{13,21})/4 + (\sum_i\sum_k Y_{i2k})/16]$

$$= \text{var}[\tfrac{3}{16}(Y_{111} + Y_{511} + Y_{911}) + \tfrac{7}{16}(Y_{13,11})$$

$$- \tfrac{3}{16}(Y_{122} + Y_{523} + Y_{924}) - \tfrac{7}{16}(Y_{13,21})$$

$$- \tfrac{3}{16}(Y_{421} + Y_{721} + Y_{10,21}) + \tfrac{3}{16}(Y_{414}$$

$$+ Y_{713} + Y_{10,12})$$

$$- \tfrac{1}{16}(\text{sum of 9 observations in first period})$$

$$+ \tfrac{1}{16}(\text{sum of 9 observations in second period})]$$

$$= \frac{224}{16^2}\sigma^2 = \frac{7}{8}\sigma^2$$

An alternative experiment (for 16 animals) to the one used here would be one in which 8 animals are used in the first period (2 per treatment) and the remaining 8 animals used in the second period. The pooled error (σ_a^2) or within treatments (period 1 + period 2) would be based on 8 degrees of freedom compared with 9 degrees of freedom in the present experiment. The variance of a treatment mean and a treatment-period mean would be $\sigma_a^2/4 (= 8\sigma_a^2/32)$ and $\sigma_a^2/2 (= 8\sigma_a^2/16)$. To assess the efficiency of the proposed design, these variances might be compared with the formulae $7\sigma^2/32$ and $7\sigma^2/16$ obtained above. However a comparison of these different variances does not indicate completely the efficiency of the present design. In the alternative design the variance σ_a^2 is computed on an inter-animal basis while in the proposed design, σ^2 is computed on an intra-animal basis. It is reasonable to assume that σ^2 would be less than σ_a^2 and this would further increase the efficiency.

References

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TABLE 1

A Simple Change-Over Design

Row (Period)	Column or Replicate					
	1	2	3	4	5	6
I	A	B	C	A	B	C
II	B	C	A	C	A	B
III	C	A	B	B	C	A

TABLE 2

A Double Change-Over Design

	Sequence or Animal Number											
Row (Period)	1	2	3	4	5	6	7	8	9	10	11	12
I	A	B	C	D	A	B	C	D	A	B	C	D
II	B	A	D	C	D	C	B	A	C	D	A	B
III	C	D	A	B	B	A	D	C	D	C	B	A
IV	D	C	B	A	C	D	A	B	B	A	D	C

TABLE 3

Two Period Design for 16 Experimental Units and Treatments A, B, C, D

Experimental Unit or Animal Number		1	2	3	4	5	6	7	8
Period	I	A	B	C	D	A	B	C	D
	II	B	C	D	A	C	D	A	B
Experimental Unit or Animal Number		9	10	11	12	13	14	15	16
Period	I	A	B	C	D	A	B	C	D
	II	D	A	B	C	A	B	C	D

TABLE 4

Allocation of Experimental Units as Given by Latin Square Arrangement

Experimental Units: 1,2,3,•••,16

		Treatments in Second Period			
		A	B	C	D
Treatments in First Period	A	13	1	5	9
	B	10	14	2	6
	C	7	11	15	3
	D	4	8	12	16

TABLE 5

Sums of Squares for Analysis of Variance

Source of Variation	d.f.	Sums of Squares
Mean	1	$Y^2_{\cdot\cdot\cdot}/2t^2$
Animals	t^2-1	$\frac{1}{2}\left[\sum_{i=1}^{t^2} Y^2_{i\cdot\cdot}\right.$ - Treatment S/S - Interaction S/S $\left.\right] - Y^2_{\cdot\cdot\cdot}/2t^2$
Periods	1	$\sum_{j=1}^{2} Y^2_{\cdot j\cdot}/t^2 - Y^2_{\cdot\cdot\cdot}/2t^2$
Treatments	t-1	$\sum_{k=1}^{t} [Y^c_{\cdot\cdot k}]^2/2t$
Periods × Treatments	t-1	$\sum_{j}\sum_{k} [Y^c_{\cdot jk}]^2/2t$ - Period S/S - Treatment S/S
Residual	$(t-1)^2$	difference
Total	$2t^2$	$\sum\sum\sum Y^2_{ijk}$

TABLE 6

Yield in an Artificial Experiment and Periods v Treatment Totals

Yields

Animal Number	Period 1		Period 2	
1	A	-3	B	3
2	B	0	C	8
3	C	7	D	15
4	D	-3	A	-5
5	A	2	C	11
6	B	4	D	18
7	C	1	A	0
8	D	6	B	10
9	A	1	D	21
10	B	8	A	6
11	C	11	B	13
12	D	10	C	18
13	A	18	A	15
14	B	17	B	21
15	C	25	C	32
16	D	8	D	22

Periods v Treatments Totals

$Y_{\cdot jk}$

Treatments		1	2	3	4	$X_{\cdot j \cdot}$
Periods	1	18	29	44	21	112
	2	16	47	69	76	208
$X_{\cdot k}$		34	76	113	97	320

Adjusting Factors for Periods v Treatments Totals

$Y^c_{\cdot(jk)}$

Treatments		1	2	3	4	$X^c_{\cdot(j\cdot)}$
Periods	1	50	53	60	45	208
	2	24	31	37	20	112
$X^c_{\cdot(\cdot k)}$		74	84	97	65	320

Adjusted Periods v Treatments Totals

$Y^c_{\cdot jk}$

Treatments		1	2	3	4	$X^c_{\cdot j \cdot}$
Periods	1	-32	-24	-16	-24	-96
	2	-8	16	32	56	96
$X^c_{\cdot\cdot k}$		-40	-8	16	32	0

TABLE 7

Sums of Squares and Adjusted Means

C.F.M. $= Y^2_{\cdot\cdot\cdot} \div 32 = 320^2 \div 32$ $= 3200$ (1 d.f.)

Total S/S $= \sum\sum\sum Y^2_{ijk}$ - C.F.M. $= 5724 - 3200$ $= 2524$ (3 d.f.)

Period S/S $= \{(Y^2_{\cdot 1\cdot} + Y^2_{\cdot 2\cdot}) \div 16\}$ - C.F.M. $= 3488 - 3200 = 288$ (1 d.f.)

Treatment S/S $= \sum \{Y^c_{\cdot\cdot k}\} \div 8$ $= 368$ (3 d.f.)

Interaction S/S $= \left[\sum_j \sum_k \{Y^c_{\cdot jk}\}^2 \div 8 \right]$ - Period S/S - Treatment S/S

$= 864 - 288 - 368 = 208$ (3 d.f.)

Animal S/S $= \frac{1}{2} \left[\left\{ \sum X^2_{i\cdot\cdot} \right\} - \text{Treatment S/S} - \text{Interaction S/S} \right] - \text{C.F.M.}$

$= \frac{1}{2}\, [10{,}192 - 368 - 208] - 3200 = 1608$ (15 d.f.)

Residual or Error S/S = $= 52$ (9 d.f.)

Adjusted Periods v Treatments Means

Treatments		1	2	3	4	$\bar{y}_{\cdot j\cdot}$
Periods	1	5	7	9	7	7
	2	5	7	15	21	13
Means $\bar{y}^c_{\cdot\cdot k}$		5	7	12	14	$10 = \bar{y}_{\cdot\cdot\cdot}$

$$\bar{y}^c_{\cdot 11} = Y^c_{\cdot 11}/4 - \bar{y}_{\cdot 1\cdot} + 2\bar{y}_{\cdot\cdot\cdot}$$