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A SSOLS(15, 14) SET AND A COMPLETE SET OF F-SQUARES FOR N = 3 × 6

By

Walter T. Federer

ABSTRACT

A sum-of-squares orthogonal set for n = 15 is constructed. A computer code for demonstrating that this set is sum-of-squares orthogonal is presented. In addition, a complete set of 59 sum-of-squares orthogonal F-squares is given. The computer code for constructing the F-squares and demonstrating their sum-of-squares orthogonality is included.

Key words: sum-of-squares orthogonality, combinatorial orthogonality, Latin square, computer code, code output.

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INTRODUCTION

In the next section, it is shown how to construct a set of 14 sum-of-squares orthogonal Latin squares of side 15, SSOLS(15, 14). This supplements the results given by Federer (2004b) for Latin squares of sides 6 and 10. In the following section, a complete set of 59 sum-of-squares orthogonal F-squares are described for n = 3 × 6 = 18 in 18 × 18 squares. A computer code is given for constructing the F-squares. The 59 F-squares in the complete set are not given but it is described how to obtain them from the given code. The output from the code is presented. These results supplement those given by Federer (2004a, 2004b) for n = 2 × 4 = 8, n = 3 × 4 = 12, and n = 3 × 5 = 15.

A SSOLS(15, 14) SET

Using the procedure described by Federer (2004b), a complete set of sum-of-squares orthogonal Latin squares may be obtained as follows. As a starting point, any 15 × 15 Latin square may be used. We shall use the following cyclically generated Latin Square:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	3	4	5	6	7	8	9	10	11	12	13	14	15	1
3	4	5	6	7	8	9	10	11	12	13	14	15	1	2
4	5	6	7	8	9	10	11	12	13	14	15	1	2	3

5	6	7	8	9	10	11	12	13	14	15	1	2	3	4
6	7	8	9	10	11	12	13	14	15	1	2	3	4	5
7	8	9	10	11	12	13	14	15	1	2	3	4	5	6
8	9	10	11	12	13	14	15	1	2	3	4	5	6	7
9	10	11	12	13	14	15	1	2	3	4	5	6	7	8
10	11	12	13	14	15	1	2	3	4	5	6	7	8	9
11	12	13	14	15	1	2	3	4	5	6	7	8	9	10
12	13	14	15	1	2	3	4	5	6	7	8	9	10	11
13	14	15	1	2	3	4	5	6	7	8	9	10	11	12
14	15	1	2	3	4	5	6	7	8	9	10	11	12	13
15	1	2	3	4	5	6	7	8	9	10	11	12	13	14

To obtain the second Latin square, move the last row of the above square to the second row position and move all other rows down one position. Doing this the following Latin square is obtained:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
15	1	2	3	4	5	6	7	8	9	10	11	12	13	14
2	3	4	5	6	7	8	9	10	11	12	13	14	15	1
3	4	5	6	7	8	9	10	11	12	13	14	15	1	2
4	5	6	7	8	9	10	11	12	13	14	15	1	2	3
5	6	7	8	9	10	11	12	13	14	15	1	2	3	4
6	7	8	9	10	11	12	13	14	15	1	2	3	4	5
7	8	9	10	11	12	13	14	15	1	2	3	4	5	6
8	9	10	11	12	13	14	15	1	2	3	4	5	6	7
9	10	11	12	13	14	15	1	2	3	4	5	6	7	8
10	11	12	13	14	15	1	2	3	4	5	6	7	8	9
11	12	13	14	15	1	2	3	4	5	6	7	8	9	10
12	13	14	15	1	2	3	4	5	6	7	8	9	10	11
13	14	15	1	2	3	4	5	6	7	8	9	10	11	12
14	15	1	2	3	4	5	6	7	8	9	10	11	12	13

Continuing this process, 14 Latin squares may be obtained, the 14^{th} one being the following one:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
3	4	5	6	7	8	9	10	11	12	13	14	15	1	2
4	5	6	7	8	9	10	11	12	13	14	15	1	2	3
5	6	7	8	9	10	11	12	13	14	15	1	2	3	4
6	7	8	9	10	11	12	13	14	15	1	2	3	4	5
7	8	9	10	11	12	13	14	15	1	2	3	4	5	6
8	9	10	11	12	13	14	15	1	2	3	4	5	6	7
9	10	11	12	13	14	15	1	2	3	4	5	6	7	8
10	11	12	13	14	15	1	2	3	4	5	6	7	8	9
11	12	13	14	15	1	2	3	4	5	6	7	8	9	10

```
12 13 14 15 1  2  3  4  5  6  7  8  9  10 11
13 14 15 1  2  3  4  5  6  7  8  9  10 11 12
14 15 1  2  3  4  5  6  7  8  9  10 11 12 13
15 1  2  3  4  5  6  7  8  9  10 11 12 13 14
2  3  4  5  6  7  8  9  10 11 12 13 14 15 1
```

To demonstrate that this set of 14 Latin squares is a sum-of-squares orthogonal set, one my use the following SAS computer code. The input variables are response (Y), row (R), column (C), and the numbers from the 14 Latin squares (L1 to L14).

```
data ssols15;
input Y R C L1 L2 L3 L4 L5 L6 L7 L8 L9 L10 L11 L12 L13 L14;
datalines;
1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
3  1  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2
7  1  3  3  3  3  3  3  3  3  3  3  3  3  3  3  3
4  1  4  4  4  4  4  4  4  4  4  4  4  4  4  4  4
5  1  5  5  5  5  5  5  5  5  5  5  5  5  5  5  5
6  1  6  6  6  6  6  6  6  6  6  6  6  6  6  6  6
5  1  7  7  7  7  7  7  7  7  7  7  7  7  7  7  7
9  1  8  8  8  8  8  8  8  8  8  8  8  8  8  8  8
0  1  9  9  9  9  9  9  9  9  9  9  9  9  9  9  9
7  1 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10
4  1 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11
3  1 12 12 12 12 12 12 12 12 12 12 12 12 12 12 12
2  1 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13
8  1 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14
7  1 15 15 15 15 15 15 15 15 15 15 15 15 15 15 15
3  2  1  2 15 14 13 12 11 10  9  8  7  6  5  4  3
7  2  2  3  1 15 14 13 12 11 10  9  8  7  6  5  4
9  2  3  4  2  1 15 14 13 12 11 10  9  8  7  6  5
0  2  4  5  3  2  1 15 14 13 12 11 10  9  8  7  6
5  2  5  6  4  3  2  1 15 14 13 12 11 10  9  8  7
9  2  6  7  5  4  3  2  1 15 14 13 12 11 10  9  8
6  2  7  8  6  5  4  3  2  1 15 14 13 12 11 10  9
3  2  8  9  7  6  5  4  3  2  1 15 14 13 12 11 10
5  2  9 10  8  7  6  5  4  3  2  1 15 14 13 12 11
4  2 10 11  9  8  7  6  5  4  3  2  1 15 14 13 12
2  2 11 12 10  9  8  7  6  5  4  3  2  1 15 14 13
7  2 12 13 11 10  9  8  7  6  5  4  3  2  1 15 14
9  2 13 14 12 11 10  9  8  7  6  5  4  3  2  1 15
5  2 14 15 13 12 11 10  9  8  7  6  5  4  3  2  1
8  2 15  1 14 13 12 11 10  9  8  7  6  5  4  3  2
2  3  1  3  2 15 14 13 12 11 10  9  8  7  6  5  4
2  3  2  4  3  1 15 14 13 12 11 10  9  8  7  6  5
2  3  3  5  4  2  1 15 14 13 12 11 10  9  8  7  6
2  3  4  6  5  3  2  1 15 14 13 12 11 10  9  8  7
2  3  5  7  6  4  3  2  1 15 14 13 12 11 10  9  8
2  3  6  8  7  5  4  3  2  1 15 14 13 12 11 10  9
3  3  7  9  8  6  5  4  3  2  1 15 14 13 12 11 10
3  3  8 10  9  7  6  5  4  3  2  1 15 14 13 12 11
3  3  9 11 10  8  7  6  5  4  3  2  1 15 14 13 12
3  3 10 12 11  9  8  7  6  5  4  3  2  1 15 14 13
3  3 11 13 12 10  9  8  7  6  5  4  3  2  1 15 14
5  3 12 14 13 11 10  9  8  7  6  5  4  3  2  1 15
```

5	3	13	15	14	12	11	10	9	8	7	6	5	4	3	2	1
5	3	14	1	15	13	12	11	10	9	8	7	6	5	4	3	2
5	3	15	2	1	14	13	12	11	10	9	8	7	6	5	4	3
7	4	1	4	3	2	15	14	13	12	11	10	9	8	7	6	5
7	4	2	5	4	3	1	15	14	13	12	11	10	9	8	7	6
7	4	3	6	5	4	2	1	15	14	13	12	11	10	9	8	7
7	4	4	7	6	5	3	2	1	15	14	13	12	11	10	9	8
4	4	5	8	7	6	4	3	2	1	15	14	13	12	11	10	9
4	4	6	9	8	7	5	4	3	2	1	15	14	13	12	11	10
3	4	7	10	9	8	6	5	4	3	2	1	15	14	13	12	11
3	4	8	11	10	9	7	6	5	4	3	2	1	15	14	13	12
3	4	9	12	11	10	8	7	6	5	4	3	2	1	15	14	13
4	4	10	13	12	11	9	8	7	6	5	4	3	2	1	15	14
5	4	11	14	13	12	10	9	8	7	6	5	4	3	2	1	15
6	4	12	15	14	13	11	10	9	8	7	6	5	4	3	2	1
6	4	13	1	15	14	12	11	10	9	8	7	6	5	4	3	2
6	4	14	2	1	15	13	12	11	10	9	8	7	6	5	4	3
3	4	15	3	2	1	14	13	12	11	10	9	8	7	6	5	4
5	5	1	5	4	3	2	15	14	13	12	11	10	9	8	7	6
5	5	2	6	5	4	3	1	15	14	13	12	11	10	9	8	7
4	5	3	7	6	5	4	2	1	15	14	13	12	11	10	9	8
5	5	4	8	7	6	5	3	2	1	15	14	13	12	11	10	9
6	5	5	9	8	7	6	4	3	2	1	15	14	13	12	11	10
4	5	6	10	9	8	7	5	4	3	2	1	15	14	13	12	11
5	5	7	11	10	9	8	6	5	4	3	2	1	15	14	13	12
5	5	8	12	11	10	9	7	6	5	4	3	2	1	15	14	13
3	5	9	13	12	11	10	8	7	6	5	4	3	2	1	15	14
4	5	10	14	13	12	11	9	8	7	6	5	4	3	2	1	15
5	5	11	15	14	13	12	10	9	8	7	6	5	4	3	2	1
6	5	12	1	15	14	13	11	10	9	8	7	6	5	4	3	2
4	5	13	2	1	15	14	12	11	10	9	8	7	6	5	4	3
5	5	14	3	2	1	15	13	12	11	10	9	8	7	6	5	4
5	5	15	4	3	2	1	14	13	12	11	10	9	8	7	6	5
6	6	1	6	5	4	3	2	15	14	13	12	11	10	9	8	7
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6	6	3	8	7	6	5	4	2	1	15	14	13	12	11	10	9
5	6	4	9	8	7	6	5	3	2	1	15	14	13	12	11	10
3	6	5	10	9	8	7	6	4	3	2	1	15	14	13	12	11
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6	6	7	12	11	10	9	8	6	5	4	3	2	1	15	14	13
5	6	8	13	12	11	10	9	7	6	5	4	3	2	1	15	14
6	6	9	14	13	12	11	10	8	7	6	5	4	3	2	1	15
5	6	10	15	14	13	12	11	9	8	7	6	5	4	3	2	1
4	6	11	1	15	14	13	12	10	9	8	7	6	5	4	3	2
5	6	12	2	1	15	14	13	11	10	9	8	7	6	5	4	3
6	6	13	3	2	1	15	14	12	11	10	9	8	7	6	5	4
7	6	14	4	3	2	1	15	13	12	11	10	9	8	7	6	5
8	6	15	5	4	3	2	1	14	13	12	11	10	9	8	7	6
3	7	1	7	6	5	4	3	2	15	14	13	12	11	10	9	8
0	7	2	8	7	6	5	4	3	1	15	14	13	12	11	10	9
1	7	3	9	8	7	6	5	4	2	1	15	14	13	12	11	10
8	7	4	10	9	8	7	6	5	3	2	1	15	14	13	12	11
6	7	5	11	10	9	8	7	6	4	3	2	1	15	14	13	12
7	7	6	12	11	10	9	8	7	5	4	3	2	1	15	14	13
7	7	7	13	12	11	10	9	8	6	5	4	3	2	1	15	14
6	7	8	14	13	12	11	10	9	7	6	5	4	3	2	1	15
7	7	9	15	14	13	12	11	10	8	7	6	5	4	3	2	1

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5	7	11	2	1	15	14	13	12	10	9	8	7	6	5	4	3
6	7	12	3	2	1	15	14	13	11	10	9	8	7	6	5	4
7	7	13	4	3	2	1	15	14	12	11	10	9	8	7	6	5
8	7	14	5	4	3	2	1	15	13	12	11	10	9	8	7	6
5	7	15	6	5	4	3	2	1	14	13	12	11	10	9	8	7
4	8	1	8	7	6	5	4	3	2	15	14	13	12	11	10	9
6	8	2	9	8	7	6	5	4	3	1	15	14	13	12	11	10
8	8	3	10	9	8	7	6	5	4	2	1	15	14	13	12	11
8	8	4	11	10	9	8	7	6	5	3	2	1	15	14	13	12
6	8	5	12	11	10	9	8	7	6	4	3	2	1	15	14	13
7	8	6	13	12	11	10	9	8	7	5	4	3	2	1	15	14
8	8	7	14	13	12	11	10	9	8	6	5	4	3	2	1	15
8	8	8	15	14	13	12	11	10	9	7	6	5	4	3	2	1
7	8	9	1	15	14	13	12	11	10	8	7	6	5	4	3	2
6	8	10	2	1	15	14	13	12	11	9	8	7	6	5	4	3
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8	8	12	4	3	2	1	15	14	13	11	10	9	8	7	6	5
9	8	13	5	4	3	2	1	15	14	12	11	10	9	8	7	6
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4	8	15	7	6	5	4	3	2	1	14	13	12	11	10	9	8
9	9	1	9	8	7	6	5	4	3	2	15	14	13	12	11	10
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6	9	3	11	10	9	8	7	6	5	4	2	1	15	14	13	12
9	9	4	12	11	10	9	8	7	6	5	3	2	1	15	14	13
9	9	5	13	12	11	10	9	8	7	6	4	3	2	1	15	14
9	9	6	14	13	12	11	10	9	8	7	5	4	3	2	1	15
8	9	7	15	14	13	12	11	10	9	8	6	5	4	3	2	1
8	9	8	1	15	14	13	12	11	10	9	7	6	5	4	3	2
7	9	9	2	1	15	14	13	12	11	10	8	7	6	5	4	3
3	9	10	3	2	1	15	14	13	12	11	9	8	7	6	5	4
4	9	11	4	3	2	1	15	14	13	12	10	9	8	7	6	5
5	9	12	5	4	3	2	1	15	14	13	11	10	9	8	7	6
6	9	13	6	5	4	3	2	1	15	14	12	11	10	9	8	7
7	9	14	7	6	5	4	3	2	1	15	13	12	11	10	9	8
8	9	15	8	7	6	5	4	3	2	1	14	13	12	11	10	9
1	10	1	10	9	8	7	6	5	4	3	2	15	14	13	12	11
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3	10	3	12	11	10	9	8	7	6	5	4	2	1	15	14	13
4	10	4	13	12	11	10	9	8	7	6	5	3	2	1	15	14
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6	10	6	15	14	13	12	11	10	9	8	7	5	4	3	2	1
1	10	7	1	15	14	13	12	11	10	9	8	6	5	4	3	2
2	10	8	2	1	15	14	13	12	11	10	9	7	6	5	4	3
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4	10	10	4	3	2	1	15	14	13	12	11	9	8	7	6	5
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6	10	12	6	5	4	3	2	1	15	14	13	11	10	9	8	7
0	10	13	7	6	5	4	3	2	1	15	14	12	11	10	9	8
1	10	14	8	7	6	5	4	3	2	1	15	13	12	11	10	8
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6	11	4	14	13	12	11	10	9	8	7	6	5	3	2	1	15
2	11	5	15	14	13	12	11	10	9	8	7	6	4	3	2	1
3	11	6	1	15	14	13	12	11	10	9	8	7	5	4	3	2

4	11	7	2	1	15	14	13	12	11	10	9	8	6	5	4	3
4	11	8	3	2	1	15	14	13	12	11	10	9	7	6	5	4
2	11	9	4	3	2	1	15	14	13	12	11	10	8	7	6	5
3	11	10	5	4	3	2	1	15	14	13	12	11	9	8	7	6
4	11	11	6	5	4	3	2	1	15	14	13	12	10	9	8	7
5	11	12	7	6	5	4	3	2	1	15	14	13	11	10	9	8
6	11	13	8	7	6	5	4	3	2	1	15	14	12	11	10	9
9	11	14	9	8	7	6	5	4	3	2	1	15	13	12	11	10
2	11	15	10	9	8	7	6	5	4	3	2	1	14	13	12	11
5	12	1	12	11	10	9	8	7	6	5	4	3	2	15	14	13
4	12	2	13	12	11	10	9	8	7	6	5	4	3	1	15	14
5	12	3	14	13	12	11	10	9	8	7	6	5	4	2	1	15
3	12	4	15	14	13	12	11	10	9	8	7	6	5	3	2	1
7	12	5	1	15	14	13	12	11	10	9	8	7	6	4	3	2
8	12	6	2	1	15	14	13	12	11	10	9	8	7	5	4	3
9	12	7	3	2	1	15	14	13	12	11	10	9	8	6	5	4
1	12	8	4	3	2	1	15	14	13	12	11	10	9	7	6	5
1	12	9	5	4	3	2	1	15	14	13	12	11	10	8	7	6
1	12	10	6	5	4	3	2	1	15	14	13	12	11	9	8	7
2	12	11	7	6	5	4	3	2	1	15	14	13	12	10	9	8
3	12	12	8	7	6	5	4	3	2	1	15	14	13	11	10	9
4	12	13	9	8	7	6	5	4	3	2	1	15	14	12	11	10
5	12	14	10	9	8	7	6	5	4	3	2	1	15	13	12	11
4	12	15	11	10	9	8	7	6	5	4	3	2	1	14	13	12
1	13	1	13	12	11	10	9	8	7	6	5	4	3	2	15	14
1	13	2	14	13	12	11	10	9	8	7	6	5	4	3	1	15
2	13	3	15	14	13	12	11	10	9	8	7	6	5	4	2	1
3	13	4	1	15	14	13	12	11	10	9	8	7	6	5	3	2
4	13	5	2	1	15	14	13	12	11	10	9	8	7	6	4	3
2	13	6	3	2	1	15	14	13	12	11	10	9	8	7	5	4
4	13	7	4	3	2	1	15	14	13	12	11	10	9	8	6	5
5	13	8	5	4	3	2	1	15	14	13	12	11	10	9	7	6
6	13	9	6	5	4	3	2	1	15	14	13	12	11	10	8	7
1	13	10	7	6	5	4	3	2	1	15	14	13	12	11	9	8
2	13	11	8	7	6	5	4	3	2	1	15	14	13	12	10	9
3	13	12	9	8	7	6	5	4	3	2	1	15	14	13	11	10
4	13	13	10	9	8	7	6	5	4	3	2	1	15	14	12	11
5	13	14	11	10	9	8	7	6	5	4	3	2	1	15	13	12
3	13	15	12	11	10	9	8	7	6	5	4	3	2	1	14	13
4	14	1	14	13	12	11	10	9	8	7	6	5	4	3	2	15
4	14	2	15	14	13	12	11	10	9	8	7	6	5	4	3	1
3	14	3	1	15	14	13	12	11	10	9	8	7	6	5	4	2
0	14	4	2	1	15	14	13	12	11	10	9	8	7	6	5	3
1	14	5	3	2	1	15	14	13	12	11	10	9	8	7	6	4
2	14	6	4	3	2	1	15	14	13	12	11	10	9	8	7	5
3	14	7	5	4	3	2	1	15	14	13	12	11	10	9	8	6
9	14	8	6	5	4	3	2	1	15	14	13	12	11	10	9	7
1	14	9	7	6	5	4	3	2	1	15	14	13	12	11	10	8
3	14	10	8	7	6	5	4	3	2	1	15	14	13	12	11	9
4	14	11	9	8	7	6	5	4	3	2	1	15	14	13	12	10
5	14	12	10	9	8	7	6	5	4	3	2	1	15	14	13	11
3	14	13	11	10	9	8	7	6	5	4	3	2	1	15	14	12
4	14	14	12	11	10	9	8	7	6	5	4	3	2	1	15	13
4	14	15	13	12	11	10	9	8	7	6	5	4	3	2	1	14
5	15	1	15	14	13	12	11	10	9	8	7	6	5	4	3	2
4	15	2	1	15	14	13	12	11	10	9	8	7	6	5	4	3
4	15	3	2	1	15	14	13	12	11	10	9	8	7	6	5	4

```
1 15  4  3  2  1 15 14 13 12 11 10  9  8  7  6  5
2 15  5  4  3  2  1 15 14 13 12 11 10  9  8  7  6
4 15  6  5  4  3  2  1 15 14 13 12 11 10  9  8  7
6 15  7  6  5  4  3  2  1 15 14 13 12 11 10  9  8
7 15  8  7  6  5  4  3  2  1 15 14 13 12 11 10  9
4 15  9  8  7  6  5  4  3  2  1 15 14 13 12 11 10
5 15 10  9  8  7  6  5  4  3  2  1 15 14 13 12 11
4 15 11 10  9  8  7  6  5  4  3  2  1 15 14 13 12
3 15 12 11 10  9  8  7  6  5  4  3  2  1 15 14 13
4 15 13 12 11 10  9  8  7  6  5  4  3  2  1 15 14
5 15 14 13 12 11 10  9  8  7  6  5  4  3  2  1 15
6 15 15 14 13 12 11 10  9  8  7  6  5  4  3  2  1
; run;
Proc glm data = ssols15;
class R C;
model Y = R C R*C;
run;
Proc GLM data = ssols15;
CLASS R C L1 L2 L3 L4 L5 L6 L7 L8 L9 L10 L11 L12 L13 L14;
MODEL Y = R C L1 L2 L3 L4 L5 L6 L7 L8 L9 L10 L11 L12 L13 L14;
RUN;
```

The edited output of the above code and data set is presented below:

Class Level Information

Class	Levels	Values
R	15	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
C	15	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Number of observations 225

Dependent Variable: Y

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	224	1102.382222	4.921349	.	.
Error	0	0.000000	.		
Corrected Total	224	1102.382222			

R-Square	Coeff Var	Root MSE	Y Mean
1.000000	.	.	4.631111

Source	DF	Type I SS	Mean Square	F Value	Pr > F
R	14	313.5822222	22.3987302	.	.
C	14	68.1155556	4.8653968	.	.
R*C	196	720.6844444	3.6769615	.	.

Source	DF	Type III SS	Mean Square	F Value	Pr > F
R	14	313.5822222	22.3987302	.	.
C	14	68.1155556	4.8653968	.	.
R*C	196	720.6844444	3.6769615	.	.

Class Level Information

Class	Levels	Values
R	15	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
C	15	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
L1	15	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
L2	15	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
L3	15	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
L4	15	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
L5	15	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
L6	15	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
L7	15	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
L8	15	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

L9	15	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
L10	15	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
L11	15	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
L12	15	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
L13	15	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
L14	15	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Number of observations 225

Dependent Variable: Y

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	224	1102.382222	4.921349	.	.
Error	0	0.000000	.		
Corrected Total	224	1102.382222			

R-Square	Coeff Var	Root MSE	Y Mean
1.000000	.	.	4.631111

Source	DF	Type I SS	Mean Square	F Value	Pr > F
R	14	313.5822222	22.3987302	.	.
C	14	68.1155556	4.8653968	.	.
L1	14	42.9155556	3.0653968	.	.
L2	14	98.0508455	7.0036318	.	.
L3	14	38.4090989	2.7435071	.	.
L4	14	67.7378861	4.8384204	.	.
L5	14	15.3524467	1.0966033	.	.
L6	14	15.3721293	1.0980092	.	.
L7	14	88.1495421	6.2963959	.	.
L8	14	60.3913440	4.3136674	.	.
L9	14	45.1210030	3.2229288	.	.
L10	14	53.3436496	3.8102607	.	.
L11	14	33.6423195	2.4030228	.	.
L12	14	105.0446419	7.5031887	.	.
L13	14	35.2341134	2.5167224	.	.
L14	14	21.9198688	1.5657049	.	.

Source	DF	Type III SS	Mean Square	F Value	Pr > F
R	14	299.2090933	21.3720781	.	.
C	14	73.7055541	5.2646824	.	.
L1	14	59.5275624	4.2519687	.	.
L2	14	55.4309926	3.9593566	.	.
L3	14	35.8996113	2.5642579	.	.
L4	14	22.9846964	1.6417640	.	.
L5	14	10.0986355	0.7213311	.	.
L6	14	60.5001035	4.3214360	.	.
L7	14	31.7963856	2.2711704	.	.
L8	14	56.7777934	4.0555567	.	.
L9	14	62.7735622	4.4838259	.	.
L10	14	46.8299131	3.3449938	.	.
L11	14	63.1029203	4.5073515	.	.
L12	14	69.1503781	4.9393127	.	.
L13	14	28.0257572	2.0018398	.	.
L14	14	21.9198688	1.5657049	.	.

As may be seen from the above analysis of variance, the sum of the Type I sums of squares for the 14 Latin square numbers is the same as the row by column interaction of squares. Hence, this set is a SSOLS(15, 14) set and is a complete set as all of the row by column sum of squares and degrees of freedom have been accounted for. As the Type III sums of squares are not the same as the Type I sums of squares, these squares are not combinatorially orthogonal but they are sum-of-squares orthogonal.

COMPLETE SET OF F-SQUARES FOR N = 3 × 6 = 18

A 6 × 6 Latin square of the same form as the above one of side 15 was used to construct the SSOLS(6, 5) set. The numbers from these five Latin squares are entered in

the following DATALINES as F11, F12, F13, F14, and F15 as INPUT variables. In addition to entering the row (R) and column (C) numbers (1, 2, ..., 18), the row numbers are entered as levels of the 3 × 6 factorial of factors A and B. The column numbers are entered as the levels of the 3 × 6 factorial for factors C and D. The response variable is entered as Y. After the INPUT line, the 59 F-squares are defined for the interactions of the factors A, B, C, and D. For example, the two F(18, 6) squares formed from the A × D interaction are F3 and F4. In order to construct the 54 F-squares other than those used as INPUT variables (F11, F12, F13, F14, F15), it is also necessary to write a series of IF ... THEN statements in order to have the correct number of levels and to construct the F-squares. The INPUT statement, the definition of the F-squares, the IF...THEN statements, the DATALINES, and the commands are given in the following SAS computer code:

```
DATA FSS3X6;
INPUT Y ROW COL A B C D F11 F12 F13 F14 F15;
F1 = A + C; F2 = A + 2*C; F1A = 2*A + C; F2A = 2*A + 2*C;
F3 = A + D; F4 = 2*A + D - 1;
F5 = F1 + D; F6 = F2 + D; F7 = F1A + D; F8 = F2A + D;
F9 = B + C; F10 = B + 2*C - 1;
F16 = F11 + C; F17 = F11 + 2*C - 1;
F18 = F12 + C; F19 = F12 + 2*C - 1;
F20 = F13 + C; F21 = F13 + 2*C - 1;
F22 = F14 + C; F23 = F14 + 2*C - 1;
F24 = F15 + C; F25 = F15 + 2*C - 1;
F26 = F1 + B; F27 = F2 + B; F28 = F1A + B; F29 = F2A + B;
F30 = F11 + A; F31 = F11 + 2*A - 1;
F32 = F12 + A; F33 = F12 + 2*A - 1;
F34 = F13 + A; F35 = F13 + 2*A - 1;
F36 = F14 + A; F37 = F14 + 2*A - 1;
F38 = F15 + A; F39 = F15 + 2*A - 1;
F40 = F11 + F1; F41 = F11 + F2; F42 = F11 + F1A; F43 = F11 + F2A;
F44 = F12 + F1; F45 = F12 + F2; F46 = F12 + F1A; F47 = F12 + F2A;
F48 = F13 + F1; F49 = F13 + F2; F50 = F13 + F1A; F51 = F13 + F2A;
F52 = F14 + F1; F53 = F14 + F2; F54 = F14 + F1A; F55 = F14 + F2A;
F56 = F15 + F1; F57 = F15 + F2; F58 = F15 + F1A; F59 = F15 + F2A;

IF F1>2 THEN F1 = A + C - 3;
IF F2>5 THEN F2 = A + 2*C - 6; IF F2>2 THEN F2 = A + 2*C - 3;
IF F1A>5 THEN F1A = 2*A + C - 6; IF F1A>2 THEN F1A = 2*A + C - 3;
IF F2A>5 THEN F2A = 2*A + 2*C - 6; IF F2A>2 THEN F2A = 2*A + 2*C - 3;
IF F3>5 THEN F3 = A + D - 6;
IF F4>5 THEN F4 = 2*A + D - 6 - 1; IF F4=-1 THEN F4 = 2*A + D=5;
IF F5>5 THEN F5 = F1 + D - 6; IF F5<0 THEN F5 = F1 + D;
IF F6>5 THEN F6 = F2 + D - 6; IF F6<0 THEN F6 = F2 + D;
IF F7>5 THEN F7 = F1A + D - 6; IF F7<0 THEN F7 = F1A + D;
IF F8>5 THEN F8 = F2A + D = 6; IF F8<0 THEN F8 = F2A + D;
IF F9>5 THEN F9 = B + C - 6; IF F10>5 THEN F10 = B + 2*C - 1 - 6;
IF F10=-1 THEN F10=B+2*C=5;
IF F16>5 THEN F16 = F11 + C - 6;
IF F17>5 THEN F17 = F11 + 2*C - 1 - 6; IF F17=-1 THEN F17 = F11 +
2*C=5;
IF F18>5 THEN F18 = F12 + C - 6;
IF F19>5 THEN F19 = F12 + 2*C - 1 - 6; IF F19=-1 THEN F19 = F12 +
2*C=5;
```

```
IF F20>5 THEN F20 = F13 + C - 6;
IF F21>5 THEN F21 = F13 + 2*C - 1 - 6; IF F21=-1 THEN F21 = F13 +
2*C=5;
IF F22>5 THEN F22 = F14 + C - 6;
IF F23>5 THEN F23 = F14 + 2*C - 1 - 6; IF F23=-1 THEN F23 = F14 +
2*C=5;
IF F24>5 THEN F24 = F15 + C - 6;
IF F25>5 THEN F25 = F15 + 2*C - 1 - 6; IF F25=-1 THEN F25 = F15 +
2*C=5;
IF F26>5 THEN F26 = F1 + B - 6; IF F26<0 THEN F26 = F1 + B;
IF F27>5 THEN F27 = F2 + B - 6; IF F27<0 THEN F27 = F2 + B;
IF F28>5 THEN F28 = F1A + B - 5; IF F28<0 THEN F28 = F1A + B;
IF F29>5 THEN F29 = F2A + B - 6; IF F29<0 THEN F29 = F2A = B;
IF F30>5 THEN F30 = F11 + A - 6;
IF F31>5 THEN F31 = F11 + 2*A - 1 - 6; IF F31=-1 THEN F31 = F11 +
2*A=5;
IF F32>5 THEN F32 = F12 + A - 6;
IF F33>5 THEN F33 = F12 + 2*A - 1 - 6; IF F33=-1 THEN F33 = F12 +
2*A=5;
IF F34>5 THEN F34 = F13 + A - 6;
IF F35>5 THEN F35 = F13 + 2*A - 1 - 6; IF F35=-1 THEN F35 = F13 +
2*A=5;
IF F36>5 THEN F36 = F14 + A - 6;
IF F37>5 THEN F37 = F14 + 2*A - 1 - 6; IF F37=-1 THEN F37 = F14 +
2*A=5;
IF F38>5 THEN F38 = F15 + A - 6;
IF F39>5 THEN F39 = F15 + 2*A - 1 - 6; IF F39=-1 THEN F39 = F15 +
2*A=5;
IF F40>5 THEN F40 = F11 + F1 - 6; IF F40<0 THEN F40 = F11 + F1;
IF F41>5 THEN F41 = F11 + F2 - 6; IF F41<0 THEN F41 = F11 + F2;
IF F42>5 THEN F42 = F11 + F1A - 6; IF F42<0 THEN F42 = F11 + F1A;
IF F43>5 THEN F43 = F11 + F2A - 6; IF F43<0 THEN F43 = F11 + F2A;
IF F44>5 THEN F44 = F12 + F1 = 6; IF F44<0 THEN F44 = F12 + F1;
IF F45>5 THEN F45 = F12 + F2 - 6; IF F45<0 THEN F45 = F12 + F2;
IF F46>5 THEN F46 = F12 + F1A - 6; IF F46<0 THEN F46 = F12 + F1A;
IF F47>5 THEN F47 = F12 + F2A - 6; IF F47<0 THEN F47 = F12 + F2A;
IF F48>5 THEN F48 = F13 + F1 - 6; IF F48<0 THEN F48 = F13 + F1;
IF F49>5 THEN F49 = F13 + F2 - 6; IF F49<0 THEN F49 = F13 + F2;
IF F50>5 THEN F50 = F13 + F1A - 6; IF F50<0 THEN F50 = F13 + F1A;
IF F51>5 THEN F51 = F13 + F2A - 6; IF F51<0 THEN F51 = F13 + F2A;
IF F52>5 THEN F52 = F14 + F1 - 6; IF F52<0 THEN F52 = F14 + F1;
IF F53>5 THEN F53 = F14 + F2 - 6; IF F53<0 THEN F53 = F14 + F2;
IF F54>5 THEN F54 = F14 + F1A - 6; IF F54<0 THEN F54 = F14 + F1A;
IF F55>5 THEN F55 = F14 + F2A - 6; IF F55<0 THEN F55 = F14 + F2A;
IF F56>5 THEN F56 = F15 + F1 - 6; IF F56<0 THEN F56 = F15 + F1;
IF F57>5 THEN F57 = F15 + F2 - 6; IF F57<0 THEN F57 = F15 + F2;
IF F58>5 THEN F58 = F15 + F1A - 6; IF F58<0 THEN F58 = F15 + F1A;
IF F59>5 THEN F59 = F15 + F2A - 6; IF F59<0 THEN F59 = F15 + F2A;

DATALINES;
10  1  1  0  0  0  0    0  0  0  0  0
12  1  2  0  0  0  1    1  1  1  1  1
13  1  3  0  0  0  2    2  2  2  2  2
14  1  4  0  0  0  3    3  3  3  3  3
15  1  5  0  0  0  4    4  4  4  4  4
16  1  6  0  0  0  5    5  5  5  5  5
17  1  7  0  0  1  0    0  0  0  0  0
```

18	1	8	0	0	1	1	1	1	1	1	1
19	1	9	0	0	1	2	2	2	2	2	2
20	1	10	0	0	1	3	3	3	3	3	3
21	1	11	0	0	1	4	4	4	4	4	4
22	1	12	0	0	1	5	5	5	5	5	5
23	1	13	0	0	2	0	0	0	0	0	0
24	1	14	0	0	2	1	1	1	1	1	1
25	1	15	0	0	2	2	2	2	2	2	2
26	1	16	0	0	2	3	3	3	3	3	3
27	1	17	0	0	2	4	4	4	4	4	4
28	1	18	0	0	2	5	5	5	5	5	5
29	2	1	0	1	0	0	1	5	4	3	2
28	2	2	0	1	0	1	2	0	5	4	3
27	2	3	0	1	0	2	3	1	0	5	4
26	2	4	0	1	0	3	4	2	1	0	5
25	2	5	0	1	0	4	5	3	2	1	0
24	2	6	0	1	0	5	0	4	3	2	1
23	2	7	0	1	1	0	1	5	4	3	2
22	2	8	0	1	1	1	2	0	5	4	3
21	2	9	0	1	1	2	3	1	0	5	4
20	2	10	0	1	1	3	4	2	1	0	5
19	2	11	0	1	1	4	5	3	2	1	0
18	2	12	0	1	1	5	0	4	3	2	1
17	2	13	0	1	2	0	1	5	4	3	2
16	2	14	0	1	2	1	2	0	5	4	3
15	2	15	0	1	2	2	3	1	0	5	4
14	2	16	0	1	2	3	4	2	1	0	5
13	2	17	0	1	2	4	5	3	2	1	0
12	2	18	0	1	2	5	0	4	3	2	1
11	3	1	0	2	0	0	2	1	5	4	3
12	3	2	0	2	0	1	3	2	0	5	4
13	3	3	0	2	0	2	4	3	1	0	5
14	3	4	0	2	0	3	5	4	2	1	0
15	3	5	0	2	0	4	0	5	3	2	1
16	3	6	0	2	0	5	1	0	4	3	2
17	3	7	0	2	1	0	2	1	5	4	3
18	3	8	0	2	1	1	3	2	0	5	4
19	3	9	0	2	1	2	4	3	1	0	5
20	3	10	0	2	1	3	5	4	2	1	0
21	3	11	0	2	1	4	0	5	3	2	1
22	3	12	0	2	1	5	1	0	4	3	2
23	3	13	0	2	2	0	2	1	5	4	3
24	3	14	0	2	2	1	3	2	0	5	4
25	3	15	0	2	2	2	4	3	1	0	5
26	3	16	0	2	2	3	5	4	2	1	0
27	3	17	0	2	2	4	0	5	3	2	1
28	3	18	0	2	2	5	1	0	4	3	2
29	4	1	0	3	0	0	3	2	1	5	4
28	4	2	0	3	0	1	4	3	2	0	5
27	4	3	0	3	0	2	5	4	3	1	0
26	4	4	0	3	0	3	0	5	4	2	1
25	4	5	0	3	0	4	1	0	5	3	2
24	4	6	0	3	0	5	2	1	0	4	3
23	4	7	0	3	1	0	3	2	1	5	4
22	4	8	0	3	1	1	4	3	2	0	5
21	4	9	0	3	1	2	5	4	3	1	0
20	4	10	0	3	1	3	0	5	4	2	1

19	4	11	0	3	1	4	1	0	5	3	2
18	4	12	0	3	1	5	2	1	0	4	3
17	4	13	0	3	2	0	3	2	1	5	4
16	4	14	0	3	2	1	4	3	2	0	5
15	4	15	0	3	2	2	5	4	3	1	0
14	4	16	0	3	2	3	0	5	4	2	1
13	4	17	0	3	2	4	1	0	5	3	2
12	4	18	0	3	2	5	2	1	0	4	3
11	5	1	0	4	0	0	4	3	2	1	5
12	5	2	0	4	0	1	5	4	3	2	0
13	5	3	0	4	0	2	0	5	4	3	1
14	5	4	0	4	0	3	1	0	5	4	2
15	5	5	0	4	0	4	2	1	0	5	3
16	5	6	0	4	0	5	3	2	1	0	4
17	5	7	0	4	1	0	4	3	2	1	5
18	5	8	0	4	1	1	5	4	3	2	0
19	5	9	0	4	1	2	0	5	4	3	1
20	5	10	0	4	1	3	1	0	5	4	2
21	5	11	0	4	1	4	2	1	0	5	3
22	5	12	0	4	1	5	3	2	1	0	4
23	5	13	0	4	2	0	4	3	2	1	5
24	5	14	0	4	2	1	5	4	3	2	0
25	5	15	0	4	2	2	0	5	4	3	1
26	5	16	0	4	2	3	1	0	5	4	2
27	5	17	0	4	2	4	2	1	0	5	3
28	5	18	0	4	2	5	3	2	1	0	4
29	6	1	0	5	0	0	5	4	3	2	1
28	6	2	0	5	0	1	0	5	4	3	2
27	6	3	0	5	0	2	1	0	5	4	3
26	6	4	0	5	0	3	2	1	0	5	4
25	6	5	0	5	0	4	3	2	1	0	5
24	6	6	0	5	0	5	4	3	2	1	0
23	6	7	0	5	1	0	5	4	3	2	1
22	6	8	0	5	1	1	0	5	4	3	2
21	6	9	0	5	1	2	1	0	5	4	3
20	6	10	0	5	1	3	2	1	0	5	4
19	6	11	0	5	1	4	3	2	1	0	5
18	6	12	0	5	1	5	4	3	2	1	0
17	6	13	0	5	2	0	5	4	3	2	1
16	6	14	0	5	2	1	0	5	4	3	2
15	6	15	0	5	2	2	1	0	5	4	3
14	6	16	0	5	2	3	2	1	0	5	4
13	6	17	0	5	2	4	3	2	1	0	5
12	6	18	0	5	2	5	4	3	2	1	0
11	7	1	1	0	0	0	0	0	0	0	0
12	7	2	1	0	0	1	1	1	1	1	1
13	7	3	1	0	0	2	2	2	2	2	2
14	7	4	1	0	0	3	3	3	3	3	3
15	7	5	1	0	0	4	4	4	4	4	4
16	7	6	1	0	0	5	5	5	5	5	5
17	7	7	1	0	1	0	0	0	0	0	0
18	7	8	1	0	1	1	1	1	1	1	1
19	7	9	1	0	1	2	2	2	2	2	2
20	7	10	1	0	1	3	3	3	3	3	3
21	7	11	1	0	1	4	4	4	4	4	4
22	7	12	1	0	1	5	5	5	5	5	5
23	7	13	1	0	2	0	0	0	0	0	0

24	7	14	1	0	2	1	1	1	1	1	1
25	7	15	1	0	2	2	2	2	2	2	2
26	7	16	1	0	2	3	3	3	3	3	3
27	7	17	1	0	2	4	4	4	4	4	4
28	7	18	1	0	2	5	5	5	5	5	5
29	8	1	1	1	0	0	1	5	4	3	2
28	8	2	1	1	0	1	2	0	5	4	3
27	8	3	1	1	0	2	3	1	0	5	4
26	8	4	1	1	0	3	4	2	1	0	5
25	8	5	1	1	0	4	5	3	2	1	0
24	8	6	1	1	0	5	0	4	3	2	1
23	8	7	1	1	1	0	1	5	4	3	2
22	8	8	1	1	1	1	2	0	5	4	3
21	8	9	1	1	1	2	3	1	0	5	4
20	8	10	1	1	1	3	4	2	1	0	5
19	8	11	1	1	1	4	5	3	2	1	0
18	8	12	1	1	1	5	0	4	3	2	1
17	8	13	1	1	2	0	1	5	4	3	2
16	8	14	1	1	2	1	2	0	5	4	3
15	8	15	1	1	2	2	3	1	0	5	4
14	8	16	1	1	2	3	4	2	1	0	5
13	8	17	1	1	2	4	5	3	2	1	0
12	8	18	1	1	2	5	0	4	3	2	1
11	9	1	1	2	0	0	2	1	5	4	3
12	9	2	1	2	0	1	3	2	0	5	4
13	9	3	1	2	0	2	4	3	1	0	5
14	9	4	1	2	0	3	5	4	2	1	0
15	9	5	1	2	0	4	0	5	3	2	1
16	9	6	1	2	0	5	1	0	4	3	2
17	9	7	1	2	1	0	2	1	5	4	3
18	9	8	1	2	1	1	3	2	0	5	4
19	9	9	1	2	1	2	4	3	1	0	5
20	9	10	1	2	1	3	5	4	2	1	0
21	9	11	1	2	1	4	0	5	3	2	1
22	9	12	1	2	1	5	1	0	4	3	2
23	9	13	1	2	2	0	2	1	5	4	3
24	9	14	1	2	2	1	3	2	0	5	4
25	9	15	1	2	2	2	4	3	1	0	5
26	9	16	1	2	2	3	5	4	2	1	0
27	9	17	1	2	2	4	0	5	3	2	1
28	9	18	1	2	2	5	1	0	4	3	2
29	10	1	1	3	0	0	3	2	1	5	4
28	10	2	1	3	0	1	4	3	2	0	5
27	10	3	1	3	0	2	5	4	3	1	0
26	10	4	1	3	0	3	0	5	4	2	1
25	10	5	1	3	0	4	1	0	5	3	2
24	10	6	1	3	0	5	2	1	0	4	3
23	10	7	1	3	1	0	3	2	1	5	4
22	10	8	1	3	1	1	4	3	2	0	5
21	10	9	1	3	1	2	5	4	3	1	0
20	10	10	1	3	1	3	0	5	4	2	1
19	10	11	1	3	1	4	1	0	5	3	2
18	10	12	1	3	1	5	2	1	0	4	3
17	10	13	1	3	2	0	3	2	1	5	4
16	10	14	1	3	2	1	4	3	2	0	5
15	10	15	1	3	2	2	5	4	3	1	0
14	10	16	1	3	2	3	0	5	4	2	1

13	10	17	1	3	2	4	1	0	5	3	2
12	10	18	1	3	2	5	2	1	0	4	3
11	11	1	1	4	0	0	4	3	2	1	5
12	11	2	1	4	0	1	5	4	3	2	0
13	11	3	1	4	0	2	0	5	4	3	1
14	11	4	1	4	0	3	1	0	5	4	2
15	11	5	1	4	0	4	2	1	0	5	3
16	11	6	1	4	0	5	3	2	1	0	4
17	11	7	1	4	1	0	4	3	2	1	5
18	11	8	1	4	1	1	5	4	3	2	0
19	11	9	1	4	1	2	0	5	4	3	1
20	11	10	1	4	1	3	1	0	5	4	2
21	11	11	1	4	1	4	2	1	0	5	3
22	11	12	1	4	1	5	3	2	1	0	4
23	11	13	1	4	2	0	4	3	2	1	5
24	11	14	1	4	2	1	5	4	3	2	0
25	11	15	1	4	2	2	0	5	4	3	1
26	11	16	1	4	2	3	1	0	5	4	2
27	11	17	1	4	2	4	2	1	0	5	3
28	11	18	1	4	2	5	3	2	1	0	4
29	12	1	1	5	0	0	5	4	3	2	1
28	12	2	1	5	0	1	0	5	4	3	2
27	12	3	1	5	0	2	1	0	5	4	3
26	12	4	1	5	0	3	2	1	0	5	4
25	12	5	1	5	0	4	3	2	1	0	5
24	12	6	1	5	0	5	4	3	2	1	0
23	12	7	1	5	1	0	5	4	3	2	1
22	12	8	1	5	1	1	0	5	4	3	2
21	12	9	1	5	1	2	1	0	5	4	3
20	12	10	1	5	1	3	2	1	0	5	4
19	12	11	1	5	1	4	3	2	1	0	5
18	12	12	1	5	1	5	4	3	2	1	0
17	12	13	1	5	2	0	5	4	3	2	1
16	12	14	1	5	2	1	0	5	4	3	2
15	12	15	1	5	2	2	1	0	5	4	3
14	12	16	1	5	2	3	2	1	0	5	4
13	12	17	1	5	2	4	3	2	1	0	5
12	12	18	1	5	2	5	4	3	2	1	0
11	13	1	2	0	0	0	0	0	0	0	0
12	13	2	2	0	0	1	1	1	1	1	1
13	13	3	2	0	0	2	2	2	2	2	2
14	13	4	2	0	0	3	3	3	3	3	3
15	13	5	2	0	0	4	4	4	4	4	4
16	13	6	2	0	0	5	5	5	5	5	5
17	13	7	2	0	1	0	0	0	0	0	0
18	13	8	2	0	1	1	1	1	1	1	1
19	13	9	2	0	1	2	2	2	2	2	2
20	13	10	2	0	1	3	3	3	3	3	3
21	13	11	2	0	1	4	4	4	4	4	4
22	13	12	2	0	1	5	5	5	5	5	5
23	13	13	2	0	2	0	0	0	0	0	0
24	13	14	2	0	2	1	1	1	1	1	1
25	13	15	2	0	2	2	2	2	2	2	2
26	13	16	2	0	2	3	3	3	3	3	3
27	13	17	2	0	2	4	4	4	4	4	4
28	13	18	2	0	2	5	5	5	5	5	5
29	14	1	2	1	0	0	1	5	4	3	2

28	14	2	2	1	0	1	2	0	5	4	3
27	14	3	2	1	0	2	3	1	0	5	4
26	14	4	2	1	0	3	4	2	1	0	5
25	14	5	2	1	0	4	5	3	2	1	0
24	14	6	2	1	0	5	0	4	3	2	1
19	14	7	2	1	1	0	1	5	4	3	2
22	14	8	2	1	1	1	2	0	5	4	3
21	14	9	2	1	1	2	3	1	0	5	4
20	14	10	2	1	1	3	4	2	1	0	5
19	14	11	2	1	1	4	5	3	2	1	0
18	14	12	2	1	1	5	0	4	3	2	1
17	14	13	2	1	2	0	1	5	4	3	2
16	14	14	2	1	2	1	2	0	5	4	3
15	14	15	2	1	2	2	3	1	0	5	4
14	14	16	2	1	2	3	4	2	1	0	5
13	14	17	2	1	2	4	5	3	2	1	0
12	14	18	2	1	2	5	0	4	3	2	1
11	15	1	2	2	0	0	2	1	5	4	3
12	15	2	2	2	0	1	3	2	0	5	4
13	15	3	2	2	0	2	4	3	1	0	5
14	15	4	2	2	0	3	5	4	2	1	0
15	15	5	2	2	0	4	0	5	3	2	1
16	15	6	2	2	0	5	1	0	4	3	2
17	15	7	2	2	1	0	2	1	5	4	3
18	15	8	2	2	1	1	3	2	0	5	4
19	15	9	2	2	1	2	4	3	1	0	5
20	15	10	2	2	1	3	5	4	2	1	0
21	15	11	2	2	1	4	0	5	3	2	1
22	15	12	2	2	1	5	1	0	4	3	2
23	15	13	2	2	2	0	2	1	5	4	3
24	15	14	2	2	2	1	3	2	0	5	4
25	15	15	2	2	2	2	4	3	1	0	5
26	15	16	2	2	2	3	5	4	2	1	0
27	15	17	2	2	2	4	0	5	3	2	1
28	15	18	2	2	2	5	1	0	4	3	2
29	16	1	2	3	0	0	3	2	1	5	4
28	16	2	2	3	0	1	4	3	2	0	5
27	16	3	2	3	0	2	5	4	3	1	0
26	16	4	2	3	0	3	0	5	4	2	1
25	16	5	2	3	0	4	1	0	5	3	2
24	16	6	2	3	0	5	2	1	0	4	3
23	16	7	2	3	1	0	3	2	1	5	4
22	16	8	2	3	1	1	4	3	2	0	5
21	16	9	2	3	1	2	5	4	3	1	0
20	16	10	2	3	1	3	0	5	4	2	1
19	16	11	2	3	1	4	1	0	5	3	2
18	16	12	2	3	1	5	2	1	0	4	3
17	16	13	2	3	2	0	3	2	1	5	4
16	16	14	2	3	2	1	4	3	2	0	5
15	16	15	2	3	2	2	5	4	3	1	0
14	16	16	2	3	2	3	0	5	4	2	1
13	16	17	2	3	2	4	1	0	5	3	2
12	16	18	2	3	2	5	2	1	0	4	3
11	17	1	2	4	0	0	4	3	2	1	5
12	17	2	2	4	0	1	5	4	3	2	0
13	17	3	2	4	0	2	0	5	4	3	1
14	17	4	2	4	0	3	1	0	5	4	2

```
15 17  5  2  4  0  4    2  1  0  5  3
16 17  6  2  4  0  5    3  2  1  0  4
16 17  7  2  4  1  0    4  3  2  1  5
18 17  8  2  4  1  1    5  4  3  2  0
19 17  9  2  4  1  2    0  5  4  3  1
20 17 10  2  4  1  3    1  0  5  4  2
21 17 11  2  4  1  4    2  1  0  5  3
22 17 12  2  4  1  5    3  2  1  0  4
23 17 13  2  4  2  0    4  3  2  1  5
24 17 14  2  4  2  1    5  4  3  2  0
25 17 15  2  4  2  2    0  5  4  3  1
26 17 16  2  4  2  3    1  0  5  4  2
27 17 17  2  4  2  4    2  1  0  5  3
28 17 18  2  4  2  5    3  2  1  0  4
29 18  1  2  5  0  0    5  4  3  2  1
28 18  2  2  5  0  1    0  5  4  3  2
27 18  3  2  5  0  2    1  0  5  4  3
26 18  4  2  5  0  3    2  1  0  5  4
25 18  5  2  5  0  4    3  2  1  0  5
24 18  6  2  5  0  5    4  3  2  1  0
23 18  7  2  5  1  0    5  4  3  2  1
22 18  8  2  5  1  1    0  5  4  3  2
21 18  9  2  5  1  2    1  0  5  4  3
20 18 10  2  5  1  3    2  1  0  5  4
19 18 11  2  5  1  4    3  2  1  0  5
18 18 12  2  5  1  5    4  3  2  1  0
17 18 13  2  5  2  0    5  4  3  2  1
16 18 14  2  5  2  1    0  5  4  3  2
15 18 15  2  5  2  2    1  0  5  4  3
14 18 16  2  5  2  3    2  1  0  5  4
13 18 17  2  5  2  4    3  2  1  0  5
12 18 18  2  5  2  5    4  3  2  1  0
 ;
PROC  GLM DATA = FSS3X6;
CLASS ROW COL A B C D;
MODEL Y = ROW COL A*C A*D A*C*D B*C B*D B*C*D A*B*C A*B*D A*B*C*D;
RUN;
PROC GLM DATA = FSS3X6;
CLASS ROW COL A  B  C  D F1 F2 F3 F4 F5 F6 F7 F8 F9 F10 F11 F12 F13 F14
F15
F16 F17 F18 F19 F20 F21 F22 F23 F24 F25 F26 F27 F28 F29 F30 F31 F32 F33
F34 F35 F36
F37 F38 F39 F40 F41 F42 F43 F44 F45 F46 F47 F48 F49 F50 F51 F52 F53 F54
F55 F56 F57
F58 F59;
MODEL Y = ROW COL F1 F2 F3 F4 F5 F6 F7 F8 F9 F10 F11 F12 F13 F14 F15
F16 F17 F18 F19 F20 F21 F22 F23 F24 F25 F26 F27 F28 F29 F30 F31 F32 F33
F34 F35 F36
F37 F38 F39 F40 F41 F42 F43 F44 F45 F46 F47 F48 F49 F50 F51 F52 F53 F54
F55 F56 F57
F58 F59 ;
RUN;
```

The edited output from the above code is given below:

Class Level Information

Class	Levels	Values
ROW	18	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
COL	18	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
A	3	0 1 2
B	6	0 1 2 3 4 5
C	3	0 1 2
D	6	0 1 2 3 4 5
F11	6	0 1 2 3 4 5
F12	6	0 1 2 3 4 5
F13	6	0 1 2 3 4 5
F14	6	0 1 2 3 4 5
F15	6	0 1 2 3 4 5

Number of observations 324

Dependent Variable: Y

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	323	8819.888889	27.306158	.	.
Error	0	0.000000	.		
Corrected Total	323	8819.888889			

R-Square	Coeff Var	Root MSE	Y Mean
1.000000	.	.	19.98148

Source	DF	Type I SS	Mean Square	F Value	Pr > F
ROW	17	79.888889	4.699346	.	.
COL	17	1.333333	0.078431	.	.
A*C	4	0.351852	0.087963	.	.
A*D	10	0.648148	0.064815	.	.
A*C*D	20	1.759259	0.087963	.	.
B*C	10	7788.537037	778.853704	.	.
B*D	25	936.111111	37.444444	.	.
B*C*D	50	2.685185	0.053704	.	.
A*B*C	20	0.981481	0.049074	.	.
A*B*D	50	2.685185	0.053704	.	.
A*B*C*D	100	4.907407	0.049074	.	.

Source	DF	Type III SS	Mean Square	F Value	Pr > F
ROW	0	0.000000	.	.	.
COL	0	0.000000	.	.	.
A*C	4	0.351852	0.087963	.	.
A*D	10	0.648148	0.064815	.	.
A*C*D	20	1.759259	0.087963	.	.
B*C	10	7788.537037	778.853704	.	.
B*D	25	936.111111	37.444444	.	.
B*C*D	50	2.685185	0.053704	.	.
A*B*C	20	0.981481	0.049074	.	.
A*B*D	50	2.685185	0.053704	.	.
A*B*C*D	100	4.907407	0.049074	.	.

Class Level Information

Class	Levels	Values
ROW	18	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
COL	18	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
A	3	0 1 2
B	6	0 1 2 3 4 5
C	3	0 1 2
D	6	0 1 2 3 4 5
F1	3	0 1 2
F2	3	0 1 2
F3	6	0 1 2 3 4 5
F4	6	0 1 2 3 4 5
F5	6	0 1 2 3 4 5
F6	6	0 1 2 3 4 5
F7	6	0 1 2 3 4 5
F8	6	0 1 2 3 4 5
F9	6	0 1 2 3 4 5
F10	6	0 1 2 3 4 5
F11	6	0 1 2 3 4 5
F12	6	0 1 2 3 4 5
F13	6	0 1 2 3 4 5
F14	6	0 1 2 3 4 5
F15	6	0 1 2 3 4 5
F16	6	0 1 2 3 4 5
F17	6	0 1 2 3 4 5

F18	6	0 1 2 3 4 5
F19	6	0 1 2 3 4 5
F20	6	0 1 2 3 4 5
F21	6	0 1 2 3 4 5
F22	6	0 1 2 3 4 5
F23	6	0 1 2 3 4 5
F24	6	0 1 2 3 4 5
F25	6	0 1 2 3 4 5
F26	6	0 1 2 3 4 5
F27	6	0 1 2 3 4 5
F28	6	0 1 2 3 4 5
F29	6	0 1 2 3 4 5
F30	6	0 1 2 3 4 5
F31	6	0 1 2 3 4 5
F32	6	0 1 2 3 4 5
F33	6	0 1 2 3 4 5
F34	6	0 1 2 3 4 5
F35	6	0 1 2 3 4 5
F36	6	0 1 2 3 4 5
F37	6	0 1 2 3 4 5
F38	6	0 1 2 3 4 5
F39	6	0 1 2 3 4 5
F40	6	0 1 2 3 4 5
F41	6	0 1 2 3 4 5
F42	6	0 1 2 3 4 5
F43	6	0 1 2 3 4 5
F44	6	0 1 2 3 4 5
F45	6	0 1 2 3 4 5
F46	6	0 1 2 3 4 5
F47	6	0 1 2 3 4 5
F48	6	0 1 2 3 4 5
F49	6	0 1 2 3 4 5
F50	6	0 1 2 3 4 5
F51	6	0 1 2 3 4 5
F52	6	0 1 2 3 4 5
F53	6	0 1 2 3 4 5
F54	6	0 1 2 3 4 5
F55	6	0 1 2 3 4 5
F56	6	0 1 2 3 4 5
F57	6	0 1 2 3 4 5
F58	6	0 1 2 3 4 5
F59	6	0 1 2 3 4 5

Number of observations 324

Dependent Variable: Y

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	323	8819.888889	27.306158	.	.
Error	0	0.000000	.		
Corrected Total	323	8819.888889			

R-Square	Coeff Var	Root MSE	Y Mean
1.000000	.	.	19.98148

Source	DF	Type I SS	Mean Square	F Value	Pr > F
ROW	17	79.888889	4.699346	.	.
COL	17	1.333333	0.078431	.	.
F1	2	0.222222	0.111111	.	.
F2	2	0.129630	0.064815	.	.
F3	5	0.329630	0.065926	.	.
F4	5	0.318519	0.063704	.	.
F5	5	0.307051	0.061410	.	.
F6	5	0.340807	0.068161	.	.
F7	5	0.302157	0.060431	.	.
F8	5	0.809244	0.161849	.	.
F9	5	0.206481	0.041296	.	.
F10	5	7788.330556	1557.666111	.	.
F11	5	79.222222	15.844444	.	.
F12	5	316.967873	63.393575	.	.
F13	5	110.172298	22.034460	.	.
F14	5	224.356602	44.871320	.	.
F15	5	205.392116	41.078423	.	.
F16	5	0.206481	0.041296	.	.
F17	5	0.330556	0.066111	.	.
F18	5	0.726375	0.145275	.	.
F19	5	0.863131	0.172626	.	.
F20	5	0.183027	0.036605	.	.
F21	5	0.149974	0.029995	.	.
F22	5	0.046954	0.009391	.	.

F23	5	0.111067	0.022213	.	.
F24	5	0.032534	0.006507	.	.
F25	5	0.035087	0.007017	.	.
F26	5	0.208672	0.041734	.	.
F27	5	0.148038	0.029608	.	.
F28	5	0.361148	0.072230	.	.
F29	5	0.263623	0.052725	.	.
F30	5	0.307407	0.061481	.	.
F31	5	0.229630	0.045926	.	.
F32	5	0.844992	0.168998	.	.
F33	5	0.744515	0.148903	.	.
F34	5	0.158997	0.031799	.	.
F35	5	0.174004	0.034801	.	.
F36	5	0.093332	0.018666	.	.
F37	5	0.064689	0.012938	.	.
F38	5	0.032309	0.006462	.	.
F39	5	0.035311	0.007062	.	.
F40	5	0.208672	0.041734	.	.
F41	5	0.148038	0.029608	.	.
F42	5	0.330583	0.066117	.	.
F43	5	0.294189	0.058838	.	.
F44	5	0.195896	0.039179	.	.
F45	5	0.561923	0.112385	.	.
F46	5	0.930581	0.186116	.	.
F47	5	0.624732	0.124946	.	.
F48	5	0.155110	0.031022	.	.
F49	5	0.239273	0.047855	.	.
F50	5	0.227258	0.045452	.	.
F51	5	0.344999	0.069000	.	.
F52	5	0.091145	0.018229	.	.
F53	5	0.110751	0.022150	.	.
F54	5	0.089807	0.017961	.	.
F55	5	0.169266	0.033853	.	.
F56	5	0.035751	0.007150	.	.
F57	5	0.038639	0.007728	.	.
F58	5	0.039353	0.007871	.	.
F59	5	0.071442	0.014288	.	.

Source	DF	Type III SS	Mean Square	F Value	Pr > F
ROW	17	4399.814289	258.812605	.	.
COL	17	342.568840	20.151108	.	.
F1	2	0.223849	0.111924	.	.
F2	2	0.040319	0.020159	.	.
F3	5	0.663102	0.132620	.	.
F4	5	0.437482	0.087496	.	.
F5	5	0.341153	0.068231	.	.
F6	5	0.175813	0.035163	.	.
F7	5	0.656978	0.131396	.	.
F8	5	0.809244	0.161849	.	.
F9	5	2543.079613	508.615923	.	.
F10	5	2703.084305	540.616861	.	.
F11	5	63.219438	12.643888	.	.
F12	5	35.825578	7.165116	.	.
F13	5	37.245415	7.449083	.	.
F14	5	35.836631	7.167326	.	.
F15	5	36.738961	7.347792	.	.
F16	5	0.261466	0.052293	.	.
F17	5	0.324456	0.064891	.	.
F18	5	0.311460	0.062292	.	.
F19	5	0.684223	0.136845	.	.
F20	5	0.069113	0.013823	.	.
F21	5	0.040491	0.008098	.	.
F22	5	0.034181	0.006836	.	.
F23	5	0.047457	0.009491	.	.
F24	5	0.022943	0.004589	.	.
F25	5	0.017783	0.003557	.	.
F26	5	0.129200	0.025840	.	.
F27	5	0.191028	0.038206	.	.
F28	5	0.333283	0.066657	.	.
F29	5	0.263623	0.052725	.	.
F30	5	1.002244	0.200449	.	.
F31	5	0.913583	0.182717	.	.
F32	5	0.483185	0.096637	.	.
F33	5	0.274809	0.054962	.	.
F34	5	0.160225	0.032045	.	.
F35	5	0.155085	0.031017	.	.
F36	5	0.035058	0.007012	.	.
F37	5	0.020488	0.004098	.	.
F38	5	0.061742	0.012348	.	.
F39	5	0.049500	0.009900	.	.
F40	5	0.431582	0.086316	.	.

F41	5	0.464104	0.092821	.	.
F42	5	1.061273	0.212255	.	.
F43	5	0.754258	0.150852	.	.
F44	5	0.162202	0.032440	.	.
F45	5	0.177185	0.035437	.	.
F46	5	0.596526	0.119305	.	.
F47	5	0.736056	0.147211	.	.
F48	5	0.126019	0.025204	.	.
F49	5	0.048535	0.009707	.	.
F50	5	0.111225	0.022245	.	.
F51	5	0.174807	0.034961	.	.
F52	5	0.023542	0.004708	.	.
F53	5	0.009067	0.001813	.	.
F54	5	0.041949	0.008390	.	.
F55	5	0.041842	0.008368	.	.
F56	5	0.043288	0.008658	.	.
F57	5	0.022963	0.004593	.	.
F58	5	0.061075	0.012215	.	.
F59	5	0.071442	0.014288	.	.

Note that the sum of the sums of squares for F1 and F2 is 0.351852 that is the same as for the interaction A × C, for F3 and F4 is 0.648148 that is the same as for the interaction A × D, for F5, F6, F7, and F8 is 1.759259 that is the same as for the A × C × D interaction, for F9 and F10 is 7,788.537037 that is the same as for the B × C interaction, etc. for all the other interactions associated with the F-squares. This demonstrates that this set of 59 F-squares is a sum of squares orthogonal set and is complete as all the degrees of freedom and sums of squares have been taken into account. The Error sum of squares and degrees of freedom are both zero.

If the set of 59 F-squares is desired, they may be obtained by inserting the following commands after the semicolon at the end of the data input and before the PROC GLM DATA = FSS3X6; statement:

```
PROC PRINT;
RUN;
```

LITERATURE CITED

Federer, W. T. (2004a). Complete sets of F-squares of order n. Utilitas Mathematica (November Issue).

Federer, W. T. (2004b). Extending the number of complete sets of SSOFSs. In the Technical Report Series of the Department of Biological Statistics and Computational Biology, Cornell University, Ithaca, New York 14853, July 2004.