The Effects of Mass Transportation During a Deliberate Release of Smallpox

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1 Abstract

Since the attack on the world trade center, in New York City on September 11th, the possibility of bio-terrorist attacks on major cities has received continuous attention. The deliberate release of smallpox, a deadly virus, still available in selected U.S. and Russian laboratories, is a source of great concern. Ring vaccination is the current official response policy in the event of smallpox deliberate release. Edward H. Kaplan, David L. Craft and, Lawrence M. Wein argue that mass vaccination is a more viable solution that would optimize resource usage and minimize the mortality rate. In this paper, we model the dynamics of a deliberate release of smallpox in an idealized model of public transportation system of a major city. The city is divided into n neighborhoods and stratified by the proportion of individuals who use public transportation. Two levels of mixing are introduced via within neighborhood and between neighborhood activity levels. Transmission between neighborhoods is driven by interactions in the mass transportation system under the assumption of proportionate mixing. We complement our theoretical work with an agent based model for
two populations of individuals with different proportions of subway and non-subway users. We explore the impact of Kaplan, Craft and, Wein vaccination policy on the spread of epidemics for both models and our initial results seem to support their recommendations.

2 Introduction

After the events of September 11th, 2001, researchers in mathematical biology, epidemiology, and other related disciplines have been concerned about the use of viruses as a tool to cause terror in the American community. This concept is known as Bioterrorism, which is defined as the deliberate release of biological agents and their consequences. The goal of this paper is to study the disease dynamics the follow a subway transportation attack in order to test a set of possible responses. The reaction of a population after or during the attack is key to control the situation. The attack could consist of an outbreak of a fatal disease such as ebola, smallpox, rubella, etc. In this paper we consider smallpox as a biological agent, which is spread mainly through the transportation system of a city, and study the efficiency of the Kaplan, Craft and Wein vaccination strategy.

The initial idea was to consider ebola because it is a very fatal disease that has caused up to 88% [11] of the infected population to die in a matter of 2 weeks. Even though ebola has such a high mortality rate, it does not seem as useful agent of biological terror, it is hard to transmit, because it is not airborne. Furthermore, ebola’s latent period is very short 2-4 days, which implies early detection. One needs to find a disease that could cause enough damage in a short amount of time and that it is easy to transmit. Smallpox is one of the most serious candidates. Even though smallpox was officially declared “eradicated” in 1980, there still exist two official repositories of smallpox virus in the world: the CDC in Atlanta and the Russian State Research Center of Virology and Biotechnology in Koltsovo, Novosibirsk. Those supplies are used for scientific research and vaccine development. Those two sources may not be the only two sources for the virus. It is known that the year that worldwide vaccination ceased, the Soviet government began growing and stockpiling large quantities of smallpox virus, specially adapted for use in bombs and missiles [12]. Thus, a deliberate release of smallpox is feasible. Even before September 11th, interest was rising in how prepared we are to face a bioterrorism attack. Now, the “unthinkable” has happened, and bolstering our smallpox vaccine supply has become a priority. There are currently about 50 million vaccine doses worldwide – with $300 \times 10^6$ the U.S. Experts say that even with an all-out manufacturing effort, it would take at least three years before there was sufficient supply to prevent an epidemic.[12]

If used as a biological weapon, smallpox represents a serious threat to civilian populations because of its case-fatality rate of 30 percent or more among unvaccinated persons and the absence of specific treatment. In a city like that of New York City
with a population size of about 8 million, and where there are about 4.3 million people take the subway weekly, it is easy to cause a fast outbreak [7]. Today smallpox has a far greater devastation potential, because routine vaccination throughout the United States ceased more than 25 years ago [9]. In a highly susceptible and mobile population like that of the US, smallpox is likely able to spread widely and rapidly. Smallpox is a viral communicable disease, that it is passed from person to person in a continuing chain of infection. It is spread by inhalation of air droplets or aerosols. This can occur whenever an individual is within a six feet radius from the infectious person. Twelve to fourteen days after infection, the patient typically becomes febrile and has severe aching pains and prostration. Some two to three days later, a papular rash develops over the face and spreads to the extremities. The rash soon becomes vesicular and later, pustular. The patient remains febrile throughout the evolution of the rash and customarily experiences considerable pain as the pustules grow and expand. Gradually, scabs form, which eventually separate, leaving pitted scars. Death usually occurs during the second week [8].

An agent-based model was used to compare mass vaccination vs. ring vaccination, and to determine the effects of transportation in a bio-terrorist attack. We divide the population into 4-neighborhoods to assimilate the transportation model. Mass vaccination and ring vaccination are introduced after a predetermined number of infected cases are reported. At time zero, 50 exposed individuals are introduced representing an initial bio-terrorist attack. We found that both vaccination strategies work better when the epidemic is detected early. Yet, it is not clear whether mass vaccination works better than ring vaccination. The difference in the resulting fatality cases with both vaccination methods is not sufficient to determine which method is better. This suggests that more detailed study is necessary before concluding what is the optimal vaccination method to implement. We noticed a non-surprising distribution of fatality cases per neighborhood. These are directly proportional to the percentage of SU per neighborhood.

3 The Model

The main purpose of this paper is to model smallpox transmission in a city with a widely used subway system, particularly when their is a deliberate released. There are many different options on how to model the contact dynamics of populations that use a transportation system. Here, we modify the model found in a paper “El transporte público y la dinámica de la tuberculosis a nivel poblacional” (The public transportation system and the dynamics of tuberculosis at a population level) [1]. A secondary objective of this project is to test various responses strategies to a deliberate release. Kaplan, Craft and Wein argues that mass vaccination is the best option because provides the best response for what defines as the worst case scenario. We hope to test his policy suggestion in a dynamic model.
In order to focus the discussion we use New York City as the basis for our project. The city is divided into \( n \) neighborhoods and within each neighborhood the population is subdivided into regular subway users (SU) and regular non-subway users (NSU). It is assumed that SU have contacts with both SU and NSU in their own neighborhood. SU may also have contact with other SU from different neighborhoods, but only when they shared a subway ride. Contacts between SU individuals from different neighborhoods outside the subway are considered negligible in the context of smallpox transmission. It is also assumed that NSU individuals have most of their contacts not only within their own neighborhood but also mostly with those living in the same neighborhood.

If a fixed number of infected individuals is introduced in the subway, then the first cases of infection would occur in the SU population. Newly infected individuals will then take the virus back to their own neighborhoods generating infections in the NSU and SU populations. Once the attack is recognized and smallpox is detected, vaccination starts. The model is constructed in a way that only a proportion of the population is vaccinated per day because the immediate vaccination of the whole population is impossible in one day. From a view point of a deliberate release, it will be assumed (later tested by simulations) that rapid spread is most likely if the disease is introduced in the subway.

Susceptibles that are vaccinated have 0.97 probability of moving into the recovered class and immunity is supposed to fast for about 5 years. Exposed individuals that are vaccinated within the first four days are vaccinated to smallpox can move into the recovered class with the same chance as the susceptible individuals. Otherwise they become infectious. There is not effective treatment for smallpox. Infected individuals have a chance of 70% of recovery [10].

The analysis of this model is complicated, and we have limited results. However we use numerical analysis to gain insight into the impact of smallpox epidemics in this system. We use matlab to run some simulations and an agent-based model which gave us an insight on different case scenarios.
Figure 1: Subway Users Model

Figure 2: Non-Subway Users Model
4 Model Equations

From the flow diagram Figure 1 we can put down the equations for subway users:

\[
\frac{dW_i}{dt} = \Lambda_i - V_i(t) - \left( \mu W_i + ql_1 \frac{W_i}{W_i + X_i} \right), \tag{1}
\]

\[
\frac{dX_i}{dt} = V_i(t) - \left( \mu X_i + \phi X_i + ql_2 \frac{X_i}{W_i + X_i} \right), \tag{2}
\]

\[
\frac{dY_i}{dt} = \phi X_i - (\mu + \alpha + d) Y_i, \tag{3}
\]

\[
\frac{dZ_i}{dt} = \alpha Y_i - \mu Z_i + ql_1 \frac{W_i}{W_i + X_i} + ql_2 \frac{X_i}{W_i + X_i}, \tag{4}
\]

\[
T_i(t) = W_i(t) + X_i(t) + Y_i(t) + Z_i(t). \tag{5}
\]

For SU: \(W(t)_i\) is the number of susceptibles at time \(t\), \(X(t)_i\) is the number of exposed at time \(t\), \(Y(t)_i\) is the number of infectious at time \(t\), and \(Z(t)_i\) is the number of recovered at time \(t\). This means that the total population of subway users is \(T(t)_i = W(t)_i + X(t)_i + Y(t)_i + Z(t)_i\).

From Figure 2 the non-subways users equations:

\[
\frac{dS_i}{dt} = A_i - B_i(t) - \left( \mu S_i + ql_1 \frac{S_i}{S_i + E_i} \right), \tag{6}
\]

\[
\frac{dE_i}{dt} = B_i(t) - \left( \mu E_i + \phi E_i + ql_2 \frac{E_i}{S_i + E_i} \right), \tag{7}
\]

\[
\frac{dI_i}{dt} = \phi E_i - (\mu + \alpha + d) I_i, \tag{8}
\]

\[
\frac{dR_i}{dt} = \alpha I_i - \mu R_i + ql_1 \frac{S_i}{S_i + E_i} + ql_2 \frac{E_i}{S_i + E_i}, \tag{9}
\]

\[
Q_i(t) = S_i(t) + E_i(t) + I_i(t) + R_i(t). \tag{10}
\]

For NSU: \(S_i(t)\) is the number of susceptibles at time \(t\), \(E(t)_i\) is the number of exposed at time \(t\), \(I(t)_i\) is the number of infectious at time \(t\), and \(R(t)_i\) is the number of recovered at time \(t\). This means that the total population of non-subway users is \(Q(t)_i = S(t)_i + E(t)_i + I(t)_i + R(t)_i\).

where \(i = 1, \ldots, n\) is the number of neighborhoods. Terms and parameters are defined in table 1.
5 Parameters of the model

The following table show the parameters and their meaning. We model interactions between individuals in the subway and non-subway environment using proportionate mixing.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_i(t)$</td>
<td>rate of incidence, new number of infections per unit of time for NSU</td>
</tr>
<tr>
<td>$V_i(t)$</td>
<td>rate of incidence, new number of infections per unit of time for SU</td>
</tr>
<tr>
<td>$\mu$</td>
<td>natural mortality rate</td>
</tr>
<tr>
<td>$d$</td>
<td>mortality rate due to smallpox</td>
</tr>
<tr>
<td>$q$</td>
<td>amount of people vaccinated at time $t$</td>
</tr>
<tr>
<td>$l_i$</td>
<td>vaccination efficacy rate</td>
</tr>
<tr>
<td>$\phi$</td>
<td>rate of infection</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>recovery rate for $I$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>is the rate at which a SU leaves the subway</td>
</tr>
<tr>
<td>$\rho$</td>
<td>is the rate at which a SU takes the subway</td>
</tr>
<tr>
<td>$a$</td>
<td>average number of contacts of the non-subway users per unit of time</td>
</tr>
<tr>
<td>$b$</td>
<td>average number of contacts of the subway users per unit of time</td>
</tr>
<tr>
<td>$\beta$</td>
<td>transmission rate</td>
</tr>
<tr>
<td>$\frac{1}{p}$</td>
<td>is the average time spent in the subway</td>
</tr>
<tr>
<td>$\frac{\sigma}{\sigma+\rho}$</td>
<td>is the proportion of time spent off the subway for a subway user</td>
</tr>
<tr>
<td>$\frac{\rho}{\sigma+\rho}$</td>
<td>is the proportion of time spent in the subway for a subway user</td>
</tr>
</tbody>
</table>

Table 1: Proportionate contacts and their definitions

The rate of infection for non users is

$$B_i(t) = \beta_i a_i S_i \left[ \frac{I_i}{T_i \left( \frac{\alpha_i}{\rho_i+\sigma_i} \right)} + \frac{Y_i \left( \frac{\alpha_i}{\rho_i+\sigma_i} \right)}{T_i \left( \frac{\alpha_i}{\rho_i+\sigma_i} \right)} \right], \quad (11)$$

and the rate of infection for subway users is

$$V_i(t) = \beta_i b_i W_i \left[ \frac{I_i}{T_i \left( \frac{\alpha_i}{\rho_i+\sigma_i} \right)} + \frac{Y_i \left( \frac{\alpha_i}{\rho_i+\sigma_i} \right)}{T_i \left( \frac{\alpha_i}{\rho_i+\sigma_i} \right)} + \sum_{j=1}^{N} \frac{Y_j \left( \frac{\rho_j}{\rho_j+\sigma_j} \right)}{T_j \left( \frac{\rho_j}{\rho_j+\sigma_j} \right)} \right], \quad (12)$$

where
\[
P_{a_{ai}} = \frac{a_i Q_i}{a_i Q_i + b_i \left( \frac{\sigma_i}{\rho_i + \sigma_i} \right) T_i},
\]
\[
P_{a_{bi}} = \frac{b_i \left( \frac{\sigma_i}{\rho_i + \sigma_i} \right) T_i}{a_i Q_i + b_i \left( \frac{\sigma_i}{\rho_i + \sigma_i} \right) T_i},
\]
\[
P_{b_{ai}} = \frac{a_i Q_i}{a_i Q_i + b_i \left( \frac{\sigma_i}{\rho_i + \sigma_i} \right) T_i},
\]
\[
P_{b_{bi}} = \frac{b_i \left( \frac{\sigma_i}{\rho_i + \sigma_i} \right) T_i}{a_i Q_i + b_i \left( \frac{\sigma_i}{\rho_i + \sigma_i} \right) T_i},
\]
\[
P_{b_{bj}} = \frac{b_j \left( \frac{\rho_j}{\rho_j + \sigma_j} \right) T_j}{\sum_{k=1}^{N} b_k \left( \frac{\rho_k}{\rho_k + \sigma_k} \right) T_k}.
\]

| \(P_{a_{ai}}\) | the proportionate mixing between non-subway user from neighborhood \(i\) and non-subway users from neighborhood \(i\) |
| \(P_{a_{bi}}\) | the proportionate mixing between non-subway user from neighborhood \(i\) and subway users from neighborhood \(i\) |
| \(P_{b_{ai}}\) | the proportionate mixing between subway user from neighborhood \(i\) and non-subway users from neighborhood \(i\) |
| \(P_{b_{bi}}\) | the proportionate mixing between subway user from neighborhood \(i\) and subway users from neighborhood \(i\) |
| \(\tilde{P}_{b_{bj}}\) | the proportionate mixing between subway user from neighborhood \(i\) and subway users from neighborhood \(j\) |
| \(P_{a_{aj}}\) | the proportionate mixing between non-subway user from neighborhood \(i\) and non-subway users from neighborhood \(j\) |
| \(P_{a_{bj}}\) | the proportionate mixing between non-subway user from neighborhood \(i\) and subway users from neighborhood \(j\) |
| \(P_{b_{ai}}\) | the proportionate mixing between subway user from neighborhood \(i\) and non-subway users from neighborhood \(j\) |
| \(P_{b_{bi}}\) | the proportionate mixing between subway user from neighborhood \(i\) and subway users from neighborhood \(j\) |

Notice that
\(P_{a_{ai}} = 0\) if \(i \neq j\) (individuals \(Q_i\) and \(Q_j\) do not have contact if \(i \neq j\)),
\(P_{a_{bj}} = 0\) if \(i \neq j\) (individuals \(Q_i\) and \(T_j\) do not have contact if \(i \neq j\)),
\(P_{a_{ai}} = \tilde{P}_{a_{ai}} > 0\) (individuals \(Q_i\) and \(Q_i\) have contact),
\(P_{a_{bi}} = \tilde{P}_{a_{bi}} > 0\) (individuals \(Q_i\) and \(T_i\) have contact outside subway),
\(P_{b_{ai}} = \tilde{P}_{a_{ai}} > 0\) (individuals \(T_i\) and \(Q_i\) have contact outside subway),
\(P_{b_{bi}} = \tilde{P}_{b_{bi}} > 0\) (individuals \(T_i\) and \(T_i\) have contact outside subway),
\(P_{b_{bj}} = \tilde{P}_{b_{bj}} > 0\) (individuals \(T_i\) and \(T_j\) have contact in the subway).
The following constraints apply
1) The sum of all conditional probabilities that individuals mix inside the subway is equal to 1
\[ \sum_{j=1}^{N} \tilde{\mathbb{P}}_{b_j} = 1 \]
2) The sum of conditional probabilities that individuals from \( Q_i \) have contacts with individuals from \( Q_i \) or individuals from \( T_i \) (outside the subway), in their neighborhood (\( i=1,2,...,N \)) satisfy
\[ \tilde{\mathbb{P}}_{a_i} + \tilde{\mathbb{P}}_{b_i} = 1 \]
\[ , \; i = 1,2,...,n \]
3) The sum of conditional probabilities that individuals from \( T_i \) have contacts with \( Q_i \) individuals or with \( T_i \) individuals off the subway, that is in their neighborhood satisfy
\[ \tilde{\mathbb{P}}_{a_i} + \tilde{\mathbb{P}}_{b_i} = 1 \]
\[ , \; i = 1,2,...,n. \]

6 Calculation of \( R_0 \) for one neighborhood

In order to determine the central behavior of the epidemic, we linearize the system at the disease free equilibrium. \( R_0 \) the number of secondary infections produced by an infected individual, is compute following [5] and [6] for one neighborhood. \( R_0 \) is found using the eigenvalues of the matrix \((\lambda I - MD^{-1})\) from our system of equations. The disease free equilibrium is:

\[ \left( \frac{\Lambda - ql_1}{\mu}, 0, 0, \frac{ql_1}{\mu}, \frac{\Lambda - ql_1}{\mu}, 0, 0, \frac{ql_1}{\mu} \right), \]

where \( \frac{1}{\mu} \) represent the average life span of the system and \( ql_1 \) represent the vaccine efficacy.

The Jacobian matrix evaluate at the disease free equilibrium is:
In this matrix we can see that we have four negative eigenvalue, hence the stability conditions must be computed from the 4 by 4 submatrix.

In other to obtain $R_0$ we need to find the largest eigenvalue from the matrix $(\lambda I - MD^{-1})$:

The characteristic equation for the determinant of $(\lambda I - MD^{-1})$ is a double quartic polynomial of the form $\lambda^4 - G\lambda^2 + H$, where:
Solving this equation for $\lambda$ positive we obtain:

$$\lambda = \left( \frac{1}{2} \left( G + \sqrt{G^2 - 4H} \right) \right)^{\frac{1}{2}}$$

Since this is the biggest eigenvalue we can define $R_0$:

$$R_0 = \left( \frac{1}{2} \left( G + \sqrt{G^2 - 4H} \right) \right)^{\frac{1}{2}} \quad (13)$$

7 $R_0$ For Two Neighborhoods

In order to analyze the $R_0$ for two neighborhoods (calculations are in the appendix) we assume that $q = 0$, $\rho_i = 0$ and $\sigma_i = 0$. For this two neighborhoods $R_0 = \max\{R_{01}, R_{02}\}$, where:

$$R_{0i} = \frac{\phi \beta_i [(a_i A_i)^2 + (b_i \Lambda_i)^2]}{(\mu + \phi)(\mu + \alpha + d)(a_i A_i + b_i \Lambda_i)(A_i + \Lambda_i)}$$

Hence $R_0$ can be rewritten as:

$$R_{0i} = \beta_i \left( \frac{\phi}{\mu + \phi} \right) \left( \frac{1}{\mu + \alpha + d} \right) \left[ a_i \left( \frac{a_i A_i}{a_i A_i + b_i \Lambda_i} \right) \left( \frac{\Lambda_i}{A_i + \Lambda_i} \right) + b_i \left( \frac{b_i A_i}{a_i A_i + b_i \Lambda_i} \right) \left( \frac{\Lambda_i}{A_i + \Lambda_i} \right) \right]$$

where:

$\frac{\phi}{\mu + \phi}$ : Proportion of individuals that goes from the exposed to the infected class.

$\frac{1}{\mu + \alpha + d}$ : Average infectious period.

$\frac{\Lambda_i}{A_i + \Lambda_i}$ : Proportion of non-subway users in neighborhood $i$. 
\[ \frac{\Delta_i}{\mu} \]: Proportion of subway users in neighborhood \( i \).

\[ \frac{\Delta_i}{\mu} \]: Proportion of contacts of non-subway users the total contacts in the neighborhood \( i \).

\[ \frac{\Delta_i}{\mu} \]: Proportion of contacts of subway users to the total contacts in the neighborhood \( i \).

\( R_0 \) has two sources, one from the non-subway users and the other from subway users.

8 Estimation of Parameters

The data was obtained from different organizations but mainly from the Center For Disease Control (CDC), and various websites which hold New York City Visitor Information.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( \mu )</th>
<th>( d )</th>
<th>( l_1 )</th>
<th>( l_2 )</th>
<th>( \phi )</th>
<th>( \alpha )</th>
<th>( \sigma )</th>
<th>( \rho )</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.0065</td>
<td>0.3</td>
<td>0.97</td>
<td>0.97</td>
<td>0.083</td>
<td>0.7</td>
<td>19-22 hrs</td>
<td>2-5 hrs</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Table 3: Estimation of Parameters

9 Simulations

9.1 Numerical Simulations

Since our model is an attempt to describe the spread of smallpox in a subway system from individuals of different neighborhoods in New York City, it is important to examine the behavior of the population as time progresses. A good approach to this is the use of computer simulations. In our project we use two different programs matlab, and Starlogo to write an agent based model. We found that the total population of NYC is 8 million. In matlab, we rescaled this population to about 8,000. According to the data we decide to make the time scale in days, with a period of 30 days. We consider two neighborhoods, where each has a NSU and a SU population. The first neighborhood deals with the 8,000 residents of NYC and the second neighborhood has 1000 individuals, which represent the tourist of NYC 1 million per day. In our programs we assign the values of the different parameters. We found that the parameters
that affect the model most are $q$, which is the amount of people vaccinated, $a$ which is average number of contacts of the non-subway users per unit of time and $b$, which is the average number of contacts of subway users per unit of time.

From Figure 3 we can see that with no vaccination, and low contact rates, smallpox is almost non-existant, especially for the NSU population. For both neighborhoods, the susceptibles almost constitute of the entire population. For the subway users, the exposed class increased and for the non-subway users it stayed almost constant. In the majority of the cases, the quantity of recovered individuals does not grow because there is no vaccination.

In the case where we have no vaccination, and higher contact rates (Figure 4), the number of susceptibles is low, and the number of exposed increases rapidly. The majority of the population becomes recovered, which is due to the fact that 30% of those who have died of smallpox. Thus, 70% survive, but depending on the population size, 30% could be a huge number, especially within a city of NYC.

Analyzing the case were the population gets vaccinated, and where we have low contact rates (Figure 5), we found that the population is divided into susceptibles and recovered. The amount of exposed and infected, for the non-subway users, is considered low and constant. However, for the subway users, the majority of the population is exposed. For the first neighborhood, the number of susceptibles is very high, but eventually decreases after the twentieth day for the SU. This shows that the virus propagates more quickly and easily in the population of SU. Having vaccination for smallpox, decreases the chances of having an uncontrolled epidemic.

In Figure 6, vaccination and higher contact rates are given. From these results, we found that even if the contact rates are high, the vaccination is effective. For both neighborhoods, there exists a low number of people infected with smallpox, and a low susceptible population. Almost everybody becomes immune, and the recovered class grows dramatically fast, due to vaccination. Finally, the exposed class grows rapidly, but after a short period of time, it reaches a steady state.

In general, we can conclude that vaccination is effective, even if the contact rates are high. It seems that vaccination is the most effective solution, but these simulation do not illustrate whether mass vaccination is better or worst than ring vaccination. Ring vaccination versus mass vaccination are more deeply discussed in the agent bases-model simulations.

In our second part of the numerical simulation we let $q = 0$ which means that we don’t have vaccination and, $\rho_1 = \rho_2$, $\sigma_1 = \sigma_2$, $\beta_1 = \beta_2$, for $R_0$ of two neighborhoods. If $b_i > a_i$ (figure 7), this is if the contact rate of the subway users is greater than the contact rate of non-subway users, the basic reproductive number $R_0(0)$ when $\rho = 0$ is greater than $R_0(\rho)$ when $\rho \neq 0$. In this case the terrorist have a big impact on the spread of smallpox. On the other hand if $b_i < a_i$, the terrorist have less activity (figure 8), then the basic reproductive number $R_0(0)$ when $\rho = 0$ is less than $R_0(\rho)$ when $\rho \neq 0$. 
Figure 3: Parameters Varying $q = 0, a_1 = 5, a_2 = 3, b_1 = 7, b_2 = 2$
Figure 4: Parameters Varying $q = 0, a_1 = 30, a_2 = 20, b_1 = 40, b_2 = 20$
Figure 5: Parameters Varying $q = 200, a_1 = 5, a_2 = 3, b_1 = 7, b_2 = 2$
Figure 6: Parameters Varying $q = 200, a_1 = 30, a_2 = 20, b_1 = 40, b_2 = 20$
Figure 7: Graphic of $R_0$ in function of $\rho$ when $a_i > b_i$
Figure 8: Graphic of $R_0$ in function of $\rho$ when $a_i < b_i$
9.2 Agent Based Model

An agent-based model was used to compare mass vaccination vs. ring vaccination, and to determine the effects of transportation in a bio-terrorist attack. In the case of a deliberate release of smallpox, the StarLogo software was used to write two codes to model the behavior of subway and non-subway users. The bio-terrorist attack consists of introducing 50 exposed individuals into a total population of 4000 individuals. We are interested in obtaining results that either verify or question Kaplan's conclusion that mass vaccination should be an alternative to the current ring vaccination policy. Both vaccination methods were implemented when a predetermined number of infected cases has been reported. The values assumed for this number are 50, 100 and 400.

In the case of two neighborhoods (fig.9), 90% of the population were assumed to be subway users in one neighborhood, and 44% in the other, representing the populations of tourists and locals respectively. Mass vaccination was found to be a better method than ring vaccination in the sense that it gets rid of the epidemic faster. Results suggest that if the attack is detected promptly, ring vaccination is a better method than mass vaccination, otherwise ring vaccination not only takes longer to control the epidemic, but also results in more fatalities. These apparently contradicting results question Kaplan's conclusion, since under different conditions and assumptions, both vaccination methods work effectively.

In the case of four neighborhoods (fig.10, fig.11), we consider the percentages for subway users per neighborhood to be 90%, 10%, 44%, and 70%. Again, mass vaccination and ring vaccination are introduced as control measures after a predetermined value of infected cases has been reported.

Similar to the two neighborhood case, both vaccination methods work well when the epidemic is detected early. We observe that the results are optimal when mass vaccination is implemented after 50 infected cases have been reported, yet the number of fatalities was higher than when ring vaccination was implemented. We attribute this to the fact that the vaccine is assumed to have a 1% chance of killing the individuals by its side-effects. It is not clear whether mass vaccination works better than ring vaccination. This suggests that more detailed study is necessary before concluding what is the best vaccination method to be used. If the epidemic is not detected early, the casualties double when ring vaccination is used instead of mass vaccination. Hence, in this case, mass vaccination is indeed a better method.

The percentage of subway users per neighborhood affects the amount of infection that takes place in each neighborhood. In general, in the case of four neighborhoods, we found the distribution of fatalities per neighborhood to be directly proportional to the percentage of SU per neighborhood (fig.11). This implies that transportation plays a major role in the spread of smallpox.
Figure 9: Graphs for 2-neighborhoods varying the population size to start mass vaccination or ring vaccination
Figure 10: Graphs for 4-neighborhoods varying the population size to start mass vaccination or ring vaccination
Figure 11: Graphs for 4-neighborhoods with population size to start vaccination is 50. Fatalities divided per neighborhood.
10 Results and Discussion

The results obtained from this paper, do not answer the initials questions completely, but do give an insight, and create more open questions. For instance, it is true that vaccination has a major impact on the spread of the virus, but which is the adequate vaccination strategy is still an open question. Ed Kaplan suggested that for the worst case scenario, mass vaccination is the most effective strategy. However, the agent based-model created for this problem, offers different results. For example, it is not so obvious that mass vaccination is more effective, and in some instances, ring vaccination seemed to be more effective. It all depends on different factors, such as the time that it takes to detect the attack. The reaction time is important because the faster the attack is detected, the faster people get vaccinated.

In the simulations, the time it takes to start vaccinating, depends on recognizing a certain number of infected individuals. The question becomes a different, and we ask, how many people need to be infected to announce an outbreak? Also, the number of individuals infected at time zero is very important. The amount of people that get vaccinated each day, can also have a different effect. The number of contacts for a NSU and a SU can make a huge difference because that number increases the chances of infection, and later, the chances of infecting susceptible individuals. Another major difference is the amount of time spent in the subway and outside the subway. The population of SU are infected more rapidly, and in this group, the epidemic spreads more quickly than that of the NSU. Depending on the different initial values, on the different parameters, and on the vaccination strategy, results vary. The problem is more complex, and depending on the different circumstances, either mass or ring vaccination can be effective.

There are some differences between Kaplan, Craft and Wein’s model and in this model, but that only shows that their result is incomplete. His assumption about the worst case scenario is not necessarily the worst because he did not incorporate many possible situations. There are many factors that have some impact on the final results, and in this case, transportation is a very important factor, because in the subway, individuals have a major potential of infection. There are many other circumstances that are not taken into consideration by them, and as a result, their conclusions are somewhat handicapped. Kaplan, Craft and Wein’s results are not incorrect, but there is a lot more work that can be done to ameliorate the quality of his results, which in turn will give a more holistic view in case of smallpox being deliberatly released.

11 Future Work

There are still many open questions that require more research. We need to provide a better interpretation of the $R_0$ for multiple neighborhoods. The next question will be
to try to find if there are multiple endemic equilibria, and analized their stability. It is possible to study the case of multiple neighborhoods via numerical simulations. The study of bifurcations is another further consideration. Another good approach will be to create a different transportation model that allows more space for flexibility; more subways, and allowing more interactions with the different neighborhoods. Finally, not many biological problems deal with systems of this type using proportionate mixing and further studies related with the topic should be considered.

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References


13 Appendix

13.1 Calculation of $R_0$ for two neighborhoods

We compute $R_0$ following the procedure suggested by Dieckman et al. 1990 [5] and Heesterbeek 1992 [6] only for two neighborhoods.

To reduce the calculation, the total take-subway population size is $T_i(t) = W_i(t) + X_i(t) + Y_i(t) + Z_i(t)$ which is governed by the differential equation $\frac{dT_i}{dt} = \Lambda_i - \mu T_i - dY_i$. The total non-take subway population size is $Q_i(t) = S_i(t) + E_i(t) + I_i(t) + R_i(t)$ which is governed by the differential equation $\frac{dQ_i}{dt} = A_i - \mu Q_i - dI_i$. We use $T_i$ and $Q_i$ as variables in place of variables $W_i$ and $S_i$, respectively. Then, the model becomes the following system.
Equations for subway users

\[
\begin{align*}
\frac{dX_i}{dt} &= V_i(t) - (\mu + \phi)X_i - \frac{q_1X_i}{T_i - Y_i - Z_i}, \\
\frac{dY_i}{dt} &= \phi X_i - (\mu + \phi + d)Y_i, \\
\frac{dZ_i}{dt} &= \alpha Y_i - \mu Z_i + q_1 + \frac{q_1(2-I_i)X_i}{T_i - Y_i - Z_i}, \\
\frac{dQ_i}{dt} &= \Lambda_i - \mu Q_i - dQ_i \\
\end{align*}
\]

Equations for non-subway users

\[
\begin{align*}
\frac{dE_i}{dt} &= B_i(t) - (\mu + \phi)E_i - \frac{q_3E_i}{Q_i - E_i - R_i}, \\
\frac{dI_i}{dt} &= \phi E_i - (\mu + \phi + d)I_i, \\
\frac{dQ_i}{dt} &= \alpha I_i - \mu Q_i + q_1 + \frac{q_1(2-\delta_i)E_i}{Q_i - E_i - R_i}, \\
\frac{dR_i}{dt} &= A_i - \mu Q_i - dQ_i \\
\end{align*}
\]

where

\[
V_i(t) = \beta_i b_i(T_i - X_i - Y_i - Z_i) \left( \frac{a_i Q_i I_i + b_i \delta_i^2 T_i Y_i}{G_i} + \sum_{i=1}^{2} b_i \theta_i Y_i \right),
\]

\[
B_i(t) = \beta_i a_i(Q_i - E_i - I_i - R_i) \left( \frac{a_i Q_i I_i + b_i \delta_i^2 T_i Y_i}{G_i} \right),
\]

and \( \delta_i = \frac{\sigma_i}{\sigma_i + \mu}, \quad \theta_i = \frac{\epsilon_i}{\epsilon_i + \mu}, \quad G_i = (a_i Q_i + b_i \delta_i T_i)(Q_i + \delta_i T_i), \quad H = \sum_{i=1}^{2} b_i \theta_i T_i. \)

The disease-free equilibrium (DFE) of our model is \( P_0 = (X_{00}, Y_{00}, Z_{00}, E_{00}, I_{00}, R_{00}, T_{00}, Q_{00}) \),

where

\[
\begin{align*}
X_{00} &= Y_{00} = 0, \quad Z_{00} = \frac{q_1}{\mu}, \quad T_{00} = \frac{\Lambda_i}{\mu}; \\
E_{00} &= I_{00} = 0, \quad R_{00} = \frac{q_1}{\mu}, \quad Q_{00} = \frac{\Lambda_i}{\mu}.
\end{align*}
\]

The linearization matrix at DFE is

\[
\begin{pmatrix}
-a_{11} & a_{12} & 0 & a_{15} & 0 & a_{18} & 0 & 0 \\
\phi & -(\mu + \alpha + d) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & a_{42} & -a_{44} & a_{45} & 0 & 0 & 0 & 0 \\
0 & 0 & \phi & -(\mu + \alpha + d) & 0 & 0 & 0 & 0 \\
0 & a_{72} & 0 & 0 & -a_{77} & a_{78} & 0 & a_{7 \times 11} \\
0 & 0 & 0 & 0 & \phi & -(\mu + \alpha + d) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & a_{10 \times 8} & -a_{10 \times 10} & a_{10 \times 11} \\
0 & 0 & 0 & 0 & 0 & 0 & \phi & -(\mu + \alpha + d)
\end{pmatrix},
\]

where

\[
\begin{align*}
a_{11} &= \mu + \phi \frac{q_1}{F_1}, \quad a_{44} = \mu + \phi \frac{q_1}{C_1}, \\
a_{77} &= \mu + \phi \frac{q_1}{F_2}, \quad a_{10 \times 10} = \mu + \phi \frac{q_1}{C_2},
\end{align*}
\]
\[ a_{12} = \beta_1 b_1 F_1 \left( \frac{b_1 \delta_1^2 \Lambda_1}{G_1} + \frac{b_1 \theta_1}{H} \right), \quad a_{15} = \beta_1 b_1 F_1 \left( \frac{a_1 A_1}{G_1} \right), \]
\[ a_{18} = \beta_1 b_1 F_1 \left( \frac{b_2 \theta_2}{H} \right), \quad a_{42} = \beta_1 a_1 C_1 \left( \frac{b_1 \delta_1^2 \Lambda_1}{G_1} \right), \]
\[ a_{45} = \beta_1 a_1 C_1 \left( \frac{a_1 A_1}{G_1} \right), \quad a_{78} = \beta_2 b_2 F_2 \left( \frac{b_2 \delta_2^2 \Lambda_2}{G_2} + \frac{b_2 \theta_2}{H} \right), \]
\[ a_{78} = \beta_2 b_2 F_2 \left( \frac{a_2 A_2}{G_2} \right), \quad a_{72} = \beta_2 b_2 F_2 \left( \frac{b_1 \theta_1}{H} \right), \]
\[ a_{10 \times 8} = \beta_2 a_2 C_2 \left( \frac{b_2 \delta_2^2 \Lambda_2}{G_2} \right), \quad a_{10 \times 11} = \beta_2 a_2 C_2 \left( \frac{a_2 A_2}{G_2} \right), \]

with

\[ F_i = \Lambda_i - q l_1, \quad C_i = A_i - q l_1, \quad \tilde{G}_i = (a_i A_i + b_i \delta_i \Lambda_i)(A_i + \delta_i \Lambda_i), \quad \tilde{H} = \sum_{i=1}^{2} b_i \theta_i \Lambda_i. \]

From Dieckman et al. 1990 and Heesterbeek 1992, the characteristic polynomial of the matrix \( MD^{-1} \) is

\[
f(\lambda) = |MD^{-1} - \lambda I| = \begin{vmatrix} -\lambda & b_{12} & 0 & b_{14} & 0 & b_{16} & 0 & 0 \\ b_{21} & -\lambda & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & b_{32} & -\lambda & b_{34} & 0 & 0 & 0 & 0 \\ 0 & 0 & b_{43} & -\lambda & 0 & 0 & 0 & 0 \\ 0 & b_{52} & 0 & 0 & -\lambda & b_{56} & 0 & a_{58} \\ 0 & 0 & 0 & 0 & b_{65} & -\lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & b_{76} & -\lambda & b_{78} & 0 \\ 0 & 0 & 0 & 0 & 0 & b_{87} & -\lambda & 0 \end{vmatrix},
\]

where
\[ b_{21} = \frac{\phi}{a_{11}}, \quad b_{43} = \frac{\phi}{a_{44}}, \quad b_{65} = \frac{\phi}{a_{10 \times 10}}, \quad b_{12} = \frac{a_{12}}{\alpha + \mu + d}, \quad b_{32} = \frac{a_{42}}{\alpha + \mu + d}, \]
\[ b_{52} = \frac{a_{72}}{\alpha + \mu + d}, \quad b_{14} = \frac{a_{14}}{\alpha + \mu + d}, \quad b_{34} = \frac{a_{44}}{\alpha + \mu + d}, \quad b_{16} = \frac{a_{16}}{\alpha + \mu + d}, \quad b_{56} = \frac{a_{76}}{\alpha + \mu + d}, \quad b_{76} = \frac{a_{10 \times 8}}{\alpha + \mu + d}, \]

Hence,

\[
f(\lambda) = \left( \left( \lambda^2 - b_{12} b_{21} \right) \left( \lambda^2 - b_{34} b_{43} \right) - b_{14} b_{21} b_{32} b_{43} \right) \left( \left( \lambda^2 - b_{56} b_{65} \right) \left( \lambda^2 - b_{78} b_{87} \right) - b_{58} b_{65} b_{76} b_{87} \right) \\
- b_{16} b_{21} b_{52} b_{65} \left( \lambda^2 - b_{34} b_{43} \right) \left( \lambda^2 - b_{78} b_{87} \right) \\
= \lambda^8 + e_2 \lambda^6 + e_1 \lambda^4 + e_0 \lambda^2 + e
where
\[ e_2 = -(b_{12}b_{21} + b_{34}b_{43} + b_{56}b_{65} + b_{78}b_{87}), \]
\[ e_1 = b_{12}b_{21}b_{34}b_{43} + (b_{12}b_{21} + b_{34}b_{43})(b_{56}b_{65} + b_{78}b_{87}) + b_{56}b_{65}b_{78}b_{87} \]
\[ -b_{56}b_{65}b_{76}b_{87} - b_{14}b_{21}b_{32}b_{43} - b_{16}b_{21}b_{52}b_{65}, \]
\[ e_0 = -b_{21}b_{43}(b_{12}b_{34} - b_{14}b_{32})(b_{56}b_{65} + b_{78}b_{87}) \]
\[ -b_{56}b_{78}(b_{56}b_{78} - b_{58}b_{76})(b_{12}b_{21} + b_{34}b_{43}) + b_{16}b_{21}b_{52}b_{65}(b_{34}b_{43} + b_{78}b_{87}), \]
\[ e = b_{21}b_{43}(b_{12}b_{34} - b_{14}b_{32})b_{65}b_{78}(b_{56}b_{78} - b_{58}b_{76}) - b_{16}b_{21}b_{34}b_{43}b_{52}b_{65}b_{78}b_{87}. \]

Noting that
\[ b_{12}b_{34} - b_{14}b_{32} = \frac{\alpha_{12}}{\mu + \alpha + d} \frac{\alpha_{34}}{\mu + \alpha + d} - \frac{\alpha_{15}}{\mu + \alpha + d} \frac{\alpha_{42}}{\mu + \alpha + d} \]
\[ = \frac{\alpha_{45}}{(\mu + \alpha + d)^2} \beta_1 b_1 F_1 \beta_1 \theta_1 \frac{1}{H} = b_{34} \frac{\beta_1 b_1 F_1 \beta_1 \theta_1}{H (\mu + \alpha + d)}, \]
and
\[ b_{56}b_{78} - b_{58}b_{76} = \frac{\alpha_{78}}{\mu + \alpha + d} \frac{\alpha_{10} x_{11}}{\mu + \alpha + d} - \frac{\alpha_{78}}{\mu + \alpha + d} \frac{\alpha_{10} x_{11}}{\mu + \alpha + d} \]
\[ = \frac{\alpha_{10} x_{11}}{(\mu + \alpha + d)^2} \beta_2 b_2 F_2 \beta_2 \theta_2 \frac{1}{H} = b_{78} \frac{\beta_2 b_2 F_2 \beta_2 \theta_2}{H (\mu + \alpha + d)} \]
\[ b_{16}b_{52} = \frac{\alpha_{45}}{(\mu + \alpha + d)^2} = \frac{1}{(\mu + \alpha + d)^2} \beta_1 b_1 F_1 \beta_1 \frac{\beta_2 b_2 F_2 \beta_2}{H} \theta_1 \theta_1 \frac{1}{H}. \]

We known \( e_0 < 0 \) and \( e = 0 \). In fact,
\[ e_0 = -b_{21}b_{34}b_{43}(b_{56}b_{65} + b_{78}b_{87}) \frac{\beta_1 b_1 F_1 \beta_1 \theta_1}{H (\mu + \alpha + d)} \]
\[ -b_{56}b_{78}b_{87}(b_{12}b_{21} + b_{34}b_{43}) \frac{\beta_2 b_2 F_2 \beta_2 \theta_2}{H (\mu + \alpha + d)} \]
\[ + b_{16}b_{21}b_{52}b_{65}(b_{34}b_{43} + b_{78}b_{87}) \]
\[ = -b_{21}b_{34}b_{43}b_{65} \left( b_{56} \frac{\beta_1 b_1 F_1 \beta_1 \theta_1}{H (\mu + \alpha + d)} - b_{16}b_{52} \right) \]
\[ -b_{12}b_{65}b_{78}b_{87} \left( b_{12} \frac{\beta_2 b_2 F_2 \beta_2 \theta_2}{H (\mu + \alpha + d)} - b_{16}b_{52} \right) \]
\[ -b_{34}b_{43}b_{78}b_{87} \left( b_{21} \frac{\beta_1 b_1 F_1 \beta_1 \theta_1}{H (\mu + \alpha + d)} + b_{65} \frac{\beta_2 b_2 F_2 \beta_2 \theta_2}{H (\mu + \alpha + d)} \right) \]
\[ = -b_{21}b_{65} \frac{(\beta_1 b_1 F_1)(\beta_2 b_2 F_2)b_1 b_2}{D_2 H (\mu + \alpha + d)^4} \left( b_{34}b_{43} \theta_1 \theta_2^2 \Lambda_2 + b_{78}b_{87} \theta_2 \theta_2^2 \Lambda_1 \right) \]
\[ -b_{34}b_{43}b_{78}b_{87} \left( b_{21} \frac{\beta_1 b_1 F_1 \beta_1 \theta_1}{H (\mu + \alpha + d)} + b_{65} \frac{\beta_2 b_2 F_2 \beta_2 \theta_2}{H (\mu + \alpha + d)} \right). \]
Then the characteristic polynomial \( f(\lambda) \) of the matrix \( MD^{-1} \) becomes

\[
f(\lambda) = \lambda^2(\lambda^6 + e_2\lambda^4 + e_1\lambda^2 + e_0) \triangleq xg(x),
\]
where, \( x = \lambda^2 \) and \( g(x) = (x^3 + e_2x^2 + e_1x + e_0) \)

\[
x_1 = -\frac{1}{3}e_2 + (S + T),
\]
\[
x_2 = -\frac{1}{2}e_2 - \frac{1}{2}(S + T) + \frac{\sqrt{3}}{2}i(S - T),
\]
\[
x_2 = -\frac{1}{3}e_2 - \frac{1}{2}(S + T) - \frac{\sqrt{3}}{2}i(S - T),
\]
where \( S = (R + \sqrt{D})^{\frac{1}{3}} \), \( T = (R - \sqrt{D})^{\frac{1}{3}} \), \( D = Q^3 + R^2 \). \( Q \) and \( R \) are given by

\[
Q = \frac{1}{9}(3e_1 - e_0^2), \quad R = \frac{1}{54}(9e_1e_2 - 27e_0 - 2e_2^3).
\]

Because \( g(0) = e_0 < 0 \) and \( g(+\infty) = +\infty \), it follows from mean value Theorem, that, there exists at least one \( x^* > 0 \) such that \( f(x^*) = 0 \). We have to determine the largest of \( x_1, x_2 \) and \( x_3 \).

If the discriminant \( D > 0 \), the first root \( x_1 \) is a positive real root and other two roots \( (x_2 \) and \( x_3 \) \) are complex conjugates. Then the basic reproduction number \( R_0 \) is equal to \( \sqrt{x_1} \). If \( D \leq 0 \), all roots are real (when \( D = 0 \), at least two roots are equal), define

\[
\theta = \cos^{-1}\left(\frac{R}{\sqrt{-Q^3}}\right), \quad 0 \leq \theta \leq \pi
\]

In terms of \( \theta \), \( S \) and \( T \) can be rewritten as \( S = \sqrt{-Q}(\cos^3 \theta - i\sin^3 \theta) \), \( T = \sqrt{-Q}(\cos^3 \theta + i\sin^3 \theta) \). Hence,

\[
x_1 = -\frac{1}{3}e_2 + (S + T) = -\frac{1}{3}e_2 + 2\sqrt{-Q}\cos \theta,
\]
\[
x_2 = -\frac{1}{3}e_2 - \frac{1}{2}(S + T) + \frac{\sqrt{3}}{2}i(S - T) = -\frac{1}{3}e_2 + 2\sqrt{-Q}\cos\left(\frac{\theta + 2\pi}{3}\right),
\]
\[
x_3 = -\frac{1}{3}e_2 - \frac{1}{2}(S + T) - \frac{\sqrt{3}}{2}i(S - T) = -\frac{1}{3}e_2 + 2\sqrt{-Q}\cos\left(\frac{\theta + 4\pi}{3}\right),
\]
\[
x_1 - x_2 = 2\sqrt{-Q}\left(\cos^3 \theta - \cos^3 \frac{\theta + 2\pi}{3}\right) = 4\sqrt{-Q}\sin \frac{\theta + \pi}{3}\sin \frac{\pi}{3} = 2\sqrt{-3Q}\sin \frac{\theta + \pi}{3},
\]
\[
x_1 - x_3 = 2\sqrt{-Q}\left(\cos^3 \theta - \cos^3 \frac{\theta + 4\pi}{3}\right) = 4\sqrt{-Q}\sin \frac{\theta + 2\pi}{3}\sin \frac{2\pi}{3} = 2\sqrt{-3Q}\sin \frac{\theta + 2\pi}{3},
\]

Noting that \( 0 \leq \theta \leq \pi \), then \( 0 \leq \theta + \frac{\pi}{3} \leq \frac{2\pi}{3} \) and \( 0 \leq \theta + \frac{4\pi}{3} \leq \pi \), then \( x_1 - x_2 \geq 0 \) and \( x_1 - x_3 \geq 0 \). In both cases \( (D \geq 0 \) and \( D < 0 \), we conclude that \( x_1 \) is the largest. Thus the basic reproduction number \( R_0 \) is \( \sqrt{x_1} \), that is

\[
R_0 = \sqrt{-\frac{1}{3}e_2 + (S + T)}.
\]
13.2 First Matlab Programs for the Simulations

tspan=[0:.5:35];
x0=[4500 40 50 10 3500 50 60 10 100 15 20 7 900 30 40 5]';

```
d=.3;
q=210;
L1=.95;
L2=.95;
phi=0.083;
alpha=.7;
Lambda1=4500;
A1=5500;
Lambda2=1000;
A2=200;
u1=.00025;
u2=.2;
sigma1=2;
sigma2=3;
rho1=22;
rho2=21;
a1=30;
a2=20;
b1=40;
b2=20;
beta1=0.77;
beta2=0.77;
```

global ppp
we wrote all the parameters as a vector
ppp = [d q L1 L2 phi alpha Lambda1 A1 Lambda2 A2 mu1 mu2 sigma1 sigma2 rho1 rho2 a1 a2 b1 b2 beta1 beta2]';

```
[t,x]=ode45('elmejor1',tspan,x0);
```

figure
subplot(221)
hold on
plot(t,x(:,1),'r+');
plot(t,x(:,2),'b*');
plot(t,x(:,3),'g');
plot(t,x(:,4),'k.');
title(['q=',num2str(q)]);
legend('S1', 'E1', 'I1', 'R1')
xlabel('time')
ylabel('Population NSU Residents')

subplot(222)
hold on
plot(t,x(:,5), 'r+')
plot(t,x(:,6), 'b*')
plot(t,x(:,7), 'g')
plot(t,x(:,8), 'k.' )
title(['q=',num2str(q)]);
legend('W1', 'X1','Y1', 'Z1')
xlabel('time')
ylabel('Population SU Residents')

subplot(223)
hold on
plot(t,x(:,9), 'r+')
plot(t,x(:,10), 'b*')
plot(t,x(:,11), 'g')
plot(t,x(:,12), 'k.' )
title(['q=',num2str(q)]);
legend('S2', 'E2', 'I2', 'R2')
xlabel('time')
ylabel('Population NSU Tourist')

subplot(224)
hold on
plot(t,x(:,13), 'r+')
plot(t,x(:,14), 'b*')
plot(t,x(:,15), 'g')
plot(t,x(:,16), 'k.' )
title(['q=',num2str(q)]);
legend('W2', 'X2', 'Y2', 'Z2')
xlabel('time')
ylabel('Population SU Tourist')
13.3 Second Matlab Program for Simulations

function retval=nuevo1(t,x)

global d q L1 L2 phi alpha Lambda1 A1 Lambda2 A2 mu1 B V sigma1 sigma2 rho1 rho2 a1 a2 b1 b2 beta1 beta2

global ppp

j = 1;
p = ppp;

d = p(j); j = j+1;
q = p(j); j = j+1; varies
L1 = p(j); j = j+1;
L2 = p(j); j = j+1;
phi = p(j); j = j+1;
alpha = p(j); j = j+1;
Lambda1 = p(j); j = j+1;
A1 = p(j); j = j+1;
Lambda2 = p(j); j = j+1;
A2 = p(j); j = j+1;
mu1 = p(j); j = j+1;
mu2 = p(j); j = j+1;
sigma1 = p(j); j = j+1;
sigma2 = p(j); j = j+1;
rho1 = p(j); j = j+1;
rho2 = p(j); j = j+1;
a1 = p(j); j = j+1;
a2 = p(j); j = j+1;
b1 = p(j); j = j+1;
b2 = p(j); j = j+1;
beta1 = p(j); j = j+1;
beta2 = p(j); j = j+1;

For the initial conditions
S1=x(1);
E1=x(2);
K1=x(3);
R1=x(4);
W1=x(5);
X1=x(6);
Y1=x(7);
Z1=x(8);
S2=x(9);
E2=x(10);
K2=x(11);
R2=x(12);
W2=x(13);
X2=x(14);
Y2=x(15);
Z2=x(16);

Q1=S1+E1+L1+R1; Total Population for non-user (Residents)
T1=W1+X1+Y1+Z1; Total Population for user (Residents)

kc1=a1*Q1; Average number of contact for non-user
kc2=b1*T1*((sigma1)./(rhol+sigma1)); Average number of contact between users off the subway

Proportionate Mixing C1=(kc1)./(kc1+kc2); non-user with non-user same neighborhood
D1=(kc2)./(kc1+kc2); non-user with user same neighborhood
H1=(kc1)./(kc1+kc2); user with non-user same neighborhood
F1=(kc2)./(kc1+kc2); user with user same neighborhood

Q2=S2+E2+L2+R2; Total Population for non-users (Tourist)
T2=W2+X2+Y2+Z2; Total Population for users (Tourist)

ec3=b1*T1*((rho1)./(rho1+sigma1));
ec4=b2*T2*((rho2)./(rho2+sigma2));

Pb1=(ec3)./(ec3+ec4);
Pb2=(ec4)./(ec3+ec4);

G1=((Pb1).*((Y1./T1)))+((Pb2).*((Y2./T2)));

Rates of Infections
bb1=(T1*((sigma1)./(sigma1+rho1)))+(Q1);
bb2=Y1*((sigma1)./(sigma1+rho1));

B1(t)-transmission rate NSU
B1=(beta1*a1*S1).*(((C1.*K1)./(bb1))+((D1.*bb2)./(bb1)));
V1(t)-transmission rate for SU
V1=(beta1*a1*W1).*(((H1*K1)/(bb1))+((F1*bb2)/(bb1))+(G1));

derivative of S1
t1=A1-B1;
t2=mu1*S1;
t3=(q*L1*S1)./(S1+E1);
cS1=t1-t2-t3;

derivative of E1
e1=B1;
e2=mu1*E1;
e3=phi*E1;
e4=(q*L2*E1)./(S1+E1);
cE1=e1-e2-e3-e4;

derivative of K1
k1=phi*E1;
k2=(mu1+alpha+d)*K1;
cK1=k1-k2;

derivative of R1
r1=alpha*K1;
r2=mu1*R1;
r3=(q*L1*S1)./(S1+E1);
r4=(q*L2*E1)./(S1+E1);
cR1=r1-r2+r3+r4;

derivative of W1
w1=Lambda1-V1;
w2=mu1*W1;
w3=(q*L1*W1)./(W1+X1);
cW1=w1-w2-w3;

derivative of X1
x1=mu1*X1;
x2=phi*X1;
x3=(q*L2*X1)./(W1+X1);
cX1=V1-x1-x2-x3;
derivative of $Y_l$
\[ y_1 = \phi \times X_l; \]
\[ y_2 = (\mu_1 + \alpha + d) \times Y_l; \]
\[ c_{Y_l} = y_1 - y_2; \]

derivative of $Z_l$
\[ z_1 = \alpha \times Y_l; \]
\[ z_2 = \mu_1 \times Z_l; \]
\[ z_3 = (q \times L_1 \times W_1) ./ (W_1 + X_1); \]
\[ z_4 = (q \times L_2 \times X_1) ./ (W_1 + X_1); \]
\[ c_{Z_l} = z_1 - z_2 + z_3 + z_4; \]

$e_{c_1} = a_2 \times Q_2$; Average number of contact for non-user
$e_{c_2} = b_2 \times T_2 \times ((\sigma_2)/(\rho_2 + \sigma_2));$ Average number of contact between users off the subway

Proportionate Mixing
\[ C_2 = (e_{c_1}) ./ (e_{c_1} + e_{c_2}); \text{ non-user with non-user same neighborhood} \]
\[ D_2 = (e_{c_2}) ./ (e_{c_1} + e_{c_2}); \text{ non-user with user same neighborhood} \]
\[ H_2 = (e_{c_1}) ./ (e_{c_1} + e_{c_2}); \text{ user with non-user same neighborhood} \]
\[ F_2 = (e_{c_2}) ./ (e_{c_1} + e_{c_2}); \text{ user with user same neighborhood} \]
\[ G_2 = (e_{c_3}) ./ (e_{c_3} + e_{c_2}); \text{ user with user in the bus within different neighborhood} \]

Rates of Infections
\[ bb_3 = (T_2 \times ((\sigma_2) ./ (\sigma_2 + \rho_2))) + (Q_2); \]
\[ bb_4 = Y_2 \times ((\sigma_2) ./ (\sigma_2 + \rho_2)); \]

$B_2(t)$-transmission rate NSU
\[ B_2 = (\beta_2 \times a_2 \times S_2) \times (((C_2 \times K_2) ./ (bb_3)) + ((D_2 \times bb_4) ./ (bb_3))); \]

$V_2(t)$-transmission rate for SU
\[ V_2 = (\beta_2 \times a_2 \times W_2) \times (((H_2 \times K_2) ./ (bb_3)) + ((F_2 \times bb_4) ./ (bb_3)) + (G_1)); \]

derivative of $S_2$
\[ s_1 = A_2 - B_2; \]
\[ s_2 = \mu_2 \times S_2; \]
\[ s_3 = (q \times L_1 \times S_2) ./ (S_2 + E_2); \]
\[ c_{S_2} = s_1 - s_2 - s_3; \]

derivative of $E_2$
\[ m_1 = B_2; \]
\[ m2 = \mu_2 \cdot E_2; \]
\[ m3 = \phi \cdot E_2; \]
\[ m4 = (q \cdot L_2 \cdot E_2) / (S_2 + E_2); \]
\[ cE2 = m1 - m2 - m3 - m4; \]

\textit{derivative of K2}
\[ h1 = \phi \cdot E_2; \]
\[ h2 = (\mu_2 + \alpha + d) \cdot K2; \]
\[ cK2 = h1 - h2; \]

\textit{derivative of R2}
\[ r5 = \alpha \cdot K2; \]
\[ r6 = \mu_2 \cdot R_2; \]
\[ r7 = (q \cdot L_1 \cdot S_2) / (S_2 + E_2); \]
\[ r8 = (q \cdot L_2 \cdot E_2) / (S_2 + E_2); \]
\[ cR2 = r5 - r6 + r7 + r8; \]

\textit{derivative of W2}
\[ w4 = \Lambda_2 - V_2; \]
\[ w5 = \mu_2 \cdot W_2; \]
\[ w6 = (q \cdot L_1 \cdot W_2) / (W_2 + X_2); \]
\[ cW2 = w4 - w5 - w6; \]

\textit{derivative of X2}
\[ x4 = \mu_2 \cdot X_2; \]
\[ x5 = \phi \cdot X_2; \]
\[ x6 = (q \cdot L_2 \cdot X_2) / (W_2 + X_2); \]
\[ cX2 = V_1 - x4 - x5 - x6; \]

\textit{derivative of Y2}
\[ y3 = \phi \cdot X_2; \]
\[ y4 = (\mu_2 + \alpha + d) \cdot Y_2; \]
\[ cY2 = y3 - y4; \]

\textit{derivative of Z2}
\[ z5 = \alpha \cdot Y_2; \]
\[ z6 = \mu_2 \cdot Z_2; \]
\[ z7 = (q \cdot L_1 \cdot W_2) / (W_2 + X_2); \]
\[ z8 = (q \cdot L_2 \cdot X_2) / (W_2 + X_2); \]
\[ cZ2 = z5 - z6 + z7 + z8; \]
retval=[cS1,cEl,cK1,cR1,cW1,cX1,cY1,cZ1,cS2,cE2,cK2,cR2,cW2,cX2,cY2,cZ2];

13.4 Codes for StarLogo Simulations

13.5 Code for 2-neighborhoods

;;;; Model for Population Dynamics in 2-neighborhoods and one subway station
;;;; People are divided in two populations bustakers and nonbustakers
;;;; An initial attack occurs in the subway station with n-exposed individuals
;;;; where only bustakers from all neighborhoods meet and as they move back into
;;;; their neighborhoods
;;;; they spread the disease
;;;; We assume that the probability of contacting a nonbustaker is the same
;;;; for all nonbustakers and bustakers, as well as
;;;; the probability of contacting a bustaker is the same for all nonbustakers
;;;; and bustakers
;;;; green - susceptible population, yellow - exposed population, red -
;;;; infected population
;;;; As people recover they exit turn blue and do not contribute to the
;;;; contact process

patches-own [vacpatch vac? vacpatchx vacpatchy neighborhood neighborhoodx
;;;; neighborhoody old-pc]
turtles-own [t-neighborhood bustaker? clock]
globals [n v susceptible exposed infected recovered numdead infect d1 d2]

;;;; PATCHES
;;;; vacpatch, vacpatchx, vacpatchy define the vaccination sectors
;;;; vac? is boolean variable for random patch vaccination purposes
;;;; neighborhood is the neighborhood the patch is in.
;;;; neighborhoodx and neighborhoody are used to calculate neighborhood
;;;; old-pc keeps track of the original color of the patch so it can change
;;;; back.

;;;; TURTLES
;;;; t-neighborhood is the neighborhood the turtle is in
;;;; bustaker is to control the percentage of the population that will take
;;;; the subway
;;;; clock is to control both the infection time and the time of exposure to
smallpox

;;;; GLOBALS
;;;; n is used as a dummy variable
;;;; v is a dummy variable for vaccination procedures
;;;; susceptible, exposed, infected and recovered are self-explanatory
;;;; numdead keeps track of the casualties due to the attack
;;;; infect is to initialize vaccination procedures
;;;; d1, d2 keep dead count per neighborhood

;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
THE PROGRAM
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
to clear
caclassificate       ;; clears all
setsusceptible 0
setnumdead 0
setrecovered 0
setinfected 0
setexposed 0
setn 0
setv 0
end

to setup
caclassificate       ;; clear all
define-vaccinepatch ;; creates vaccination sectors
define-neighborhood ;; defines neighborhoods and subway
create-people ;; creates n people
setclock 0 ;; starts clock on exposed individuals
setsusceptible inipop ;; susceptible population at time 0
setnumdead 0 ;; dead count at time 0
setrecovered 0 ;; recovered count at time 0
setinfected 0 ;; infected count at time 0
setexposed nexposed ;; exposed at time 0
init-graph ;; initializes graphics
setv 0 ;; loop clock for vaccination
setvac? FALSE
setinfect 0
to go
  if (time > 0) [wait time]
  elseif ((stop-bus = 0) or (infect <= ninfected)) [move-people]
  [move-people2] ;; defines movement of people
  up-clock
  get-infectious ;; infection
  get-exposed ;; smallpox exposure

  if (vaccinate-1 = 1) [if (infect > ninfected) [patch-vaccinate1]]
  if (vaccinate-no = 1) [if (infect > ninfected) [patch-vaccinate2]]
  if (vaccinate-3 = 1) [if (infect > ninfected) [patch-vaccinate3]]

  ;;if (vaccinate-1 = 1) [patch-vaccinate1] ;; vaccinates Everybody per vacpatch horizontally once
  ;;if (vaccinate-2 = 1) [patch-vaccinate2] ;; vaccinates Everybody per randomly selected vacpatch once
  ;;if (vaccinate-3 = 1) [patch-vaccinate3] ;; vaccinates Everybody in patch when first infected case is detected

  count
  update-graph

  if (infected + exposed < 1) [stopbutton2]
end

to count
  setsusceptible count-turtles-with [color = green]
  setexposed count-turtles-with [color = yellow]
  setinfected count-turtles-with [color = red]
end

to count-dead
  if (t-neighborhood = 1) [tsetd1 d1 + 1]
  if (t-neighborhood = 2) [tsetd2 d2 + 1]
end
to define-neighborhood
  if (ycor > 0) [setneighborhood 1] ;; defines neighborhood 1
  if (ycor <= 0) [setneighborhood 2] ;; defines neighborhood 2
  if (xcor > -1 * n-var) and (xcor < n-var) and (ycor > -1 * n-var) and (ycor < n-var) [setpc grey setneighborhood 3] ;; defines neighborhood 3
end

to up-clock
  if (color = yellow) [setclock clock + 1] ;; tic toc
  if (color = red) [setclock clock + 1] ;; tic toc
end

to move-people
  rt random 180
  lt random 180
  ifelse ((t-neighborhood = (neighborhood-at dx dy)) or ((bustaker?) and ((neighborhood-at dx dy) = 3))) [fd 1] [rt 180]
end

to move-people2
  rt random 180
  lt random 180
  ifelse ((neighborhood-at 0 0) = (neighborhood-at dx dy)) [fd 1] [lt 180]
end

to create-people
  crt inipop ;; creates n people at time 0
  fd random 100 ;; all people randomly move forward up to 100 steps at time 0
  setc green ;; all green for susceptible
repeat 100 [if ((neighborhood-at 0 0) = 3) [fd random 100]]

if (ycor > 0) [sett-neighborhood 1] ;; defines neighborhood 1
if (ycor <= 0) [sett-neighborhood 2] ;; defines neighborhood 2
repeat 100 [if ((neighborhood-at 0 0) != t-neighborhood) [fd random 100]]

;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

; CONTROL THE % OF SUBWAY TAKERS PER NEIGHBORHOOD;
setbustaker? FALSE
if ((random 100) < per-bt-1) and (t-neighborhood = 1) [setbustaker? TRUE]
if ((random 100) < per-bt-2) and (t-neighborhood = 2) [setbustaker? TRUE]
if who < nexposed [setc yellow setbustaker? true setxcor 0 setycor 0 rt random 360 fd random 15] ;;make exposed people by an initial attack of smallpox
end

;;;;; Individuals have 85% chance of becoming exposed due to contacts with infected individuals
to get-exposed
if (((color-at 0 0) = red) and ((random 100) < 85)) [if (color = green) [setc yellow]]
end

to get-infectious
if (clock > latent-time and color = yellow) [tsetinfect (infect + 1) setc red setclock 0]
if (clock > infectious-time and color = red) [ifelse ((random 100) < 30) [tsetnumdead numdead + 1 die] [tsetrecovered recovered + 1 setc blue]]
end

;;;;;; Vaccination Procedures

;;;;;;;;;;
to patch-vaccinate1
if (v <= vaccinepatch ^ 2) [if (vacpatch = v) [to-vaccinate] setv (v + 1)]
end
to patch-vaccinate2
setv (random (1 + vaccinepatch ^ 2) + 1)
if (vacpatch = v)[if (vac? = FALSE) [setvac? TRUE to-vaccinate]]
end
to patch-vaccinate3
if (color = red and random 4000 = who) [tsetv (vacpatch-at 0 0)]
if (vacpatch = v)[if (vac? = FALSE) [setvac? TRUE to-vaccinate]]
end
to to-vaccinate
if ((vacpatch-at 0 0) = v) [
  if ((color = green) and ((random 100) < 97)) [ifelse ((random 100) < 1) [tsetnumdead numdead + 1 count-dead die] [tsetrecovered recovered + 1 setc blue]] ;; vaccine has 97% chance of immunizing susceptible individuals 1% chance of killing individuals due to secondary effects of vaccine
  if ((color = yellow) and (clock <= 4) and ((random 100) < 97)) [ifelse ((random 100) < 1) [tsetnumdead numdead + 1 count-dead die] [tsetrecovered recovered + 1 setc blue]] ;; vaccine has 97% chance of immunizing exposed individuals if time of exposure is less than 4 days (loops), 1% chance of killing individuals due to secondary effects of vaccine
]
end
to define-vaccinepatch
  setn -1 * vaccinepatch ;; start negative
  repeat vaccinepatch [if (xcor >= ((screen-edge-x I vaccinepatch) * n)) [setvacpatchx (n + vaccinepatch) * vaccinepatch / 2] setn n + 2] ;; this marks off the vaccination sectors horizontally.
  setn -1 * vaccinepatch
  repeat vaccinepatch [if (ycor >= ((screen-edge-y / vaccinepatch) * n)) [setvacpatchy ((n + vaccinepatch) / 2)] setn n + 2] ;; this marks off the vaccination sectors vertically
  setvacpatch vacpatchx + vacpatchy + 1 ;; add them up and you get a unique neighborhood (uses powers to make this work)
  scale-pc brown (vacpatch + 1) vaccinepatch (vaccinepatch ^ 2) ;; shade them blue to tell them apart
  setold-pc pc ;; set the old color to this color (the color we come back to if we change back from yellow)
end
begin

to init-graph

    pw1    
clearplot

    setplot-yrange 0 inipop
    auto-plot-on

    setplotwindow-name "Population_Dynamics"
    setplot-xlabel "TIME"
    setplot-ylabel "POPULATION"

    pp1
    setppc green
    setpen-name "Susceptible"

    pp2
    setppc yellow
    setpen-name "Exposed"

    pp3
    setppc red
    setpen-name "Infected"

    pp4
    setppc blue
    setpen-name "Recovered"

    pp5
    setppc violet
    setpen-name "Casualties"

    pw2    
clearplot

    setplot-yrange 0 20
    auto-plot-on

    setplotwindow-name "Casulties_per_Neighborhood"
    setplot-xlabel "TIME"
    setplot-ylabel "Casualties"

end
pp1
setppc green
setpen-name "N1

pp2
setppc pink
setpen-name "N2

end

to update-graph
pw1
pp1
plot susceptible
pp2
plot exposed
pp3
plot infected
pp4
plot recovered
pp5
plot numdead

pw2
pp1
plot d1
pp2
plot d2

end

:;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;; Debugers
:;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
:;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
:;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

to show-neighborhoods
    if (neighborhood = 1) [setpc white]
if (neighborhood = 2) [setpc black]
if (neighborhood = 3) [setpc gray]
end

to show-vacpatch
scale-pc brown (vacpatch + 1) vaccinepatch (vaccinepatch ^ 2);; shade them blue to tell them apart
if (neighborhood = 3) [setpc gray]
end

to mark-people
if (t-neighborhood = 1) [setc pink]
if (t-neighborhood = 2) [setc green]
end

to mark-bustakers
if bustaker? [setc orange]
end

13.6 Code for 4-neighborhoods

;; Model for Population Dynamics in 4-neighborhoods and one subway station
;; People are divided in two populations bustakers and nonbustakers
;; An initial attack occurs in the subway station with n-exposed individuals
;; where only bustakers from all neighborhoods meet and as they move back into
;; their neighborhoods
;; they spread the disease
;; We assume that the probability of contacting a nonbustaker is the same
;; for all nonbustakers and bustakers, as well as
;; the probability of contacting a bustaker is the same for all nonbustakers
;; and bustakers
;; green - susceptible population, yellow - exposed population, red -
;; infected population
;; As people recover they exit turn blue and do not contribute to the
;; contact process

patches-own [vacpatch vac? vacpatchx vacpatchy neighborhood neighborhoodx
neighborhoody old-pc]
turtles-own [t-neighborhood bustaker? clock]
globals [n v susceptible exposed infected recovered numdead infect d1 d2 d3 d4]

;; PATCHES
;; vacpatch, vacpatchx, vacpatchy define the vaccination sectors
;; vac? is boolean variable for random patch vaccination purposes
;; neighborhood is the neighborhood the patch is in.
;; neighborhoodx and neighborhoody are used to calculate neighborhood
;; old-pc keeps track of the original color of the patch so it can change back.

;;; TURTLES
;;; t-neighborhood is the neighborhood the turtle is in
;;; bustaker is to control the percentage of the population that will take
the subway
;;; clock is to control both the infection time and the time of exposure to
smallpox

;;; GLOBALS
;;; n is used as a dummy variable
;;; v is a dummy variable for vaccination procedures
;;; susceptible, exposed, infected and recovered are self-explanatory
;;; numdead keeps track of the casualties due to the attack
;;; d1, d2, d3, d4 keep track of dead count per neighborhood

;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;; THE PROGRAM
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

; to clear
ca ;;; clears all
setsusceptible 0
setnumdead 0
setrecovered 0
setinfected 0
setexposed 0
setn 0
setv 0
end
to setup
ca
define-vaccinepatch
define-neighborhood
create-people
set-clock 0
set-susceptible inipop
set-num-dead 0
set-recovered 0
set-infected 0
set-exposed nexposed
init-graph
setv 0
setvac? FALSE
set-infect 0
set-d1 0
set-d2 0
set-d3 0
set-d4 0
end

to go
if (time > 0) [wait time]
ifelse ((stop-bus = 0) or (infect <= ninfected)) [move-people]
[move-people2] ;; defines movement of people
up-clock
get-infectious ;; infection
procedure
get-exposed ;; smallpox exposure
procedure

if (vaccinate-1 = 1) [if (infect > ninfected) [patch-vaccinate1]]
if (vaccinate-no = 1) [if (infect > ninfected) [patch-vaccinate2]]
if (vaccinate-3 = 1) [if (infect > ninfected) [patch-vaccinate3]]

;;;;if (vaccinate-1 = 1) [patch-vaccinate1] ;; vaccinates
everybody per vacpatch horizontally once
;;;;if (vaccinate-2 = 1) [patch-vaccinate2] ;; vaccinates
everybody per randomly selected vacpatch once
;;;;if (vaccinate-3 = 1) [patch-vaccinate3] ;; vaccinates
everybody in patch when first infected case is detected

count
update-graph

if (infected + exposed < 1) [stopbutton2]
end

to count
set-susceptible count-turtles-with [color = green]
set-exposed count-turtles-with [color = yellow]
set-infected count-turtles-with [color = red]
end

to count-dead
if (t-neighborhood = 1) [tsetd1 d1 + 1]
if (t-neighborhood = 2) [tsetd2 d2 + 1]
if (t-neighborhood = 3) [tsetd3 d3 + 1]
if (t-neighborhood = 4) [tsetd4 d4 + 1]
end

to define-neighborhood
if (ycor > 0 and xcor > 0) [set-neighborhood 1] ;; defines neighborhood 1
if (ycor > 0 and xcor <= 0) [set-neighborhood 2] ;; defines neighborhood 2
if (ycor <= 0 and xcor <= 0) [set-neighborhood 3] ;; defines neighborhood 3
if (ycor <= 0 and xcor > 0) [set-neighborhood 4] ;; defines neighborhood 4
if (xcor > -1 * n-var) and (xcor < n-var) and (ycor > -1 * n-var) and (ycor < n-var) [setpc grey set-neighborhood 5] ;; defines neighborhood 5
end

to up-clock
if (color = yellow) [setclock clock + 1] ;; tic toc
if (color = red) [setclock clock + 1] ;; tic toc
end

to move-people
   rt random 180
   lt random 180
   ifelse ((t-neighborhood = (neighborhood-at dx dy)) or ((bustaker?) and ((neighborhood-at dx dy) = 5))) [fd 1] [rt 180]
end
to move-people2
  rt random 180
  lt random 180
  ifelse ((neighborhood-at 0 0) = (neighborhood-at dx dy)) [fd 1] [lt 180] end

to create-people
  crt inipop ;; creates n people at time 0
  fd random 100 ;; all people randomly move forward up to 100 steps at time 0
  setc green ;; all green for susceptible

repeat 100 [if ((neighborhood-at 0 0) = 5 ) [fd random 100]]

if (ycor > 0 and xcor > 0) [sett-neighborhood 1] ;; defines neighborhood 1
if (ycor > 0 and xcor <= 0) [sett-neighborhood 2] ;; defines neighborhood 2
if (ycor <= 0 and xcor <= 0) [sett-neighborhood 3] ;; defines neighborhood 3
if (ycor <= 0 and xcor > 0) [sett-neighborhood 4] ;; defines neighborhood 4

repeat 100 [if ((neighborhood-at 0 0) != t-neighborhood ) [fd random 100]]

;;;;;;;;;;;;;;;CONTROL THE % OF SUBWAY TAKERS PER NEIGHBORHOOD;;;;;;;;;;;;;;;.

setbustaker? FALSE
if (((random 100) < per-bt-1) and (t-neighborhood = 1) [setbustaker? TRUE]
if (((random 100) < per-bt-2) and (t-neighborhood = 2) [setbustaker? TRUE]
if (((random 100) < per-bt-3) and (t-neighborhood = 3) [setbustaker? TRUE]
if (((random 100) < per-bt-4) and (t-neighborhood = 4) [setbustaker? TRUE]

if who < nexposed [setc yellow setbustaker? true setxcor 0 setycor 0 rt random 360 fd random n-var] ;;make exposed people by an initial attack of smallpox

end
Individuals have 85% chance of becoming exposed due to contacts with infected individuals

to get-exposed
if (((color-at 0 0) = red) and ((random 100) < 85)) [if (color = green) [setc yellow]]
end

to get-infectious
if (clock > latent-time and color = yellow) [tsetinfect (infect + 1) setc red setclock 0]
if (clock > infectious-time and color = red) [ifelse ((random 100) < 30) [tsetnumdead numdead + 1 count-dead die] [tsetrecovered recovered + 1 setc blue]]
end

;;;;;;;;;;;;;;;;;;; Vaccination Procedures
;;;;;;;;;;;;;;;;;;
;;;;;;;;;;;;;;;;;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

to patch-vaccinate1
if (v <= vaccinepatch - 2) [if (vacpatch = v) [to-vaccinate] setv (v + 1)]
end

to patch-vaccinate2
setv (random (1 + vaccinepatch ^ 2) + 1)
if (vacpatch = v) [if (vac? = FALSE) [setvac? TRUE to-vaccinate]]
end

to patch-vaccinate3
if (color = red and random 4000 = who) [tsetv (vacpatch-at 0 0)]
if (vacpatch = v) [if (vac? = FALSE) [setvac? TRUE to-vaccinate]]
end

to to-vaccinate
if ((vacpatch-at 0 0) = v) [if ((color = green) and ((random 100) < 97)) [ifelse ((random 100) < 1) [tsetnumdead numdead + 1 count-dead die] [tsetrecovered recovered + 1 setc blue]] ;; vaccine has 97% chance of immunizing susceptible individuals 1%
chance of killing individuals due to secondary effects of vaccine
if ((color = yellow) and (clock <= 4) and ((random 100) < 97)) [ifelse
((random 100) < 1) [tsetnumdead numdead + 1 count-dead die] [tsetrecovered
recovered + 1 setc blue]] ;; vaccine has 97% chance of immunizing exposed
individuals if time of exposure is less than 4 days (loops), 1% chance of
killing individuals due to secondary effects of vaccine
]
end
to define-vaccinepatch
setn -1 * vaccinepatch ;; start negative
repeat vaccinepatch [if (xcor >= ((screen-edge-x / vaccinepatch) * n))
[setvacpatchx (n + vaccinepatch) * vaccinepatch / 2] setn n + 2] ;; this
marks off the vaccination sectors horizontally.
setn -1 * vaccinepatch
repeat vaccinepatch [if (ycor >= ((screen-edge-y / vaccinepatch) * n))
[setvacpatchy ((n + vaccinepatch) / 2)] setn n + 2] ;; this marks off the
the vaccination sectors vertically
setvacpatch vacpatchx + vacpatchy + 1 ;; add them up and you get a unique
neighborhood (uses powers to make this work)
scale-pc brown (vacpatch + 1) vaccinepatch (vaccinepatch ^ 2) ;; shade them
blue to tell them apart
setold-pc pc ;; set the old color to this color (the color we come back to
if we change back from yellow)
end

Graphics

to init-graph
pw1
clearplot
dsetplot-yrange 0 inipop
auto-plot-on
setplotwindow-name "Population_Dynamics"
setplot-xlabel "TIME"
setplot-ylabel "POPULATION"

pp1
setppc green
setpen-name "Susceptible"

pp2
setppc yellow
setpen-name "Exposed"

pp3
setppc red
setpen-name "Infected"

pp4
setppc blue
setpen-name "Recovered"

pp5
setppc violet
setpen-name "Casualties"

pw2
clearplot
setplot-yrange 0 30
auto-plot-on
setplotwindow-name "Casualties_per_Neighborhood"
setplot-xlabel "TIME"
setplot-ylabel "Casualties"

pp1
setppc green
setpen-name "N1"

pp2
setppc pink
setpen-name "N2

pp3
setppc violet
setpen-name "N3

pp4
setppc brown
setpen-name "N4

end

to update-graph
pw1
pp1
plot susceptible
pp2
plot exposed
pp3
plot infected
pp4
plot recovered
pp5
plot numdead

pw2
pp1
plot d1
pp2
plot d2
pp3
plot d3
pp4
plot d4

end

;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;; Debugers
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
to show-neighborhoods
  if (neighborhood = 1) [setpc white]
  if (neighborhood = 2) [setpc black]
  if (neighborhood = 3) [setpc white]
    if (neighborhood = 4) [setpc black]
  if (neighborhood = 5) [setpc gray]
end

to show-vacpatch
  scale-pc brown (vacpatch + 1) vaccinepatch (vaccinepatch - 2) ;; shade them blue to tell them apart
  if (neighborhood = 5) [setpc gray]
end

to mark-people
  if (t-neighborhood = 1) [setc green]
  if (t-neighborhood = 2) [setc pink]
    if (t-neighborhood = 3) [setc violet]
  if (t-neighborhood = 4) [setc brown]
end

to mark-bustakers
  if bustaker? [setc orange]
end