MOUSE IN THE HOUSE:
LOOKING AT THE SPREAD OF
HANTAVIRUS IN HOUSES THROUGH
THE DEER MOUSE POPULATION

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Brandon J. Brown
University of California, Irvine

Edgar Cabral
University of California, Irvine

Tiffany R. Hegg
Mesa State College, Colorado

Carlos Castillo-Garsow
Cornell University

Baojun Song
Cornell University

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Abstract

The Sin Nombre Virus is part of the Bunyaviridae family that causes hantavirus pulmonary syndrome. The deer mouse, the primary host of Sin Nombre Virus, supports a prevalence of about 25% in its adult population. Since deer mice are typically found in fields, homes, and barns, we examine the risk of infection Sin Nombre Virus poses on humans by looking at the dynamics of the deer mouse population as it moves through homes and barns in rural areas within western Colorado. Hence, the barn and the house are our epidemiological units and, consequently, it is initially assumed that each unit is in one of three infestation states, that is, at zero, low or high mouse infestation. The threshold that governs the likelihood of an epidemiological outbreak is computed. Explicit spatial simulations of small communities that involve the movement of mice and their seasonally driven reproductive capacities are carried out. The impacts of control measures are tested in the stochastic frameworks.
1 Introduction

In 1993 an outbreak of unexplained acute respiratory distress syndrome (UARDS) struck the Four Corners area of the United States. The first reported cases came from healthy young adults who became sick and rapidly died at local hospitals. The unknown disease caused a high percentage of deaths to those who had become infected [8]. The common border of Colorado, New Mexico, Arizona and Utah defines the Four Corners where the cases were found. Only weeks after the discovery of the disease, the Center of Disease Control and Prevention (CDC) were able to link UARDS to an unknown type of hantavirus. The disease was subsequently termed hantavirus pulmonary syndrome (HPS). Hantavirus belongs to the family Bunyaviridae that are responsible for HPS and hemorrhagic fever with renal syndrome (HFRS). From previous knowledge of hantaviruses, including Hantaan, researchers knew that the disease was transmitted to people by contact with rodents. This lead to an extensive effort to trap all different types of rodents within the Four Corners area and detect the rodent with the antibodies to the strain of hantavirus in question [7].

Among all types of rodents trapped, the deer mouse (Peromyscus maniculatus) was found to be the principle reservoir to the previously unknown strain of hantavirus. The deer mouse population is abundant in North America where a potential for extensive outbreaks of HPS exists. The deer mouse is found in rural and semi-rural areas, in barns, homes and other buildings. Researchers believe that the virus is being passed from the deer mouse to humans from the contact made in these settings. Approximately 25% of the deer mice trapped were found to be infected with hantavirus. Other mice were also found to be infected, but in lesser quantities [8].

The Four Corners strain of hantavirus was found to be Sin Nombre virus (SNV). The virus was discovered by looking at samples of tissue taken years before from patients with UARDS. The earliest known case of SNV was from a man from Green River, Utah in 1959 [3]. Hantaviruses, including SNV, are classified as "emerging viruses" because of their tendency to appear in new populations unexpectedly [7]. There have been no known occurrences of human to human spread of hantavirus. All humans who come in contact with an infected mouse's excrement are susceptible[8].

Figure 1: A photograph of SNV taken by scientists at the CDC with electron microscope[4]

The virus is spread to humans via inhalation of particles of dried excrement including feces, urine and saliva of the deer mouse. Most cases of SNV occur in patients whom have
worked in an area where they were sweeping, vacuuming or working with soils and have moved the dried excrement around to create a dust that was inhaled. The early symptoms of HPS include fever, fatigue and muscle aches. Other symptoms may include headaches, nausea, abdominal pain, dizziness and chills. With only limited information, it has been shown that symptoms may develop between 1 and 5 weeks after possible exposure. About four to ten days after the initial stage of the illness, the late symptoms of HPS can occur. The lungs fill with fluid causing a dyspnea (fluid fills lungs) and coughing [5]. Patients with HPS also have thrombocytopenia (a sudden decrease in the number of blood platelet levels) and leukocytosis (increase level of white blood cells) leading to pulmonary capillary leakage. Most deaths of HPS are due to acute shock and cardiac complications [6]. The fatality rate of those infected with HPS is about 30% with about 285 cases reported in the United States [4].

Researchers have discovered that there are several hantaviruses that cause HPS. The Bayou virus is carried by the rice rat and was discovered when a Louisiana man was infected. A Florida man came down with HPS from another hantavirus, the Black Creek Canal virus carried by the cotton rat. An SNV-like virus was found in New York and named New York-1, which is carried by the white-footed mouse. There have been other occurrences of hantavirus in Argentina, Brazil, Canada, Chile, Paraguay, and Uruguay [8][4].

There is no known cure for SNV. Emergency facilities may be able to treat the early symptoms of hantavirus, but are unable to treat the virus itself [15].

### 2 Deer Mouse

The deer mouse (*Peromyscus maniculatus*) is found in North America and is distributed from Alaska and Canada southward to central Mexico. Within this range, the deer mouse is absent from the southeastern United States and some coastal areas of Mexico. The range of the deer mouse includes many different biomes which include: alpine habitats, northern boreal forest, desert, grassland, brushland, southern montane woodland, and arid upper tropical habitats [11].

![Deer mouse](image)

**Figure 2: Deer mouse [9]**

The deer mouse is between 11.9 and 22.2 centimeters (cm) long. They have fur that is a grayish to reddish brown on their dorsal side and white on their ventral side. The deer
mouse also has a finely haired tail that is bicolored. The half of the tail closest to their body is dark and the other half is distinguishably lighter. Their ears are large, round and mostly hairless. They have large and bulging eyes. The most distinct characteristic of the deer mouse is their bicolored tail, which is used to identify the species [9].

The deer mouse is sexually mature at the age of six weeks. Reproduction occurs year round except for in the winter and other unfavorable conditions. Their gestation period ranges from 23 days to 31 days and a litter size from between one and eleven pups [11]. Ninety percent of the litter die in the in the first four weeks of life because of high vulnerability when they are young [12]. This leaves only one or two survivors that reach sexual maturity. Litter size increases with each birth until the fifth litter where it decreases thereafter. The young are weaned in about 30 days and usually leave the nest and become independent of their mother. The expected lifespan of a deer mouse is between 1 and 2 years if they survive the initial 3 months.

During the winter season, the deer mouse stays with the nest and rarely travels a far distance except to get more food. In the summer season, the deer mouse will travel a greater distance, up to a three-acre radius to reproduce and gather food. The deer mouse has no restrictions as to where it travels and is always exploring new territory for food and shelter. The length of time they remain in one area depends on the finding of a high quantity of food and a suitable place to build a nest [14]. The deer mouse is primarily nocturnal. Animals such as owls, snakes and various mammals are their biggest threat. These animals prey mostly on the young, which is the primary reason the fatality rate in young mice is so high.

Population densities of deer mice fluctuate throughout the year. The total population reaches an apex in the fall, which is the end of the breeding season. At this point the population density reflects the number of young. As the young begin to die in the winter, the total population begins to decrease and hits a low point in the spring. The percentage of adult deer mice throughout the year goes from low in the fall to a peak in the spring [1]. These seasonal patterns have an effect on the spread of the hantavirus through the deer mouse population.

Mouse behaviors in particular lead to occurrences that are typically unpredictable. Until recently, scientists have had little understanding of behaviors that mice demonstrate. Scientists have found that it is more likely to have a group of deer mice in a building than one or two. Deer mice tend to stay where they have food and can build a nest where they live in groups. With living conditions ideal, more deer mice tend to stay together in a nest with up to five adult deer mice in a single nest. The young from a nest are forced to leave after they reach sexual maturity. The deer mice will stay in a single area until they have a reason to leave mainly, a lack of food or death. Mice are found in scarce numbers in some areas because of a lack of these ideal conditions. This is when deer mice generally pass through barns and homes and generally stay for an abbreviated amount of time unless other conditions arise [12].

The deer mouse acquires the disease by contact with another infected mouse. They become infected through exposure to the dust particles from feces, urine and saliva. It is unknown if the mother mouse passes the virus to her young. Studies have shown that the virus is found predominantly in adult male deer mice. There is a strong correlation between the number of scars from fighting on male mice and antibodies to hantavirus. Many mice
are suspected to be infected through this activity [13]. Seroconversion, where a mouse goes from susceptible to infected, indicates when a mouse is shedding the most virus. The virus is shed more towards late winter because the mice are more prone to infection in the winter season [12]. The deer mouse is not affected by the virus and only acts as a carrier of the disease.

The tendency for deer mice to move into homes, barns and other outdoor buildings has made many people in the rural community take notice. With one in four mice infected with SNV, many homeowners are going to great lengths to rid the areas of the deer mouse. Researchers, including Dr. Rick Douglass, advise to mouse proof your home by eliminating holes larger than 0.5” in diameter and placing all food in mouse proof containers. Trapping mice or using poison could lead to a greater infestation; the mouse density in a home depends on the treatment used [12]. In buildings where mouse proofing is not possible, extra precautions should be taken into consideration so the infection is not passed.

In our study, we are looking at the density of mice in the western half of Colorado. This area has a high percentage of mice infected and there have been numerous cases of humans coming down with HPS. In our deterministic model we are finding at what point the mouse population will no longer exist in the buildings and at what point the mouse population will always persist. The classes of buildings include susceptible or mouse free homes, low infested homes with 1-2 mice, and high infested homes with 3-4 mice. Also, we include in our model a class of susceptible or mouse free barns, low infestation or barns with 1-5 mice, and high infested barns with 5-15 mice. In our stochastic simulation we will look at the movement of mice between buildings and show the changes of class of each building. This model will take into account fluctuations in population density through seasons and also birth and death rates of mice. We will look at the extermination rates and how to control the deer mouse population to limit the number of mice humans come in contact with.
3 Deterministic Model

Our approach to the study of the transmission dynamics of hantavirus is two fold. First, we develop a rather simple deterministic model with houses and barns as the epidemiological units, to gain some basic understanding of the process under the simplest scenario. Secondly, we derive a detailed stochastic model that takes into account the reproductive cycle and seasonally-dependent fecundity of the mice (section 4). Hence, in this section we introduce the baseline deterministic model and analyze it. Our equations include susceptible, low and highly infested houses (H) and barns (B). The equations that describe our model are as follows:

\[
S'_H = -\beta S_H \left[ \frac{L_H + L_B + q(H_H + H_B)}{K_H + K_B} \right] + \delta _2 H_H + \delta_1 L_H, \\
L'_H = \beta S_H \left[ \frac{L_H + L_B + q(H_H + H_B)}{K_H + K_B} \right] - \gamma L_H + \phi H_H - \delta_1 L_H, \\
H'_H = \gamma L_H - \phi H_H - \delta_3 H_H, \\
S'_B = -\beta S_B \left[ \frac{L_H + L_B + q(H_H + H_B)}{K_H + K_B} \right] + \delta_2 H_B + \delta_1 L_B, \\
L'_B = \beta S_B \left[ \frac{L_H + L_B + q(H_H + H_B)}{K_H + K_B} \right] - \gamma L_B + \phi H_B - \delta_1 L_B, \\
H'_B = \gamma L_B - \phi H_B - \delta_2 H_B, \\
K'_H = S'_H + L'_H + H'_H, \\
K'_B = S'_B + L'_B + H'_B,
\]

where the state variables and parameters are defined in figure 3 and table 1.

Figure 3: Flowchart for Deterministic Model
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>rate at which houses/barns get infected with mice</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>rate at which mice levels in a house/barn go from low to high</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>extermination rate of low houses/barns</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>extermination rate of high houses/barns</td>
</tr>
<tr>
<td>$\phi$</td>
<td>rate at which the mice levels in a house/barn go from high to low</td>
</tr>
<tr>
<td>$S_H$</td>
<td>the number of houses with no mice</td>
</tr>
<tr>
<td>$S_B$</td>
<td>the number of barns with no mice</td>
</tr>
<tr>
<td>$L_H$</td>
<td>the number of houses with a low infestation of mice, between 1-2 mice</td>
</tr>
<tr>
<td>$L_B$</td>
<td>the number of barns with a low infestation of mice, between 1-5 mice</td>
</tr>
<tr>
<td>$H_H$</td>
<td>the number of houses with a high infestation of mice, between 3-4 mice</td>
</tr>
<tr>
<td>$H_B$</td>
<td>the number of barns with a high infestation of mice, between 6-15 mice</td>
</tr>
<tr>
<td>$K_H$</td>
<td>the total number of houses within the population, a constant number</td>
</tr>
<tr>
<td>$K_B$</td>
<td>the total number of barns within the population, a constant number</td>
</tr>
<tr>
<td>$q$</td>
<td>A constant between 0 and 1 that is used to show that a susceptible house is more likely to be infected from a low infested building than a highly infested building</td>
</tr>
</tbody>
</table>

Table 1: Parameters for our Equations

In our model we are looking at the spread of mice between houses and barns by examining the infestation of mice within these areas. The sole difference we are examining between houses and barns is that barns on average carry more mice than houses. This is because barns have more access to food and shelter for mice and they are able to nest without detection for long periods of time. Mice are usually less likely to be found in a house because of a limited amount of food and shelter. From the six original equations, we were able to reduce our model down to three equations. We did this by combining the susceptible, low and high classes of barns and houses and labeling them $X$, $Y$, and $Z$, respectively. This is because the equations for houses and barns in the deterministic model are exactly the same. Hence letting,

\[
X = S_H + S_B, \\
Y = L_H + L_B, \\
Z = H_H + H_B,
\]

which leads to the modified model:

\[
X' = -\beta X \frac{Y + qZ}{X + Y + Z} + \delta_1 Y + \delta_2 Z, \\
Y' = \beta X \frac{Y + qZ}{X + Y + Z} - \delta_1 Y - \gamma Y + \phi Z, \\
Z' = \gamma Y - (\delta_2 + \phi)Z, \\
K' = X' + Y' + Z',
\]

with its flow diagram in figure 4.

Since $X + Y + Z = K$, we can substitute $K - Y - Z$ for $X$, and reduce the model to only two equations, namely,

\[
Y' = \beta (K - Y - Z) \frac{Y + qZ}{K} - (\delta_1 + \gamma) Y + \phi Z, \\
Z' = \gamma Y - (\delta_2 + \phi)Z
\]
These equations will be the baseline for finding the equilibrium in the following sections. We will refer to this as our final model for our calculations.

3.1 Calculating $R_0$

From the disease free equilibrium, we can calculate $R_0$ with some algebraic manipulation. We can see just by looking at our final model that our disease free equilibrium exists at $Y=Z=0$. The Jacobian of the disease free equilibrium is as follows:

$$J(0) = \begin{pmatrix} \beta - (\delta_1 + \gamma) & \phi + \beta q \\ \gamma & -(\delta_2 + \phi) \end{pmatrix}$$

Taking the determinant we get:

$$\text{Det} = (\delta_1 + \gamma - \beta)(\delta_2 + \phi) - \gamma(\phi + \beta q)$$

We want a positive determinant to guarantee the disease free equilibrium is stable, so we have:

$$(\delta_2 + \phi)(\delta_1 + \phi - \beta) - \gamma(\beta q + \phi) > 0$$

$$(\delta_2 + \phi)(\delta_1 + \gamma - \beta) > \gamma(\beta q + \phi)$$

$$(\delta_2 + \phi)(\delta_1 + \gamma) > \gamma(\beta q + \phi) + \beta(\delta_2 + \gamma)$$

$$1 > \frac{\beta(\phi + \delta_2 + q\gamma) + \gamma\phi}{(\delta_1 + \gamma)(\phi + \delta_2)}$$
Now, we need a negative trace. From calculating a positive determinant we found that \((\delta_2 + \phi)(\delta_1 + \gamma - \beta)\) is positive. This means that \((\delta_1 + \gamma - \beta)\) is also positive, so its opposite is negative: \(\beta - (\delta_1 + \phi)\) and this explains that the trace \(\beta - (\delta_1 + \gamma) - (\delta_2 + \phi)\) is negative. This ensures stability, so now we use some algebraic manipulation to get our \(R_0\).

\[
R_0 = \frac{\beta(\phi + \delta_2 + q\gamma) + \gamma\phi}{(\delta_1 + \gamma)(\phi + \delta_2)}
\]

We simplified this equation to get an \(R_0\) that could be easily interpreted.

\[
R_0 = \frac{\beta}{\delta_1 + \gamma} + \frac{\gamma}{(\delta_1 + \gamma)(\phi + \delta_2)}\phi + \frac{\gamma}{(\gamma + \delta_1)(\phi + \delta_2)}\beta q
\]  

(18)

Here is our interpretation of the values in \(R_0\):

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{1}{\delta_1 + \gamma})</td>
<td>average infectious period in low class</td>
</tr>
<tr>
<td>(\frac{\beta}{\delta_1 + \gamma})</td>
<td>average number of new infections made by the low class</td>
</tr>
<tr>
<td>(\frac{\gamma}{\delta_1 + \gamma})</td>
<td>proportion of infected buildings moving from low class to high class</td>
</tr>
<tr>
<td>(\frac{\beta q}{\delta_1 + \phi})</td>
<td>average number of new infections made, given that the house reaches the high class</td>
</tr>
<tr>
<td>(\phi)</td>
<td>risk of infection from high class to low class</td>
</tr>
<tr>
<td>(\beta)</td>
<td>infection rate of the low class</td>
</tr>
<tr>
<td>(\frac{1}{\phi + \delta_2})</td>
<td>average infectious period in the high class</td>
</tr>
</tbody>
</table>

Table 2: Parameter Definitions for \(R_0\)

\(R_0\) is the number of secondary houses and barns infected by a typical house or barn during the infectious period.

### 3.2 Stability of Steady States

In proving the global stability of the endemic and infection free states, we first prove that all possible trajectories point within the domain of our final model. We then need to use the Dulac Function to prove there are no closed trajectories. We use the Poincaré-Bendixon theory to show where the disease free equilibrium and endemic equilibrium are stable in terms of \(R_0\). For our triangle to be a positive invariant set, if we start our trajectory within the triangle(T>0)(see figure 5), we stay in it forever. With our final model:

\[
\begin{align*}
Y' &= \beta(K - Y - Z) \left(\frac{Y + qZ}{K}\right) - (\delta_1 + \gamma)Y + \phi Z \\
Z' &= \gamma Y - (\delta_2 + \phi)Z
\end{align*}
\]
Figure 5: The possible Y and Z values for stability \(\{Y, Z\} | 0 \leq Y \leq K, 0 \leq Z \leq K\)

For the Y plane, (2) when \(Y = 0\) we get \(\beta(K - Z)(\frac{qZ}{K}) + \phi Z > 0\). This tells us that the Y plane has a positive trajectory.

For the Z plane, (1) when \(Z = 0\) we get \(\gamma Y > 0\). This tells us that the Z plane has a positive trajectory.

Now, for \(K(3)\), we add up our Y and Z equations to get:

\[
\beta(K - Y - Z)(\frac{Y + qZ}{K}) - (\delta_1 + \gamma)Y + \phi Z + \gamma Y - (\delta_2 + \phi)Z
\]

We know that \(K = Y + Z\) so this makes the first part of our Y equation equal to zero. This leaves us with

\[-(\delta_1 + \gamma)Y + \phi Z + \gamma Y - (\delta_2 + \phi)Z\]

and by adding like terms we get \(-\delta K < 0\) which is a negative trajectory. So now we know that the trajectory always points towards the inside of the triangle, in other words the triangle is invariant under the flow. This means that there must be a point within the triangle which is globally asymptotically stable.

Now that we know the trajectories, we will use the Dulac Function to determine that there are no closed trajectories [19]. We look for a \((Df, Dg)\) that gives:

\[
\frac{\partial(Df)}{\partial y} + \frac{\partial(Dg)}{\partial z} < 0
\]

We substitute \(f = Y'\) and \(g = Z'\) and \(D = \frac{1}{Y + qZ}\) as one of our many possible D to prove that there are no closed trajectories. In fact,

\[
fD = \frac{f}{Y + qZ} = \frac{\beta(K - Y - Z)}{K} - (\delta_1 + \gamma)\frac{Y}{Y + qZ} + \frac{\phi Z}{Y + qZ}
\]

\[
gD = \frac{g}{Y + qZ} = \frac{\gamma Y}{Y + qZ} - (\delta_2 + \phi)\frac{Z}{Y + qZ}
\]

We add the partial derivatives of \(Df\) and \(Dg\) to show that there are no closed trajectories, namely:
\[
\frac{\partial Df}{\partial y} + \frac{\partial Dg}{\partial z} = -\frac{\beta}{K} - (\delta_1 + \gamma) \frac{qZ}{(Y + qZ)^2} - \frac{\phi Z}{(Y + qZ)^2} - \frac{\gamma Y q}{(Y + qZ)^2} - (\delta_2 + \phi) \frac{Y}{(Y + qZ)^2} < 0
\]

Since there are no closed trajectories, all trajectories go to the equilibrium points. This means that we can use the Poincaré-Bendixson theory to completely analyze the equilibria. According to this theory, if \( R_0 < 1 \), \((0,0)\) is globally asymptotically stable. If \( R_0 > 1 \), then our endemic equilibrium point is globally stable and lies within the triangle [19].

If \( R_0 < 1 \), this means that in the long run, the disease dies out. If \( R_0 > 1 \) the endemic equilibrium is always positive and the disease persists and in the long run the values of \( Y \) approach \( Y^* \) and the values of \( Z \) approach \( Z^* \), the positive endemic state.

The endemic state can be explicitly computed by setting (16) equal to zero and solving for \( Z, Z = \frac{\gamma Y}{\delta_2 + \phi} \). From (15) we get that

\[
\beta(K - Y - \frac{\gamma Y}{\delta_2 + \phi})(\frac{Y - q(\frac{\gamma Y}{\delta_2 + \phi})}{K}) - (\delta_1 + \gamma)Y + \phi(\frac{\gamma Y}{\delta_2 + \phi}) = 0
\]

Since \( Y > 0 \), we can divide by \( Y \) to get:

\[
\beta(K - 1 - \frac{\gamma}{\delta_2 + \phi})Y(\frac{1 + q(\frac{\gamma}{\delta_2 + \phi})}{K}) + (-\delta_1 + \gamma) + (\frac{\gamma \phi}{\delta_2 + \phi}) = 0
\]

and solve for \( Y \), this gives us our \( Y^* \). Since \( Z^* = (\frac{\gamma}{\delta_2 + \phi})Y^* \) then:

\[
Y^* = \frac{K(R_0 - 1)(\delta_1 + \gamma)}{(1 + \frac{\gamma}{\delta_2 + \phi})\beta(1 + \frac{\gamma}{\delta_2 + \phi})}
\]

\[
Z^* = \frac{\gamma K(R_0 - 1)(\delta_1 + \gamma)}{(\delta_2 + \phi)(1 + \frac{\gamma}{\delta_2 + \phi})\beta(1 + \frac{\gamma}{\delta_2 + \phi})}
\]

A positive and globally stable equilibrium whenever \( R_0 \) is greater than 1.
4 Stochastic Process

4.1 Introduction

In attempting to find the most efficient way to control the deer mouse population, we will run spatial simulations of mice as they move between neighboring houses and barns. We began with a spatial community of 100 buildings consisting of 66 barns and 34 houses, with a total of 170 deer mice in the community. We then used two different distributions of the mice among the community that were common occurrences in real life to determine if the control of populations generated from different distributions needed to be approached differently. In the first distribution the mice were evenly spread among the buildings, with a total of 32 susceptible, 27 low, and 41 high risk buildings. In the second distribution a high risk cluster of buildings at or near their carrying capacity was placed in the center of the community, with 75 susceptible, 10 low, and 15 high risk buildings. In the cluster, 5 houses had 4 mice, 10 barns were infested with 14 mice, while 7 barns and 3 houses near the cluster had one mouse. These conditions are intended to model real-life situations where a whole community has a moderate rodent problem or where a handful of houses are completely neglected and cause a problem for the whole community.

Using these two different scenarios, we attempt to find the most efficient way to exterminate and prevent further mouse infestation over a 20 year period (under these two sets of initial conditions). The scenarios will look at ways to control the population of deer mice and number of high risk buildings using different extermination methods. These scenarios include: one basic extermination rate for the whole community, exterminating in only one season (summer, fall, or winter), and exterminating in only barns. Spring is omitted because we assumed that since this period has the highest growth rate, it would be the most inefficient time to exterminate (a few practice simulations proved our assumption to be true). After these rates are determined, the minimum successful rate will be determined for exterminating only in barns and only in the season which was most efficient. Since houses have much smaller carrying capacities and consist of only 1/3 of the spatial community, the minimum successful rate for exterminating in only houses was not determined. However, exterminating in houses with the same effort as the minimum successful rate in barns will be simulated to compare the two. We will then compare these minimum successful extermination rates to the estimated current extermination rate to observe the amount of added effort that needs to occur to control the mice population. By comparing these scenarios using both initial conditions, we will be able to find the lowest rate of extermination to control further infestation of mice and spread of the hantavirus.

The birth and death rates of mice are also important factors to take into account in our model. In the spring and summer, mouse populations are increasing as reproduction rates are at their peak. In the winter, deer mice do not reproduce because of the unfavorable conditions, where very few of their young have a chance of survival. These changes lead to oscillations in the deer mouse population over periods of time and are a necessary element to include in the simulation. The birth and death rates oscillate with the seasons but stay near constant if observed yearly [1]. This last fact is an important factor in approximating the current extermination rate that occurs.

When speaking of extermination rates, it is important to explain the significance
of the rates used in our simulations. Our extermination rate is not in the form of houses exterminated per year. This would not be a realistic rate given that the frequency of extermination in buildings is dependent on the owners becoming aware of a mice problem and subsequently hiring an exterminator. These events are dependent on the number of mice that are in the house and the willingness of the owner to spend money on the extermination. Our extermination rate for our simulation then is the rate at which a homeowner becomes aware of a mice problem in their building multiplied by the probability that the owner hires an exterminator. From this information, one can see that our simulation with a cluster of high risk houses should have more frequent exterminations, since these buildings have mice near their carrying capacity. If we were to use a rate of only exterminations per year, these factors would not be taken into account and the number of mice in each house would not be a factor in the frequency of extermination. However, the problem with our rate is that it is difficult to interpret. The probability in which a person hires an exterminator and the rate of finding mice are not numbers that can be easily determined. The only way to give significance to this rate is to compare it to another rate that is meaningful. Therefore before running any simulations we attempted to find the extermination rate that is currently occurring. In Western Colorado and many other places where there is a large amount of mice, the population of deer mice remains constant. Although there are fluctuations due to weather conditions and seasons, over a large span of time the population remains constant. Therefore we ran simulations to find the extermination rate that would yield a constant deer mice population, and it was found to be 0.17. In order to give significance to our results, they will be compared to this rate of 0.17 to find how much more effort must be placed on extermination to control the deer mouse population.

![Graphical Display of Mouse Population Dynamics](image)

Figure 7: A Graphical Display of Mouse Population Dynamics with Reference to SNV [1]

### 4.2 Process

Our simulation models a typical neighborhood with a combination of one hundred houses and barns in a ten by ten matrix. The matrix is labeled 0 or red for barns and 1 or blue for houses, where the ratio is 2:1 respectively. This matrix shows the location and distribution of houses and barns while taking into account our initial conditions. The second matrix records the number of mice in each building and shows the change in status of the building
as time elapses. A blue patch labels our susceptible houses and barns with no deer mice. A yellow patch indicates a low number of mice, 1-2 or 1-5 deer mice for houses and barns respectively. A red patch indicates a high number of mice, with 3-4 deer mice for houses and 5-15 deer mice for barns. After a house or barn has been exterminated it will go back into the susceptible class. The maximum values for each building represent the carrying capacity of the houses and barns. We assume that the populations do not exceed these numbers because mice either move out or die. Our simulation looks at different scenarios for our initial conditions. A graph is made to show the percentage of buildings that are in the susceptible, low or high infested class over time.

To determine which extermination rates are successful, we developed some requirements for a successful extermination rate. A rate is successful if at least 75% of the buildings have no mice (the susceptible category) and no more than 1% of the buildings have a high amount of mice. If 75% of buildings are mouse-free, this means that for the average building, only 1 out of 4 neighbors have mice. This gives a very small chance that the building will develop a mice problem because of neighboring buildings.

Simulations were run 100 times at a length of 20 years for each scenario, and the results were graphed. Then the mean result for each scenario was found and graphed. The minimum extermination rate, which resulted in at least 75% of buildings susceptible and less than 1% of buildings as high risk, is recorded. After the most efficient season is determined, we will run 100 simulations in which extermination occurs only in this season and only in barns. To be able to make adequate comparisons about the efficiency of each rate—and to compare to our current rate of 0.17—all rates are manipulated to find an index that is equivalent to the amount of work exerted had extermination occurred in all buildings and all seasons. For example, if the successful rate in the winter is 1.0, the amount of work done in this case is equivalent to an index of 0.25 when extermination occurs year round, since extermination is only occurring for 1/4 of the year. Similarly, an extermination rate of 1.0 when only barns are exterminated is equivalent in effort to an index of 0.667 when there is extermination in all buildings (since barns account for 2/3 of the buildings in our spatial community).

4.3 Results

There were no major differences in treating the set of houses that have evenly distributed mice compared to the clustered population. Although the clustered population did have slightly lower rates in a few simulations, there was no difference that would be considered significant. The differences were between 1.5 and 6.4%. The small discrepancy is due to the fact that if the high risk barns are spotted and exterminated early, the number of mice will drop quickly. However, the similarity in numbers shows that this is a rare occurrence and that the clustered population behaves similar to the evenly distributed population.

For the seasons, the best time to exterminate is in the winter or fall. With an evenly distributed population the extermination rate was 1.25 per year (with and index of 0.3125), while the rates for the clustered population were 1.25 (0.3125 index) for fall and 1.175 (0.29375) for winter. According to the mean results, winter was slightly more effective in the extermination, but not by much. Summer had a higher rate of 1.325 (0.33125) for both distributions of mice.
The minimum successful rate for exterminating only barns was 0.375 (0.25 index). Although the minimum successful extermination rate of only houses was not specifically tested, a simulation of only houses was done with the rate of 0.75 and an index (0.25) equal to that of the minimum extermination rate for barns. This left 29 buildings at a high risk, 64 buildings at a low risk, and only 7 susceptible buildings that had no mice. This shows that exterminating only in houses is not an effective way to prevent mice infestation without a high number of houses being exterminated which would not result in a realistic or cost effective solution. This should be a warning to homeowners who take good care of their homes and neglect their barns to start paying more attention to the prevalence of mice in their barns.

However, the most successful rate was exterminating just barns and only in the winter. This rate was equivalent to the extermination of just barns—0.375—even though there was only extermination in the winter. Its index is 0.0625, even lower than the current extermination rate. This is a very significant finding because it shows that the amount of effort currently being exerted is more than enough to control the deer mouse population. In fact, it is more than twice the effort that is needed. For those people who choose to exterminate at a constant rate throughout the year, they will have to exert almost twice the effort that they are currently exerting, and over 4 times the effort if they were exterminating only barns in the winter. Exterminating in the summer and spring are inefficient because of the higher deer mouse population. A great deal of these mice are young and will not survive until the winter, which means effort is wasted killing mice that will naturally die in a short period of time. By killing in the winter the number of adults who would reproduce in the spring would be much lower and hence the expected number of offspring (including grandchildren) would be lowest. Also, since there is a higher population, there are many areas that will only have one mouse and will end up being exterminated. This causes inefficiency in the effort to control the deer mouse population. Exterminating houses also creates inefficiency because it decreases the average number of mice that are killed per extermination, since barns are capable of carrying more than 3 times the number of mice than houses. Therefore for those people who do not have the money to exterminate all of their buildings, it would be better to use the money to exterminate the mouse population in a barn, since it most likely has more mice and can serve as an initial home to mice who will later infest houses. Also, it would be better to focus on exterminating in winter, when these exterminations will be more effective.
### Table 3: Chart of Extermination Rates for our Different Scenarios

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Evenly Distributed Population</th>
<th>Index</th>
<th>Clustered Population</th>
<th>Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extermination all seasons, every building</td>
<td>0.325</td>
<td>0.325</td>
<td>0.32</td>
<td>0.32</td>
</tr>
<tr>
<td>Extermination only in Barns</td>
<td>0.375</td>
<td>0.25</td>
<td>0.375</td>
<td>0.25</td>
</tr>
<tr>
<td>Extermination only in the Summer</td>
<td>1.325</td>
<td>0.33125</td>
<td>1.325</td>
<td>0.33125</td>
</tr>
<tr>
<td>Extermination only in the Fall</td>
<td>1.25</td>
<td>0.3125</td>
<td>1.225</td>
<td>0.30625</td>
</tr>
<tr>
<td>Extermination only in the Winter</td>
<td>1.25</td>
<td>0.3125</td>
<td>1.175</td>
<td>0.29375</td>
</tr>
<tr>
<td>Extermination only in Barns in the Winter</td>
<td>0.375</td>
<td>0.0625</td>
<td>0.375</td>
<td>0.0625</td>
</tr>
</tbody>
</table>

![Graph of the Mean for a Typical Successful Simulation](image_url)

**Figure 8:** Graph of the Mean for a Typical Successful Simulation
5 Discussion

The contact rate of humans and SNV is unknown, so we decided to model hantavirus prevalence in houses and barns through the deer mouse population. From our deterministic model we were able to calculate the time when the mouse populations approach extinction and when the population grows to epidemic levels. Since we know that 25% of deer mice are infected with the hantavirus, we know something about the prevalence of the virus in different population sizes. We assume that the number of deer mice in a large population that have the SNV is going to be greater than the number in a small population. The number of infected mice in the small population will only be a fraction of that in the large population. Since the number of deer mice is directly affected by the reproductive number, we know that these assumptions can be made. Houses and barns only differ in the value of their parameters, so we were able to reduce our model of six equations to two to analyze hantavirus prevalence.

Mouse behaviors are an important factor in the model. Without the fluctuations of births and death rates in different seasons, there would not be an ideal time to exterminate and thus no reason to model different extermination rates and different seasons. Few mice that live in buildings seek shelter in fields and few who seek shelter in fields try to move into buildings [12]. In both the deterministic model and the stochastic simulations, we chose to concentrate on mice that live in buildings and move between them. These mice make nests within a house or barn where they live with groups of about 5 mice and where they find an abundant food supply. The mice will leave their nest only when there is a lack of food or when they are being exterminated.

From the stochastic simulation we determined that by focusing on exterminating only in winter and focusing on barns, we can greatly reduce the risk of hantavirus infection with the same (or even smaller) amount of effort that we are currently exerting. In attempting to control the deer mouse population, the problem is not a lack of effort, it is that the effort is used in inefficient manners, such as exterminating in summer or exterminating houses.
Focusing on barns means that more mice will be killed per extermination, since they have more than 3 times the carrying capacity of a house. Exterminating only in the winter assures us that most efforts are on adult mice who will reproduce, not young mice who will probably die naturally. If the people of areas such as western Colorado start using these techniques to begin exterminating more efficiently, they can be very successful in virtually eliminating the risk of being infected with the hantavirus.

6 References

References


7 Appendix

Figure 10: Our Estimation of the Current Extermination Rate where Deer Mouse Population is Constant

Figure 11: Even Distribution with Extermination Rate of 0.3
Figure 12: Even Distribution with Extermination of Houses

Figure 13: Spatial Configuration of Houses and Barns
Figure 14: Even Distribution of Mice in a Spatial Community

Figure 15: Cluster Distribution of Mice in a Spatial Community
function [data,initial]=mouse(n,T,step,ext,display)
%set up the nxn matrix G here
[G,ID]=initpop;
%set up identifying nxn matrix ID here: house=0 barn=1
count=0;
%create matrix that saves the changes through one iteration
m=n,
risk=zeros(n,m);
%sum(sum(G));
LB=5; %low cutoff for a barn
LH=2; %low cutoff for a house
c=0;
t=0;
%event0=0;
%event1=0;
%event2=0;
%event3=0;
%event4=0;
%event5=0;
%event6=0;
%event7=0;
initial=sum(sum(G));
data=[0,0,0,0,0];
datat=[0,0,0,0,0];

%k=sum(sum(ID)) *4+(100- sum(sum(ID))) *1.5;
if display==1
    mapcorrect=ones(n,m);
    clmG=[0 0.1]
        colormap for G: blue,yellow,red
    1 1 0.25
    1 0 0],
    clmID=[.75 0 0]
        colormap for ID: blue,red
    0 0 0.75],
figure(1)
image(ID+mapcorrect), colormap(clmID);  %displays figure 1

figure(2)
map=image(G+mapcorrect); colormap(clmG);  %displays figure 2
set(map,'EraseMode','none');
end

while (t<T) & (sum(sum(G))>0)
    eventlist=0;
    found=0;
    t0=t-fix(t);
    datat=[0,0,0,0,0];
    % set up seasonal birth and death rates
    if t0>=0 & t0<0.25  %spring
        b=3.5;
        mu=0.5;
    elseif t0>=.25 & t0<.5  %summer
        b=3;
        mu=0.5;
    elseif t0>=.5 & t0<.75  %fall
        b=1;
    end

mu = 2;
else if 10 >= 0.75 & 10 <= 1.0
b = 0;
mu = 1.2;
end
mulh = (b - mu) / 4; % density dependent death rate for house
mulb = (b - mu) / 1.5; % for barn
for row = 1:n
  for col = 1:m
    up = row - 1; % Setting up the neighbors
down = row + 1;
left = col - 1;
right = col + 1;
    if up <= 0 % North
      up = n;
    end
    if down > n % South
      down = 1;
    end
    if left <= 0 % West
      left = m;
    end
    if right > m % East
      right = 1;
    end
N = G(up, col);
W = G(row, left);
E = G(row, right);
S = G(down, col);

% set up different rates for barns and houses
if G(row, col) > 0 % if the house or barn has mice
  if ID(row, col) = 1 % patch is a house
    d = 9 / 20 * G(row, col);
    R = b * G(row, col) + (mu + mulh * G(row, col)) * G(row, col) + 4 * d * G(row, col) + G(row, col) * ext;
pd = (mu + mulh * G(row, col)) * G(row, col) * R;
cutoff = LH;
  else % patch is a barn
    d = 9 / 20 * G(row, col);
    R = b * G(row, col) + (mu + mulb * G(row, col)) * G(row, col) + 4 * d * G(row, col) + G(row, col) * ext;
pd = (mu + mulb * G(row, col)) * G(row, col) * R;
cutoff = LB;
  end
if rand(1) > exp(-R * step); % probability that an event occurs
  pd = G(row, col) * d / R;
pb = G(row, col) * b / R;
pect = G(row, col) * ext / R;
x = rand(1);
  if x >= 0 & x < pd
    count(row, col) = count(row, col) - 1; % mouse moves up
  count(up, col) = count(up, col) + 1;
end
%event1; %event1 = event1 + 1;
end
if x >= pd & x < 2 * pd
count(row, col) = count(row, col) - 1; %mouse moves down
count(0, col) = count(col, 0) + 1;
%event2 = 2;
%event2 = event2 + 1;
end
if x >= 2 * pd & x < 3 * pd
count(row, col) = count(row, col) - 1; %mouse moves right
count(row, right) = count(row, right) + 1;
%event3 = 3;
%event3 = event3 + 1;
end
if x >= 3 * pd & x < 4 * pd
count(row, col) = count(row, col) - 1; %mouse moves left
0, col) = count(0, col) + 1;
%event4 = 4;
%event4 = event4 + 1;
end
if x >= 4 * pd & x < 4 * pd + pb %mouse is born
0, col) = count(0, col) + 1;
%event5 = 5;
%event5 = event5 + 1;
end
if x >= 4 * pd + pb & x < 4 * pd + pb + pdt %mouse dies
0, col) = count(0, col) - 1;
%event6 = 6;
%event6 = event6 + 1;
end
if x >= 4 * pd + pb + pdt & x <= 4 * pd + pb + pdt + ext
0, col) = 0; %house/barn is exterminated
%event7 = 7;
%event7 = event7 + 1;
end
end
end

%if (sum(sum(count)) < (sum(sum(G))) & (found == 0)
%[row, col, event];
%found = 1;
%end

if count(row, col) == 0 %house/barn is susceptible
risk(row, col) = 0;
elseif count(row, col) <= cutoff %house/barn is low
risk(row, col) = 1;
else %house/barn is high
risk(row, col) = 2;
end
```matlab
if risk(row,col)==0
    data(2)=data(2)+1; % saving number of susceptibles, low, and high
elseif risk(row,col)==1
    data(3)=data(3)+1;
else
    data(4)=data(4)+1;
end
end
end
if display==1
    set(map,'CData',risk+map_correct);
drawnow;
end
G=count; % updates G, time, number of iterations
$t=t+step;
c=c+1;
data(1)=c;
data(5)=sum(sum(G));
data(c,:)=data;
end
figure(3)
subplot(2,1,1)
plot(data(:,1)*step,data(:,2),'b');
hold on;
plot(data(:,1)*step,data(:,3),'g');
plot(data(:,1)*step,data(:,4),'y');

subplot(2,1,2)
plot(data(:,1)*step,data(:,5),'k');
hold on;
% risk;
% count;
% data;
% sum(sum(G));
% $k=\text{event1 + event2 + event3 + event4 + event5 + event6 + event7}$
% $\text{event}=[0 1 2 3 4 5 6 7]$;
% $\text{event0 event1 event2 event3 event4 event5 event6 event7}$;

Figure 16: Code for Stochastic Model
```
8 Acknowledgments

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