GENDEX--DEX MODULE FOR CONSTRUCTING D-OPTIMAL TREATMENT DESIGNS

BY

Walter T. Federer
Department of Biometrics, Cornell University,

Nam-Ky Nguyen
Designcomputing, and

Sophonie Nshinyabakobeje
Department of Biometrics, Cornell University

ABSTRACT

DEX is a program for constructing D-optimal or near-optimal 2-level fractional factorial treatment designs (FFDs) of resolution III or V and response surface treatment designs (RSDs). DEX can also augment an existing design with additional runs. DEX uses the Federov exchange algorithm. Three model options are available. A main-effect model option produces D-optimal or near D-optimal 2-level factor designs and linear effects of 3-level factors. The interaction model option adds the interaction of 2-level factors and/or the interaction of the linear effect of 3-level factors with other factors to the main-effect model. The full model option adds the quadratic effect of 3-level factors to the interaction model option as for response surface designs. Several examples are presented to illustrate the output for each of the models and for various combinations of 2-level and 3-level factors. An example of augmenting an existing design with additional runs is also presented.

Keywords: Parameter vector, Linear effect, Quadratic effect, Interaction effect, Design matrix, Runs, Factors, Fractional factorial, Response surface design.

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INTRODUCTION

DEX is a program for constructing D-optimal or near-optimal 2-level fractional factorial treatment designs (FFDs) of resolution III or V and response surface treatment designs (RSDs) when 3-level factors are used. DEX can also augment an existing design with additional runs. DEX uses the Federov exchange algorithm described in Nguyen and Miller (1994) and Miller and Nguyen (1992). DEX is one of the ten modules in the Gendex toolkit (See Nguyen, 2001).

The number of 2-level factors m, that may be accommodated with DEX is $0 \leq m \leq 15$ and the number of 3-level factors that may be selected is $0 \leq m \leq 9$. The number of runs n, that may be selected is $p \leq n \leq 128$ where p is the number of parameters in the model. Three different model selections are main-effect model, interaction, and full model. Selecting the main-effect model results in observations at the 0 (-1) and 2 (+1) levels and there are no observations at the 1 (0) level for 3-level factors. The linear effect of a 3-level factor is the parameter. The usual main effect of a 2-level factor is the parameter. Selecting the interaction model adds the interaction of the linear effect of 3-level factors with other 3-level linear effects or with the 2-level factors. Selecting the full model adds combinations of the middle level 0 to the factor levels and the quadratic effect of the 3-level factors.

The following examples illustrate the various types of designs that may be generated using the DEX module. An example of augmenting a design with additional runs is presented; the various steps for doing this are given in detail.

STEPS TO GENERATE A TREATMENT DESIGN

It is assumed that the DEX module has been placed in folder named Gendex on the C:\ drive as explained in Federer *et al.* (2001). To open this module, go to the MS-DOS Prompt directory and change directory to C:\Gendex> with the CD\Gendex command. The output from a DEX run then will be saved as DEX.HTM in this directory. The following steps are used to obtain the design:

1. Type DEX or java -cp C:\Gendex Dex (last command case sensitive), return.
2. The first prompt is "Choose the number of 3-level factors:". Select a number from from 0 to 9. Click OK.
3. The second prompt is "Choose the number of 2-level factors:". Select a number from from 0 to 15. Click OK.
4. The third prompt is "Choose a model:". The three choices are "main-effect". "interaction", and "full model". Click on selected model and on OK.
5. The fourth prompt is "Choose the number of runs:". Select from lowest value, which is the number of parameters p up to 128. Select number and click OK.
6. The fifth prompt is "Enter a random seed:". Any number may be entered or the entry may be left blank. Click OK.
7. The sixth prompt is "Enter the number of tries:". Any number may be entered or the entry may be left blank. Click OK.

8. When the output appears on the screen, click OK at bottom of screen and a note will appear describing the design just created and that it has been saved as DEX.HTM.
9. In Windows Explorer, one needs to highlight DEX.HTM and right click on mouse and on REFRESH to obtain the latest DEX.HTM.

If a copy of the design is desired, go to Windows Explorer and click on the Gendex folder in the C:\drive. Highlight the file DEX.HTM and select PRINT. To PASTE the file in a document, from FILE select OPEN, under EDIT click on SELECT ALL and then on COPY. Go to target document and select PASTE under EDIT to place the output in the desired document.

OUTPUT FOR A TREATMENT DESIGN

The output for the DEX module contains the following:

1. The values of the number of factors m, the number of parameters p, and the number of runs n.
2. The number of tries, try #, to obtain the design.
3. The random seed number used.
4. The number of iterations, # of iterations, for the try.
5. The determinant of **X'X**, det, where **X** is the matrix corresponding to the p parameters for the model selected.
6. The std. det.
7. Next, det which is the m^{th} root of the correlation matrix.
8. Trace V which is the determinant of inverse(**X'X**).
9. Factors level which lists the levels of the m factors.
10. The **X'X** matrix.
11. The inverse of **X'X**.

EXAMPLES

A number of examples are given below to illustrate various aspects of treatment designs created by the DEX module.

Example 1: one 3-level and two 2-level factors in n = 12 runs, main effect model

The first example illustrates the effect of selecting the main-effect model for one 3-level and two 2-level factors. Following the above steps, the output below was obtained.

DEX 3.0: Construct D-optimal fractional factorial and response surface designs
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Note: design for m=3, p=4, and n=12.

try #	1
seed	1007063520660
# of iterations	1
det	20736.0
std. det	1.0
det*	1
trace V	0.3333

Factor levels:

-1	-1	-1
-1	-1	1
-1	-1	1
-1	1	-1
-1	1	-1
-1	1	1
1	-1	-1
1	-1	-1
1	-1	1
1	1	-1
1	1	1
1	1	1

X'X

12	0	0	0
	12	0	0
		12	0
			12

inverse(X'X)

0.0833	0	0	-0
	0.0833	0	-0
		0.0833	-0
			0.0833

Note: DEX used 0.06 seconds.
Note: this software is licensed to AV Biometrics.

Note that even though one 3-level factor was selected, only two levels (0 and 2 or -1 and +1) are included in the Factor levels for 3-level factors to obtain the linear effect. Selecting the main-effect model option, uses only two levels for each factor selected. The determinant of **X'X** is $12^4 = 20,736$. The trace V = 4(0.0833) = 0.3333. The Factor levels given are optimal as indicated by the * after det and this is an orthogonal design. The **X** matrix for this design is obtained by adding a column of ones to the design listed under factor levels above and is::

1	-1	-1	-1
1	-1	-1	1
1	-1	-1	1
1	-1	1	-1
1	-1	1	-1

1	-1	1	1
1	1	-1	-1
1	1	1	-1
1	1	-1	1
1	1	1	-1
1	1	1	1
1	1	1	1

Example 2: one 3-level and two 2-level factors in n = 12 runs, interaction model selected.

This example illustrates the effect of the interaction model for one 3-level factor and two 2-level factors in 12 runs. The output obtained was:

DEX 3.0: Construct D-optimal fractional factorial and response surface designs
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Note: design for m=3, p=7, and n=12.

try #	1
seed	1007069352150
# of iterations 3	
det	2.5165824E7
std. det	0.70233196
det*	0.9508
trace V	0.6458

Factor levels:

-1	-1	-1
-1	-1	-1
-1	-1	1
-1	-1	1
-1	1	-1
-1	1	1
-1	1	1
1	-1	-1
1	-1	1
1	1	-1
1	1	1
1	1	1

X'X

12	-2	0	2	2	0	2
	12	2	0	0	2	0
		12	2	-2	0	2
			12	0	-2	0
				12	2	0
					12	2
						12

inverse(X'X)

0.0938	0.0156	0	-0.0156	-0.0156	0	-0.0156
	0.0911	-0.0156	-0.0026	-0.0026	-0.0156	0.0026
		0.0938	-0.0156	0.0156	0	-0.0156
			0.0911	-0.0026	0.0156	0.0026
				0.0911	-0.0156	0.0026
					0.0938	-0.0156
						0.0911

Note: DEX used 0.11 seconds.
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Listing the factors as A, B, and C, the parameter vector is **B'** = [mean AL B C AL*B AL*C B*C], where AL is the linear effect of factor A. Selecting the interaction model option again uses only two levels for each factor. The difference from the main effect option is that the levels of the 3-level factor occur unequally, i.e., 7 and 5. The **X** matrix for this design is:

1	-1	-1	-1	1	1	1
1	-1	-1	-1	1	1	1
1	-1	-1	1	1	-1	-1
1	-1	-1	1	1	-1	-1
1	-1	1	-1	-1	1	-1
1	-1	1	1	-1	-1	1
1	-1	1	1	-1	-1	1
1	1	-1	-1	-1	-1	1
1	1	-1	1	-1	1	-1
1	1	1	-1	1	-1	-1
1	1	1	1	1	1	1
1	1	1	1	1	1	1

Factor B has an equal number of the two levels but factor C does not. The last three columns are obtained as the product of coefficients in columns, 2 and 3, 2 and 4 and 3 and 4, respectively.

Example 3: one 3-level and two 2-level factors in n = 12 runs, full model selected.

For this example, the same selection as for the previous two examples was used except that the full model was selected. The output obtained was:

DEX 3.0: Construct D-optimal fractional factorial and response surface designs
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Note: design for m=3, p=8, and n=12.

try # 1
seed 1007062292800
# of iterations 0
det 2.8311552E7
std. det 0.06584362
det* 0.7117

trace V 1.25

Factor levels:

-1	-1	-1
-1	-1	1
-1	1	-1
-1	1	1
0	-1	-1
0	-1	1
0	1	-1
0	1	1
1	-1	-1
1	-1	1
1	1	-1
1	1	1

X'X

12	0	0	0	0	0	0	8
	8	0	0	0	0	0	0
		12	0	0	0	0	0
			12	0	0	0	0
				8	0	0	0
					8	0	0
						12	0
							8

inverse(X'X)

0.25	0	0	0	0	0	0	-0.25
	0.125	0	0	0	0	0	-0
		0.0833	0	0	0	0	-0
			0.0833	0	0	0	-0
				0.125	0	0	-0
					0.125	0	-0
						0.0833	-0
							0.375

Note: DEX used 0.22 seconds.
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The **X** matrix for this design is

1	-1	-1	-1	1	1	1	1
1	-1	-1	1	1	-1	-1	1
1	-1	1	-1	-1	1	-1	1
1	-1	1	1	-1	-1	1	1
1	0	-1	-1	0	0	1	0
1	0	-1	1	0	0	-1	0
1	0	1	-1	0	0	-1	0
1	0	1	1	0	0	1	0
1	1	-1	-1	-1	-1	1	1
1	1	-1	1	-1	1	-1	1
1	1	1	-1	1	-1	-1	1
1	1	1	1	1	1	1	1

The parameter vector is **B'** = [mean AL B C AL*B AL*C B*C AQ], where AL and AQ are the linear and quadratic effects of factor A. The last column is obtained as the square of the elements in column 2. These parameters are those used for a response surface design.

Example 4: 3^3, full model, n = 18 runs

Three factors, m = 3, at 3-levels each with n = 18 runs is used to show the number of times a level of a factor appears in the factor combinations, using the full model option. The p = 10 parameters are the mean, the linear and quadratic effects of the m = 3 factors, and the three linear by linear interactions of the three factors. The number times a level appears varies from factor to factor and level to level. Note that there are only 4 instead of 6 occurrences of 0 for the first and third factors and only 3 occurrences of the 0 level for factor 2. For factor 2, 1 occurs eight times and -1 occurs seven times. 1 and -1 each occur seven times for the first and third factors.

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Note: design for m=3, p=10, and n=18.

try #	1
seed	1007395812760
# of iterations	1
det	1.52707072E9
std. det	4.2769493E-4
det*	0.4604
trace V	2.2335

Factor levels:

-1	-1	-1
-1	-1	-1
-1	-1	1
-1	0	0
-1	1	-1
-1	1	-1
-1	1	1
0	-1	1
0	0	-1
0	1	0
0	1	1
1	-1	-1
1	-1	0
1	-1	1
1	0	1
1	1	-1
1	1	0
1	1	1

X'X

18	0	1	0	0	3	0	14	15	14

14	0	3	0	-1	0	0	0	-1
	15	0	0	0	0	0	1	0
		14	0	-1	0	-1	0	0
			12	0	2	0	0	0
				11	0	3	2	3
					12	0	0	0
						14	12	11
							15	12
								14

inverse(X'X)

0.6309	-0.0133	-0.0238	-0.0133	0	-0.0036	0	-0.2223	-0.2732	-0.2223
	0.0771	0.001	-0.0167	0	0.0046	0	-0.0068	-0.0015	0.0244
		0.0689	0.001	0	-0.0015	0	0.0183	-0.0098	0.0183
			0.0771	0	0.0046	0	0.0244	-0.0015	-0.0068
				0.0857	0	-0.0143	0	0	-0
					0.1002	0	-0.0224	0.0261	-0.0224
						0.0857	0	0	-0
							0.3442	-0.0516	0.0004
								0.4196	-0.0516
									0.3442

Note: DEX used 0.28 seconds.
Note: this software is licensed to AV Biometrics.

The parameter vector **B'** = [mean AL BL CL AL*BL AL*CL BL*CL AQ BQ CQ]. The **X** matrix for this design is

1	-1	-1	-1	1	1	1	1	1	1
1	-1	-1	-1	1	1	1	1	1	1
1	-1	-1	1	1	-1	-1	1	1	1
1	-1	0	0	0	0	0	1	0	0
1	-1	1	-1	1	-1	-1	1	1	1
1	-1	1	-1	-1	1	-1	1	1	1
1	-1	1	1	-1	-1	1	1	1	1
1	0	-1	1	0	0	-1	0	1	1
1	0	0	-1	0	0	0	0	0	1
1	0	1	0	0	0	0	0	1	0
1	0	1	1	0	0	1	0	1	1
1	1	-1	-1	-1	-1	1	1	1	1
1	1	-1	0	-1	0	0	1	1	0
1	1	-1	1	-1	1	-1	1	1	1
1	1	0	1	0	1	0	1	0	1
1	1	1	-1	1	-1	-1	1	1	1
1	1	1	0	1	0	0	1	1	0
1	1	1	1	1	1	1	1	1	1

Example 5: m = 5 2-level factors, n = 20 runs, main-effect model

This example and the next two demonstrate the effect of model change for a five 2-level factor design in n = 20 runs. The output for the main-effect model is:

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Note: design for m=5, p=6, and n=20.

try #	4
seed	-1138098773
# of iterations	2
det	6.4E7
std. det	1.0
det*	1
trace V	0.3

Factor levels:

-1	-1	-1	-1	1
-1	-1	-1	-1	1
-1	-1	-1	1	1
-1	-1	1	1	-1
-1	-1	1	1	-1
-1	1	-1	-1	-1
-1	1	-1	1	-1
-1	1	1	-1	-1
-1	1	1	-1	1
-1	1	1	1	1
1	-1	-1	-1	-1
1	-1	-1	-1	-1
1	-1	1	-1	1
1	-1	1	1	-1
1	-1	1	1	1
1	1	-1	1	-1
1	1	-1	1	1
1	1	-1	1	1
1	1	1	-1	-1
1	1	1	-1	1

X'X

20	0	0	0	0	0
	20	0	0	0	0
		20	0	0	0
			20	0	0
				20	0
					20

inverse(X'X)

0.05	0	0	0	0	-0
	0.05	0	0	0	-0
		0.05	0	0	-0
			0.05	0	-0
				0.05	-0
					0.05

Note: DEX used 0.22 seconds.
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The **X** matrix for this design is obtained by adding a column of ones to the design listed under Factor levels. The parameter vector is **B'** = [mean A B C D E].

Example 6: m = 5 2-level factors, n = 20 runs, interaction model

This example demonstrates the effect of changing the model from a main-effect model in Example 5 to the interaction model. The output is:

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Note: design for m=5, p=16, and n=20.

try #	1
seed	1008078627890
# of iterations	0
det	2.951479E20
std. det	0.45035997
det*	0.9514
trace V	0.875

Factor levels:

-1	-1	-1	-1	-1
-1	-1	-1	1	1
-1	-1	1	-1	-1
-1	-1	1	-1	1
-1	-1	1	1	-1
-1	1	-1	-1	1
-1	1	-1	1	-1
-1	1	1	-1	-1
-1	1	1	1	1
1	-1	-1	-1	-1
1	-1	-1	-1	1
1	-1	-1	1	-1
1	-1	1	-1	-1
1	-1	1	-1	1
1	-1	1	1	1
1	1	-1	-1	-1
1	1	-1	1	1
1	1	1	-1	1
1	1	1	1	-1
1	1	1	1	1

X'X

20	2	-2	2	-2	0	0	0	0	2	0	4	2	0	2	2
	20	0	0	0	2	-2	2	-2	0	2	2	0	2	4	0
		20	0	4	2	2	2	2	0	2	-2	0	2	0	0
			20	0	2	2	2	2	4	-2	2	0	-2	0	0
				20	2	2	2	2	0	2	-2	0	2	0	0
					20	0	4	0	2	0	0	-2	0	2	-2
						20	0	4	2	0	0	2	0	-2	2

20	0	2	0	0	-2	0	2	-2
	20	2	0	0	2	0	-2	2
		20	-2	2	0	-2	0	0
			20	0	2	4	2	2
				20	2	0	2	2
					20	2	0	4
						20	2	2
							20	0
								20

inverse(X'X)

0.0547	-0.0039	0.0039	-0.0039	0.0039	-0	0	-0	0	-0.0039	0	-0.0078	-0.0039	0	-0.0039	-0.0039
	0.0547	-0	-0	-0	-0.0039	0.0039	-0.0039	0.0039	-0	-0.0039	-0.0039	0	-0.0039	-0.0078	0
		0.0547	-0	-0.0078	-0.0039	-0.0039	-0.0039	-0.0039	-0	-0.0039	0.0039	-0	-0.0039	-0	-0
			0.0547	-0	-0.0039	-0.0039	-0.0039	-0.0039	-0.0078	0.0039	-0.0039	0	0.0039	-0	0
				0.0547	-0.0039	-0.0039	-0.0039	-0.0039	-0	-0.0039	0.0039	-0	-0.0039	-0	-0
					0.0547	-0	-0.0078	0	-0.0039	-0	-0	0.0039	-0	-0.0039	0.0039
						0.0547	0	-0.0078	-0.0039	0	0	-0.0039	0	0.0039	-0.0039
							0.0547	0	-0.0039	-0	-0	0.0039	-0	-0.0039	0.0039
								0.0547	-0.0039	0	0	-0.0039	0	0.0039	-0.0039
									0.0547	0.0039	-0.0039	0	0.0039	-0	0
										0.0547	0	-0.0039	-0.0078	-0.0039	-0.0039
											0.0547	-0.0039	0	-0.0039	-0.0039
												0.0547	-0.0039	0	-0.0078
													0.0547	-0.0039	-0.0039
														0.0547	0
															0.0547

Note: DEX used 0.39 seconds.
Note: this software is licensed to AV Biometrics.

The **X** matrix for this design is obtained in the usual manner for a 2-level resolution V fractional replicate. The parameter vector is **B'** = [mean A B C D E AB AC AD AE BC BD BE CD CE DE]. Note that the number of combinations at the different levels is unequal for factors A,B, C, and D but equal for factor E.

Example 7: m = 5 2-level factors, n = 20 runs, full model

For this example, the full model is selected. This results in the same type of design and same parameter model as in Example 6. The output is:

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Note: design for m=5, p=16, and n=20.

try #	1
seed	1008078627890
# of iterations	0
det	2.951479E20
std. det	0.45035997
det*	0.9514
trace V	0.875

Factor levels:

-1	-1	-1	-1	-1
-1	-1	-1	1	1
-1	-1	1	-1	-1
-1	-1	1	-1	1
-1	-1	1	1	-1
-1	1	-1	-1	1
-1	1	-1	1	-1
-1	1	1	-1	-1
-1	1	1	1	1
1	-1	-1	-1	-1
1	-1	-1	-1	1
1	-1	-1	1	-1
1	-1	1	-1	-1
1	-1	1	-1	1
1	-1	1	1	1
1	1	-1	-1	-1
1	1	-1	1	1
1	1	1	-1	1
1	1	1	1	-1
1	1	1	1	1

X'X

20	2	-2	2	-2	0	0	0	0	2	0	4	2	0	2	2
	20	0	0	0	2	-2	2	-2	0	2	2	0	2	4	0
		20	0	4	2	2	2	2	0	2	-2	0	2	0	0
			20	0	2	2	2	2	4	-2	2	0	-2	0	0
				20	2	2	2	2	0	2	-2	0	2	0	0
					20	0	4	0	2	0	0	-2	0	2	-2
						20	0	4	2	0	0	2	0	-2	2
							20	0	2	0	0	-2	0	2	-2
								20	2	0	0	2	0	-2	2
									20	-2	2	0	-2	0	0
										20	0	2	4	2	2
											20	2	0	2	2
												20	2	0	4
													20	2	2
														20	0
															20

inverse(X'X)

0.0547	-0.0039	0.0039	-0.0039	0.0039	-0	0	-0	0	-0.0039	0	-0.0078	-0.0039	0	-0.0039	-0.0039
	0.0547	-0	-0	-0	-0.0039	0.0039	-0.0039	0.0039	-0	-0.0039	-0.0039	0	-0.0039	-0.0078	0
		0.0547	-0	-0.0078	-0.0039	-0.0039	-0.0039	-0.0039	-0	-0.0039	0.0039	-0	-0.0039	-0	-0
			0.0547	-0	-0.0039	-0.0039	-0.0039	-0.0039	-0.0078	0.0039	-0.0039	0	0.0039	-0	0
				0.0547	-0.0039	-0.0039	-0.0039	-0.0039	-0	-0.0039	0.0039	-0	-0.0039	-0	-0
					0.0547	-0	-0.0078	0	-0.0039	-0	-0	0.0039	-0	-0.0039	0.0039
						0.0547	0	-0.0078	-0.0039	0	0	-0.0039	0	0.0039	-0.0039
							0.0547	0	-0.0039	-0	-0	0.0039	-0	- 0.0039	0.0039
								0.0547	-0.0039	0	0	-0.0039	0	0.0039	-0.0039
									0.0547	0.0039	-0.0039	0	0.0039	-0	0
										0.0547	0	-0.0039	-0.0078	-0.0039	-0.0039
											0.0547	-0.0039	0	-0.0039	-0.0039
												0.0547	-0.0039	0	-0.0078
													0.0547	-0.0039	-0.0039
														0.0547	0
															0.0547

Note: DEX used 0.39 seconds.
Note: this software is licensed to AV Biometrics.

The **X** matrix for this design is the same as for Example 6.

Example 8: m = 3 3-level factors, n = 18 runs, main effect model

This example is used to show the effect of model selection for 3-level factor designs. The parameter vector for the main-effect model is **B'** = [mean AL BL CL], where these are the linear effects for the three factors A, B, and C. The output is:

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Note: design for m=3, p=4, and n=18.

try #	1
seed	1008080646900
# of iterations	4
det	102400.0
std. det	0.97546107
det*	0.9938
trace V	0.225

Factor levels:

-1	-1	-1
-1	-1	-1
-1	-1	1
-1	-1	1
-1	-1	1
-1	1	-1
-1	1	-1
-1	1	-1
-1	1	1
1	-1	-1
1	-1	-1
1	-1	-1
1	-1	1
1	1	-1
1	1	-1
1	1	1
1	1	1
1	1	1

X'X

18	0	0	-2
	18	2	0
		18	0
			18

inverse(X'X)

0.0562	0	-0	0.0062
	0.0562	-0.0062	-0
		0.0562	-0
			0.0562

Note: DEX used 0.11 seconds.
Note: this software is licensed to AV Biometrics.

The **X** matrix for this design is obtained by adding a column of ones to the design under Factor levels. Note that the middle level 0, is not included as one of the combinations.

Example 9: m = 3 3level factors, n = 18 runs, interaction model

This example uses the same values of m and n as in Example 8 except that the interaction model is selected. The output is:

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Note: design for m=3, p=7, and n=18.

try #	1
seed	1008080798110
# of iterations	4
det	5.5364813E8
std. det	0.90432864
det*	0.9857
trace V	0.3996

Factor levels:

-1	-1	-1
-1	-1	-1
-1	-1	1
-1	-1	1
-1	1	-1
-1	1	-1
-1	1	1
-1	1	1
1	-1	-1
1	-1	-1
1	-1	-1
1	-1	1
1	-1	1
1	-1	1
1	1	-1
1	1	-1
1	1	1
1	1	1

X'X

18	2	-2	0	-2	0	0

18	-2	0	-2	0	0
	18	0	2	0	0
		18	0	2	-2
			18	0	0
				18	-2
					18

inverse(X'X)

0.0573	-0.0052	0.0052	0	0.0052	-0	- 0
	0.0573	0.0052	0	0.0052	-0	- 0
		0.0573	0	-0.0052	-0	-0
			0.0568	0	-0.0057	0.0057
				0.0573	-0	-0
					0.0568	0.0057
						0.0568

Note: DEX used 0.05 seconds.
Note: this software is licensed to AV Biometrics.

The **X** matrix for this design is obtained in the usual manner by using products of the coefficients listed under Factor levels above. The parameter vector **B'** = [mean AL BL CL AL*BL AL*CL BL*CL].

Example 10: m = 3 3-level factors, n = 18 runs, full model

Using the same values of m = 3 and n = 18 as in the previous two examples, the output for the full model is:

DEX 3.0: Construct D-optimal fractional factorial and response surface designs
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Note: design for m=3, p=10, and n=18.

try #	1
seed	1008083233280
# of iterations	1
det	1.52707072E9
std. det	4.2769493E-4
det*	0.4604
trace V	2.2335

Factor levels:

-1	-1	-1
-1	-1	-1
-1	-1	1
-1	0	0
-1	0	1
-1	1	-1
-1	1	0
-1	1	1
0	-1	0
0	0	-1

0	1	1
1	-1	-1
1	-1	-1
1	-1	1
1	0	1
1	1	-1
1	1	0
1	1	1

X'X

18	-1	0	0	0	0	3	15	14	14
	15	0	0	0	0	0	-1	0	0
		14	3	0	0	-1	0	0	-1
			14	0	0	-1	0	-1	0
				12	2	0	0	0	0
					12	0	0	0	0
						11	2	3	3
							15	12	12
								14	11
									14

inverse(X'X)

0.6309	0.0238	-0.0133	-0.0133	0	0	-0.0036	-0.2732	-0.2223	-0.2223
	0.0689	-0.001	-0.001	0	0	0.0015	0.0098	-0.0183	-0.0183
		0.0771	-0.0167	0	0	0.0046	-0.0015	-0.0068	0.0244
			0.0771	0	0	0.0046	-0.0015	0.0244	-0.0068
				0.0857	-0.0143	0	0	-0	-0
					0.0857	0	0	-0	-0
						0.1002	0.0261	-0.0224	-0.0224
							0.4196	-0.0516	-0.0516
								0.3442	0.0004
									0.3442

Note: DEX used 0.11 seconds.
Note: this software is licensed to AV Biometrics.

The **X** matrix for this design is obtained by adding the squared elements of the factor levels to the X matrix of Example 9. The parameter vector **B'** = [mean AL BL CL AL*BL AL*CL BL*CL AQ BQ CQ] where the last three terms represent the quadratic effect of factors A, B, and C.

Example 11: An existing design of 11 rows and 3 columns augmented with one additional run

The original design is the first 11 rows of the design following Factor levels in the following output. One additional run is to be added. The procedure for obtaining this output with one additional run follows:

1. In DOS and the directory C:\Gendex>, type notepad, return.

2. An open screen appears. Type in the existing design. Here it was the first 11 rows of the design under Factor levels below. Under FILE, select SAVE AS. Save this file in C:\Gendex as DEX.TXT. Close notepad.
3. In C:\Gendex, type dex, return.
4. At the prompt, DEX.TXT has 11 rows and 3 columns. Click YES to include these rows in the new design.
5. At the prompt, Choose the number of 3-level factors:, choose any of the numbers.
6. At the prompt, Choose the model:, main-effect model was selected.
7. At the prompt choose the number of runs, the choices are from p - n up to 128. Selecting any number less than 1 reproduces the original design. The number 1 was selected to add one run to the original design.
8. At the prompt, Enter a random number:, any number could be entered or it could be left blank and the program selects a random number. Click OK.
9. At the prompt, Enter the number of tries:, any number could be entered or it could be left blank and the program selects a random number. Click OK
10. When the output appears on the screen, click OK at bottom of screen to save the output as DEX.HTM in the C:\Gendex directory.

The output for the design is:

DEX 3.0: Construct D-optimal fractional factorial and response surface designs
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Note: design for m=3, p=4, and n=12.

try #	1
seed	1008166806430
# of iterations	0
det	19008.0
std. det	0.9166667
det*	0.9785
trace V	0.3485

Factor levels:

-1	-1	-1
-1	-1	1
-1	1	-1
-1	1	1
1	-1	-1
1	-1	1
1	1	-1
1	1	1
-1	-1	1
1	1	-1
-1	1	1
1	-1	1

X'X

12	0	0	2
	12	0	-2

12	-2
	12

inverse(X'X)

0.0859	-0.0025	-0.0025	-0.0152
	0.0859	0.0025	0.0152
		0.0859	0.0152
			0.0909

Note: the first 11 runs of the designs are protected runs.
Note: DEX used 0.05 seconds.
Note: this software is licensed to AV Biometrics.

The **X** matrix is obtained by adding a column of ones to the design listed under Factor levels. The parameter vector is **B'** = [mean A B C].

COMMENTS

The above examples illustrate the type of treatment designs that can be obtained using the DEX module of the Gendex toolkit. These are D-optimal or near D-optimal designs. This module together with the FFD and NOA modules provide a versatile set of tools for obtaining fractional replicates of a factorial and response surface designs. An investigator may wish to obtain the desired treatment design by selecting one created from each of the modules.

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