ADDENDUM TO BU1500M: "COMPLETE SET OF F-SQUARES"

BU-1536 -M **October, 2000**

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ABSTRACT

A new property, sums of squares orthogonality (FSSO), is defined. It is used to demonstrate that a complete set of F-squares with FSSO property for the number six can be constructed via the method proposed by Federer (2000). Preliminary investigations for the numbers 10, 12, and 15 indicate that a complete set of F-squares with the FSSO property can be conducted for n = (2 × any prime number) and for n = 3 × 2^s. This is the first time that a complete set of F-squares for n = 6 with the FSSO property has been constructed.

Keywords: Factorial, Orthogonality, Method of Construction, Sums of squares, Single degree of freedom contrast.

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INTRODUCTION

A method for constructing a complete set of F-squares for a mixture of prime numbers has been described by Federer (2000). As amplification of the method and for further insight into the construction procedure, we look at the complete set of orthogonal single degree of freedom contrasts. The idea is that this may lead to a complete set of F-squares with desirable properties. A new concept called *sums of squares orthogonality* (FSSO) is introduced as an alternative to *combinatorial orthogonality*. We show that FSSO can be achieved for a square of side n = 6. A set of F-squares is obtained by using single degree of freedom contrasts. This is not the same set as obtained by the method of Federer (2000). The 6 × 6 square is studied in detail whereas only preliminary investigations are given for n = 10, 12, and 15.

The method of construction, FMOC, described by Federer (2000) is:

Given an n × n square, where n is a product of prime numbers, partition n into its multiples. Then obtain an ANOVA partitioning of degrees of freedom for the factorial formed by the prime numbers. For each main effect and interaction form the corresponding F-square as for prime numbers except that modulo the largest number is used.

The method is illustrated with several examples.

ADL:	1 0-1 1 0-1	1 0-1 1 0-1	1 0-1 1 0-1	-1 0 1-1 0 1	-1 0 1-1 0 1	-1 0 1-1 0 1
ADQ:	-1 2-1-1 2-1	-1 2-1-1 2-1	-1 2-1-1 2-1	1-2 1 1-2 1	1-2 1 1-2 1	1-2 1 1-2 1
FAD:	2 1 0 2 1 0	2 1 0 2 1 0	2 1 0 2 1 0	0 1 2 0 1 2	0 1 2 0 1 2	0 1 2 0 1 2
	0 1 2 0 1 2	0 1 2 0 1 2	0 1 2 0 1 2	1 2 0 1 2 0	1 2 0 1 2 0	1 2 0 1 2 0

ACDL:	-1 0 1 1 0-1	-1 0 1 1 0-1	-1 0 1 1 0-1	1 0-1-1 0 1	1 0-1-1 0 1	1 0-1-1 0 1
ACDQ:	1-2 1-1 2-1	1-2 1-1 2-1	1-2 1-1 2-1	-1 2-1 1-2 1	-1 2-1 1-2 1	-1 2-1 1-2 1
FACD:	0 1 2 2 1 0	0 1 2 2 1 0	0 1 2 2 1 0	2 1 0 0 1 2	2 1 0 0 1 2	2 1 0 0 1 2
	0 1 2 1 2 0	0 1 2 1 2 0	0 1 2 1 2 0	1 2 0 2 0 1	1 2 0 2 0 1	1 2 0 2 0 1

BLC:	1 1 1-1-1-1	0 0 0 0 0 0	-1-1-1 1 1 1	1 1 1-1-1-1	0 0 0 0 0 0	-1-1-1 1 1 1
BQC:	-1-1-1 1 1 1	2 2 2-2-2-2	-1-1-1 1 1 1	-1-1-1 1 1 1	2 2 2-2-2-2	-1-1-1 1 1 1
FBC:	2 2 2 0 0 0	1 1 1 1 1 1	0 0 0 2 2 2	0 0 0 2 2 2	1 1 1 1 1 1	0 0 0 2 2 2
	0 0 0 1 1 1	1 1 1 2 2 2	2 2 2 0 0 0	0 0 0 1 1 1	1 1 1 2 2 2	2 2 2 0 0 0

BLDL:	1 0-1 1 0-1	0 0 0 0 0 0	-1 0 1-1 0 1	1 0-1 1 0-1	0 0 0 0 0 0	-1 0 1-1 0 1
BQDL:	-1 0 1-1 0 1	2 0-2 2 0-2	-1 0 1-1 0 1	-1 0 1-1 0 1	2 0-2 2 0-2	-1 0 1-1 0 1
FBDL:	2 1 0 2 1 0	1 1 1 1 1 1	0 1 2 0 1 2	2 1 0 2 1 0	1 1 1 1 1 1	0 1 2 0 1 2
	0 1 2 0 1 2	1 2 0 1 2 0	2 0 1 2 0 1	0 1 2 0 1 2	1 2 0 1 2 0	2 0 1 2 0 1

BLDQ:	-1 2-1-1 2-1	0 0 0 0 0 0	1-2 1 1-2 1	-1 2-1-1 2-1	0 0 0 0 0 0	1-2 1 1-2 1
BQDQ:	1-2 1 1-2 1	-2 4-2-2 4-2	1-2 1 1-2 1	1-2 1 1-2 1	-2 4-2-2 4-2	1-2 1 1-2 1
FBDQ:	0 2 0 0 2 0	1 1 1 1 1 1	2 0 2 2 0 2	0 2 0 0 2 0	1 1 1 1 1 1	2 0 2 2 0 2
	0 2 1 0 2 1	1 0 2 1 0 2	2 1 0 2 1 0	0 2 1 0 2 1	1 0 2 1 0 2	2 1 0 2 1 0

BLCDL:	-1 0 1 1 0-1	0 0 0 0 0 0	1 0-1-1 0 1	-1 0 1 1 0-1	0 0 0 0 0 0	1 0-1-1 0 1
BQCDL:	1 0-1-1 0 1	-2 0 2 2 0-2	1 0-1-1 0 1	1 0-1-1 0 1	-2 0 2 2 0-2	1 0-1-1 0 1
FBCDL:	0 1 2 2 1 0	1 1 1 1 1 1	2 1 0 0 1 2	0 1 2 2 1 0	1 1 1 1 1 1	2 1 0 0 1 2
	0 1 2 1 2 0	1 2 0 2 0 1	2 0 1 1 2 0	0 1 2 1 2 0	1 2 0 2 0 1	2 0 1 0 1 2

BLCDQ:	1-2 1-1 2-1	0 0 0 0 0 0	-1 2-1 1-2 1	1-2 1-1 2-1	0 0 0 0 0 0	-1 2-1 1-2 1
BQCDQ:	-1 2-1 1-2 1	2-4 2-2 4-2	-1 2-1 1-2 1	-1 2-1 1-2 1	2-4 2-2 4-2	-1 2-1 1-2 1
FBCDQ:	2 0 2 0 2 0	1 1 1 1 1 1	0 2 0 2 0 2	2 0 2 0 2 0	1 1 1 1 1 1	0 2 0 2 0 2
	0 2 1 1 0 2	1 0 2 2 1 0	2 1 0 0 2 1	0 2 1 1 0 2	1 0 2 2 1 0	2 1 0 0 2 1

ABLC:	-1-1-1 1 1 1	0 0 0 0 0 0	1 1 1-1-1-1	1 1 1-1-1-1	0 0 0 0 0 0	-1-1-1 1 1 1
ABQC:	1 1 1-1-1-1	-2-2-2 2 2 2	1 1 1-1-1-1	-1-1-1 1 1 1	2 2 2-2-2-2	-1-1-1 1 1 1
FABC:	0 0 0 2 2 2	1 1 1 1 1 1	2 2 2 0 0 0	2 2 2 0 0 0	1 1 1 1 1 1	0 0 0 2 2 2
	0 0 0 1 1 1	1 1 1 2 2 2	2 2 2 0 0 0	1 1 1 2 2 2	2 2 2 0 0 0	0 0 0 1 1 1

ABLDL:	-1 0 1-1 0 1	0 0 0 0 0 0	1 0-1 1 0-1	1 0-1 1 0-1	0 0 0 0 0 0	-1 0 1-1 0 1
ABQDL:	1 0-1 1 0-1	-2 0 2-2 0 2	1 0-1 1 0-1	-1 0 1-1 0 1	2 0-2 2 0-2	-1 0 1-1 0 1
FABDL:	0 1 2 0 1 2	1 1 1 1 1 1	2 1 0 2 1 0	2 1 0 2 1 0	1 1 1 1 1 1	0 1 2 0 1 2
	0 1 2 0 1 2	1 2 0 1 2 0	2 0 1 2 0 1	1 2 0 1 2 0	2 0 1 2 0 1	0 1 2 0 1 2

ABLDQ:	1-2 1 1-2 1	0 0 0 0 0 0	-1 2-1-1 2-1	-1 2-1-1 2-1	0 0 0 0 0 0	1-2 1 1-2 1

ABQDQ:	-1 2-1-1 2-1	2-4 2 2-4 2	-1 2-1-1 2-1	1-2 1 1-2 1	-2 4-2-2 4-2	1-2 1 1-2 1
FABDQ:	2 0 2 2 0 2	1 1 1 1 1 1	0 2 0 0 2 0	0 2 0 0 2 0	1 1 1 1 1 1	2 0 2 2 0 2
	0 2 1 0 2 1	1 0 2 1 0 2	2 1 0 2 1 0	1 0 2 1 0 2	2 1 0 2 1 0	0 2 1 0 2 1

ABLCDL:	1 0-1-1 0 1	0 0 0 0 0 0	-1 0 1 1 0-1	-1 0 1 1 0-1	0 0 0 0 0 0	1 0-1-1 0 1
ABQCDL:	-1 0 1 1 0-1	2 0-2-2 0 2	-1 0 1 1 0-1	1 0-1-1 0 1	-2 0 2 2 0-2	1 0-1-1 0 1
FABCDL:	2 1 0 0 1 2	1 1 1 1 1 1	0 1 2 2 1 0	0 1 2 2 1 0	1 1 1 1 1 1	2 1 0 0 1 2
	0 1 2 1 2 0	1 2 0 2 0 1	2 0 1 0 1 2	1 2 0 2 0 1	2 0 1 0 1 2	0 1 2 1 2 0

ABLCDQ:	-1 2-1 1-2 1	0 0 0 0 0 0	1-2 1-1 2-1	1-2 1-1 2-1	0 0 0 0 0 0	-1 2-1 1-2 1
ABQCDQ:	1-2 1-1 2-1	-2 4-2 2-4 2	1-2 1-1 2-1	-1 2-1 1-2 1	2-4 2-2 4-2	-1 2-1 1-2 1
FABQCD:	0 2 0 2 0 2	1 1 1 1 1 1	2 0 2 0 2 0	2 0 2 0 2 0	1 1 1 1 1 1	0 2 0 2 0 2
	0 2 1 1 0 2	1 0 2 2 1 0	2 1 0 0 2 1	1 0 2 2 1 0	2 1 0 0 2 1	0 2 1 1 0 2

As can be seen from the above, the method of constructing F-squares using the single degree of freedom contrasts, SDOFM, requires additional work. F-squares were not obtained in some cases. Since each set used for an F-square with three symbols has two degrees of freedom, it should be possible to find some method for constructing F-squares using SDOFM.

Let us now turn our attention to the method of constructing F-squares put forth by Federer (2000). We shall use a numerical example to illustrate some of the properties of the complete set. Federer (1955, Problems, page 5) gives an example of a randomized complete block design with six replicates and six treatments. The data set was originally studied by Bartlett (1936) and later by Federer (1970). The 36 observations will be used as if there were six rows (replicates) and six columns (treatments). The data are presented in Table 1 along with the designation of the 19 F-squares making up a complete set of F-squares for the number six. The FMOC was used to construct these F-squares. A SAS/GLM code is used as the format for presenting this information. The output for this code is given in the APPENDIX.

The sum of squares for each main effect and interaction of the four-factor factorial is given in Table 2. The sum of squares for each corresponding F-square constructed by the FMOC is given also. Note that the Type I sum of squares for any factorial effect is *identical* to that for the corresponding F-square or F-squares. The FSSO property may be stated as

When the sum of squares for any main effect or interaction from a complete factorial is the same as that for the corresponding F-square or F-squares, this property is denoted as sum of squares orthogonality, FSSO.

For every main effect and interaction in Table 2, we note that FSSO holds. Therefore, the 19 F-squares constitute a *complete set* of F-squares for the number six, and the set has the FSSO property.

THE CASE FOR n = 2 × 5 = 10

F-squares with two, F_2, and with five, F_5, symbols may be constructed for the number ten. One may construct (i) one F_2 square and two F_5 squares for rows and for columns, respectively. Alternatively, (ii) one F_5 square and five F_2 squares. Using FMOC for case (i), we obtain

FA	0	0	0	0	0	1	1	1	1	1
FB	0	1	2	3	4	0	1	2	3	4
FAB	0	1	2	3	4	1	2	3	4	0

For a 2 × 5 factorial with ten observation, we note that the above set has the FSSO property.

Or, using the orthogonal polynomial coefficients for linear, L, quadratic, Q, cubic, 3, and quartic, 4, and the SDOFM, we obtain for case (ii):

FA	0	0	0	0	0	1	1	1	1	1
FB	0	1	2	3	4	0	1	2	3	4
BLA	2	1	0	-1	-2	-2	-1	0	1	2
FBLA	1	1	1	0	0	0	0	0	1	1
BQA	-2	1	2	1	-2	2	-1	-2	-1	2
FBQA	0	1	1	1	0	1	0	0	0	1
B3A	-1	2	0	-2	1	1	-2	0	-1	2
FB3A	0	1	0	0	1	1	0	1	1	0
B4A	1	-4	6	-4	1	-1	4	-6	4	-1
FB4A	1	0	1	0	1	0	1	0	1	0

A fold-over design for each of the FAXB squares was used in order to obtain equal frequency of ones and zeros. For the numerical example above, we note that this set is also FSSO.

THE CASE FOR n = 3 × 4 = 3 × 2 × 2 = 12

For the number 12, we may construct one F_3 square and three F_4 squares, four F_3 squares and three F_2 squares, or one F_2 square and two F_6 squares. For the first case with FMOC,

FA	0	0	0	0	1	1	1	1	2	2	2	2
FB	0	1	2	3	0	1	2	3	0	1	2	3
FAB	0	1	2	3	1	2	3	0	2	3	0	1
FA2B	0	1	2	3	2	3	0	1	0	1	2	3

This set is not FSSO. In order for this set to be FSSO, the last four numbers of FA2B, 0123, need to be replaced by 1230. Doing this makes the set FSSO.

For the second set of F-squares using FMOC, we have

FA	0	0	0	0	1	1	1	1	2	2	2	2
FB	0	0	1	1	0	0	1	1	0	0	1	1
FC	0	1	0	1	0	1	0	1	0	1	0	1
FD	1	0	0	1	1	0	0	1	1	0	0	1
FAB	0	0	1	1	1	1	2	2	2	2	0	0
FAC	0	1	0	1	1	2	1	2	2	0	2	0
FAD	1	0	0	1	2	1	1	2	0	2	2	0

This set is FSSO for an example.
We use FMOC to form the third set as follows:

FA	0	0	0	0	0	0	1	1	1	1	1	1
FB	0	1	2	3	4	5	0	1	2	3	4	5
FAB	0	1	2	3	4	5	1	2	3	4	5	0

This set is also FSSO, but the complete set of F-squares with this partitioning for a 12 × 12 square would not be possible since six is not a prime number.

THE CASE FOR n = 3 × 5 = 15

One F_3 square and three F_5 squares may be formed for the number 15. For this case and FMOC, the following F-squares are formed:

FA	0	0	0	0	0	1	1	1	1	1	2	2	2	2	2
FB	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4
FAB	0	1	2	3	4	1	2	3	4	0	2	3	4	0	1
FA2B	0	1	2	3	4	2	3	4	0	1	4	0	1	2	3

The level for factor A is doubled modulo 5 to obtain FA2B. This set is FSSO for an example with 15 observations.

COMMENTS AND A CONJECTURE

From the results for n = 6, 10, 12, and 15, it appears that complete sets of FSSO F-squares are available for all n = 2 × prime number and for all 3×2^s. This leads to the following conjectured theorem:

Theorem: *For all n = 2 × prime number and for all 3×2^s, a complete set of F-squares with the FSSO property exists.*

Since combinatorial orthogonality of F-squares appears not to be possible for non-prime numbers, another property

Table 1. SAS/GLM code, data, and 19 F-squares.

```
data  fsquare;
input number rep treat A B AB C D CD AC AD ACD BC BD BD2 BCD BCD2
      ABC ABD ABD2 ABCD ABCD2 ;
datalines;
92  1  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
66  1  2  0  0  0  0  1  1  0  1  1  0  1  2  1  2  0  1  2  1  2
19  1  3  0  0  0  0  2  2  0  2  2  0  2  1  2  1  0  2  1  2  1
29  1  4  0  0  0  1  0  1  1  0  1  1  0  0  1  1  1  0  0  1  1
16  1  5  0  0  0  1  1  2  1  1  2  1  1  2  2  0  1  1  2  2  0
25  1  6  0  0  0  1  2  0  1  2  0  1  2  1  0  2  1  2  1  0  2
60  2  1  0  1  1  0  0  0  0  0  0  1  1  1  1  1  1  1  1  1  1
46  2  2  0  1  1  0  1  1  0  1  1  1  2  0  2  0  1  2  0  2  0
35  2  3  0  1  1  0  2  2  0  2  2  1  0  2  0  2  1  0  2  0  2
10  2  4  0  1  1  1  0  1  1  0  1  2  1  1  2  2  2  1  1  2  2
11  2  5  0  1  1  1  1  2  1  1  2  2  2  0  0  1  2  2  0  0  1
 5  2  6  0  1  1  1  2  0  1  2  0  2  0  2  1  0  2  0  2  1  0
46  3  1  0  2  2  0  0  0  0  0  0  2  2  2  2  2  2  2  2  2  2
81  3  2  0  2  2  0  1  1  0  1  1  2  0  1  0  1  2  0  1  0  1
17  3  3  0  2  2  0  2  2  0  2  2  2  1  0  1  0  2  1  0  1  0
22  3  4  0  2  2  1  0  1  1  0  1  0  2  2  0  0  0  2  2  0  0
16  3  5  0  2  2  1  1  2  1  1  2  0  0  1  1  2  0  0  1  1  2
 9  3  6  0  2  2  1  2  0  1  2  0  0  1  0  2  1  0  1  0  2  1
120 4  1  1  0  1  0  0  0  1  1  1  0  0  0  0  0  1  1  1  1  1
59  4  2  1  0  1  0  1  1  1  2  2  0  1  2  1  2  1  2  0  2  0
43  4  3  1  0  1  0  2  2  1  0  0  0  2  1  2  1  1  0  2  0  2
15  4  4  1  0  1  1  0  1  0  1  2  1  0  0  1  1  2  1  1  2  2
10  4  5  1  0  1  1  1  2  0  2  0  1  1  2  2  0  2  2  0  0  1
 2  4  6  1  0  1  1  2  0  0  0  1  1  2  1  0  2  2  0  2  1  0
49  5  1  1  1  2  0  0  0  1  1  1  1  1  1  1  1  2  2  2  2  2
64  5  2  1  1  2  0  1  1  1  2  2  1  2  0  2  0  2  0  1  0  1
25  5  3  1  1  2  0  2  2  1  0  0  1  0  2  0  2  2  1  0  1  0
24  5  4  1  1  2  1  0  1  0  1  2  2  1  1  2  2  0  2  2  0  0
 8  5  5  1  1  2  1  1  2  0  2  0  2  2  0  0  1  0  0  1  1  2
 7  5  6  1  1  2  1  2  0  0  0  1  2  0  2  1  0  0  1  0  2  1
134 6  1  1  2  0  0  0  0  1  1  1  2  2  2  2  2  0  0  0  0  0
60  6  2  1  2  0  0  1  1  1  2  2  2  0  1  0  1  0  1  2  1  2
52  6  3  1  2  0  0  2  2  1  0  0  2  1  0  1  0  0  2  1  2  1
20  6  4  1  2  0  1  0  1  0  1  2  0  2  2  0  0  1  0  0  1  1
28  6  5  1  2  0  1  1  2  0  2  0  0  0  1  1  2  1  1  2  2  0
11  6  6  1  2  0  1  2  0  0  0  1  0  1  0  2  1  1  2  1  0  2
run ;
proc glm data = fsquare;
class rep treat;
model number = rep treat;
run;
proc glm data = fsquare;
  class A B C D ;
  model number = A B A*B C D C*D A*C A*D A*C*D B*C B*D B*C*D
  A*B*C A*B*D A*B*C*D ;
run ;
proc glm data = fsquare;
class A B C D AB CD AC AD ACD BD BD2 BCD BCD2 ABC ABD ABD2
  ABCD ABCD2 BC;
model number = A B AB C D CD AC AD ACD BC BD BD2  BCD BCD2
ABC ABD ABD2 ABCD ABCD2 ;
RUN; QUIT;
```

Table 2. ANOVAs for four-factor factorial and for the 19 F-squares given in Table 1.

Source of variation	D.F.	Sum of squares	F-square	D.F	Sum of squares
Total	36	85,312.00			
Mean	1	49,580.44			
Row(Rep)	5	2,375.22			
A	1	441.00	FA	1	441.00
B	2	1,283.56	FB	2	1,283.56
A×B	2	650.67	FAB	2	650.67
Column(Tr)	5	26,196.22			
C	1	17,777.78	FC	1	17,777.78
D	2	5,783.39	FD	2	5,783.39
C×D	2	2,635.06	FCD	2	2,635.06
Row×Column	25	7,160.11			
A×C	1	729.00	FAC	1	729.00
A×D	2	522.17	FAD	2	522.17
A×C×D	2	624.50	FACD	2	624.50
B×C	2	360.22	FBC	2	360.22
B×D	4	843.11	FBD	2	707.72
			FBD2	2	135.39
B×C×D	4	805.44	FBCD	2	338.39
			FBCD2	2	467.06
A×B×C	2	612.67	FABC	2	612.67
A×B×D	4	763.67	FABD	2	487.50
			FABD2	2	276.17
A×B×C×D	4	1,899.33	FABCD	2	1,356.17
			FABCD2	2	543.17

needs to be used if it is desired to obtain complete sets of F-squares for non-primes. As is well-known, sums of squares orthogonality has desirable properties.

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APPENDIX

The output for the code given in Table 1 is:

```
                General Linear Models Procedure
                    Class Level Information
                   Class     Levels     Values
                   REP            6     1 2 3 4 5 6
                   TREAT          6     1 2 3 4 5 6
               Number of observations in data set = 36


Dependent Variable: NUMBER
                             Sum of            Mean
Source              DF      Squares          Square   F Value      Pr >
Model               10  28571.44444      2857.14444      9.98    0.0001
Error               25   7160.11111       286.40444
Corrected Total     35  35731.55556
    R-Square              C.V.         Root MSE           NUMBER Mean
    0.799614          45.60221         16.92349              37.11111

Dependent Variable: NUMBER

Source            DF       Type I SS    Mean Square   F Value     Pr > F
REP                5      2375.22222      475.04444      1.66     0.1815
TREAT              5     26196.22222     5239.24444     18.29     0.0001

Source            DF     Type III SS    Mean Square   F Value     Pr > F
REP                5      2375.22222      475.04444      1.66     0.1815
TREAT              5     26196.22222     5239.24444     18.29     0.0001
```

Class Level Information

Class	Levels	Values
A	2	0 1
B	3	0 1 2
C	2	0 1
D	3	0 1 2

Number of observations in data set = 36

Dependent Variable: NUMBER

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	35	35731.55556	1020.90159	.	.
Error	0	.	.		
Corrected Total	35	35731.55556			

R-Square	C.V.	Root MSE	NUMBER Mean
1.000000	0	0	37.11111

Dependent Variable: NUMBER

Source	DF	Type I SS	Mean Square	F Value	Pr > F
A	1	441.00000	441.00000	.	.
B	2	1283.55556	641.77778	.	.
A*B	2	650.66667	325.33333	.	.
C	1	17777.77778	17777.77778	.	.
D	2	5783.38889	2891.69444	.	.
C*D	2	2635.05556	1317.52778	.	.
A*C	1	729.00000	729.00000	.	.
A*D	2	522.16667	261.08333	.	.
A*C*D	2	624.50000	312.25000	.	.
B*C	2	360.22222	180.11111	.	.
B*D	4	843.11111	210.77778	.	.
B*C*D	4	805.44444	201.36111	.	.
A*B*C	2	612.66667	306.33333	.	.
A*B*D	4	763.66667	190.91667	.	.
A*B*C*D	4	1899.33333	474.83333	.	.

Source	DF	Type III SS	Mean Square	F Value	Pr > F
A	1	441.00000	441.00000	.	.
B	2	1283.55556	641.77778	.	.
A*B	2	650.66667	325.33333	.	.
C	1	17777.77778	17777.77778	.	.
D	2	5783.38889	2891.69444	.	.
C*D	2	2635.05556	1317.52778	.	.
A*C	1	729.00000	729.00000	.	.
A*D	2	522.16667	261.08333	.	.
A*C*D	2	624.50000	312.25000	.	.
B*C	2	360.22222	180.11111	.	.
B*D	4	843.11111	210.77778	.	.
B*C*D	4	805.44444	201.36111	.	.
A*B*C	2	612.66667	306.33333	.	.
A*B*D	4	763.66667	190.91667	.	.
A*B*C*D	4	1899.33333	474.83333	.	.

Class Level Information

Class	Levels	Values
A	2	0 1
B	3	0 1 2
C	2	0 1
D	3	0 1 2
AB	3	0 1 2
CD	3	0 1 2
AC	2	0 1
AD	3	0 1 2
ACD	3	0 1 2
BD	3	0 1 2
BD2	3	0 1 2
BCD	3	0 1 2
BCD2	3	0 1 2
ABC	3	0 1 2
ABD	3	0 1 2
ABD2	3	0 1 2
ABCD	3	0 1 2
ABCD2	3	0 1 2
BC	3	0 1 2

Number of observations in data set = 36

Dependent Variable: NUMBER

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	35	35731.55556	1020.90159	.	.
Error	0	.	.		
Corrected Total	35	35731.55556			

R-Square	C.V.	Root MSE	NUMBER Mean
1.000000	0	0	37.11111

Dependent Variable: NUMBER

Source	DF	Type I SS	Mean Square	F Value	Pr > F
A	1	441.00000	441.00000	.	.
B	2	1283.55556	641.77778	.	.
AB	2	650.66667	325.33333	.	.
C	1	17777.77778	17777.77778	.	.
D	2	5783.38889	2891.69444	.	.
CD	2	2635.05556	1317.52778	.	.
AC	1	729.00000	729.00000	.	.
AD	2	522.16667	261.08333	.	.
ACD	2	624.50000	312.25000	.	.
BC	2	360.22222	180.11111	.	.
BD	2	707.72222	353.86111	.	.
BD2	2	135.38889	67.69444	.	.
BCD	2	338.38889	169.19444	.	.
BCD2	2	467.05556	233.52778	.	.
ABC	2	612.66667	306.33333	.	.
ABD	2	487.50000	243.75000	.	.
ABD2	2	276.16667	138.08333	.	.
ABCD	2	1356.16667	678.08333	.	.
ABCD2	2	543.16667	271.58333	.	.

Dependent Variable: NUMBER

Source	DF	Type III SS	Mean Square	F Value	Pr > F
A	1	441.00000	441.00000	.	.
B	2	474.50000	237.25000	.	.
AB	2	155.16667	77.58333	.	.
C	1	17777.77778	17777.77778	.	.
D	2	2541.16667	1270.58333	.	.
CD	2	1040.16667	520.08333	.	.
AC	1	729.00000	729.00000	.	.
AD	2	734.00000	367.00000	.	.
ACD	2	624.50000	312.25000	.	.
BC	2	338.00000	169.00000	.	.
BD	2	104.00000	52.00000	.	.
BD2	2	182.00000	91.00000	.	.
BCD	2	9.50000	4.75000	.	.
BCD2	2	463.50000	231.75000	.	.
ABC	2	612.66667	306.33333	.	.
ABD	2	547.16667	273.58333	.	.
ABD2	2	291.16667	145.58333	.	.
ABCD	2	1356.16667	678.08333	.	.
ABCD2	2	543.16667	271.58333	.	.