Dynamics of HIV/AIDS in Core Groups in the Presence of a Transient Population

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Abstract

We investigate the role of a transient population of male individuals who interact with a core group of prostitutes in the dynamics of a sexually transmitted disease. We use a stochastic model to follow the evolution of the individuals of each population, in terms of the disease. The analysis is concentrated in the equilibrium of the process.

We adapt this model to a city on the U.S.-Mexico border, where some of the parameters have already been estimated. In this way, we seek to provide a method of evaluating the effect of U.S. border policy in the transmission of diseases to the U.S.

1 Introduction

Generally, models built for the study of transmitted disease dynamics consider closed populations, where the number of total individuals is kept constant through time. These populations are divided into several groups according to the type of the disease, but generally into susceptibles, infected, latent and recovered. The mixing can be homogeneous or heterogeneous. In homogeneous mixing, the degree of interaction is the same for the whole population. In the case of heterogeneous mixing, there is a core group where the interactions leading to disease are more intense. The individuals of this group are therefore efficient transmitters of the disease (Hethcote and York 1984).

In our model, we have considered an open population, where there are individuals in transit, entering the system and leaving after a certain amount of time. This model can be

applied to cases where there are temporary residents, such as in border cities, refugee camps and hotels. Within the system, there is a core group with which the transient population may interact actively before leaving the system. In both groups, there are susceptible, latent and infected individuals.

What we study here is the evolution of a sexually transmitted disease, HIV/AIDS, both in a core group formed by prostitutes and in a transient population of Mexicans emigrating to the U.S.. Our objective is to study the dynamics of the disease in terms of the number of infected individuals who leave the system heading towards different destinations, and the proportion of infected individuals in the core group in the endemic equilibria. We assume that different policies have an impact on the migration dynamics in a border city. The model proposed may therefore be a tool for evaluating these policies in terms of the dynamics of the disease. Since most of the surveys regarding the prostitute and the migrant population in the U.S.-Mexican border were done in the city of Tijuana (Baja California, Mexico), we have used the parameter values from these surveys for the adjustment of the model.

2 The Spread of the Disease

AIDS is a terminal sexually transmitted disease associated with the HIV virus. The interval between diagnosis and death is variable. In developed countries, about 50% of the patients die 18 months after the disease has been diagnosed and almost 80% die within 36 months. In Africa and Haiti the survival rate is lower, possibly because the disease is diagnosed in later stages. The mechanisms of propagation of HIV are sexual relations, blood transfusion and perinatal transmission Chin. Lwanga, Mann 1990, cited in Rangel 1997), and needle exchange among others, with higher probability of transmission through sexual intercourse.

The World Health Organization (WHO) has described the spread of this disease in nine regions of the world and five patterns of transmission of HIV/AIDS (Valdespino Gómez et al. 1995a):

a. Transmission in some regions of Africa - sub-Saharan: It is estimated that over a half of the world cases of AIDS have occurred in this region. The disease is mainly transmitted heterosexually and there is a high perinatal transmission rate. AIDS constitutes a significant cause of death among both children and adults.

b. Transmission in the U.S., Western Europe and Australasia: The most important sources of transmission are homosexual males and intravenous drug addicts. Heterosexual trans-

mission is moderately increasing. In urban areas, AIDS is a significant cause of death in 20 to 40 years old adults.

c. Transmission in Latin America and the Caribbean: Heterosexual transmission has increased in some countries of the Caribbean (Haiti, Dominican Republic), Central America (Honduras) and South America (Brazil). The prevalence of HIV infection in pregnant women in these countries is 1% to 2%. The primary sources of transmission are also homosexuals and intravenous drug addicts.

d. Transmission in Southeast Asia: This region shows the greatest growth rate of the disease in the last years, with an estimation of 2.5 million HIV-infected people. The transmission occurs mainly through intravenous drug use and heterosexual contacts.

e. Rest of the world: The regions with the least transmission of HIV until now are the Far East and the Pacific area of the Asian Continent, central Asia and the countries of Eastern Europe and North Africa.

In some parts of the world, such as the U.S. and Western Europe, the prevalence of cases has stabilized in the last years, which indicates that the number of new AIDS cases equals the number of deaths. In other regions, such as Southeast Asia, however, there is currently exponential growth in the number of AIDS cases (Valdespino Gómez et al. 1995a).

In developed countries, the pattern of transmission of HIV is changing. Infection due to heterosexual contact is rapidly increasing in women; health officials expect that over 13,000,000 women will be infected by the year 2000 (Boletín epidemiológico fronterizo 1994, cited in Rangel 1997).

3 Prostitution and AIDS in a Border City: The Case of Tijuana

In the beginnings of this century, a moralist movement arose in the U.S. that spread along most of the country (Rangel 1997). In 1911, bars and horse bets were banned in the U.S.. It was then that in Tijuana the number of cantines, liquor stores and night clubs increased. In this way, Tijuana became the "spiritual and organic relief" of the U.S. citizens, that allowed them to maintain their moralist campaign in their country (Rangel 1997). In 1920, the race tracks attracted so many U.S. that the U.S. government closed the border from 6:00 pm to 8:00 am. The effects of U.S. policy caused great social problems for Tijuana, reflected in the

increasing number of crimes. In 1921, the municipal government of Tijuana decided to close all the types of businesses described above. Later on, these businesses flourished again with greater strength. Tijuana became a refuge for revolutionaries, outlaws, agricultural workers expelled by the U.S. due to the Great Depression, and Mexicans who were seduced by the city (Murrieta and Hernandez 1991, cited in Rangel 1997). From 1942 to 1952, Tijuana increased its population by 250%.

Tijuana is a dynamic border city near the U.S., with 747,381 inhabitants of which 374,632 are women and 372,749 are men (XI Censo General de Población y Vivienda 1990, cited in Rangel 1997). Migration from Mexico to the U.S. has been significant since the end of last century, although with different characteristics through time: differences in origins and destinations, incorporation of a greater proportion of women to the migration flow, increase in temporary and definite migration (Rangel 1997).

The results of one survey in 1994 (Encuesta sobre migración en la Frontera Norte) show that the time spent by the migrant population from the south with destination in the border cities, is in great part short, since in 60% of the cases they stay for less than a month, and only 8% stay for over 6 months. The average length of time expected for the migrant population whose destination is the U.S. is around 25 weeks. Of those who intend to go to the U.S., 40.1% expect to remain as long as possible or even stay, and 33% expect to remain between 1 and 6 months (Rangel 1997).

Prostitution and migration relate in two ways. In the first place, the migrant workers become the main customers of the service, and secondly 84% of the prostitutes in Tijuana come from other states of the country (Rangel 1997). According to Barrón (cited in Rangel 1997), there are approximately 15,000 prostitutes in the city.

The municipality of Tijuana has specific social factors that favor the conditions for a greater exposure to HIV (Rangel 1993, cited in Rangel 1997). These are:

- a high rate of growth generated mostly by a large immigration flow;
- the social and demographic exchange between the cities of Tijuana, San Diego and Los Angeles, and
- the lack of educational programs aimed at preventing the disease.

HIV/AIDS is a recent disease in Mexico. We can conclude that the cases associated with blood transmission originated in Europe and were caused by European blood derivatives, while the first cases of sexual transmission came from the U.S. Endogenous cases have been

documented since 1987. Nevertheless, on the Mexican side of the border with California HIV/AIDS is transmitted in two ways: through interactions between the permanent residents of Tijuana, and through interactions between the transient population and the permanent residents (Rangel and Izazola 1995, cited in Rangel 1997).

Several studies of HIV seroprevalence have been made in the city of Tijuana (Rangel 1997). One corresponds to a study conducted by the Mexican Health Agency during 1987 and 1988 in six cities of Mexico. The prevalence of HIV found among prostitutes was 0.05% (unpublished data), and the same prevalence was found in another study conducted in 1988 (Guereña et al. 1991, cited in Rangel 1997). In a study made in 1990, 2,000 prostitutes were surveyed who were subject to periodic sanitary control, of whom 0.04% were found to be infected (data from the records of Servicios Médicos Municipales, cited in Rangel 1997). In 1990, the INDRE (Instituto Nacional de Diagnóstico y Referencia Epidemiológica de la Secretaría de Salud) conducted a survey, where they found that the prevalence of infection with HIV in female prostitutes in Tijuana was of 0.5% (Rangel 1997). This coincides with data from Valdespino-Gómez et al. (1995b), where prostitutes were surveyed in Mexico and the prevalence of HIV was of 0.5%.

4 The Model

In our model, male individuals enter the city and stay for a short period of time, and may or may not have contacts with the core group of female prostitutes. Female prostitutes enter the city, but stay for longer periods of time, forming the core group with which male individuals interact. This dynamic is reflected in the diagram of Figure 1.

The paramaters represent, respectively,

λ_x: rate of entry of female prostitutes to the city,

λ_y: rate of entry of males to the city,

μ_x: rate of exit of prostitutes from the core group,

μ_y: rate of exit of male individuals from the city,

β: rate of contacts of a male customer with a prostitute,

k: number of contacts per prostitute per day,

P_y: proportion of infected males that arrive in the city,

P_x: proportion of infected female prostitutes that arrive in the city,

π: proportion of infected female prostitutes in the system.

In our model of HIV/AIDS transmission, there are three types of compartments into which individuals can be classified: susceptible (S), latent (E) and infected (I). The first group corresponds to those individuals who have not contracted the disease and, therefore, if exposed to it, can be infected. The latent stage corresponds to the period immediately after effective transmission of the disease (generally a few days), during which the latent individuals cannot in turn transmit the disease (Longini et al. 1989). The infected individuals are those who have already gone through the latent period and are now capable of transmitting the disease to other susceptible individuals. They enter this stage at a rate δ.

In the case where there is a contact between male customers and female prostitutes, the probability of infection depends not only on the probability of coming into contact with an infective individual, but also on the rate of transmission per contact, which differs if the susceptible individual is the customer or the prostitute. In this way, we have two different transmission rates, which are:

θ_F: transmission rate of HIV from females to males,

θ_M: transmission rate of HIV from males to females.

Male individuals enter the city at a rate λ_y and stay for a mean time of $E[S_y] = \frac{1}{\mu_y}$, i.e. the exit rate is μ_y. From this, we infer that the average number of individuals in the system (the city) has a Poisson distribution with parameter $E[S_y] \cdot \lambda_y = \frac{\lambda_y}{\mu_y}$.

The rate of contacts of a male customer with a prostitute is β; the mean time between contacts is exponential with parameter β. So, if the time to the next contact is T, the probability of contact in one given day is $P(T < 1) = 1 - e^{-\beta}$. The mean number of individuals that have contact with prostitutes in a given day is the product of the mean number of individuals and the probability of contact in one day, $\left(1 - e^{-\beta}\right) \frac{\lambda_y}{\mu_y}$.

This value relates to the number of prostitutes in the core group. The rate at which prostitutes arrive in the city is λ_x, and the rate at which they leave is μ_x, i.e. they stay for a mean time of $E[S_x] = \frac{1}{\mu_x}$. The average number of prostitutes in the city is equal to the average number of sexual contacts per day divided by the number of contacts per prostitute

per day, k. But the number of sexual contacts per day is equal to $\left(1-e^{-\beta}\right)\frac{\lambda_y}{\mu_y}$, so the number of female prostitutes at a given time, $\frac{\lambda_x}{\mu_x}$, must be equal to $\left(1-e^{-\beta}\right)\frac{\lambda_y}{k\mu_y}$. If λ_y, μ_y and μ_x are known and a prostitute has an average of k contacts per day, then the rate at which prostitutes arrive in the city, λ_x, is the following:

$$\lambda_x = \frac{\left(1-e^{-\beta}\right)\lambda_y\mu_x}{k\mu_y}$$

At the equilibrium, there is an average of (λ_y/μ_y) male individuals in the system. A proportion P_y of these individuals may be infected with the HIV virus. These individuals can transmit the disease as soon as they arrive in the system. The proportion of non-infected male individuals that arrive in the city, $(1-P_y)$, acquire the infection in the city (i.e., before leaving the system) with a probability η. Since the events of being infected and leaving the city occur at random, the probability of one of them taking place before the other is given by the quotient of the rate at which one event occurs with the sum of the rates of both events. The probability of infection for susceptible male individuals in our model is therefore given by

$$\eta = \frac{\beta\pi\theta_F}{\beta\pi\theta_F + \mu_y},$$

which corresponds to the probability of being infected before leaving the city. Consequently, the proportion of infected customers is

$$P_y + (1-P_y)\frac{\beta\pi\theta_F}{\beta\pi\theta_F + \mu_y}.$$

Once any individual is infected, he or she enters a latent stage. During this stage, the individual cannot infect. In this way, the probability of infecting susceptible individuals depends on whether the infected individual stays in the system long enough to advance to the infectious stage. Therefore, for male customers, the probability of infecting a susceptible female prostitute depends on

$$\frac{\delta}{\delta + \mu_y},$$

and for the female prostitutes, the probability of infecting a susceptible male is

$$\frac{\delta}{\delta + \mu_x}.$$

In our model, the time spent by a prostitute in the system is always longer than the latent period, so we assume that the probability of entering the infectious stage before leaving the system is 1, because μ_x is very small compared to δ.

The rate at which new infected females are recruited in the system is the result of infection of susceptible females due to contact with infected males (given that they already entered the infectious period in the case of male individuals infected within the system), the rate at which infected prostitutes enter the system, and the rate at which infected prostitutes leave the system. The first aspect can be modeled by the following term:

$$\frac{\lambda_y}{\mu_y}\left(P_y + (1-P_y)\frac{\beta\pi\theta_F}{\beta\pi\theta_F+\mu_y}\left(\frac{\delta}{\delta+\mu_y}\right)\right)\beta(1-\pi)\theta_M.$$

In this expression, the proportion of infected male customers calculated above times the total number of customers at a given time gives the number of infected male customers at a given time. The probability that male individuals infected in the system enter the infectious period before leaving is added at this point, multiplying the proportion of male infected in the system. $\beta(1-\pi)\theta_M$ is the rate of transmission of the disease from the infected male individuals to female susceptibles. The rate at which infected female prostitutes enter the system is given by $\lambda_x P_x$, and the rate at which prostitutes leave the system is given by the total number of prostitutes at a certain time times the proportion of those who are infected (π) and the rate of exit from the system (μ_x). But the total number of prostitutes has been calculated above by $\frac{\lambda_x}{\mu_x}$, so the rate at which prostitutes leave the system is $\lambda_x\pi$.

Finally, the average rate of change in the infected female population is given by the following expression:

$$\frac{\lambda_y}{\mu_y}\left(P_y + (1-P_y)\frac{\beta\pi\theta_F}{\beta\pi\theta_F+\mu_y}\left(\frac{\delta}{\delta+\mu_y}\right)\right)\beta(1-\pi)\theta_M + \lambda_x P_x - \lambda_x\pi.$$

5 Numerical analysis

For the parameters in our model, we used values found for the city of Tijuana and other cities with similar characteristics in Mexico.

λ_x: the value of this parameter was calculated based on the rate of arrival and exit of males, the contact rates with the prostitutes and the exit rate of prostitutes.

λ_y: this parameter was estimated in 700 individuals per day (Velasco-Hernández 1997).

μ_x: rate of exit of prostitutes from the core group and was calculated as shown above. We found different values for this parameter in the literature we consulted, so we assumed a value of approximately 4 years.

μ_y: the rate of exit of male individuals was calculated based on the time spent by the individuals in the system, which was estimated in 14 days (Velasco-Hernandez 1997).

β: the contact rate of a male customer with a prostitute was assumed to be 1 every 14 days.

k: it was assumed that prostitutes have 3 contacts with male individuals per day.

P_y: the value for the proportion of infected males that arrive in the city was taken from the prevalence of HIV/AIDS in the male population of Mexico (Valdespino-Gómez et al. 1995a) which is of 0.05%.

P_x: the value for the proportion of infected female prostitutes that arrive in the city was also taken from the prevalence of HIV/AIDS in the female prostitute population of Mexico (Rangel 1997, Valdespino-Gómez et al. 1995b), and is 0.5% in this case,

θ_F: the transmission rate of HIV from females to males was calculated based on the proportion of prostitutes who use condoms (80%), considering the degree of protection that they confer (estimated in 60%) and the particular risk of transmission of the disease from females to males (0.01),

θ_M: the transmission rate of HIV from males to females was also calculated based on the proportion of males who use condoms (which we assume equal to that of prostitutes who use condoms), considering the degree of protection that they confer and the particular risk of transmission of the disease from male to females (0.10),

δ: the length of the latent period, 7 days, was taken from Longini et al. (1989).

With these values, we calculated the average rate of change at the endemic state for the proportion of infected female individuals in the core group and for the corresponding number of infected male individuals in the system (see Appendix I). From the two sets of results, the biological significant solutions are the positive values for each variable, and approximately 17% of the female prostitutes are infected in the core group, with a corresponding 0.09% of the infected male individuals at a given moment.

6 Sensitivity Analysis

As we mentioned before, the ultimate goal of our model is to serve as a tool for evaluating different policies. The most significant parameter related to the application of these policies

is the mean time of permanence of the male individuals in the city, *i.e.* $1/\mu_y$. In Figure 2, we show the variation of the proportion of infected female prostitutes in the equilibrium, and the probability with which the male population is infected in the city, related to the mean time the male population stays in the city. We have evaluated this variation considering different contact rates.

Our results show a dramatic increase on the proportion of infected female prostitutes as the mean time spent by male individuals in the system increases. The contact rate of male individuals with prostitutes is also a factor that increases the proportion of infected females in the core group. The shape of the curve is almost logistic, although further study is needed to confirm this.

In terms of the probability that male individuals are infected in the system, the increase is not as dramatic as the above case, but it is still significant. As the contact rate of male individuals with prostitutes increases, the probability of contracting the disease is higher.

These results are only realistic near the origin, within the first 30 days that the male customers spend in the city. As this period increases, we might expect a different behavior of male customers in terms of the rate of contact with the female prostitutes in the core group.

7 Simulations

To study the dynamics of HIV/AIDS in the core group in the presence of a transient population we study the number of infected individuals in both subpopulations. Since each subpopulation is divided into susceptibles, latent and infected, we can study their evolution through time using computer simulations.

The following figures show the dynamics of the individuals in the city. Let $x(t)$ and $y(t)$ be the number of individuals in the core group and in the transient population, respectively, at time t. The variables $x_-(t)$ and $y_-(t)$ are the susceptibles, $x_+(t)$ and $y_+(t)$ are the infected and $x_l(t)$ and $y_l(t)$ are the latent individuals at time t. Individuals arrive in the city at certains rates. They can be susceptible or infected. The individuals that leave the system can be susceptible, infected or latent. When individuals of both subpopulations interact, those that get infected enter the latent category of the respective subpopulation and, after a certain amount of time they either enter the infected class or leave the city (see Figure 1).

The following are the events that induce changes in the number of individuals in the different classes which implies changes in the whole population at a certain time t. Using the

parameters previously described, we define the rates at which these events occur.

Event	Rate
$x_+(t+h) = x_+(t) + 1,\ x_l(t+h) = x_l(t)$	$\lambda_x P_x$
$x_+(t+h) = x_+(t) + 1,\ x_l(t+h) = x_l(t) - 1$	$\delta x_l(t)$
$x_+(t+h) = x_+(t) - 1]$	$\mu_x x_+(t)$
$x_-(t+h) = x_-(t) + 1$	$\lambda_x(1 - P_x)$
$x_-(t+h) = x_-(t) - 1,\ x_l(t+h) = x_l(t)$	$\mu_x x_-(t)$
$x_l(t+h) = x_l(t) + 1,\ x_-(t+h) = x_-(t) - 1$	$\theta_M \beta y_+(t)(x_-(t)/x(t))$
$x_l(t+h) = x_l(t) - 1,\ x_+(t+h) = x_+(t)$	$\mu_x x_l(t)$
$y_+(t+h) = y_+(t) + 1,\ y_l(t+h) = y_l(t)$	$\lambda_y P_y$
$y_+(t+h) = y_+(t) + 1,\ y_l(t+h) = y_l(t) - 1$	$\delta y_l(t)$
$y_+(t+h) = y_+(t) - 1$	$\mu_y y_+(t)$
$y_-(t+h) = y_-(t) + 1$	$\lambda_y(1 - P_y)$
$y_-(t+h) = y_-(t) - 1,\ y_l(t+h) = y_l(t)$	$\mu_y y_-(t)$
$y_l(t+h) = y_l(t) + 1,\ y_-(t+h) = y_-(t) - 1$	$\theta_F \beta y_-(t)(x_+(t)/x(t))$
$y_l(t+h) = y_l(t) - 1,\ y_+(t+h) = y_+(t)$	$\mu_y y_l(t)$

Using these rates and the values that we found in the literature, we wrote computer programs using Matlab to best simulate the behavior of both populations. One of the programs generated the time at which the events happened in a continous form. The execution time is very high, thus we decided to make the time discrete (days), generating the number of events of each type for each day using a poisson distribution. The execution time was lowered substantially, but not enough. It is time expensive to generate the number of events using the poisson distribution when the rates are large. In the final program we used both a normal and a poisson distribution to generate the number of events. The normal distribution is faster when the rates are large.

Figure 3 is a graph of one of the computer simulations. The value of the parameters we used are the same as those used to find the average rate of change in the proportion of infected female prostitutes in the core group. The number of individuals in both populations vary around a respective mean value which is the one that we found using the equation described before. This mean value can be clearly seen from the graph for the male population. For the prostitutes this conclusion is not as clear. We can expect to see the oscillation around the mean value if the simulation is run for a longer period of time, but this is not biologically feasible. We cannot assume that the conditions will remain the same for a long period of time.

The number of infected prostitutes is higher than the number of infected males. This is

expected since the prostitutes stay in the city for a longer period of time and the rate of infection from a males to females is greater than the rate of infection from females to males.

The distribution for both populations is also shown below (Figures 4 and 5). These graphs were generated using the simulation previously shown. We are assuming that in this stochastic process the populations will not go extint. This assumption is based on the results that we obtained related to the mean value that exists for both populations. The males distribution has a bell shape similar to a poisson distribution, which can be expected. This probably can be achived also for the prostitutes population if, as we said before, the simulation is run for a longer period of time.

8 Conclusions

From the sensitivity analysis that we carried out, we conclude that the longer the male population remains in the city, the higher the probability of transmission of the disease both in the core group and the transient population. This conclusion is also in some measure intuitive, but we seek to provide with this model a way of estimating in what degree this probability of transmission of HIV/AIDS varies. In this way, we want to show how certain policies and socio-economic factors can favor the spread of HIV/AIDS in both the U.S. and Mexico, by causing an increase in the mean time spent by the immigrant male population in the border city.

As a by-product of the construction of this model, we observed the lack of complete surveys and data collection. Given the importance in controlling the spread of HIV/AIDS, we strongly advise to conduct detailed surveys for the better study of the dynamics of this disease in these systems, to be able to design and enforce the most appropriate policies.

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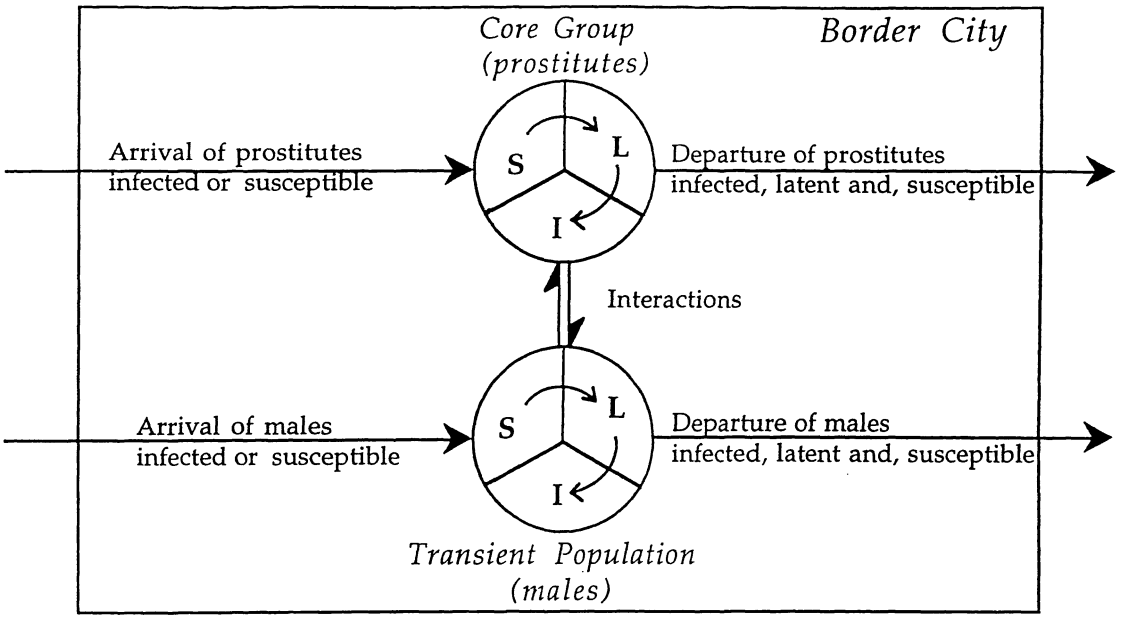

Figure 1. Conceptual model of HIV transmission through prostitution in a border town.

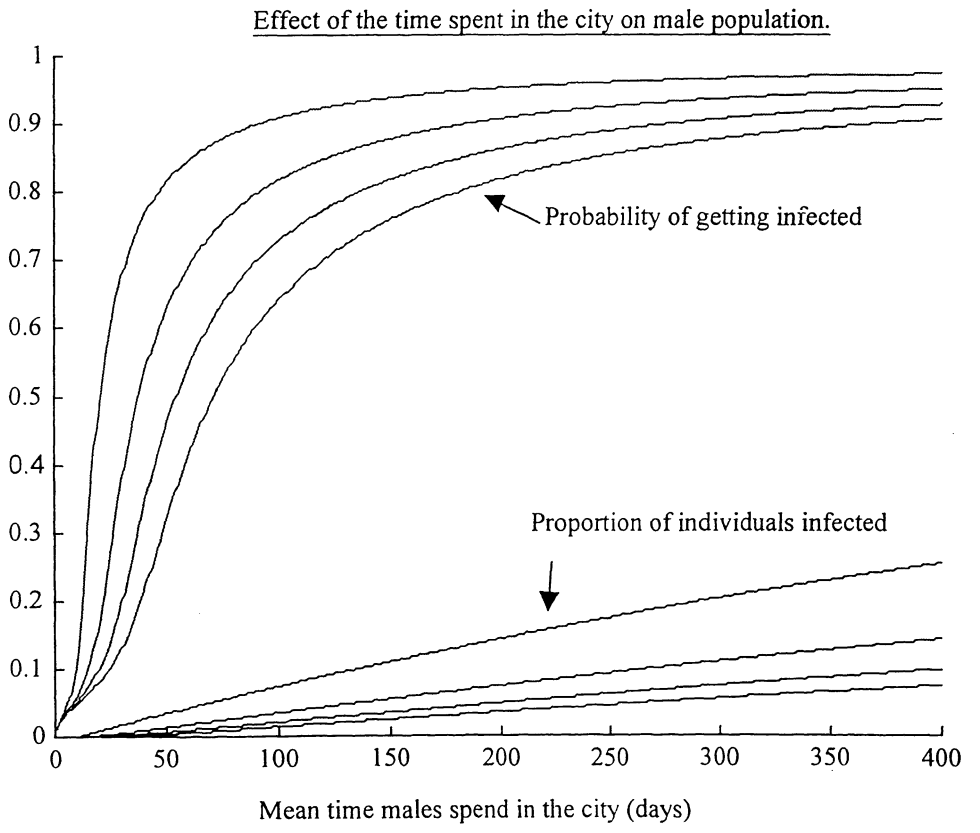

Figure 2. Variation in the proportion of female prostitutes infected at equilbrium.

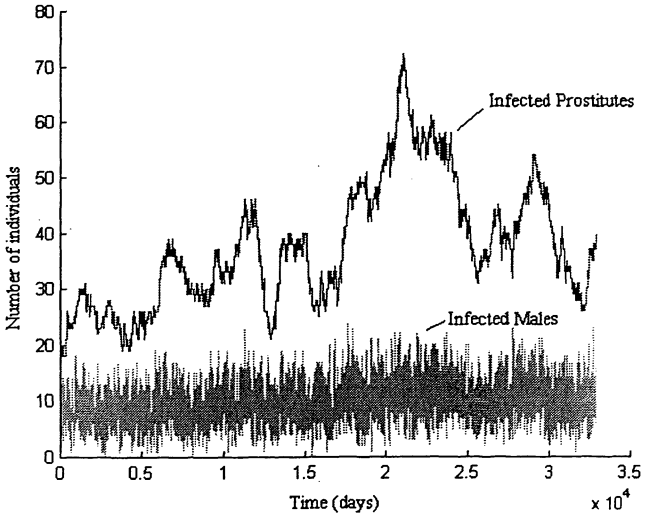

Figure 3. Infected Population.

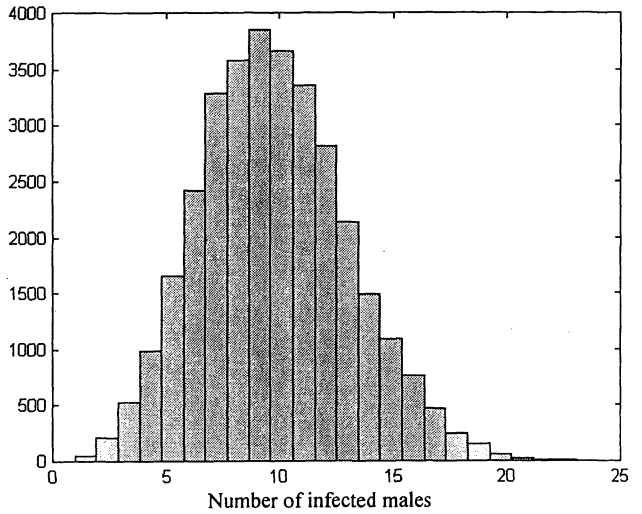

Figure 4. Infected Males Distribution

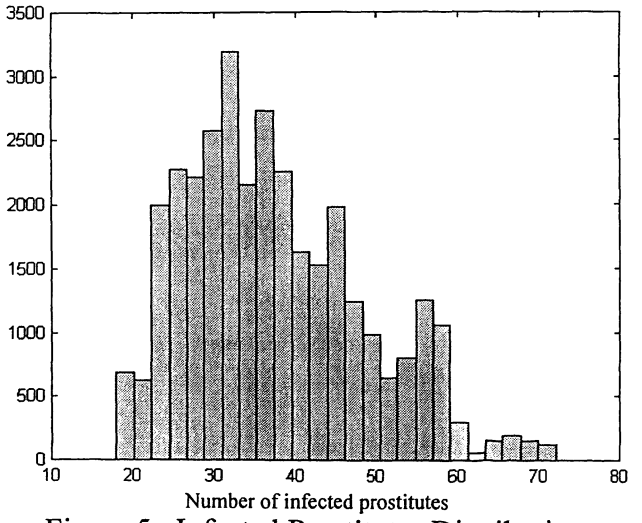

Figure 5. Infected Prostitutes Distribution

Appendix

$$\boldsymbol{f[x\_] := \frac{\lambda_y}{\mu_y}\Big(P_y + (1-P_y)\frac{\beta x\theta_f}{\beta x\theta_f + \mu_y}\frac{\delta}{\delta + \mu_y}\Big)\beta(1-x)\theta_m + \lambda_x(P_x - x)}$$

$$\delta = 1/7$$

$$P_x = 0.005$$

$$P_y = 0.0005$$

$$\beta = 1/14$$

$$\theta_f = (1/100)0.3 * 0.8 + 0.2 * (1/100) = 0.0044$$

$$\theta_m = (1/10)0.3 * 0.8 + 0.2 * (1/10) = 0.044$$

$$\lambda_y = 700$$

$$\mu_x = 1/(4 * 365) = \frac{1}{1460}$$

$$\mu_y = 1/14$$

$$k = 3$$

$$\boldsymbol{\lambda_x = (((1 - \exp[-\beta])\lambda_y\mu_x)/(k\mu_y)) = \frac{490}{219}(1 - \frac{1}{e^{1/14}})}$$

$$\text{Solve}[\{f[x] == 0\}, x]$$

$$\{\{x \to -1.04123\}, \{x \to 0.17058\}\}$$

$$h = \frac{\lambda_y}{\mu_y} = 9800.$$

$$\frac{\lambda_x}{\mu_x} = 225.195$$

$$\boldsymbol{g[x\_] := \Big(P_y + (1-P_y)\frac{\beta 0.1\theta_f}{\beta 0.1\theta_f + \mu_y}\Big) = 0.000939587}$$