A SURVEY OF LOTTO IN THE U.S.A.

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Abstract

Information is summarized from a 1991-2 survey of the lotteries (of the form known as Lotto) operating in many states of the USA and in a few overseas countries.

INTRODUCTION

Recent years have seen an increasing spread of various versions of Lotto throughout the individual states of the U.S.A.; and in overseas countries, too. Almost all these versions are adaptations of the Genoese lottery discussed by Bellhouse (1991). Earlier origins, suggests Morton (1990), go back some two thousand years, and can be traced to the Han Dynasty in China. Whatever the origins, the current forms have a player buy a ticket containing r (usually 6) numbers from the first n (often 54) integers. Prizes are determined by the r numbers shown on the r ping pong (or other) balls selected at random, without replacement, from an urn containing such balls numbered 1 through n. We call those r numbers the draw, and refer to such a lottery as r/n Lotto.

A player's ticket wins the jackpot if its r numbers are the same as the draw. The size of the jackpot is determined by two factors. First, is the amount of money put into the jackpot by the Lotto operator, usually some percentage of the total ticket sales. Second, is the possibility that more than one ticket will have been sold bearing the same r numbers as the draw, whereupon all holders of such tickets share the jackpot. We call this the parimutuel effect.

In addition to the jackpot prize, there are prizes for tickets that contain some fewer than r of the numbers in the draw. A ticket having w such numbers is described as having w "hits". For w = r, the prize is the jackpot (or a share of it); and for some limited values of w (less than r), prizes are awarded for all tickets having w hits. And here the parimutuel effect is very dilutory of prizes. For example, in the 6/n Lotto in New York, a prize is awarded for every ticket containing any one of the possible ${}^6C_5 = 6$ different ways[1] of having 5 hits; likewise for any of the ${}^6C_4 = 15$ different ways of having 4 hits. The pool of money for prizes for 5 hits is much less than the jackpot, and although the pool for 4 hits is usually more than that for 5 (see Tables 4 and 5) the parimutuel effect often reduces the prize per winning ticket to a relatively small amount (e.g., \$1,500 for 5 hits and \$25 for 4 hits).

The parimutuel effect affects just the size of the prize that a winning ticket gets, it does not affect the probability of a ticket winning a prize, i.e., of having w hits. And, of course, the parimutuel effect cannot be quantified until after the draw has been made and the actual number of winning tickets has been counted. Furthermore, the total number of tickets sold is affected by the publicized size of the jackpot, the rate of ticket sales increasing rapidly when the jackpot is known to be getting large, say \$15,000,000 or greater. And of course, the total number of tickets sold is not restricted by any pre-set upper limit; it is limited solely by the time available for selling (four days in New York State for the Wednesday drawing and three for the Saturday draw). All this means that one cannot calculate the expected prize value for a ticket. With some states, though, information is available as to how many cents from each \$1 of sales is retained for operating the lottery and how many cents are allocated to each prize pool (see Table 4 and 5). Then, for example, if 11c is allocated to the 4-hit prize pool, as in New York, the prize for each *sold* ticket that has 4 hits will be

$$\$(.11)(\text{total number of all tickets sold}) \Big/ (\text{total number of 4-hit tickets sold}) .$$

This would be the 4-hit prize that would have to be multiplied by the probability of a ticket having 4 hits as part of calculating the expected prize from a ticket. And clearly, this 4-hit prize cannot itself be derived until after sales have ceased and the draw has been made.

In addition to prizes being awarded for $w < r$ hits, for some w, some states also award prizes for w hits in combination with also hitting some other random drawing, in most cases the draw of a "supplementary" or $(r+1)$th number from the $n-r$ cards not in the draw of r/n Lotto. And in at least one case (Iowa) that supplementary draw is a draw from a pack of 52 playing cards; and in another it is a draw of one number from all n numbers of r/n Lotto.

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[1] The number of ways of choosing s out of t different things is represented by ${}^tC_s$; and ${}^tC_s = t!/s!(t-s)!$ for t! being factorial t, namely $t! = t(t-1)(t-2)\cdots 3(2)1$.

DEFINITION: A TICKET

In r/n Lotto we define each combination of r numbers out of the possible n numbers as a "ticket". This definition is necessary because in the U.S.A. $1 buys a slip of paper which, in some states, shows one ticket (as just defined), whereas in other states it shows two tickets. In almost all cases where the odds of winning are published, they are the odds based on $1, so that in the two-tickets-for-a-dollar states the published odds appear more favorable than in the one-ticket states. To simplify discussion, and particularly for comparing different lotteries, we consider all probabilities on a per-ticket basis, thus ignoring the cost of a ticket; we also ignore the parimutuel effect because, as already explained, it cannot be quantified before the draw.

A possible matter of concern is, for example, the fact that a ticket with 5 hits also has five different sets of 4 hits. But it wins only one prize, that for 5 hits. And this is true in general: a winning ticket wins only one prize, that for the largest number of hits that it contains. Thus in dealing with a ticket's prize-winning possibilities we need consider only the probability of its winning a single prize, that for its total number of hits.

THE SURVEY

To an occasional buyer of Lotto tickets, New York's 1987 changing from 6/48 Lotto to 6/54 Lotto might be though inconsequential. It is not. It greatly reduces the probability of a ticket winning the jackpot, from .000,000,0815 to .000,000,0387 or, equivalently, from 1 chance in (approximately) 12.3 million to 1 in 25.8 million, a reduction of 52.5%. This prompted the idea of gathering information on lotteries in all the states of the U.S.A. to see what differences there are, and to what extent some lotteries might be kinder (more "user friendly", perhaps) than others.

There are, of course, many forms of lottery available. The one which generates much larger prizes than most others is r/n Lotto. Among those others are those of picking an integer between 000 and 999, often called "Pick 3" or, similarly, "Pick 4". And there are the scratch-off games where one buys a ticket and on it scratches off the covering from a set of symbols, certain combinations of which result in cash prizes.

This paper is confined to r/n Lotto (and its variant Keno) as offered in most states of the U.S.A. and the District of Columbia, and in Barbados, Canada, France, Germany, New Zealand, Ontario and South Australia. The survey was designed primarily to gather information about Lotto in the U.S.A. but then it was thought to be interesting to have comparison with a few overseas places, the choice of which was entirely a consequence of personal contacts. For all of these locales we use the word "states" generically, simply for the sake of easy reference. And attention is directed to the probabilities of winning, particularly the extent to which these probabilities vary among the states. Some of the variations in lotto that affect these probabilities, as well as the prize structures, are also discussed. After all, it is the prizes and the probabilities of winning them that appeal to the ticket buyer. Not included are the multitude of newspaper stories about lotteries, many of which discuss the ups and downs in lottery popularity, the size and dispersion of profits, the occurrence of "very unlikely" events (such as winning two big prizes in one game) and stories about "systems" for winning. The best of the latter is, of course, the Australian syndicate which, in early 1992, bought one of nearly every one of the 7,059,052 possible tickets in the 6/44 Lotto in Virginia, at a time when its jackpot was some $29 million. That state has since limited the buying of tickets, because some sales outlets had temporarily closed when asked for 200,000 tickets: printing them was no brief task, and local potential buyers were sorely displeased at being shut out.

To gather information a letter was sent in August 1991 to a statistician in each state enclosing $1 for a ticket and asking for information (to be returned in the enclosed stamped addressed envelope) on the nature of the local lotto. As far as is known, no prizes were won, although one reply did include the message "Your ticket won nothing"; this earned the response "Yours was the only state where that happened".

The first datum arising from these letters was the non-response rate. One year after mailing the letters there was no reply from 22% (all statisticians!). And responses during that year had come with varying degrees of promptness. This was to be expected, and resulted in an inability to get an exact picture of all state lotteries at the same point in time – especially since, from occasional newspaper articles, it is evident that from time to time states are making changes in their lotteries. Thus, the data shown here represent the situation during the twelve months ending August 1992. With benefit of hindsight, writing for information directly to the lottery commission in each state might well have produced better data (and saved the $1 from each letter!). Data would probably have come more promptly. They would have been current at more or less the same time and, for information on prize structure, they would probably have been more complete than what is shown in Tables 4 and 5. However, hindsight is always akin to perfect vision. Nevertheless, although there will have been changes since August 1992 (e.g., Texas at that time was soon to start a lottery), it seems reasonable to think that the general picture given by the collected data will not be greatly affected by those changes.

Information gained from the survey is summarized in six tables. Table 1 shows the states that did not have lotto as of August 1992. Tables 2 (for 6/n Lotto) and 3 (for 5/n Lotto) show states grouped by n and by the hits that win a prize. Tables 4 (for 6/n Lotto) and 5 (for 5/n Lotto) show (the somewhat) sparse information gathered on prizes; and Table 6 summarizes the varieties of Keno, which is a variation of Lotto, that are available in New York, Michigan and Oregon. Comments about these tables follow.

Table 1 This is just a simple list of the states that did not have Lotto as of August 1992.

Table 2 For 6/n Lotto this is a table that groups states by two features of their lotto. First by n: values of n are in the left-most column of the table, in decreasing order, starting with 54 at the top of the table. Also shown is the number N (to the nearest 1,000) for the probability being 1/N of a given ticket winning (or sharing) the jackpot. Thus N, which equals ${}^{n}C_{6}$, the number of different combinations of 6 things available from n, is also the possible number of different tickets that could be sold in 6/n Lotto.

A first observation from Table 2 is that some states have more than one form of lotto. More interesting is the wide range of values of n, from 54 to 25. Although the probability of winning a jackpot is, in all cases, extremely small ($1/{}^{n}C_{6}$) it is greatly affected by n. For example, the probability of a ticket winning (or, more correctly, sharing – though perhaps with no one) the jackpot is 145 times as large in 6/25 Lotto as in 6/54 Lotto. Of course, both probabilities are small: 1 out of 177,100 (= .000,005,647) in 6/25 Lotto and 1 in 25,827,165 (= .000,000,039) in 6/54. Equations (7) in the appendix show a recurrence relationship for calculating these (and other) probabilities, with values for n = 54, 45, 39 and 25 displayed in Table A1 of the appendix. Equations (8) and Table A2 show similar details for 5/n Lotto.

The second classification of the states is according to their list of hits that win prizes. In this regard, the states fall into three main groups, which are the column headings of Table 2: 6, 5, 4 (i.e., prizes given for 6, 5 or 4 hits), 6, 5, 4, 3 and 6, 5^+, 5, 4, 3. In the latter case 5^+ means 5 hits in the usual sense together with a hit on the seventh or supplementary number, usually a number drawn at random from the 54−6 = 48 numbers not in the jackpot-winning draw.

The three column headings account for all but five of the states in the survey. Each of these five has a prize-winning list of hits that is a slight variant of one of the column headings, and so it is included in that column, preceded by its prize-winning list of hits. For example, for 6/54 Lotto, we see that Delaware (and other states) has a prize list of 6, 5 and 4 hits. In contrast, New York's list is 6, 5, 4 and 3^+, which is considered a variant of 6, 5, 4, 3 and thus is entered in the column so labeled; but because it is a variant, its list, 6, 5, 4, 3^+, is shown there too. In this manner there is one variant in the 6, 5, 4 column (Arizona, with n = 47) and three variants in the 6, 5^+, 5, 4, 3 column (Pennsylvania with n = 48, New Zealand with n = 40, and Iowa with n = 39).

Listed under Delaware in Table 1 are seven other states: they and Delaware all have the same form of Lotto, namely 6/54 with prizes for 6, 5 or 4 hits. Underneath that list of eight states are two numbers: (2) and [1500]. The (2) indicates that $1 always buys two tickets (as defined earlier). The [1500] means that in that form of Lotto the probability that a particular ticket will win a prize is 1/1500. By using the 6/54 section of Table A1 of the appendix this probability is calculated as

$$\frac{1+288+16920}{25{,}827{,}165} = .000{,}000{,}039 + .000{,}011{,}151 + .000{,}655{,}124$$

$$= .000{,}666{,}315 = 1/1500.8 \doteq 1/1500\ .$$

(The use of commas in decimal numbers less than unity is unorthodox but helpful when dealing with very small but non-zero numbers.)

The [665] for New York is considerably smaller than the [1500] for Delaware *et al.*; i.e., the probability of a prize is larger. This is because New York gives prizes not only for 6, 5 and 4 hits but also for 3^+ hits, where 3^+ means 3 hits plus a hit on the supplementary number. In this case the probability that a particular ticket will win a prize is the .000,666,315 just calculated for 6, 5 or 4 hits, plus the probability of 3^+ hits. Hence, on using Table A1 for 6/54 Lotto again, the total probability of a prize is

$$.000{,}666{,}315 + \frac{345{,}920}{25{,}827{,}165} \times \frac{3}{48} = .001{,}503{,}418 = 1/665\ .$$

In this calculation, the 3/48 is the probability for a ticket of size 6 that has 3 hits in the draw to also have a hit with one of its other members on the supplementary number.

Notice in Table 2 that the numbers $[\cdot]$ decrease, for given n, in going from the 6, 5, 4 column to the 6, 5, 4, 3 column. This is because in the former, fewer prizes are given than in the latter and so the probability of winning a prize is smaller and hence $[\cdot]$ is larger. The same is true in moving from the 6, 5, 4 column to the 6, 5^+, 5, 4, 3 column because insofar as the probability of winning a prize is concerned the 6, 5, 4, 3 and the 6, 5^+, 5, 4, 3 columns are equivalent, for the same n. (Variants in the columns are exceptions; e.g., Pennsylvania.)

One can also notice that $[\cdot]$ decreases as n increases.

Table 3 For 5/n Lotto this table is laid out in a manner similar to Table 2 and with the same use of numbers $(\cdot)$ and $[\cdot]$.

Different from Table 2 is the instance in Table 3 of N increasing with n decreasing. In Table 2 N always decreases as n decreases. The opposite of this in Table 3 is when n goes from 52 to 45, and N changes from approximately 2 to 54 million. This occurs because although for n = 52, N = ${}^{52}C_5$ = 2,598,960 and ${}^{45}C_5$ = 1,221,757, the latter is not N for the states that have 5/45 Lotto. What they have is a variant of 5/45, which uses a supplementary number chosen from all of the integers 1-45. Thus N is $45\,{}^{45}C_5$ = 54,979,155, shown as 54,979,000 in Table 3. This variant of Lotto advertises itself as "Powerball".

Tables 4 and 5 Table 4 (for 6/n Lotto) and Table 5 (for 5/n Lotto) show data on the prize structure garnered from the back of purchased tickets or from lottery commission information sent by respondents. These are the data that would be more complete if gathered directly from lottery commissions. Even then the form of the data would not be uniform because the states do have different prize structures.

The organization of Tables 4 and 5 is, as in Tables 2 and 3, for decreasing n as one goes down each table; and the states are listed in approximately the same sequence as in Tables 2 and 3. But now there is only one listing of prize-winning hits, as headings to the columns. For each state, a blank in a column means that information was not obtained; a dash in a line where there are other entries means that that column (as a prize-winning number of hits) does not apply; i.e., a prize is not given for that number of hits; e.g., in Table 4 the dashes in the first four lines means that no prize is given for 5^+ hits.

For most states the available data consists of how each $1 of sales revenue is distributed, first for operating Lotto and then to each of the prize pools. For example, in Delaware *et al.*, 55¢ of each $1 of sales goes for operation, 38¢ to the jackpot, and 2¢ and 5¢, respectively, to the 5-hit and 4-hit pools. Some states report a lower limit for one or more prizes, some report average prizes (Ave.), some report exact prizes and some report estimated prizes (Est.). The abbreviations used to indicate the various forms of reporting are shown in the right-hand column of Table 4. They also apply to Table 5.

Table 6 This table is for a variation of Lotto known as Keno. It is available in one of three different forms in New York, Michigan and Oregon. The basic form of Keno is that it is r/n Lotto with tickets of size $t < r$ and prizes for $w \leq t$ hits. So it differs from Lotto in that ticket size, t, differs from r, indeed is usually much less than r: and, r and n in Keno are usually much larger than in Lotto. Keno in New York and in Michigan is of this basic form with one small difference between them. In both of these states $n = 80$ with ticket size $t = 10$ and prizes for 10, 9, 8, 7, 6 and 0 hits; in New York it is 20/80 whereas Michigan has 22/80. For these two forms of Keno the top part of Table 6 shows the prizes (all fixed, no jackpot) in dollar amounts, and it also shows N for the probability $p_w = 1/N$ f having w hits.

In Oregon the Keno is based on 20/80 Lotto as in New York, but whereas New York has just one ticket size $t = 10$, Oregon allows ticket sizes $t = 10, 9, \cdots, 3, 2, 1$. For each ticket size there are prizes for w hits where $w = t$; and for $t \geq 3$ for a limited number of values of w less than t. Thus for $t = 1$ and 2 there are prizes for $w = t$; for $t = 3$ there are prizes for $w = t$ and $t-1$; for $w = 4$ or 5 prizes go for $w = t$, $t-1$ and $t-2$; and so on. For all ticket sizes t, the values of N for $1/N = p_w$ for $w \leq t$ are shown in Table 6, and the dollar prizes (all fixed, no jackpot) for prize-winning hits are also shown.

The right-most column of Table 6 shows the expected return (to the nearest cent) on \$1 purchase (in all cases for a single ticket). A noticeable feature of this column is that this expected return in New York or Michigan is much less than Oregon (except ticket size 2 in Oregon is equivalent to New York's Keno).

The next-to-right-most column in Table 6 shows N for 1/N being the probability of a cash prize. Because of rounding error both in this column and in the columns that show N for $p_w = 1/N$ there will not always be exact correspondence between the relevant N values and N as shown in Table 6. For example for $t = 3$, the N-values are 72 for $w = 3$ and 7 for $w = 2$. But

$$\tfrac{1}{72} + \tfrac{1}{7} = .156{,}746{,}032 = 1/6.379 \neq 1/6.3$$

as shown. Similarly the expected returns as shown will not always have exact correspondence: e.g., for $t = 4$

$$\$50\left(\tfrac{1}{330}\right) + \$5\left(\tfrac{1}{23}\right) + \$1\left(\tfrac{1}{5}\right) = \$.5689 \neq 58 \text{ cents}$$

as shown. It is the rounding error in the denominators 330, 23 and 5 that lead to this inexactness, but it is an inexactitude of no practical importance.

CONCLUSIONS

1. The available forms of Lotto and accompanying prize structure vary greatly among the states of the U.S.A. and the six other places included in the survey.

2. The probability that a particular ticket will win a prize can be calculated, and differs greatly from place to place; but the expected return on a ticket cannot be calculated because of the parimutuel effect.

3. The Lotto having the highest probability of winning a prize is the 6/39 Lotto in Iowa, that probability being 1 in 28 (see Table 2). This is basic 6/39 Lotto, for $1 per ticket plus the option for a further $1 to have as a supplementary number, so to speak, a card (e.g., the five of hearts) from a deck of 52 playing cards; and it has a long list of prize-winning hits (Table 2) although with mostly small prizes (Table 4).

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Table 1

STATES NOT HAVING LOTTO

as of August 1992

Alabama	North Carolina
Alaska	North Dakota
Arkansas	Oklahoma
Georgia	South Carolina
Hawaii	Tennessee
Mississippi	Texas
Nebraska	Utah
Nevada	Wyoming
New Mexico	

Table 2

6/n LOTTO

STATES GROUPED BY n AND BY HITS THAT WIN A PRIZE

NOTES:

In the column headings, for example,

6, 5, 4 means tickets with 6 or 5 or 4 hits win a prize.

3^+ means tickets with 3 hits and some supplementary draw (usually a 7'th number) win a prize.

For all locations listed immediately above pairs of numbers such as (2) [1500]

(t) means t tickets cost $1 (t = 2 or 1).

[M] means 1/M = probability of a ticket winning *some* prize.

n of 6/n Lotto	N (to nearest 1000) for probability 1/N of 6 hits (Jackpot)	Number of hits that win a prize: 6,5,4 and one variant	6,5,4,3 and one variant	$6,5^+,5,4,3$ and three variants
54	25,827,000	Delaware Idaho Iowa Illinois Oregon R. Island S. Dakota W. Virginia (2) [1500]	$6,5,4,3^+$ New York (2) [665]	
53	22,957,000			California (1) [67]
49	13,984,000	Maryland (2) [1013]		Florida Massachusetts Canada (1) [50] Kentucky Washington Wisconsin (2) [50] France Germany [50]
48	12,271,000		Missouri (2) [50]	$6,5^+,5,4^+,4,3^+$ Pennsylvania (2) [415]
47	10,738,000	Michigan (1) [885]	Ohio (1) [47]	

Table 2 (Continued)

n of 6/n Lotto	N (to nearest 1000) for probability 1/N of 6 hits (Jackpot)	Number of hits that win a prize		
		6,5,4 and one variant	6,5,4,3 and one variant	6,5$^+$,5,4,3 and three variants
46	9,367,000	New Jersey (1) (784]		
45	8,145,000			Ontario S. Australia (2) [42]
44	7,059,000	Louisiana (1) [655]	Connecticut Virginia (1) [39]	
			Oregon (2) [39]	
42	5,246,000	6,5$^+$,5,4 Arizona (1) [542]	Colorado Massachusetts (1) [34]	
40	3,838,000		Maine New Hampshire Vermont (1) [30]	6,5$^+$,5,4,3$^+$ New Zealand (2) [200]
39	3,263,000	D. of Columbia (1) [401]		6,5$^+$,5,4$^+$,4,3$^+$,3,2$^+$,1$^+$,0$^+$ Iowa (1) [28]
36	1,948,000	Wisconsin (2) [290]		
34	1,108,000	Kansas (1) [230]		
25	177,000	West Virginia (1) [48]		

Table 3

5/n LOTTO

STATES GROUPED BY n AND BY HITS THAT WIN A PRIZE

NOTES:

In the column headings, for example,

5, 4, 3 means tickets with 5 or 4 or 3 hits win a prize.

3^+ means tickets with 3 hits and also with a separate selection hitting the "Powerball" draw of one number from 1–45.

For all locations listed immediately above numbers such as (1) [235]

(t) means t tickets cost $1 (t = 2 or 1)1.

[M] means 1/M = probability of a ticket winning *some* prize.

n of 5/n Lotto	N (to nearest 1000) for probability 1/N of 5 hits (Jackpot)	Number of hits that win a prize: 5,4,3	$\underbrace{5^+,5,4^+,4,3^+,3,2^+,1^+0^+}$	5,4,3,2
52	2,599,000	Washington (1) [235]		
45*	54,979,000*		Indiana Minnesota Missouri Wisconsin (2) [32]	
38	502,000			Ohio (1) [8]
37	436,000	Montana (2) [85]		
35	325,000	Illinois Massachusetts (1) [72]		
34	278,000	Barbados (1/2) [66]		
32	201,000	Idaho (1) [55]		

* This is 5/45 Lotto plus, if the player wishes, selection of any number from 1–45. Hence $N = 45\left({}^{45}C_5\right) = 54{,}979{,}155$. It is called "Powerball".

Table 4. 6/n LOTTO: INFORMATION ON PRIZE POOLS

n of 6/n Lotto	Locale	Amount retained from each $1 of sales	Prizes: $ amount or cents allocated from each $1 sale — Number of Hits: 6	5^+	5	4^+	4	3^+	3	Abbreviations
54	Delaware									
	Idaho	55¢	38¢	–	2¢	–	5¢	–	–	– in a line having other entries, means that column is not applicable.
	Iowa									
	New York	61¢	20¢	–	4¢	–	11¢	4¢	–	
	Illinois	50¢	35¢	–	5¢	–	10¢	–	–	
	Oregon									
	R. Island									
	S. Dakota	55¢	38¢	–	2¢	–	5¢	–	–	
	W. Virginia									
53	California					–		–	$\nless$ $5	$\nless$ = not less than
49	Maryland	50¢	39¢	–	4¢	–	7¢	–	–	
	Florida									
	Massachusetts		$\nless$ $1m	Ave$50K	Ave$3K	–	Ave$100	–	Free tkt.	$1m = $1,000,000
	Canada									Ave = average
	Kentucky									$50K = $50,000
	Washington		J		Ave$780	–	Ave$30	–	$3	J = Jackpot
	Wisconsin	47¢	J	–	$500	–	$30	–	$2	
	France									
	Germany									
48	Missouri	50¢	J	–		–		–	Free tkt.	Tkt. = ticket
	Pennsylvania	51¢	29¢	3¢	4¢	5¢	5¢	3¢	–	
47	Michigan									
	Ohio		$\nless$ $4m	–	2.75¢	–	8.6¢	–	Spec tkt.	
46	New Jersey									
45	Ontario	50¢	22¢	3¢	9¢	–	16¢	–	$5	
	S. Australia	40¢	16¢	5¢	8¢	14¢	17¢	–	–	
44	Louisiana	52¢	36¢ $\nless$ $1m	–	5¢ $1500	–	7¢ $50	–	–	
	Connecticut			–		–		–	$3	
	Virginia		Est.$1m	–	Est.$1K	–	Est.$50	–	Free tkt.	Est. = Estimated
	Oregon									
42	Arizona	53¢	27¢	5¢	6¢	–	9¢	–	–	
	Colorado	50¢		–		–		–		
	Massachusetts	50¢		–	$5K	–	$75	–	Free tkt.	
40	N. Zealand	40¢	21¢	3¢	7¢	–	17¢	12¢	–	
	Maine									
	N. Hampshire	50¢	34¢	–	$1K	–	$40	–	Free tkt.	
	Vermont									
39	D. of Columbia		$50K	–		–		–	–	
	Iowa[1]		J	$20K	$600	$2K	$30	$200	Free tkt.	
36	Wisconsin		$250K	–	$500	–	$25	–	–	
34	Kansas	55¢	37¢	–	3¢	–	5¢	–	–	
25	W. Virginia		25K	–	Ave.$255	–	Ave. $10	–	–	

[1] Iowa also has prizes for 2^+: $20 and for 1^+ or 0^+: $10.

Table 5

5/n LOTTO

INFORMATION ON PRIZES

n of 5/n Lotto	Locale	Amount retained from each $1 of sales	Prizes: $ amount, or cents allocated from each $1 sale*									
			Number of Hits									
			5^+	5	4^+	4	3^+	3	2^+	2	1^+	0^+
52	Washington		–	$100K	–	$3K	–	$30	–	–	–	–
45	Indiana, Minnesota, Missouri, Wisconsin		$\nless$ $2m	$100K	$5K	$100	–	$5	$5	–	$2	$1
38	Ohio		–	$100K	–	$250	–	$10	–	$1	–	–
37	Montana		–	$\nless$ $20K	–	$200	–	$5	–	–	–	–
35	Illinois	50¢	–	30¢	–	10¢	–	10¢	–	–	–	–
	Massachusetts	44¢	–	$100K	–	$250	–	$10	–	–	–	–
34	Barbados											
32	Idaho	50¢	–		–		–		–	–	–	–

* See right-most column in Table 4 for abbreviations.

Table 6

ODDS AND PRIZES IN KENO IN THREE STATES

Ticket size (t)	Number of Hits (w): 10	9	8	7	6	5	4	3	2	1	0	N for probability of a cash prize = 1/N	Expected return (to nearest cent) on $1
			NEW YORK: t = 10. The draw is 20/80										
10	8,900,000	160,000	7,400	620	87	19	7	3	3	6	22	17	
	$ 500,000	$ 6,000	$ 300	$40	$10						$4		50¢
			MICHIGAN: t = 10. The draw is 22/80										
10	2,500,000	57,000	3,100	310	52	14	6	4	4	7	31	18	
	$ 250,000	$ 2,500	$ 250	$25	$ 7						$1		47¢
			OREGON: t = 10, or 9, or 8, or···2, or 1. The draw is 20/80										
10	8,900,000	160,000	7,400	620	87	19	7	3	3	6	22	9.0	
	$ 100,000	$ 4,000	$ 400	$50	$10	$2					$5		62¢
9		140,000	31,000	1,400	175	31	9	3	4	4	16	6.5	
		$ 25,000	$ 2,500	$200	$25	$4	$1						62¢
8			230,000	6,200	420	55	12	5	3	4	11	9.8	
			$ 10,000	$500	$50	$10	$2						61¢
7				41,000	1,400	120	19	6	3	3	8	4.2	
				$5,000	$125	$15	$2	$1					62¢
6					7,800	320	35	8	3	3	6	6.2	
					$1,500	$50	$5	$1					62¢
5						1,550	82	12	4	2	4	10.3	
						$400	$15	$2					61¢
4							330	23	5	2	3	4.1	
							$50	$5	$1				58¢
3								72	7	2	2	6.3	
								$25	$2				63¢
2									17	3	2	17	
									$10				60¢
1										4	1.3	4.0	
										$2			50¢

Note: In each entry the $ number is the prize, and the plain number is N for 1/N = probability p_w, of having w hits.

APPENDIX: SOME COMBINATORICS

The combinatoric properties of Lotto are easily derived and are available in numerous places. Nevertheless, it is appropriate, for some degree of completeness, to have some of the results here.

A.1 Lotto

The number of possible tickets in r/n Lotto is the number of combinations of r things chosen from n, which we represent by

$$ {}^{n}C_{r} = \frac{n!}{r!(n-r)!} \,. \tag{1}$$

From this the probability of a ticket containing the draw, which we shall denote by $P_J(r, n)$, is

$$P_J(r, n) = \text{probability of a ticket winning the Jackpot} = 1 \Big/ {}^{n}C_{r} = r!(n-r)! \Big/ n! \,. \tag{2}$$

For the lesser prizes, the number of possible tickets that can have exactly w hits, for a given w, shall be denoted $N_w(r, n)$. Whatever the draw is, it has r numbers, within which there are ${}^{r}C_{w}$ sets of w numbers. For each of these sets there are ${}^{n-r}C_{r-w}$ sets of $r-w$ numbers that are not in the draw. Therefore the number of possible tickets having w hits (and hence $r-w$ non-hits) is, as is implicit in Bellhouse (1991),

$$N_w(r, n) = {}^{r}C_{w} \; {}^{n-r}C_{r-w} \,; \tag{3}$$

and clearly $N_r(r, n) = 1$. Thus, for $w \leq r$, the probability of a ticket having exactly w hits is, for $w = r, r-1, \cdots, 2, 1, 0$,

$$P_w(r, n) = N_w(r, n) \Big/ {}^{n}C_{r} = {}^{r}C_{w} \; {}^{n-r}C_{r-w} \Big/ {}^{n}C_{r} \,. \tag{4}$$

Notice that for $w = r$ this is $P_J(r, n)$ of (2). And, of course, we can observe the well-known result that

$$\sum_{w=0}^{r} N_w(r, n) = \sum_{w=0}^{r} {}^{r}C_{w} \; {}^{n-r}C_{r-w} = {}^{n}C_{r} \,, \tag{5}$$

so that, as one would expect,

$$\sum_{w=0}^{r} P_w(r, n) = 1 \,.$$

An algebraic proof of (5), in contrast to the familiar combinatoric proof, is that ${}^{r}C_{w}$ is the coefficient of x^w in $(1+x)^r$ and ${}^{n-r}C_{r-w}$ is the coefficient of x^{r-w} in $(1+x)^{n-w}$. Therefore $N_w(r, n)$ is the coefficient of x^r in $(1+x)^n$ when the latter is factored as $(1+x)^w(1+x)^{n-w}$. Hence summing over all values of w gives ${}^{n}C_{r}$, the coefficient of x^r in $(1+x)^n$.

A recurrence relationship between $N_w(r, n)$ and $N_{w-1}(r, n)$ is, from (3)

$$N_{w-1}(r, n) = \frac{w(n-2r+w)}{(r-w+1)^2} N_w(r, n) \qquad \text{for } w = r, r-1, \cdots, 1, 0 \ . \tag{6}$$

This, along with $N_r(r, n) = 1$, provides convenient computing formulae. Thus for $r = 6$, and with dropping the parenthetical (r, n) to simplify notation,

$$\begin{array}{ll} N_6 = 1 & N_3 = \frac{4}{9}(n-8)N_4 \\ N_5 = 6(n-6) & N_2 = \frac{3}{16}(n-9)N_3 \\ N_4 = 1.25(n-7)N_5 & N_1 = .08(n-10)N_2 \\ & N_0 = \frac{1}{36}(n-11)N_1 \ . \end{array} \tag{7}$$

And for $r = 5$ we have N'_w written for $N_w(5, n)$ as

$$\begin{array}{ll} N'_5 = 1 & N'_2 = \frac{1}{3}(n-7)N_3 \\ N'_4 = 5(n-5) & N'_1 = \frac{1}{8}(n-8)N_2 \\ N'_3 = (n-6)N_4 & N'_0 = .04(n-9)N_1 \ . \end{array} \tag{8}$$

Values of (7) for N_w, and of $p_w = N_w \big/ {}^nC_6$ for 6/n Lotto are shown in Table A1 for $n = 54$, 48, 32 and 25, and for all $w \leq 6$; and using (8), values of N'_w and $p_w = N'_w \big/ {}^nC_5$ for 5/n Lotto are shown in Table A2 for $n = 45$ and 32, and for all $w \leq 5$.

A.2 Keno

A game known as Keno, available in several states, is that of buying tickets of size t, meaning that they have t numbers on them, for $t \leq r$. Winning tickets are those having w hits for $w \leq t$. Let $N_w(t, r, n)$ be the number of possible tickets of size t that have w hits. In using an argument similar to that of deriving (3) we now have, for given w, ${}^tC_w$ winning tickets for tickets of size t. And for each to be a winning ticket the draw must have $r-w$ numbers from among the $n-t$ numbers that are not on the ticket. Therefore

$$N_w(t, r, n) = {}^tC_w \, {}^{n-t}C_{r-w} \ . \tag{9}$$

A recurrence formula for this, analogous to (6), is

$$N_{w-1}(t, r, n) = \frac{w(n-t-r+w)}{(t-w+1)(r-w+1)} N_w(t, r, n) \tag{10}$$

which does, of course, reduce to (6) when $t = r$. Special cases of (9) are that (9) reduces to $N_w(r, n)$ of

(3) for t = r, to ${}^{n-t}C_{r-t}$ for t = w, and to unity for t = w = r. And, for each value of t,

$$\sum_{w=0}^{t} N_w(t,\, r,\, n) = \sum_{w=0}^{t} {}^{t}C_w\, {}^{n-t}C_{r-w} = {}^{n}C_r\,, \tag{11}$$

as may be proven by exactly the same argument as used following (5).

From (9) and (11) we then have, comparable to (4)

$$\begin{aligned} P_w(t,\, r,\, n) &= \text{probability that a ticket of size t } (\leq r) \text{ has w } (\leq t) \text{ hits} \\ &= N_w(t,\, r,\, n) \Big/ {}^{n}C_r \\ &= {}^{t}C_w\, {}^{n-t}C_{r-w} \Big/ {}^{n}C_r\,. \end{aligned} \tag{12}$$

An alternative derivation of (12) is that a ticket can have w hits in one of ${}^{r}C_w$ ways; and for each, the ticket must also have t−w numbers that are among the n−r that are not in the draw. This can be done in ${}^{n-r}C_{t-w}$ ways. Therefore the number of winning tickets is ${}^{r}C_w\, {}^{n-r}C_{t-w}$. But the number of possible tickets of size t is ${}^{n}C_t$. Hence

$$P_w(t,\, r,\, n) = {}^{r}C_w\, {}^{n-r}C_{t-w} \Big/ {}^{n}C_t\,.$$

This equals (12) because

$$\begin{aligned} {}^{r}C_w\, {}^{n-r}C_{t-w} \Big/ {}^{n}C_t &= \frac{r!}{w!(r-w)!}\ \frac{(n-r)!}{(t-w)!(n-r-t+w)!}\ \frac{t!(n-t)!}{n!} \\ &= \frac{t!}{w!(t-w)!}\ \frac{(n-t)!}{(r-w)!(n-r-t+w)!}\ \frac{r!(n-r)!}{n} \\ &= {}^{t}C_w\, {}^{n-t}C_{r-w} \Big/ {}^{n}C_r\,. \end{aligned}$$

A.3 Other Considerations

Another version of Lotto that is intermediate between Lotto itself and Keno is that of buying tickets of size $t \leq r$, as in Keno, but where such a ticket wins only if it has t hits. Details for this, akin to (6), are given in Searle (1992).

A further set of combinatoric considerations is that of establishing the number of tickets in r/n Lotto that can contain different patterns of consecutive numbers; e.g., consecutive numbers, or exactly 5 consecutive numbers and one other number, and so on. Morton (1987) deals with this in some detail.

Table A1

6/n LOTTO

NUMBERS OF POSSIBLE TICKETS AND PROBABILITIES OF A TICKET HAVING w HITS, FOR n = 54, 48, 32 AND 25.

6/54 Lotto

w No. of hits	N_w No. of possible different tickets	$p_w = N_w/{}^{54}C_6$ = probability a ticket has w hits (to 9 dec. pts.)	N for $p_w = \frac{1}{N}$
6	1	.000,000,039(12)	25,827,165
5	288	.000,011,157	89,678
4	16,920	.000,655,124	1,526
3	345,920	.013,393,650	75
2	2,918,700	.113,008,919	9
1	10,273,824	.397,791,395	3
0	12,271,512	.475,139,722	2
Total	25,827,165	1.000,000,000	

6/48 Lotto

w No. of hits	N_w No. of possible different tickets	$p_w = N_w/{}^{48}C_6$ = probability a ticket has w hits (to 9 dec. pts.)	N for $p_w = \frac{1}{N}$
6	1	.000,000,081(49)	12,271,512
5	252	.000,020,535	48,696
4	12,915	.001,052,438	950
3	229,600	.018,710,001	53
2	1,678,950	.136,816,881	7
1	5,104,008	.415,923,319	2
0	5,245,786	.427,476,745	2
Total	12,271,512	1.000,000,000	

6/32 Lotto

w No. of hits	N_w No. of possible different tickets	$p_w = N_w/{}^{32}C_6$ = probability a ticket has w hits (to 9 dec. pts.)	N for $p_w = \frac{1}{N}$
6	1	.000,000,513	1,947,792
5	180	.000,092,412	10,821
4	6,525	.003,349,947	299
3	81,200	.041,688,230	22
2	411,075	.211,046,662	5
1	855,036	.438,977,057	2
0	593,775	.304,845,179	3
Total	1,947,792	1.000,000,000	

6/25 Lotto

w No. of hits	N_w No. of possible different tickets	$p_w = N_w/{}^{25}C_6$ = probability a ticket has w hits (to 9 dec. pts.)	N for $p_w = \frac{1}{N}$
6	1	.000,005,647	177,100
5	114	.000,643,704	1,553
4	2,565	.014,483,342	69
3	19,380	.109,429,701	9
2	58,140	.328,289,102	3
1	69,768	.393,946,923	2
0	27,132	.153,201,581	6
Total	177,100	1.000,000,000	

Table A2

5/n LOTTO

NUMBERS OF POSSIBLE TICKETS AND PROBABILITIES OF A TICKET HAVING w HITS, FOR n = 45 AND 32.

5/45 Lotto

w No. of hits	N_w No. of possible different tickets	$p_w = N_w/{}^{45}C_5$ = probability a ticket has w hits (to 9 dec. pts.)	N for $p_w = \frac{1}{N}$
5	1	.000,000,818	1,221,759
4	200	.000,163,698	6,108
3	7,800	.006,384,238	156
2	98,800	.080,867,012	12
1	456,950	.374,009,932	3
0	658,008	.538,574,302	2
Total	1,221,759	1.000,000,000	

5/32 Lotto

w No. of hits	N_w No. of possible different tickets	$p_w = N_w/{}^{32}C_5$ = probability a ticket has w hits (to 9 dec. pts.)	N for $p_w = \frac{1}{N}$
5	1	.000,004,966	201,376
4	135	.000,670,388	1,491
3	3,510	.017,430,081	57
2	29,250	.145,250,675	7
1	87,750	.935,752,026	2
0	80,730	.400,891,860	2
Total	201,376	1.000,000,000	