

Studies in Theoretical Epidemiology

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Abstract. In this brief introductory note, we describe the scientific environment that—in part—has motivated and instigated recent theoretical advances in the fields of theoretical/mathematical epidemiology, immunology, ecology, demography, and sociology. The papers described in this brief note will appear in one of the volumes that collects the proceedings of the First World Congress in Nonlinear Analysis held in Tampa, Florida, 1992.

The transmission dynamics of diseases, genetic characteristics, or cultural traits can be studied at different spatial, temporal, or organizational scales. The epidemiological/immunological articles in this volume concentrate on issues related to *organizational* scale. Two issues relevant to the articles in this chapter are: (i) how much organizational detail such as age structure, immune response, or genetic variability, must be included in epidemiological models? and (ii) how do we model relevant detail? In fact, by selecting a particular level of organization and a particular model, the investigator may have decided *a priori* what is and what is not relevant. This problem, although often discussed in the biological and social sciences, is not seriously addressed in the epidemiological literature.

The social consequences of the HIV/AIDS epidemic have forced researchers to consider the robustness of, among other things, HIV incidence predictions to changes in organizational or aggregational schemes. These studies have supported the need to increase our understanding not only of who is mixing with whom but what these mixing patterns are at different organizational scales. One cannot study the effect of mixing patterns on disease dynamics without data. The need to collect longitudinal data on mixing behavior at several levels of sociological organization has also resulted from studying the effects of mixing patterns in mathematical models. Several of the articles discussed in this chapter address these and related issues. We briefly describe them here.

Lubkin and Castillo-Chavez look at the problem of mixing/pair formation in a setting that includes but it is not confined to epidemiological phenomena. Approaches common to the transmission dynamics of sexually-transmitted diseases are used to study the transmission dynamics of cultural traits such as language and religion. Their modelling approach is based on the pair-formation framework developed by Busenberg and Castillo-Chavez [1,2,3]. A simple example, their maternal determination model, in which a trait is transmitted/inherited primarily through females is discussed in some detail.

Hsu Schmitz and Carlos Castillo-Chavez estimate from real data the mixing structure of a population. Cross-sectional data from a survey on dating among college students [4] is used to determine the probability of dating individuals from all the classes. However, there is a problem that needs to be addressed, namely that the surveyed population is not closed. In fact, individuals from the target/surveyed population interact very frequently with members of a population that cannot be surveyed. A solution to this problem is discussed.

Denise Kirschner looks at models for the transmission dynamics of HIV in a population of heterosexually-active individuals. She begins to develop new approaches for estimating an individual's infectivity, that is, (in her definition) the probability of becoming infected with HIV if a fixed number of different sexual partners is assumed.

Hyman and Li introduce a multigroup epidemiological model with nonrandom mixing. A properly formulated model requires a balance law or a group reversibility condition, e.g., the total average rate of mixing among individuals of group i with individuals of group j equals the total average rate of mixing among individuals of group j with individuals of group i . Hyman and Li build these conditions into their model in a novel way and explore the consequences of nonrandom or biased mixing in a model with discrete and continuous risk categories.

Brauer addresses heterogeneity but at a different level. He incorporates—in a crude manner—the effects of the immune system (a different organizational scale) into epidemiological models for communicable diseases. He uses the age of infection as a marker for disease infectivity in a very general setting. In addition, he begins to analyze three classes of models under very general assumptions. Mathematical problems for further study are suggested.

Harnevo studies the role of heterogeneity at the level of the cell. She introduces a model for the dynamics of the interactions between one strain of HIV/AIDS and the strain-specific response of the immune system. She addresses questions that may explain such factors as the observed variability in infectivity (usually a function of the age of infection) among HIV-infected individuals and the mechanisms behind HIV progression.

Velasco-Hernández and Castillo-Chavez show the flexibility of the mixing/pair formation framework of Busenberg and Castillo-Chavez by using it to model the dynamics of vector-host disease transmission and also food web dynamics. The framework is not used in its present form but is modified to take into account search and handling times, which are very important in vector-host and predator-prey interactions. The group reversibility property (total rate of contacts/mixing between host and vectors or predator and prey) is preserved in this formulation. However, by taking into account handling and search times it is possible to incorporate different modes of transmission. This is very appropriate for vector-transmitted diseases such as Chagas' disease.

Finally, Castillo-Chavez, Fridman and Luo apply the mixing framework of Busenberg and Castillo-Chavez but in a stochastic model.

As in any collection of works, this volume does not include the contributions of many very active investigators. This is quite unfortunate but unavoidable. Some of their work is reviewed by the contributors to this epidemiological series. Gaps may be filled by looking at the recently edited volumes by Castillo-Chavez [5], Jewell, Dietz, and Farewell [6], and the books by Anderson and May [7] and Hethcote and Van Ark [8].

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