"SCALING LAWS IN HUMAN SEXUAL ACTIVITY"

Stephen P. Blythe
Department of Statistics and Modelling Science
University of Strathclyde, Glasgow G1 1XH Scotland

and

Carlos Castillo-Chavez
Biometrics Unit and Center for Applied Mathematics, 341 Warren Hall
Cornell University, Ithaca, NY 14853 USA

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Stephen P. Blythe
Statistics and Modelling Science
Livingstone Tower
University of Strathclyde
Glasgow G4, Scotland

Carlos Castillo-Chavez
Biometrics Unit and Center for Applied Mathematics
341 Warren Hall
Cornell University
Ithaca, NY 14853-7801, U.S.

Abstract

Sexual activity surveys have shown the existence of an approximate relationship of the form $\sigma^2 = a\mu^b$ between the variance (σ^2) and the mean (μ) of the number of sexual partners per unit of time. We use a simple probabilistic model of a closed homosexual population to show that such scaling laws may arise naturally as a result of the processes of pair formation and dissolution.

Recently Anderson and May[1] observed that there is a striking approximate relationship of the form $\sigma^2 = a\mu^b$ between the variance (σ^2) and the mean (μ) of the number of sexual partners taken per unit of time, in surveys of many different types, and a variety of populations and locations. Until now no explanation of this scaling law has been provided by any of the models currently used to describe sexual activity (particularly with respect to the sexual transmission of HIV). We use a simple probabilistic model of a closed homosexual population to show that such scaling laws may arise naturally as a result of the processes of pair formation and dissolution. Our analytic results suggest that approximate variance/mean scaling with exponent close to 4 may be the norm at very low activity levels (which here means low average probabilities of pair formation and dissolution per unit time), and stochastic simulations provide some justification for the view that the observed exponent (b=3.231) is the result of increased uncertainty at slightly higher mean levels.

We assign to the ith individual unique values s_i and f_i, the probabilities per unit time of that individual initiating dissolution of a pair (if paired), and of seeking a new partner (if single), respectively. The $\{s_i\}$ and $\{f_i\}$ are drawn from some given distribution on (0,1). At each time step, the sequence of events is: (i) separation of (some) pairs occurs, and the individuals return to the singles pool; (ii) each single individual "decides" whether or not to look for a new partner; (iii) individuals who are looking are paired off at random (in the simple model used here). No changes occur until the start of the next time interval.

The asymptotic rate of acquisition of new partners in the model is

$$r(F,S) = (1/F + 1/S - 1)^{-1}, \tag{1}$$

where F and S are asymptotic averages of pair formation and separation rates in the populations of singles and pairs, respectively. Their exact form will vary depending on model assumptions, and in fact may often not be calculable analytically. For this model S is the mean squared value of the $\{s_i\}$ among paired people, and F is the mean of $\{f_i\}$ among single people. These need not equal the equivalent statistics for $\{s_i\}$ and $\{f_i\}$ for the population as a whole.

Expanding[2] r(F,S) around the value $\overline{r} = r(\mu_F, \mu_S)$, where μ_F and μ_S are the expected values of F and S respectively, we can obtain approximate expressions for the mean and variance of the rate of new partner acquisition,

$$\mu_r = \overline{r}(\alpha\overline{r}^2 - \beta\overline{r} + 1), \tag{2}$$

$$\sigma^2{}_r = \alpha\, \overline{r}^4, \tag{3}$$

where $\alpha = \sigma^2{}_F/\mu^4{}_F + \sigma^2{}_S/\mu^4{}_S$ and $\beta = \sigma^2{}_F/\mu^3{}_F + \sigma^2{}_S/\mu^3{}_S$, with $\sigma^2{}_F$ and $\sigma^2{}_S$ the variances of F and S respectively. Covariance terms are dropped because in the model described there is no correlation between the $\{s_i\}$ and the $\{f_i\}$. Equations (2) and (3) indicate a fourth order scaling of variance with mean as $\bar{r}$ tends to zero, for small enough α and β. A scaling "law" such as Anderson and May[1] report can arise where the values of F and S observed in the various samples are not large and the variances associated with them are moderate.

To test the validity of this approximation, we implemented a stochastic simulation of the model outlined above, with a variety of different population sizes, and distributions from which the $\{s_i\}$ and $\{f_i\}$ were randomly drawn. Fig 1 illustrates how a scaling law can arise. In this case the $\{s_i\}$ and $\{f_i\}$ were drawn from identical Beta distributions, with mean values 0.035 to 0.167 and variance 0.0011 to 0.0107. The resulting scaling relationship between mean and variance of numbers of partners per unit time (fitted line in Fig 1) is of the form $\sigma^2{}_r = a\mu_r{}^b$ with a = 0.41, and b = 1.67. It is possible to find many scaling patterns with different exponents. We speculate that the exact value observed in Ref 1 reflects a restrictive set of population parameter values. We are currently looking at other mixing and pair formation/dissolution models in order to determine whether or not "realistic" mixing structures give rise to scaling patterns similar to that reported in Ref 1.

References

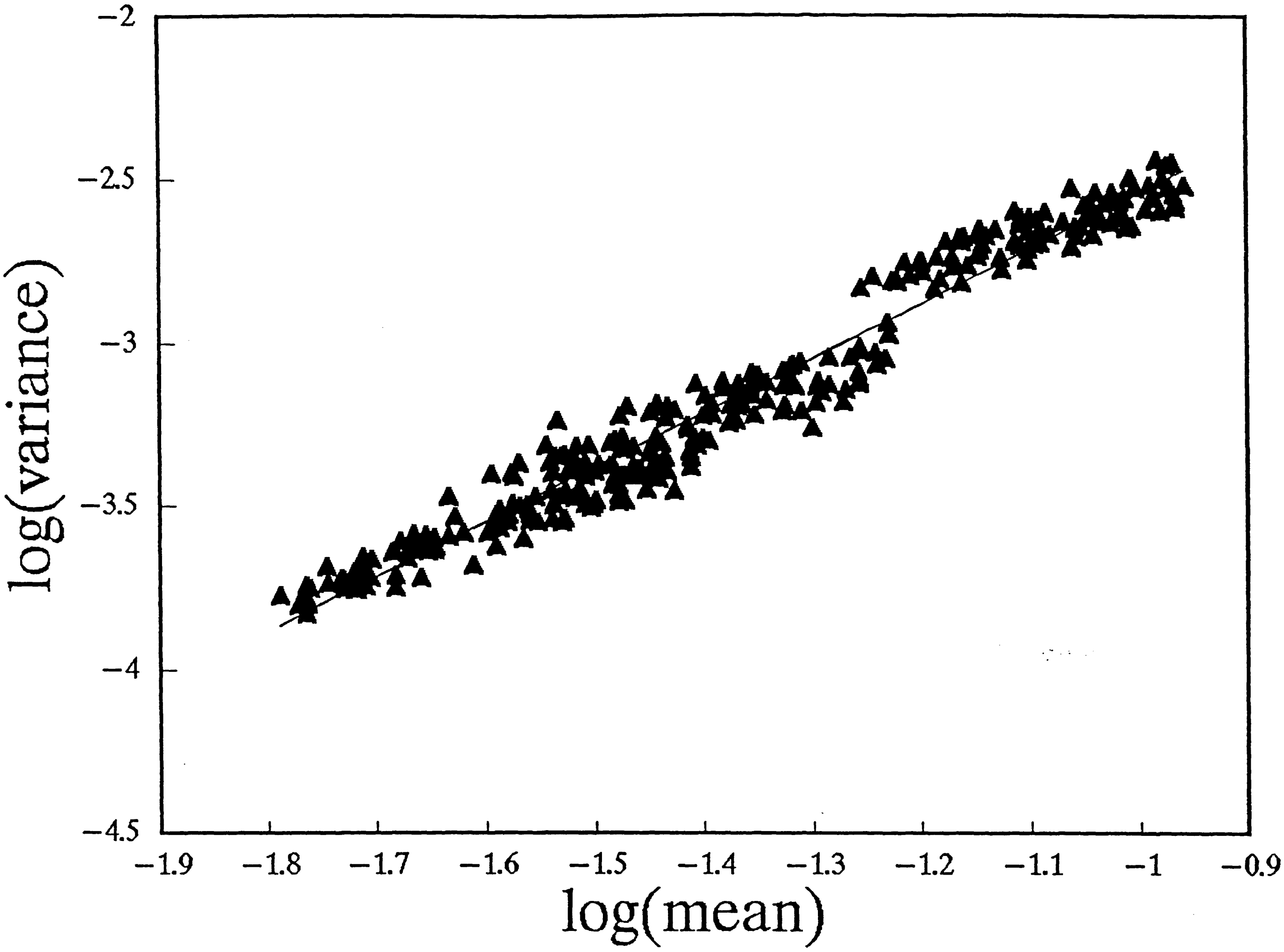
1. Anderson RM & RM May (1988) *Nature* 333, 514-519

2. Mood AM, Graybill FA & DC Boes (1974) *Introduction to the theory of Staistics*, third edition McGraw-Hill New York

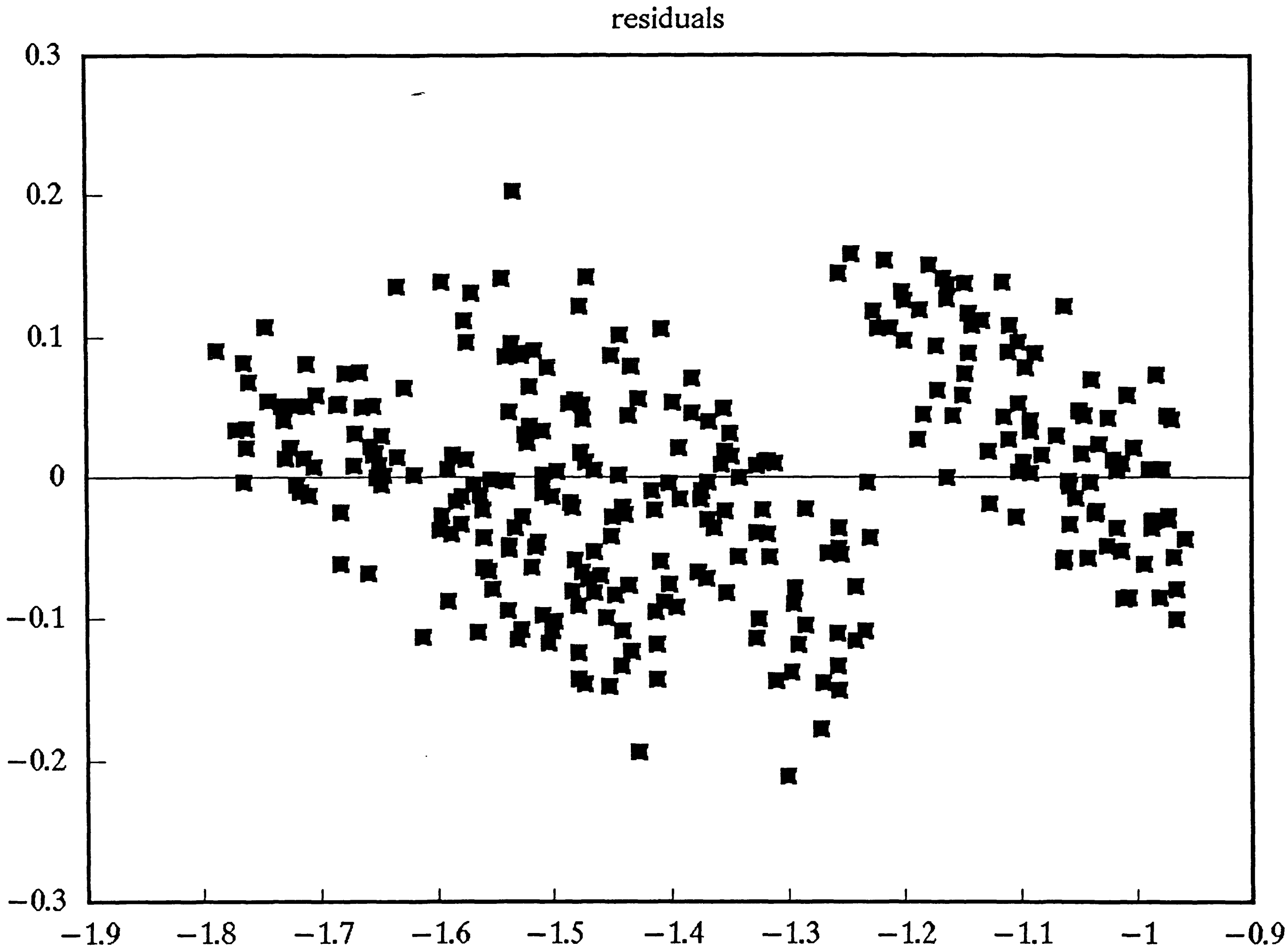
FIGURE CAPTION

Figure 1. A scaling law from the simulation model. Log variance and log mean of number of partners acquired per unit time are approximately related by $y = -0.869 + 1.675x$; $r^2 = 0.9624$, $n = 299$.

Figure 2. Residuals for the regression shown in Figure 1.



-2
-2.5
-3
-3.5
-4
-4.5
log(variance)
-1.9
-1.8
-1.7
-1.6
-1.5
-1.4
-1.3
-1.2
-1.1
-1
-0.9
log(mean)


residuals
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
−1.9
−1.8
−1.7
−1.6
−1.5
−1.4
−1.3
−1.2
−1.1
−1
−0.9