Mixing Patterns and Multiple Endemic States in Models of AIDS

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Abstract

A two group model for the spread of HIV, in a heterogeneous mixing homosexually active population is presented. We look breifly at the influence of mixing parameters and contact parameters on the bifurcation and hence in the reproductive number. More studies of the sensitivity of the model behavior to the parameter values, are needed in order to determine whether or not multiple endemic equilibria are possible for realistic epidemiological and sociological parameters when a specific family of biased mixing functions is utilized.

I. Introduction

In studying models of the transmission of AIDS, it is essential to take account of several features that do not occur in classical epidemic models. The population cannot be taken constant, because of the long incubation periods before symptoms occur and because the disease causes great mortality. The classical models usually assume that there is a single homogeneously mixing population, which makes them inappropriate for AIDS. Members of the at-risk population for AIDS vary widely in frequency and type of sexual activity, sharing of intravenous needles, social dynamics, and so on. For a discussion of these issues one may, for example, see Anderson et al. (1986), Anderson (1988), Castillo-Chavez (1989), and the references cited therein.

A number of authors have formulated and studied models in which the assumption of homogeneous mixing is abandoned. These include Anderson et al. (1986), Hyman and Stanley (1988, 1989), Anderson (1988), Jacquez et al. (1988, 1989), Sattenspiel (1987), Sattenspiel and Simon (1988), and others. Castillo-Chavez et al. (1989a,b) and Huang et al. (1989), formulated a multiple group model containing an arbitrary number of subpopulations, each of which is considered to be homogeneous as to sexual behavior, susceptibility, etc. The resulting model was then analyzed using methods of stability and bifurcation theory. In Section 2 of this paper we review the formulation of two multiple group models. The assumptions about the patterns of mixing between subpopulations are given and the principal theorems are stated and their implications discussed.

In Section 3, we report on parameter studies for the model of Section 2 in the two-group case. These are designed to suggest how the values of key bifurcation parameters depend on the biological and mixing parameters, for a range of (possibly realistic) values of the latter, that is, we begin a sensitivity analysis of the bifurcation parameters as function of the mixing framework.

2. A Multiple Group Model

We assume that there are n sexually active subpopulations, each of which is divided into three epidemiological classes: S_i is the number of uninfected but susceptible individuals, I_i is the number of infected (and infectious) individuals who have not yet developed AIDS, and A_i is the number of individuals with AIDS. We assume that individuals that have developed AIDS are sexually inactive, and hence $T_i = S_i + I_i$ denotes the sexually active population in group i. This is a simplifying and somewhat unrealistic assumption (see Longini et al. 1989) that could be modified. Λ_i denotes the constant recruitment rate of susceptibles into class S_i, μ denotes the rate of removal from sexual activity for all causes other than AIDS, d_i denotes the disease-induced mortality in class A_i, and α_i denotes the removal rate from class I_i. Further, λ_{ij} denotes the proportion of contacts needed for passing the virus from a group i infective to a group j susceptible, and C_i denotes the average number of sexual contacts per unit time of a group i individual. In general, C_i may be a function of the size and availability of partners in all the groups, but in the present analysis each C_i is assumed to be constant. In an alternate interpretation, C_i may be regarded as the number of partners per unit time, and λ_{ij} as the average transmission coefficient per partner.

In order to complete the model, we let $p_{ij}(t)$ denote the fraction of contacts by a group i individual that are made with individuals in group j. Under these assumptions, the rate of infection of susceptibles in group i is given by

$$B_i(t) = S_i(t)\, C_i \sum_{j=i}^{n} \lambda_{ij}\, p_{ij}(t) \frac{I_j(t)}{T_j(t)} \quad . \tag{1}$$

The mixing coefficients $p_{ij}(t)$ are not arbitrary, since they must satisfy the following conditions for all times $t \geq 0$.

$$\begin{array}{lll} p_{ij} \geq 0 & & (i, j = 1, \ldots, n) \\ \sum_{j=1}^{n} p_{ij} = 1 & & (i = 1, \ldots, n) \\ C_i T_i C_j T_j = 0 & \rightarrow & p_{ij} = p_{ji} = 0 \\ C_i T_i p_{ij} = C_j T_j p_{ji} & & (i, j = 1, \ldots, n) \quad . \end{array} \tag{2}$$

The last equation in (2) is a conservation equation, which states that the total number of contacts by all individuals in group i that are with individuals in group j is equal to the total number of contacts by all individuals in group j that are with individuals in group i. A particular choice of the p_{ij} that satisfies (2) is so-called preferred mixing, which has been used by Nold (1980), Hethcote and Yorke (1984), Jacquez et al. (1988), Blythe and Castillo-Chavez (1989), and Castillo-Chavez and Blythe (1989). In this case we take

$$p_{ij} = \begin{cases} f_i + (1-f_i) \dfrac{C_i(1-f_i)T_i}{N(T_1, \ldots, T_n)}, & i = j \\[2ex] (1-f_i) \dfrac{C_j(1-f_j)T_j}{N(T_1, \ldots, T_n)}, & i \neq j \end{cases} \tag{3}$$

where $i, j = 1, \ldots, n$, and

$$N(T_1, \ldots, T_n) = \sum_{k=1}^{n} (1-f_k)\, T_k C_k \quad .$$

The dependence of the T_j and p_{ij} on t has not been indicated in these formulas, for notational simplicity. In this definition, f_i denotes the fraction of a group i individual's contacts that are reserved for individuals in group i, while the remaining fraction, $1 - f_i$, are assumed to be distributed according to proportional mixing within all n groups. These fractions are assumed to be constants, and when all $f_i = 0$, this mixing function reduces to the so-called proportional mixing function.

It is also possible to formulate a one-sex model in which a sexual activity level s is associated with each person, and with a mixing function p(s,r), where p(s,r) is the fraction of partnerships of an individual of activity level s that are with individuals having activity level. r. This idea has been developed by Blythe and Castillo-Chavez (1989), who developed and compared several such mixing functions that satisfy the necessary constraints. Busenberg and Castillo-Chavez (1989) have obtained a characterization of all possible mixing functions. Dietz and Hadeler (1988) have proposed models in which the formation and dissolution of pairs of partnerships are explicitly included in their models.

With the above definitions, it is shown in Castillo-Chavez (1989a) that the dynamic equations for the model take the following form.

$$\frac{dS_i}{dt} = \Lambda_i - S_i \left[\frac{\theta_i I_i}{T_i} + \frac{1}{N(T)} \sum_{j=1}^{n} l_{ij} I_j\right] - \mu S_i$$

$$\frac{dI_i}{dt} = S_i \left[\frac{\theta_i I_i}{T_i} + \frac{1}{N(T)} \sum_{j=1}^{n} l_{ij} I_j\right] - \mu(\sigma_i + 1)\, I_i \quad . \tag{4}$$

In these equations, $T_j = S_j + I_j$, $\theta_i = f_i\, \lambda_{ij} C_i$, $r_i = C_i(1-f_i)$, $l_{ij} = C_i(1-f_i)C_j\,(1-f_j)\,\lambda_{ij} = r_i r_j \lambda_{ij}$, $\sigma_i = \alpha_i/\mu$, and

$$N(T) = \sum_{k=1}^{n} C_k T_k (1-f_k) = \sum_{k=1}^{n} r_k T_k \; .$$

We observe that this model does not incorporate a realistic incubation period distribution, however, this could be easily done by adding extra compartments (see Castillo-Chavez et al. 1989a). Since we are concerned, in this paper, with the sensitivity of the bifurcation parameter to the mixing parameter we concentrate on the simplest model.

To state our bifurcation results, we introduce the following notation. let Q be the matrix

$$Q = \operatorname{diag}\left[\frac{\theta_i}{\sigma_i + 1}\right] + \operatorname{diag}\left[\frac{\Lambda_i}{k(\sigma_i + 1)}\right] L \quad . \tag{5}$$

Were $L = (l_{ij})$, $k = \sum_{k=1}^{n} r_k A_k$. Observe that there is a disease-free equilibrium of the system, where $I_i = 0$, and $S_i = \Lambda_i/\mu$ $(i=,\ldots,n)$. The first result is as follows:

Let $H(\mu) = Q-\mu E$ where E is the identity matrix, and let $M(H(\mu)) = \sup\{\mathrm{Re}\ z: \det(zE-H(\mu)) = 0\}$. Then there exists μ_0 such that

$$M(H\ (\mu)) \begin{cases} < 0 \text{ if } \mu > \mu_0 \\ = 0 \text{ if } \mu = \mu_0 \\ > 0 \text{ if } \mu < \mu_0 \end{cases}$$

Then the disease-free equilibrium is asymptotically stable if $\mu > \mu_0$ and unstable if $\mu < \mu_0$. This result can be stated in terms of the "basic reproductive number," R_o, which is not readily computed since in general it is a nonlinear function of the parameters (for a recent discussion see Dickmann et al. 1989): the disease-free equilibrium is locally asymptotically stable if $R_o < 1$ and unstable if $R_o > 1$.

Our next results concern the bifurcation of a positive equilibrium (endemic disease state) at $\mu = \mu_o$. To describe these results, denote a right eigenvector of Q corresponding to the eigenvalue μ_o, $I_o = (I_{o1},\ldots,I_{on})$. We have shown the existence of a matrix function $G(\mu, I)$ such that I_o is a right eigenvector of $G(\mu_o,0)$ with zero eigenvalue. We let I_o^* denote a right eigenvector of the transpose $G^T(\mu_o, 0)$ with zero eigenvalue, for which the inner product of I_o and I_o^* is one. Further, we let

$$h(\mu_o) = \sum_{j=1}^{n} x_j\, I_{oj}\, I_{oj}^* \tag{6}$$

where the x_j are determined from the following relations:

$$x_i = \mu_o \left[\frac{K I_{oi}\{\mu_o(\sigma_i+1)^2 - \theta_i\sigma_i\}}{\Lambda_i^2} - \left(\sum_{j=1}^{n} r_j\sigma_j I_{oj} \right) \frac{\mu_o(\sigma_i+1)-\theta_i}{\Lambda_i} \right].$$

We have shown the following:

2. If $h(\mu_o) \neq 0$, then μ_o is a bifurcation point. Specifically, if $h(\mu_o) > 0$ ($h(\mu_o) < 0$), then there is an $\epsilon > 0$ and there are unique continuously differentiable vector functions S and I mapping $(\mu_o-\epsilon, \mu_o] \to R_+^n$ ($[\mu_o, \mu_o+\epsilon) \to R_+^n$) such that $S(\mu_o) = (\Lambda_1,\ldots,\Lambda_n)/\mu_o$, $I(\mu_o) = 0$, and $(S(\mu), I(\mu))$ is a strictly positive endemic equilibrium for $\mu \neq \mu_o$. Furthermore, this endemic equilibrium is locally asymptotically stable for each μ in $(\mu_o-\epsilon, \mu_o)$ (unstable for each μ in $(\mu_o, \mu_o+\epsilon)$).

We see from the above result just stated that the direction of the bifurcation at μ_o, and the stability of the bifurcating branch, depend on the sign of the quantity $h(\mu_o)$. Specifically, we have that:

3. If $h(\mu_o) \neq 0$, then for each μ in $(0, \mu_o)$, the system (4) has at least one positive endemic equilibrium. Furthermore, if $h(\mu_o) < o$ and if $\epsilon > 0$ is sufficiently small, then for each μ in $(\mu_o, \mu_o+\epsilon)$, the system (4) has at least two positive equilibria.

Examples of systems for which there are two positive equilibria are included in Huang (1989), Huang et al. (1989) and Castillo-Chavez et al. (1989a). In Section 3 below, we examine briefly the dependence of $\mu_o$and $h(\mu_o)$ on the parameters of the model. The analysis provided in the next section provides our first steps towards a more complete study (i.e. sensitivity anlysis) of the effect of the mixing structure on the qualitative dynamics of multi-group epidemic models.

3. Preliminary Parameter Studies for the Two-Group Case

As a specific application of the general model presented in Section 2, we consider the following two-group case for HIV transmission. We will illustrate it with data from the sexually active San Francisco homosexual male population. We assign individuals from this group to one of two sub-populations according to their relative levels of sexual activity. Equations (4) and all of the subsequent equilibria analysis apply for $i = 1, 2$ and hence the specific formulae will not be repeated. Using the approach of Section 2, we calculate μ_o and $h(\mu_o)$ for plausible ranges of several system parameters to demonstrate how the epidemic depends on the values of these parameters. As previously discussed, these quantities, μ_o and $h(\mu_o)$, are crucial for determining the existence and stability of various equilibria.

We refer here to the San Francisco population because of the quality of data available. This region has served as the focus for a substantial body of research concerning the levels and patterns of sexual activity among homosexual populations. This research includes recent field studies directed primarily towards the AIDS epidemic as well as previous efforts which have since proven relevant, such as the highly publicized San Francisco City Clinic Cohort Study regarding the transmission of Hepatitis B. However, we remark that the results of this section are only for illustrative puposes and that no specific conclusions, regarding the dynamics of AIDS in this population, can be drawn.

The values selected for the parameters that compose the matrices Q and G^T from Section 2, which are crucial for the calculation of μ_o and $h(\mu_o)$, have been taken directly from various literature when possible. Unfortunately, however, good estimates for several parameters are not available, and hence some of the estimates used are quite unreliable. The mean length of the infective period, the time between initial HIV infection and progression into "full-blown" AIDS, is approximately 10 years (Quinn, 1989) and apparently does not depend on the level of sexual activity of the infected individual. This implies that the removal rates from the infected classes of both groups are equivalent, hence we take $\alpha_1=\alpha_2=\frac{1}{10}$. We assume that the average length of an individual's active sex-life is 30 years. Thus, $\mu=\frac{1}{30}$ is the rate of removal from the sexually active population due to any cause other than AIDS. From Section 2, $\sigma_i=\frac{\sigma_i}{\mu}$, which implies that $\sigma_1=\sigma_2=(\frac{1}{10})(\frac{30}{1})=3$. Choices for T_i, C_i, f_i, and the matrix of transmission coefficients, λ_{ij}. are described in detail below. All other parameters can then be derived from these using the formulae of Section 2.

Several studies have suggested a significant variation in the level of sexual activity, represented in the model by the parameters C_i, among homosexual populations. For instance, during interviews conducted in 1986 at STD clinics in London, a few individuals reported having over 700 different partners per year although the mean number reported was only 51.36 (Anderson, Medley, May, and Johnson, 1987). Data from the San Francisco area suggests that at any one time, 10% of the male homosexual population has an average number of partners per year that is ten times that of the remaining 90% (Hethcote, 1989). Because of this variation we use the parameter C_i as the distinguishing characteristic by which to divide the population into a small, extremely active group T_1 and a larger, less active group T_2. The estimated size of the active male homosexual population in the San Francisco region is 56,000. Therefore we employ the criteria $C_1>C_2$, $T_1<T_2$, and $T_1+T_2=56{,}000$ for each selection of values for the size and activity level of both sub-populations.

The specific values of C_i and T_i used for calculating μ_0 and $h(\mu_0)$ depend largely on context since C_i and the transmission coefficients λ_{ij} can be expressed in terms of either sexual partners or sexual contacts, see Section 2. In practice, each of these units has distinct advantages with regard to parameter selection. The mean number of sexual contacts per unit time for individuals in homosexual populations may be difficult to measure since the number of contacts per partnership is so variable. However, several authors have published reliable data for the number of partners per unit time. This data provides us with accurate approximations for C_1 and C_2 in terms of partners but not contacts. Unfortunately, quite the opposite proves true for the transmission coefficients λ_{ij}. We have some idea for the probability of transmission per contact between an infected individual and a susceptible but not for the probability per partnership, as it depends on the number of contacts per partnership which is extremely variable and difficult to estimate. In the next sub-sections we state parameters in terms of either partners or contacts depending upon the purpose of each individual calculation.

3.1 Proportional vs. Preferential Mixing

The model of Section 2 allows for two forms of mixing between individuals in the population. The first, proportional mixing, assumes that these individuals choose their partners at random from the general population. The second, preferential mixing, predicts that individuals of certain characteristics will tend to choose partners with the same characteristics. To study the effect each type of mixing pattern might have on the spread of the epidemic, we analyze the system for equilibria by calculating μ_o and $h(\mu_o)$ over a range of the parameter f_i.

The value of f_i relates to factors such as partner choice and mixing patterns, and therefore, the parameters are selected in terms of partners. The population is divided according to the assumption that at any one time, Group 1 individuals only make up 10% of the overall homosexual population but are ten times as active as Group 2 individuals. This implies that T_1=5,600, T_2=50,400, and $C_1=10C_2$. The mean number of partners per year for the entire population is given by $\frac{T_1C_1+T_2C_2}{T_1+T_2}$. The values of C_1 and C_2 are chosen to correspond to data from McKusick et al. (1985) for the mean number of partners per year of the San Francisco male homosexual population in 1982, 1983, and 1984 and are displayed in Table 1. The chosen transmission coefficients in terms of probability of transmission per partnership are $\lambda_{ij} = 0.1(i,j=1,2)$ for these calculations. These selections are believed to be realistic although there is not strong evidence to support them.

The parameter f_i represents the fraction of a group i individual's partnerships that are reserved exclusively for individuals from the same group. Therefore $f_i = 0$ implies random partner selection or proportional mixing and $0 < f_i \leq 1$ indicates preferential mixing. We choose for this preliminary study $f_1=f_2$, although this is not necessarily the case. We calculate $\mu_o$and $h(\mu_o)$ for the range [0,1] for $f_1 = f_2 = f$.

For each choice of f we find that $\mu<\mu_o$ and $h(\mu_o)>0$. According to the analysis by Huang (1989) and Huang et al. (1989ab), this implies several things regarding the status of the epidemic. For one $\mu<\mu_o$. demonstrates the instability of the disease-free steady state. Further, these calculations imply that there exists an endemic equilibrium and that μ_o is a bifurcaton point, with μ the bifurcation parameter. Therefore the magnitude of μ_o has a qualitative relationship with the severity of the epidemic. Figure 1 shows the value of μ_o for various f's and three different choices of C_i, each corresponding to data from 1982 through 1984 as described in Table 1.

The results indicate that the fraction of partnerships reserved for preferential mixing varies directly with the difference between μ and the bifurcation point μ_o , that is with the nature of the stability of the disease-free equilibrium.

From Figure 1 we see that as $f = f_1 = f_2$ increases, the value of μ_o increases, and μ_o is farther from μ. Thus the endemic state is more robust and probably the reproductive number is larger. That is, as we move away from proportional mixing towards within-group mixing, we may get, under appropriate initial conditions, a larger first initial outbreak. Also, we note that as the value of $(C_1T_1 + C_2T_2)/T$ decreases (81.6, 57.6, 31.2 as in Table 1), the value of μ_o decreases, which suggests a probably reduction in the reproductive number. This change in behavior, due to selective removal of sexually active undividend, reduces the rate of spread of the epidemic. There are two issues here: the pattern of mixing and the mean number of partners. Both are important in determining the severity of the epidemic.

3.2 Recent Reductions in the Level of Sexual Activity

As the data from McKusick et al. suggests, the level of sexual activity of the San Francisco homosexual population has steadily decreased in the last decade (this might be a direct response of the population to fear induced by the AIDS epidemic). We would expect, however, that such a decrease in sexual activity would cause a parallel decrease in HIV transmission and thus reduce the severity of the epidemic. The results presented in Figure 1 support this hypothesis since the decrease in C_1 and C_2 in the calculations for 1982 through 1984 correspond with a substantial decrease in the magnitude of the bifurcation point (and consequently of the reproductive number, although we do not know the exact magnitude of this change).

Although these results seem to indicate a positive trend with respect to a reduction in HIV transmission, we must exercise caution when basing any conclusion upon them. The decrease in the average number of sexual partners per year does not necessarily imply a reduction in the number of sexual contacts. The probability of HIV transmission per contact between an infected and a susceptible individual which depends on biological factors is assumed constant in the model, while the probability of transmission per partnership which depends on the number of contacts per partnership will change if the average number of contacts per partner changes. Thus although the level of sexual activity in terms of partners may have decreased, the transmission coefficient may potentially increase or remain constant. To investigate this possibility, we compute μ_o and $h(\mu_o)$ again for C_1 and C_2 corresponding to the data from 1982, 1983, 1984 with variable λ_{ij} which increases from 0.1 to 0.18 in the time period. The results are recorded in Figure 2. We observe that the value of the μ_o decreases at a very slow rate (a little faster in the preferred mixing case). We note however, that these results have not taken into account some of the recent epidemiological considerations. For example, we have ignored variable infectivity (i.e. the reported two infectivity peaks). Variable infectivity may change the above conclusions, by making a change in partnership an even more risky activity than that of increasing the number of contacts per partner. For a model that incorporate variable infectivity and for references on variable infectivity data see

Thieme and Castillo-Chavez (1989). Further, we must be careful drawing conclusions from these results since the selection of the transmission coefficients is arbitrary.

3.3 The Transmission Coefficients

The AIDS epidemic resembles the spread of another disease, Hepatitis B, in that both have similar principal modes of transmission. Research on Hepatitis B suggests that the probability of transmission per sexual contact between an infected individual and a susceptible individual is slightly less than the probability of transmission during a needle-stick to a susceptible with a contaminated needle. The probability of HIV transmission in such a needle–stick situation has been estimated at 1 in 250. For lack of better data regarding the transmission coefficient and based on the above result for Hepatitis B, we assume that the probability of HIV transmission per sexual contact is in the neighborhood of $\frac{1}{250}$ (Curran, 1989). However, the probability of transmission per contact is dependent on several variable factors, including the nature of the contact, for instance insertive vs. receptive or anal vs. oral, and the practice of "safe-sex" methods such as condom use, and time since infection. Recently, the media has launched several extensive campaigns to encourage such "safe–sex" practices to combat the AIDS epidemic. Although field studies have not yet had time to confirm the effect of these campaigns, it is reasonable to assume that such efforts have had at least limited success. It is expected that an increase in condom use and other similarly motivated practices will reduce the probability of transmission per sexual contact. Calculations of μ_o and $h(\mu_o)$ can be used to demonstrate the effect this might have on the epidemic with regard to the existence and stability of non-negative equilibria.

Since the model is now placed in terms of sexual contacts, however, we must adapt our choices for C_1 and C_2 which were previously measured in partners per year. This task proves difficult since most data regarding levels of sexual activity focus on frequency of partner change rather than frequency of intercourse. We arbitrarily assume that one fourth of the population averages three times as many contacts per year as the remaining three fourths which has a mean of 200 per year. this implies that $T_1 = 14{,}000$, $T_2 = 42{,}000$, $C_1 = 600$, and $C_2 = 200$.

With these values for the system parameters, the various equilibria are analyzed for a range of λ_{ij} from $\frac{1}{250}$ to $\frac{1}{1000}$ and the results recorded in Figure 3. The values for the transmission coefficients are presented in decreasing order since the probability of transmission is most likely decreasing as AIDS awareness increases with time. For each λ_{ij}, $\mu < \mu_o$ and $h(\mu_o) > 0$ which implies that once again the disease–free equilibrium is unstable, an endemic steady-state exists and is stable, and μ_o gives the point of bifurcation.

3.4 Concluding Remarks

We have presented a multigroup model with only two groups, and have looked briefly at the influence of mixing parameters and contact parameters on the bifurcation parameter and hence on the reproductive number. We have taken equal transmission parameters (all λ_{ij} equal) and equal mixing parameters ($f_1 = f_2$). In reality, this may not be the case. For example, if group 1 practices receptive anal sex and group 2 practices insertive anal sex, then $\lambda_{12} \neq \lambda_{21}$, and perhaps $f_1 \neq f_2$. In an extreme instance, it is known that it is possible to have more than one endemic equilibrium. More studies of the sensitivity of the model behavior to the parameter values λ_{ij}, f_i are needed in order to determine whether or not multiple endemic equilibria are possible for realistic epidemiological and sociological parameters.

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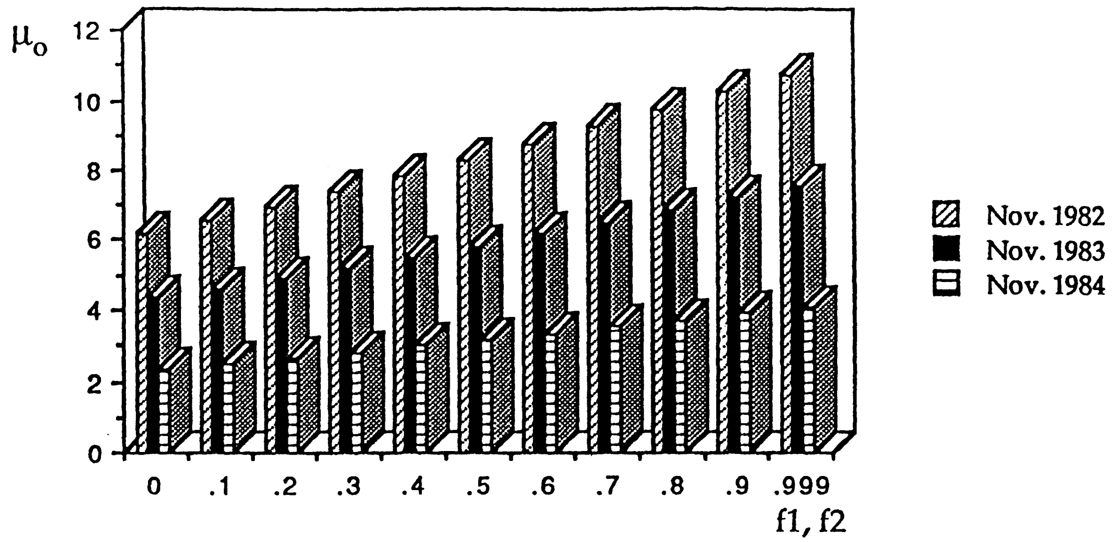

Fig. 1. Calculation of μ_o for proportional and preferential mixing patterns. In each case, $\mu_o > \mu = .033$ which indicates that the disease-free equilibrium is unstable. Further calculations show $h(\mu_o)>0$ which implies that an endemic steady state exists and μ_o is a bifurcation point.

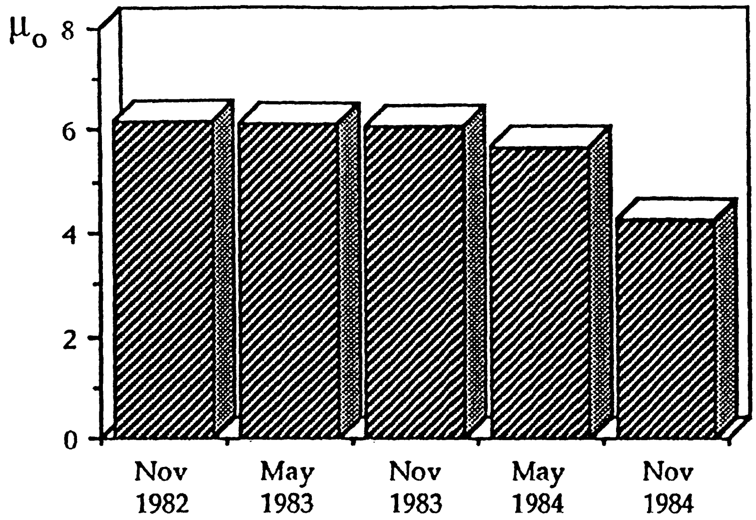

Fig. 2. Calculations of μ_o for choices of C1 and C2 which correspond to data reported for 1982 through 1984 using proportional mixing and variable transmission coefficients as described in the text.

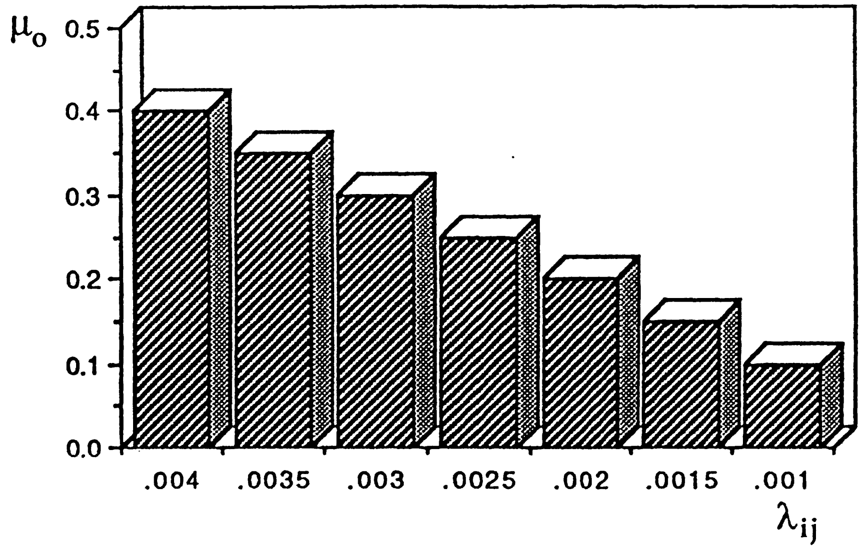

Fig. 3. Calculations of μ_o for a decreasing range of the transmission coefficients, λ_{ij}. Again, $\mu_o > \mu = 1/30$ demonstrates the instability of the disease-free equilibrium.

Date	Mean # of partners/year	T_1	C_1	T_2	C_2
Nov 1982	81.6	5600	430	50400	43
Nov 1983	57.6	5600	303	50400	30.3
Nov 1984	31.2	5600	164	50400	16.4

Table 1. The mean number of partners per year of homosexual individuals visiting STD clinics in San Francisco (McKusick et al, 1985) and the values of the parameters T_i and C_i chosen to fit this data.